

SEVENTH EDITION

PRINCIPLES OF

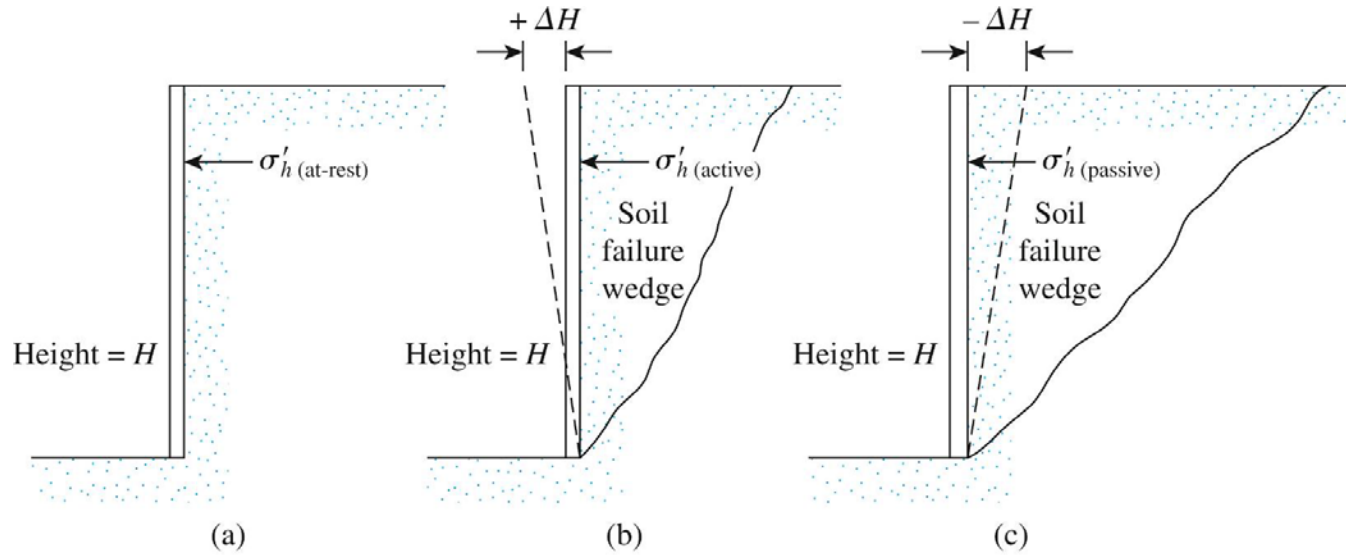
# FOUNDATION ENGINEERING



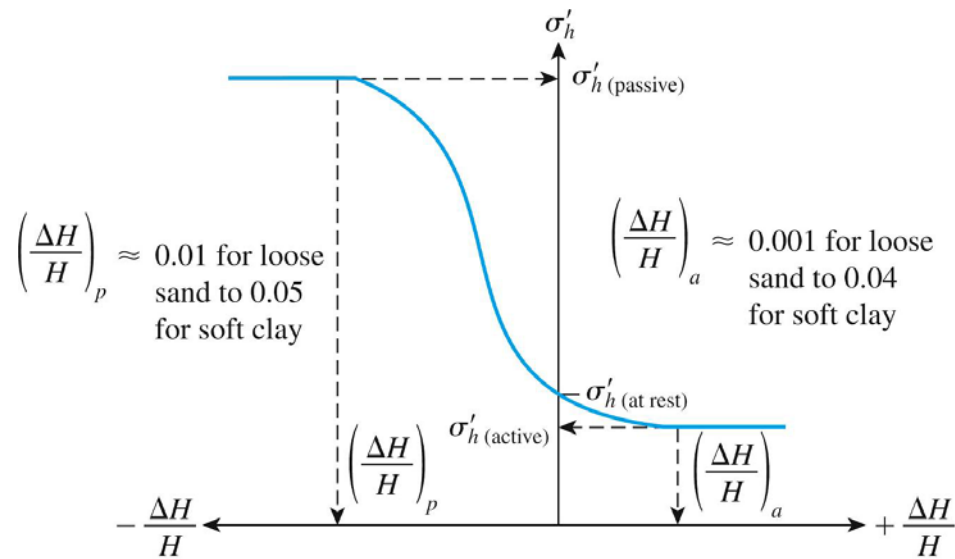
BRAJA DAS

## Chapter 7: Lateral Earth Pressure

# Variation of Lateral Earth Pressure



**Figure 7.1** Nature of lateral earth pressure on a retaining wall



**Figure 7.2** Nature of variation of lateral earth pressure at a certain depth

# Lateral Earth Pressure at Rest

$$s = c' + \sigma' \tan \phi'$$

$c'$  = cohesion

$\phi'$  = effective angle of friction

$\sigma'$  = effective normal stress

$\sigma'_0 = q + \gamma z$  vertical subsurface stress

$\sigma_h = K_0 \sigma'_0 + u$  lateral pressure

$u$  = pore pressure

$K_0$  = coefficient of at - rest pressure

$$K_0 \approx 1 - \sin \phi' \quad (\text{normally consolidated clay})$$

$$K_0 = (1 - \sin \phi')^{OCR^{\sin \phi'}} \quad (\text{overconsolidated clay})$$

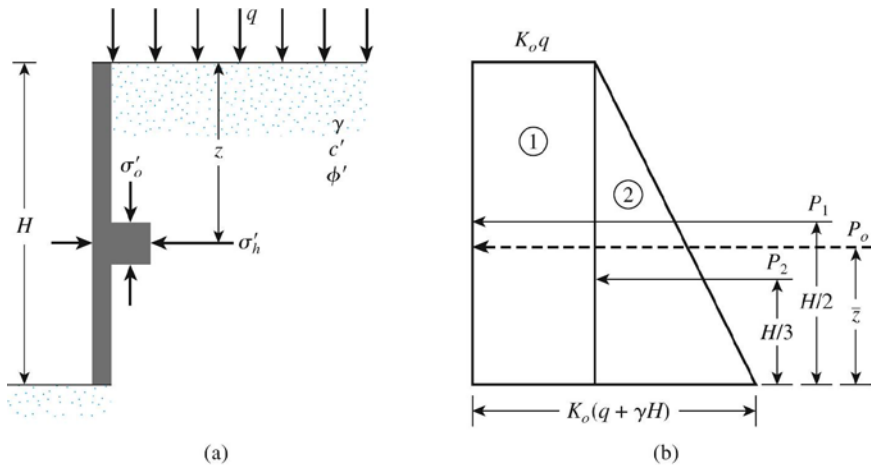
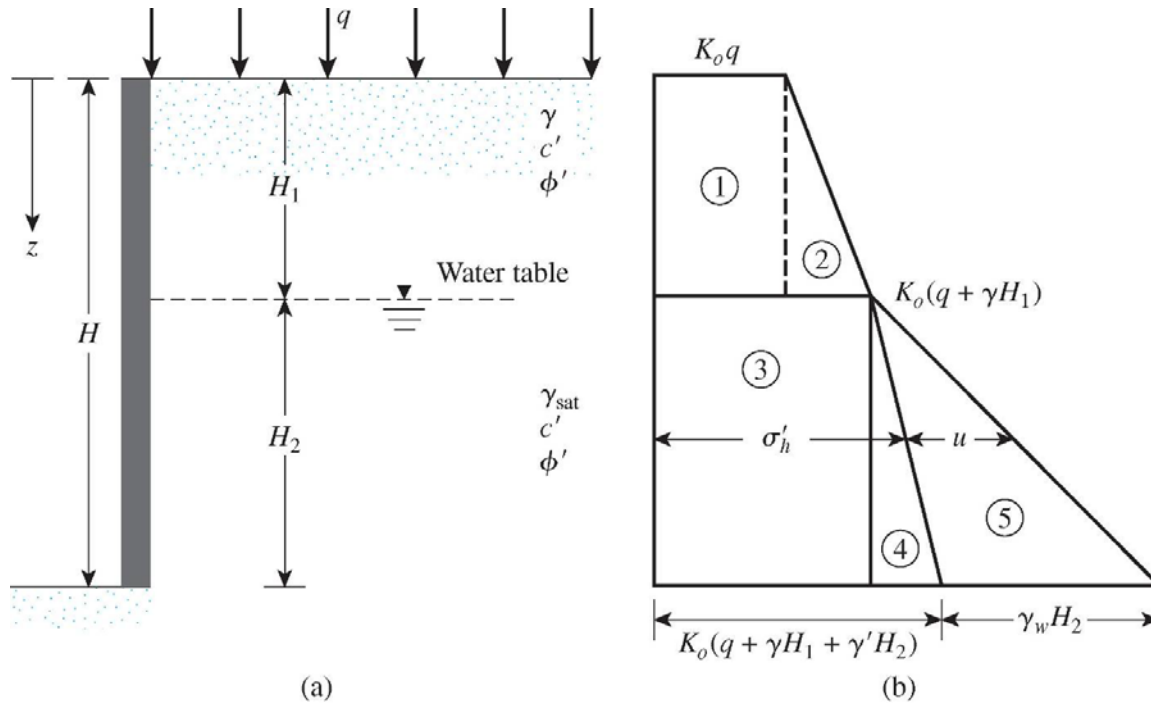


Figure 7.3 At-rest earth pressure

$$P_0 = P_1 + P_2$$

$$P_0 = K_0 q H + \frac{1}{2} K_0 \gamma H^2$$

# Lateral Earth Pressure at Rest

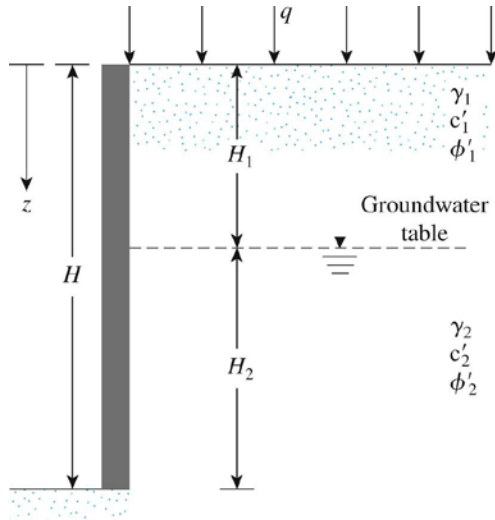


**Figure 7.4** At-rest earth pressure with water table located at a depth  $z < H$

$$P_0 = A_1 + A_2 + A_3 + A_4 + A_5$$

$$P_0 = K_0 q H_1 + \frac{1}{2} K_0 \gamma H_1^2 + K_0 (q + \gamma H_1) H_2 + \frac{1}{2} K_0 \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

