Solutions Chapter 8

8.1. Compute the Darcy velocity and seepage velocity for water flowing through a sand column with the following characteristics:

$K = 10^{-4}$ cm/s,

$dh/dl = 0.01$,

Area $= 75$ cm^2,

$n = 0.20$.

Use Darcy's Law:

$$v = -K\frac{dh}{dl}$$

$$= \left(10^{-4} cm / s\right)\left(0.01\right)$$

$$v = 10^{-6} cm / s$$

Then, the seepage velocity is

$$v_s = v / n$$

$$= \left(10^{-6} cm / s\right) / \left(0.20\right)$$

8.2. The average water table elevation has dropped 5 ft due to the removal of 100,000 ac-ft from an unconfined aquifer over an area of 75 mi^2. Determine the storage coefficient for the aquifer.

By definition:

$S = \Delta V / (\Delta h)(\text{unit vol.})(\text{unit area})$

$= (100000 \text{ac-ft}) / (5\text{ft})(75\text{mi}^2)(640\text{ac/mi}^2)$

$\boxed{S = 0.42}$

8.3. A confined aquifer is 50 m thick and 0.5 km wide. Two observation wells are located 1.4 km apart in the direction of flow. Head in well 1 is 50.0 m and in well 2 it is 42 m. Hydraulic conductivity K is 0.7 m/day.

a) What is the total daily flow of water through the aquifer?

b) What is the height z of the piezometric surface 0.5 km from well 1 and 0.9 km from well 2?

a) $Q = -KA\frac{dh}{dl}$

= (-0.7m/day)(50m)(0.5km)(42m-50m) / 1.4km

Q = 100m^3/day

b) Since this is a confined aquifer, the piezometric surface is linear.

$$\frac{z_1 - z}{z_1 - z_2} = \frac{x_1 - x}{x_1 - x_2}$$

$$\frac{50 - z}{50 - 42} = \frac{0.5}{1.4}$$

$$z = 47.1m$$

8.4. A well with a diameter of 18 in. penetrates an unconfined aquifer that is 100 ft thick. Two observation wells are located at 100 ft and 235 ft from the well, and the measured drawdowns are 22.2 ft and 21 ft, respectively. Flow is steady and the hydraulic conductivity is 1320 gpd/ft^2. What is the steady-state rate of discharge from the well?

Steady radial flow to an unconfined aquifer is governed by Eq. 8.42.

$$h_1 = h - s_1^1 = 100ft - 22.2ft = 77.8ft$$

$$h_2 = h - s_2^1 = 100ft - 21ft = 79ft$$

$$Q = \pi K\left(h_2^2 - h_1^2\right) / \ell n\left(r_2 / r_1\right)$$

$$= (\pi)\left(1320 gpd / ft^2\right)\left[(79ft)^2 - (77.8ft)^2\right] / \ell n(235ft / 100ft)$$

$$Q = 0.91 mgd = 1.41 cfs$$

8.5. Two piezometers are located 1000 ft apart with the bottom located at depths of 50 ft and 350 ft, respectively, in a 400-ft-thick unconfined aquifer. The depth to the water table is 50 ft in the deeper piezometer and 40 ft in the shallow one. Assume that hydraulic conductivity is 0.0002 ft/s.

a) Use the Dupuit equation to calculate the height of the water table midway between the piezometers.

b) Find the flow rate per unit thickness for a section midway between the wells.

a) From Example 8.2, we have:

$$h^2 = h_0^2 - \left(h_0^2 - h_L^2\right)x/L$$

If we choose the head in the shallow piezometer as h_0 and the head in the deeper piezometer as h_L, we get:

$$h_0 = 400 - 40 = 360\,ft$$

$$h_L = 400 - 50 = 350\,ft$$

$$h^2 = (360\,ft)^2 - \left[(360\,ft)^2 - (350\,ft)^2\right](500\,ft/1000\,ft)$$

$$h^2 = 126{,}050\,ft^2$$

$$h = 355.04\,ft$$

b) Again, from Example 8.2 we have:

$$q = K\left(h_0^2 - h_L^2\right)/2L$$

$$= (0.0002\,ft/s)\left[(360\,ft)^2 - (350\,ft)^2\right]/(2)(1000\,ft)$$

$$q = 7.10\times10^{-4}\,cfs/ft$$

8.6. In a fully penetrating well, the equilibrium drawdown is 30 ft measured at r = 100 ft from the well, which pumps at a rate of 20 gpm. The aquifer is unconfined with K = 20 ft/day, and the saturated thickness is 100 ft. What is the steady-state drawdown at the well (r = 0.5 ft) for this aquifer?

From Eq. 8.42

$$Q = \pi K \frac{h_2^2 - h_1^2}{\ln\left(\frac{r_2}{r_1}\right)} \qquad h_1 = b - s_1^1 = 100ft - 30ft$$

$$r_1 = 100ft$$

$$(20gal/\min)\left(\frac{192.5ft^3/day}{1gal/\min}\right) = \pi(20ft/day)\frac{h_2^2 - (100ft - 30ft)^2}{\ln(-5ft/100ft)}$$

$$h_2 = 67.64ft$$

So the drawdown is 100ft – 67.64ft so $\boxed{s^1 = 32.36ft}$ when r = 0.5ft

8.7. A soil sample 6 in. in diameter and 1 ft long is placed in a falling head permeameter. The falling-head tube diameter is 1 in. and the initial head is 6 in. The head falls 1 in. over a 2-hr period. Calculate the hydraulic conductivity.

Using Eq. 8.10, we have:

$$K = \frac{r^2 L}{r_c^2 t} \ln\left(\frac{h_1}{h_2}\right)$$

$$= \frac{(0.5in)^2 (1ft)}{(3in)^2 (2hr)} \ln\left(\frac{6in}{5in}\right)$$

$$= \left(2.53 \times 10^{-3} ft / hr\right)\left(24hr / day\right)$$

$$K = 6.08 \times 10^{-2} ft / day$$

8.8. A constant head permeameter containing very fine-grained sand has a length of 12 cm and a cross-sectional area of 30 cm^2. With a head of 10 cm, a total of 100 ml of water is collected in 25 min. Find the hydraulic conductivity.

