PART III: DESIGN PROVISIONS AND EQUATIONS

3.1-GENERAL

3.1.2-Net Section Area

3.1.2.1 Specific provisions pertaining to notches in bending members are given in section 3.2.3. Provisions for calculation of actual shear stresses parallel to grain in notched bending members are given in section 3.4.4. Section 3.6.3 concerning compression parallel to grain provides for the use of gross section area when the reduced section of a column does not occur in the critical part of the length that is most subject to potential buckling.

The effect of eccentric loading of compression members can be evaluated using the equations given in Section 15.4 of the Specification.

3.1.2.2 A specific criterion for determining when staggered fasteners in adjacent rows should be considered in the same critical section was first introduced in the 1960 edition. The criterion was expressed in terms of a minimum spacing of fasteners parallel to grain in the same row, or less than 8 fastener diameters. This provision, which was continued to the 1986 edition, assumed symmetrical placement of fasteners in the joint. However, the requirement provides no minimum parallel to grain distance between fasteners in adjacent rows if a non-symmetrical staggered fastener pattern is used. To avoid possible misapplication when non-uniform patterns are used, the provision has been changed in the 1991 edition to require staggered or offset fasteners in adjacent rows to be considered in the same critical section if the parallel to grain distance between them is less than 4 diameters.

3.1.2.3 The limiting parallel to grain distance of one or less connector diameters between staggered split ring or shear plate connectors in adjacent rows that is used to determine net area of critical section has been in the Specification since 1960. The limit should be applied to the parallel to grain offset or stagger of rings or plates in adjacent rows.

The alternate procedure for checking the adequacy of net section at split ring and shear plate connector joints in compression members and sawn lumber tension members given in Appendix A.12 assumes that the net section area has been reduced for the cross-sectional area of any knots occurring within 1/2 a connector diameter of the critical section (62,65,178). When this criterion is met, actual tension stress parallel to grain (f_t) and actual compression stress parallel to grain (f_c) values are checked against bearing design values parallel to grain (F_g') (see Table 2A). The use of clear wood compression parallel to grain design values for checking net section of sawn lumber tension members joined with connectors is based on test results that show the concentration of stresses in the net section caused failure in tension at levels approximately equal to the maximum compressive strength of the material parallel to grain (62,159,178,183).

The alternate procedure for checking the net section at split ring and shear plate connections was first introduced for tension members in the 1944 edition of the Specification and subsequently extended to compression members in the 1951 edition. Prior to 1982, the clear wood allowable bearing design values parallel to grain used to check the net section were obtained through use of connector net section factors tabulated for connector species groups (178). Use of the special connector species group values was discontinued in the 1982 edition in favor of the species specific bearing design values parallel to grain (see Table 2A).

3.1.3-Connections

Inadequate design of the connections between members is a frequent cause of unsatisfactory performance in wood structures. Important connection designs include those between beams and columns, between members of built-up trusses, between roof structural members and vertical supporting walls, and between load-bearing walls and foundations.

Particular attention should be given to the design of joints involving multiple fasteners and to those subject to moment forces. Only fastener types having the same general load-slip or stiffness characteristics should be employed in the same joint (see Commentary for Section 7.1.1.1).

Requirements for (i) design and fabrication of connections so as to insure all members at the location of the joint carry their proportional load and (ii) use of symmetrical members and fasteners at connections unless induced moments resulting from unsymmetrical design are taken into account have been included in various forms in the Specification since the first edition. Section 3.1.3 in the 1991 edition consolidates two separate provisions relating to this subject in the 1986 and earlier editions. Also, the reference to stresses induced by unsymmetrical arrangement of members and/or fasteners as "secondary stresses" has been discontinued to avoid confusion with the common usage of this term in engineering practice. Lapped joints have been referenced as examples of unsymmetrical connection design to add emphasis to the need to consider the bending moments induced in such configurations (see Commentary for 7.1.2). The effects of eccentricity of loads at critical net section also are referenced in paragraph 3.1.2.1 of the Specification.

Joists attached to one side of a built-up girder made of two or more pieces of dimension lumber of the appropriate width is a common design configuration where the connection between the members of the girder and between the girder and the joist are important to insure the each carries its proportional share of the load. If joists are attached to or bear on ledger strips attached to the outside member of the girder only, members of the girder should be bolted, clinched nailed or otherwise connected to each other such that all will deflect the same amount under the distributed load from the intersecting joists.

3.1.4-Time Dependent Deformations

Consideration of time dependent deformations in built-up members of structural frames was first introduced in the 1960 edition of the Specification in the context that the arrangement of connections between the leaves or sections of a member shall provide for equal inelastic deformation of the components. The provision was generalized and made advisory in the 1977 edition.

One type of application being addressed by 3.1.4 is the use of a leaf or flange to strengthen or stiffen a single main member in a truss without increasing the size of other members in the same plane. Mechanically fastening additional layers of material to a compression web to form an I or T section, or to otherwise increase the least dimension of the web, is such a practice (179). Because the components of such built-up members do not provide full composite action, judgment must be used to establish the level of contribution of the layers of material attached to the main member. Creep effects should be considered in making this assessment.

3.1.5-Composite Construction

Structural composites of lumber and other materials utilize the superior characteristics of each to obtain desirable structural efficiencies and/or extended service life. Timber-concrete bridge decks, timber-steel flitch beams and plywood-lumber stress-skin panels and box beams are such composites. Proven design procedures for timber-concrete beams and timber-steel members are available in wood engineering handbooks and textbooks (83,179). Detailed design and fabrication information for plywood-lumber structural components are available from the American Plywood Association (11). The American Institute of Timber Construction provides design information for composites involving glued laminated timber (4).

3.2-BENDING MEMBERS-GENERAL

3.2.1-Span of Bending Members

Use of the clear span plus one-half the required bearing length at each reaction to establish the design span for simple bending members has been a provision of the Specification since 1944. This definition provides a reasonable design standard for applying the assumption of knife edge supports used in the derivation of general equations for moment and reactions.

Prior to 1986, the span of continuous bending members was taken as the distance between the centers of supports over which the beam was continuous. To avoid unrealistic moment determinations where wider supports than those required for bearing were used, and to provide consistent treatment of all bending members, the definition of span length for continuous bending members was first made the same as simple members in the 1986 edition. For completeness and clarity, cantilevered members were included under the provision at the same time.

When determining shear forces on bending members resisting uniform loads, the provisions of 3.4.3 and 4.4.2 with regard to loads at or near the supports apply.

3.2.2-Lateral Distribution of Concentrated Load

Generally all designs involving multiple parallel bending members which are loaded through transverse elements such as flooring, decking or sheathing are capable of some lateral distribution of a concentrated load on one member to adjacent members on either side. The repetitive member factor for dimension lumber in paragraph 4.3.4 indirectly and partially accounts for such load redistribution. Comprehensive computer design methodology is available to establish the extent of load redistribution or sharing that is obtained in light frame wood floor systems (125). Such methodology is based on the stiffness, size, spacing and spans of the joist and panel members in the system and the rigidity of the connections between them.

The lateral distribution of concentrated loads is particularly important to obtain efficient design of bending members in structures such as bridges and warehouse or industrial buildings where heavy wheel loads are involved. Easily applied methods for determining the maximum moment and maximum shear in bending members subject to concentrated wheel loads are given in section 15.1 of the Specification. These methods, which are based on the thickness of the flooring or decking involved (two to six inches thick) and the spacing of the beams or stringers, have long been used in timber bridge design (2). The procedures have been verified through test and shown to be generally conservative, particularly when the portion of the load distributed to adjacent members is 40 percent or less (53).

3.2.3-Notches

3.2.3.1 Avoidance of notches in bending members has been a recommendation of the Specification since the earliest editions. Notches are a special problem in bending members due to the stress concentrations occurring at the corners and the difficulty of calculating the effects of shear and perpendicular to grain stresses occurring at such locations. The reduction in these stress concentrations that can result by using gradually tapered rather than square corner notches (62) was formally noted in the Specification in the 1977 edition.

The assumption that a notch having a depth of up to 1/6 the bending member depth and a length up to 1/3 the bending member depth has little practical effect on bending member stiffness (57,62) has been a provision of the Specification since the 1944 edition. For example, the reduction in stiffness of a 2×4 on 6 ft. span, a 2×8 on 12 ft. span and a 2×12 on a 16 ft. span of such a notch at midspan, when calculated by integration of the elastic curve for the variable cross-section bending member under a uniform load, is 1.9, 2.0 and 2.3 percent respectively. Research has shown that the effect of notches on bending member stiffness is somewhat greater than that indicated by the notch dimensions, and can be approximated by using a notch length equal to the actual length of the notch plus twice the depth of the notch (65,66,120). When this approximation is applied to the 1/6 depth by 1/3 length notch used in the example bending members above, the reduction in stiffness of the 2×4, 2×8 and 2×12 bending members is 3.8, 3.9 and 4.6 percent respectively, or about twice the effect obtained using the actual notch dimensions.

3.2.3.2 Prior to 1977, the Specification provided for the use of the net section at the notch for determining the bending strength of a notched bending member. This provision was based on early research which indicated that use of the net section at the notch was a sufficiently conservative design basis for commercial grades of sawn lumber (57,62). It was recognized even at that time that stress concentrations at the corners of the notch caused lowered proportional limit loads and caused failure to begin at lower loads than those expected from an unnotched bending member having a depth equal to the net depth of the notched bending member (57,62).

In the 1977 edition, as a result of field experience and new research related to crack propagation, the use of the net section procedure for determining induced bending moment in notched bending members was discontinued and specific notch limitations were established for different bending member sizes. These new provisions were continued in the 1986 and 1991 editions. The field performance history considered included (i) large bending members end-notched to the quarter points of the span which exhibited splitting and tension perpendicular to grain separations at relatively low loads; and (ii) the long record of satisfactory performance of light frame construction joists notched using good practice recommendations. Fracture mechanics research also confirmed and quantified the propensity of cracks to develop at square-cornered notches at relatively low bending loads (119,120,172). Narrow slit notches (3/32 inch long) were found to cause greater strength reductions than wide (greater than two inches long) notches of the same depth. The interaction of size and crack propagation has been characterized, with crack initiation increasing in proportion to the square root of the bending member depth for a given relative notch depth and constant induced bending and shear stress (66).

The allowance of notches on both the tension and compression sides of two and three inch thick sawn lumber bending members up to 1/6 the depth of the member in the outer thirds of the span is consistent with good practice recommendations for light frame construction (91,126). The satisfactory field performance of notched joists meeting these limitations, without use of the net section at the notch to determine actual stress, is attributed in part to the fact that tabulated bending design values (F_b) for the dimension grades of lumber already include section reductions for edge knots ranging from 1/6 to 1/2 the depth of the member. The restriction that only end notches are permitted in the tension side of nominal four inch and thicker sawn lumber bending members is based on experience with larger bending members and fracture mechanics analyses, as well as consideration of the shrinkage stresses that occur in such members when seasoning in service. Such stresses contribute to the perpendicular to grain stress conditions existing at the notch corners.

Tension perpendicular to grain stresses also occur with shear stresses at end notches to make a bending member more susceptible to splitting at the corner of such notches. The design provisions for shear in end notched bending members given in 3.4.4 include a magnification factor to account for this condition. The limitation on end notches in sawn lumber bending members to 1/4 or less the bending member depth is a good practice recommendation that also reflects experience and the effects of shrinkage stresses. Restricting the length of rectangular end notches, measured from the end of the bending member, to the depth of the member is advisable.

3.2.3.3 Prior to 1977, notching provisions for glued laminated timber and sawn lumber were the same. As a result of field experience with notched large glued laminated bending members and the difficulty of accurately quantifying stress concentration effects, application of sawn lumber notch provisions to glued laminated timber bending members was discontinued in the 1977 and subsequent editions of the Specification. The designer has the responsibility of determining if glued laminated timber bending members should be notched and how load carrying capacity should be calculated. Current good engineering practice is to avoid all notching of such bending members on the tension side except where notching at the supports is necessary. This end notching is generally limited to the smaller bending members and to notch depths not exceeding 1/10 of the bending member depth (4). The methods of 3.4.4 are used to calculate actual shear stresses parallel to grain (f_v) of tension side end notches in glued laminated timber members (4).

3.3-BENDING MEMBERS-FLEXURE

3.3.3-Beam Stability Factor, C_L

Background

Equations, verified experimentally, for calculating critical lateral buckling loads of any length and thickness of wood bending members under various loading and end fixity conditions were available as early as 1931 (184). However, in reflection of designer preference, the early editions of the Specification addressed the subject of lateral buckling or twisting of bending members by referencing approximate bending member depth to thickness rules (Section 4.4.1) for determining when and what type of lateral support should be provided (62). In 1968, general provisions for determining the adequacy of lateral support of bending members to prevent lateral buckling or displacement of the compression face were incorporated in the Specification for glued laminated timber bending members. The provisions were extended to sawn lumber bending members in the 1977 edition.

The procedures used to adjust bending design values for slenderness factor that were introduced in the Specification in 1968, and continued through the 1986 edition, were based on Canadian research involving verification of the applicability of lateral buckling theory to wood bending members and the development of simplified adjustment procedures for slenderness effects (86). The verification tests involved 3/4 inch wide simply supported and cantilever bending members up to 8 inches in depth and from 24 to 204 inches in length. The tests confirmed that the following simplified general equations based on a modulus of elasticity to modulus of rigidity ratio (E/G) of 16 gave reasonably good estimates of critical stress:

$$f_b = \frac{1.20E}{S^2} \qquad \text{(C3.3-1)}$$

where:

f_b = critical bending stress
E = modulus of elasticity

and

$$S^2 = \frac{\dfrac{T\ell_u d}{b^2}}{1 - C\left(\dfrac{d}{\ell_u}\right)\left(1 + \dfrac{2b}{3d}\right)} \qquad \text{(C3.3-2)}$$

with

T, C = constants depending on type of load and support condition
ℓ_u = unsupported length
d, b = depth and breadth of bending member

The critical bending stress equation is similar to that for long columns except with the Euler constant of 0.822 replaced by 1.20 and ℓ/d replaced by $\ell d/b^2$, the latter ratio being similar to the comparable ratio for steel bending members of $\ell d/bt$. Where t is defined as the flange or web thickness of the steel bending member.

Based on test results and steel design practice in England, the general equation was applied to critical stresses above the proportional limit by assuming the transition stress-strain curve for bending above the proportional limit was the same shape as that for compression. Thus, applying the established intermediate column formula defining this region for compression loading, the level at which inelastic beam buckling began was established as 2/3 of the tabulated bending design value F_b. Also analogous to column design provisions in the 1977 to 1986 editions of the Specification, an S value of 10 was selected as the level below which the full allowable bending design value could be used. Thus long, intermediate and short beam buckling criteria were established comparable to long, intermediate and short columns.

The equation for S was further simplified for design use by defining

$$S^2 = \frac{\ell_e d}{b^2} \qquad \text{(C3.3-3)}$$

and an effective length

$$\ell_e = 1.15\,\ell_u + 3d \qquad \text{(C3.3-4)}$$

for loads applied to the top of the bending member. For loads applied at the center of gravity of the bending member, the $3d$ term in the ℓ_e equation is deleted; and for loads applied on the bottom of the bending member the $3d$ term is replaced with $-d$. The 1.15 factor in the equation for ℓ_e represents a 15 percent increase in the actual ℓ_u to account for less than full torsional restraint at lateral support points (86).

Based on the foregoing analyses and simplifications, bending design values adjusted for slenderness factor (F_b') incorporated in the 1968 and 1977 editions of the Specification were:

Slenderness factor

$$C_s = \sqrt{\frac{\ell_e d}{b^2}} \qquad \text{(C3.3-5)}$$

For $C_s \leq 10$

$$F_b' = F_b \qquad \text{(C3.3-6)}$$

For $11 < C_s \leq C_k$

$$F_b' = F_b\left[1 - \frac{1}{3}\left(\frac{C_s}{C_k}\right)^4\right] \qquad \text{(C3.3-7)}$$

in which

$$C_k = \sqrt{3E/5F_b} \qquad \text{(C3.3-8)}$$

For $C_s > C_k$

$$F_b' = \frac{0.40\,E}{\left(C_s\right)^2} \qquad \text{(C3.3-9)}$$

The Euler buckling constant for long bending members of 0.40 used in the 1968 and 1977 editions of the Specification, corresponded to a factor of safety of 3.0 on E. This constant was changed in the 1982 edition to 0.438 to correspond to the factor of safety of 2.74 associated with the Euler buckling constant of 0.30 for long columns. (Note: The beam constant of 0.438 = the column constant of 0.30 times the ratio 1.20/0.822). The constant in the formula for C_k defining the slenderness factor above which inelastic stresses begin to occur also was changed at the same time from $\sqrt{3/5}$, or 0.775, to 0.811 to reflect the same change in factor of safety basis. In addition, the 1982 and 1986 editions incorporated changes in the specific equations for ℓ_e as noted in the specific commentary given under 3.3.3.5.