## Practice Problem # 1



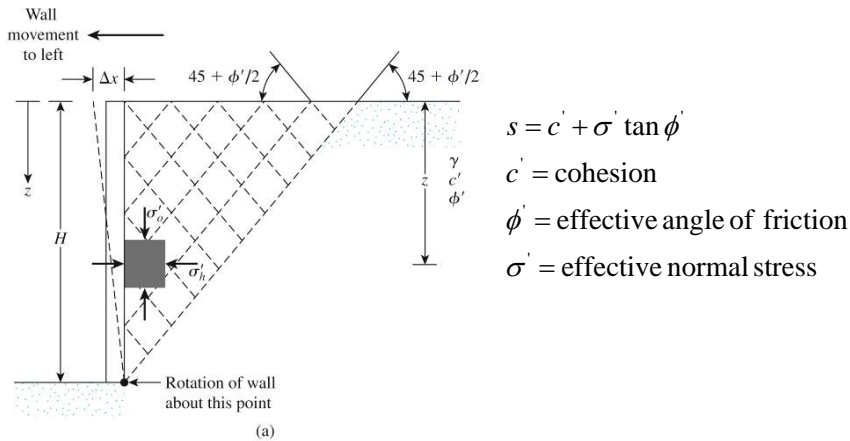
Use the following values to determine the at - rest lateral earth force per unit length of the wall. Also find the location of the resultant.

$H = 5\text{m}$ ;  $H_1 = 2\text{m}$ ;  $H_2 = 3\text{m}$ ;  $\gamma = 15.5\text{ kN/m}^3$ ;  $\gamma_{\text{sat}} = 18.5\text{ kN/m}^3$

$\phi' = 34^\circ$ ;  $c' = 0$ ;  $q = 20\text{ kN/m}^2$ ; and  $\text{OCR} = 1$ .

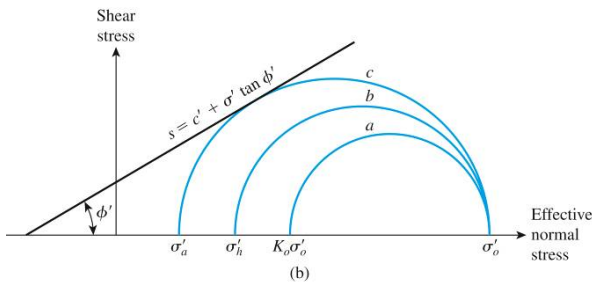
Figure P7.2

# Rankine Active Earth Pressure



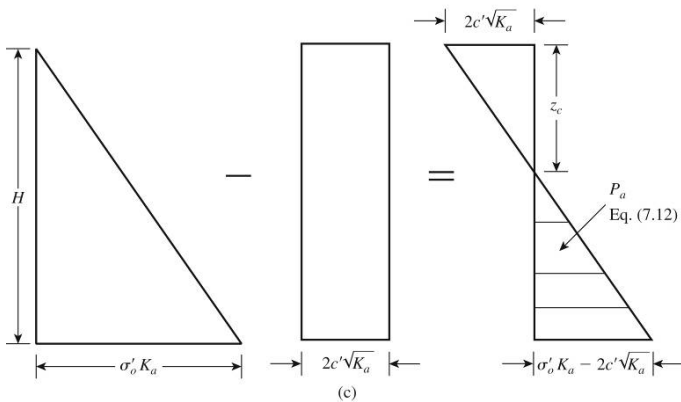
$$\sigma'_1 = \sigma'_3 \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

Major principal stress,  $\sigma'_1 = \sigma'_0$     Minor principal stress,  $\sigma'_2 = \sigma'_a$



$$\sigma'_a = \sigma'_0 \tan^2 \left( 45 - \frac{\phi'}{2} \right) - 2c' \tan \left( 45 - \frac{\phi'}{2} \right) = \sigma'_0 K_a - 2c' \sqrt{K_a}$$

$$K_a = \tan^2 \left( 45 - \frac{\phi'}{2} \right) = \text{Rankine active - pressure coefficient}$$



$$\gamma z_c K_a - 2c' \sqrt{K_a} = 0 \quad \Rightarrow \quad z_c = \frac{2c'}{\gamma \sqrt{K_a}} = \text{Depth of tensile crack}$$

$$P_a = \frac{1}{2} \gamma H^2 K_a - 2c' H \sqrt{K_a} \quad (\text{before tensile crack occurs})$$

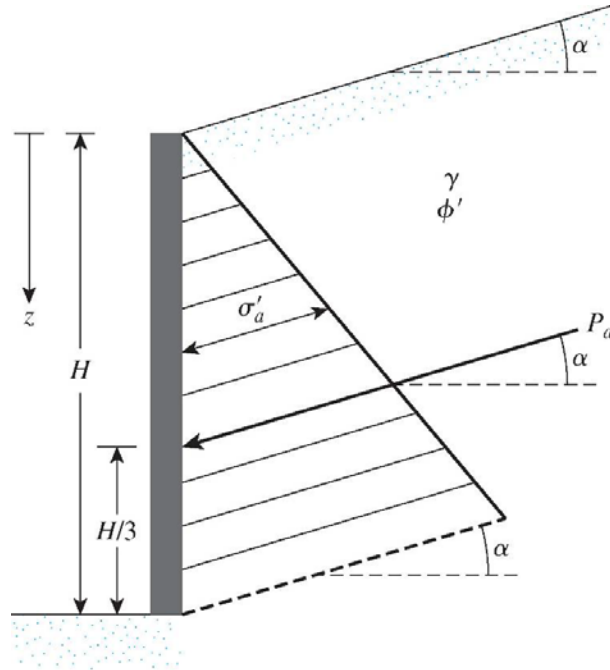
$$P_a = \frac{1}{2} (H - z_c) (\gamma H K_a - 2c' \sqrt{K_a}) \quad (\text{after tensile crack occurs})$$

Figure 7.6 Rankine active pressure



# Generalized Case for Rankine Active Pressure

(Granular Backfill with vertical back face)



**Figure 7.10** Notations for active pressure—Eqs. (7.19), (7.20), (7.21)

$$P_a = \frac{1}{2} \gamma H^2 K_a$$

$$K_a = \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi'}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi'}}$$

# Generalized Case for Rankine Active Pressure

(Granular Backfill with vertical back face)

**Table 7.1** Values of  $K_a$  [Eq. (7.19)]

$\alpha$ (deg)	$\phi'$ (deg) $\rightarrow$												
	28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.2174
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.2175
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.2177
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.2181
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.2186
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.2192
6	0.3676	0.3531	0.3389	0.3253	0.3120	0.2992	0.2868	0.2747	0.2631	0.2518	0.2409	0.2303	0.2200
7	0.3701	0.3553	0.3410	0.3272	0.3138	0.3008	0.2883	0.2761	0.2644	0.2530	0.2420	0.2313	0.2209
8	0.3730	0.3580	0.3435	0.3294	0.3159	0.3027	0.2900	0.2778	0.2659	0.2544	0.2432	0.2325	0.2220
9	0.3764	0.3611	0.3463	0.3320	0.3182	0.3049	0.2921	0.2796	0.2676	0.2560	0.2447	0.2338	0.2233
10	0.3802	0.3646	0.3495	0.3350	0.3210	0.3074	0.2944	0.2818	0.2696	0.2578	0.2464	0.2354	0.2247
11	0.3846	0.3686	0.3532	0.3383	0.3241	0.3103	0.2970	0.2841	0.2718	0.2598	0.2482	0.2371	0.2263
12	0.3896	0.3731	0.3573	0.3421	0.3275	0.3134	0.2999	0.2868	0.2742	0.2621	0.2503	0.2390	0.2281
13	0.3952	0.3782	0.3620	0.3464	0.3314	0.3170	0.3031	0.2898	0.2770	0.2646	0.2527	0.2412	0.2301
14	0.4015	0.3839	0.3671	0.3511	0.3357	0.3209	0.3068	0.2931	0.2800	0.2674	0.2552	0.2435	0.2322
15	0.4086	0.3903	0.3729	0.3564	0.3405	0.3253	0.3108	0.2968	0.2834	0.2705	0.2581	0.2461	0.2346
16	0.4165	0.3975	0.3794	0.3622	0.3458	0.3302	0.3152	0.3008	0.2871	0.2739	0.2612	0.2490	0.2373
17	0.4255	0.4056	0.3867	0.3688	0.3518	0.3356	0.3201	0.3053	0.2911	0.2776	0.2646	0.2521	0.2401
18	0.4357	0.4146	0.3948	0.3761	0.3584	0.3415	0.3255	0.3102	0.2956	0.2817	0.2683	0.2555	0.2433
19	0.4473	0.4249	0.4039	0.3842	0.3657	0.3481	0.3315	0.3156	0.3006	0.2862	0.2724	0.2593	0.2467
20	0.4605	0.4365	0.4142	0.3934	0.3739	0.3555	0.3381	0.3216	0.3060	0.2911	0.2769	0.2634	0.2504
21	0.4758	0.4498	0.4259	0.4037	0.3830	0.3637	0.3455	0.3283	0.3120	0.2965	0.2818	0.2678	0.2545
22	0.4936	0.4651	0.4392	0.4154	0.3934	0.3729	0.3537	0.3356	0.3186	0.3025	0.2872	0.2727	0.2590
23	0.5147	0.4829	0.4545	0.4287	0.4050	0.3832	0.3628	0.3438	0.3259	0.3091	0.2932	0.2781	0.2638
24	0.5404	0.5041	0.4724	0.4440	0.4183	0.3948	0.3731	0.3529	0.3341	0.3164	0.2997	0.2840	0.2692
25	0.5727	0.5299	0.4936	0.4619	0.4336	0.4081	0.3847	0.3631	0.3431	0.3245	0.3070	0.2905	0.2750