Equation 8.7 gives:

$$K = VL/(Ath)$$

$$= (100ml)(12cm)/(30cm^2)(25\min)(10cm)(1ml/1cm^3)$$

$$= (0.16cm/\min)(60s/\min)$$

$$K = 2.7 \times 10^{-3} cm/s$$

8.9. Bull Creek flows through and completely penetrates a confined aquifer 10 ft thick, as shown in Fig. P8.9. The flow is reduced in the stream by 16 cfs between two gaging stations located 4 miles apart along the creek. On the west side of the creek, the piezometric contours parallel the bank and slope toward Bull Creek at S = 0.0004 ft/ft. The piezometric contours on the east side of the creek slope away from the channel at a slope of 0.0006 ft/ft. Using Darcy's law and the continuity equation, compute the transmissivity of the aquifer along this section of Bull Creek. Note the flow area to the creek is 10 ft thick and 4 miles wide.

From continuity, we have

$$Q_{in} - Q_{out} = \Delta Q$$

Due to the slope of the piezometric surface, we assume the flow into the stream from the aquifer occurs on the west and flow from the stream into the aquifer occurs on the east, or:

$$Q_{in} = KA\left(\frac{dh}{dl}\right)w$$

$$Q_{out} = KA\left(\frac{dh}{dl}\right)e$$

Thus,

$$KA\left(\frac{dh}{dl}\right)w - KA\left(\frac{dh}{dl}\right)e = -16cfs$$

$$K(4mi)(10ft)(0.0004ft/ft - 0.0006ft/ft)(5280ft/mi) = -16cfs$$

$$K = 3.79 \times 10^{-1} ft/s$$

$$T = Kb$$

$$= (3.79 \times 10^{-1} ft/s)(10ft)$$

$$T = 3.79 ft^2/s$$

8.10. Three geologic formations overlie one another with the characteristics listed below. A constant-velocity vertical flow field exists across the three formations. The hydraulic head is 75 ft at the top of the formations and 59 ft at the bottom, with a datum located at the bottom of the three units. Calculate the hydraulic head at the two internal boundaries.

$b_1 = 20$ ft $K_1 = 20$ ft/day

$b_2 = 10$ ft $K_2 = 0.20$ ft/day

$b_3 = 25$ ft $K_3 = 0.030$ ft/day

First we need to find the equivalent value for K of the system.

$$K_2 = \frac{\Sigma z_i}{\Sigma \frac{z_i}{K_i}} = \frac{(20ft + 10ft + 25ft)}{\frac{20ft}{20ft/day} + \frac{10ft}{0.2ft/day} + \frac{25ft}{0.3ft/day}} = 0.6219 ft/day = K_2$$

Next, find the velocity through the entire system.

$$V = -K_2 \frac{dh}{dl} = -(0.06219 ft/day)\left(\frac{59ft - 75ft}{20ft + 10ft + 25ft}\right) = 0.01809 ft/day = V$$

Then use this velocity to find the change in head across each layer.

$$V = -K\frac{dh}{dl} = -K\frac{(h_2 - h_1)}{b}$$

$$h_2 = \frac{-vb}{K} + h_1$$

So at the bottom of layer 1, we get:

$$h_1 = 75ft - \frac{(0.0189ft/day)(20ft)}{20ft/day} = 74.98ft = h_1$$

At the bottom of the second layer, we get:

$$h_2 = 74.98ft - \frac{(0.0189ft/day)(10ft)}{0.2ft/day} = 74.08ft = h_2$$

To check out calculations, we continue:

$$h_3 = 74.08ft - \frac{(0.01809ft/day)(25ft)}{0.03ft/day} = 59.0ft = h_3$$

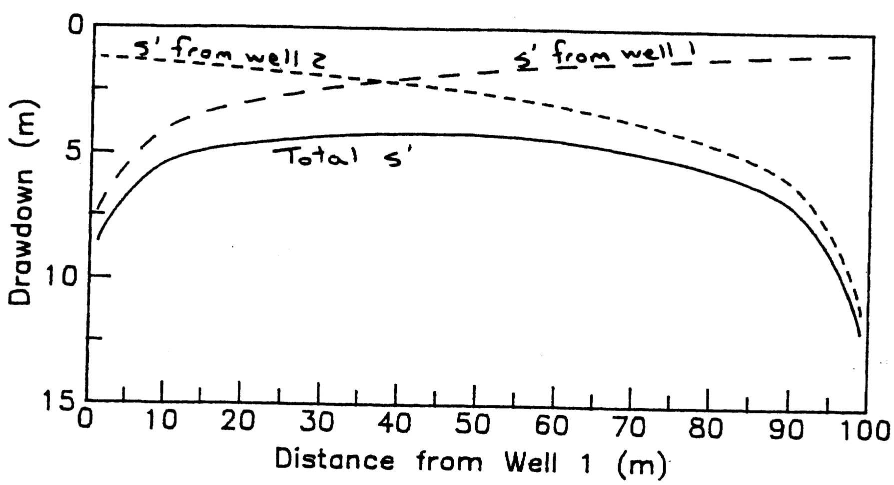
8.11. Two wells are located 100 m apart in a confined aquifer with a transmissivity $T = 2 \cdot 10^{-4}$ m^2/s and storativity $S = 7 \cdot 10^{-5}$. One well to the west is pumped at a rate of 6.6 m^3/hr and the other to the east at a rate of 10.0 m^3/hr. Plot drawdown as a function of distance along the line joining the wells at 1 hr after the pumping starts.