In the 1991 edition, the short, intermediate and long beam equations for adjustment of bending design values for slenderness factor were replaced with a single continuous beam equation comparable to that introduced for columns in section 3.7.1. This change and further modifications and additions to the ℓ_e equations made in the 1991 edition are addressed in the Commentary for 3.3.3.5 and 3.3.3.8.

3.3.3.1 When the breadth of the bending member is equal to or greater than the depth, lateral buckling of the bending member is not a factor and bending member capacity is limited by the applicable bending design value adjusted for factors other than slenderness (57,86).

3.3.3.2 The approximate depth to breadth rules for determining lateral support requirements for sawn lumber bending members given in 4.4.1 are alternate provisions to those of 3.3.3. The lateral support guidelines represented by the approximate rules have proven satisfactory in service for more than 40 years. The provisions of 3.3.3.2 and 4.4.1 may not give equivalent combinations of lateral support and allowable bending design values. Specific span and loading conditions should be checked to compare the relative restrictiveness of each.

3.3.3.3 When the compression edge of a bending member is continuously supported along its length and end bearing points are also restrained from rotation or displacement, lateral buckling under loads inducing compressive stresses in the supported edge is not a concern. However, the possibility of stress reversal, such as that associated with wind loadings, should be fully considered to assure that what is the tension side of the bending member under the predominant loading case also is adequately supported to carry any expected compressive forces. Bending members with very large depth to breath ratios, such that perpendicular to grain buckling of the member along its depth (web buckling) may be a factor, should be avoided.

3.3.3.4 The unsupported length ℓ_u of a bending member laterally supported at points of end bearing and loaded through uniformly spaced purlins or similar framing members that are appropriately connected to the top face of the bending member, or the side of the bending member by hangers or ledgers, to prevent lateral translation is the distance between such purlins or framing members (85).

3.3.3.5 Formulas for determining the effective span length, ℓ_e, from the unsupported length, ℓ_u, in the 1968 and 1977 editions of the Specification were for an ℓ_u/d ratio of 17. Formulas for other ℓ_u/d ratios were added in 1982. To correct a misinterpretation of a recommendation of the original researchers that resulted in unnecessarily conservative ℓ_e values, the formulas for determining ℓ_e were revised and generalized to apply to any ℓ_u/d ratio in the 1986 edition. Also included in this edition were new formulas for single span and cantilever beams with any load condition.

The 1991 edition continues the ℓ_e formulas from the 1986 edition but limits ℓ_e values for relatively short, deep bending members to those for bending members having an ℓ_u/d of 7. This limit was added because use of the general equations resulted in unrealistically large ℓ_e values for certain short, deep bending members and the fact that this beam length to depth ratio was the lower limit of the experimental data used to verify the methodology (86). In addition, the 1991 edition provides ℓ_e formulas for single span beams with various numbers of equally spaced and equal concentrated loads where lateral support occurs at each load point. These formulas, based on analysis of the effect of equally spaced purlins on beam buckling (85), show the increased capacity that is obtained by having lateral support at the point of application of a concentrated load.

The constants in the formulas for effective length in Table 3.3.3 include the 15 percent increase in ℓ_u for incomplete torsional restraint at lateral support points. The formulas given in the table apply to the condition of loads applied to the top of the bending member, the most conservative loading case. Formulas given in the footnote for load conditions not covered by the formulas in the body of the table represent the most limiting formula for the ℓ_u/d range from those given for specified load conditions.

3.3.3.6 The beam slenderness ratio, R_B, calculated as:

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} \qquad \text{(C3.3-10)}$$

is comparable to the slenderness ratio for solid columns, ℓ_e/d, in terms of its effect on allowable design strength.

3.3.3.7 Limiting the beam slenderness ratio of bending members, R_B, to a maximum value of 50 is a good practice recommendation that has been a requirement in the beam stability provisions of the Specification since 1968 to preclude the design of bending members with high buckling potential. For example, a 2×16 bending member having an unsupported length (ℓ_u) of 16 feet has a slenderness ratio of approximately 50. The limit was originally recommended to parallel the limit on the ℓ_e/d slenderness ratio for columns of 50 (86).

3.3.3.8 The single beam stability factor equation is applicable to all bending member slenderness ratio values (R_B) and replaces the short, intermediate and long beam equations given in previous editions for determining the effects of slenderness on allowable bending design values. The equation is of the same form as the continuous equation for calculating the column stability factor in 3.7.1.5. This column equation was first proposed in alternate form as a new method of determining the buckling stress of columns of any material in the elastic and inelastic range by a Finnish researcher in 1956 (226). The column equation has as its upper limit the tabulated compression design value parallel to grain associated with a crushing mode of failure at small ℓ_e/d ratios and the critical buckling design value (Euler column buckling stress) at large ℓ_e/d ratios. Continuity between these extremes of slenderness is obtained by assuming a curvilinear stress-strain relationship where the degree of nonlinearity caused by inelasticity, nonuniform material structure and initial eccentricity can be modeled through a single parameter "c". This continuous column equation form has been applied to bending members through substituting tabulated bending design values for tabulated compression parallel to grain (crushing) design values, critical (Euler) beam buckling design values for critical (Euler) column buckling design values, and selection of a value of "c" considered representative of bending member behavior. The general beam stability factor equation thus becomes

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{2c} - \sqrt{\left[\frac{1 + (F_{bE}/F_b^*)}{2c}\right]^2 - \frac{F_{bE}/F_b^*}{c}} \qquad \text{(C3.3-11)}$$

where:

F_b^* = tabulated bending design value multiplied by all applicable adjustment factors except C_{fu}, C_V, and C_L, psi

F_{bE} = critical buckling design value for bending members, psi

$$= \frac{K_{bE}E'}{R_B^2}$$

K_{bE} = Euler buckling coefficient for beams

c = nonlinear parameter for beams = 0.95

The 0.95 value for the parameter "c" is based on the generally satisfactory experience with bending members designed using the slenderness factor adjustment provisions included in previous editions of the Specification. A value of 0.95 for the parameter gives C_L values reasonably similar to equivalent values obtained from the previous specified short, intermediate and long beam equations.

For visually graded lumber, the Euler buckling coefficient for beams, K_{bE}, is 0.438. As shown by the above equation, the critical (Euler) beam buckling design value, F_{bE}, and the adjusted bending design value used in the beam stability factor equation are the same as the comparable values in the slenderness factor equations in the 1986 and earlier editions (see Equations C3.3-5 - C3.3-9). As shown in Figure C3.3-1, the continuous beam stability factor equation (Equation C3.3-11) gives lower values of C_L in the intermediate slenderness ratio ranges than comparable values obtained from earlier editions of the Specification.

Beam stability factors obtained from the equation in the 1991 edition are in general agreement with comparable values developed using the beam buckling equations developed in 1931 (57,184) when the latter are entered with a 15 percent increase of ℓ_u, an E/G ratio of 16 is assumed, and a 2.74 reduction factor on E is used (see Commentary under 3.3.3 for discussion of these assignments). For example, a 2x14 bending member with F_b of 1200 psi, E of 1,800,000 psi with an unsupported length of 168 inches and carrying a uniformly distributed load has a C_L of 0.347 based on 3.3.3.8 of the 1991 Specification and an equivalent C_L of 0.357 based on the 1931 methodology. For the same bending member and span carrying a concentrated load at the center, the C_L from 3.3.3.8 is 0.400 compared to a value of 0.426 based on the 1931 methodology. The differences between the two methodologies is attributed to the conservative simplifications made in the formulas for ℓ_e (86) and the use of the more conservative continuous beam equation for the long beam examples rather than direct application of the Euler beam buckling equation.

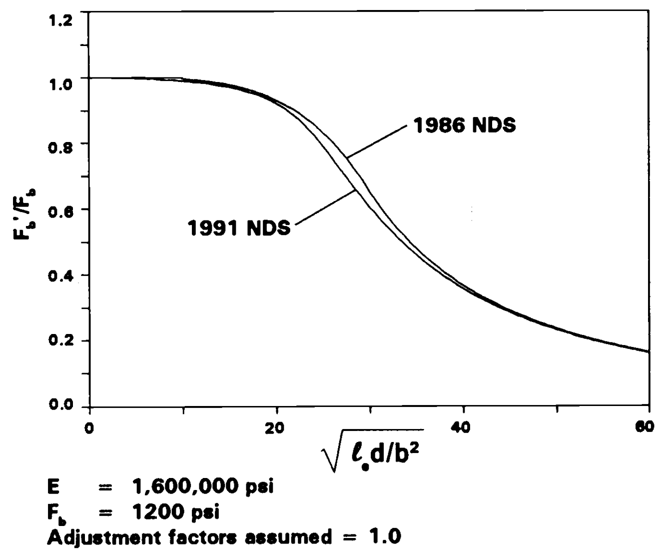


Figure C3.3-1 Beam Stability Equation Comparison

For sawn lumber bending members, the C_L equation of 3.3.3.8 is entered with an F_b^* adjusted for all C factors except C_L and C_{fu}. The latter is the flatwise use adjustment factor, which is applied independently to the bending design value when lumber is used in the flatwise orientation. A bending member is not subject to lateral buckling in this orientation since the breadth of the member is greater than its depth, therefore, C_L = 1.0 (see Commentary section 3.3.3.1). Thus, size adjustment factors for sawn lumber, C_F, are to be applied simultaneously with the beam stability factor, C_L. This is a more conservative practice than that used in earlier editions of the Specification, wherein size adjustments for sawn lumber bending members over 12 inches in depth were not applied simultaneously with slenderness factor adjustment. The previous approach was based on transfer of the original application of the slenderness factor methodology for glued laminated timber in the 1968 edition to large sawn lumber bending members. In the 1991 edition, the practice of not applying volume (size) adjustments,

C_V, simultaneously with the beam stability factor, C_L, is continued for glued laminated timber. This continuation is based on design experience and the position that beam buckling is associated with compression stresses whereas tabulated bending design values for glued laminated bending members are limited by the strength of the tension laminations.

The 0.438 value for the Euler buckling coefficient for beams is applicable to visually graded lumber and machine evaluated lumber (MEL). It includes a reduction of 2.74 on tabulated modulus of elasticity values and is related to the equivalent Euler buckling coefficient for columns of 0.30 by the ratio 1.2/0.822 (see background discussion in Commentary for 3.3.3). The modulus of elasticity of visually graded sawn lumber and MEL is considered to have a coefficient of variation of 0.25, unless a different coefficient of variation is specifically included on the MEL grade mark. The 2.74 factor thus represents an approximate 5 percent lower exclusion value on pure bending modulus of elasticity (1.03 times tabulated E values) and a 1.66 factor of safety.

For glued laminated timber, machine stress rated lumber (MSR) or other products having a coefficient of variation in modulus of elasticity of 0.11 or less, the Euler buckling coefficient for beams is 0.609. This also represents an approximate 5 percent lower exclusion value on pure bending modulus of elasticity and a factor of safety of 1.66. The 0.609 coefficient is related to the 0.438 coefficient for visually graded lumber by the ratio

$$\frac{[1 - 0.11(1.645)]}{[1 - 0.25(1.645)]} = 1.39$$

Examples C3.3-1 and C3.3-2 illustrate the use of beam stability provisions for bending members. Other design considerations for bending members (i.e., shear, compression perpendicular to grain, etc.) are not covered in these examples, but are addressed in other sections of the Commentary.

3.4-BENDING MEMBERS - SHEAR

3.4.1-Strength in Shear Parallel to Grain (Horizontal Shear)

3.4.1.1 Shear strength perpendicular to the grain, alternatively termed cross-grain or vertical shear, refers to shear stresses in the radial-tangential plane tending to cut the wood fibers perpendicular to their long axis. The strength of wood in this plane is very high relative to shear strength parallel to grain, or horizontal shear, which refers to shear stresses in the longitudinal-radial or longitudinal-tangential plane tending to slide one fiber past another along their long axes. As both parallel and perpendicular to grain shear occur simultaneously in structural wood bending members, parallel to grain shear strength is always the limiting case. Tabulated shear design values, F_v, are horizontal or parallel to grain shear stresses.

Shear in the tangential-longitudinal or radial-longitudinal plane tending to roll one fiber over another perpendicular to their long axes is termed rolling shear. Such shear, which occurs in structural plywood applications as shear in the plane of the plies, is not a design consideration in most lumber or timber product applications.

3.4.1.2 The limitation on application of shear design provisions to solid flexural members such as sawn lumber, glued laminated timber and mechanically laminated timber bending members was added to the Specification in the 1986 edition. Built-up components, such as trusses, were specifically excluded because of field experience that indicated the procedures might not be adequate for shear design of top-hung parallel chord trusses and similar components that contained load-bearing web and top chord connections near points of support. Because of the difficulty of making a general determination for all truss designs of the effects of stress concentrations and embedded metal connectors at such locations, and of the applicability of the general practice of ignoring loads close to supports, shear design at supports for proprietary built-up components is required to be based on testing, theoretical analysis and/or documented experience related to specific design configurations.

3.4.2-Shear Design Equations

Actual shear stress parallel to grain, f_v, in a circular bending member may be determined as:

$$f_v = \frac{4V}{3A} \qquad \text{(C3.4-1)}$$

where:

V = shear force

A = cross-sectional area of circular member

3.4.3-Shear Force

3.4.3.1 (a) Ignoring loads within a bending member depth of the support face for purposes of calculating shear force has been a provision of the Specification since the 1944 edition. The allowance assumes such loads are carried directly to the support by diagonal compression through the depth of the

Example C3.3-1

A Select Structural Southern Pine 4×16 beam on a 20 ft span supports a hoist located at the center of the span. Determine the maximum allowable load on the hoist (including its weight) based on bending. Assume normal load duration. Lateral support is provided at the ends only.

F_b = 1900 psi E = 1,800,000 psi (Table 4B)
C_F = (0.9)(1.1) C_D = 1.0 A = 53.38 in^2 S = 135.7 in^3

Beam Stability Factor C_L (3.3.3)

$F_b^* = F_bC_DC_F = (1900)(1.0)(0.9)(1.1) = 1880$ psi
$\ell_u/d = (20)(12)/(15.25) = 15.7 > 7$
$\ell_e = 1.37\ell_u + 3d$ (Table 3.3.3)
$= 1.37(20)(12) + 3(15.25) = 374.6$ in.

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(374.6)(15.25)}{(3.5)^2}} = 21.6$$

$K_{bE} = 0.438$

$$F_{bE} = \frac{K_{bE}E'}{R_b^2} = \frac{(0.438)(1,800,000)}{(21.6)^2} = 1691 \text{ psi}$$

$$C_L = \frac{1+(F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1+(F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$= \frac{1+1691/1880}{1.9} - \sqrt{\left[\frac{1+1691/1880}{1.9}\right]^2 - \frac{1691/1880}{0.95}}$$

$= 0.770$

Allowable Bending Design Value, F_b' (Table 2.3.1)

$F_b' = F_bC_DC_LC_F = (1900)(1.0)(0.770)(0.9)(1.1)$
$= 1448$ psi

Maximum Moment

Assume density of beam = 37.5 lb/ft^3
weight of beam = (37.5)(53.38/144) = 13.9 lb/ft
hoist plus payload = P
$M_{max} = P\ell/4 + w\ell^2/8$
$= (P)(20)(12)/4 + (13.9)(20)^2(12)/8$
$= 60P + 8340$

Maximum Allowable Load

$M_{allow.} = F_b'S$
Substituting,
$P = (F_b'S - 8340)/60$
$= ((1448)(135.7) - 8340)/60$
$= 3136$ lb ~ 3100 lb

Total allowable concentrated load = 3100 lb (hoist plus payload)

Example C3.3-2

A Select Structural Southern Pine 2×14 beam on a 16 ft span carries five 500 lb (DL+SL) concentrated loads from purlins spaced at 32 in. on center (1/6 points). Determine if the member is adequate for bending. Lateral support is provided at the purlins and the supports.