# Generalized Case for Rankine Active Pressure

(Vertical backface with  $c' - \phi'$  soil backfill)

$$\sigma'_a = \gamma z K'_a = \gamma z K'_a \cos \alpha$$

Depth of tensile crack :

$$z_c = \frac{2c'}{\gamma} \sqrt{\frac{1 + \sin \phi'}{1 - \sin \phi'}}$$

**Table 7.2** Values of  $K'_a$

$\phi'$ (deg)	$\alpha$ (deg)	$\frac{c'}{\gamma z}$			
		0.025	0.05	0.1	0.5
15	0	0.550	0.512	0.435	-0.179
	5	0.566	0.525	0.445	-0.184
	10	0.621	0.571	0.477	-0.186
	15	0.776	0.683	0.546	-0.196
20	0	0.455	0.420	0.350	-0.210
	5	0.465	0.429	0.357	-0.212
	10	0.497	0.456	0.377	-0.218
	15	0.567	0.514	0.417	-0.229
25	0	0.374	0.342	0.278	-0.231
	5	0.381	0.348	0.283	-0.233
	10	0.402	0.366	0.296	-0.239
	15	0.443	0.401	0.321	-0.250
30	0	0.305	0.276	0.218	-0.244
	5	0.309	0.280	0.221	-0.246
	10	0.323	0.292	0.230	-0.252
	15	0.350	0.315	0.246	-0.263

(Active pressure is inclined at an angle  $\alpha$  with the horizontal)

## Practice Problem #2

Given the height of the retaining wall,  $H$  is 21 ft; the backfill is a saturated clay with  $\phi = 0^0$ ;  $c = 630$  psf;  $\gamma_{\text{sat}} = 113$ pcf.

- (a) Determine the Rankine active pressure distribution diagram behind the wall.
- (b) Determine the depth of tensile crack,  $z_c$ .
- (c) Estimate the Rankine active force per foot length of the wall before and after the occurrence of the tensile crack.

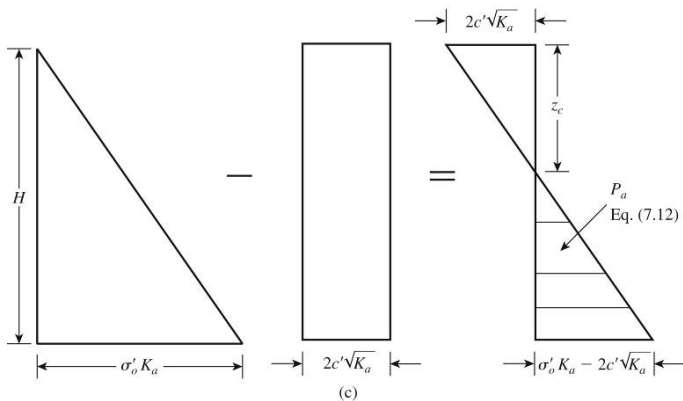
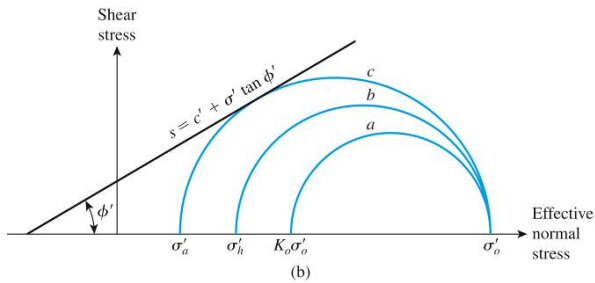
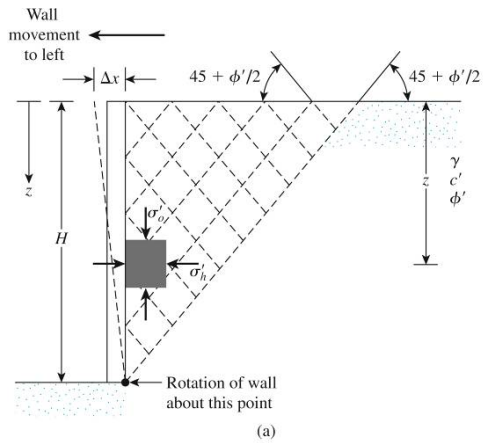
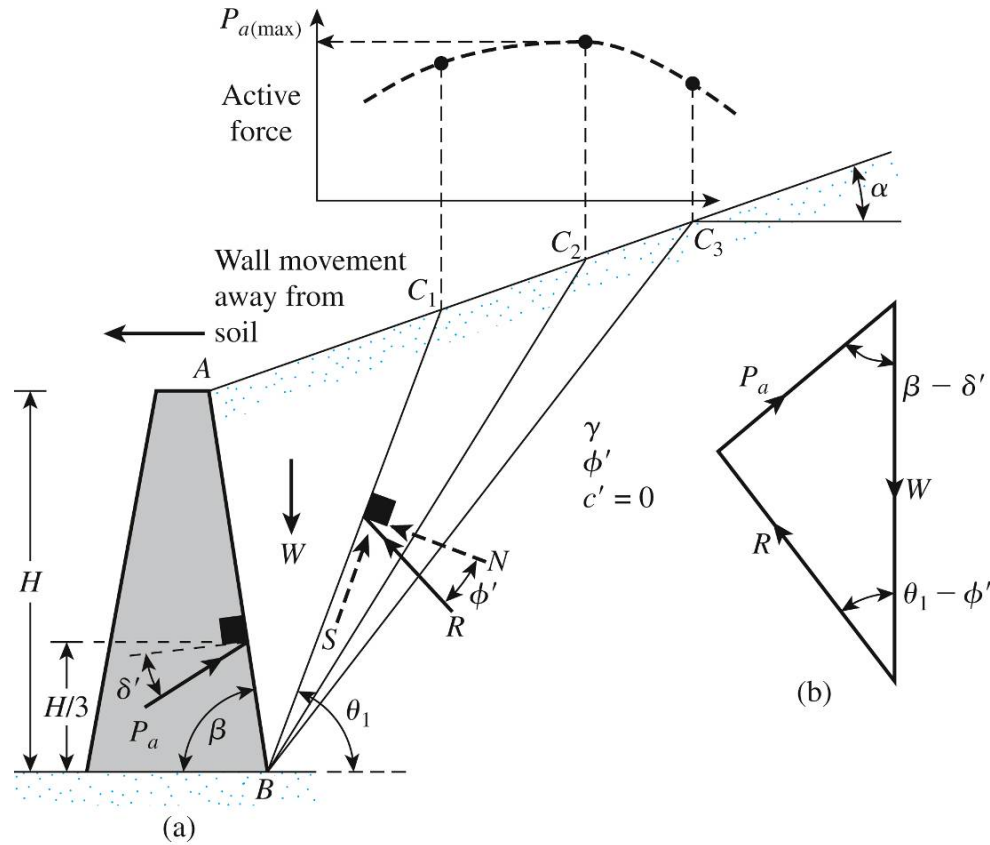


Figure 7.6 Rankine active pressure

# Coulomb's Active Earth Pressure



**Figure 7.12** Coulomb's active pressure

$$P_a = \frac{1}{2} K_a \gamma H^2$$

$K_a$  = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2(\beta + \phi')}{\sin^2 \beta \sin(\beta - \delta') \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' - \alpha)}{\sin(\beta - \delta') \sin(\alpha + \beta)}} \right]^2}$$

# Coulomb's Active Earth Pressure

**Table 7.3** Values of  $K_a$  [Eq. (7.26)] for  $\beta = 90^\circ$  and  $\alpha = 0^\circ$

$\phi'$ (deg)	$\delta'$ (deg)					
	0	5	10	15	20	25
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2098	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

# Coulomb's Active Earth Pressure

*Table 7.4* Values of  $K_a$  [from Eq. (7.26)] for  $\delta' = \frac{2}{3}\phi'$

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)					
		90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
5	28	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302
	29	0.3431	0.3845	0.4311	0.4843	0.5461	0.6190
	30	0.3295	0.3709	0.4175	0.4707	0.5325	0.6056
	31	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	32	0.3039	0.3451	0.3916	0.4447	0.5067	0.5800
	33	0.2919	0.3329	0.3792	0.4324	0.4943	0.5677
	34	0.2803	0.3211	0.3673	0.4204	0.4823	0.5558
	35	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	36	0.2583	0.2987	0.3446	0.3975	0.4594	0.5330
	37	0.2479	0.2881	0.3338	0.3866	0.4484	0.5221
	38	0.2379	0.2778	0.3233	0.3759	0.4377	0.5115
	39	0.2282	0.2679	0.3131	0.3656	0.4273	0.5012
	40	0.2188	0.2582	0.3033	0.3556	0.4172	0.4911
	41	0.2098	0.2489	0.2937	0.3458	0.4074	0.4813
42	0.2011	0.2398	0.2844	0.3363	0.3978	0.4718	
10	28	0.1927	0.2311	0.2753	0.3271	0.3884	0.4625
	28	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	29	0.3548	0.4007	0.4528	0.5128	0.5831	0.6672
	30	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	31	0.3259	0.3713	0.4230	0.4826	0.5526	0.6365
	32	0.3123	0.3575	0.4089	0.4683	0.5382	0.6219
	33	0.2993	0.3442	0.3953	0.4545	0.5242	0.6078
	34	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	35	0.2748	0.3190	0.3696	0.4283	0.4976	0.5810
	36	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	37	0.2522	0.2957	0.3456	0.4037	0.4726	0.5558
	38	0.2415	0.2846	0.3342	0.3920	0.4607	0.5437
	39	0.2313	0.2740	0.3231	0.3807	0.4491	0.5321
	40	0.2214	0.2636	0.3125	0.3697	0.4379	0.5207
41	0.2119	0.2537	0.3021	0.3590	0.4270	0.5097	
42	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990	
15	28	0.4065	0.4585	0.5179	0.5868	0.6685	0.7670

*(continued)*

# Coulomb's Active Earth Pressure

**Table 7.4** (Continued)

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)					
		90	85	80	75	70	65
20	29	0.3881	0.4397	0.4987	0.5672	0.6483	0.7463
	30	0.3707	0.4219	0.4804	0.5484	0.6291	0.7265
	31	0.3541	0.4049	0.4629	0.5305	0.6106	0.7076
	32	0.3384	0.3887	0.4462	0.5133	0.5930	0.6895
	33	0.3234	0.3732	0.4303	0.4969	0.5761	0.6721
	34	0.3091	0.3583	0.4150	0.4811	0.5598	0.6554
	35	0.2954	0.3442	0.4003	0.4659	0.5442	0.6393
	36	0.2823	0.3306	0.3862	0.4513	0.5291	0.6238
	37	0.2698	0.3175	0.3726	0.4373	0.5146	0.6089
	38	0.2578	0.3050	0.3595	0.4237	0.5006	0.5945
	39	0.2463	0.2929	0.3470	0.4106	0.4871	0.5805
	40	0.2353	0.2813	0.3348	0.3980	0.4740	0.5671
	41	0.2247	0.2702	0.3231	0.3858	0.4613	0.5541
	42	0.2146	0.2594	0.3118	0.3740	0.4491	0.5415
	28	0.4602	0.5205	0.5900	0.6714	0.7689	0.8880
	29	0.4364	0.4958	0.5642	0.6445	0.7406	0.8581
	30	0.4142	0.4728	0.5403	0.6195	0.7144	0.8303
	31	0.3935	0.4513	0.5179	0.5961	0.6898	0.8043
	32	0.3742	0.4311	0.4968	0.5741	0.6666	0.7799
	33	0.3559	0.4121	0.4769	0.5532	0.6448	0.7569
	34	0.3388	0.3941	0.4581	0.5335	0.6241	0.7351
	35	0.3225	0.3771	0.4402	0.5148	0.6044	0.7144
	36	0.3071	0.3609	0.4233	0.4969	0.5856	0.6947
	37	0.2925	0.3455	0.4071	0.4799	0.5677	0.6759
38	0.2787	0.3308	0.3916	0.4636	0.5506	0.6579	
39	0.2654	0.3168	0.3768	0.4480	0.5342	0.6407	
40	0.2529	0.3034	0.3626	0.4331	0.5185	0.6242	
41	0.2408	0.2906	0.3490	0.4187	0.5033	0.6083	
42	0.2294	0.2784	0.3360	0.4049	0.4888	0.5930	

# Coulomb's Active Earth Pressure

**Table 7.5** Values of  $K_a$  [from Eq. (7.26)] for  $\delta' = \phi'/2$

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)					
		90	85	80	75	70	65
0	28	0.3264	0.3629	0.4034	0.4490	0.5011	0.5616
	29	0.3137	0.3502	0.3907	0.4363	0.4886	0.5492
	30	0.3014	0.3379	0.3784	0.4241	0.4764	0.5371
	31	0.2896	0.3260	0.3665	0.4121	0.4645	0.5253
	32	0.2782	0.3145	0.3549	0.4005	0.4529	0.5137
	33	0.2671	0.3033	0.3436	0.3892	0.4415	0.5025
	34	0.2564	0.2925	0.3327	0.3782	0.4305	0.4915
	35	0.2461	0.2820	0.3221	0.3675	0.4197	0.4807
	36	0.2362	0.2718	0.3118	0.3571	0.4092	0.4702

# Coulomb's Active Earth Pressure

**Table 7.5** (Continued)

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)						
		90	85	80	75	70	65	
5	37	0.2265	0.2620	0.3017	0.3469	0.3990	0.4599	
	38	0.2172	0.2524	0.2920	0.3370	0.3890	0.4498	
	39	0.2081	0.2431	0.2825	0.3273	0.3792	0.4400	
	40	0.1994	0.2341	0.2732	0.3179	0.3696	0.4304	
	41	0.1909	0.2253	0.2642	0.3087	0.3602	0.4209	
	42	0.1828	0.2168	0.2554	0.2997	0.3511	0.4177	
	28	0.3477	0.3879	0.4327	0.4837	0.5425	0.6115	
	29	0.3337	0.3737	0.4185	0.4694	0.5282	0.5972	
	30	0.3202	0.3601	0.4048	0.4556	0.5144	0.5833	
	31	0.3072	0.3470	0.3915	0.4422	0.5009	0.5698	
	32	0.2946	0.3342	0.3787	0.4292	0.4878	0.5566	
	33	0.2825	0.3219	0.3662	0.4166	0.4750	0.5437	
	34	0.2709	0.3101	0.3541	0.4043	0.4626	0.5312	
	35	0.2596	0.2986	0.3424	0.3924	0.4505	0.5190	
	36	0.2488	0.2874	0.3310	0.3808	0.4387	0.5070	
	37	0.2383	0.2767	0.3199	0.3695	0.4272	0.4954	
	38	0.2282	0.2662	0.3092	0.3585	0.4160	0.4840	
	39	0.2185	0.2561	0.2988	0.3478	0.4050	0.4729	
40	0.2090	0.2463	0.2887	0.3374	0.3944	0.4620		
41	0.1999	0.2368	0.2788	0.3273	0.3840	0.4514		
42	0.1911	0.2276	0.2693	0.3174	0.3738	0.4410		
10	28	0.3743	0.4187	0.4688	0.5261	0.5928	0.6719	
	29	0.3584	0.4026	0.4525	0.5096	0.5761	0.6549	
	30	0.3432	0.3872	0.4368	0.4936	0.5599	0.6385	
	31	0.3286	0.3723	0.4217	0.4782	0.5442	0.6225	
	32	0.3145	0.3580	0.4071	0.4633	0.5290	0.6071	
	33	0.3011	0.3442	0.3930	0.4489	0.5143	0.5920	
	34	0.2881	0.3309	0.3793	0.4350	0.5000	0.5775	
	35	0.2757	0.3181	0.3662	0.4215	0.4862	0.5633	
	36	0.2637	0.3058	0.3534	0.4084	0.4727	0.5495	
	37	0.2522	0.2938	0.3411	0.3957	0.4597	0.5361	
	38	0.2412	0.2823	0.3292	0.3833	0.4470	0.5230	
	39	0.2305	0.2712	0.3176	0.3714	0.4346	0.5103	
	40	0.2202	0.2604	0.3064	0.3597	0.4226	0.4979	
	41	0.2103	0.2500	0.2956	0.3484	0.4109	0.4858	
	42	0.2007	0.2400	0.2850	0.3375	0.3995	0.4740	
	15	28	0.4095	0.4594	0.5159	0.5812	0.6579	0.7498
		29	0.3908	0.4402	0.4964	0.5611	0.6373	0.7284
		30	0.3730	0.4220	0.4777	0.5419	0.6175	0.7080
31		0.3560	0.4046	0.4598	0.5235	0.5985	0.6884	
32		0.3398	0.3880	0.4427	0.5059	0.5803	0.6695	
33		0.3244	0.3721	0.4262	0.4889	0.5627	0.6513	
34		0.3097	0.3568	0.4105	0.4726	0.5458	0.6338	
35		0.2956	0.3422	0.3953	0.4569	0.5295	0.6168	

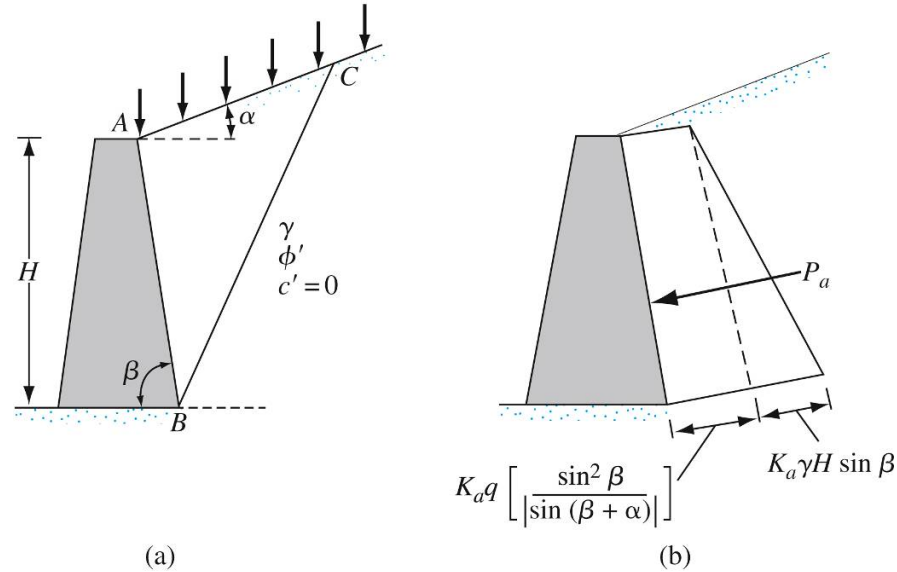
# Coulomb's Active Earth Pressure

**Table 7.5** (Continued)

$\alpha$ (deg)	$\phi'$ (deg)	$\beta$ (deg)					
		90	85	80	75	70	65
20	36	0.2821	0.3282	0.3807	0.4417	0.5138	0.6004
	37	0.2692	0.3147	0.3667	0.4271	0.4985	0.5846
	38	0.2569	0.3017	0.3531	0.4130	0.4838	0.5692
	39	0.2450	0.2893	0.3401	0.3993	0.4695	0.5543
	40	0.2336	0.2773	0.3275	0.3861	0.4557	0.5399
	41	0.2227	0.2657	0.3153	0.3733	0.4423	0.5258
	42	0.2122	0.2546	0.3035	0.3609	0.4293	0.5122
	28	0.4614	0.5188	0.5844	0.6608	0.7514	0.8613
	29	0.4374	0.4940	0.5586	0.6339	0.7232	0.8313
	30	0.4150	0.4708	0.5345	0.6087	0.6968	0.8034
	31	0.3941	0.4491	0.5119	0.5851	0.6720	0.7772
	32	0.3744	0.4286	0.4906	0.5628	0.6486	0.7524
	33	0.3559	0.4093	0.4704	0.5417	0.6264	0.7289
	34	0.3384	0.3910	0.4513	0.5216	0.6052	0.7066
	35	0.3218	0.3736	0.4331	0.5025	0.5851	0.6853
	36	0.3061	0.3571	0.4157	0.4842	0.5658	0.6649
	37	0.2911	0.3413	0.3991	0.4668	0.5474	0.6453
	38	0.2769	0.3263	0.3833	0.4500	0.5297	0.6266
	39	0.2633	0.3120	0.3681	0.4340	0.5127	0.6085
	40	0.2504	0.2982	0.3535	0.4185	0.4963	0.5912
	41	0.2381	0.2851	0.3395	0.4037	0.4805	0.5744
	42	0.2263	0.2725	0.3261	0.3894	0.4653	0.5582

