Class

4.28: PROBLEM DEFINITION

Situation:

Kerosene is accelerated upward in vertical pipe.
$S = 0.81$, $a_z = 0.3\text{g}$.

Find:

Pressure gradient required to accelerate flow (lbf/ft^3).

Properties:

$\gamma = 62.4\,\text{lbf/ft}^3$.

PLAN Apply Euler's equation.

SOLUTION Applying Euler's equation in the z direction.

$$\begin{aligned}
\frac{\partial(p+\gamma z)}{\partial z} &= -\rho a_z = -\frac{\gamma}{g}\times 0.30g \\
\frac{\partial p}{\partial z}+\gamma &= -0.30\gamma \\
\frac{\partial p}{\partial z} &= \gamma(-1-0.30) \\
&= 0.81\left(62.4\,\text{lbf/ft}^3\right)(-1.30) \\
&\boxed{\frac{\partial p}{\partial z} = -65.7\ \text{lbf/ft}^3}
\end{aligned}$$

Class

4.30: PROBLEM DEFINITION

Situation:

A piston and water accelerating upward at 0.5g.
$a = 0.5\text{g}$, $z = 2\,\text{ft}$.

Find:

Pressure in water column (psfg).

Properties:

$\rho = 62.4\ \text{lbm/ft}^3$, $\gamma = 62.4\,\text{lbf/ft}^3$

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation

$$\rho a_\ell = -\frac{\partial}{\partial \ell}(p + \gamma z)$$

Let ℓ be positive upward.

$$\begin{aligned}
\rho(0.5g) &= -\frac{\partial p}{\partial \ell} - \gamma\frac{\partial z}{\partial \ell} \\
(\frac{\gamma}{g})(0.5g) &= -\frac{\partial p}{\partial \ell} - \gamma(1) \\
\frac{\partial p}{\partial \ell} &= -\gamma(0.5+1) = -1.5\gamma
\end{aligned}$$

Thus the pressure decreases upward at a rate of 1.5γ. The pressure at the top is atmospheric. At a depth of 2 ft.:

$$\begin{aligned}
p_2 &= (1.5\gamma)(2) = 3\gamma \\
&= 3\text{ ft.} \times 62.4\ \text{lbf/ft}^3 \\
& \boxed{p_2 = 187\ \text{psfg}}
\end{aligned}$$

C

4.32: PROBLEM DEFINITION

Situation:

Water accelerates in a horizontal pipe.

$a_s = 6\,\mathrm{m/s^2}$, $\rho = 1000\,\mathrm{kg/m^3}$.

Find:

Pressure gradient $(\mathrm{N/m^3})$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation with no change in elevation

$$\begin{aligned} \frac{\partial p}{\partial s} &= -\rho a_s \\ &= -1,000\,\mathrm{kg/m^3} \times 6\,\mathrm{m/s^2} \\ &\boxed{\frac{\partial p}{\partial s} = -6,000\ \mathrm{N/m^3}} \end{aligned}$$

1a5s

4.33: PROBLEM DEFINITION

Situation:

Water accelerated from rest in horizontal pipe.
$L = 100\,\text{m}$, $D = 30\,\text{cm}$, $a_s = 5\,\text{m/s}^2$.

Find:

Pressure at upstream end (kPa).

Properties:

$\rho = 1000\,\text{kg/m}^3$, $p_{downstream} = 90\,\text{kPa}$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation with no change in elevation

$$
\begin{aligned}
\frac{\partial p}{\partial s} &= -\rho a_s \\
&= -1,000\,\text{kg/m}^3 \times 5\,\text{m/s}^2 \\
&= -5,000\ \text{N/m}^3 \\
p_{\text{downstream}} - p_{\text{upstream}} &= \frac{\partial p}{\partial s}\Delta s \\
p_{\text{upstream}} &= 90,000\,\text{Pa} + \left(5,000\,\text{N/m}^3\right)(100\,\text{m}) \\
&= 590,000\ \text{Pa, gage} \\
&\boxed{p_{\text{upstream}} = 590\ \text{kPa, gage}}
\end{aligned}
$$

C

4.37: PROBLEM DEFINITION

Situation:

Closed tank is full of liquid.
$L = 3\,\text{ft}$, $H = 4\,\text{ft}$, $a_x = 0.9g$.
$a_\ell = 1.5g$, $S = 1.2$.

Find:

(a) $p_C - p_A$ (psf).
(b) $p_B - p_A$ (psf).

Properties:

$\rho = 1.94\,\text{slug/ft}^3$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation. Take ℓ in the z-direction.

$$-\frac{dp}{d\ell} - \gamma\frac{d\ell}{d\ell} = \rho a_l$$

$$\begin{aligned}
\frac{dp}{d\ell} &= -\rho(g + a_\ell) \\
&= -1.2\left(1.94\,\text{slug/ft}^3\right)\left(32.2\,\text{ft/s}^2 - 1.5\left(32.2\,\text{ft/s}^2\right)\right) \\
&= 37.5\ \text{psf/ft} \\
p_B - p_A &= -37.5\ \text{psf/ft} \times 4\,\text{ft} \\
&\boxed{p_B - p_A = -150\ \text{psf}}
\end{aligned}$$

Take ℓ in the x-direction. Euler's equation becomes

$$\begin{aligned}
-\frac{dp}{dx} &= \rho a_x \\
p_C - p_B &= \rho a_x L \\
&= 1.2 \times 1.94\,\text{slug/ft}^3 \times 0.9g \times 3\,\text{ft} \\
&= 202.4\ \text{psf} \\
p_C - p_A &= p_C - p_B + (p_B - p_A) \\
p_C - p_A &= 202.4 - 150 \\
&\boxed{p_C - p_A = 52.4\ \text{psf}}
\end{aligned}$$

C

4.59: PROBLEM DEFINITION

Situation:

A water jet fires vertically from a nozzle.