F_b = 1900 psi E = 1,800,000 psi (Table 4B)
C_F = 0.9 C_D = 1.15 A = 19.88 in^2 S = 43.89 in^3

Beam Stability Factor C_L (3.3.3)

$F_b^* = F_bC_DC_F = (1900)(1.15)(0.9) = 1967$ psi
$\ell_u = 32$ in.
$\ell_e = 1.73\ell_u = 1.73(32) = 55.4$ in. (Table 3.3.3)

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(55.4)(13.25)}{(1.5)^2}} = 18.1$$

$K_{bE} = 0.438$

$$F_{bE} = \frac{K_{bE}E'}{R_b^2} = \frac{(0.438)(1,800,000)}{(18.1)^2} = 2418 \text{ psi}$$

$$C_L = \frac{1+(F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1+(F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$= \frac{1+2418/1967}{1.9} - \sqrt{\left[\frac{1+2418/1967}{1.9}\right]^2 - \frac{2418/1967}{0.95}}$$

$= 0.886$

Allowable Bending Design Value, F_b' (Table 2.3.1)

$F_b' = F_bC_DC_LC_F = (1900)(1.15)(0.886)(0.9)$
$= 1742$ psi

(cont.)

Example C3.3-2 (cont.)

Maximum Moment

Assume density of beam = 37.5 lb/ft^3

weight of beam = (37.5)(19.88/144) = 5.2 lb/ft

Purlin loads = P (five at 1/6 points) = 500 lb

$$M_{max} = P(0.75\ell) + w\ell^2/8$$
$$= (500)(0.75)(16)(12) + (5.2)(16)^2(12)/8$$
$$= 72{,}000 + 1{,}988 = 73{,}988 \text{ in-lb}$$

Actual Bending Stress, f_b

$$f_b = M_{max}/S$$
$$= (73{,}988)/(43.89)$$
$$= 1686 \text{ psi} < F_b' = 1742 \text{ psi} \quad \textbf{ok}$$

2x14 member satisfies NDS bending criteria

bending member. The clarification that such practice is permitted only for bending members fully supported on one surface and loads applied to the opposite surface, the condition obtained with solid single structural members, was added in the 1982 edition.

3.4.3.1 (b) Earliest editions of the Specification placed the critical moving load at three beam depths from the support. This procedure was based on shear analysis of checked or end-split sawn lumber bending members that shows part of the shear force near the support is carried to the support by the upper and lower portions of the bending member independently of the portion of the bending member at the neutral axis (see Appendix E of the Specification). The general shear stress equation does not account for such a redistribution of stress. Beginning with the 1960 edition, placement of the critical moving load was changed to one beam depth from the support to cover the general case of shear force for any material. The special case of critical moving loads in checked or end-split lumber bending members is treated separately in the Specification (see Section 4.4.2).

3.4.3.2 This provision of the Specification establishes a limiting condition for use of the two-beam shear force equations for sawn lumber given in 4.4.2. The criterion requires the actual shear stress parallel to grain, f_v, calculated in accordance with 3.4.2 and 3.4.3.1 using conventional shear force values to be equal to or lower than the maximum allowable shear design value for an unsplit or unchecked member of the species and grade of lumber being considered. Such stresses are equivalent to twice the shear design values parallel to grain tabulated in the Supplement to the Specification. This same limitation was included in the 1986 edition.

Interpretation of how the two-beam provisions of the Specification are applied differed in the 1982 and earlier editions from that in the 1986 and 1991 editions. However, these early provisions also contained limits on the maximum shear design value parallel to grain that could be used. From 1944 through the 1973 editions, the shear design value parallel to grain for an equivalent unsplit member also was the upper limit. In the 1977 and 1982 editions a limit equivalent to 75 percent of the tabulated shear design value parallel to grain was employed. The two-beam provisions and associated limitations in the current Specification were first published in the 1986 edition after reevaluation of the original two-beam research and review of early interpretative analyses (see Commentary for 4.4.2).

3.4.4-Shear Design for Notched Bending Members

3.4.4.1 The equation for calculating actual shear stress parallel to grain in rectangular bending members containing end notches on the tension face has been a provision of the Specification since the 1944 edition. In this calculation, the actual shear stress determined by entering the normal shear equation with the depth of the bending member above the notch is increased by the ratio of original (depth away from the notch) to notched depth. Thus for a given shear force and depth of notched member, shear capacity is reduced as the amount of material below the notch is increased. This behavior, verified by tests of bending members of clear wood of two species notched to various depths (158), is related to the concentration of tension and shear stresses occurring at the reentrant corner of the notch.

The equation is applicable to any rectangular bending member notched on the tension face. Because of the reduction in shear strength associated with the stress concentrations occurring at the notch, shear capacity is more likely to govern overall bending member capacity of notched beams with span to depth ratio of 12 or less than equivalent unnotched bending members. End notches in sawn lumber bending members are limited to 1/4 the beam depth (see 3.2.3.2). Limiting the length of rectangular end notches (measured from the end of the bending member) to the original depth of the member is advisable.

Recent research on tension-side end notched beams has examined the combined effects of loading type, end support, and beam and notch variables such as beam height, fractional notch depth, notch radius, and notch location (235). Results indicate that the effects of these factors can be represented by a material parameter established from destructive tests of notched beams. The strength equation developed from this research provides design criteria for beams notched on the tension face that explicitly consider notch geometries and the relative proportion of moment and shear at the notch sections. Current provisions of the Specification do not provide for this type of analysis. Designers who use the alternative new methodology have the responsibility for establishing its appropriateness and adequacy for particular design cases.

Shear strength of bending members is less affected by rectangular end notches on the compression face than on the tension face. Tests have shown that the actual shear stress parallel to grain in bending members end-notched on the compression face may be determined from the following equation (62):

$$f_v = \frac{3V}{2bg} \qquad \text{(C3.4-2)}$$

where:

V = shear force

b = width of bending member

g = $[d - (d - d_n)(e/d_n)]$

with

d = depth of bending member

d_n = depth of bending member below the notch

e = distance between end of notch and face of support

For a bending member with a compression face end-notch of one-quarter the beam depth extending one-half the depth of the bending member from the support, the value of the effective depth "g" in the foregoing equation is $(15/18)d$. For a tension face end-notch of one-quarter the beam depth, the effective beam depth from Equation 3.4-3 is $(9/16)d$.

In all cases, the shear force, V, used in the calculation of actual shear stress parallel to grain in end-notched bending members shall not be based on the alternate two-beam shear provisions of 4.4.2 for sawn lumber and shall include all loads located within a beam depth from the face of the support. These limitations were first introduced in the 1977 and 1982 editions, respectively.

3.4.4.2 The equation for calculating actual shear stress parallel to grain in members of circular cross section end-notched on the tension face is a new provision of the Specification. This equation parallels that for end-notched rectangular bending members with the area of the circular member (A_n) at the notch replacing the width (b) and depth at the notch (d_n) in the equation for the rectangular beam equation as shown below.

rectangular cross section:

$$f_v = \frac{3V}{2bd_n}\left(\frac{d}{d_n}\right) \qquad \text{(C3.4-3)}$$

circular cross section:

$$f_v = \frac{3V}{2A_n}\left(\frac{d}{d_n}\right) \qquad \text{(C3.4-4)}$$

where:

d_n = depth of bending member above the notch

A_n = cross-sectional area of circular bending member at notch

and

other symbols as defined under Equation C3.4-2

Although it is known that the boundary conditions assumed in the derivation of the standard VQ/Ib equation are not met by unnotched circular cross sections, it has been shown that shear stresses parallel to grain near the neutral axis of an unnotched circular member that are calculated using this standard theory, or $4V/3A$, are within 5 percent of actual stresses (149).

The equation for end-notched members of circular cross section (Equation C3.4-4) is an approximate expression that is considered to provide for reasonable allowable shear loads. The cross-sectional area of a circular bending member above a notch, A_n, may be calculated from the formula:

$$A_n = \frac{d^2}{4}(\pi - \alpha_1 + \cos\alpha_1 \sin\alpha_1) \qquad \text{(C3.4-5)}$$

where:

d = diameter of unnotched circular bending member

α_l = one-half the angle subtending the chord located at $d/2$ minus the notch depth (c) below the unnotched center of the circular bending member
= $\cos^{-1}(1 - 2c/d)$

For a circular member containing a 1/4 depth (1/2 radius) end notch, the approximate equation, C3.4-4, without the application of the magnification factor of d/d_n results in actual shear stresses parallel to grain that are 8 percent greater than those determined through application of the conventional VQ/Ib theory to truncated circular sections. Also, because of the curvature of the member in the perpendicular plane, the stress concentrations occurring at end notches in members of circular cross section are viewed as somewhat less severe than those in end-notched rectangular sections. Thus the provisions of 3.4.4.2, which utilize the same magnification factor as that used for notches in rectangular members, are judged a conservative basis for shear design of end-notched circular bending members.

3.4.4.3 Procedures used to calculate actual shear stresses parallel to grain in bending members of other than rectangular or circular cross section containing end notches on the tension face should account for any effects of stress concentrations that may occur at reentrant corners.

3.4.4.4 Use of tapered rather than squared end notches have been shown by test to greatly reduce the magnification factor for stress concentrations represented by the d/d_n factor in the shear stress equation for end notched members in 3.4.4.1. Rounding of the cut to the center of the bending member effectively eliminates the effect of stress concentrations and the actual shear stress parallel to grain is reduced to that which occurs with a bending member equivalent to the depth (d_n) of the notched bending member (158).

3.4.5-Shear Design for Bending Members at Connections

3.4.5.1 Provisions for calculating actual shear stress parallel to grain in bending members at connections were made progressively more conservative between the 1944 and 1977 editions of the Specification but have remained essentially unchanged since that time. In the 1944 edition, a 50 percent increase in tabulated shear design values parallel to grain, F_v, was allowed for shear design associated with connections and use of two-beam shear procedures (see 4.4.2) was permitted.

In 1947, actual shear stress parallel to grain at all connections was required to be calculated as

$$f_v = \frac{V}{bd_e} \qquad \text{(C3.4-6)}$$

where d_e was the depth of the member less the distance from the unloaded edge to the edge or center of the nearest fastening, the same effective distance as defined in the present edition. In the 1957 edition, the equation was changed to

$$f_v = \frac{3V}{2bd_e} \qquad \text{(C3.4-7)}$$

In 1973, the 50 percent increase in tabulated shear design values parallel to grain was limited to only those connections which were at least five times the depth of the member from its end. Additionally, the actual shear stress parallel to grain determined on the basis of the full cross section at such joints was required to be equal to or less than the tabulated shear design value parallel to grain without the 50 percent increase.

In 1977, a modified equation for calculating actual shear stress parallel to grain at connections located less than five times the depth of the member from its end was introduced. This equation:

$$f_v = \frac{3V}{2bd_e}\left(\frac{d}{d_e}\right) \qquad \text{(C3.4-8)}$$

is similar to that for end-notched rectangular bending members where the ratio d/d_e is comparable to the magnification factor d/d_n. The disallowance of the 50 percent increase in tabulated shear design values parallel to grain when checking actual shear at connections less than five times the depth of the member from its end was not changed with introduction of the new equation.

Also in 1977, use of the alternate two-beam shear provisions for determining V in sawn lumber at connections at any location was specifically excluded. Use of the general provision allowing loads within a beam depth from the support to be neglected in shear at connections was specifically disallowed in 1982.

Provisions for shear design at connections, as revised through the 1982 edition, have been carried forward to the 1991 edition.

The 50 percent increase in tabulated shear design values parallel to grain allowed in some joint details is considered to be based on the judgment that the stress concentrations present at such joints are of lower magnitude than those assumed (4/9 reduction on block shear specimen values) in the establishment of tabulated shear design values parallel to grain. The more restrictive provisions introduced for shear design at connections since 1944, including the limitation on application of the increase to tabulated shear design values parallel to grain and the addition of the d/d_e stress magnification factor, reflect conservative assessments of (i) field experience with large bending members and (ii) the effects of shrinkage or potential splitting and excessive tension perpendicular to grain stresses at connections, particularly those near the ends of the members.

Although neither the provisions of 3.4.3.1 allowing loads within a beam depth from the support to be ignored nor the two-beam provisions of 4.4.2 are applicable to shear design at connections, an allowable shear design value parallel to grain, F_v', for members with limited splits, shakes or checks (C_H of 2.0) may be used for connections in sawn lumber bending members. This adjusted allowable shear design value ($F_v \times C_H$) is cumulative with the 50 percent increase in tabulated shear design values parallel to grain provided in 3.4.5.1 (b) when checking the actual shear stress from Equation 3.4-6 (Commentary Equation C3.4-8); as well as when checking the actual shear stress based on the gross section area of the member as required by 3.4.5.1 (b).

When the ratio d/d_e exceeds 0.67 for a connection located five or more times the depth of any member from its end, the actual shear stress parallel to grain, f_v, based on gross section will limit the design rather than the actual stress based on Equation 3.4-6 and the related 50 percent increase in the allowable shear design value parallel to grain. However, in most loading situations, the gross section actual shear stress parallel to grain at locations five or more times the depth of the bending member from its end will not be more limiting than the maximum actual shear stress occurring at supports.

Examples C3.4-1 and C3.4-2 illustrate the use of shear design provisions for bending members at connections.

3.4.5.2 Bending members supported by concealed or partially hidden hangers whose installation involves kerfing or notching of the member are designed for shear using the notched bending member provisions of 3.4.4. This requirement was introduced in the Specification in the 1973 edition.

3.5-BENDING MEMBERS - DEFLECTION

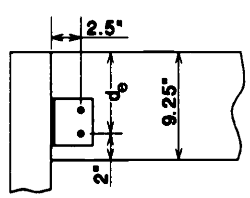
3.5.1-Deflection Calculations

Allowable Modulus of Elasticity. Tabulated modulus of elasticity design values, *E*, in the Specification for sawn lumber, glued laminated timber and round timber are average values for the species and grade combinations designated. Individual pieces within each such combination will have modulus of elasticity values both higher and lower then the tabulated mean value.

Tabulated modulus of elasticity values are considered to contain a shear deflection component equivalent to that occurring in a rectangular bending member on a span-depth ratio of 21 under uniformly distributed load. Assuming a modulus of elasticity to modulus of rigidity ratio (*E/G*) of 16, pure bending modulus of elasticity may be taken as 1.03 times the tabulated value. Standard methods for adjusting modulus of elasticity to other load and span-depth conditions are available (21).

Example C3.4-1

A No. 1 Douglas Fir-Larch 2×10 beam, supported by two 1/2-in. bolts in a clip angle connection attached to a girder, has an end reaction of 650 lb (DL+LL). Assume a normal load duration. Check shear in the member at the connection. The connection meets NDS criteria for bolt load, spacing and end and edge distance. The bolt on the unloaded edge is 2" from the edge of the member.



F_v = 95 psi (Table 4A)
C_D = 1.0
b = 1.5 in.
d = 9.25 in.

Allowable Shear Design Value Parallel to Grain, F_v' (Table 2.3.1)

$F_v' = F_v C_D$ = (95)(1.0) = 95 psi

Actual Shear Stress Parallel to Grain at Connection, f_v (3.4.5.1)

end distance to bolt = 2.5 in. < 5d = 5(9.25) = 46.25 in.
therefore, Equation 3.4-5 controls

edge distance to bolt = 2.0 in. > 1.5d = 1.5(0.5) = 0.75 in.
therefore, **ok** (8.5.3)

d_e = d - distance from unloaded edge to nearest bolt
= 9.25 - 2.0 = 7.25 in.

V = reaction = 650 lbs *(cont.)*

Example C3.4-1 (cont.)

$$f_v = \frac{3V}{2bd_e}\left(\frac{d}{d_e}\right)$$

$$= \frac{(3)(650)}{(2)(1.5)(7.25)}\left(\frac{9.25}{7.25}\right)$$

$= 114$ psi $> F_v' = 95$ psi **ng**

At this point, member size could be increased or the clip angle connection repositioned. Reposition connection so that the bottom bolt is 1.25 in. from the unloaded edge.