The Thesis equations (Eqs. 8.49 and 8.50) are used to develop values of s^1 for values of r for each well. These values are superimposed to obtain the drawdown curve, where r is the distance from well 1.

r (m)	si (m)
1	7.35
10	3.98
20	2.97
30	2.39
40	1.98
50	1.67
60	1.42
70	1.25
80	1.08
90	0.91
99	0.80

r (m)	si (m)
99	11.13
90	6.03
80	4.50
70	3.63
60	3.00
50	2.53
40	2.16
30	1.89
20	1.64
10	1.37
1	1.21

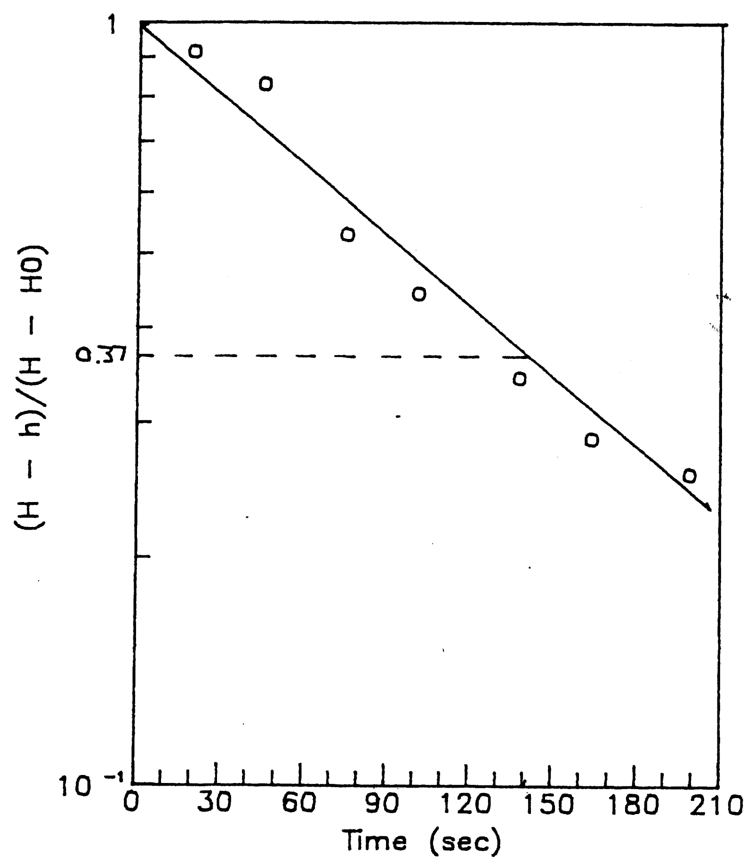
Thus, the drawdown curve is:



8.12. At a waste site, a Hvorslev slug test was performed in a confined aquifer with a piezometer intake length of 20 ft and a radius of 1 in. The radius of the rod was 0.68 in. The following recovery data for the well were observed. Given that the static water level is 7.58 ft and H_0 = 6.88 ft, calculate the hydraulic conductivity.

TIME (*s*)	20	45	75	101	138	164	199
h (ft)	6.94	7.00	7.21	7.27	7.34	7.38	7.40

First we plot values of $(H - h / H - H_0)$ vs. t on semilog paper.



A straight line is drawn as a "best fit" to the data. The value T_0 at $(H - h / H - H_0) = 0.37$ is read as 141 sec.

From Eq. 8.61, we get:

$$K = r^2 \ln(L / R) / 2LT$$

$$= (lin)^2 \ln\left(20ft / \tfrac{1}{12} ft\right) / (2)(20ft)(1415)(12in / ft)$$

$$K = 8.10 \times 10^{-5} in / s$$

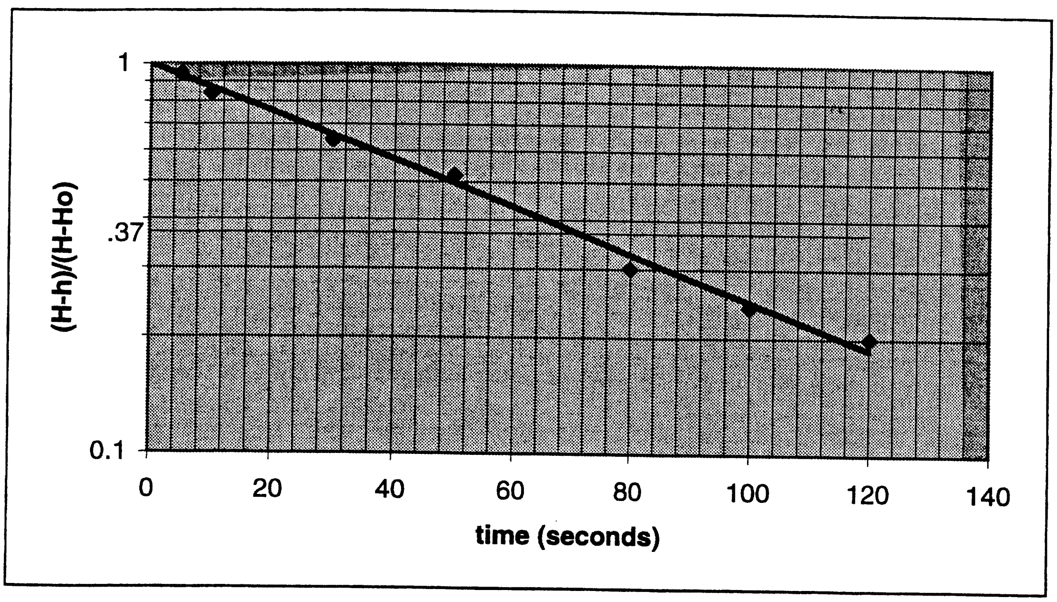
OR

$$K = 6.75 \times 10^{-6} ft / s$$

8.13. A well casing with a radius of 2 in. is installed through a confining layer into a formation with a thickness of 10 ft. A screen with a radius of 2 in. is installed in the casing. A slug of water is injected, raising the water level by 0.5 ft initially. Given the following recorded data for head decline, find the values of T and K for this aquifer, using Hvorslev analysis. Assume $H_0 = 0.0$ and $H = 0.5$ ft after injection. Assume $L = 15$ ft.

First, we plot the values of $(H - h / H - H_0)$ vs. t on semilog paper:

TIME (s)	5	10	30	50	80	100	120
h (ft)	0.47	0.42	0.32	0.26	0.15	0.12	0.10
$(H-h)/(H-h_0)$	0.94	0.84	0.64	0.52	0.30	0.24	0.20



A straight line is drawn as a "best fit" to the data. The value T_0 at $(H - h / H - H_0) = 0.37$ is read as 71 seconds.

From equation 8.61, we get:

$$K = r^2 \ln(L / R) / 2LT_0$$

$$= (2in)^2 \ln\left(15ft \times \frac{12in}{1ft} / 2in\right) / 2\left(15ft \times \frac{12in}{1ft}\right)(71s)$$

$$7.04 \times 10^{-4} in / s$$

OR

$$K = 5.87 \times 10^{-5} ft / s$$

$$T = Kb$$

$$= (5.87 \times 10^{-5} ft / s)(10 ft)$$

$$T = (5.87 \times 10^{-4} ft / s)$$

8.14. A small municipal well was pumped for 2 hr at a rate of 15.75 liters/s (0.556 cfs). An observation well was located 50 ft from the pumping well and the following data were recorded. Using the Theis method outlined in Example 8.6, compute *T* and *S*.

t (min)	0	1	2	3	5	7	9
s' (ft)	0	1.5	4.0	6.2	8.5	10.0	12.0
t (min)	12	15	20	40	60	90	120
s' (ft)	13.7	14.9	17.0	21.7	23.1	26.0	28.0

Values of s^1 vs. r^2/t are plotted on log-log paper at the same scale as a plot of w(u) vs. u. A convenient match point is found and the following values are read

$$r^2/t = 100 ft^2/\min$$

$$s^1 = 10.75 ft$$

$$u = 4\times 10^{-2}$$

$$w(u) = 2.7$$

Then, $T = Qw(u)/4\pi s^1$

$$= (15.75 l/s)(1 ft^3/28.317 l)(2.7)/(4)(\pi)(10.75 ft)$$

$$T = 1.11\times 10^{-2} ft^2/s$$

and $S = 4Tu/(r^2/t)$

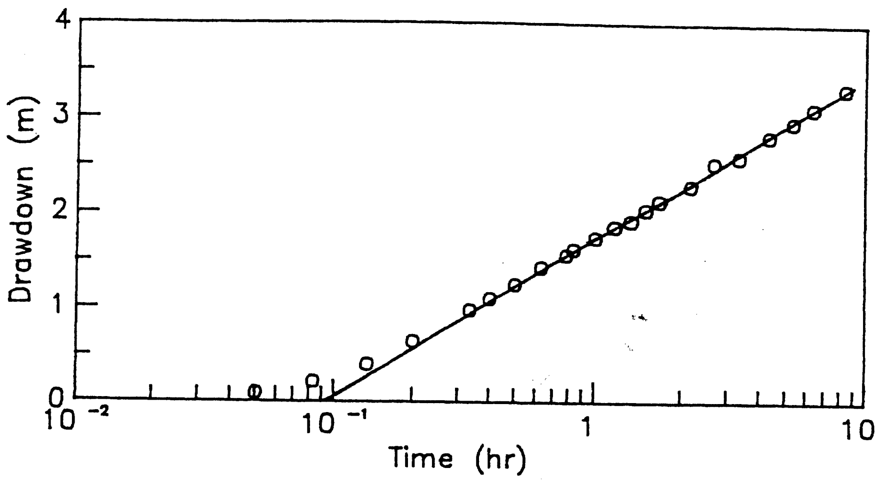
$$= (4)(1.11\times 10^{-2} ft^2/s)(4\times 10^{-2})/(100 ft^2/\min)(1\min/60s)$$

$$S = 1.07\times 10^{-3}$$

8.15. A well in a confined aquifer is pumped at a rate of 833 liters/min (1199.5 m^3/day) for a period of over 8 hr. Time-drawdown data for an observation well located 250 m away are given below. The aquifer is 5 m thick. Use the Cooper-Jacob method to find values of *T*, *K*, and *S* for this aquifer.

TIME (hr)	DRAWDOWN (m)
0.050	0.091
0.083	0.214
0.133	0.397
0.200	0.640
0.333	0.976
0.400	1.100
0.500	1.250
0.630	1.430
0.780	1.560
0.830	1.620
1.000	1.740
1.170	1.860
1.333	1.920
1.500	2.040
1.670	2.130
2.170	2.290
2.670	2.530
3.333	2.590
4.333	2.810
5.333	2.960
6.333	3.110
8.333	3.320

First, we plot s^1 vs. t on a semilog scale:



A straight line is fit through the later data to obtain a value t_0 for $s^1 = 0$. Δs^1 is measured over 1 log cycle of t.

$$t_0 = 0.09 hr$$

$$\Delta s^1 = 1.74m - 0.10m = 1.64m$$

Using Eqs. 8.57 and 8.58, we get:

$$T = 2.3Q / 4\pi\Delta s^1$$

$$= 2.3(833l / \min) / (4)(\pi)(1.64m)(1l / 0.001m^3)$$

$$= (9.3 \times 10^{-2} m^2 / \min)(60 \min/ hr)(24hr / day)$$

$$\boxed{T = 134m^2 / day}$$

$$K = T / b$$

$$= (134m^2 / day) / (5m)$$

$$\boxed{K = 26.8m / day}$$

$$S = 2.25Tt_0 / r^2$$

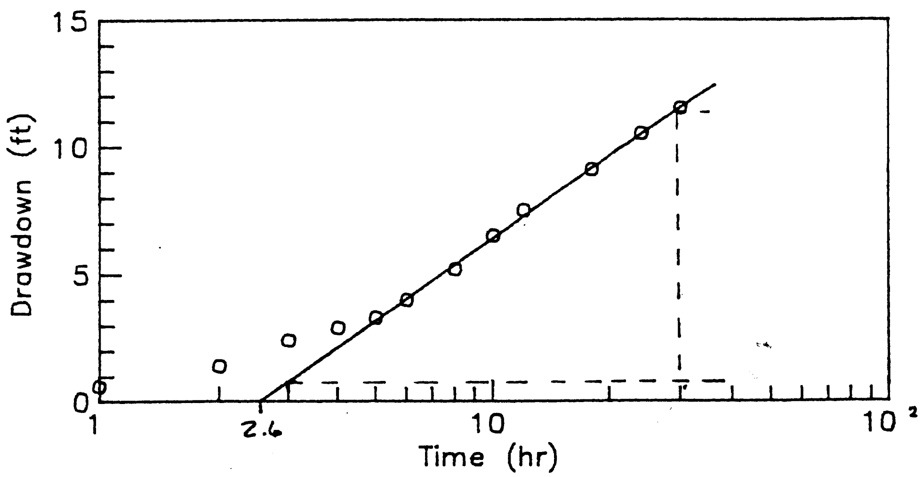
$$= \left[(2.25)\left(9.3\times10^{-2} m^2 / \min\right)(0.09hr)/(250m)^2\right](60\min/hr)$$

$$\boxed{S = 3.01\times10^{-7}}$$

8.16. Drawdown was observed in a well located 100 ft from a pumping well that was pumped at a rate of 1.11 cfs (498 gpm) for a 30-hr period. Use the Cooper-Jacob method to compute *T* and *S* for this aquifer.

