Solutions Chapter 4

4.1. An inflow hydrograph is measured for a cross section of a stream. Compute the outflow hydrograph at a point five miles downstream using the Muskingum method. Assume $K = 12$ hr, $x = 0.15$, and outflow equals inflow initially. Plot the inflow and outflow hydrographs.

We have K = 12 hr, x = 0.15, and Δt = 6 hr. Substituting these into Eqs. 4.7 through 4.10, we get:

$D = K - Kx + 0.50\Delta t$

$= 12 - (12)(0.15) + (0.5)(6)$

$D = 13.2$

$C_0 = (-Kx + 0.5\Delta t) / D$

$= [(-12)(0.15) + (0.5)(6)] / 13.2$

$C_0 = 0.0909$

$C_1 = (Kx + 0.5\Delta t) / D$

$= [(12)(0.15) + (0.5)(6)] / 13.2$

$C_1 = 0.3636$

$C_2 = (K - Kx - 0.5\Delta t) / D$

$= [12-(12)(0.15) - (0.5)(6)] / 13.2$

$C_2 = 0.5455$

As a check, we add C_0, C_1, and C_2:

$C_0 + C_1 + C_2 = 0.0909 + 0.3636 + 0.5455 = 1.00$

We then use these values in Eq. 4.6 to compute the outflow hydrograph:

$O_2 = C_0I_2 + C_1I_1 + C_2O_1$

Or

$O_i = C_0I_i + C_1I_{i-1} + C_2O_{i-1}$

$O_i = 0.0909I_i + 0.3636I_{i-1} + 0.5455O_{i-1}$

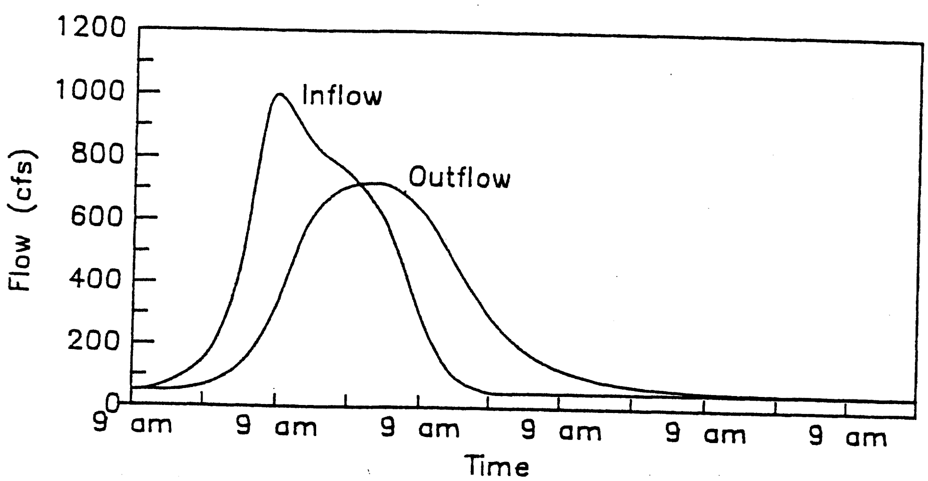
At i = 1, O_i = 50 cfs. Then,

$O_2 = (0.0909)(75) + (0.3636)(50) + (0.5455)(50)$

$O_2 = 52$ cfs

4.1 (cont)

TIME	I_i	I_{i-1}	O_i	O (cfs)
9:00 A.M.	50	-	-	50
3:00 P.M.	75	50	50	52
9:00 P.M.	150	75	52	69
3:00 A.M.	450	150	69	133
9:00 A.M.	1000	450	133	327
3:00 P.M.	840	1000	327	618
9:00 P.M.	750	840	618	711
3:00 A.M.	600	750	711	715
9:00 A.M.	300	600	715	636
3:00 P.M.	100	300	636	465
9:00 A.M.	50	100	465	294
3:00 P.M.	50	50	294	183
9:00 A.M.	50	50	183	123
3:00 P.M.	50	50	123	90
9:00 A.M.	50	50	90	72
3:00 P.M.	50	50	72	62
9:00 A.M.	50	50	62	56
3:00 P.M.	50	50	56	54
9:00 A.M.	50	50	54	52
3:00 P.M.	50	50	52	51
9:00 A.M.	50	50	51	51
3:00 P.M.	50	50	51	50
9:00 A.M.	50	50	50	50



4.2. Using the Muskingum method, route the following inflow hydrograph assuming (a) $K = 4$ hr, $x = 0.1$, and (b) $K = 2$ hr, $x = 0.3$. Plot the inflow and outflow hydrographs for each case assuming initial outflow equals initial inflow.

TIME (hr)	INFLOW (m^3/s)
0	0
2	5
4	25
6	50
8	35
10	21
12	13
14	7.5
16	2.5
18	0

(a) $K = 4\text{hr}, x = 0.1$

$D = K - Kx + 0.5\Delta t = 4 - (4)(0.1) + (0.5)(2) = 4.60$

$C_0 = (-Kx + 0.5\Delta t) / D = (-4(0.1) + 0.5(2)) / 4.6 = 0.13$

$C_1 = (Kx + 0.5\Delta t) / D = (4(0.1) + 0.5(2)) / 4.6 = 0.30$

$C_2 = (K - Kx - 0.5\Delta t) / D = (4 - 4(0.1) - 0.5(2)) / 4.6 = 0.57$

Find outflows using these coefficients and the equations below

$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 = (0.13)(5) + (0.3)(0) + (0.57)(0) = 0.7$

Assuming: $O_1 = I_1 = O$

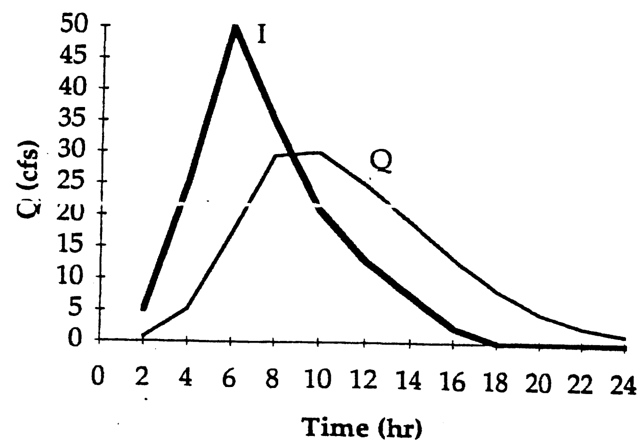
Similarly, $O_3 = C_0 I_3 + C_1 I_2 + C_2 O_2$

4.2. (cont)

The table and figure below summarize the results of the process:

Time (hr)	Inflow (cfs)	Outflow (cfs)
0	0	0.0
2	5	0.7
4	25	5.2
6	50	17.0
8	35	29.4
10	21	30.0
12	13	25.1
14	7.5	19.1
16	2.5	13.4
18	0	8.3
20	0	4.7
22	0	2.7
24	0	1.5

Figure P4.2.(a)



(b) $K = 2$ hr, $x = 0.3$

$D = K - Kx + 0.5\Delta t = 2 - 2(0.3) + 0.5(2) = 2.4$

$C_0 = (-Kx + 0.5\Delta t) / D = (-2(0.3) + (0.5)2) / 2.4 = 0.17$

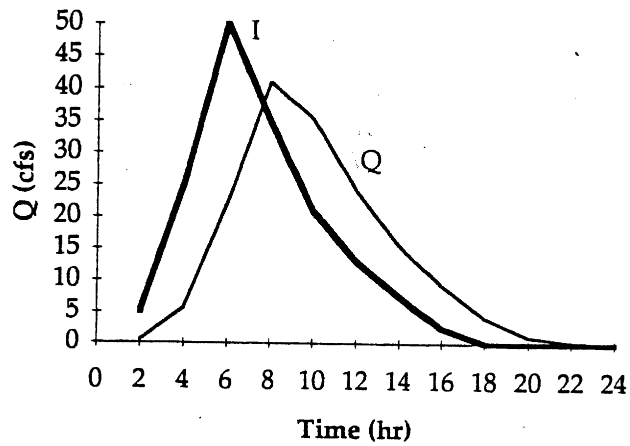
$C_1 = (Kx + 0.5\Delta t) / D = (2(0.3) + (0.5)2) / 2.4 = 0.67$

$C_2 = (K - Kx - 0.5\Delta t) / D = (2\text{-}2(0.3) - (0.5)(2)) / D = 0.16$

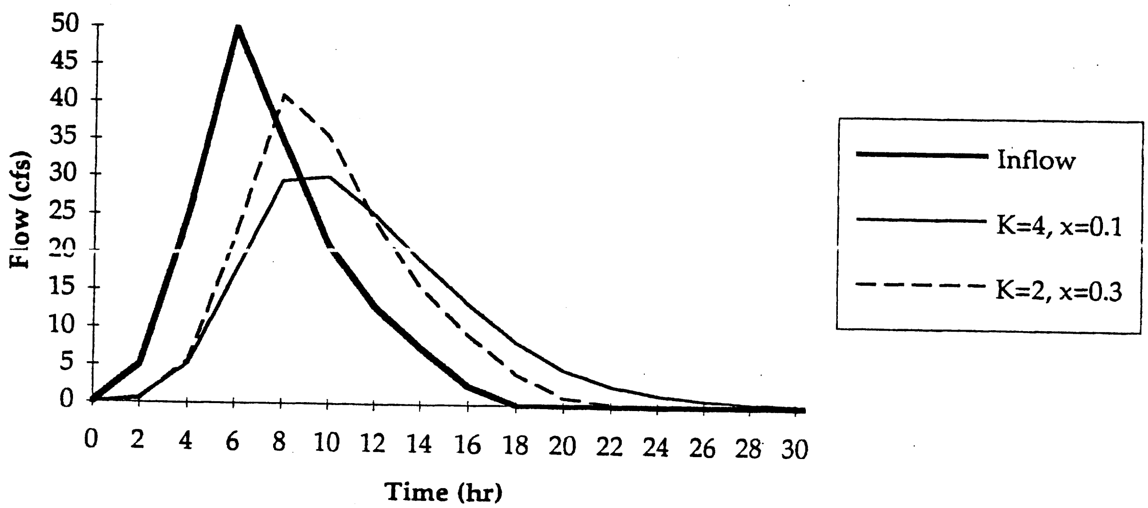
4.2. (cont)

Time (hr)	Inflow (cfs)	Outflow (cfs)
0	0	0.0
2	5	0.5
4	25	5.6
6	50	22.0
8	35	41.0
10	21	35.4
12	13	24.2
14	7.5	15.6
16	2.5	9.2
18	0	4.1
20	0	1.1
22	0	0.3
24	0	0.1

Figure P4.2.(b)

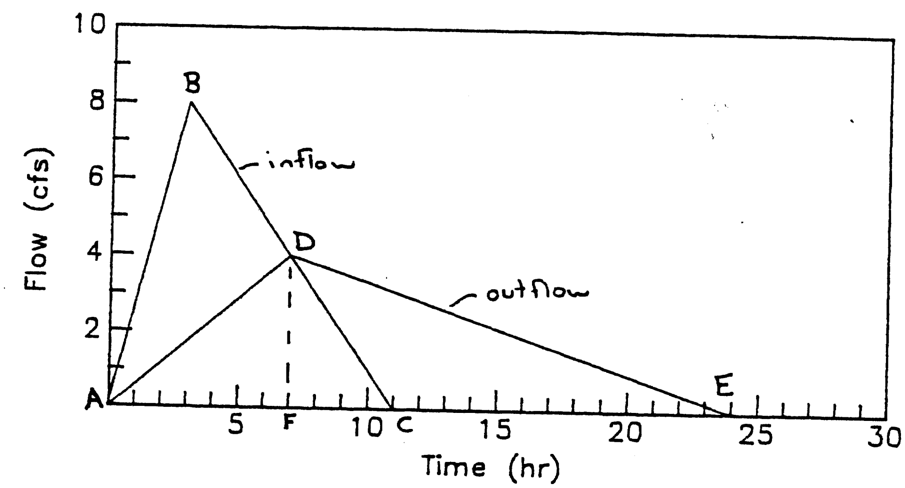


Hydrograph Comparison



4.3. A detention pond needs to be designed with a total capacity of 30 ac-in. of storage. The inflow hydrograph for the pond is given in Fig. P4.3. Assume that the pond is initially 50% full and outflow will cease when the pond is again 50% full. Graphically determine the peak outflow from the pond assuming a linear rise and fall in the outflow hydrograph. Draw the outflow hydrograph. At what time does outflow cease?

