Diffy Q

#1

Separable: $dy/dx = g(x)\ f(y)$; $y' = g(x)\ f(y)$

$\int g(x) = \int f(y)$

Linear: $dy/dx + P(x)y = Qx$; $y' + P(x)y = Qx$

$\mu(x) = e^{\int P(x)dx}$ $\quad \mu y = \int \mu Q\, dx + c \Rightarrow y = \frac{1}{\mu}\int \mu Q\, dx + C$ $\quad C = y(0)$

Exact: $M(x,y)dx + N(x,y)dy = \emptyset$ if $M_y = N_x$

If exact: $F = \int M dx + h(y)$; $h'(y) = N - \frac{\partial}{\partial y}\int M\, dx$

or $F = \int N dy + h(x)$; $h'(x) = M - \frac{\partial}{\partial x}\int N\, dy$

If not exact test for both x & y dependancies

#2

test for x dependancy $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$; test for y $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$

$\mu(x) = e^{\int \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)/N}$

$\mu(y) = e^{\int \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)/M}$

$\int \psi_x = \int \tilde{M} dx \Rightarrow \psi = \int \tilde{M} dx + h(y) \Rightarrow \psi_y = \frac{\partial}{\partial y}$

$\psi_y = \tilde{N} dy \Rightarrow \psi = \int \tilde{N} dx + h(x) \Rightarrow \psi_x = \frac{\partial}{dx}$

Homo Gen : $\frac{dy}{dx} = G(y/x)$ Let $v = y/x$; $\frac{dy}{dx} = v + x\frac{dv}{dx}$

if in the form $\frac{dy}{dx} = G(ax+by)$ Let $z = ax+by$; $\frac{dz}{dx} = a_x + b_y\frac{dy}{dx}$

Bernoulli : $\frac{dy}{dx} + P(x)y = Q(x)y^n$ set eq. $y^{-n}\frac{dy}{dx} + P(x)\frac{y}{y^{-n}} = Q(x)$

let $z = y^{n-1}$ [Note from $P(x)\frac{y}{y^{-n}}$]

then solve as Linear Equation.

!! do not forget to revert z to original

$\frac{dz}{dy} = \text{der.}\, y^{n-1}\frac{dy}{dx}$

Linear Coefficient : $(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2)dy = 0$

test $a_1b_1 \neq a_2b_2$; $x = u+h$, $y = v+k$; input $a_1h + b_1k + c_1 = 0$ & $a_2h + b_2k + c_2 = 0$ find h&k

input h&k with $x = u+h$ & $y = v+k$; make equation $\frac{dv}{du} = \frac{a_1u + b_2v}{a_2u + b_2v} \Rightarrow \frac{a_1 + b_2 v/u}{a_2 + b_2 v/u}$

then sub $z = (v/u)$ & $\frac{dv}{du} = z + u\left(\frac{dz}{du}\right)$!! Sub v/u back for z & Sub y & x

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Roots

$b^2-4ac>\emptyset$	$b^2-4ac=\emptyset$	$b^2-4ac<\emptyset$
real		complex
$y=C_1e^{r_1t}+C_2e^{r_2t}$	$y=C_1e^{r_1t}+C_2te^{r_1t}$	$e^{\alpha t}(C_1\cos\beta t+C_2\sin\beta t)$
		$\alpha=-b/2a$; $\beta=\frac{\sqrt{4ac-b^2}}{2a}$

Undetermined Coefficents

$ay''+by'+cy=g(x)$; $g(x)=Ct^me^{rt}$ or $Ct^me^{\alpha t}\cos\beta t$

$g(x)=Ct^me^{rt}$

$y_p=t^s(A_mt^m+\cdots A_1t^1+A_0)e^{rt}$ this r matches to orginal equation r (Auxilary Equation)

$r=$ simple match	$r=$ double match	$r\neq$ does not match
$s=1$	$s=2$	$s=\emptyset$

$Ct^me^{\alpha t}\cos\beta t$ or $\sin\beta t$

$$y_p=t^s(A_mt^m+A_{m-1}t^{m-1}\cdots A_0)e^{\alpha t}\cos\beta t+t^s(A_mt^m\cdots A_0)e^{\alpha t}\sin\beta t$$

$s=0$ if $\alpha+\beta i\neq$ root of auxillary

$s=1$ if $\alpha+\beta i=$ root of auxillary

$y''+4y=8\sin 2t$

$r^2+4=0 \Rightarrow r=\pm 2i$

($8\sin 2t$) Here $\alpha=\emptyset$ & $\beta=2$ which $\Rightarrow \alpha=\frac{-b}{2a}=0$ $\beta=\frac{\sqrt{0^2-4(1)(4)}}{2}=\frac{\sqrt{-16}}{2}=2i$