Check edge distance requirements (8.5.3)

required unloaded edge distance = 1.5D
= (1.5)(0.5) = 0.75 in. < 1.25 in. **ok**

Recompute f_v

d_e = d - distance from unloaded edge to nearest bolt
= 9.25 - 1.25 = 8.0 in.

$$f_v = \frac{3V}{2bd_e}\left(\frac{d}{d_e}\right)$$

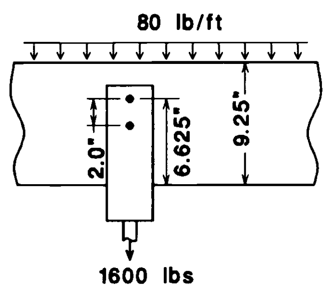
$$= \frac{(3)(650)}{(2)(1.5)(8.0)}\left(\frac{9.25}{8.0}\right)$$

$= 94$ psi $< F_v' = 95$ psi **ok**

Member satisfies NDS shear criteria at the connection if the bottom bolt is located 1.25 in. from the unloaded edge

Example C3.4-2

A Select Structural Southern Pine 4×10 beam, on a span of 141 in., supports a uniform load of 80 lb/ft (DL+LL) plus two equal concentrated loads of 1600 lb (LL) at the one-third points from tension stirrups supporting a catwalk. The tension stirrups are connected to the member with a steel U-plate with two 1/2 in. bolts at each connection as shown. Check shear in the member at one of the connections. Assume 3 in. supports and normal load duration.



Check Edge Distance Requirements (8.5.3)

At each U-plate connection the two bolts are spaced 2.0 in. apart with the bottom bolt located at the neutral axis (see 8.5.3.3), or 9.25/2 = 4.625 in. from the bottom edge, and the top bolt at 4.625 + 2.0 = 6.625 in. from the bottom edge, or 9.25 - 6.625 = 2.625 from the top edge.

Requirements:

Loaded edge = $4D$ = 4(0.5) = 2.0 in. < 4.625 in. **ok**
Unloaded edge = $1.5D$ = 1.5(0.5) = 0.75 in. < 2.625 in. **ok**
Spacing between bolts = $4D$ = 4(0.5) = 2.0 in. = 2.0 in. **ok**

F_v = 90 psi C_D = 1.00 (Table 4B)
b = 3.5 in. d = 9.25 in.

Allowable Shear Design Value Parallel to Grain, F_v' (Table 2.3.1)

$F_v' = F_v C_D C_H = (90)(1.0) = 90$ psi

Actual Shear Stress at Connection, f_v (3.4.5.1)

end distance to connection = 141/3 + 1/2 support length = 47 + 3.0/2 = 48.5 in. > $5d$ = 5(9.25) = 46.25 in. therefore, Equation 3.4-6 controls

d_e = d – distance from unloaded edge to nearest bolt
= 9.25 - 2.625 = 6.625 in.

$V_{at\ connection} = V_{support} - wx = P + w\ell/2 - wx$
= 1600 + (80)(141/12)/2 - (80)(47/12) = 1757 lb

$$f_v = \frac{3V}{2bd_e} = \frac{(3)(1757)}{(2)(3.5)(6.625)}$$

$= 113.7$ psi $< 1.5F_v' = 1.5(90) = 135$ psi **ok**

Check Shear in Gross Section

$V_{at\ connection}$ = 1757 lb

$$f_v = \frac{3V}{2bd} = \frac{(3)(1757)}{(2)(3.5)(9.25)}$$

$= 81.4$ psi $< F_v' = 90$ psi **ok**

Member satisfies NDS shear criteria at the connections

The use of average modulus of elasticity values for the deflection design of wood bending members has been general practice for more than 50 years. Experience has shown that such values provide an adequate measure of the immediate deflection of bending members used in normal wood structural applications. It should be noted that a reduced modulus of elasticity value is used in beam stability analyses. The reduction is incorporated in the Euler buckling coefficient for beams, K_{bE}, and is equivalent to that incorporated in the buckling coefficient for columns (see Commentary for 3.3.3).

Floor Joist or Beam Deflection. In the particular case of floor joists or beams, maximum allowable spans are commonly associated with a deflection limitation of $\ell/360$ under uniform live load, or

$$\frac{\ell}{360} = \frac{5w\ell^4}{384EI} \qquad \text{(C3.5-1)}$$

For constant w and E, this standard criterion requires that I change in proportion to ℓ^3 as the span increases. This means that the deflection under a constant concentrated load P applied at the center of the span of a joist or beam having the same E, or

$$\Delta_c = \frac{P\ell^3}{48EI} \qquad \text{(C3.5-2)}$$

will remain constant as span increases. Thus, for a given species and grade combination and constant uniform load, there is no change in the absolute deflection associated with a given concentrated load as span is increased if the $\ell/360$ criterion is met.

Notwithstanding the concentrated load deflection limit imposed by the $\ell/360$ requirement, the deflection performance of floors subjected to walking traffic loads and designed to meet the $\ell/360$ criterion for code specified uniform load may not be acceptable to some owners or occupants, particularly where beam spans exceed 14 feet. A comprehensive study of home owner response to levels of floor performance conducted in Canada found that deflection under a concentrated load is the best available measure of a floor's acceptability (142). The study indicated that an increasingly lower absolute deflection under a constant concentrated load was required as spans increase above 10 feet if the range of preferences of most home owners was to be covered. The use by many builders in the United States of joists a size deeper than those required by the code for a given span also is indicative of such performance preferences.

A comprehensive computer design methodology for light frame wood floor systems, which accounts for the composite action of sheathing and framing members joined with varying connector stiffness, is available that provides optional spans that are shorter than those permitted by the $\ell/360$ criterion in order to meet the stiffness performance desired by many users (125). Differences between the concentrated load deflection permitted by the optional criterion and the $\ell/360$ criterion increase as the span increases. For example, the $\ell/360$ criterion imposes the same absolute deflection limit for a fixed concentrated load applied to joists used on spans of 10, 12 and 20 feet (see Commentary related to Equation C3.5-2). The optional methodology limits such concentrated load deflections at spans of 12 feet and 20 feet to 72 percent and 38 percent, respectively, of that for a 10 foot span.

In view of the foregoing, it should be understood that the use of the $\ell/360$ deflection criterion for uniform load and the modulus of elasticity values tabulated in the Specification do not necessarily provide dynamic floor performance that will be found acceptable to all users. Such performance must be evaluated under other criteria.

Critical Applications. In certain applications, such as in uses of closely engineered structural components on long spans, deflection may be a critical design consideration. If a maximum immediate deflection under load must be assured, use of a reduced modulus of elasticity value may be appropriate. Such values may be developed using the following coefficients of variation:

	COV_E
Visually graded sawn lumber and machine evaluated lumber (MEL)	0.25
Machine stress-rated sawn lumber (MSR)	0.11
Glued laminated timber (six or more laminations)	0.10

Machine evaluated lumber (MEL) is assumed to have the foregoing coefficient of variation (COV_E) unless a different coefficient is specifically included in the grade mark.

Reducing average modulus of elasticity design values tabulated in the Specification for a species-grade combination by the product of the average value and 1.0 or 1.65 times the applicable coefficient of variation provides an estimate of the modulus of elasticity that is nominally exceeded by 84 percent or 95 percent, respectively, of the individual pieces in the combination.

Such estimated reductions are given by the following formulas:

$$E_{0.16} = E - E(1.0)(COV_E) \qquad \text{(C3.5-3)}$$

$$E_{0.05} = E - E(1.65)(COV_E) \qquad \text{(C3.5-4)}$$

3.5.2-Long Term Loading

The modulus of elasticity values tabulated in the specification provide a measure of the immediate deflection of a member that occurs when a load is applied. If the load is sustained, the member will exhibit a slow but continual increase in deflection over time (creep). At moderate to low levels of sustained stress and under stable environmental conditions, the rate of creep will decrease over time. Such creep can be described (73,87) by a power function of the form

$$\Delta_{cr} = At^B \qquad \text{(C3.5-5)}$$

where:

Δ_{cr} = relative creep

$= \dfrac{\text{(total deflection - immediate deflection)}}{\text{immediate deflection}}$

t = time

A, B = constants

Where creep is decreasing over time, total creep occurring in a specific period of time is approximately proportional to stress level (168,221). Total bending creep increases with increase in moisture content (44,176) and temperature (154); and is greater under variable compared to constant relative humidity conditions (154). Creep deflection that is increasing at a constant rate should be considered a possible danger signal; and when creep deflection is increasing at an increasing rate, imminent failure is indicated (13,18,176,221).

Code specified maximum wind, snow and live loads are pulse type loadings with low frequency of occurrence. Thus creep deflection is not a significant factor in most situations. Where dead loads or sustained live loads represent a relatively high percentage of total design load, creep may be a design consideration. In such situations, total deflection under long term loading is estimated by increasing the immediate deflection associated with the long term load component by 1.5 for seasoned lumber or glued laminated timber or by 2.0 for unseasoned lumber. Use of twice the initial deflection as a basis for accounting for deformation associated with long-term loads has long been recognized (13). This guideline was introduced in the 1971 edition as a general criterion where it was necessary to limit long-time deflection. The guideline was revised to 1.5 for seasoned and 2.0 for unseasoned material in the 1977 edition.

3.6-COMPRESSION MEMBERS - GENERAL

3.6.2-Column Classifications

3.6.2.1 Simple solid columns are defined as single piece members or those made of pieces properly glued together to form a single member. Such glued members are considered to have the grain of all component pieces oriented in the same direction and to be made with a phenolic, resorcinol or other rigid adhesive. The performance of columns made using elastomeric adhesives are not covered by the provisions of the Specification except where it has been established that the adhesive being used possesses strength and creep properties comparable to those of standard rigid adhesives.

3.6.2.2 Design provisions for spaced columns have been a part of the Specification since 1944.

3.6.2.3 Built-up mechanically laminated columns are not designed as solid columns. The strength of such columns is the sum of the strengths of the individual laminations except where specific additional procedures apply. The provisions of 15.3 for the design of bolted or nailed built-up mechanically laminated columns are new to the Specification. Previous editions referenced other methods (62).

3.6.3-Strength in Compression Parallel to Grain

Calculated actual compression stress parallel to grain, f_c, based on net section area is required not to exceed the tabulated compression design value parallel to grain, F_c, multiplied by all applicable adjustment factors except the column stability factor, C_P. This provision is included to cover the case where the net or reduced section does not occur at the column length location most subject to potential buckling; and actual compression stress, f_c, at such points are calculated on the basis of gross section area. Whether f_c at the critical buckling location is based on net or gross section area, it is to be understood that this stress shall not exceed the tabulated compression parallel to grain design value multiplied by all applicable adjustment factors, including the column stability factor, C_P.

Prior to 1977, the Specification provided that calculated allowable compression design values parallel to grain which included the effect of slenderness ratio, ℓ_e/d, could be adjusted by the load duration factor. This early practice was based on the assessment that the reduction factor incorporated in the Euler buckling coefficient for columns, K_{cE}, accounted for any long-time loading effects and that increases could be made for short-time loads without encroaching on the factor of safety (62). The practice began to be interpreted by some users to mean that modulus of elasticity design values were subject to the same load duration adjustments as strength design values. To avoid such misinterpretations and to clearly separate load duration factors for strength from creep effects or deflection under long term loading, the 1977 and subsequent editions of the Specification have provided for application of load duration adjustments only to tabulated compression design values parallel to grain. Thus no adjustments for load duration are embedded in column stresses limited by stiffness or buckling potential. This is evidenced by the column stability factor, C_P, computation procedure of 3.7.1.5 and the separation of this factor from the load duration factor, C_D.

3.6.4-Compression Members Bearing End to End

Bearing stresses occurring at the ends of compression members are limited by bearing design values parallel to grain (F_g) and differ from compression design values parallel to grain (F_c) which are applicable to columns. The latter include the effects of knots and other permitted grade characteristics while the former are based on clear wood properties. As provided in 3.10.1, use of bearing design values parallel to grain is predicated on the presence of adequate lateral support to resist moments at the joint.

3.6.6-Column Bracing

Alternative methods of bracing columns supporting trusses or beams are described in Appendix A.9 of the Specification. Unless fixity at the column base or diaphragm action of walls and connecting roof sheathing is provided, columns resisting lateral loads on the side walls should be braced in the direction of the trusses by knee braces (179) or by extending the column to the top chord of a sufficiently deep parallel chord truss. If the sheathing material does not provide adequate resistance to end-wall lateral loads, cross or knee bracing between column members should be provided. Related information on bracing of trusses is given in Appendix A.10.

Designing vertical cross bracing and horizontal struts between trusses to withstand a horizontal compression load equivalent to 10 percent of the stress (load) on the bottom chord of the truss divided by the product of the number of lines (along the span of the truss) of cross bracing and the number of cross braces in each line has been recommended (179).

3.6.7-Lateral Support of Arches, Studs and Compression Chords of Trusses

Provisions for determining slenderness ratio, ℓ_e/d, of laterally supported arches and compression chords have been part of the Specification since the first edition. These provisions were removed from the body of the Specification and placed in the Appendix in the 1982 edition. At this time the guidelines were revised to recognize lateral support provided by purlins (members spaced more than 24 inches apart) as well as roof joists and expanded to include studs used in light frame construction.

The provisions of Appendix A.11 provide that where roof joists or purlins are used between arches or compression chords, the larger of (i) the slenderness ratio based on the depth of the arch or chord and its length or (ii) the ratio based on the breadth of the beam and the distance between purlins or joists is used in the calculation of column stability factor, C_P. Where the depth of the arch or chord is used in the slenderness ratio calculation, the effective length, ℓ_e, is the length between points of lateral support and/or points of contraflexure on the deflection curve in the plane of the depth dimension. Thus the slenderness ratio based on depth is that represented by the ratio ℓ_{e1}/d_1 in Figure 3F of the Specification. The ratio based on the breadth of the arch or chord and the distance between purlins or joists is that represented by the ratio ℓ_{e2}/d_2 in Figure 3F of the Specification.

Use of the depth of the arch or compression chord in the determination of ℓ/d rather than the breadth or thickness of the member was limited to roof joists prior to the 1982 edition. In these earlier editions, purlins (members spaced more than 24 inches apart) were specifically excluded from this provision. The spacing of roof joists (24 inches or less) was considered such that when these members were used, the ℓ_{e1}/d_1 ratio rather than the ℓ_{e2}/d_2 ratio was the limiting case. In the 1982 and subsequent editions, buckling about both the strong and weak axis of the arch or chord is specifically examined and therefore the lateral support provided by purlins as well as roof joists is taken into account.

When continuous decking or sheathing is installed on the top of the arch or compression chord, it is long standing practice to use the ratio of the depth of the arch or chord and the length between points of lateral support and/or points of contraflexure in the plane of the depth dimension, or ℓ_{e1}/d_1, as the slenderness ratio.

Use of the depth of the stud as the least dimension in calculating the slenderness ratio in determining the axial load-carrying capacity of normally sheathed or clad light frame wall systems also is long standing practice. Experience has shown that any code allowed thickness of gypsum board, hardwood plywood or other interior finish adequately fastened directly to studs will provide adequate lateral support of the stud across its thickness irrespective of the type or thickness of exterior sheathing and/or finish used.

3.7-SOLID COLUMNS

3.7.1-Column Stability Factor, C_P

3.7.1.2 Some standard practices for determining effective column length are given in Appendix A.11 of the Specification. In general, the effective length of a column is the distance between points of support that prevent lateral displacement of the member in the plane of buckling, and/or between points of contraflexure on the deflection curve. It is common practice in wood construction to assume most column end conditions to be pin connected (translation fixed, rotation free) even though in many cases some partial rotational fixity is present. Where the end conditions in the plane of buckling are significantly different than translation fixed and rotation free, adjustment of actual column lengths by the recommended coefficients, K_e, in Appendix G of the Specification are permitted.

As shown in Table G-1 of Appendix G, the recommended coefficients are larger than the theoretical values for all cases where rotational restraint of one or both ends of the column is assumed. This conservatism is introduced in recognition that full fixity is generally not realized in practice. The recommended values of K_e are the same as those used in steel design (3) except for the sixth case (rotation and translation fixed one end, rotation free and translation fixed other end) where a more conservative coefficient (20 percent larger than the theoretical value) is specified. The level of conservatism used in the sixth case is equivalent to that used for the third case (rotation and translation fixed both ends).

3.7.1.4 The limitation on the slenderness ratio of solid columns in permanent structures to 50, which has been a provision of the Specification since 1944, is a good practice that precludes the use of column designs susceptible to potential buckling from slight eccentricities in loading or nonuniform cross sectional properties. The ℓ_e/d limit of 50 is comparable to the ℓ/r limit of 200 (ℓ/d of 58) used in steel design (3).

Allowing a temporary ℓ_e/d ratio of 75 during construction is a new provision in the 1991 edition. This allowance is based on 15 years of satisfactory experience with temporary bracing of trusses installed in accordance with truss industry standards (185); recognition that in most cases the assembly will carry only dead loads until load distributing and racking resisting sheathing elements are installed; and experience with a similar provision in steel design. In the latter regard, an ℓ/r ratio of 300 (ℓ/d of 87), or 50 percent higher than the permanent design maximum of 200 (3) is permitted during construction with cold-formed steel structural members (8). The allowable load on a column with an ℓ_e/d ratio of 75 is approximately 45 percent that of an equivalent column with an ℓ_e/d ratio of 50.

3.7.1.5

Background. Design provisions in the Specification for wood columns prior to 1991 generally were based on formulas for three classes of wood columns, defined in terms of slenderness ratio, ℓ_e/d, as follows:

Short columns: $\ell_e/d \leq 11$

$$F_c' = F_c \qquad \text{(C3.7-1)}$$

Intermediate columns: $11 < \ell_e/d \leq K$

$$F_c' = F_c\left[1 - \frac{1}{3}\left(\frac{\ell_e/d}{K}\right)^4\right] \qquad \text{(C3.7-2)}$$

in which

$$K = K_{cI}\sqrt{E/F_c} \qquad \text{(C3.7-3)}$$

Long columns: $\ell_e/d > K$

$$F_c' = \frac{K_{cE}E}{(\ell_e/d)^2} \qquad \text{(C3.7-4)}$$

where:

F_c = tabulated compression parallel to grain design values adjusted for condition of use

E = tabulated modulus of elasticity design value adjusted for condition of use

K_{cE} = Euler buckling coefficient for columns

K_{cI} = intermediate column coefficient

$$= \sqrt{\frac{3K_{cE}}{2}}$$

In the above formulas, K is the minimum ℓ_e/d ratio at which the column will behave as an Euler column. This is defined as occurring at a stress of $2F_c/3$, the assumed elastic limit. At K, the 4th power intermediate column equation and the Euler formula are tangent.

The foregoing column formulas, established in 1930, were based on tests of one hundred and sixty 12 inch by 12 inch by 24 foot Douglas fir and southern pine timbers representing a full range of density and grade; and were supported by the findings of earlier column research (133). The results of the large column tests showed that limiting Euler column behavior to stresses below $2F_c/3$ was a conservative assumption for all but green or unseasoned material. Dry or seasoned columns showed Euler behavior extending to stresses over $4F_c/5$, which was consistent with the elastic limit for that material. Although assuming an elastic limit of 80 percent of ultimate would have resulted in use of an 8th power parabolic equation for intermediate columns, the 4th power equation was used in order to cover all seasoning conditions (133). The large column tests also showed that ℓ_e/d ratios of 11 or below had a negligible effect on column capacity.

Provisions in Earlier Editions. Column design provisions in the 1944 edition of the Specification were based on the short, intermediate and long column equations using constants of

K_{cE} (Euler buckling coefficient for columns) = 0.329

K_{cI} (intermediate column coefficient) = 0.702

These coefficients incorporate the wartime increase (20 percent) in permanent load design values authorized by the War Production Board (see Commentary for 2.3.2). The coefficient of 0.329 included a 2.5 reduction or safety factor on the Euler constant for rectangular sections of 0.822.

In 1953, the buckling coefficient in the long column equation, K_{cE}, was reduced to 0.300 to remove the war-time increase and to reflect the application of the normal loading concept (10 percent increase over permanent load values) to long column stresses (see Commentary for 3.6.3). Simplified column design procedures also were introduced in the 1953 edition. These new methods provided for use of the lower of the Euler buckling stress or the short column stress. Although the simplified procedure resulted in stresses up to 15 percent higher than those calculated by the intermediate column equation for intermediate slenderness ratios (62), the simplification was considered reasonable based on (i) the assessment that the property values used in the short and intermediate column equations had factors of safety of approximately four compared to a factor of safety of three for those used in the long column equation (57); (ii) that columns used in dry conditions of service had a higher elastic limit than that assumed by the intermediate column equation (133); and (iii) the low probability of having limiting strength reducing grade characteristics present at the location most subject to potential buckling. The increases in 1991 tabulated compression parallel to grain design values, F_c, for dimension lumber based on the results of extensive testing of in-grade full size pieces is supportive of these earlier assessments.

The column design provisions of the 1953 edition were carried forward essentially unchanged until 1977 when the intermediate column formula was reinstated. Structural columns in buildings designed using the simplified provisions had performed satisfactorily for more than 20 years. However, a change to a more conservative procedure was considered appropriate in view of the use being made of the Specification in certain industrial design applications. Particular attention was given to use of the provisions of the Specification in the design of wood cooling towers which involved subjecting spliced columns continuously to full design loads in a hot water environment. Although the column design and related procedures of the Specification were not tied to field problems encountered with one tower design, the use of the Specification for such a specialized and severe application indicated the more conservative design methodology should be reinstated.

The intermediate column equation was reintroduced in the 1977 edition with a K_{cI} coefficient of 0.671 to correspond to the Euler buckling coefficient, K_{cE}, of 0.300 used in the long column equation. In addition, load duration factors for columns were limited only to values of F_c, thus disallowing any adjustment to column stresses controlled by buckling (see Commentary for 3.6.3). The 1977 provisions were continued in the 1982 and 1986 editions.

New Continuous Column Formula. The single column stability factor equation in 3.7.1.5 of the 1991 edition of the Specification is applicable to all slenderness ratios, ℓ_e/d, and replaces the short, intermediate and long column equations given in earlier editions. In addition to facilitating column and beam-column design, the new continuous column equation takes into account research that shows the 4th power intermediate column formula, which was based on tests of 12 by 12 inch columns, overestimates the strength of columns made with lumber of two inch nominal thickness in the intermediate ℓ_e/d range (37,94,109,130,232).

The new column equation is

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} \qquad \text{(C3.7-5)}$$

where:

F_c^* = tabulated compression design value multiplied by all applicable adjustment factors except C_P

F_{cE} = critical buckling design value for compression members

$= \dfrac{K_{cE}E'}{(\ell_e/d)^2}$

K_{cE} = Euler buckling coefficient for columns

c = nonlinear parameter for columns

The foregoing equation was first proposed in alternate form as a new method of determining the buckling stress of columns of any material in the elastic and inelastic range by a Finnish researcher in 1956 (226). The equation has as its upper limit the allowable compression parallel to grain design value associated with a crushing mode of failure (F_c) at very small ℓ_e/d ratios and the critical (Euler) buckling design value (F_{cE}) at large ℓ_e/d ratios. Continuity between these extremes of slenderness, where crushing and buckling interact, is obtained by assuming a curvilinear stress-strain relationship where the degree of nonlinearity caused by inelasticity, nonuniform material structure and initial eccentricity can be modeled through a single parameter "c". In this modeling, the slope of the stress-strain relationship is considered proportional to level of stress (226), with the rate of change of slope being constant.

Evaluation of the applicability of the proposed new procedure to short and intermediate in-grade wood columns in 1964 showed that when the parameter "c" was empirically established from the stress-strain relationship of very short columns (ℓ_e/d of 2.5), the equation could provide a good approximation of column strength if the short prism tests adequately characterized the properties and nonuniformities of the longer columns (143). It subsequently was found that establishment of the parameter "c" by empirical fitting of the equation to column strength data resulted in a predictive equation that closely followed test results at all ℓ/d ratios (229,232,233). A significant advantage of the methodology is that by selecting column test material representative of the nonuniform properties across the cross section and along the length that are associated with permitted grade characteristics such as knots, slope of grain and warp, the combined effects of these variables on column behavior are included in the resultant value of "c" (233).

The critical buckling design value used in the continuous column equation is based on the same Euler buckling coefficients, K_{cE}, as those used in the 1986 and previous editions: 0.300 for visually graded lumber and 0.418 for products having a coefficient of variation in modulus of elasticity of 0.11 or less. The 0.300 factor includes the Euler constant for rectangular sections (0.822) and a reduction of 2.74 on tabulated modulus of elasticity values. Assuming a coefficient of variation of 0.25 in the modulus of elasticity for visually graded sawn lumber or machine evaluated lumber (MEL), the 2.74 factor represents an approximate 5 percent lower exclusion value on pure bending modulus of elasticity (1.03 times tabulated E values) and a 1.66 factor of safety.

For glued laminated timber and machine stress rated lumber (MSR) or other products having a coefficient of variation in modulus of elasticity of 0.11 or less, the Euler buckling coefficient of 0.418 also represents an approximate 5 percent lower exclusion value on pure bending modulus of elasticity with a factor of safety of 1.66. The 0.418 coefficient is related to the 0.300 coefficient for visually graded lumber by the ratio

$$\frac{[1 - 0.11(1.645)]}{[1 - 0.25(1.645)]} = 1.39$$

A "c" value of 0.95 in the continuous column equation gives column stresses close to those obtained with the short, intermediate and long column equations used in previous editions (see Figure C3.7-1). Comparing the values of "c" specified for use in the new equation of 0.80 for sawn lumber, 0.85 for round timber piles and 0.90 for glued laminated timber with this baseline value indicates the relative reduction in column values resulting from adoption of the new design equation. The 0.80 factor for lumber has been established from column tests of dimension lumber (232,233) and, based on early large timber tests (133), is considered to be a conservative value for posts and timbers and beams and stringers. The values of "c" for piles and glued laminated timber are based on assessment of the sizes of these products relative to dimension lumber and the relative degree of warp and nonuniformity in density and grain slope that can occur as a result of permitted grade characteristics.

Example C3.7-1 illustrates the use of the column stability factor in calculating the allowable compression design value parallel to grain.

Comparison with other Column Equations. The column stability factor equation of 3.7.1.5 is somewhat similar to the column equation used in the British Standard Code of Practice CP112 and included in the 1988 draft standard of Eurocode No. 5 (45,140). As given in the British Code, this equation has the form

$$F_c' = \frac{F_c + [1 + n(\ell_e/d)]\,F_{cE}}{2} - \sqrt{\frac{F_c + n(\ell_e/d)\,F_{cE}^{\,2}}{4} - F_c F_{cE}} \qquad \text{(C3.7-6)}$$

where:

F_c = tabulated compression parallel to grain design value

F_{cE} = critical buckling design value for compression members

$= \dfrac{K_{cE}\,E'}{(\ell_e/d)^2}$

n = eccentricity factor

= 0.006928

Whereas Equation of C3.7-5 is based on the Engesser tangent modulus theory and assumes a curvilinear stress-strain curve, the European equation (C3.7-6) assumes linear elastic behavior to ultimate load, linear interaction between axial load and bending failure and an initial eccentricity of axial load (232). The eccentricity factor is assumed to represent nonuniformity and initial lack of straightness of the member due to grade characteristics (140). The value of 0.006928 for the factor is an average based on column tests of various grades. At the level of design stresses, the European (British Code of Practice) column equation gives results approximately the same (generally within +/-5 percent) as those obtained from 3.7.1.5 with "c" equal to 0.80 (see Figure C3.7-1). As previ-

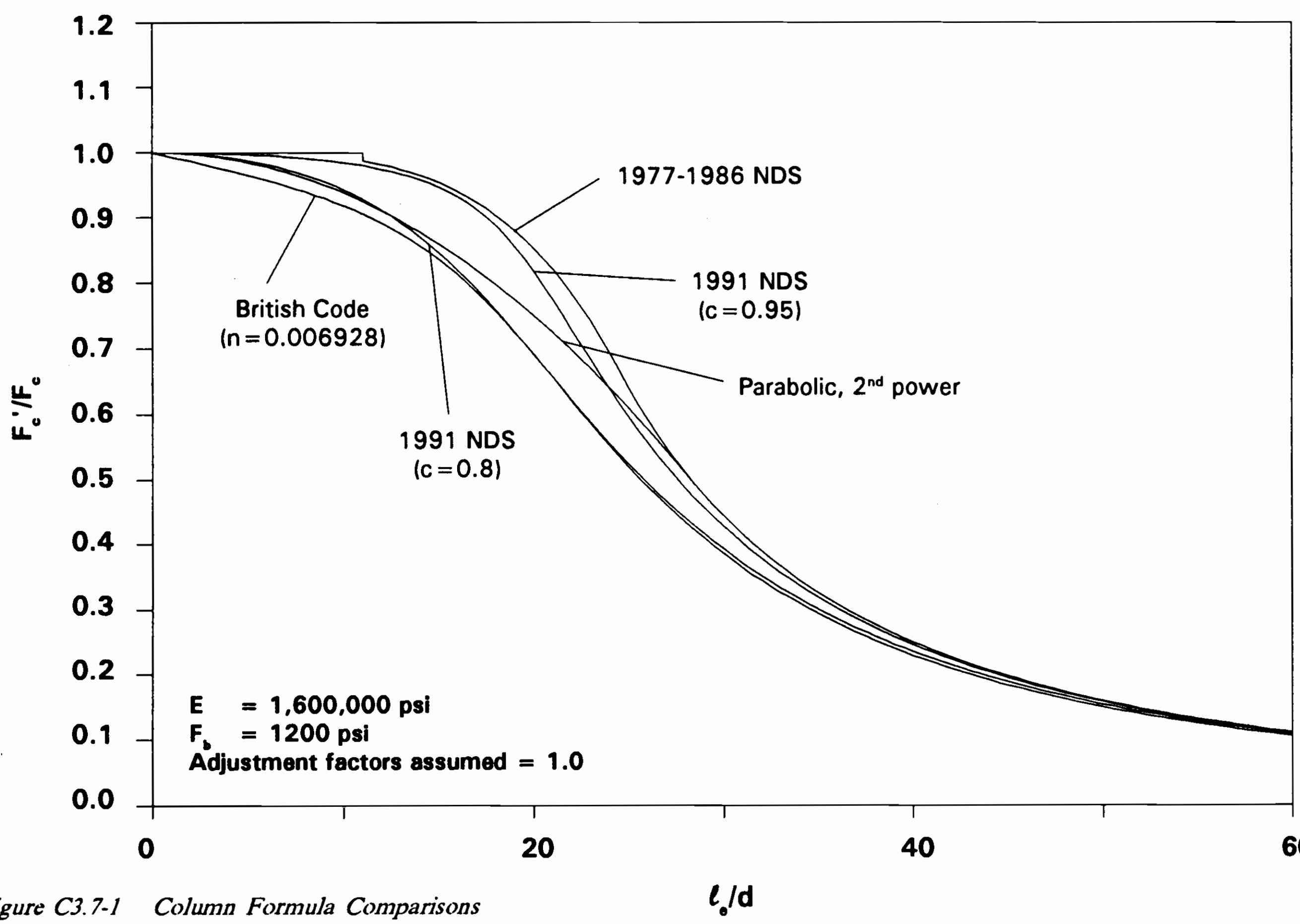
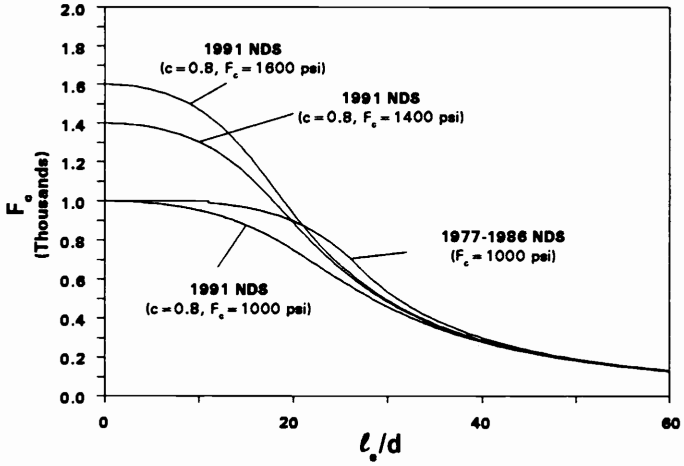


Figure C3.7-1 Column Formula Comparisons ously noted, when the value "c" is based on column tests of material representative of commercial grades, the effects of small eccentricities associated with crook or other warp as well as with nonuniform placement of knots and variable slopes of grain are embedded in the "c" value.

The design equation of 3.7.1.5 also is compared in Figure C3.7-1 with the results obtained with the short, intermediate and long column equations given in the 1977-1986 editions, and with the 2nd power parabolic equation for inelastic buckling proposed in the 1920's for wood (216) and presently used in steel design (3, 153). The 2nd power parabolic is tangent with the Euler curve at $F_c/2$. It can be seen from Figure C3.7-1 that the new continuous equation with "c" of 0.80 gives significantly lower stresses than those obtained from the 1977-1986 equations at slenderness ratios less than about 35. For various E/F_c ratios at the design level, the maximum difference between the two criterion is about 21 percent. The new equation also gives lower values than the 2nd power parabolic equation in the intermediate and low Euler slenderness ratio range.