TIME (hr)	1	2	3	4	5	6	8	10	12	18	24	30
DRAWDOWN (ft)	0.6	1.4	2.4	2.9	3.3	4.0	5.2	6.5	7.5	9.1	10.5	11.5

Values of s^1 vs. t are plotted to obtain values for t_0 and Δs^1:



$$t_0 = 2.6hr$$

$$\Delta s^1 = 9.20 ft$$

$$T = 2.3Q / 4\pi\Delta s^1$$

$$= (2.3)(1.11cfs) / (4)(\pi)(9.20 ft)$$

$$= (2.21 \times 10^{-2} ft^2 / s)(60s / \min)(60\min / hr)(24hr / day)$$

$$\boxed{T = 1908 ft^2 / day}$$

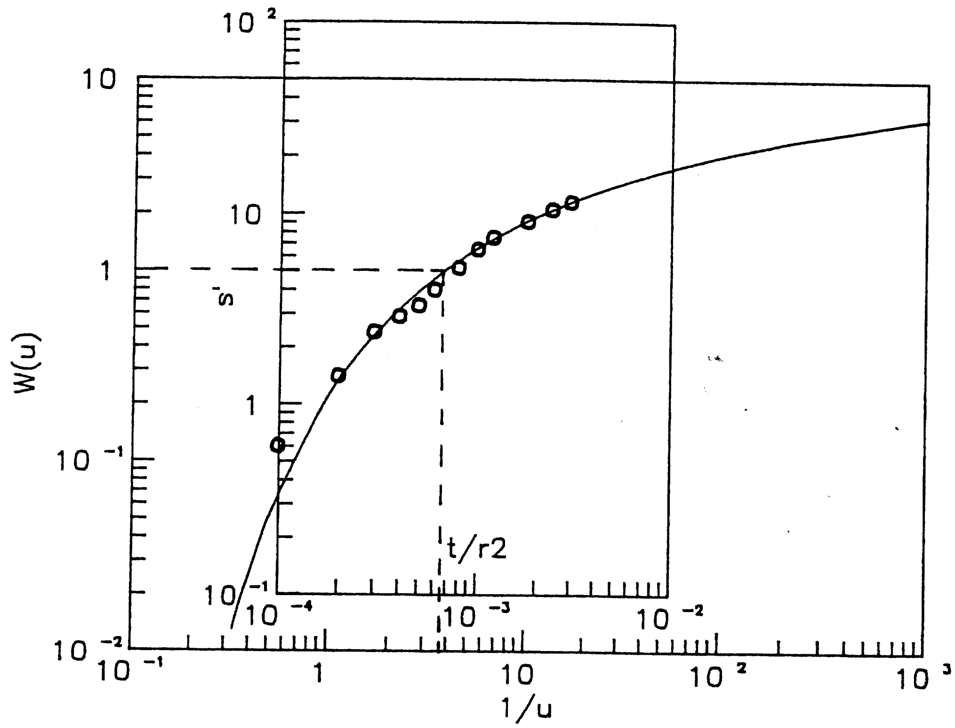
$S = 2.25Tt_0 / r^2$

$= (2.25)(1908\,ft^2 / day)(2,6hr)(1day / 24hr) / (100\,ft)^2$

$\boxed{S = 4.65 \times 10^{-2}}$

8.17. Repeat problem 8.16 using the Theis method.

Values of s^1 vs. t/r^2 are plotted and fit to a plot of w(u) vs. 1/u:



A match point is chosen and values obtained for:

$$w(u) = 1$$

$$s^1 = 5\,ft$$

$$t / r^2 = 6.7 \times 10^{-4} hr / ft^2$$

$$1 / u = 3.8$$

Then,

$$T = QW(u) / 4\pi s^1$$

$$= (1.11 cfs)(1) / (4)(\pi)(5 ft)$$

$$= (1.77 \times 10^{-2} ft^2 / s)(60 s / \min)(60 \min / hr)(24 hr / day)$$

$$\boxed{T = 1526 ft^2 / day}$$

$$S = 4T(t / r^2) / (1 / u)$$

$$= (4)(1526 ft^2 / day)(1 day / 24 hr)(6.7 \times 10^{-4} hr / ft^2) / 3.8$$

$$\boxed{S = 4.49 \times 10^{-2}}$$

8.18. Refer to Fig. 8.9b, which shows a flow net under a dam section. Note the values of head for the two sides of the dam, and compute the seepage or flow rate through the dam if the dam is 120 ft long with $K =$ 20 ft/day.