We know that the outflow hydrograph will peak at the intersection of the inflow and outflow hydrographs. We also know that the available volume for storage is equal to the area between the two hydrographs and must be equal to 50% of 30 ac-in or 15 ac-in. The outflow hydrograph will have the following shape:



To find the actual location of point D, we make use of the knowledge that the area of triangle

ABD = 15 ac-in and that $\Delta ABD = (1/2)(11)(8) = 44$ cfs-hr

Then

$$\Delta ADC = \Delta ABC - \Delta ABD = (44 \text{ cfs-hr}) - (15 \text{ ac-in})(1 \text{ cfs-hr/ac-in}) = 29 \text{ cfs-hr}$$

But

$$\Delta ADC = \tfrac{1}{2} bh = (1/2)(11)(y_d) \Rightarrow y_d = 5.27 \text{ cfs}$$

4.3. (cont) So the peak outflow is 5.27 cfs. To find the time at which the peak occurs we must find The point Q = 5.27 cfs on the line between B and C. We write the equation of this line as:

$$Y = -x + 11$$

At $y_d = 5.27$, $x_d = 5.73$. Thus, point D is located at (5.73, 5.27). To determine the time at which outflow ceases, we make use of the fact that the area of triangle DEC must also be equal to 15 ac-in or 15 cfs-hr. Since we know x_d, we can find the area of ΔDCF:

$$\Delta DCF = 1/2bh = (1/2)(11-5.73)(5.27) = 13.89 \text{ cfs-hr}$$

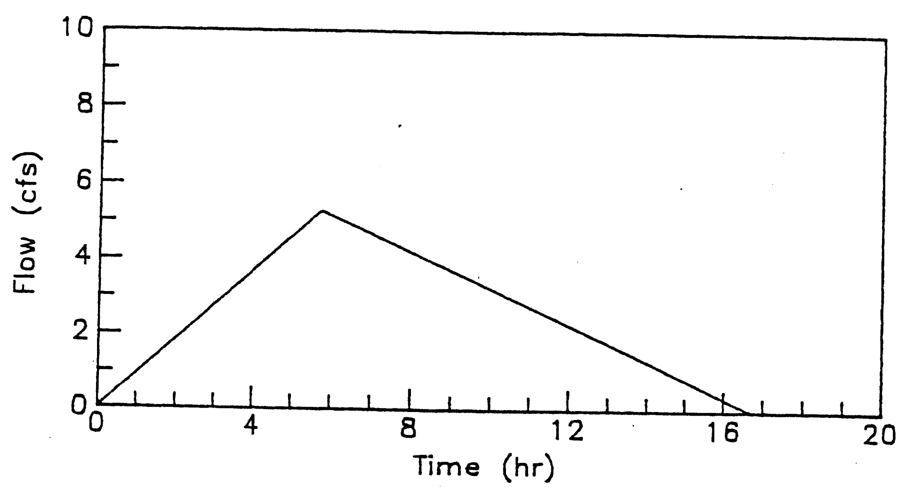
Then

$$\Delta DEF = 13.89 + 15 = 28.89 \text{ cfs-hr.}$$

But

$$\Delta DEF = \tfrac{1}{2}\, bh = (1/2)(EF)(5.27) \Rightarrow EF = 10.96 \Rightarrow x = x_d + 10.96 = 16.7 \text{ hr}$$

Thus the time at which outflow peaks is 16.7 hr and the actual shape of the outflow hydrograph is:



4.4. An inflow hydrograph is given for a reservoir that has a weir-spillway outflow structure. The flow through the spillway is governed by the equation

$$Q = 3.75\ Ly^{3/2}\ \text{(cfs)},$$

where L is the length of the weir and y is the height of the water above the spillway crest. The storage in the reservoir is governed by

$$S = 300y\ \text{(ac-ft)}.$$

Using Δt = 12 hr, L = 15 ft, and $S_0 = Q_0$ = O, route the inflow hydrograph through the reservoir using the storage indication method.

First, we develop a table of $2S_n/\Delta t + Q$ versus Q.

y (ft)	**Q (cfs)**	**S (ac-ft)**	**S (cfs-hr)**	**2S/Δt + Q (cfs)**
0	0	0	0	0
0.2	5.03	60	720	125
0.4	14.23	120	1440	254
0.6	26.14	180	2160	386
0.8	40.25	240	2880	520
1	56.25	300	3600	656
1.2	73.94	360	4320	794
1.4	93.18	420	5040	933
1.6	113.8	480	5760	1074
1.8	135.8	540	6480	1216
2	159.1	600	7200	1359
2.2	183.6	660	7920	1504
2.4	209.1	720	8640	1649
2.6	235.8	780	9360	1796
2.8	263.6	840	10080	1944
3	292.3	900	10800	2092
3.2	322	960	11520	2242
3.4	352.7	1020	12240	2393
3.6	384.2	1080	12960	2544
3.8	416.7	1140	13680	2697
4	450	1200	14400	2850
4.2	484.2	1260	15120	3004
4.4	519.2	1320	15840	3159
4.6	555	1380	16560	3315
4.8	591.5	1440	17280	3472
5	628.9	1500	18000	3629

4.4. (cont)

Next, Eq. 2.13 is solved for the right hand side. This value is used to find a value of Q. For example:

$$Q_1 = 0,\ s_1 = 0,\ (2S_1/\Delta t - Q_1) = 0$$

$$(I_1 + I_2) + (2S_1/\Delta t - Q_1) = (25_2/\Delta t - Q_2)$$

$$(40 + 35) + (0 - 0) = 75$$

For $(2S/\Delta t + Q) = 75$, $Q = 3.02$. The values are tabulated as:

Time (hr)	I_i (cfs)	$I_i + I_{i+1}$ (cfs)	$2S_{i+1}/\Delta t + Q_i$ (cfs)	$2S_{i+1}/\Delta t + Q_{i+1}$ (cfs)	Q_i (cfs)
12	40				0
		75	0	75	
24	35				3.0
		72	69	141	
36	37				6.0
		162	129	291	
48	125				17.5
		465	256	721	
60	340				64.6
		915	592	1507	
72	575				184.1
		1297	1139	2436	
84	722				361.6
		1462	1713	2175	
96	740				522.8
		1413	2129	3542	
108	673				608.3
		1129	2325	3454	
120	456				587.4
		706	2279	2985	
132	250				480.0
		390	2025	2415	
144	140				357.3
		150	1701	1851	
156	10				246.0
		10	1359	1369	
168	0				160.8
		0	1048	1048	
180	0				110.0
		0	828	828	
192	0				78.7
		0	671	671	
204	0				58.1
		0	555	555	
216	0				44.3
		0	466	466	
228	0				34.6
		0	397	397	
240	0				27.3

4.5. A reservoir has a linear *S-Q* relationship of

$$S = KQ,$$

where $K = 1.21$ hr. The inflow hydrograph for a storm event is given in the table.

a) Develop a simple recursive relation using the continuity equation and *S-Q* relationship for the linear reservoir [i.e., $aQ_2 = bQ_1 + c\bar{I}$, where a, b, and c are constants and $\bar{I} = (I_1 + I_2)/2$].

b) Storage route the hydrograph through the reservoir using $\Delta t = 1$ hr.

c) Explain why the shape of storage-discharge relations is usually not linear for actual reservoirs.

a) The continuity equation is:

$$\text{In} - \text{Out} = \Delta S/\Delta t$$

Or

$$(I_i + I_{i+1})(\Delta t/2) - (O_i + O_{i+1})(\Delta t/2) = \Delta S = S_{i+1} - S_i$$

Substituting for S = KQ and rearranging yields:

$$(I_i + I_{i+1})(\Delta t/2) - (O_i + O_{i+1})(\Delta t/2) = K\,(O_{i+1} - O_i)$$

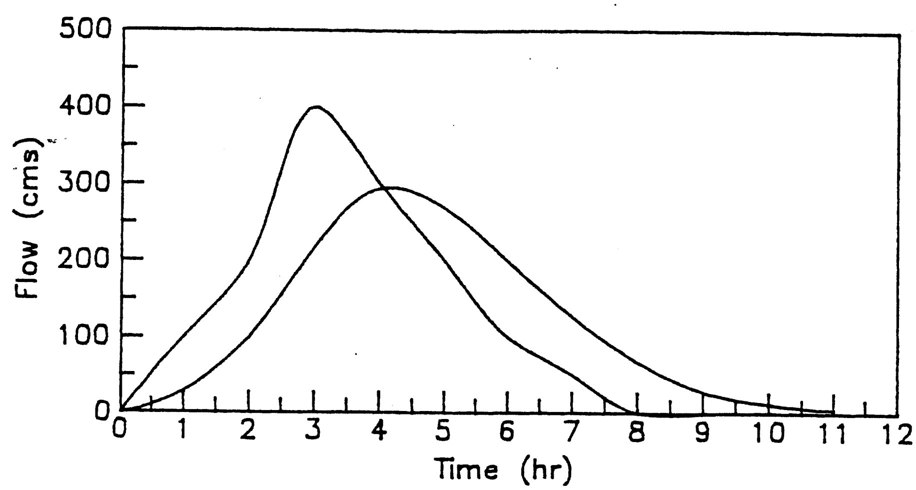
$$(K/\Delta t + 0.5)O_{i+1} = I + (K/\Delta t - 0.5)O_i$$

b) For K = 1.21 and $\Delta t = 1$, the equation found in part (a) becomes:

$$1.71\;O_{i+1} = I + (0.71)\;O_i$$

4.5. (cont)

Time (hrs)	I (m^3/s)	I (m^3/s)	0.71 O_i (m^3/s)	1.71 O_{i+1} (m^3/s)	O (m^3/s)
0	0				0
		50	0	50	
1	100				29.2
		150	21	171	
2	200				99.9
		300	71	371	
3	400				216.9
		350	154	504	
4	300				294.7
		250	209	459	
5	200				268.6
		150	191	341	
6	100				199.2
		75	142	271	
7	50				126.6
		25	90	115	
8	0				67.2
		0	48	48	
9	0				27.9
		0	20	20	
10	0				11.6
		0	8	8	
11	0				4.8



c) Storage is rarely uniform with depth since few reservoirs are uniform in shape. Most outflow structures have flow relations which are a function of depth raised to a power. Both of these facts lead to a non-linear storage-discharge relation.

4.6. Given the reservoir with a storage-discharge relationship governed by the equation

$$S = KQ^{3/2},$$

route the inflow hydrograph for problem 4.5 using storage routing techniques and a value of K = 1.21 for Q in m^3/s and S in m^3/s-hr. Discuss the differences in the outflow hydrograph for this reservoir and for the reservoir of problem 4.5. Use Δt = 1 hr.