For visually graded dimension lumber in a number of the more important species groups, the reduction in column stresses associated with the new column equation will be partially offset by new higher compression parallel to grain design values, F_c, based on in-grade test results. This is illustrated in Figure C3.7-2 where column design values from 3.7.1.2 based on $1.40F_c$ and $1.60F_c$ (representative of the increases reflected in the new design values) are compared with those from 1982 and 1986 column equations based on $1.00F_c$, assuming no change in E.



E = 1,600,000 psi
Adjustment factors assumed = 1.0

Figure C3.7-2 Effect of New Design Values on Column Formulas

3.7.1.6 Continuous exposure to elevated temperature and moisture in combination with continuous application of full design loads is an example of a severe service condition. Particularly when such design environments are coupled with design uncertainties, such as end fixity or stiffness of unsupported spliced connections, use of a reduced K_{cE} value should be considered. Included in such evaluations should be the possibility of eccentric application of the axial load and

Example C3.7-1

A No. 2 Spruce-Pine-Fir 2×6 interior bearing wall stud, 91.5 in. long, sheathed on both sides with gypsum board, carries dead load + snow load from the roof. Determine C_p and the allowable compressive stress, F_c' for the stud. Assume 16" on center spacing.

$F_c = 1100$ psi $E = 1{,}400{,}000$ psi (Table 4A)

$C_F = 1.1 \quad C_D = 1.15$

Column Stability Factor C_p (3.7.1)

$F_c^* = F_c C_D C_F = (1100)(1.15)(1.1) = 1392$ psi

$\ell_{e2}/d_2 = (0)/(1.5) = 0$

$\ell_{e1}/d_1 = (91.5)/(5.5) = 16.64$ (controls)

$\ell_e/d = 16.65 < 50$ **ok**

$K_{cE} = 0.3 \quad c = 0.8$

$$F_{cE} = \frac{K_{cE}E'}{(\ell_e/d)^2} = \frac{(0.3)(1{,}400{,}000)}{(16.64)^2} = 1517 \text{ psi}$$

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$= \frac{1+1517/1392}{2(0.8)} - \sqrt{\left[\frac{1+1517/1392}{2(0.8)}\right]^2 - \frac{1517/1392}{0.8}}$$

$= \mathbf{0.72}$

Allowable Compression Design Value, F_c' (2.3.1)

$F_c' = F_c C_D C_P C_F = (1100)(1.15)(0.72)(1.1)$

$= \mathbf{1002}$ **psi**

the need to design the member as a beam-column (see section 15.4 of the Specification).

3.7.2-Tapered Columns

Use of a minimum diameter (dimension) plus one-third the difference between the minimum and maximum diameters (dimensions) as the representative dimension of a tapered column for determining effect of slenderness ratio was a general provision of the Specification from the 1944 through the 1986 editions. The rule was considered to give the diameter of a round column that could be used in the long column (Euler) equation to prevent buckling (57). This representative dimension was limited to no more than 1.5 times the minimum diameter or dimension until the 1960 edition when this upper limit was dropped.

Equation 3.7-2 in Section 3.7.2 for determining the representative dimension of tapered columns for specific support conditions is new in the 1991 edition. It was added following new analyses (47) that showed the general one-third rule was overly conservative for some end support conditions but unconservative for another. The use of a dimension 1/3 the length from the smaller end was estimated to give a buckling load that was 35 percent too low for a tapered column fixed at the large end and unsupported at the small end, and a load that was 16 percent too low for a tapered column simply supported (translation fixed) at both ends. Alternatively, the 1/3 rule was shown to give buckling loads that were 13 percent too high for a tapered column fixed on the small end and unsupported on the large end. These estimates were for a minimum to maximum diameter (dimension) ratio of 0.70. The new equation provides more realistic estimates of column performance for these specific support conditions while the general 1/3 equation is retained for other support arrangements.

The support conditions referenced in 3.7.2 are related to the end condition modes given in Appendix G of the Specification as follows:

Fixed:	rotation fixed - translation fixed
Unsupported:	rotation free - translation free
Simply supported:	rotation free - translation fixed

The one end fixed - one end free or simply supported conditions referenced in 3.7.2 correspond to the fifth and sixth buckling mode cases in Appendix G. The condition of both ends simply supported corresponds to the fourth case. Values for the constant "a", given under "Support Conditions" in 3.7.2, are considered applicable when the ratio of minimum to maximum diameter equals or exceeds 1/3 (47).

The effective length factor, K_e, from Appendix G is used in conjunction with the representative dimension (equivalent prism) when determining the stability factor, C_P, for tapered columns. It is to be noted that the actual compression stress parallel to grain, f_c, based on the minimum dimension of the column is not to exceed F_c adjusted for all applicable adjustment factors except C_P.

3.7.3-Round Columns

The required size of a round column may be determined by first designing a square column having the same taper and then using a round member diameter that will give the same cross-sectional area as the square. This procedure is based on the equivalence of the bending and compression load-carrying capacities of round and square wood members having the same cross-section area (see Commentary for 2.3.8).

3.8-TENSION MEMBERS

3.8.2 Average strength values for tension perpendicular to grain that are available in reference documents (62,66) apply to small, clear specimens that are free of shakes, checks and other seasoning defects. Such information indicates that tension design values perpendicular to grain of clear, check and shake free wood may be considered to be about one-third the shear design value parallel to grain of comparable quality material of the same species (20). However, because of undetectable ring shake and checking and splitting that can occur as a result of drying in service, very low strength values for the property can be encountered in commercial grades of lumber. For this reason, no sawn lumber tension design values perpendicular to grain have been published in the Specification. In the 1982 edition, cautionary provisions about the use of design configurations which induce such stresses were introduced. Avoidance of these design configurations wherever possible is now required. Particular attention should be given in the design of bending member connections to have incoming vertical loads applied above the neutral axis or distributed across the entire cross-section through hangers or other means.

If perpendicular to grain tension stresses are not avoidable, use of stitch bolts or other mechanical reinforcement to take such loads is to be considered. When such a solution is used, care should be taken to insure the reinforcement itself does not cause splitting of the member as a result of drying in service (4). In any case, it is to be understood that the designer is responsible for avoiding the introduction of tension perpendicular to grain stresses or to assuring that the methods and practices used to account for such stresses are adequate.

Radial stresses are induced in curved, pitch tapered and certain other shapes of glued laminated timber beams. Such beams are made of dry material which is controlled for quality at the time of manufacture. The Specification has provided allowable radial tension design values perpendicular to grain for curved glued laminated timber bending members since 1944. The present radial tension design values perpendicular to grain given in 5.2.2 were established in 1968 and have been shown to be adequate by both test (30,155) and experience.

3.9-COMBINED BENDING AND AXIAL LOADING

3.9.1-Bending and Axial Tension

The linear interaction equation for combined bending and tension stresses

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1 \qquad \text{(C3.9-1)}$$

where:

F_b^* = the tabulated bending design value multiplied by all applicable adjustment factors except slenderness, C_L

has been a provision of the Specification since 1944. Theoretical analyses and experimental results show the equation yields conservative results (229). It can be shown that the effect of moment magnification, which is not included in the equation, serves to reduce the effective bending ratio rather than increase it, as is the case for combined bending and axial compression.

Where eccentric axial tension loading is involved, the moment associated with the axial load should be added to the actual bending stress (f_b') induced by the bending load, or $f_b = f_b' + 6Pe/bd^2$, when applying the interaction equation. In this case e would carry the sign appropriate to the direction of eccentricity: plus when the moment associated with the axial load is acting in the same direction as that associated with the bending load and minus when opposite.

Where biaxial bending occurs with axial tension, the equation should be expanded to

$$\frac{f_t}{F_t'} + \frac{f_{b1}}{F_{b1}^*} + \frac{f_{b2}}{F_{b2}^*} \leq 1 \qquad \text{(C3.9-2)}$$

where the subscripts indicate the principal axes and:

F_{b1}^*, F_{b2}^* = the tabulated bending design values for the two directions multiplied by all applicable adjustment factors except C_L

Tabulated bending design values, F_b, are not adjusted for slenderness, C_L, in the bending-tension interaction equation because the tension load acts to reduce buckling propensity and the combined stress is not the critical buckling condition. Buckling is checked separately through the second equation given in 3.9.1 or

$$\frac{f_b - f_t}{F_b^{**}} \leq 1 \qquad \text{(C3.9-3)}$$

where:

F_b^{**} = is the tabulated bending design value multiplied by all applicable adjustment factors including C_L (excluding C_V)

Although inferred in earlier editions, the specific requirement that actual net bending stress (stress on the compression face of the member) not exceed the allowable bending design value adjusted for slenderness was added to the Specification in the 1982 edition. The moment effect of any eccentric axial load is to be included in f_b when the buckling check is made. Where biaxial bending is involved, actual net bending stress in each direction, (f_{b1}-f_t) and (f_{b2}-f_t) is checked against the slenderness adjusted design value, F_{b1}^{**} and F_{b2}^{**}, for that direction.

Load Duration Adjustments. The allowable design values, F_t', F_b^* and F_b^{**} used in the bending-tension interaction equations are to be adjusted for load duration. The Specification specifically permits the use of the same C_D factor for both properties based on the shortest load duration in the combination of loads, even though the load of shortest duration may be associated with only one of the actual stresses. For example, if the actual tension stress parallel to grain is a result of roof snow load and dead load and the actual bending stress is associated with floor live load and dead load, a C_D of 1.15 (assumed applicable snow load duration) may be used to obtain both F_t' and F_b^* or F_b^{**} for use in the interaction equations. However, the design must also be evaluated without the snow load using a C_D of 1.0 for both properties, and without both snow and live loads using a C_D of 0.9 for both properties, to assure these conditions also meet the acceptance criteria of the interaction equations.

The provision for use of a load duration factor, C_D, associated with the shortest duration of load in a combination of loads is based on field performance and long-standing practice in the design of trusses and glued laminated timber arches (4). Example C3.9-1 illustrates the use of these load duration provisions in a combination of loads.

For other applications, a different load duration adjustment may be used with each property, F_t', F_b^* and F_b^{**}, in the equation depending on the nature of the load or loads that are responsible for the actual stress associated with that property. For example, if the maximum actual tension stress on a member is a result of a roof snow load and the actual bending stress is a result of a permanent load, a C_D of 1.15 (assumed applicable snow load duration) is used to obtain F_t' and a C_D of 0.9 to obtain F_b^* or F_b^{**}. The factor associated with the longest total load duration is not applied to both properties when evaluating the maximum actual stresses.

3.9.2-Bending and Axial Compression

Background

Design provisions for members subject to bending and axial compression were essentially the same in the 1986 and earlier editions of the Specification except for location in the body of the standard or the appendix and in specific application requirements. These provisions provided two general interaction equations, one for short and one for long columns, as shown below

For $\ell_e/d \leq 11$:

$$\frac{f_c}{F_c'} + \frac{f_b}{F_b'} \leq 1 \qquad \text{(C3.9-4)}$$

For $\ell_e/d \geq K$:

$$\frac{f_c}{F_c'} + \frac{f_b}{F_b' - f_c} \leq 1 \qquad \text{(C3.9-5)}$$

in which:

$$K = 0.671\sqrt{E/F_c} \qquad \text{(C3.9-6)}$$

For intermediate columns, $11 < \ell_e/d < K$, (see Commentary for 3.7.1.5), allowable loads were determined through linear interpolation between those established for ℓ_e/d of 11 and those for ℓ_e/d of K. The foregoing two linear interaction equations were developed from analysis of test results of clear Sitka spruce columns subject to combined bending and concentric and eccentric axial compression loads (135). These tests indicated (223) that, when the additional moments resulting from member deflection where taken into account, the strength of members subject to combined stress could be closely estimated by the relationship

$$\frac{P/A}{C} + \left(\frac{M/S}{F}\right)^2 = 1 \qquad \text{(C3.9-7)}$$

where:

P/A = applied axial stress

M/S = applied flexural stress, including additional moments associated with deflection of the member

C,F = ultimate strengths in flexure and compression

For short columns, where lateral deflections are small, it was concluded that the equation could be conservatively applied at the design stress level by dropping the exponent on the flexure ratio term (132,223). The resulting linear equation, the one used in the 1986 and earlier editions of the Specification, is comparable to that used with other structural materials (161,223).

In the case of long columns, however, neither the original nor the simplified form of the equation describing test results were applicable to design conditions where the additional moments associated with deflections due to eccentric axial and bending loads are not readily calculated. To obtain a useable equation that would account for these additional moments, the assumption was made that the ultimate strength of long columns under combined compression and bending loads was equal to the bending strength of the members (223). Under this assumption, which was shown to be reasonable by Sitka spruce column test data (135,223), the minimum bending strength available under combined loading to resist actual bending stress is the bending strength of the material less the long-column (Euler) buckling stress (223). For the general case where actual compression stress parallel to grain, f_c, is less than the Euler buckling stress, the available bending strength becomes the allowable bending design value less the actual compressive stress parallel to grain, thus giving rise to the general equation

$$\frac{f_c}{F_c'} + \frac{f_b}{F_b' - f_c} \leq 1 \qquad \text{(C3.9-8)}$$

Formulas for eccentric axial loads in combination with bending and concentric axial loads were developed following the same general assumption, and further

Example C3.9-1

A No. 2 Southern Pine 2×8, used as the bottom chord of a 28-ft roof truss (14 ft between panel points), is subject to a uniform dead load of 10 psf (4 ft truss spacing), as well as tension forces (assuming pinned connections) of 1680 lb from roof wind loads, 1680 lb from roof live loads and 1560 lb from dead loads. Check the adequacy of the member between panel points for bending and tension for all load combinations.

F_b = 1200 psi E = 1,600,000 psi (Table 4B)

F_t = 650 psi C_F = 1.0 A = 10.88 in^2 S = 13.14 in^3

Load Case 1: DL+RLL+WL, C_D = 1.6

Tension (3.8.1)

$F_t' = F_tC_DC_F = (650)(1.6)(1.0) = 1040$ psi (2.3.1)

Tension force in chord, T = 1560+1680+1680 = 4920 lb

$f_t = T/A_g = 4920/10.88 = 452$ psi < $F_t' = 1040$ psi **ok**

Bending (3.3)

$F_b^* = F_bC_DC_F = (1200)(1.6)(1.0) = 1920$ psi (3.9.1)

$\ell_u/d = (14)(12)/(7.25) = 23.2 > 7$

$\ell_e = 1.63\ell_u + 3d = 1.63(14)(12) + 3(7.25) = 295.6$ in.

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(295.6)(7.25)}{(1.5)^2}} = 30.9 \qquad K_{bE} = 0.438$$

$$F_{bE} = \frac{K_{bE}E'}{R_b^2} = \frac{(0.438)(1,600,000)}{(30.9)^2} = 736 \text{ psi}$$

$$C_L = \frac{1+(F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1+(F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$= \frac{1+736/1920}{1.9} - \sqrt{\left[\frac{1+736/1920}{1.9}\right]^2 - \frac{736/1920}{0.95}} = 0.372$$

$F_b^{**} = F_bC_DC_LC_F = (1200)(1.6)(0.372)(1.0)$ (3.9.1)
$= 715$ psi

$M_{max} = w\ell^2/8 = (10)(4)(14)^2(12)/8 = 11,760$ in-lb

$f_b = M_{max}/S = 11,760/13.14$
$= 895$ psi < $F_b^* = 1920$ psi **ok**

Combined Bending and Axial Tension (3.9.1)

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} = \frac{452}{1040} + \frac{895}{1920} = 0.90 < 1.0 \quad \textbf{ok}$$

$$\frac{f_b - f_t}{F_b^{**}} = \frac{895-452}{715} = 0.62 < 1.0 \quad \textbf{ok}$$

Load Case 2: DL+RLL, C_D = 1.25

Tension (3.8.1)

$F_t' = F_tC_DC_F = (650)(1.25)(1.0) = 812.5$ psi (2.3.1)

Tension force in chord, T = 1560+1680 = 3240 lb

$f_t = 3240/10.88 = 298$ psi < $F_t' = 812.5$ psi **ok**

Bending (3.3)

$F_b^* = F_bC_DC_F = (1200)(1.25)(1.0) = 1500$ psi

$F_{bE} = 736$ psi

$$C_L = \frac{1+736/1500}{1.9} - \sqrt{\left[\frac{1+736/1500}{1.9}\right]^2 - \frac{736/1500}{0.95}} = 0.470$$

$F_b^{**} = F_bC_DC_LC_F = (1200)(1.25)(0.470)(1.0) = 705$ psi

$f_b = 895$ psi < $F_b^* = 1500$ psi **ok**

Combined Bending and Axial Tension (3.9.1)

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} = \frac{298}{812.5} + \frac{895}{1500} = 0.97 < 1.0 \quad \textbf{ok}$$

$$\frac{f_b - f_t}{F_b^{**}} = \frac{895-298}{705} = 0.85 < 1.0 \quad \textbf{ok}$$

Load Case 3: DL only, C_D = 0.9

Tension (3.8.1)

$F_t' = F_tC_DC_F = (650)(0.9)(1.0) = 585$ psi (2.3.1)

Tension force in chord, T = 1560 lb

$f_t = 1560/10.88 = 143$ psi < $F_t' = 585$ psi **ok**

Bending (3.3)

$F_b^* = F_bC_DC_F = (1200)(0.9)(1.0) = 1080$ psi

$F_{bE} = 736$ psi

$$C_L = \frac{1+736/1080}{1.9} - \sqrt{\left[\frac{1+736/1080}{1.9}\right]^2 - \frac{736/1080}{0.95}} = 0.628$$

$F_b^{**} = F_bC_DC_LC_F = (1200)(0.9)(0.628)(1.0) = 678$ psi

$f_b = 895$ psi < $F_b^* = 1080$ psi **ok**

Combined Bending and Axial Tension (3.9.1)

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} = \frac{143}{585} + \frac{895}{1080} = 1.07 > 1.0 \quad \textbf{ng}$$

$$\frac{f_b - f_t}{F_b^{**}} = \frac{895-143}{678} = 1.11 > 1.0 \quad \textbf{ng}$$

(cont.)

