MATH 2107-601 **EXAM #2** NAME: ______________________
(Print)

THURSDAY

NOVEMBER 1, 2001 NAME: ______________________
(Signature)

[7:30 p.m. - 8:45 p.m.] STUDENT I. D.: ______________________

**INSTRUCTIONS:** There are 2 parts to this exam, the main part plus an optional **EXTRA CREDIT** part (on the last page). The main part is worth 100 points while the extra credit part is worth 10 points. But any score over 100 will be truncated to 100.

Clarity of exposition (including proper spelling and punctuation is an integral part of a correct solution to any problem. In particular, **DO NOT PUT EQUAL SIGNS BETWEEN THINGS THAT ARE NOT EQUAL.** But do put them where they belong.

It is necessary to show all your work.

**GOOD LUCK!**

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**PLEASE SIGN THE FOLLOWING STATEMENT:**
On my honor, I declare that the work that follows is entirely my own. With regard to all the questions on this exam, I have neither given nor received help from anyone [including myself, say, via any type of cheat sheet or device (e.g., cell phone)]. Nor have I used a calculator of any sort (i.e., regular or programmable).

NAME: ______________________
(Signature)

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**PLEASE DO NOT WRITE BELOW THIS LINE:**

| | | |
|---|---|---|
| 1. a. ____ | 5. a. ____ | **EXTRA-CREDIT:** |
| b. ____ | b. ____ | 1. ____ |
| 2. a. ____ | c. ____ | 2. ____ |
| b. ____ | d. ____ | **TOTALS**: |
| 3. ____ | e. ____ | Column 3: ____ |
| 4. a. ____ | f. ____ | Column 2: ____ |
| b. ____ | g. ____ | Column 1: ____ |
| c. ____ | h. ____ | **Grand Total**: ____ |
| | i. ____ | **Grade:** ____ |
| Total: ____ | Total: ____ | |

1. (a) Let $f : \mathbb{R} \to \mathbb{R}$ be a function and let $a \in \mathbb{R}$. Prove that

$f$ is continuous at $a$ whenever $f'(a)$ exists.

[10 points]

(b) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ and a point $a \in \mathbb{R}$ such that $f$ is continuous at $a$ but $f'(a)$ does <u>not</u> exist.

**[To get credit, you must give at least some indication of why $f$ is continuous at $a$ and why $f'(a)$ does not exist.]**

[5 points]

2. (a) State the **Chain Rule**. Be sure to state the hypotheses as well as the conclusion.

**ANSWER**:

**The Chain Rule**:

**Hypotheses**:

[1 point]

**Conclusions**:

[4 points]

(b) Complete the following:

[5 points: 1 each]

$(F \cdot S)' =$

$\left(\frac{T}{B}\right)' =$

$(c \cdot F)' =$

$(F + G)' =$

$(F^n)' =$

3. Assume that the equation

$$x^2 + xy + +y^2 = 12$$

defines $y$ as a differentiable function of $x$. Use <u>implicit differentiation</u> to find $\frac{dy}{dx}$. Then find the equation of the straight line tangent to the graph of $x^2 + xy + +y^2 = 12$ at the point $(2,2)$.

[15 points]

**SOLUTION**:

4. For each of the following, use the various rules that we have developed [not the definition] to compute $f'(x)$ when f(x) is given.

Do **NOT** simplify your answers.

**NOTE**: DO NOT COMPUTE $f'(x)$ DIRECTLY FROM THE DEFINITION!

[15 points: 5 each]

a. $f(x) = \sqrt[6]{\dfrac{x^8 + 1}{x^4 + 1}}$

b. $f(x) = \dfrac{1 + \sin x}{\cos x}$

c. $f(x) = (x^8 + x^4 + 2^8)^{100} \cdot (x^{1/7} - 2)$

5. For each of the following, use the various rules that we have developed [not the definition] to compute $f'(x)$ when f(x) is given.

Do **NOT** simplify your answers.

**NOTE**: DO NOT COMPUTE $f'(x)$ DIRECTLY FROM THE DEFINITION!

[45 points: 5 each]

a. $f(x) = \cos[\csc^6(4x) - 4]$

b. $f(x) = \sin(x^7) + \sin^7 x$

c. $f(x) = \dfrac{x}{1 - x} + \dfrac{1}{x - 1}$

d. $f(x) = x^4 \cdot \tan \dfrac{1}{x^3}$

5. (Continued):

e. $f(x) = \sec x \cdot \sin x$

f. $f(x) = \sec [\ \sin x\ ]$

g. $f(x) = \dfrac{8x^3 + 7}{4x^2 + 3}$

h. $f(x) = \sqrt[2]{\cot(4x) \cdot \tan(4x)}$

i. $f(x) = [\cot (7x^6)]^5$

**EXTRA-CREDIT**: [**10 points**: **But**, **any score over 100 will be truncated to 100**.]

1. *If* $y = \cos[\cos(\cos x)]$, then

$$\frac{dy}{dx} =$$

[5 points]

**SOLUTION**:

2. Let y be a differentiable function of x satisfying

$$\sin(x - 4y) = x^2 \cdot \sin y \ .$$

Then

$$\frac{dy}{dx} =$$

[5 points]

**SOLUTION**: