HW #4 - 4.4, 4.9, 4.11, 4.14, 4.21

4.4) $1 - e^{(-\eta_{0.1}/3.5)} = 0.10$

$\eta_{0.1} = 0.3688 \text{ hours} = \underline{\underline{22.13 \text{ minutes}}}$

4.9) (a) $\int_0^2 k x^2(2-x)\,dx = k\frac{4}{3} = 1$

therefore $\underline{\underline{k = \frac{3}{4}}}$

(b) $P(0.5 < S < 1.5) = \underline{\underline{0.6880}}$

(c) $P(S > 1) = \underline{\underline{0.6875}}$

4.11) $P(T > 1) = e^{(-1/43.3)} = \underline{\underline{0.9770}}$

4.14) a) $E(x) = \int_{-1}^{1} x\left(\frac{1}{2} - \frac{x}{4}\right) dx = \int_{-1}^{1} \frac{x}{2} - \frac{x^2}{4} dx$

$$= \frac{x^2}{4} - \frac{x^3}{12}\Big|_{-1}^{1} = \left(\frac{1}{4} - \frac{1}{12}\right) - \left(\frac{1}{4} + \frac{1}{12}\right)$$

$$= \frac{2}{12} - \frac{4}{12} = \underline{\underline{-\frac{1}{6}}}$$

$$E(x^2) = \int_{-1}^{1} x^2\left(\frac{1}{2} - \frac{x}{4}\right) dx = \int_{-1}^{1} \frac{x^2}{2} - \frac{x^3}{4} dx$$

$$= \frac{x^3}{6} - \frac{x^4}{16}\Big|_{-1}^{1} = \left(\frac{1}{6} - \frac{1}{16}\right) - \left(-\frac{1}{6} - \frac{1}{16}\right)$$

$$= \frac{1}{6} - \frac{1}{16} + \frac{1}{6} + \frac{1}{16} = \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$$

$$V(x) = E(x^2) - [E(x)]^2 = \frac{1}{3} - \left(-\frac{1}{6}\right)^2 = \underline{\underline{\frac{11}{36}}}$$

4.14 b) $f(x) = \left(\frac{1}{2}\right)\sin x, \ 0 \le x \le \pi; \ f(x) = 0$ elsewhere

$$E(x) = \int_0^{\pi} \frac{x}{2}\sin x\,dx = \frac{1}{2}\left(-x\cos x + \sin x\right)\Big|_0^{\pi}$$

$$= \frac{1}{2}\left(-\pi(-1) + 0\right) - \frac{1}{2}(0) = \underline{\underline{\frac{\pi}{2}}}$$

$$E(x^2) = \int_0^{\pi} \frac{x^2}{2}\sin x\,dx = \frac{1}{2}\left[-x^2\cos x + 2x\sin x + 2\cos x\right]\Big|_0^{\pi}$$

$$= \frac{1}{2}\left[-\pi^2(-1) + 2(\pi)(0) + 2(-1)\right] - \frac{1}{2}\left[-0 + 0 + 2\right]$$

$$= \frac{1}{2}\left[\pi^2 - 2\right] \qquad -1$$

$$= \frac{\pi^2}{2} - 2 = \underline{\underline{\frac{\pi^2 - 4}{2}}}$$

$$V(x) = E(x^2) - \left[E(x)\right]^2$$

$$= \frac{\pi^2 - 4}{2} - \left(\frac{\pi}{2}\right)^2$$

$$= \frac{\pi^2 - 4}{2} - \frac{\pi^2}{4} = \frac{2\pi^2 - 8 - \pi^2}{4}$$

$$= \frac{\pi^2 - 8}{4} = \underline{\underline{\frac{\pi^2}{4} - 2}}$$

4.14 (c) $f(x) = 3(1-x)^2,\ 0 < x < 1;\ f(x) = 0$ elsewhere

$$E(x) = \int_0^1 3x(1-x)^2\,dx = \int_0^1 3x - 6x^2 + 3x^3\,dx$$

$$= \frac{3x^2}{2} - 2x^3 + \frac{3x^4}{4}\Big|_0^1$$

$$= \left(\frac{3}{2} - 2 + \frac{3}{4}\right) - (0) = \frac{1}{4} = \underline{\underline{0.25}}$$

$$E(x^2) = \int_0^1 3x^2(1-x)^2\,dx = \int_0^1 3x^2 - 6x^3 + 3x^4\,dx$$

$$= \frac{3x^3}{3} - \frac{6x^4}{4} + \frac{3x^5}{5}\Big|_0^1$$

$$= \left(1 - \frac{6}{4} + \frac{3}{5}\right) - (0) = \underline{\underline{0.1}}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= 0.1 - (.25)^2 =$$

$$= \underline{\underline{.0375}}$$

4.14 (d) $f(x) = x \quad 0 \le x \le 1$

$f(x) = 2-x \quad 1 \le x \le 2$

$f(x) = 0$ elsewhere

$$E(x) = \int_0^1 x^2\,dx + \int_1^2 2x - x^2\,dx$$

$$= \frac{x^3}{3}\Big|_0^1 + \left(x^2 - \frac{x^3}{3}\right)\Big|_1^2$$

$$= \left(\frac{1}{3} - 0\right) + \left[\left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)\right]$$

$$= \frac{1}{3} + \left[\frac{4}{3} - \frac{2}{3}\right] = \frac{1}{3} + \frac{2}{3} = \underline{\underline{1}}$$