Example C3.9-1 (cont.)

At this point, the web configuration might be changed, member size could be increased or a higher grade selected. Try No. 1 Dense Southern Pine. Recheck the controlling case of dead load only.

F_b = 1650 psi F_t = 875 psi E = 1,800,000 psi

Recheck Load Case 3: DL only, C_D = 0.9

Tension (3.8.1)

$F_t' = F_t C_D C_F = (875)(0.9)(1.0) = 787.5$ psi (2.3.1)

Tension force in chord, T = 1560 lb

$f_t = 1560/10.88 = 143 \text{ psi} < F_t' = 787.5$ psi **ok**

Bending (3.3)

$F_b^* = F_b C_D C_F = (1650)(0.9)(1.0) = 1485$ psi

$$F_{bE} = \frac{K_{bE}E'}{R_b^2} = \frac{(0.438)(1{,}800{,}000)}{(30.9)^2} = 826 \text{ psi}$$

$$C_L = \frac{1+826/1485}{1.9} - \sqrt{\left[\frac{1+826/1485}{1.9}\right]^2 - \frac{826/1485}{0.95}} = 0.527$$

$F_b^{**} = F_b C_D C_L C_F = (1650)(0.9)(0.527)(1.0) = 783$ psi

$f_b = 895 \text{ psi} < F_b^* = 1485$ psi **ok**

Combined Bending and Axial Tension (3.9.1)

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} = \frac{143}{787.5} + \frac{895}{1485} = 0.78 < 1.0 \quad \textbf{ok}$$

$$\frac{f_b - f_t}{F_b^{**}} = \frac{895 - 143}{783} = 0.96 < 1.0 \quad \textbf{ok}$$

No. 1 Dense Southern Pine 2x8 satisfies NDS criteria for combined bending and axial tension.

assuming a sinusoidal deflection curve in which deflection is proportional to stress (132,223). Such interaction equations were included in the Appendix of the Specification in the 1986 and all earlier editions.

Prior to the 1971 edition, only the short column interaction curve was included in the body of the Specification; with reference being made to the more exact equations for intermediate and long columns given in the Appendix. In the 1971 edition, application of the short column equation in the body of the Specification was limited to columns with $\ell_e/d < K$. In the 1977 edition, the long column equation was brought into the body of the Specification and the factor J was introduced to facilitate interpolation of allowable loads for intermediate columns, as shown below

$$\frac{f_c}{F_c'} + \frac{f_b}{F_b' - Jf_c} \le 1 \qquad \text{(C3.9-9)}$$

where:

$$J = \frac{\ell_e/d - 11}{K - 11} \qquad \text{(C3.9-10)}$$

and

$$0 \le J \le 1$$

In 1982, specific wording was added to the Specification to require checking the effect of stress interaction about both principal axes. In 1986, this provision was extended to define what slenderness values should be used in the calculation of F_c', F_b', and J. The extension required the coupling of beam buckling propensity due to loads acting on one face of the member with column buckling propensity perpendicular to the plane of bending.

New Bending and Axial Compression Interaction Equation. Although the beam-column interaction criteria in the 1986 and earlier editions have provided for satisfactory performance, it was recognized that (i) the linear equations were developed without direct consideration of beam buckling, (ii) the adjustment of moment to account for the interaction of axial load and deflection is not consistent with more recent theoretical analyses, and (iii) the equations do not address bending about both principal axes. The new interaction equation given in 3.9.2 of the Specification corrects these limitations and closely describes recent beam-column test data for in-grade lumber as well as similar earlier data for clear wood material (229,230).

For the case of bending load applied to the narrow face of the member and concentric axial compression load, the new interaction equation reduces to

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{bl}}{F_{bl}'(1 - f_c/F_{cEl})} \le 1 \qquad \text{(C3.9-11)}$$

where:

f_c = actual compression stress parallel to grain

f_{bl} = actual edgewise bending stress

F_c' = allowable compression design value parallel to grain including adjustment for largest slenderness ratio

F_{bl}' = allowable bending design value including adjustment for slenderness ratio

F_{cEl} = critical buckling design value in the plane of bending

$$= \frac{K_{cE}E'}{\left(\ell_{el}/d_l\right)^2}$$

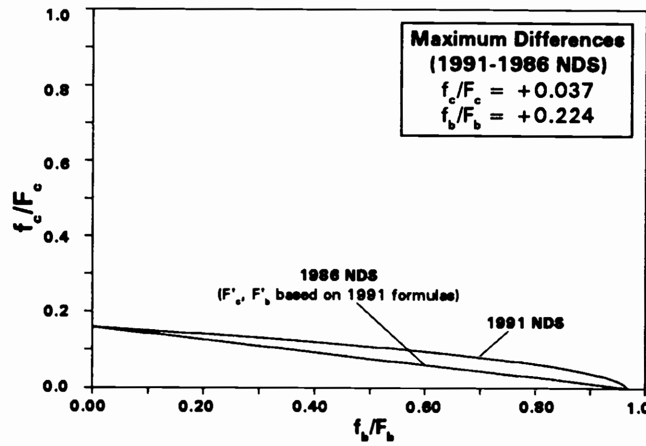
in which:

d_l = dimension of wide face

ℓ_{el} = distance between points of support restraining buckling in the plane of bending

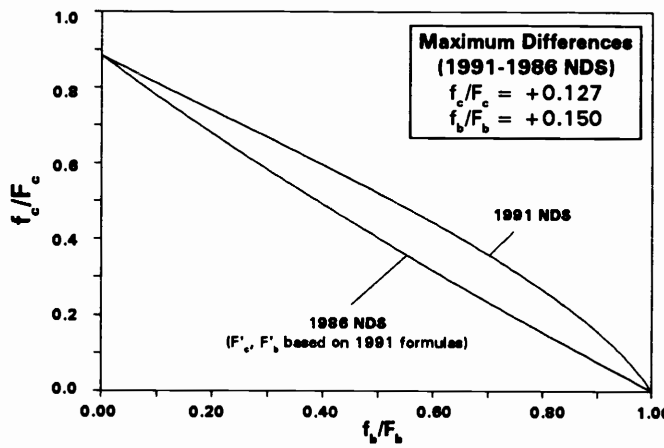
In the new equation the ratio of actual to allowable compression stress is squared where previously the exponent on this term was one. This change is based on tests of southern pine, spruce-pine-fir and western hemlock 2×4 and 2×6 southern pine short beam-columns (94,228,229). The moment magnification factor in the denominator of the bending ratio term of the new equation is $(1 - f_c/F_{cEl})$. This factor in the 1986 and earlier editions was $(1 - f_c/F_b')$ for the case of long columns ($J = 1$). The new magnification term, which is consistent with that used for other structural materials, is based on theoretical analysis and confirmed by test results for intermediate and long beam-columns made with the same species and sizes of lumber used to establish the first ratio term in the equation (229,230). The magnification factor in the new equation is larger than the equivalent adjustment used in earlier editions when (i) the higher bending strength grades are involved, (ii) the slenderness ratio for bending is small and (iii) the column slenderness ratio in the plane of bending is large.

A comparison of the new interaction equation with that used previously for the case of a bending load applied on the narrow face and a concentric axial compression load is shown in Figure C3.9-1 for different beam and column slenderness ratios and different bending and column strength combinations. The ordinate (f_c/F_c) and abscissa (f_b/F_b) ratios in this figure are based on tabulated compression and bending design values, F_c and F_b, both unadjusted for slenderness ratio. Thus the values of the ratios for pure compression are less than 1.00 and are limited by the slenderness ratio for the member being used in the example. Similarly, the values of the ratios for pure bending are less than 1.00 when the 1986 slenderness factor, C_s, for the member used in the example exceeds



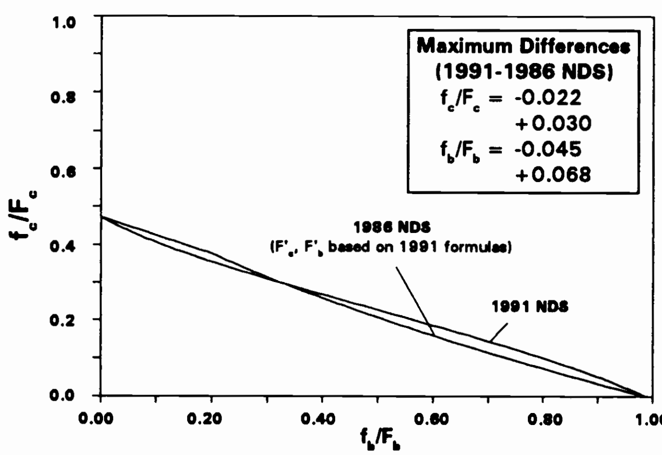
2x6; Axial Load and Concentrated Load on Narrow Face; E = 1,400,000 psi; F_b = 875 psi; F_c = 1100 psi; ℓ_1/d_1 = 13.1; ℓ_2/d_2 = 48; C_s = 16.6

(a)



2x6; Axial Load and Concentrated Load on Narrow Face; E = 1,400,000 psi; F_b = 875 psi; F_c = 1100 psi; ℓ_1/d_1 = 13.1; ℓ_2/d_2 = 0; C_s = 0

(b)



2x4; Axial Load and Concentrated Load on Narrow Face; E = 1,400,000 psi; F_b = 1300 psi; F_c = 1100 psi; ℓ_1/d_1 = 24; ℓ_2/d_2 = 26.2; C_s = 10

(c)

Figure C3.9-1 Comparison of Beam-Column Interaction Equations for 1986 and 1991 NDS Editions zero. The maximum differences shown in the comparisons are the maximum difference between the 1991 and 1986 f_c/F_c ratio for any value of the f_b/F_b ratio; and the maximum difference between the 1991 and 1996 f_b/F_b ratios for any value of the f_c/F_c ratio. It can be seen from Figure C3.9-1 that for the loading case being used (bending load on narrow face and concentric axial compression load), and at the level of design stresses, generally larger allowable loads are obtained with the new interaction equation than those obtained from the equation given in earlier editions. In these comparisons, the same values of F_c' and F_b' are used in both the 1991 and 1986 interaction equations in order to illustrate the different allowable design values obtained from each, independent of the changes made in column and beam slenderness provisions in the 1991 edition.

The complete interaction equation in 3.9.2 provides for any combination of axial concentric compression load applied either on the narrow or wide face. The equation facilitates design of members subjected to these load combinations and accounts for the amplification, which was not previously considered, of the bending moment associated with load on the wide face due to bending load on the narrow face. The general interaction equation is shown below with the moment magnification factors identified by the symbol C_m.

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}}{C_{m1}F_{b1}'} + \frac{f_{b2}}{C_{m2}F_{b2}'} \le 1 \quad \text{(C3.9-12)}$$

where:

f_c = actual compression stress parallel to grain

f_{b1} = actual edgewise bending stress

f_{b2} = actual wideface bending stress

F_c' = allowable compression design value including adjustment for largest slenderness ratio

F_{b1}' = allowable bending design value, including adjustment for slenderness ratio, for load applied to the narrow face

F_{b2}' = allowable bending design value, including adjustment for slenderness ratio, for load applied to the wide face

C_{m1} = moment magnification factor = $1 - f_c/F_{cE1}$

C_{m2} = moment magnification factor = $1 - f_c/F_{cE2} - (f_{b1}/F_{bE})^2$

in which:

F_{cE1} = critical (Euler) buckling design value for compression member in plane of bending from edgewise load

$$= \frac{K_{cE}E'}{\left(\ell_{e1}/d_1\right)^2}$$

F_{cE2} = critical (Euler) buckling design value for compression member in plane of bending from wideface load

$$= \frac{K_{cE}E'}{\left(\ell_{e2}/d_2\right)^2}$$

and

d_1 = dimension of wide face

d_2 = dimension of narrow face

ℓ_{e1} = distance between points of support restraining buckling in plane of bending from edgewise load

ℓ_{e2} = distance between points of support restraining buckling in plane of bending from wideface load

F_{bE} = critical buckling design value for bending member

The third term, $(f_{b1}/F_{bE})^2$, of C_{m2} represents the amplification of f_{b2} from f_{b1}. This term is based on theoretical analysis and certain simplifying assumptions (230). The appropriateness of the term is closely verified by the early beam-column tests made on clear Sitka spruce (135,230). The C_{m2} equation conservatively models cantilever and multispan beam-columns subject to biaxial loads (231).

For bending loads on the narrow and wide faces only, the equation reduces to

$$\frac{f_{b1}}{F_{b1}'} + \frac{f_{b2}}{C_{m2}F_{b2}'} \le 1 \quad \text{(C3.9-13)}$$

where:

C_{m2} = moment magnification factor = $1 - (f_{b1}/F_{bE})^2$

For a concentric axial load and bending load on the wide face, the equation reduces to

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b2}}{C_{m2}F_{b2}'} \le 1 \quad \text{(C3.9-14)}$$

where:

C_{m2} = moment magnification factor = $1 - f_c/F_{cE2}$

In all cases, F_c' is to be determined in accordance with the provisions of 3.6.3 and 3.7.1; and F_{b1}' and F_{b2}' are to be determined in accordance with 3.3.1 and 3.3.3. The load duration factor, C_D, used in the determination of these allowable values may be that associated with the shortest load duration in any combination of loads. Alternatively, the value of C_D associated with each load (axial compression, narrow face bending, wide face bending) may be used for the allowable design value associated with that load. The Commentary for 3.9.1 on load duration factor adjustments for the case of bending and axial tension also applies to bending and axial compression.

Examples C3.9-2 and C3.9-3 illustrate the use of the bending and axial compression interaction equation

Example C3.9-2

A No. 1 Southern Pine 2×6 beam-column carries axial compression loads of 840 lb (snow) and 560 lb (dead) plus a 25 psf wind load on the narrow face. Column spacing is 4 ft. and column length is 10 ft. Lateral support is provided at the ends and on the narrow face throughout the length of the member. Check the adequacy of the member for bending and compression for all load combinations.