Time (hr)	Inflow (m^3/s)
0	0
1	100
2	200
3	400
4	300
5	200
6	100
7	50
8	0

From the storage equation given,

$$S = KQ^{3/2} \quad , \quad K = 1.21$$

We can obtain the storage-discharge relationship and the storage-indication curve:

For example, take Q = 50 cms:

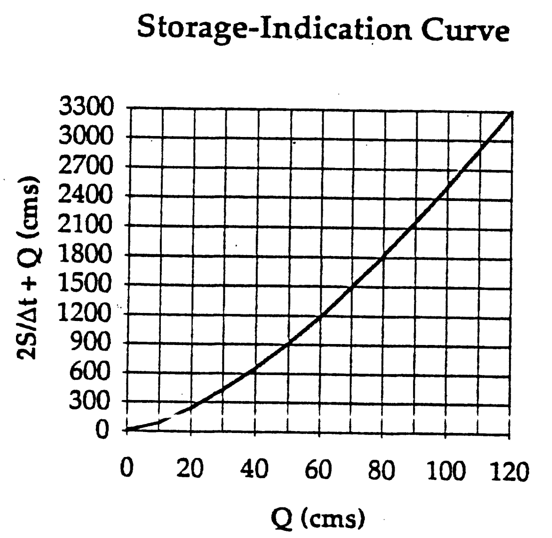
$$S = 1.21\ (50)^{3/2} = (1.21)(353.55) = 427.80 \text{ cms-hr}$$

$$2S/\Delta t + Q = [2\ (427.80) \text{ cms-hr} / 1 \text{ hr}] + 50 \text{ cms} = 906 \text{ cms}$$

4.6. (cont)

The following table and graph show these computations for a range of outflows:

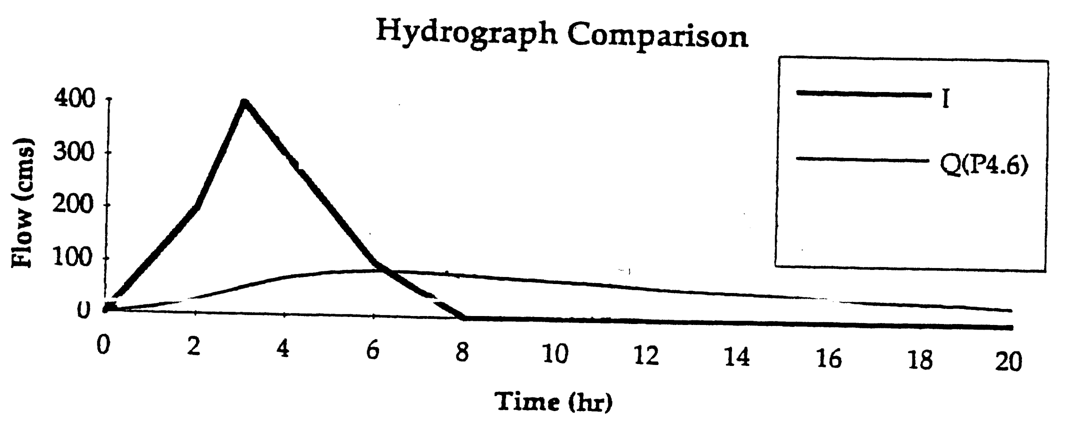
Storage (cms - hr)	Outflow (cms)	$2S/\Delta t + Q$ (cms)
0.00	0	0
38.26	10	87
108.23	20	236
198.82	30	428
306.11	40	652
427.80	50	906
562.36	60	1185
708.65	70	1487
865.81	80	1812
1033.12	90	2156
1210.00	100	2520
1395.96	110	2902
1590.59	120	3301



4.6. (cont)

Then using storage-indication, we obtain a new outflow hydrograph for comparison to problem

Time (hr)	In (cms)	In + I(n+1)	(2Sn/Δt - Qn)	2Sn/Δt + Q(n+1)	Qn (cms)
0	0	100	0		0
1	100	300	78	100	11
2	200	600	324	378	27
3	400	700	822	924	51
4	300	500	1380	1522	71
5	200	300	1716	1880	82
6	100	150	1844	2016	86
7	50	50	1824	1994	85
8	0	0	1710	1874	82
9	0	0	1556	1710	77
10	0	0	1412	1556	72
11	0	0	1276	1412	68
12	0	0	1150	1276	63
13	0	0	1034	1150	58
14	0	0	926	1034	54
15	0	0	824	926	51
16	0	0	730	824	47
17	0	0	646	730	42
18	0	0	566	646	40
19	0	0	492	566	37
20	0	0	426	492	33



There is a drastic reduction in the peak outflow for this S-Q relationship. The Linear reservoir does not reduce the peak by such an amount.

4.7. Blue Hole Lake has a spillway structure with the following relationship:

$$Q = 3.75\ Ly^{3/2},$$

where L = 10 m is the length of the weir and y is the height of the water above the spillway crest. The storage in the reservoir is governed by the relation

$$S = 1.5 \times 10^6 y.$$

Flow is measured in m^3/s, length and depth are measured in m, and storage is measured in m^3. Using a time step of 1 hr and assuming that the initial outflow is equal to zero, route the following inflow hydrograph through the reservoir using a Runge-Kutta program for routing calculations. Initial storage is zero.

TIME (hr)	INFLOW (m^3/s)
0	0
1	200
2	300
3	500
4	450
5	400
6	300
7	200
8	100
9	50
10	0

4.7. (cont) The program must have a table of flow versus depth and a table of

y (m)	Q (m^3/s)
0.00	0.00
0.25	4.69
0.50	13.26
0.75	24.36
1.00	37.50
1.25	52.41
1.50	68.89
1.75	86.81
2.00	106.07
2.25	126.56
2.50	148.23
2.75	171.01
3.00	194.86
3.25	219.71
3.50	245.55
3.75	272.32
4.00	300.00

y (m)	A (m^2)
0	1.5×10^6
10	1.5×10^6

The values are input when prompted for. Note that the units must correspond to those prompted for. The table of output values is:

TIME (hrs)	INFLOW (cms)	HEAD (m)	AREA (sq m)	dH (m)	OUTFLOW (cms)
.0	.00	.00	1500000.0	.00	.00
1.0	200.00	.24	1500000.0	.24	4.44
2.0	300.00	.80	1500000.0	.57	27.13
3.0	500.00	1.64	1500000.0	.84	79.15
4.0	450.00	2.51	1500000.0	.86	148.92
5.0	400.00	3.10	1500000.0	.59	204.74
6.0	300.00	3.40	1500000.0	.31	235.71
7.0	200.00	3.43	1500000.0	.03	238.39
8.0	100.00	3.24	1500000.0	-.19	218.56
9.0	50.00	2.93	1500000.0	-.31	188.15
10.0	.00	2.58	1500000.0	-.35	155.28
11.0	.00	2.24	1500000.0	-.34	125.86
12.0	.00	1.97	1500000.0	-.27	103.55
13.0	.00	1.74	1500000.0	-.23	86.12
14.0	.00	1.55	1500000.0	-.19	72.51
15.0	.00	1.39	1500000.0	-.16	61.64
16.0	.00	1.25	1500000.0	-.14	52.62
17.0	.00	1.14	1500000.0	-.12	45.59
18.0	.00	1.03	1500000.0	-.10	39.51
19.0	.00	.95	1500000.0	-.09	34.61
20.0	.00	.87	1500000.0	-.08	30.51

4.8. A small rectangular parking lot drains into a rectangular channel along its lower edge. Using a kinematic wave program, determine the flow into the channel from the parking lot. Assume that the parking lot may be modeled in two reaches of the same length with a time step of 5 min. The rainfall data and pertinent characteristics are given in Fig. P4.8 and in the following list.

Overland flow characteristics:

Dimensions of lot: 50 ft × 4000 ft

Overland flow length: 50 ft

Overland slope: 0.06 ft/ft

Manning's n value = 0.3

Imperviousness = 100%

Δx = 25 ft

Δt = 5 min

The overland flow only option is chosen at the beginning of the program. A total simulation time of 1 hour was used to generate the following output:

SUMMARY OUTFLOW HYDROGRAPHS

TIME (sec)	OVERLAND FLOW (cfs)	CHANNEL FLOW (cfs)
.00	.000	.0000
300.00	.307E-03	.0000
600.00	.161E-02	.0000
900.00	.159E-02	.0000
1200.00	.271E-03	.0000
1500.00	.124E-03	.0000
1800.00	.755E-04	.0000
2100.00	.513E-04	.0000
2400.00	.371E-04	.0000
2700.00	.281E-04	.0000
3000.00	.219E-04	.0000
3300.00	.176E-04	.0000
3600.00	.143E-04	.0000

4.9. The parking lot of problem 4.8 drains into a rectangular channel with the characteristics given in the following list. Using kinematic wave, determine the outflow from the channel due to overland flow from the parking lot.

Channel characteristics:

Length = 4000 ft

Slope = 0.003 ft/ft

Manning's n value = 0.025

Bottom width = 2 ft

Δx = 80 ft

Δt = 5 min

The overland and channel flow option is chosen at the beginning of the program. The Results are:

TIME (sec)	OVERLAND FLOW (cfs)	CHANNEL FLOW (cfs)
.00	.000	.0000
300.00	.307E-03	.0385
600.00	.161E-02	.8136
900.00	.159E-02	2.1717
1200.00	.271E-03	2.2619
1500.00	.124E-03	2.0586
1800.00	.755E-04	1.7399
2100.00	.513E-04	1.4077
2400.00	.371E-04	1.1141
2700.00	.281E-04	.8753
3000.00	.219E-04	.6888
3300.00	.176E-04	.5458
3600.00	.143E-04	.4368

4.10. A storm event occurred on Falls Creek Watershed that produced a rainfall pattern of 5 cm/hr for the first 10 min, 10 cm/hr in the second 10 min, and 5 cm/hr in the next 10 min. The watershed is divided into three subbasins (see Fig. P4.10) with the unit hydrographs as given in the following table. Subbasins *A* and *B* had a loss rate of 2.5 cm/hr for the first 10 min and 1.0 cm/hr thereafter. Subbasin *C* had a loss rate of 1.0 cm/hr for the first 10 min and 0 cm/hr thereafter. Using a simple lag routing method (time shift the hydrograph by K), determine the storm hydrograph at point 2. Assume a lag time K of 20 min.

First we must develop the storm hydrographs for each subbasin. Since the rainfall and the unit hydrographs are identical for both subbasin A and B, we will compute the storm hydrograph for subbasin A.

$P_1 = (5\text{cm/hr} - 2.5\text{cm/hr})(1/6\text{ hr}) = 0.417\text{cm}$

$P_2 = (10\text{cm/hr} - 1\text{cm/hr})(1/6\text{ hr}) = 1.5\text{cm}$

$P_3 = (5\text{cm/hr} - 1\text{cm/hr})(1/6\text{ hr}) = 0.667\text{cm}$

Time (min)	U_A	P_1U_A	P_2U_A	P_3U_A	$Q_A = Q_B$ (m^3/s)
0	0	0	-	-	0
10	5	2.09	0	-	2.09
20	10	4.17	7.5	0	11.67
30	15	6.26	15	3.33	24.59
40	20	8.34	22.5	6.67	37.51
50	25	10.43	30	10	50.43
60	20	8.34	37.5	13.33	59.17
70	15	6.26	30	16.67	52.93
80	10	4.17	22.5	13.33	40
90	5	2.09	15	10	27.09
100	0	0	7.5	6.67	14.17
110	-	-	0	3.33	3.33
120	-	-	-	0	0

For subbasin C:

$P_1 = (5\text{cm/hr} - 1\text{cm/hr})(1/6\text{ hr}) = 0.667\text{cm}$

$P_2 = (10\text{cm/hr})(1/6\text{ hr}) = 1.667\text{cm}$

$P_3 = (5\text{cm/hr})(1/6\text{ hr}) = 0.833\text{cm}$

4.10. (cont)

Time (min)	U_C	P_1U_C	P_2U_C	P_3U_C	Q_C (m^3/s)
0	0	0	-	-	0
10	16.7	11.13	0	-	11.13
20	33.4	22.27	27.83	0	50.10
30	50.0	33.33	55.67	13.92	102.92
40	33.4	22.27	83.33	27.83	133.43
50	16.7	11.13	55.67	41.67	108.47
60	0	0	27.83	27.83	55.66
70	-	-	0	13.92	13.92
80	-	-	-	0	0

Next, to get the storm hydrograph at point 2, we must combine the hydrographs from A and B at point 1, route this combined hydrograph through the reach 1-2, and combine it with the hydrograph from C to point 2. To combine A and B we simply take $Q_A + Q_B$ or $2Q_A$. To route this, we simply lag by 20 min.

Time (min)	Q_C (m^3/s)	$2Q_A$ lagged (m^3/s)	Q_2 (m^3/s)
0	0	-	0.00
10	11.13	-	11.13
20	50.1	0	50.10
30	102.92	4.18	107.10
40	133.43	23.34	156.77
50	108.47	49.18	157.65
60	55.66	75.02	130.68
70	13.92	100.86	114.78
80	0	118.34	118.34
90	-	105.86	105.86
100	-	80	80.00
110	-	54.18	54.18
120	-	28.34	28.34
130	-	6.66	6.66
140	-	0	0.00

4.11. Repeat problem 4.10 using Muskingum routing methods instead of simple lag routing. Discuss the differences between the two storm hydrographs. Use $x = 0.2$, $K = 20$ min, and $\Delta t = 10$ min.