# Coulomb's Active Earth Pressure

Surcharge =  $q$



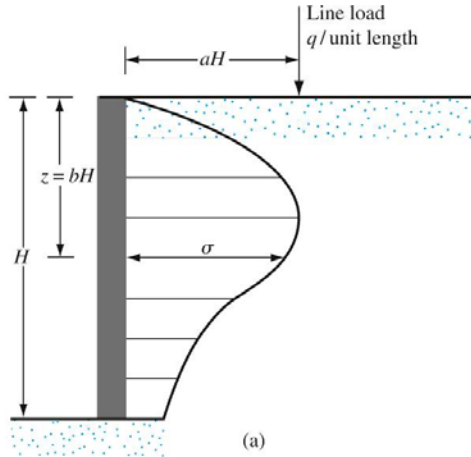
**Figure 7.13** Coulomb's active pressure with a surcharge on the backfill

$$P_a = \frac{1}{2} K_a \gamma_{eq} H^2 \qquad \gamma_{eq} = \gamma + \left[ \frac{\sin \beta}{\sin(\beta + \alpha)} \right] \left( \frac{2q}{H} \right)$$

$K_a$  = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2(\beta + \phi')}{\sin^2 \beta \sin(\beta - \delta') \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' - \alpha)}{\sin(\beta - \delta') \sin(\alpha + \beta)}} \right]^2}$$

# Lateral Earth Pressure due to Surcharge



$$\sigma = \frac{2q}{\pi H} \frac{a^2 b}{(a^2 + b^2)^2}$$

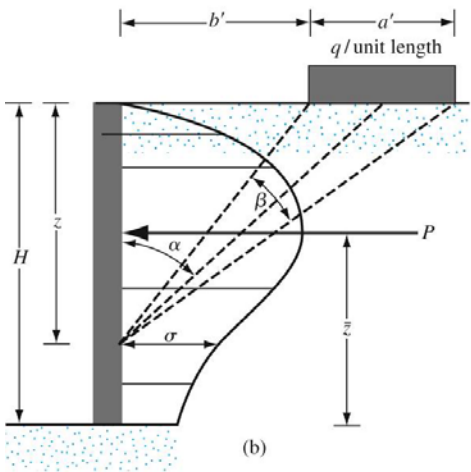
where  $\sigma$  = horizontal stress at depth,  $z = bH$

$$\sigma = \frac{4q}{\pi H} \frac{a^2 b}{(a^2 + b^2)^2} \quad (\text{for } a > 0.4)$$

$$\sigma = \frac{q}{H} \frac{0.203b}{(0.16 + b^2)^2} \quad (\text{for } a \leq 0.4)$$

$$\sigma = \frac{q}{\pi} (\beta - \sin \beta \cos 2\alpha)$$

$$\sigma = \frac{2q}{\pi} (\beta - \sin \beta \cos 2\alpha)$$



$$P = \frac{q}{90} [H(\theta_2 - \theta_1)]$$

$$\theta_1 = \tan^{-1} \left( \frac{b'}{H} \right) \quad (\text{degree})$$

$$\theta_2 = \tan^{-1} \left( \frac{a' + b'}{H} \right) \quad (\text{degree})$$

$$\bar{z} = H - \left[ \frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)} \right]$$

**Figure 7.14** Lateral earth pressure caused by (a) line load and (b) strip load

where,  $R = (a' + b')^2 (90 - \theta_2)$        $Q = b'^2 (90 - \theta_1)$

# Active Earth Pressure for Earthquake Conditions

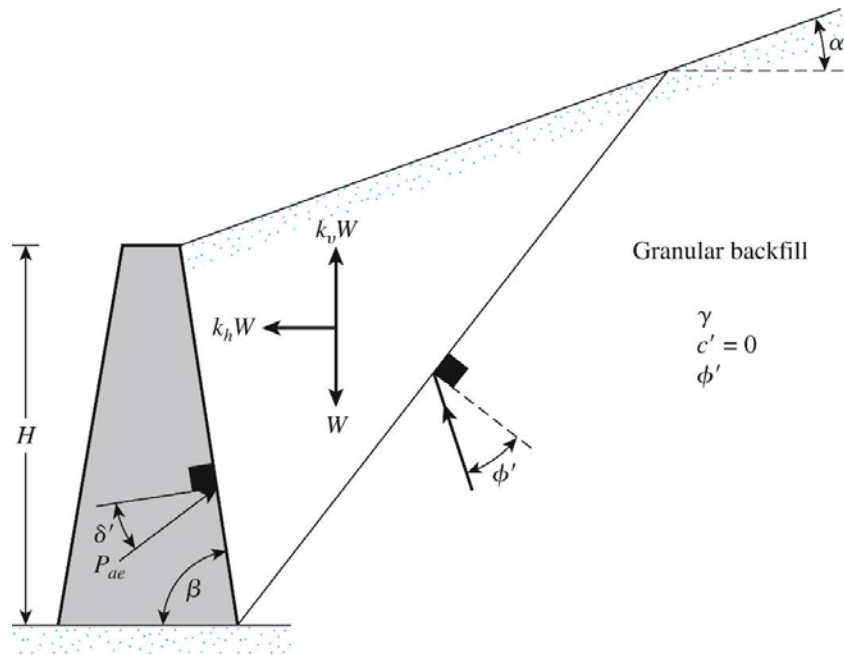
$$k_h = \frac{\text{horizontal earthquake acceleration component}}{\text{acceleration due to gravity, } g}$$

$$k_v = \frac{\text{vertical earthquake acceleration component}}{\text{acceleration due to gravity, } g}$$

$$P_{ae} = \frac{1}{2} \gamma H^2 (1 - k_v) K_{ae}$$

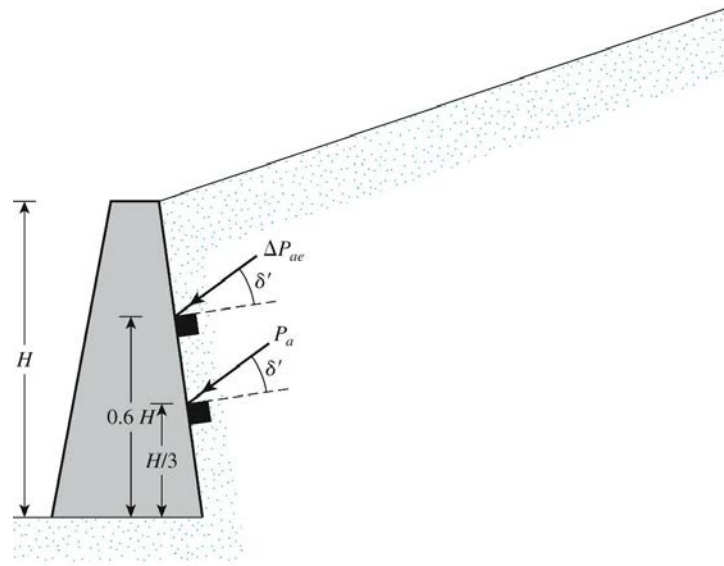
$$K_{ae} = \frac{\sin^2(\phi' + \beta - \theta')}{\cos \theta' \sin^2 \beta \sin(\beta - \theta' - \delta') \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' - \theta' - \alpha)}{\sin(\beta - \delta' - \theta') \sin(\alpha + \beta)}} \right]^2}$$

$$\theta' = \tan^{-1} \left[ \frac{k_h}{(1 - k_v)} \right]$$



**Figure 7.15** Derivation of Eq. (7.42)

# Active Earth Pressure for Earthquake Conditions



**Figure 7.16** Determining the line of action of  $P_{ae}$

Step 1: Calculate  $P_{ae}$  
$$P_{ae} = \frac{1}{2} \gamma H^2 (1 - k_v) K_{ae}$$

Step 2: Calculate  $P_a$  
$$P_a = \frac{1}{2} K_a \gamma H^2$$

Step 3: Calculate  $\Delta P_{ae} = P_{ae} - P_a$

Step 4: Assume that  $P_a$  acts at a distance of  $H/3$  from the bottom of the wall

Step 5: Assume that  $\Delta P_{ae}$  acts at a distance of  $0.6H$  from the bottom of the wall

Step 6: Calculate the location of the resultant as :

$$\bar{z} = \frac{(0.6H)(\Delta P_{ae}) + \left(\frac{H}{3}\right)(P_a)}{P_{ae}}$$