$V = 20\,\text{ft/s}$.

Find:

Height jet will rise.

PLAN

Apply the Bernoulli equation from the nozzle to the top of the jet. Let point 1 be in the jet at the nozzle and point 2 at the top.

SOLUTION

Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = p_2 = 0$ gage

$$\begin{aligned}
V_1 &= 20\text{ ft/s} \\
V_2 &= 0 \\
0 + \frac{(20\,\text{ft/s})^2}{2g} + z_1 &= 0 + 0 + z_2 \\
z_2 - z_1 &= h = \frac{400\,\text{ft}^2/\text{s}^2}{64.4\,\text{ft/s}^2} \\
& \boxed{h = 6.21\text{ ft}}
\end{aligned}$$

Class

4.62: PROBLEM DEFINITION

Situation:

Kerosene flows through a contraction section and a pressure is measured between pipe and contraction section.

$V_2 = 10\,\mathrm{m/s}$.

Find:

Velocity in upstream pipe (m/s).

Properties:

Table A.4: $\rho = 814\,\mathrm{kg/m^3}$.

$T = 20\,^\circ\mathrm{C}$, $\Delta p = 20\,\mathrm{kPa}$.

SOLUTION

Apply the Bernoulli equation between pipe (1) and contraction section (2)

$$\begin{aligned} p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} &= p_2 + \gamma z_2 + \rho\frac{V_2^2}{2} \\ p_{z1} + \rho\frac{V_1^2}{2} &= p_{z2} + \rho\frac{V_2^2}{2} \end{aligned}$$

The pressure gage measures the difference in piezometric pressure, $p_{z1} - p_{z2} = 20$ kPa. Rewrite the Bernoulli equation for V_1

$$\begin{aligned} \rho\frac{V_1^2}{2} &= \rho\frac{V_2^2}{2} - (p_{z1} - p_{z2}) \\ V_1 &= \sqrt{V_2^2 - \frac{2(p_{z1} - p_{z2})}{\rho}} \end{aligned}$$

The density of kerosene at 20°C is 814 kg/m^3.Solving for V_1

$$\begin{aligned} V_1 &= \sqrt{(10\,\mathrm{m/s})^2 - \frac{2(20,000\,\mathrm{kPa})}{(814\,\mathrm{kg/m^3})}} \\ &\boxed{V_1 = 7.13\,\mathrm{m/s}} \end{aligned}$$

Class

4.64: PROBLEM DEFINITION

Situation:

A glass tube with 90° bend inserted into a stream of water.
$V = 4\,\mathrm{m/s}$.

Find:

Rise in vertical leg above water surface (m).

PLAN

Apply the Bernoulli equation.

SOLUTION

Hydrostatic equation (between stagnation point and water surface in tube)

$$\frac{p_s}{\gamma} = h + d$$

where d is depth below surface and h is distance above water surface.

Bernoulli equation (between free stream and stagnation point)

$$\begin{aligned} \frac{p_s}{\gamma} &= d + \frac{V^2}{2g} \\ h + d &= d + \frac{V^2}{2g} \\ h &= \frac{V^2}{2g} \end{aligned}$$

$$\begin{aligned} h &= \frac{(4\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} \\ & \boxed{h = 0.815\,\mathrm{m}} \end{aligned}$$

Class

4.67: PROBLEM DEFINITION

Situation:

A flow-metering device is described in the problem.
$V_2 = 2V_1$, $\Delta h = 10\,\text{cm}$.

Find:

Velocity at station 2 (m/s).

Properties:

$\rho = 1.2\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation and the manometer equation.

SOLUTION

Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_t}{\gamma}$$

Manometer equation

$$\begin{aligned}
p_1 + 0.1 \times 9810 - \overbrace{0.1 \times 1.2 \times 9.81}^{\text{neglect}} &= p_t \\
p_t - p_1 &= 981\ \text{N/m}^2 = \frac{\rho V_1^2}{2} \\
V_1^2 &= \frac{2\,(981\,\text{N/m}^2)}{1.2\,\text{kg/m}^3} \\
V_1 &= 40.4\,\text{m/s} \\
V_2 &= 2V_1 \\
&\boxed{V_2 = 80.8\,\text{m/s}}
\end{aligned}$$

C

4.96: PROBLEM DEFINITION

Situation:

An outlet pipe from a reservoir.
$V = 6\,\mathrm{m/s}$, $h = 15\,\mathrm{m}$.

Find:

Pressure at point A (kPa).

Assumptions:

Flow is irrotational.

PLAN

Apply the Bernoulli equation.

SOLUTION

Bernoulli equation. Let point 1 be at reservoir surface.

$$\begin{aligned}
\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 &= \frac{p_A}{\gamma}+\frac{V_A^2}{2g}+z_A \\
0+0+15 &= \frac{p_A}{9810\,\mathrm{N/m^3}}+\frac{(6\,\mathrm{m/s})^2}{2\times 9.81\,\mathrm{m/s^2}}+0 \\
p_A &= (15\,\mathrm{m}-1.83\,\mathrm{m})\left(9810\,\mathrm{N/m^3}\right) \\
p_A &= 129{,}200\ \mathrm{Pa,\ gage} \\
&\boxed{p_A = 129\ \mathrm{kPa,\ gage}}
\end{aligned}$$