From Figure 8.9b, note that n = 17, m = 5, and H = 12ft. Eq. 8.19 says that:

$$Q^1 = \frac{KMH}{h}$$

$$= \frac{(20 ft / day)(5)(12 ft)}{17}$$

$$= 70.59 ft^2 / day$$

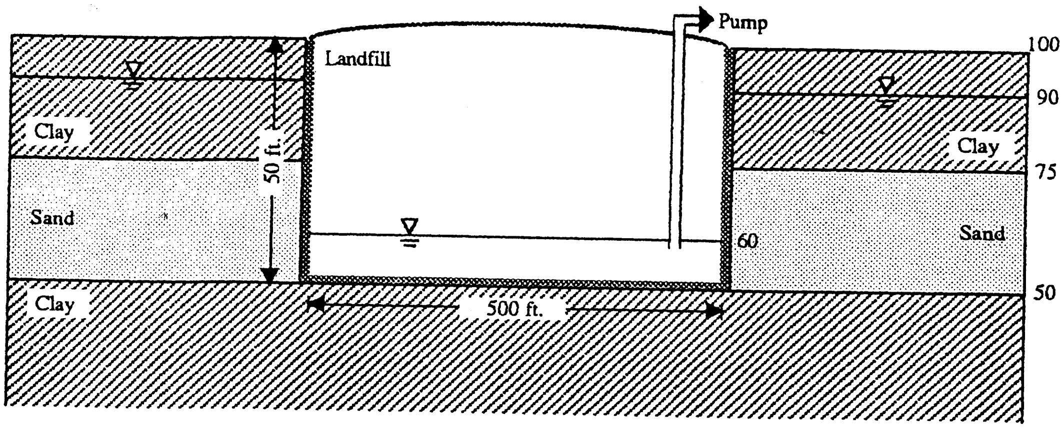
So,

$$Q = (70.59 ft^2 / day)(120 ft)$$

$$\boxed{Q = 8,470.6 ft^3 / day}$$

8.19 A landfill liner is laid at elevation 50 ft msl (mean sea level) on top of a good clay unit. A clean sand unit extends from elevation 50 ft to elevation 75 ft, and another clay unit extends up to the surface, located at elevation 100 ft. The landfill can be represented by a square with length of 500 ft on a side and a vertical depth of 50 ft from the surface. The landfill has a 3-ft-thick clay liner with $K = 10^{-7}$ cm/s around the sides and bottom, as shown in Fig. P8.19. The regional ground water level for the confined sand ($K = 10^{-2}$ cm/s) is located at a depth of 10 ft below the surface, or elevation = 90 ft.

How much water will have to be continuously pumped from the landfill to keep the potentiometric surface at 60 ft elevation (msl) within the landfill? Assume mostly horizontal flow through the clay liner.



$$Q = qA = K\frac{\Delta h}{\Delta l}A$$

$$K_{CLAY} = 10^{-7}\,cm/s$$

$$K_{SAND} = 10^{-2}\,cm/s$$

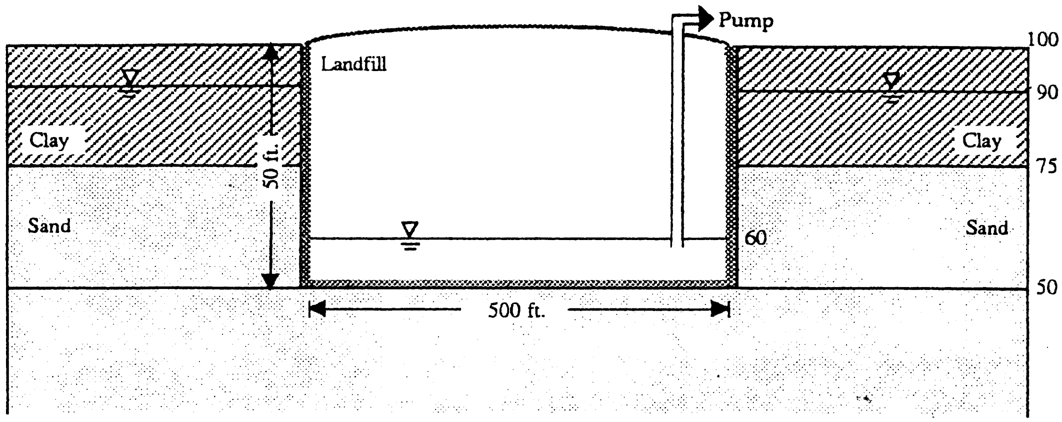
Compute flow into landfill = flow to be pumped:

$$Q = \left(10^{-7}\,cm/s\right)*\left(\frac{90-60}{3}\frac{ft}{ft}\right)\left[(75-60)(500)(4)\,ft^2\right]\left(\frac{1ft}{30.48cm}\right)$$

$$+\left(10^{-7}\,cm/s\right)*\left(\frac{90-60}{3}\frac{ft}{ft}\right)\left[(60-50)(500)(4)\,ft^2\right]\left(\frac{1ft}{30.48cm}\right)$$

$$Q = 1.64\times10^{-3}\,cfs = 1062\,gpd$$

8.20. Repeat problem 8.19 if the clean sand unit extends below the landfill to an elevation of 25 ft msl. The clay liner exists around and on the bottom of the landfill. Consider Darcy's law across both the sides and bottom of the clay liner.



$$Q = qA = K\frac{\Delta h}{\Delta l}A$$

$$K_{CLAY} = 10^{-7}\,cm/s$$

$$K_{SAND} = 10^{-2}\,cm/s$$

Compute flow into landfill = flow to be pumped:

$$Q = \left(10^{-7}\,cm/s\right) * \left(\frac{90-60}{3}\frac{ft}{ft}\right)\left[(75-60)(500)(4)\,ft^2\right]\left(\frac{1ft}{30.48cm}\right)$$

$$+\left(10^{-7}\,cm/s\right) * \left(\frac{90-60}{3}\frac{ft}{ft}\right)\left[(60-50)(500)(4)\,ft^2\right]\left(\frac{1ft}{30.48cm}\right)$$

$$+\left(10^{-7}\,cm/s\right) * \left(\frac{90-60}{3}\frac{ft}{ft}\right)(500)^2\left(\frac{1ft}{30.48cm}\right)$$

$$Q = 9.84\times10^{-3}\,cfs = 6357\,gpd$$

8.21. A well located at $x = 0, y = 0$ injects water into an aquifer at $Q = 1.0$ cfs and observation wells are located along the x-axis at $x = 10$, 50, 150, and 300 ft away from the injection well. The confined aquifer with thickness of 10 ft has $T = 3200$ sq ft/day and $S = 0.005$. The injection is affected by the presence of a linear river located at $x = 300$ ft east of the injection well. Compute the head buildup along the x-axis at the observation well locations after 6 hr of injection.