# Active Earth Pressure for Earthquake Conditions

**Table 7.6** Values of  $K_{ae}$  [Eq. (7.43)] for  $\beta = 90^\circ$  and  $k_v = 0$

$k_h$	$\delta'$ (deg)	$\alpha$ (deg)	$\phi'$ (deg)				
			28	30	35	40	45
0.1	0	0	0.427	0.397	0.328	0.268	0.217
0.2			0.508	0.473	0.396	0.382	0.270
0.3			0.611	0.569	0.478	0.400	0.334
0.4			0.753	0.697	0.581	0.488	0.409
0.5			1.005	0.890	0.716	0.596	0.500
0.1	0	5	0.457	0.423	0.347	0.282	0.227
0.2			0.554	0.514	0.424	0.349	0.285
0.3			0.690	0.635	0.522	0.431	0.356
0.4			0.942	0.825	0.653	0.535	0.442
0.5			—	—	0.855	0.673	0.551
0.1	0	10	0.497	0.457	0.371	0.299	0.238
0.2			0.623	0.570	0.461	0.375	0.303
0.3			0.856	0.748	0.585	0.472	0.383
0.4			—	—	0.780	0.604	0.486
0.5			—	—	—	0.809	0.624
0.1	$\phi'/2$	0	0.396	0.368	0.306	0.253	0.207
0.2			0.485	0.452	0.380	0.319	0.267
0.3			0.604	0.563	0.474	0.402	0.340
0.4			0.778	0.718	0.599	0.508	0.433
0.5			1.115	0.972	0.774	0.648	0.522
0.1	$\phi'/2$	5	0.428	0.396	0.326	0.268	0.218
0.2			0.537	0.497	0.412	0.342	0.283
0.3			0.699	0.640	0.526	0.438	0.367
0.4			1.025	0.881	0.690	0.568	0.475
0.5			—	—	0.962	0.752	0.620
0.1	$\phi'/2$	10	0.472	0.433	0.352	0.285	0.230
0.2			0.616	0.562	0.454	0.371	0.303
0.3			0.908	0.780	0.602	0.487	0.400
0.4			—	—	0.857	0.656	0.531
0.5			—	—	—	0.944	0.722
0.1	$\frac{2}{3}\phi'$	0	0.393	0.366	0.306	0.256	0.212
0.2			0.486	0.454	0.384	0.326	0.276
0.3			0.612	0.572	0.486	0.416	0.357
0.4			0.801	0.740	0.622	0.533	0.462
0.5			1.177	1.023	0.819	0.693	0.600
0.1	$\frac{2}{3}\phi'$	5	0.427	0.395	0.327	0.271	0.224
0.2			0.541	0.501	0.418	0.350	0.294
0.3			0.714	0.655	0.541	0.455	0.386
0.4			1.073	0.921	0.722	0.600	0.509
0.5			—	—	1.034	0.812	0.679
0.1	$\frac{2}{3}\phi'$	10	0.472	0.434	0.354	0.290	0.237
0.2			0.625	0.570	0.463	0.381	0.317
0.3			0.942	0.807	0.624	0.509	0.423
0.4			—	—	0.909	0.699	0.573
0.5			—	—	—	1.037	0.800

## Practice Problem # 3

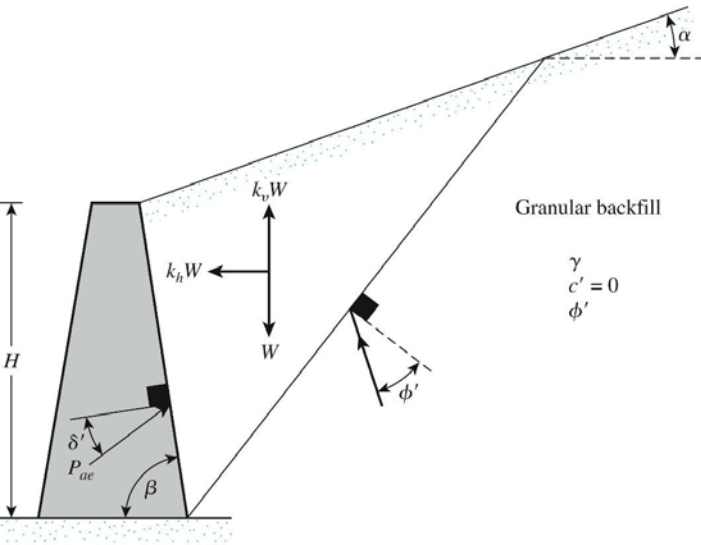


Figure 7.15 Derivation of Eq. (7.42)

Given :  $H = 6\text{m}$ ;  $\gamma = 17\text{kN/m}^3$ ;  $\phi' = 35^\circ$ ;  $\delta' = 17.5^\circ$ ;  $c' = 0$ ;  $\alpha = 10^\circ$ ; and  $\beta = 90^\circ$ .

Determine the Coulomb's active force for earthquake condition ( $P_{ae}$ ) per meter length of the wall and the location and direction of the resultant. Given  $k_h = 0.2$  and  $k_v = 0$ .

# Active Pressure for Wall Rotation about the top: Braced Cut

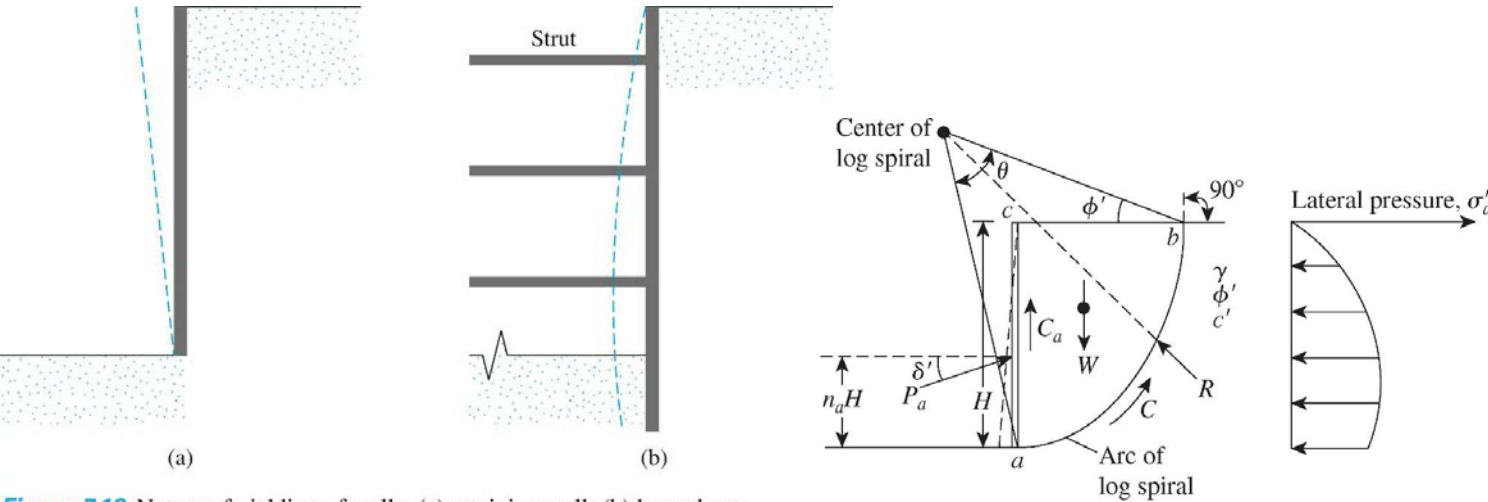


Figure 7.18 Nature of yielding of walls: (a) retaining wall; (b) braced cut

Figure 7.19 Braced cut analysis by general wedge theory: wall rotation about top

$$r = r_0 e^{\theta \tan \phi'}$$

1. Weight of the wedge,  $W$
2. Resultant of the normal and shear forces along  $ab$ ,  $R$
3. Cohesive force along  $ab$ ,  $C$
4. Adhesive force along  $ac$ ,  $C_a$
5.  $P_a$ , which is the force acting a distance  $n_a H$  from the bottom of the wall and is inclined at an angle  $\delta'$  to the horizontal

$$\text{Adhesive force, } C_a = c'_a H$$

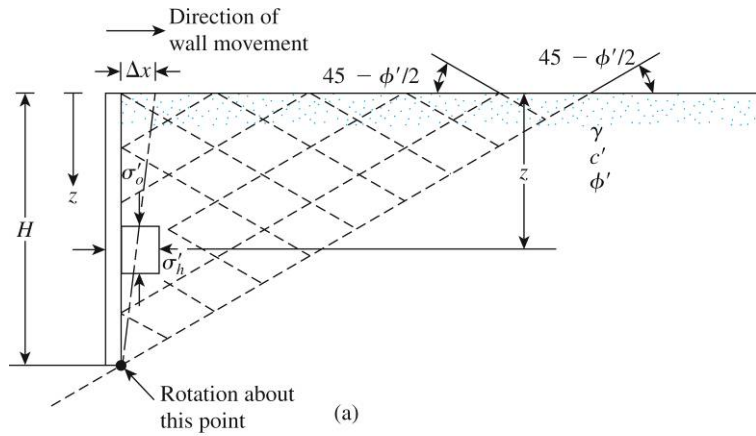
# Active Pressure for Wall Rotation about the top: Braced Cut

**Table 7.7** Active Pressure for Wall Rotation—General Wedge Theory  
(Granular Soil Backfill)

Soil friction angle, $\phi'$ (deg)	$\delta'/\phi'$	$P_a/0.5 \gamma H^2$			
		$n_a = 0.3$	$n_a = 0.4$	$n_a = 0.5$	$n_a = 0.6$
25	0	0.371	0.405	0.447	0.499
	1/2	0.345	0.376	0.413	0.460
	2/3	0.342	0.373	0.410	0.457
	1	0.344	0.375	0.413	0.461
30	0	0.304	0.330	0.361	0.400
	1/2	0.282	0.306	0.334	0.386
	2/3	0.281	0.305	0.332	0.367
	1	0.289	0.313	0.341	0.377
35	0	0.247	0.267	0.290	0.318
	1/2	0.231	0.249	0.269	0.295
	2/3	0.232	0.249	0.270	0.296
	1	0.243	0.262	0.289	0.312
40	0	0.198	0.213	0.230	0.252
	1/2	0.187	0.200	0.216	0.235
	2/3	0.190	0.204	0.220	0.239
	1	0.197	0.211	0.228	0.248
45	0	0.205	0.220	0.237	0.259
	1/2	0.149	0.159	0.171	0.185
	2/3	0.153	0.164	0.176	0.196
	1	0.173	0.184	0.198	0.215