The hydrographs for each subarea were found in Problem 4.10. Using K = 20min, x = 0.2, and Δt = 10min, we get

$D = K - Kx + \Delta t/2 = 20 - (20)(0.2) + (10)/2 = 21$

$C_0 = (-Kx + \Delta t/2)/D = [(-20)(0.2) + (10)/2]/21 = 0.0476$

$C_1 = (Kx + \Delta t/2)/D = [(20)(0.2) + (10)/2]/21 = 0.4286$

$C_2 = (K - Kx - \Delta t/2)/D = [20 - (20)(0.2) - (10)/2]/21 = 0.5238$

Then the combined hydrograph at point 1 is routed to point 2.

Time (min)	I_i (m^3/s)	I_{i-1} (m^3/s)	O_{i-1} (m^3/s)	O_i (m^3/s)
0.00	0.00	-	-	0.00
10.00	4.18	0.00	0.00	0.20
20.00	23.34	4.18	0.20	3.01
30.00	49.18	23.34	3.01	13.92
40.00	75.02	49.18	13.92	31.94
50.00	100.86	75.02	31.94	53.69
60.00	118.34	100.86	53.69	76.98
70.00	105.86	118.34	76.98	96.08
80.00	80.00	105.86	96.08	99.51
90.00	54.18	80.00	99.51	88.99
100.00	28.34	54.18	88.99	71.18
110.00	6.66	28.34	71.18	49.75
120.00	0.00	6.66	49.75	28.91
130.00		0.00	28.91	15.14
140.00			15.14	7.93
150.00			7.93	4.16
160.00			4.16	2.18
170.00			2.18	1.14
180.00			1.14	0.60
190.00			0.60	0.31
200.00			0.31	0.16
210.00			0.16	0.09
220.00			0.09	0.04
230.00			0.04	0.02
240.00			0.02	0.01
250.00			0.01	0.01
260.00			0.01	0.00

4.11. (cont) This routed hydrograph is then combined with the hydrograph from subbasin C to get the storm hydrograph at point C.

Time (min)	From 1 (m^3/s)	Q_C (m^3/s)	Q_2 (m^3/s)
0.00	0.00	0	0
10.00	0.20	11.13	11.33
20.00	3.01	50.1	53.11
30.00	13.92	102.92	116.84
40.00	31.94	133.43	165.37
50.00	53.69	108.47	162.16
60.00	76.98	55.66	132.64
70.00	96.08	13.92	110
80.00	99.51	0	99.51
90.00	88.99		88.99
100.00	71.18		71.18
110.00	49.75		49.75
120.00	28.91		28.91
130.00	15.14		15.14
140.00	7.93		7.93
150.00	4.16		4.16
160.00	2.18		2.18
170.00	1.14		1.14
180.00	0.60		0.6
190.00	0.31		0.31
200.00	0.16		0.16
210.00	0.09		0.09
220.00	0.04		0.04
230.00	0.02		0.02
240.00	0.01		0.01
250.00	0.01		0.01
260.00	0.00		0

The Muskingum route produces a higher peak at an earlier time and a longer recession time. Since the peak is attenuated in the Muskingum routing, there is not a double peak on this storm hydrograph.

4.12. The storage equation has been given as

$$S_i = K[xI_i + (1 - x)O_i]$$

and the continuity equation as

$$\bar{I} = \bar{O} + \Delta S/\Delta t.$$

Given that I and O are the average inflow and outflow within the time period and ΔS is the change in the storage, derive the Muskingum river routing equation:

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1,$$

where I_2 and O_2 refer to the inflow and outflow at the end of the time period and I_1 and O_1 refer to those values at the beginning of the time period. Verify the equations for C_0, C_1, and C_2 given in Section 4.2.

We know that

$$\Delta S = S_2 - S_1,$$

$$S_2 = K\,[xI_2 + (1\text{-}x)\,O_2], \text{ and}$$

$$S_1 = K\,[xI_1 + (1\text{-}x)\,O_1].$$

So we have

$$\Delta S = K\,[x\,(I_2 - I_1) + (1\text{-}x)\,(O_2 - O_1)].$$

Substituting this into the continuity equation yields:

$$I = O + K\,[x\,(I_2 - I_1) + (1 - x)\,(O_2 - O_1)] / \Delta t$$

Or

$$\tfrac{1}{2}(I_1 + I_2) = \tfrac{1}{2}(O_1 + O_2) + K\,[x\,(I_2 - I_1) + (1 - x)\,(O_2 - O_1)] / \Delta t$$

Isolating O_2 on the left hand side gives:

$$(K - Kx + \Delta t/2)\,O_2 = (-Kx + \Delta t/2)\,I_2 + (Kx + \Delta t/2)\,I_1 + (K - Kx - \Delta t/2)\,O_1$$

or

$$O_2 = \frac{(-kx + \Delta t/2)}{(k - kx + \Delta t/2)} I_2 + \frac{(kx + \Delta t/2)}{(k - kx + \Delta t/2)} I_1 + \frac{(k - kx - \Delta t/2)}{(k - kx + \Delta t/2)} O_1$$

Thus

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$$

Where

$$D = K - Kx + \Delta t/2$$

4.12. (cont)

$C_0 = (-Kx + \Delta t/2) / D$

$C_1 = (Kx + \Delta t/2) / D$

$C_2 = (K - Kx - \Delta t/2) / D$

4.13. Develop a storage routing program in Excel, and repeat Example 4.5. Compare the results when the inflows are doubled.

a) **S-I Chart**

	A	B	C	D	E	F	G	H	I	J	K
	h (ft)	Trap bottom (ft)	Trap top (ft)	Trap height (ft)	Trap area (ft^2)	increm vol (ft^3)	tot storage (ft^3)	Q orifice (cfs)	Q wier (cfs)	Q total (cfs)	2S/dt + Q (cfs)
3	0	80.0	B3-60	120.0	C3*D3+(B3-C3)*D3/2	0.0	0.0	0.9*3.14159*1.5^2/4*(2*32.2*A3)^0.5	0.0	I3+H3	2*H4/10/60+K4
4	1	88.9	B4-60	D3+8	C4*D4+(B4-C4)*D4/2	(E4+E3)/2*(A4-A3)	G3+F4	0.9*3.14159*1.5^2/4*(2*32.2*A4)^0.5	0.0	I4+H4	2*G4/10/60+J4
5	2	97.9	B5-60	D4+8	C5*D5+(B5-C5)*D5/2	(E5+E4)/2*(A5-A4)	G4+F5	0.9*3.14159*1.5^2/4*(2*32.2*A5)^0.5	0.0	I5+H5	2*G5/10/60+J5
6	3	106.8	B6-60	D5+8	C6*D6+(B6-C6)*D6/2	(E6+E5)/2*(A6-A5)	G5+F6	0.9*3.14159*1.5^2/4*(2*32.2*A6)^0.5	0.0	I6+H6	2*G6/10/60+J6

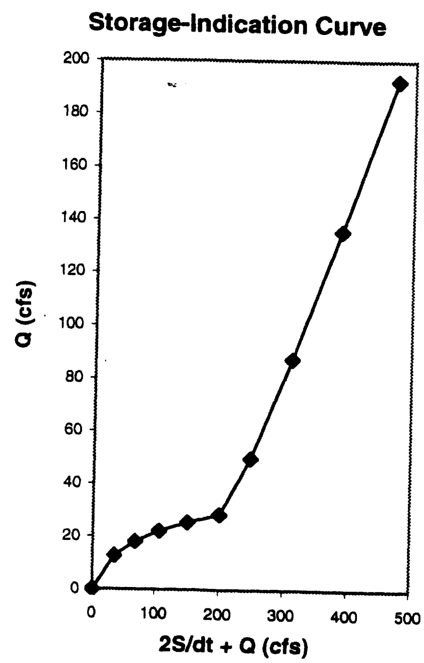
Inflow-Outflow Calculations

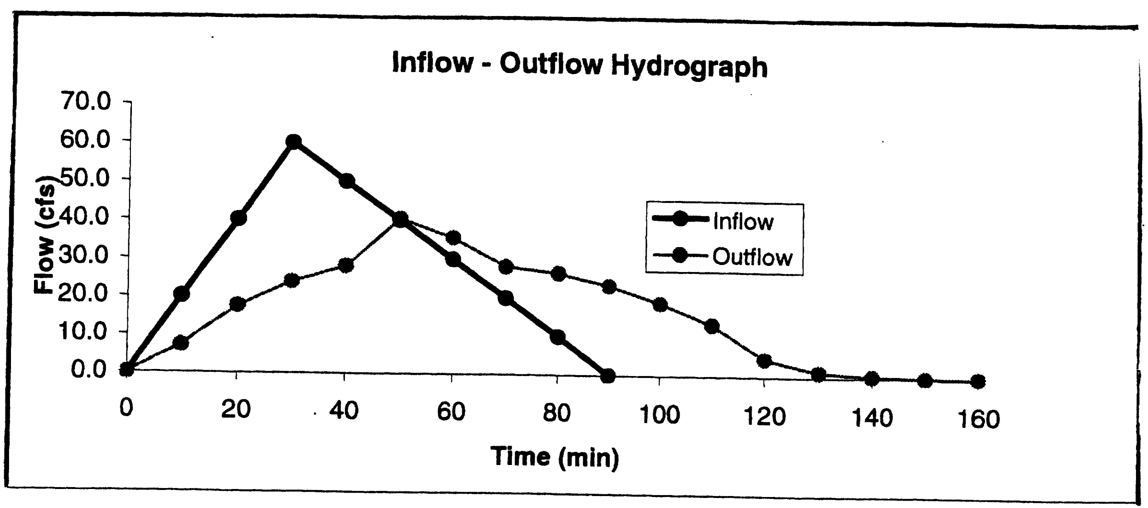
	A	B	C	D	F	G
	TIME (min)	In (cfs)	In + In+1 (cfs)	2Sn/dt - Qn (cfs)	2Sn+1/dt + Qn+1 (cfs)	Qn
12	0	0	B12+B13	0.0		0.0
13	10	20	B13+B14	E13-2*F13	C12+D12	from S-I chart
14	20	40	B14+B15	E14-2*F14	C13+D13	from S-I chart
15	30	60	B15+C16	E15-2*F15	C14+D14	from S-I chart

h (ft)	Trap bottom (ft)	Trap top (ft)	Trap height (ft)	Trap area (ft^2)	increm vol (ft^3)	total storage (ft^3)	Q orifice (cfs)	Q wier (cfs)	Q total (cfs)	2S/dt + Q (cfs)
0	80.0	20.0	120.0	6000.0	0.0	0.0	0.0	0.0	0.0	0.0
1	88.9	28.9	128.0	7539.2	6769.6	6769.6	12.8	0.0	12.8	35.3
2	97.9	37.9	136.0	9234.4	8386.8	15156.4	18.0	0.0	18.0	68.6
3	106.8	46.8	144.0	11059.2	10146.8	25303.2	22.1	0.0	22.1	106.5
4	115.8	55.8	152.0	13041.6	12050.4	37353.6	25.5	0.0	25.5	150.0
5	124.7	64.7	160.0	15152.0	14096.8	51450.4	28.5	0.0	28.5	200.0
5.5	127.0	67.0	162.0	15714.0	7716.5	59166.9	29.9	19.9	49.8	247.0
6	129.2	69.2	164.0	16268.8	7995.7	67162.6	31.3	56.2	87.4	311.3
6.5	131.4	71.4	166.0	16832.4	8275.3	75437.9	32.5	103.2	135.7	387.2
7	133.7	73.7	168.0	17421.6	8563.5	84001.4	33.8	158.9	192.7	472.7

TIME (min)	In (cfs)	In + In+1 (cfs)	2Sn/dt - Qn (cfs)	2Sn+1/dt + Qn+1 (cfs)	Qn
0	0.0	20.0	0.0		0.0
10	20.0	60.0	5.6	20.0	7.2
20	40.0	100.0	30.4	65.6	17.6
30	60.0	110.0	82.4	130.4	24.0
40	50.0	90.0	136.2	192.4	28.1
50	40.0	70.0	145.4	226.2	40.4
60	30.0	50.0	144.4	215.4	35.5
70	20.0	30.0	138.0	194.4	28.2
80	10.0	10.0	114.8	168.0	26.6
90	0.0	0.0	77.8	124.8	23.5
100	0.0	0.0	39.8	77.8	19.0
110	0.0	0.0	12.8	39.8	13.5
120	0.0	0.0	3.6	12.8	4.6
130	0.0	0.0	1.0	3.6	1.3
140	0.0	0.0	0.2	1.0	0.4
150	0.0	0.0	0.0	0.2	0.1
160	0.0	0.0	0.0	0.0	0.0
170	0.0	0.0	0.0	0.0	0.0
180	0.0	0.0	0.0	0.0	0.0