$$E(x^2) = \int_0^1 x^3\,dx + \int_1^2 2x^2 - x^3\,dx$$

$$= \frac{x^4}{4}\Big|_0^1 + \left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_1^2$$

$$= \left(\frac{1}{4} - 0\right) + \left[\left(\frac{16}{3} - 4\right) - \left(\frac{2}{3} - \frac{1}{4}\right)\right]$$

$$= \frac{1}{4} + \frac{4}{3} - \frac{5}{12} = \frac{3+16-5}{12} = \frac{14}{12}$$

$$= 1\tfrac{1}{6} = \underline{1.167}$$

$$V(x) = E(x^2) - [E(x)]^2 = 1.167 - (1)^2 = \underline{\underline{0.167}}$$

4.14 (e) $f(x) = (1+x)$, $-1 \le x \le 0$

$f(x) = (1-x)$; $0 \le x \le 1$

$f(x) = 0$, elsewhere

$$E(x) = \int_{-1}^{0} x(1+x)\,dx + \int_{0}^{1} x(1-x)\,dx$$

$$= \int_{-1}^{0} x + x^2\,dx + \int_{0}^{1} x - x^2\,dx = \left(\frac{x^2}{2} + \frac{x^3}{3}\right)\Bigg|_{-1}^{0} + \frac{x^2}{2} - \frac{x^3}{3}\Bigg|_{0}^{1}$$

$$= \left[(0) - \left(\frac{1}{2} - \frac{1}{3}\right)\right] + \left[\left(\frac{1}{2} - \frac{1}{3}\right) - 0\right] = \underline{\underline{0}}$$

$$E(x^2) = \int_{-1}^{0} x^2(1+x)\,dx + \int_{0}^{1} x^2(1-x)\,dx$$

$$= \int_{-1}^{0} x^2 + x^3\,dx + \int_{0}^{1} x^2 - x^3\,dx$$

$$= \left(\frac{x^3}{3} + \frac{x^4}{4}\right)\Bigg|_{-1}^{0} + \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Bigg|_{0}^{1}$$

$$= \left[(0) - \left(-\frac{1}{3} + \frac{1}{4}\right)\right] + \left[\left(\frac{1}{3} - \frac{1}{4}\right) - (0)\right]$$

$$= \frac{2}{12} = \frac{1}{6} = \underline{\underline{.167}}$$

$$V(x) = E(x^2) - \left(E(x)\right)^2$$

$$= .167 - 0 = \underline{\underline{.167}}$$

4.14 (f) $f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty$

$f(x) = \frac{1}{2} e^{x}, \quad -\infty < x < 0$

$f(x) = \frac{1}{2} e^{-x}, \quad 0 < x < \infty$

$$E(x) = \int_{-\infty}^{0} \frac{1}{2} x e^{x}\, dx + \int_{0}^{\infty} \frac{1}{2} x e^{-x}\, dx$$

$$= \frac{1}{2}\Big[(x-1)e^{x}\Big]\Big|_{-\infty}^{0} + \frac{1}{2}\Big[+(-x-1)e^{-x}\Big]\Big|_{0}^{\infty}$$

$$= \frac{1}{2}\Big[\text{[redacted]}\; x e^{x} - e^{x}\Big]\Big|_{-\infty}^{0} + \frac{1}{2}\Big[-\frac{x}{e^{x}} - \frac{1}{e^{x}}\Big]\Big|_{0}^{\infty}$$

$$= \frac{1}{2}\Big[(0-1)-(0-0)\Big] + \frac{1}{2}\Big[(-0-0)-(-0-1)\Big]$$

$$= -\frac{1}{2} + \frac{1}{2} = \underline{\underline{0}}$$

$$E(x^2) = \int_{-\infty}^{0} \frac{1}{2} x^2 e^{x}\, dx + \int_{0}^{\infty} \frac{1}{2} x^2 e^{-x}\, dx$$

$$= \frac{1}{2}\Big[x^2 e^{x} - 2(x-1)e^{x}\Big]\Big|_{-\infty}^{0} + \frac{1}{2}\Big[-x^2 e^{-x} + 2(-x-1)e^{-x}\Big]\Big|_{0}^{\infty}$$

$$= \frac{1}{2}\Big[x^2 e^{x} - 2xe^{x} + 2e^{x}\Big]\Big|_{-\infty}^{0} + \frac{1}{2}\Big[-x^2 e^{-x} - 2xe^{-x} - 2e^{-x}\Big]_{0}^{\infty}$$

$$= \frac{1}{2}\Big[(0-0+2)-(0-0+0)\Big] + \frac{1}{2}\Big[(-0-0-0)-(0-0-2)\Big]$$

$$= \underline{\underline{2}} \qquad V(x) = E(x^2) - (E(x))^2 = 2 - 0 = \underline{\underline{2}}$$

4.21)

$$E(P_{ro}) = (C_2 - 2C_1) + C_1 P(T > k), \text{ where}$$

$$C_1 = 6;\ C_2 = 10;\ P(T > K) = e^{(-k/1000)}$$

(a) $K = 700$

$$E(P_{ro}) = -2 + 6P(T > K)$$

$$= -2 + 6e^{(-700/1000)} = \underline{\underline{0.9795}}$$

$K = 900$

$$E(P_{ro}) = -2 + 6e^{(-900/1000)} = \underline{\underline{0.4394}}$$

(b) $E(P_{ro}) = -2 + 6e^{(-K/1000)} = 0$, therefore

$$K = 1000 \ln 3 = \underline{\underline{1098.6 \text{ hours}}}$$