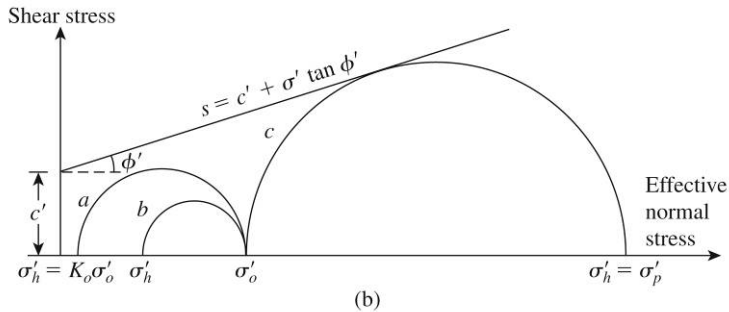
# Rankine Passive Earth Pressure



$$\sigma'_p = \sigma'_o \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

$K_p$  = Rankine passive earth pressure coefficient

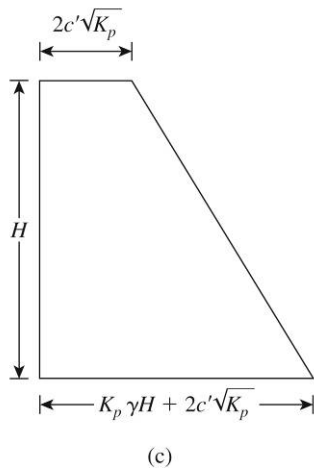
$$= \tan^2 \left( 45 + \frac{\phi'}{2} \right)$$



$$\sigma'_p = \sigma'_o K_p + 2c' \sqrt{K_p}$$

At  $z = 0$ ,  $\sigma'_o = 0$  and  $\sigma'_p = 2c' \sqrt{K_p}$

At  $z = H$ ,  $\sigma'_o = \gamma H$  and  $\sigma'_p = \gamma H K_p + 2c' \sqrt{K_p}$



Passive force per unit length of the wall :

$$P_p = \frac{1}{2} \gamma H^2 K_p + 2c' H \sqrt{K_p}$$

Figure 7.23 Rankine passive pressure

# Rankine Passive Earth Pressure: Vertical backfill and inclined backfill

## (a) Granular Soil ( $c'=0$ )

$$\sigma'_p = \gamma z K_p$$

$$P_p = \frac{1}{2} \gamma H^2 K_p$$

$$K_p = \cos \alpha \frac{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi'}}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi'}}$$

**Table 7.8** Passive Earth Pressure Coefficient  $K_p$  [from Eq. (7.67)]

$\downarrow \alpha$ (deg)	$\phi'$ (deg) $\rightarrow$						
	28	30	32	34	36	38	40
0	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10	2.551	2.775	3.022	3.295	3.598	3.937	4.316
15	2.284	2.502	2.740	3.003	3.293	3.615	3.977
20	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25	1.434	1.664	1.894	2.135	2.394	2.676	2.987

# Rankine Passive Earth Pressure: Vertical backfill and inclined backfill

(b)  $c'-\phi'$  soil

$$\sigma'_a = \gamma z K'_p = \gamma z K'_p \cos \alpha$$

$$K'_p = \frac{1}{\cos^2 \phi'} \left\{ 2 \cos^2 \alpha + 2 \left( \frac{c'}{\gamma z} \right) \cos \phi' \sin \phi' + \sqrt{4 \cos^2 \alpha (\cos^2 \alpha - \cos^2 \phi') + 4 \left( \frac{c'}{\gamma z} \right)^2 \cos^2 \phi' + 8 \left( \frac{c'}{\gamma z} \right) \cos^2 \alpha \sin \phi' \cos \phi'} \right\}$$

**Table 7.9** Values of  $K'_p$

$\phi'$ (deg)	$\alpha$ (deg)	$c'/\gamma z$			
		0.025	0.050	0.100	0.500
15	0	1.764	1.829	1.959	3.002
	5	1.716	1.783	1.917	2.971
	10	1.564	1.641	1.788	2.880
	15	1.251	1.370	1.561	2.732
20	0	2.111	2.182	2.325	3.468
	5	2.067	2.140	2.285	3.435
	10	1.932	2.010	2.162	3.339
	15	1.696	1.786	1.956	3.183
25	0	2.542	2.621	2.778	4.034
	5	2.499	2.578	2.737	3.999
	10	2.368	2.450	2.614	3.895
	15	2.147	2.236	2.409	3.726

**Table 7.9** (Continued)

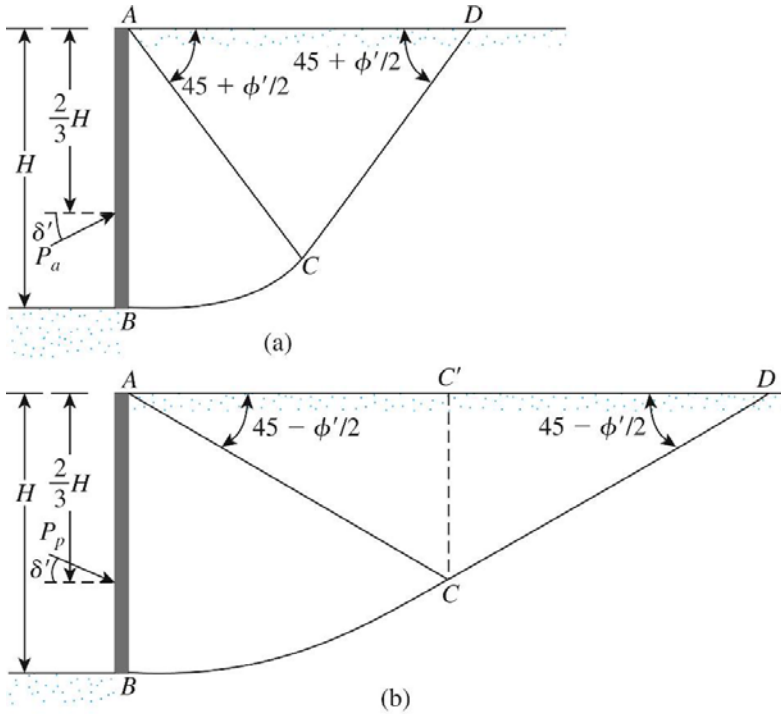
$\phi'$ (deg)	$\alpha$ (deg)	$c'/\gamma z$			
		0.025	0.050	0.100	0.500
30	0	3.087	3.173	3.346	4.732
	5	3.042	3.129	3.303	4.674
	10	2.907	2.996	3.174	4.579
	15	2.684	2.777	2.961	4.394



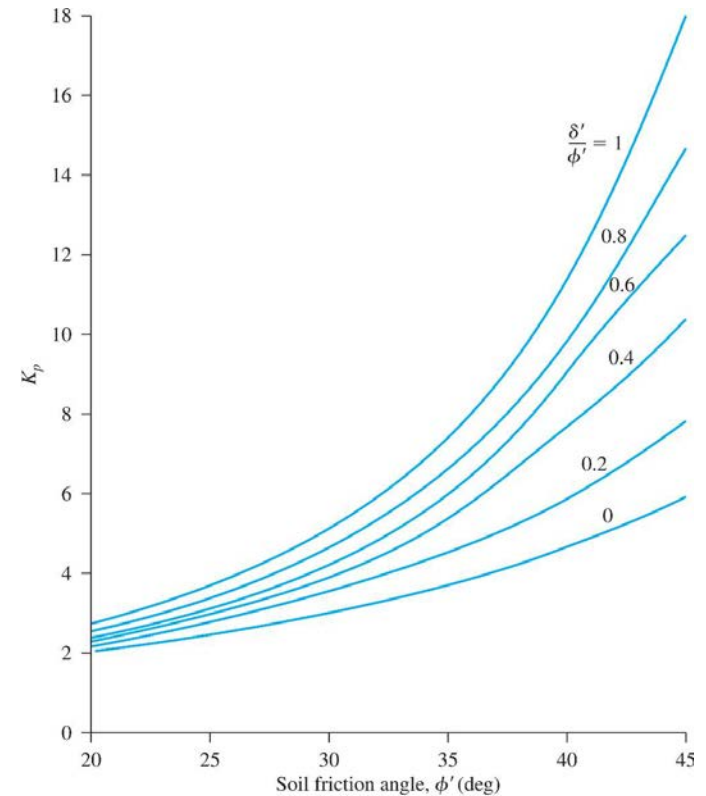
# Passive Pressure solutions by Shields & Tolunay (1973)

$$P_p = \frac{1}{2} K_p \gamma H^2$$

- 1) Horizontal granular backfill ( $c' = 0$ )
- 2) Method of slices
- 3) Logarithmic Spiral arc



**Figure 7.26** Nature of failure surface in soil with wall friction: (a) active pressure; (b) passive pressure



**Figure 7.27**  $K_p$  based on Shields and Tolunay's analysis

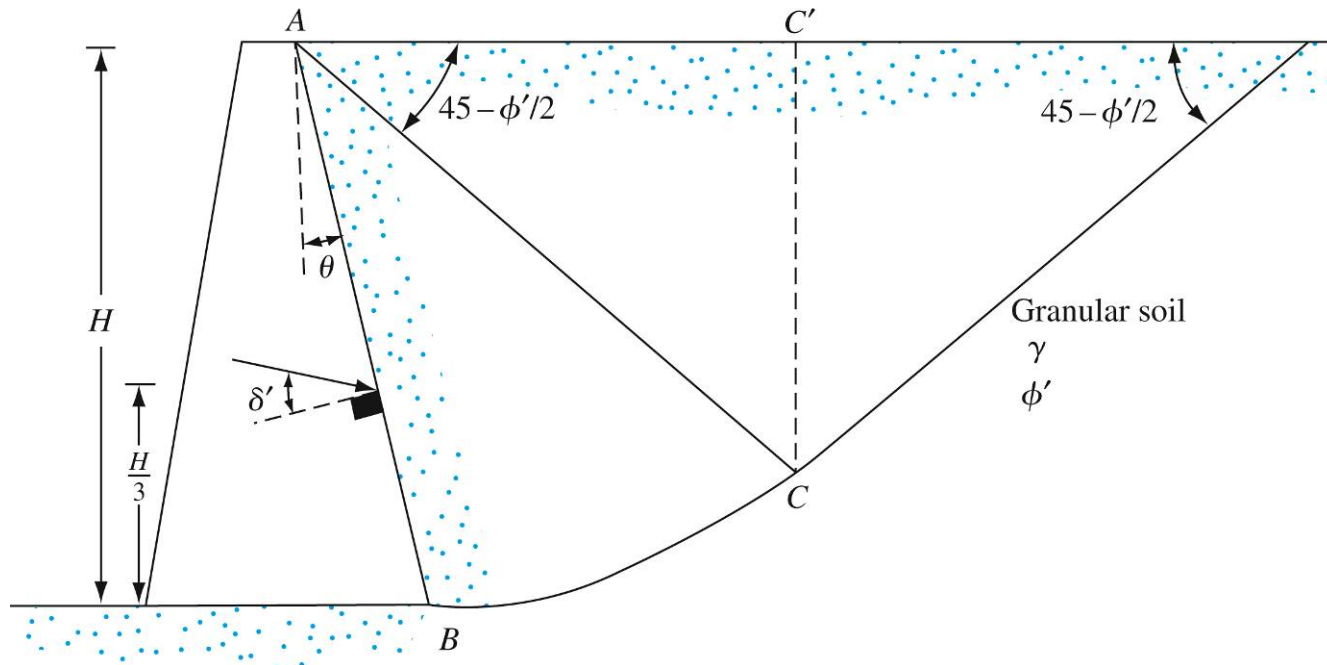
## Solution by Method of Triangular Slices (Zhu & Qian, 2000)

$$K_p = K_{p(\delta'=0)}R$$

$K_p$  = passive earth - pressure coefficient for a given value of  $\theta, \delta', \phi'$