Using a spreadsheet solution thesis method:

$T = 133$ ft^2/hr (= 3200 ft^2/day)

$S = 0.005$

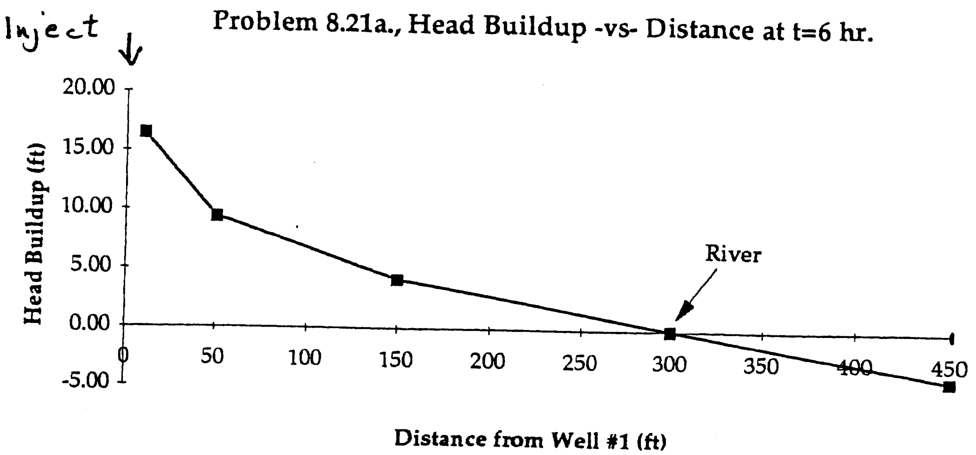
$Q_1 = 3600$ cu ft/hr (= 1.0 cfs) injecting

$Q_2 = -3600$ cu ft/hr (= 1.0 cfs) pumping

$T = 6$ hr

Well #1 (injecting)				Well #2 (pumping)				Total Drawdown	
r1	u	W(u)	s1'	r2	u	W(u)	s2'	X (ft from #1)	s'=s1'+s2' (ft)
10	0.0002	8.19	17.59	590	0.5439	0.51	-1.10	10	16.50
50	0.0039	4.97	10.68	550	0.4727	0.59	-1.28	50	9.41
150	0.0352	2.81	6.03	450	0.3164	0.87	-1.86	150	4.17
300	0.1406	1.52	3.27	300	0.1406	1.52	-3.27	300	0.00
450	0.3164	0.87	1.86	150	0.0352	2.81	-6.03	450	-4.17

Problem 8.21a., Head Buildup -vs- Distance at t=6 hr.



8.22. Repeat Dupuit Example 8.3 for the case where net recharge W = 10 cm/yr. Repeat the example for the case where W = 0.

For $W = 10cm/yr = 2.740\times10^{-4} m/day$

$$L = 1000m,\ K = 0.5m/day,\ h_0 = 20m,\ h_L = 18m$$

At $x = d,\ q = 0$

$$0 = \frac{K}{2L}\left(h_0^2 - h_L^2\right) + W\left(d - \frac{L}{2}\right)$$

$$d = \frac{L}{2} - \frac{K}{2WL}\left(h_0^2 - h_L^2\right)$$

$$d = \frac{1000m}{2} - \frac{(0.5m/day)\left(20^2 - 18^2\right)m^2}{(2)\left(2.740\times10^{-4} m/day\right)(1000)m}$$

$$d = 500m - 69.3m = 430.7m = d$$

At $x = d,\ h = h_{max}$

$$h_{max}^2 = h_0^2 + \frac{\left(h_L^2 - h_0^2\right)}{L}d + \frac{Wd}{K}(L - d)$$

$$= 20^2 + \frac{\left(18^2 - 20^2\right)(430.7)}{1000} + \frac{\left(2.740\times10^{-4}\right)(430.7)(1000 - 430.7)}{0.5}$$

$$h_{max}^2 = 400 - 32.7 + 134.4 = 501.67m^2$$

$$h_{max} = 22.4m$$

For discharge into River 1, set x = 0m:

$$q = \frac{K}{2L}\left(h_0^2 - h_L^2\right) + W\left(x - \frac{L}{2}\right)$$

$$= \frac{0.5}{2000}\left(20^2 - 18^2\right) + 2.74\times10^{-4}\left(0 - \frac{1000}{2}\right)$$

$0.019 + (-0.137) = -0.118m^2/day$ into River 1 = q

Note: The "-" sign indicates direction of flow.

For discharge into River 2, set x = L:

8.22. (cont)

$$= \frac{0.5}{2000}\left(20^2 - 18^2\right) + 2.74\times10^{-4}\left(1000 - \frac{1000}{2}\right)$$

$$= 0.019 + 0.137 = 0.156 m^2/day \text{ into River 2} = q$$

For W = 0, use: $q = \frac{K}{2L}\left(h_0^2 - h_L^2\right) \Leftarrow$ Dupuite without recharge.

Without recharge, the Dupuite equation indicates that all flow will occur from larger head to smaller head, ie. From h_0 to h_L. No water divide will occur between the rivers.

For discharge into River 1, set x = 0:

$$q = \frac{K}{2L}\left(h_0^2 - h_L^2\right) + W\cancelto{0}{(x - \frac{L}{2})}$$

$$q = \frac{0.5}{2000}\left(20^2 - 18^2\right) = 0.019 m^2/day \text{ out of River 1.}$$

For discharge into River 2, set x = L:

$$q = \frac{K}{2L}\left(h_0^2 - h_L^2\right) + W\cancelto{0}{(x - \frac{L}{2})}$$

$$q = \frac{0.5}{2000}(76) = 0.019 m^2/day \text{ into River 2}$$

8.23. Use the Theis method equations to characterize the behavior of a confined aquifer that is homogeneous and isotropic with $T = 500$ m^2/day and $S = 1 \times 10^{-5}$. A single well is pumped at 2500 m^3/day.

a) Compute the drawdown 75 m away from the well at t = 1, 10, 100, 1000, and 10,000 minutes after pumping began.

b) Compute the drawdown after 1.0 day of pumping at locations r = 2, 5, 10, 50, 200, and 1000 m from the pumping well.