4.13. (cont)

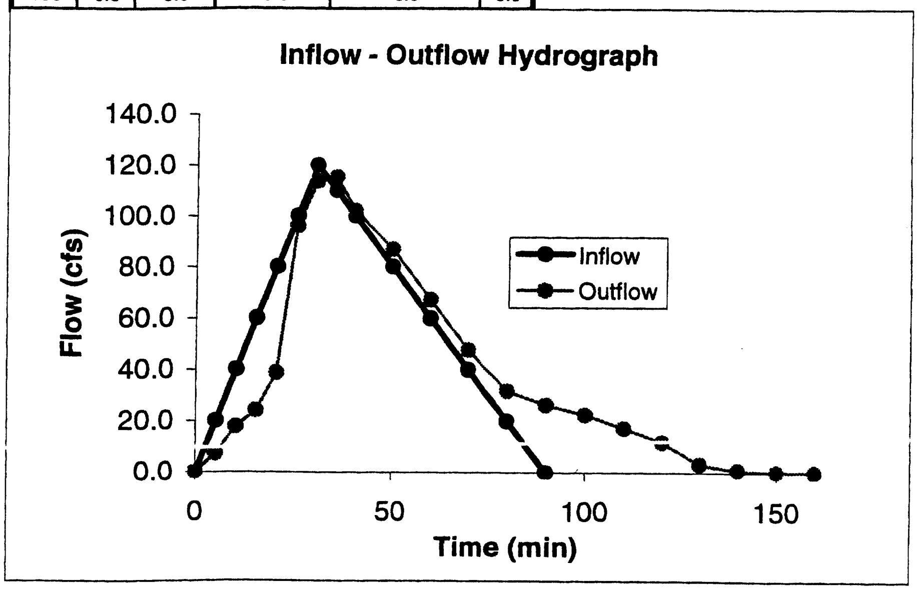




b) Note that the inflow is much greater than the storage capacity of the reservoir overwhelming it. Thus, a 10-minute time step gives faulty results, so we used a 5-minute time step during the period of increasing inflow. Also, note that the S-1 curve used in part A was used again.

4.13. (cont)

TIME (min)	In (cfs)	In + In+1 (cfs)	2Sn/dt - Qn (cfs)	2Sn+1/dt + Qn+1 (cfs)	Qn
0	0.0	20.0	0.0		0.0
5	20.0	60.0	5.6	20.0	7.2
10	40.0	100.0	30.4	65.6	17.6
15	60.0	140.0	82.4	130.4	24.0
20	80.0	180.0	145.0	222.4	38.7
25	100.0	220.0	132.6	325.0	96.2
30	120.0	230.0	125.2	352.6	113.7
35	110.0	210.0	124.4	355.2	115.4
40	100.0	180.0	130.2	334.4	102.1
50	80.0	140.0	136.6	310.2	86.8
60	60.0	100.0	142.4	276.6	67.1
70	40.0	60.0	147.0	242.4	47.7
80	20.0	20.0	143.6	207.0	31.7
90	0.0	0.0	111.0	163.6	26.3
100	0.0	0.0	66.2	111.0	22.4
110	0.0	0.0	32.0	66.2	17.1
120	0.0	0.0	8.8	32.0	11.6
130	0.0	0.0	2.4	8.8	3.2
140	0.0	0.0	0.6	2.4	0.9
150	0.0	0.0	0.2	0.6	0.2
160	0.0	0.0	0.0	0.2	0.1
170	0.0	0.0	0.0	0.0	0.0
180	0.0	0.0	0.0	0.0	0.0



4.14. Develop a Muskingum routing program in Excel, and repeat problem 4.2. Compare the results.

Sample Muskingum Excel Formulas

	A	B	C
	Inputs:		
2		X ==>	0.3
3		K ==>	2
4		dt ==>	2
5		O1 ==>	0
6	Coefficients:		
7		C0 =	(((0.5*$C$4) - ($C$3*C2))/$C$10)
8		C1 =	(($C$3*C2 + 0.5*C4)/$C$10)
9		C2 =	((((1-$C$2)*$C$3) - 0.5*C4)/ $C$10)
10		D =	((1-$C$2)*$C$3) + 0.5*$C$4
11		C0+C1+C2=	SUM($C$7:$C$9)
12	Input Inflow Hydrograph:		
13	Time	Inflow	Outflow
14	0	0	$C$5
15	2	5	($C$7*B15) + ($C$8*B14) + ($C$9*C14)
16	4	25	($C$7*B16) + ($C$8*B15) + ($C$9*C15)
17	6	50	($C$7*B17) + ($C$8*B16) + ($C$9*C16)
18	8	35	($C$7*B18) + ($C$8*B17) + ($C$9*C17)
19	10	21	($C$7*B19) + ($C$8*B18) + ($C$9*C18)
20	12	13	($C$7*B20) + ($C$8*B19) + ($C$9*C19)
21	14	7.5	($C$7*B21) + ($C$8*B20) + ($C$9*C20)
22	16	2.5	($C$7*B22) + ($C$8*B21) + ($C$9*C21)
23	18	0	($C$7*B23) + ($C$8*B22) + ($C$9*C22)
24	20	0	($C$7*B24) + ($C$8*B23) + ($C$9*C23)
25	22	0	($C$7*B25) + ($C$8*B24) + ($C$9*C24)
26	24	0	($C$7*B26) + ($C$8*B25) + ($C$9*C25)

4.14. (cont)

Inputs:

	x ==>	0.1	
	K ==>	4	K and dt must have the same units.
	dt ==>	2	
	O1 ==>	0	

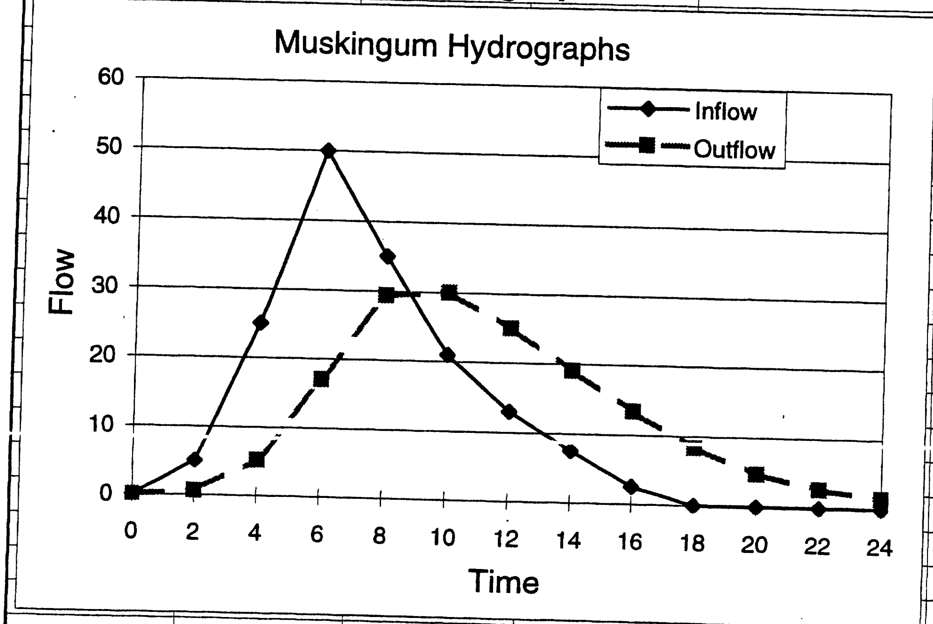
Coefficients:

	C0 =	0.1304	
	C1 =	0.3043	
	C2 =	0.5652	
	D =	4.6	
	C0+C1+C2=	1	Must equal one.

Input Inflow Hydrograph:

Time	Inflow	Outflow
0	0	0.00
2	5	0.65
4	25	5.15
6	50	17.04
8	35	29.42
10	21	30.02
12	13	25.05
14	7.5	19.10
16	2.5	13.40
18	0	8.34
20	0	4.71
22	0	2.66
24	0	1.51

Computed Outflow Hydrograph:



4.14. (cont)

Inputs:

	x ==>	0.3	
	K ==>	2	K and dt must have the same units.
	dt ==>	2	
	O1 ==>	0	

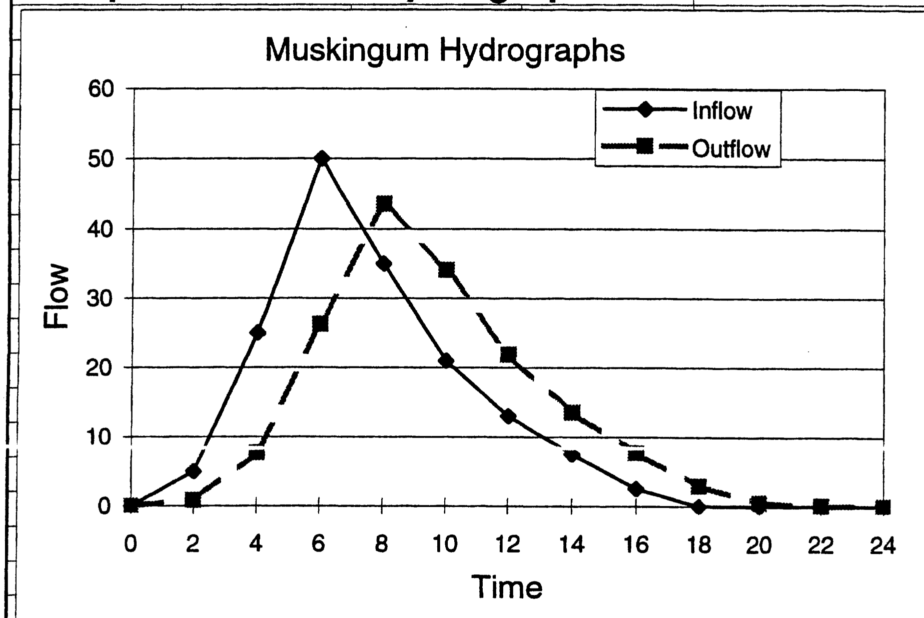
Coefficients:

	C0 =	0.1667	
	C1 =	0.6667	
	C2 =	0.1667	
	D =	2.4	
	C0+C1+C2=	1	Must equal one.

Input Inflow Hydrograph:

Time	Inflow	Outflow
0	0	0.00
2	5	0.83
4	25	7.64
6	50	26.27
8	35	43.55
10	21	34.09
12	13	21.85
14	7.5	13.56
16	2.5	7.68
18	0	2.95
20	0	0.49
22	0	0.08
24	0	0.01

Computed Outflow Hydrograph:



4.15. Determine the outflow hydrograph given the inflow hydrograph below. Use Muskingum routing, taking $K = 2$ hr, $x = 0.2$, and $\Delta t = 1$ day. From the inflow and outflow relation computed, investigate the effects of different values of x, $x = 0.1$ and 0.3, and graph weighted discharge vs. storage. (See Example 4.4.)

K = 2 hr, x = 0.2, Δt = 1 hr

$D = K - Kx + \Delta t/2 \quad = 2 - 2(0.2) + (0.5)(1) =$

$C_0 = (-Kx + \Delta t/2) / D \quad = (-2(0.2) + (0.5)(1)) / 2.1 = 0.05$

$C_1 = (Kx + \Delta t/2) / D \quad = (2(0.2) + (0.5)(1)) / 2.1 = 0.43$

$C_2 = (K - Kx - \Delta t/2) / D \quad = (2 - 2(0.2) - (0.5)(1)) / 2.1 = 0.52$

Using these values, we can obtain the inflow – outflow tables. (i.e. the first three columns.)