$K_{p(\delta'=0)}$  =  $K_p$  for a given value of  $\theta, \phi'$ , and  $\delta' = 0$

$R$  = modification factor which is a function of  $\phi', \theta, \delta'/\phi'$



**Figure 7.28** Passive pressure solution by the method of triangular slices

(Note:  $BC$  is arc of a logarithmic spiral)

# Solution by Method of Triangular Slices (Zhu & Qian, 2000)

**Table 7.11** Variation of  $K_{p(\delta' = 0)}$  [see Eq. (7.73) and Figure 7.28]\*

$\phi'$ (deg)	$\theta$ (deg)						
	30	25	20	15	10	5	0
20	1.70	1.69	1.72	1.77	1.83	1.92	2.04
21	1.74	1.73	1.76	1.81	1.89	1.99	2.12
22	1.77	1.77	1.80	1.87	1.95	2.06	2.20
23	1.81	1.81	1.85	1.92	2.01	2.13	2.28
24	1.84	1.85	1.90	1.97	2.07	2.21	2.37
25	1.88	1.89	1.95	2.03	2.14	2.28	2.46
26	1.91	1.93	1.99	2.09	2.21	2.36	2.56
27	1.95	1.98	2.05	2.15	2.28	2.45	2.66
28	1.99	2.02	2.10	2.21	2.35	2.54	2.77
29	2.03	2.07	2.15	2.27	2.43	2.63	2.88
30	2.07	2.11	2.21	2.34	2.51	2.73	3.00
31	2.11	2.16	2.27	2.41	2.60	2.83	3.12
32	2.15	2.21	2.33	2.48	2.68	2.93	3.25
33	2.20	2.26	2.39	2.56	2.77	3.04	3.39
34	2.24	2.32	2.45	2.64	2.87	3.16	3.53
35	2.29	2.37	2.52	2.72	2.97	3.28	3.68
36	2.33	2.43	2.59	2.80	3.07	3.41	3.84
37	2.38	2.49	2.66	2.89	3.18	3.55	4.01
38	2.43	2.55	2.73	2.98	3.29	3.69	4.19
39	2.48	2.61	2.81	3.07	3.41	3.84	4.38
40	2.53	2.67	2.89	3.17	3.53	4.00	4.59
41	2.59	2.74	2.97	3.27	3.66	4.16	4.80
42	2.64	2.80	3.05	3.38	3.80	4.34	5.03
43	2.70	2.88	3.14	3.49	3.94	4.52	5.27
44	2.76	2.94	3.23	3.61	4.09	4.72	5.53
45	2.82	3.02	3.32	3.73	4.25	4.92	5.80

# Solution by Method of Triangular Slices (Zhu & Qian, 2000)

**Table 7.12** Variation of  $R$  [Eq. (7.73)]

$\theta$ (deg)	$\delta'/\phi'$	$R$ for $\phi'$ (deg)			
		30	35	40	45
0	0.2	1.2	1.28	1.35	1.45
	0.4	1.4	1.6	1.8	2.2
	0.6	1.65	1.95	2.4	3.2
	0.8	1.95	2.4	3.15	4.45
	1.0	2.2	2.85	3.95	6.1
5	0.2	1.2	1.25	1.32	1.4
	0.4	1.4	1.6	1.8	2.1
	0.6	1.6	1.9	2.35	3.0
	0.8	1.9	2.35	3.05	4.3
	1.0	2.15	2.8	3.8	5.7

**Table 7.12** (Continued)

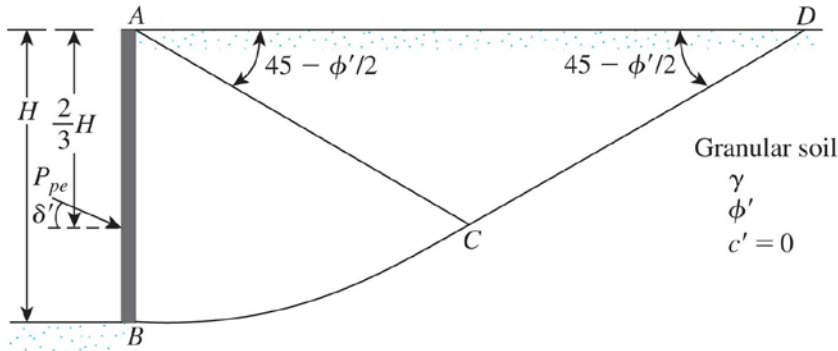
$\theta$ (deg)	$\delta'/\phi'$	$R$ for $\phi'$ (deg)			
		30	35	40	45
10	0.2	1.15	1.2	1.3	1.4
	0.4	1.35	1.5	1.7	2.0
	0.6	1.6	1.85	2.25	2.9
	0.8	1.8	2.25	2.9	4.0
	1.0	2.05	2.65	3.6	5.3
15	0.2	1.15	1.2	1.3	1.35
	0.4	1.35	1.5	1.65	1.95
	0.6	1.55	1.8	2.2	2.7
	0.8	1.8	2.2	2.8	3.8
	1.0	2.0	2.6	3.4	4.95
20	0.2	1.15	1.2	1.3	1.35
	0.4	1.35	1.45	1.65	1.9
	0.6	1.5	1.8	2.1	2.6
	0.8	1.8	2.1	2.6	3.55
	1.0	1.9	2.4	3.2	4.8

# Passive Pressure under Earthquake Conditions

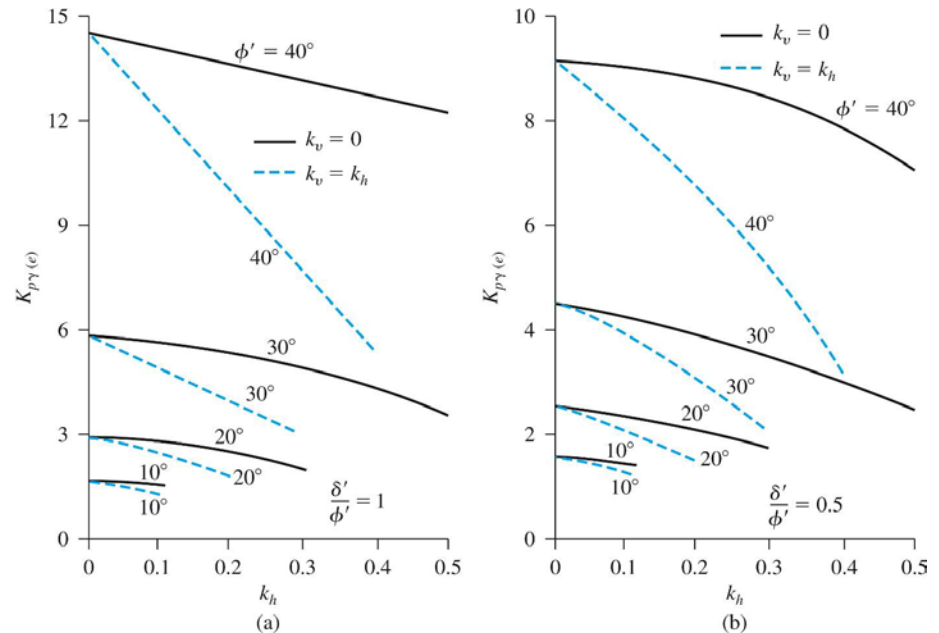
$$P_{pe} = \left[ \frac{1}{2} \gamma H^2 K_{p\gamma(e)} \right] \frac{1}{\cos \delta'}$$

(Subba Rao & Choudhury, 2005)

$K_{p\gamma(e)}$  = passive earth - pressure coefficient in the normal direction to the wall

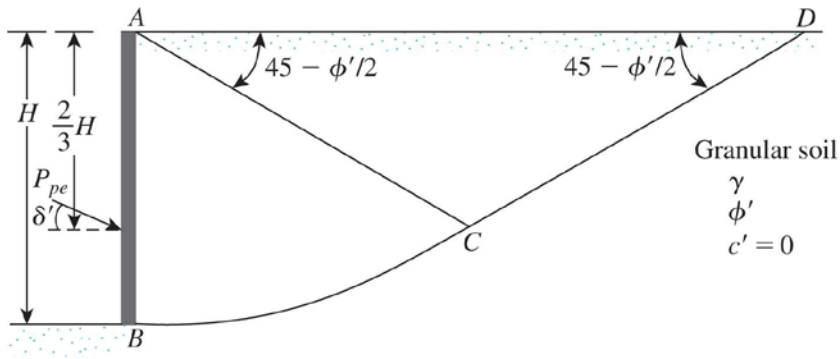


**Figure 7.29** Nature of failure surface in soil considered in the analysis to determine  $P_{pe}$



**Figure 7.30** Variation of  $K_{p\gamma(e)}$ : (a)  $\frac{\delta'}{\phi'} = 1$ , (b)  $\frac{\delta'}{\phi'} = 0.5$

## Practice Problem # 4



**Figure 7.29** Nature of failure surface in soil considered in the analysis to determine  $P_{pe}$

Consider a 4 m high retaining wall with a vertical crack and horizontal granular backfill.

Given :  $\gamma = 18 \text{ kN/m}^3$ ;  $\phi' = 40^\circ$ ;  $c' = 0$ ;  $\delta' = 20^\circ$ ;  $k_v = 0$ , and  $k_h = 0.2$

Determine the passive force  $P_{pe}$  per unit length of the wall taking the earthquake effect into consideration.