a) Eq. 8.50 gives $u = \frac{r^2 s}{4Tt}$

$$= \frac{(75m)^2 (1\times10^{-5})}{4(500m^2/d)\left(1day/24hrs * \frac{1hr}{60\min}\right)}$$

$$u = \frac{0.405}{4t}$$

Eq. 8.51 gives values of w(u)

Eq. 8.49 gives drawdown in terms of w(u):

$$s^1 = \frac{Q}{4\pi T} W(u)$$

$$= \frac{2500m^3/day}{4\pi(500m^2/day)} W(u)$$

$$= 0.3979W(u)$$

Thus, for r = 75 m,

t	1	10	100	1000	10000
u	0.0405	0.00405	4.05 x 10^{-4}	4.05 x 10^{-5}	4.05 x 10^{-6}
w(u)	2.669	4.936	7.235	9.537	11.84
s^1	1.062	1.964	2.879	3.795	4.711

(u = .405/t)
(Eq. 8.51)
(s^1 = .3979 w(u))

b) Eq. 8.50 gives $u = \frac{r^2 s}{4Tt}$

$$= \frac{r^2\left(1\times10^{-5}\right)}{4\left(500m^2/d\right)\left(1day\right)}$$

$$u = 5\times10^{-9} r^2$$

Eq. 8.51 gives values of W(u)

Eq. 8.49 gives drawdown in terms of w(u):

$$s^1 = \frac{Q}{4\pi T} W(u)$$

$$= \frac{2500m^3/day}{4\pi\left(500m^2/day\right)} W(u)$$

$$= .3979 W(u)$$

Thus, for t = 1 day;

r	**2**	**5**	**10**	**50**	**200**	**1000**
u (u = 5 x 10 r)	2 x 10^{-8}	1.25 x 10^{-7}	5 x 10^{-7}	1.25 x 10^{-5}	2 x 10^{-4}	0.005
w(u) (Eq. 8.51)	17.15	15.32	13.93	10.71	7.940	4.73
s^1 (s^1 = 0.3979 w(u))	6.82	6.10	5.54	4.26	3.16	1.88

8.24. A well at a distance d from an impermeable boundary pumps at a flow rate Q. The head and any point (x, y) is given by the following equation:

$$h(x,y)=\frac{Q}{2\pi T}[\ln(r_1 r_2)]+C,$$

where C is a constant, r_1 is the straight line distance from the well to the point (x, y), and r_2 is the distance from the image well to the point (x, y). The y-axis lies along the impermeable boundary. Use Darcy's law to show that the flow across the boundary (y-axis) is indeed zero.

At (x.y) $$h=\frac{Q}{2\pi Kb}\ln(r_1\times r_2)+C$$

$$r_1=\left\{(x-d)^2+y^2\right\}^{1/2}$$

$$r_2=\left\{(x+d)^2+y^2\right\}^{1/2}$$

$$h=\frac{Q}{2\pi Kb}\ln\left[\left\{(x-d)^2+y^2\right\}^{1/2}\left\{(x+d)^2+y^2\right\}^{1/2}\right]+C$$

$$=\frac{Q}{4\pi Kb}\ln\left[\left\{(x-d)^2+y^2\right\}\left\{(x+d)^2+y^2\right\}^{1/2}\right]+C$$

if $\frac{dh}{dx}=0$ (ie. Hydraulic gradient)

and if $q=-K\frac{dh}{dx}$

then there is no flow when $\frac{dh}{dx}=0$, so we need to prove that $\left.\frac{dh}{dx}\right|_{x=0}=0$

$$\frac{dh}{dx}=\frac{Q}{4\pi Kb}\times\frac{d}{x}(\ln u)$$

where $u=\left\{(x-d)^2+y^2\right\}^{1/2}\left\{(x+d)^2+y^2\right\}^{1/2}$

$$=\frac{Q}{4\pi Kb}\frac{1}{u}\frac{du}{dx}$$

$$=\frac{Q}{4\pi Kb}\frac{1}{\left[\left\{(x-d)^2+y^2\right\}\left\{(x+d)^2+y^2\right\}\right]}\times\frac{du}{dx}$$

$$u=\left\{(x-d)^2+y^2\right\}\left\{(x+d)^2+y^2\right\}$$

$$\frac{du}{dx}=2(x-d)\left\{(x+d)^2+y^2\right\}+\left\{(x-d)^2+y^2\right\}2(x+d)$$

at x = 0, $$\frac{du}{dx}=-2d\left(d^2+y^2\right)+\left(d^2+y^2\right)2d$$

$$=0$$

at x = 0 $$\frac{dh}{dx}=\frac{Q}{4\pi Kb}\frac{1}{\left[\left\{(x-d)^2+y^2\right\}\left\{(x+d)^2+y^2\right\}\right]}\times\frac{du}{dx}$$

$$=\frac{Q}{4\pi Kb}\frac{1}{\left[\left\{(x-d)^2+y^2\right\}\left\{(x+d)^2+y^2\right\}\right]}\times 0$$

$$= 0$$

if $\left.\frac{dh}{dx}\right|_{x=0}=0$ then there is no flow across boundary.

8.25. A fully penetrating well pumps from a confined aquifer of thickness 20 m and K = 10 m/day. The radius of the well is 0.25 m and the recorded pump rate in the well is 100 m^3/day at steady state. Assume that the radius of influence of the well is 1250 m. Compute the drawdown at the well if it is located 100 m from an impermeable boundary (see problem 8.24).

Using the equation from problem 8.24:

$$h(x,y) = \frac{Q}{2\pi T}\left[\ln\left(r_1, r_2\right)\right] + C$$

where T = Kb

Find C by using h = 0 at $r_1 = 1250m$:

$$0 = \frac{100m^3 / day}{2\pi\left(10m/1day \times 20m\right)}\left[\ln\left(1250m \times 1450m\right)\right] + C$$

$$C = -1.1467$$

Then find h at $r_1 = 0.25m,\ r_2 = 199.75m$

$$h = \frac{100m^3 / day}{2\pi\left(10m/1day \times 20m\right)}\left[\ln\left(0.25m \times 199.75\right)\right] + (-1.1467)$$

$h = -0.836m$ = drawdown at the well.