To investigate the effects of differing "x" values, find the respective values for $x \cdot I + (1 - x) \cdot Q$ for

x = 0.1, x = 0.2, and x = 0.3

For example, when x = 0.1 at I = 750, Q = 456

$x \cdot I + (1 - x) \cdot Q = (0.1)(750) + (1 - 0.1)(456) = 485$ cms

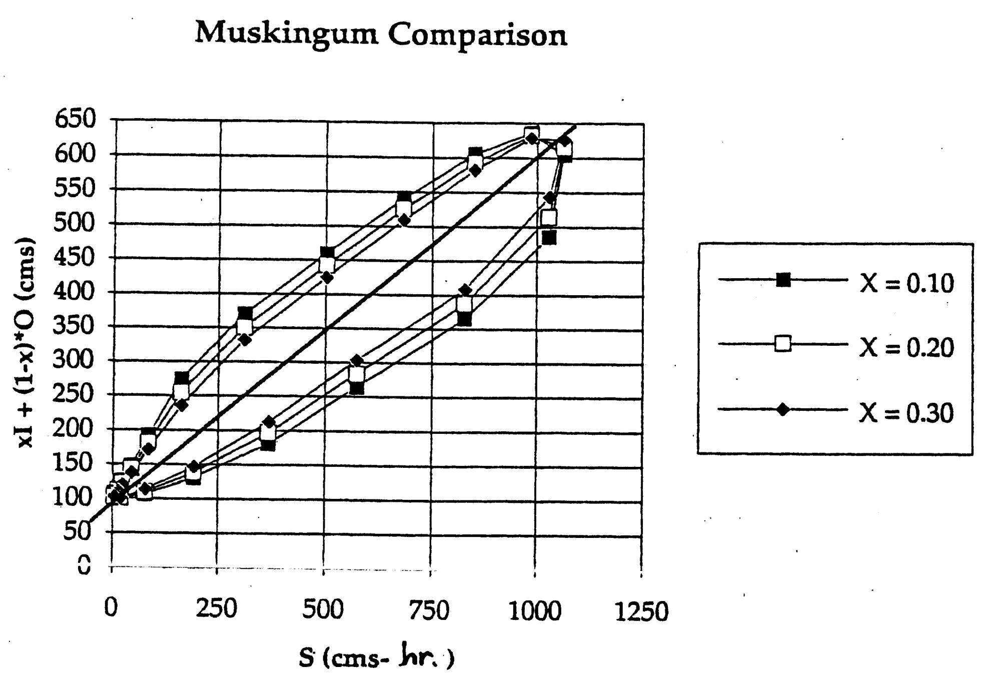
A plot of weighted discharge $x \cdot I + (1 - x) \cdot Q$ - vs - Storage should yield the correct K and x values for recomputing the outflow solution. The x-value is revealed as the most "linear" plot, and the K-value is found as the inverse-slope of the best fit line through the plot.

The following table and graph summarizes these computations:

Note that varying x has little effect on the results.

4.15. (cont)

Time (hr.)	Inflow (cms)	Outflow (cms)	Storage (cms- hr.)	Weighted discharge (cms) X = 0.10	X = 0.20	X = 0.30
1	100	100	19	100	100	100
2	140	102	77	106	110	113
3	200	123	193	131	138	146
4	320	165	368	181	196	212
5	440	245	573	264	284	303
6	560	343	829	365	387	408
7	750	456	1029	485	514	544
8	700	593	1063	604	615	625
9	600	639	985	635	632	628
10	500	616	849	604	593	581
11	400	556	683	540	525	509
12	300	477	501	459	442	424
13	200	388	310	369	350	332
14	100	294	162	274	255	236
15	100	201	85	191	181	171
16	100	153	45	148	143	137
17	100	128	23	125	122	119
18	100	115	12	113	112	110
19	100	108	6	107	106	105
20	100	104	4	104	103	103

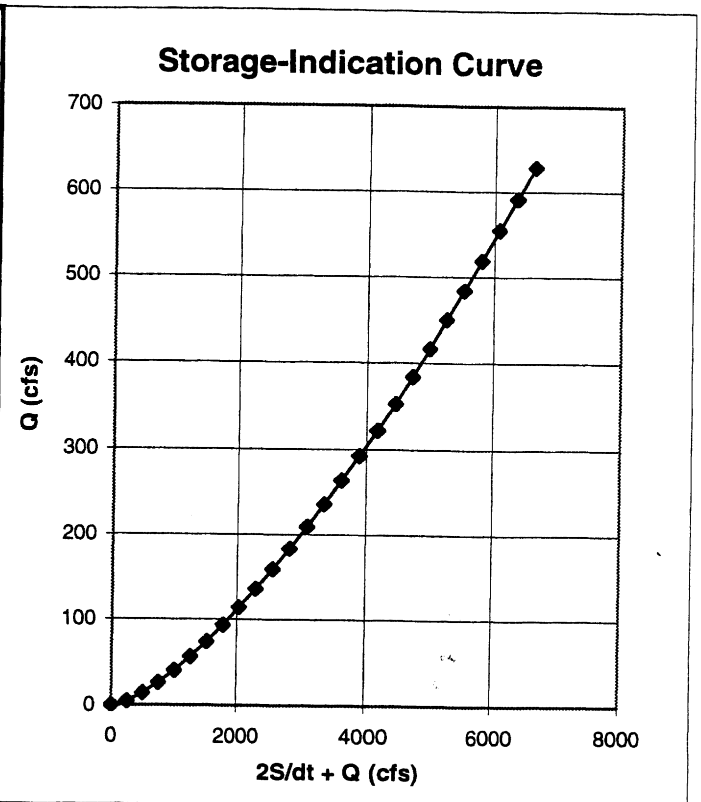


$$m = \frac{\Delta y}{\Delta x} = \frac{600-350}{1000-500} = \frac{250}{500} = 0.5 \text{ cms/cms-hr}$$

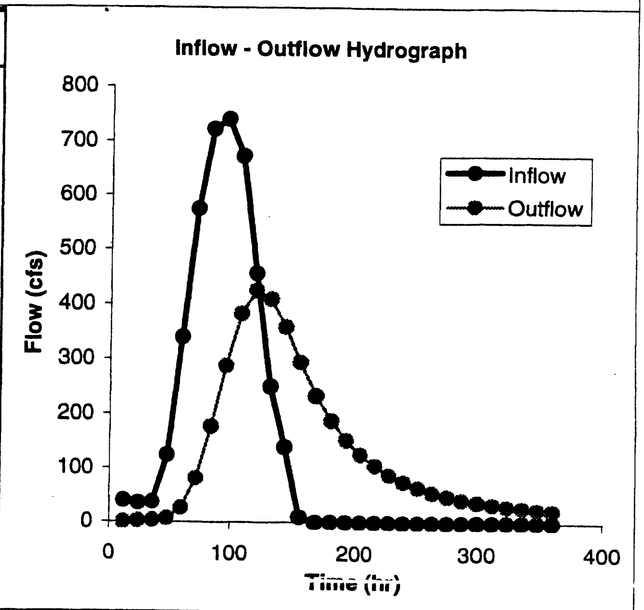
$K = 1/m = 1/0.5 = 2hr$; K is the same

4.16. Repeat problem 4.4 but double the storage volume and use a storage indication routing program written in Excel.

y (ft)	Q (cfs)	S (ac-ft)	S (cfs-hr)	2S/dt +Q (cfs)
0	0.0	0.0	0.0	0.0
0.2	5.0	120.0	1440.0	245.0
0.4	14.2	240.0	2880.0	494.2
0.6	26.1	360.0	4320.0	746.1
0.8	40.2	480.0	5760.0	1000.2
1	56.3	600.0	7200.0	1256.3
1.2	73.9	720.0	8640.0	1513.9
1.4	93.2	840.0	10080.0	1773.2
1.6	113.8	960.0	11520.0	2033.8
1.8	135.8	1080.0	12960.0	2295.8
2	159.1	1200.0	14400.0	2559.1
2.2	183.6	1320.0	15840.0	2823.6
2.4	209.1	1440.0	17280.0	3089.1
2.6	235.8	1560.0	18720.0	3355.8
2.8	263.5	1680.0	20160.0	3623.5
3	292.3	1800.0	21600.0	3892.3
3.2	322.0	1920.0	23040.0	4162.0
3.4	352.6	2040.0	24480.0	4432.6
3.6	384.2	2160.0	25920.0	4704.2
3.8	416.7	2280.0	27360.0	4976.7
4	450.0	2400.0	28800.0	5250.0
4.2	484.2	2520.0	30240.0	5524.2
4.4	519.2	2640.0	31680.0	5799.2
4.6	555.0	2760.0	33120.0	6075.0
4.8	591.5	2880.0	34560.0	6351.5
5	628.9	3000.0	36000.0	6628.9



TIME (hr)	In (cfs)	In + In+1 (cfs)	2Sn/dt - Qn (cfs)	2Sn+1/dt + Qn+1 (cfs)	Qn
12	40.0	75.0	0.0		0.0
24	35.0	72.0	72.0	75.0	1.5
36	37.0	162.0	138.0	144.0	3.0
48	125.0	465.0	285.8	300.0	7.1
60	340.0	915.0	698.4	750.8	26.2
72	575.0	1297.0	1451.4	1613.4	81.0
84	722.0	1462.0	2394.4	2748.4	177.0
96	740.0	1413.0	3279.4	3856.4	288.5
108	673.0	1129.0	3926.4	4692.4	383.0
120	456.0	706.0	4205.4	5055.4	425.0
132	250.0	390.0	4093.4	4911.4	409.0
144	140.0	150.0	3766.4	4483.4	358.5
156	10.0	10.0	3326.4	3916.4	295.0
168	0.0	0.0	2868.4	3336.4	234.0
180	0.0	0.0	2492.4	2868.4	188.0
192	0.0	0.0	2186.4	2492.4	153.0
204	0.0	0.0	1933.4	2186.4	126.5
216	0.0	0.0	1721.4	1933.4	106.0
228	0.0	0.0	1543.4	1721.4	89.0
240	0.0	0.0	1391.4	1543.4	76.0
252	0.0	0.0	1260.4	1391.4	65.5
264	0.0	0.0	1147.4	1260.4	56.5
276	0.0	0.0	1048.4	1147.4	49.5
288	0.0	0.0	962.4	1048.4	43.0
300	0.0	0.0	886.4	962.4	38.0
312	0.0	0.0	818.4	886.4	34.0
324	0.0	0.0	758.4	818.4	30.0
336	0.0	0.0	704.4	758.4	27.0
348	0.0	0.0	656.4	704.4	24.0
360	0.0	0.0	612.4	656.4	22.0



4.17. Develop a new flood routing method for rectangular cross sections based on the Muskingum and storage indication techniques. Instead of using the Muskingum storage equation $S = f(K, x, I, Q)$, use Manning's equation in the form where a and m are constants. Assume prismatic channel conditions at each time step, and derive the necessary equation for flood routing in a given rectangular channel with length L, inflows I_1 and I_2, channel width B, and outflows Q_1 and Q_2. Manning's equation takes the form

$$Q = ay^m.$$

We assume a wide rectangular channel where $B >> y$

Then

$$A = By$$

$$P = 2y + B \approx B$$

$$R = A/P \approx y$$

Manning's equation becomes

$$Q = (1.49/n)(A)(R)^{2/3}\sqrt{S_o}$$

$$= (1.49/n)(By)(y)^{2/3}\sqrt{S_o}$$

$$Q = (1.49/n)(B\sqrt{S_o})(y)^{5/3}$$

Setting

$$a = (1.49/n)(B\sqrt{S_o})$$

$$m = 5/3$$

Then

$$Q = ay^m$$

Storage is given by:

$$S = ByL$$

From continuity, we have:

$$\frac{I_1 + I_2}{2} - \frac{Q_1 + Q_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

4.17. (cont)

$$\frac{I_1 + I_2}{2} - \frac{ay_1^{m} + ay_2^{m}}{2} = \frac{By_2L - By_1L}{\Delta t}$$

Rearranging to solve for y_1, gives:

$$BLy_2 + (\Delta ta/2)y_2^{m} = BLy_1 - (\Delta ta/2)y_1^{m} + (\Delta t/2)(I_1 + I_2)$$

where all terms are known but y_2. We solve the right hand side at time step 1, then solve for y_2.

Q_2 is found from $Q_2 = ay_2^{m}$. This process is then repeated until the routing is complete.

4.18. How does the method in Problem 4.17 compare with the kinematic wave routing method described in this chapter? What hydraulic conditions are necessary for kinematic wave assumptions to be valid for overland flow? What are advantages and disadvantages of using kinematic channel routing compared with solving the Saint Venant equations?