F_b = 1650 psi E = 1,700,000 psi (Table 4B)

F_c = 1750 psi C_F = 1.0 A = 8.25 in^2 S = 7.563 in^3

Load Case 1: DL+SL+WL, C_D = 1.6

Compression (3.6, 3.7)

$F_c^* = F_cC_DC_F = (1750)(1.6)(1.0) = 2800$ psi (3.7.1.5)

$\ell_2/d_2 = 0$ (fully supported)

$K_e = 1.0$ (assume pin-pin end conditions) (App. G)

$\ell_e/d = K_e\ell_1/d_1 = (1.0)(10)(12)/(5.5) = 21.8 < 50$

$K_{cE} = 0.3$

$$F_{cE} = \frac{K_{cE}E'}{(\ell_e/d)^2} = \frac{(0.3)(1,700,000)}{(21.8)^2} = 1073 \text{ psi}$$

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$= \frac{1+1073/2800}{2(0.8)} - \sqrt{\left[\frac{1+1073/2800}{2(0.8)}\right]^2 - \frac{1073/2800}{0.8}}$$

$= 0.346$

$F_c' = F_cC_DC_FC_P = (1750)(1.6)(1.0)(0.346) = 969$ psi

Axial Force, P = 560+840 = 1400 lb

$f_c = P/A_g = 1400/8.25$

$= 170$ psi $< F_c' = 969$ psi, F_{cE} **ok**

Bending (3.3)

$F_b' = F_bC_DC_LC_F = (1650)(1.6)(1.0)(1.0) = 2640$ psi

$M_{max} = w\ell^2/8 = (25)(4)(10)^2(12)/8 = 15,000$ in-lb

$f_b = M_{max}/S = 15,000/7.563$

$= 1983$ psi $< F_b' = 2640$ psi **ok**

Combined Bending and Axial Compression (3.9.2)

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}}{F_{b1}'(1 - f_c/F_{cE1})} \le 1.0$$

$$= \left(\frac{170}{969}\right)^2 + \frac{1983}{2640(1-170/1073)} = 0.92 < 1.0 \quad \textbf{ok}$$

Load Case 2: DL+SL, C_D = 1.15

Compression (3.6, 3.7)

$F_c^* = F_cC_DC_F = (1750)(1.15)(1.0) = 2013$ psi (3.7.1.5)

$F_{cE} = 1073$ psi

$$C_P = \frac{1+1073/2013}{2(0.8)} - \sqrt{\left[\frac{1+1073/2013}{2(0.8)}\right]^2 - \frac{1073/2013}{0.8}}$$

$= 0.456$

$F_c' = F_cC_DC_FC_P = (1750)(1.15)(1.0)(0.456)$ (2.3.1)

$= 918$ psi

Axial Force, P = 560+840 = 1400 lb

$f_c = P/A_g = 1400/8.25$

$= 170$ psi $< F_c' = 918$ psi, F_{cE} **ok**

Load Case 3: DL only, C_D = 0.9

Compression (3.6, 3.7)

$F_c^* = F_cC_DC_F = (1750)(0.9)(1.0) = 1575$ psi (3.7.1.5)

$F_{cE} = 1073$ psi

$$C_P = \frac{1+1073/1575}{2(0.8)} - \sqrt{\left[\frac{1+1073/1575}{2(0.8)}\right]^2 - \frac{1073/1575}{0.8}}$$

$= 0.548$

$F_c' = F_cC_DC_FC_P = (1750)(0.9)(1.0)(0.548)$ (2.3.1)

$= 863$ psi

Axial Force, P = 560 lb

$f_c = P/A_g = 560/8.25$

$= 68$ psi $< F_c' = 863$ psi, F_{cE} **ok**

Load case 1 (DL+SL+WL) controls

No. 1 Southern Pine 2×6 satisfies NDS criteria for combined bending and axial compression.

Example C3.9-3

Assuming pinned connections for purposes of illustration, a 3 ft section (between panel points) of a No. 2 Southern Pine 4×2, top chord of a gable end, parallel chord truss is subjected to axial compression forces of 300 lb (DL) and 600 lb (SL), plus concentrated loads of 60 lb (DL) and 120 lb (SL) applied at the center of the 3 ft span on the wide face, and a concentrated load of 120 lb (WL) at the center of the span on the narrow face. Lateral support is provided at the ends only. Check the adequacy of the member for bending and compression for all load combinations.

F_b = 1500 psi F_c = 1650 psi (Table 4B)
E = 1,600,000 psi C_F = 1.0 C_{fu} = 1.1
A = 5.25 in^2 S_x = 3.063 in^3 S_y = 1.313 in^3

Load Case 1: DL+SL+WL, C_D = 1.6

Compression (3.6, 3.7, 3.9.2)

$F_c^* = F_cC_DC_F = (1650)(1.6)(1.0) = 2640$ psi (3.7.1.5)
$\ell_{e1}/d_1 = (3)(12)/(3.5) = 10.29 < 50$
$\ell_{e2}/d_2 = (3)(12)/(1.5) = 24 < 50$ Controls (3.7.1.3)
$K_{cE} = 0.3$

$$F_{cE1} = \frac{K_{cE}E'}{\left(\ell_{e1}/d_1\right)^2} = \frac{(0.3)(1,600,000)}{(10.29)^2} = 4537 \text{ psi}$$

$$F_{cE2} = \frac{K_{cE}E'}{\left(\ell_{e2}/d_2\right)^2} = \frac{(0.3)(1,600,000)}{(24)^2} = 833 \text{ psi}$$

F_{cE2} controls for C_P

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$= \frac{1+833/2640}{2(0.8)} - \sqrt{\left[\frac{1+833/2640}{2(0.8)}\right]^2 - \frac{833/2640}{0.8}} = 0.292$$

$F_c' = F_cC_DC_FC_P = (1650)(1.6)(1.0)(0.292) = 770$ psi
Axial Force, P = 300+600 = 900 lb (DL+SL)
$f_c = P/A = 900/5.25$
$= 171$ psi $< F_c' = 770$ psi, F_{cE1}, F_{cE2} **ok**

Narrow Face Bending (load parallel to wide face) (3.3)

$F_b^* = F_bC_DC_F = (1500)(1.6)(1.0) = 2400$ psi (3.3.3.8)
ℓ_u = 3 ft d_1 = 3.5 in.
$\ell_u/d = (3)(12)/(3.5) = 10.3 > 7$
$\ell_e = 1.37\ell_u + 3d$ (Table 3.3.3)
$= 1.37(3)(12) + 3(3.5) = 59.8$ in.

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(59.8)(3.5)}{(1.5)^2}} = 9.65$$

$K_{bE} = 0.438$

$$F_{bE} = \frac{K_{bE}E'}{R_b^2} = \frac{(0.438)(1,600,000)}{(9.65)^2} = 7526 \text{ psi}$$

$$C_L = \frac{1+(F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1+(F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$= \frac{1+7526/2400}{1.9} - \sqrt{\left[\frac{1+7526/2400}{1.9}\right]^2 - \frac{7526/2400}{0.95}}$$

$= 0.978$

Alternatively, using the approximate rules of 4.4.1.2, with $d/b = 2$, $C_L = 1.0$ according to 3.3.3.2; however the calculated C_L of 0.978 will be used here (see 4.4.1.1).

$F_{b1}' = F_bC_DC_LC_F = (1500)(1.6)(0.978)(1.0) = 2347$ psi
Bending Load, P = 120 lb (WL)
$M_{max} = P\ell/4 = (120)(3)(12)/4 = 1080$ in-lb
$f_{b1} = M_{max}/S_x = 1080/3.063$
$= 353$ psi $< F_{b1}' = 2347$ psi, F_{bE} **ok**

Wide Face Bending (load parallel to narrow face) (3.3)

Since $d < b$ (1.5 < 3.5 in.), $C_L = 1.0$ (3.3.3.1)
$F_{b2}' = F_bC_DC_LC_FC_{fu} = (1500)(1.6)(1.0)(1.0)(1.1)$
$= 2640$ psi
Bending Load, P = 60+120 = 180 lb (DL+SL)
$M_{max} = P\ell/4 = (180)(3)(12)/4 = 1620$ in-lb
$f_{b2} = M_{max}/S_y = 1620/1.313$
$= 1234$ psi $< F_{b2}' = 2640$ psi **ok**

Combined Bending and Axial Compression (3.9.2)

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_{b1}}{F_{b1}'(1-f_c/F_{cE1})} + \frac{f_{b2}}{F_{b2}'(1-f_c/F_{cE2}-(f_{b1}/F_{bE})^2)}$$

$$\left(\frac{171}{770}\right)^2 + \frac{353}{2347(1-171/4537)} + \frac{1234}{2640(1-171/833-(353/7526)^2)}$$

$= 0.796 < 1.0$ **ok**

Load Case 2: DL+SL, C_D = 1.15

Compression (3.6, 3.9.2)

$F_c^* = F_cC_DC_F = (1650)(1.15)(1.0) = 1898$ psi (3.7.1.5)
$F_{cE2} = 833$ psi since ℓ_{e2}/d_2 controls

(cont.)

Example C3.9-3 (cont.)

$$C_P = \frac{1+833/1898}{2(0.8)} - \sqrt{\left[\frac{1+833/1898}{2(0.8)}\right]^2 - \frac{833/1898}{0.8}} = 0.389$$

$F_c' = F_c C_D C_F C_P = (1650)(1.15)(1.0)(0.389) = 738$ psi

Axial Force, $P = 300+600 = 900$ lb (DL+SL)

$f_c = P/A = 900/5.25$

$= 171$ psi $< F_c' = 738$ psi, F_{cE2} **ok**

Wide Face Bending (no narrow face bending) (3.3)

Since $d < b$ (1.5 < 3.5 in.), $C_L = 1.0$ (3.3.3.1)

$F_{b2}' = F_b C_D C_L C_F C_{fu} = (1500)(1.15)(1.0)(1.0)(1.1)$

$= 1898$ psi

Bending Load, $P = 60+120 = 180$ lb (DL+SL)

$M_{max} = P\ell/4 = (180)(3)(12)/4 = 1620$ in-lb

$f_{b2} = M_{max}/S_y = 1620/1.313$

$= 1234$ psi $< F_{b2}' = 1898$ psi **ok**

Combined Bending and Axial Compression (3.9.2)

$$\left(\frac{171}{738}\right)^2 + 0 + \frac{1234}{1898(1 - 171/833 - 0)} = 0.872 < 1.0 \quad \textbf{ok}$$

Load Case 3: DL only, $C_D = 0.9$

Compression (3.6, 3.7, 3.9.2)

$F_c^* = F_c C_D C_F = (1650)(0.9)(1.0) = 1485$ psi (3.7.1.5)

$F_{cE2} = 833$ psi controls for C_P

$$C_P = \frac{1+833/1485}{2(0.8)} - \sqrt{\left[\frac{1+833/1485}{2(0.8)}\right]^2 - \frac{833/1485}{0.8}} = 0.475$$

$F_c' = F_c C_D C_F C_P = (1650)(0.9)(1.0)(0.475) = 705$ psi

Axial Force, $P = 300$ lb (DL)

$f_c = P/A = 300/5.25$

$= 57$ psi $< F_c' = 705$ psi, F_{cE2} **ok**

Wide Face Bending (no narrow face bending) (3.3)

Since $d < b$ (1.5 < 3.5 in.), $C_L = 1.0$ (3.3.3.1)

$F_{b2}' = F_b C_D C_L C_F C_{fu} = (1500)(0.9)(1.0)(1.0)(1.1)$

$= 1485$ psi

Bending Load, $P = 60$ lb (DL)

$M_{max} = P\ell/4 = (60)(3)(12)/4 = 540$ in-lb

$f_{b2} = M_{max}/S_y = 540/1.313$

$= 411$ psi $< F_{b2}' = 1485$ psi **ok**

Combined Bending and Axial Compression (3.9.2)

$$\left(\frac{57}{705}\right)^2 + 0 + \frac{411}{1485(1 - 57/833 - 0)} = 0.304 < 1.0 \quad \textbf{ok}$$

Load case 2 (DL+SL) controls

No. 2 Southern Pine 2x4 satisfies NDS criteria for combined bending and axial compression.

including use of load duration provisions for the shortest duration of load in a combination of loads.

3.10-DESIGN FOR BEARING

3.10.1-Bearing Parallel to Grain

Tabulated bearing design values parallel to grain, F_g, are based on clear wood properties and represent typical quality material present across various product grades. For certain of the higher grades of sawn lumber 4 inches or less in thickness, tabulated compression design values parallel to grain, F_c, may exceed F_g values for the species. Such F_c values are based on in-grade lumber tests and reflect the presence of a higher quality of clear wood material in such grades than that characteristic of the species as a whole. Where end grain bearing is a design consideration, actual bearing stress parallel to grain, f_g, shall not exceed the appropriate allowable bearing design value parallel to grain, F_g', regardless of the value of F_c for the grade. This criterion represents a continuation of design practice that has been part of the Specification since 1944.

Examples of end-grain bearing configurations are end-to-end compression chord segments laterally supported by splice plates, butt end-bearing joints in individual laminations of mechanically laminated truss chords, roof-tied arch heel connections, notched chord truss heel joints, and columns supporting beams. Prior to 1953, all end bearings were required to be on metal plates or straps or on metal inserts. This requirement was modified in the 1953 and subsequent editions to permit direct end-to-end bearing of wood surfaces when the actual bearing stress parallel to grain is less than or equal to 75 percent of the allowable bearing design value parallel to grain ($f_g \leq 3/4 F_g'$), abutting end surfaces are parallel and appropriate lateral support is provided. The required use of a metal plate or equivalent strength material as an insert in higher loaded end-to-end bearing joints is to assure a uniform distribution of load from one member to another.

3.10.2-Bearing Perpendicular to Grain

Ignoring any non-uniform distribution of bearing stress that may occur at the supports of a bending member as a result of the deflection or curvature of that member under load is long standing design practice. This practice was first addressed in the Specification in the 1977 edition.

3.10.3-Bearing at an Angle to Grain

The equation for calculating the allowable compressive stress on an inclined surface, F_θ', has been a provision of the Specification since 1944. Developed from tests on Sitka spruce in 1921, the general applicability of the equation has been confirmed by more recent tests on other species (57,77,84).

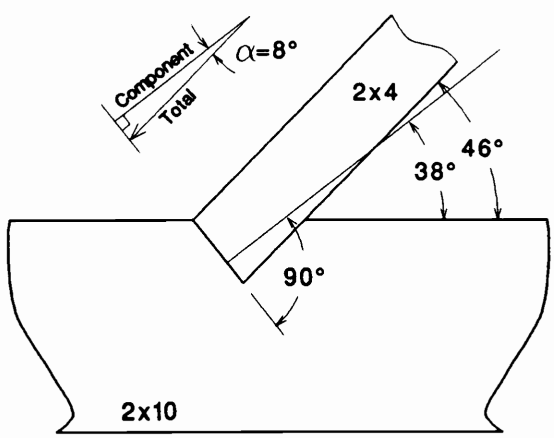
The equation

$$F_\theta' = \frac{F_g' F_{c\perp}'}{F_g' \sin^2\theta + F_{c\perp}' \cos^2\theta} \qquad \text{(C3.10-1)}$$

applies when the inclined or loaded surface is at right angles to the direction of load. The equation has limits of F_g' when the angle between direction of grain and direction of load, θ, is 0° and $F_{c\perp}'$ when this angle is 90°. When the force is at an angle other than 90° to the inclined surface, the equation is entered with θ equal to the angle between the direction of grain and the direction of the load component that is normal to the surface. In this case the allowable stress obtained from the equation, F_θ', is that component of the total force that is perpendicular to the surface. Example C3.10-1 illustrates the use of these provisions. Stresses on both inclined surfaces in a notched member should be checked if the limiting case is not apparent.

Example C3.10-1

A 2×10 member is loaded in bearing at an angle to grain from a 2×4, such that the angle between the perpendicular (to 2×10 bearing surface) component of the resultant force and the direction of grain (2×10) is 38°, while the angle between the total load direction and grain direction is 46°. For F_g' = 1670 psi and $F_{c\perp}'$ = 410 psi, determine F_θ' and the total allowable load based on bearing in the 2×10.



Allowable Bearing Design Value Parallel to Grain, F_θ' (3.10.3)

For Equation 3.10-1, θ = 38°

$$F_\theta' = \frac{F_g' F_{c\perp}'}{F_g' \sin^2\theta + F_{c\perp}' \cos^2\theta} \qquad \text{(Eq. 3.10-1)}$$

$$= \frac{(1670)(410)}{(1670)\sin^2(38) + (410)\cos^2(38)}$$

$= \mathbf{771\ psi}$

Total Allowable Load, P_{total}

α = angle between total force and component perpendicular to 2×10 bearing surface
= 46° - 38° = 8°

$A_{bearing} = bd(1/\cos\alpha)$
$= (1.5)(3.5)(1/\cos(8°))$
$= 5.30\ in^2$

$P_{\perp\ to\ 2x10\ b.s.} = F_\theta' A_{bearing}$
$= (771)(5.30)$
$= 4088$ lb

$P_{total} = (P_{\perp\ to\ 2x10\ b.s.})(1/\cos\alpha)$
$= (4088)(1/\cos(8°))$
= **4128 lb**