The method of Problem 4.17 is similar to kinematic wave routing except that values are evaluated at either end of a channel reach instead of being evaluated in a grid along the channel. For overland flow, the kinematic wave number is greater than 10 for small depths and $Fr < 2$. Kinematic wave routing is easier than dynamic wave routing but may not be as accurate for cases of steep slope or for long rivers with backwater effects.

4.19. Equation (4.47) and Manning's equation (Eq. 4.48) can be combined to develop a second form of the kinematic wave equation (Eq. 4.78), but in terms of Q.

a) Prove that the equation

$$\frac{\partial Q}{\partial x} + a\beta Q^{\beta-1}\frac{\partial Q}{\partial t} = q$$

is another form of the kinematic wave equation.

b) What is the values of a and β from Manning's equation?

a) EQ. 4.47 $\Rightarrow \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q$

EQ. 4.48 $\Rightarrow Q = \alpha_c A^{m_c}$ where $m_c = B^{-1}$

EQ. 4.78 $\Rightarrow q = \frac{\partial A}{\partial t} + \alpha m_c A^{m_c - 1}\frac{\partial A}{\partial x} = q$

First, solve the given kinematic wave equation and show equivalent:

Method 1:

$$Q = \alpha_c A^{m_c}$$

$$\frac{\partial Q}{\partial x} = \frac{\partial Q}{\partial A}\frac{\partial A}{\partial t}$$

$$\frac{\partial Q}{\partial x} = \alpha_c m_c A^{m_c - 1}\frac{\partial A}{\partial x}$$

Substitute into EQ. 4.47:

$$q = \frac{\partial A}{\partial t} + \alpha_c m_c A^{m_c - 1}\frac{\partial A}{\partial x}$$

(EQ. 4.78)

Method 2:

$$Q = \alpha_c A^{m_c}$$

$A = \alpha Q^{\beta}$ Since $\beta = m_c^{-1}$, $\alpha = \alpha_c^{-\beta}$

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial Q}\frac{\partial Q}{\partial t}$$

$$= \alpha\beta Q^{\beta - 1}\frac{\partial Q}{\partial t}$$

Substitute into EQ. 4.47

$$q = \frac{\partial Q}{\partial x} + \alpha\beta Q^{\beta - 1}\frac{\partial Q}{\partial t}$$

(Equivalent Form)

4.19. (cont)

b) Solve Manning's Equation to the form:

$$Q = \alpha_c A^{\beta^{-1}}$$

Manning's:

$$= \frac{1}{n} A \left(\frac{A}{p} \right)^{2/3} \sqrt{S_o}$$

$$= \frac{\sqrt{S_o}}{np^{2/3}} \square A^{5/3}$$

$$Q = \alpha_c A^{\beta^{-1}}$$

$$\beta^{-1} = 5/3$$

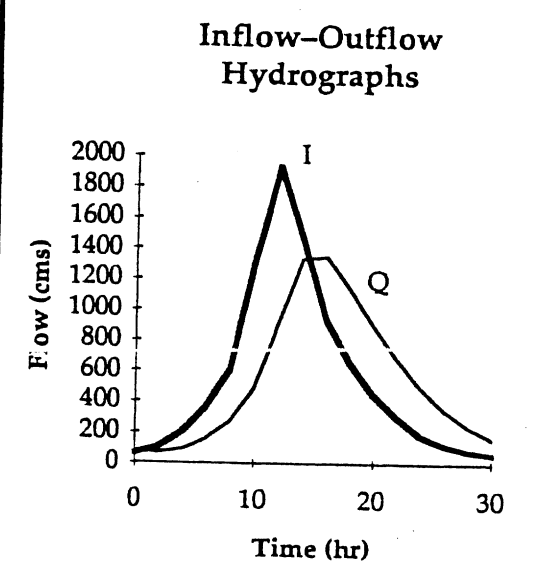
$$\beta = 3/5$$

4.20. Develop a spreadsheet for Muskingum routing using the given inflow hydrograph through the river reach, where $K = 4$ hr, $x = 0.15$, and $\Delta t = 2$ hr.

The following spreadsheet shows the equations used in computing the finalized solution directly below the first:

	A	B	C	D	E
1	Time	Inflow	Outflow		Coeff. Determination
2	0	60	60	K =	4
3	2	100	=$E$6*B3+$E$7*B2+$E$8*C2	x =	0.15
4	4	200	=$E$6*B4+$E$7*B3+$E$8*C3	Δt =	2
5	6	360	=$E$6*B5+$E$7*B4+$E$8*C4	D =	=E2-E2*E3+0.5*E4
6	8	600	=$E$6*B6+$E$7*B5+$E$8*C5	Co =	=(-E2*E3+0.5*E4)/E5
7	10	1310	=$E$6*B7+$E$7*B6+$E$8*C6	C1 =	=(E2*E3+0.5*E4)/E5
8	12	1930	=$E$6*B8+$E$7*B7+$E$8*C7	C2 =	=(E2-E2*E3-0.5*E4)/E5
9	14	1460	=$E$6*B9+$E$7*B8+$E$8*C8		
10	16	930	=$E$6*B10+$E$7*B9+$E$8*C9		
11	18	650	=$E$6*B11+$E$7*B10+$E$8*C10		
12	20	440	=$E$6*B12+$E$7*B11+$E$8*C11		
13	22	300	=$E$6*B13+$E$7*B12+$E$8*C12		
14	24	180	=$E$6*B14+$E$7*B13+$E$8*C13		
15	26	120	=$E$6*B15+$E$7*B14+$E$8*C14		
16	28	80	=$E$6*B16+$E$7*B15+$E$8*C15		
17	30	60	=$E$6*B17+$E$7*B16+$E$8*C16		

Time (hr)	Inflow (cms)	Outflow (cms)	Coeff. Determination	
0	60	60	K =	4 hr.
2	100	64	x =	0.15
4	200	89	Δt =	2 hr.
6	360	154	D =	4.40
8	600	270	Co =	0.09
10	1310	484	C1 =	0.36
12	1930	916	C2 =	0.55
14	1460	1334		
16	930	1343		
18	650	1130		
20	440	893		
22	300	674		
24	180	493		
26	120	345		
28	80	239		
30	60	165		



Note that Q_p lags I_p by approximately 4 hours, which is K.

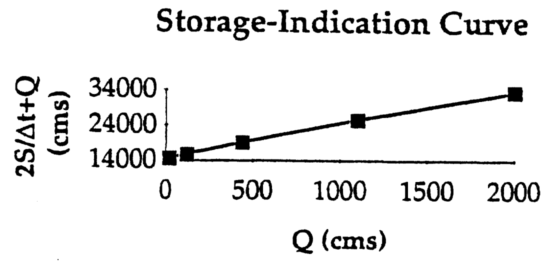
4.21. A reservoir has the storage-discharge relationship below.

Route the inflow hydrograph in problem 4.20 through the reservoir, assuming an initial storage of 52×10^6 m^3 of water.

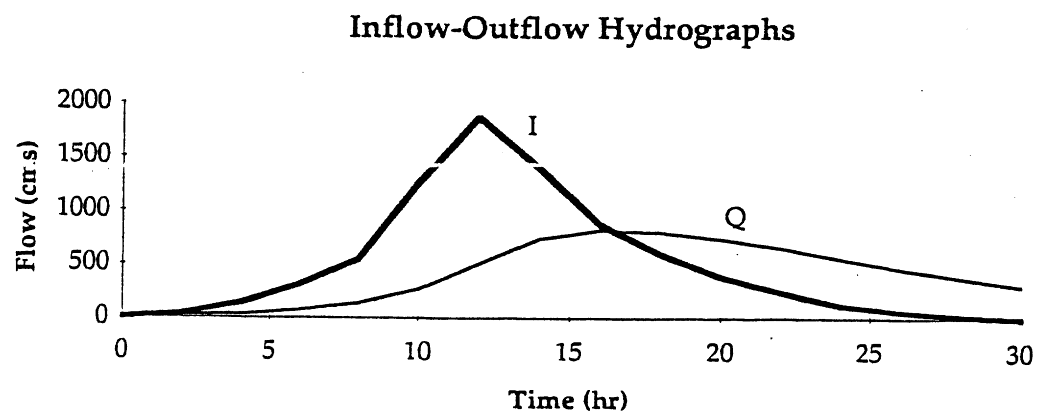
STORAGE (10^6 m^3)	DISCHARGE (m^3/s)
52	20
56	120
67.5	440
88	1100
113	2200

Using the Storage-Indication Method:

S X 10^6	Discharge (cms)	2S/Δt + Q (cms)
52	20	14464
56	120	15676
67.5	440	19190
88	1100	25544
113	2000	33389

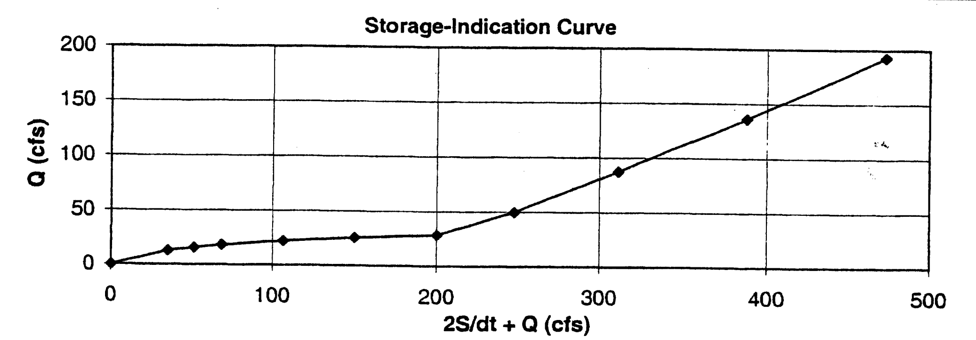


Time	In	In + I(n+1)	(2Sn/Δt - Qn)	(2Sn/Δt + Q(n+1))	Q(n)
0	60	160	14424		20
2	100	300	14534	14584	25
4	200	560	14734	14834	50
6	360	960	15134	15294	80
8	600	1910	15774	16094	160
10	1310	3240	17184	17684	250
12	1930	3390	19304	20424	560
14	1460	2390	21094	22694	800
16	930	1580	21754	23484	865
18	650	1090	21614	23334	860
20	440	740	21104	22704	800
22	300	480	20424	21844	710
24	180	300	19684	20904	610
26	120	200	19074	19984	455
28	80	140	18394	19274	440
30	60	60	17784	18534	375

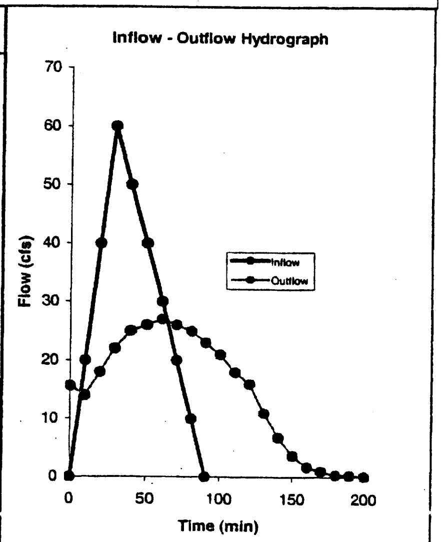


4.22. Create a spreadsheet using storage-indication routing for the storage-discharge relation and inflow hydrograph from Example 4.5. Rework the example, noting an initial depth in the reservoir of 1.5 ft.

h (ft)	Trap bottom (ft)	Trap top (ft)	Trap height (ft)	Trap area (ft^2)	increm vol (ft^3)	total storage (ft^3)	Q orifice (cfs)	Q wier (cfs)	Q total (cfs)	2S/dt + Q (cfs)
0	80	20	120	6000	0	0	0	0	0	0
1	88.9	28.9	128	7539.2	6769.6	6769.6	12.763138	0	12.7631	35.32847
1.5	93.4	33.4	132	8368.8	3977	10746.6	15.631588	0	15.6316	[illegible]
2	97.9	37.9	136	9234.4	4400.8	15147.4	18.049803	0	18.0498	68.54114
3	106.8	46.8	144	11059.2	10146.8	25294.2	22.106404	0	22.1064	106.4204
4	115.8	55.8	152	13041.6	12050.4	37344.6	25.526277	0	25.5263	150.0083
5	124.7	64.7	160	15152	14096.8	51441.4	28.539245	0	28.5392	200.0106
5.5	127	67	162	15714	7716.5	59157.9	29.932213	19.86077	49.793	246.986
6	129.2	69.2	164	16268.8	7995.7	67153.6	31.263176	56.17473	87.4379	311.2832
6.5	131.4	71.4	166	16832.4	8275.3	75428.9	32.539746	103.1996	135.739	387.169
7	133.7	73.7	168	17421.6	8563.5	83992.4	33.76809	158.8861	192.654	472.6289



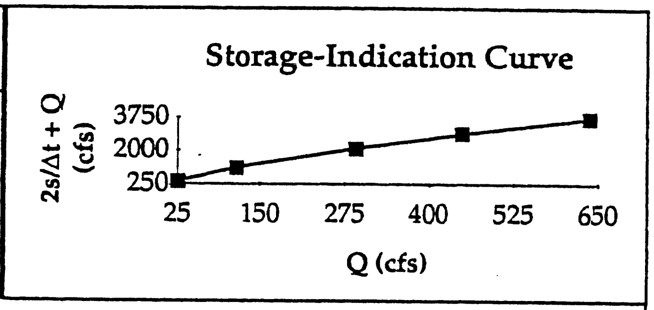
TIME (min)	In (cfs)	In + In+1 (cfs)	2Sn/dt - Qn (cfs)	2Sn+1/dt + Qn-1 (cfs)	Qn
0	0	20	20		16
10	20	60	12	40	14
20	40	100	12	72	18
30	60	110	36	112	22
40	50	90	68	146	25
50	40	70	96	158	26
60	30	50	106	166	27
70	20	30	112	156	26
80	10	10	104	142	25
90	0	0	92	114	23
100	0	0	68	92	21
110	0	0	50	68	18
120	0	0	32	50	16
130	0	0	18	32	11
140	0	0	10	18	7
150	0	0	5	10.19	4
160	0	0	3	4.590	2
170	0	0	1	2.790	1.0
180	0	0	1	1.190	0.4
190	0	0	0	0.790	0.3
200	0	0	0	0.390	0.1



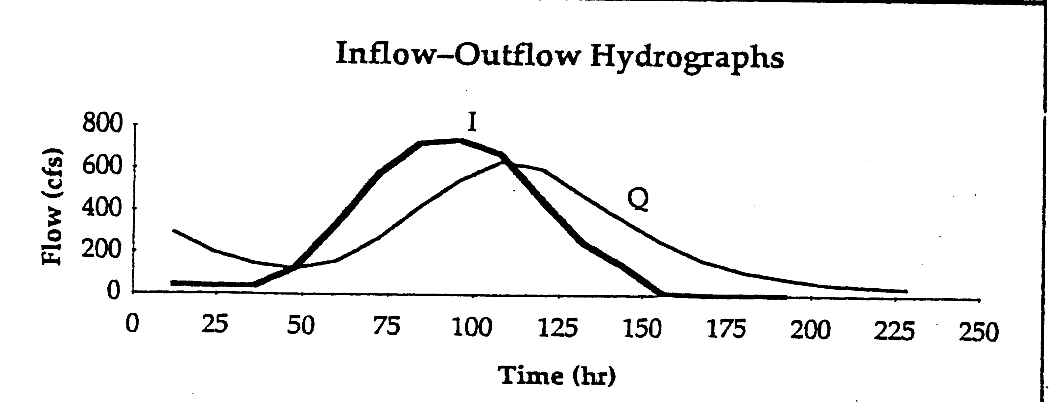
4.23. Repeat problem 4.4 for the case that the reservoir is partially full of water with initial height above the weir of $y = 3$ ft. Assume vertical walls (i.e., constant surface area for all depths).

Referring to Problem 4.4, $Q = 3.75 * L * y^{3/2}$ and $S = 300 * y$ where $L = 15$ ft. and $y_0 = 3\,ft.$ For Storage-Indication, $\Delta t = 12$ hours.

S (ac-ft)	Discharge (cfs)	2S/Δt + Q (cfs)
180	26	386
480	114	1074
900	292 ✓	2092 ✓
1200	450	2850
1512	637	3666



Time	In	In + I(n+1)	(2Sn/Δt - Qn)	(2Sn/Δt + Q(n+1))	Q(n)
12	40	75 +	1508 →		292 ← INITIAL OUTFLOW
24	35	72 +	1187 →	1583	198
36	37	162	971	1259	144
48	125	465	887	1133	123
60	340	915	1036	1352	158
72	575	1297	1419	1951	266
84	722	1462	1876	2716	420
96	740	1413	2238	3338	550
108	673	1129	2377	3651	637
120	456	706	2302	3506	602
132	250	390	2058	3008	475
144	140	150	1724	2448	362
156	10	10	1370	1874	252
168	0	0	1052	1380	164
180	0	0	830	1052	111
192	0	0	670	830	80
204	0	0	556	670	57
216	0	0	464	556	46
228	0	0	388	464	38



4.24. Repeat problem 4.7 for the case where the reservoir has variable area (and therefore a variable storage-depth relation) and is at an initial depth of h = 12 m. A detailed topographic analysis has indicated the relationship between depth, flow rate, and surface area in the reservoir. Note that the surface area of the reservoir changes with depth according to the table below.

To perform Runga-Kutta routing to this reservoir, we must first develop a head-discharge and a head-area relationship. Based on $Q = 3.75Ly^{3/2}$ and $A = 200000h^{0.7}$ where L= 10m, y = (h - 10)m and A in m^2.

When prompted by the Runga-Kutta Program in Appendix E, enter these values. The output is as follows:

h (m)	Q (cms)	A (m^2)
0.00	0.00	0
5.00	0.00	617034
10.00	0.00	1002374
11.00	37.50	1071531
12.00	106.07	1138825
13.00	194.86	1204454
14.00	300.00	1268585
15.00	419.26	1331355

Time (hr)	Inflow (cms)	Head (m)	Area (sq m)	Δh (m)	Outflow (cms)
0	0	12.00	1138825	0.00	106.07
1	200	11.99	1138355	-0.01	105.59
2	300	12.39	1164667	0.40	141.03
3	500	13.09	1210410	0.70	204.63
4	450	13.77	1253787	0.68	275.74
5	400	14.13	1276861	0.36	315.72
6	300	14.21	1281581	0.08	324.69
7	200	14.02	1269920	-0.19	302.54
8	100	13.64	1245400	-0.38	261.99
9	50	13.16	1214969	-0.47	212.10
10	0	12.67	1183041	-0.49	168.89
11	0	12.23	1153679	-0.45	126.17
12	0	11.88	1130618	-0.35	97.71
13	0	11.60	1111663	-0.28	78.39
14	0	11.37	1096245	-0.23	62.68
15	0	11.18	1083776	-0.19	49.98
16	0	11.03	1073743	-0.15	39.75
17	0	10.91	1065298	-0.12	34.12
18	0	10.80	1057783	-0.11	30.05
19	0	10.70	1051124	-0.10	26.43
20	0	10.62	1045233	-0.09	23.24

4.25. Develop a spreadsheet for a 4-km river reach with the upstream hydrograph given below. Using a subreach length Δx of 2 km, determine the hydrograph at the end of the reach according to the Muskingum-Cunge method. The channel is trapezoidal (2:1 side slope) with bottom width of 10 m. Assume $S_0 = 0.001$, $\Delta t = 30$ min, and $c = 1.47$ m/s.

TIME (min)	FLOW (m^3/s)
0	0
30	4
60	14
90	27
120	30
150	29
180	27
210	24
240	18
270	12
300	8
330	5
360	3
390	1
420	0

Given: $L = 4000m$, $S_0 = 0.001$, $\Delta x = 2000m$, $\Delta t = 1800s$, $c = 1.47m/s$

Using: $$Q = \left[\frac{\left(10 + 2y^2\right)^{5/3}}{\left(10 + 2\sqrt{5}y\right)^{2/3}} \right] * 0.79,$$

Iterate to find "y" knowing $Q_p = 30cms$

Depth is found to be $y = 2.05m$

4.25. (cont) Top width is $w = 18.2m$

We now find the intermediate values:

$$D1 = \frac{Q_p}{2S_0 w} = \frac{30cms}{2 \cdot 0.001 \cdot 18.2m} = 824.2 \, {m^2}/{s}$$

$$x = \frac{1}{2} - \left(\frac{D1}{c\Delta x}\right) = 0.5 - \left(\frac{824.2 \, {m^2}/{s}}{1.47 \, {m}/{s} \cdot 2000m}\right) = 0.2197$$

$$K = \frac{\Delta x}{c} = \frac{2000m}{1.47 \, {m}/{s}} = 1361s$$

$$D = K(1-x) + \frac{1}{2}\Delta t = 1361(1-0.2197) + \frac{1}{2}(1800) = 1962s$$

Using these values, calculate the Cunge coefficients:

$$C1 = (Kx + 0.5\Delta t)/D = (1361 \cdot 0.2197 + 0.5 \cdot 1800)/1962 = 0.6111$$

$$C2 = \left(\frac{1}{2}t - Kx\right)/D = (0.5 \cdot 1800 - 1361 \cdot 0.2197)/1962 = 0.3064$$

$$C3 = \left(K(1-x) - \frac{1}{2}\Delta t\right)/D = ((1361 \cdot 0.7803) - 0.5 \cdot 1800)/1962 = 0.0824$$

$C4 = 0$ (No lateral flow) $\qquad C1 + C2 + C3 = 1 \Leftrightarrow check$

Then:

$$Q_{j+1}^{n+1} = C_1 Q_j^n + C_2 Q_j^{n+1} + C_3 Q_{j+1}^n + C_4$$

Note: For $t = 0$ min. at any j, $Q = 0$ cms

For $n = 0, j = 0$

$$Q_1^1 = C_1 Q_0^0 + C_2 Q_0^1 + C_3 Q_1^0$$

$$= 0.6111 \cdot 0 + 0.3064 \cdot 4 + 0.0824 \cdot 0 = 1.23cms$$

For $n = 1, j = 0$;

$$Q_1^2 = C_1 Q_0^1 + C_2 Q_1^2 + C_3 Q_1^1$$

$$Q_1^2 = 0.6111 \cdot 4 + 0.3064 \cdot 14 + 0.0824 \cdot 1.23 = 6.84cms$$

4.25. (cont)

For $n = 0, j = 1$

$$Q_2^1 = C_1 Q_1^0 + C_2 Q_1^1 + C_3 Q_2^0$$

$$Q_2^1 = 0.6111 \cdot 0 + 0.3064 \cdot 1.23 + 0.0824 \cdot 0 = 0.38 cms$$

For $n = 1, j = 1$

$$Q_2^2 = C_1 Q_1^1 + C_2 Q_1^2 + C_3 Q_2^1$$

$$Q_2^2 = 0.6111 \cdot 1.23 + 0.3064 \cdot 6.84 + 0.0824 \cdot 0.38 = 2.87 cms$$

The process is continued through $j = 2$, $n = 15$ to obtain the values in the following table. Note that $\Delta t = 30$ min. and $\Delta x = 2$ *km*.

The hydrographs are summarized and graphed below:

Time (min)	Flow (cms)	(n+1)Δt (min)	Δx 0 km	Δx 2 km	Δx 4 km
0	0.0	0	0.0	0.00	0.00
30	4.0	30	4.0	1.23	0.38
60	14.0	60	14.0	6.84	2.87
90	27.0	90	27.0	17.39	9.74
120	30.0	120	30.0	27.13	19.75
150	29.0	150	29.0	29.46	27.23
180	27.0	180	27.0	28.42	28.96
210	24.0	210	24.0	26.20	27.79
240	18.0	240	18.0	22.34	25.15
270	12.0	270	12.0	16.52	20.79
300	8.0	300	8.0	11.15	15.22
330	5.0	330	5.0	7.34	10.32
360	3.0	360	3.0	4.58	6.74
390	1.0	390	1.0	2.52	4.13
420	0.0	420	0.0	0.82	2.13
450	0.0	450	0.0	0.07	0.70
480	0.0	480	0.0	0.01	0.10

Hydrograph Comparison