Related Commercial Resources

CHAPTER 10

COOLING AND FREEZING TIMES OF FOODS

PRESERVATION of food is one of the most significant applications of refrigeration. Cooling and freezing food effectively reduces the activity of microorganisms and enzymes, thus retarding deterioration. In addition, crystallization of water reduces the amount of liquid water in food and inhibits microbial growth (Heldman 1975).

Most commercial food and beverage cooling and freezing operations use air-blast convection heat transfer; only a limited number of products are cooled or frozen by conduction heat transfer in plate freezers. Thus, this chapter focuses on convective heat transfer.

For air-blast convective cooling and freezing operations to be cost-effective, refrigeration equipment should fit the specific requirements of the particular cooling or freezing application. The design of such refrigeration equipment requires estimation of the cooling and freezing times of foods and beverages, as well as the corresponding refrigeration loads.

Numerous methods for predicting the cooling and freezing times of foods and beverages have been proposed, based on numerical, analytical, and empirical analysis. Selecting an appropriate estimation method from the many available methods can be challenging. This chapter reviews selected procedures available for estimating the air-blast convective cooling and freezing times of foods and beverages, and presents examples of these procedures. These procedures use the thermal properties of foods, discussed in Chapter 9.

THERMODYNAMICS OF COOLING AND FREEZING

Cooling and freezing food is a complex process. Before freezing, sensible heat must be removed from the food to decrease its temperature to the initial freezing point of the food. This initial freezing point is somewhat lower than the freezing point of pure water because of dissolved substances in the moisture within the food. At the initial freezing point, a portion of the water within the food crystallizes and the remaining solution becomes more concentrated, reducing the freezing point of the unfrozen portion of the food further. As the temperature decreases, ice crystal formation increases the concentration of the solutes in solution and depresses the freezing point further. Thus, the ice and water fractions in the frozen food, and consequently the food's thermophysical properties, depend on temperature.

Because most foods are irregularly shaped and have temperature-dependent thermophysical properties, exact analytical solutions for their cooling and freezing times cannot be derived. Most research has focused on developing semianalytical/empirical cooling and freezing time prediction methods that use simplifying assumptions.

COOLING TIMES OF FOODS AND BEVERAGES

Before a food can be frozen, its temperature must be reduced to its initial freezing point. This cooling process, also known as precooling or chilling, removes only sensible heat and, thus, no phase change occurs.

Air-blast convective cooling of foods and beverages is influenced by the ratio of the external heat transfer resistance to the internal heat transfer resistance. This ratio (the Biot number) is

$$\mathrm{Bi} = hL/k \qquad (1)$$

where h is the convective heat transfer coefficient, L is the characteristic dimension of the food, and k is the thermal conductivity of the food (see Chapter 9). In cooling time calculations, the characteristic dimension L is taken to be the shortest distance from the thermal center of the food to its surface. Thus, in cooling time calculations, L is half the thickness of a slab, or the radius of a cylinder or a sphere.

When the Biot number approaches zero ($\mathrm{Bi} < 0.1$), internal resistance to heat transfer is much less than external resistance, and the lumped-parameter approach can be used to determine a food's cooling time (Heldman 1975). When the Biot number is very large ($\mathrm{Bi} > 40$), internal resistance to heat transfer is much greater than external resistance, and the food's surface temperature can be assumed to equal the temperature of the cooling medium. For this latter situation, series solutions of the Fourier heat conduction equation are available for simple geometric shapes. When $0.1 < \mathrm{Bi} < 40$, both the internal resistance to heat transfer and the convective heat transfer coefficient must be considered. In this case, series solutions, which incorporate transcendental functions to account for the influence of the Biot number, are available for simple geometric shapes.

Simplified methods for predicting the cooling times of foods and beverages are available for regularly and irregularly shaped foods over a wide range of Biot numbers. In this chapter, these simplified methods are grouped into two main categories: (1) those based on f and j factors, and (2) those based on equivalent heat transfer dimensionality. Furthermore, the methods based on f and j factors are divided into two subgroups: (1) those for regular shapes, and (2) those for irregular shapes.

Cooling Time Estimation Methods Based on *f* and *j* Factors

All cooling processes exhibit similar behavior. After an initial lag, the temperature at the thermal center of the food decreases exponentially (Cleland 1990). As shown in Figure 1, a cooling curve depicting this behavior can be obtained by plotting, on semilogarithmic axes, the fractional unaccomplished temperature difference Y versus time. Y is defined as follows:

$$Y = \frac{T_m - T}{T_m - T_i} = \frac{T - T_m}{T_i - T_m} \qquad (2)$$

where T_m is the cooling medium temperature, T is the product temperature, and T_i is the initial temperature of the product.

This semilogarithmic temperature history curve consists of an initial curvilinear portion, followed by a linear portion. Empirical

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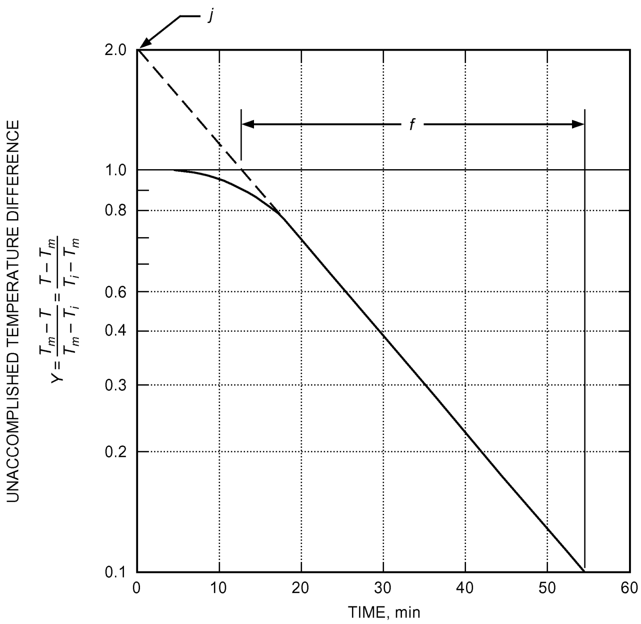

Fig. 1 Typical Cooling Curve

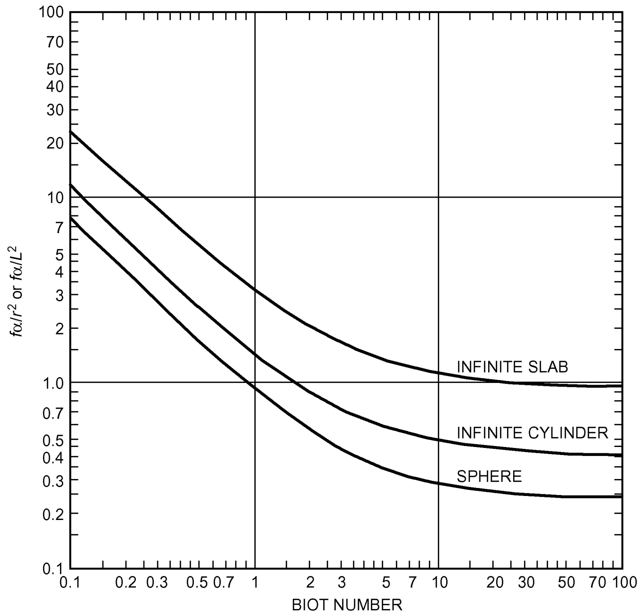

Fig. 2 Relationship Between $f\alpha/r^2$ and Biot Number for Infinite Slab, Infinite Cylinder, and Sphere

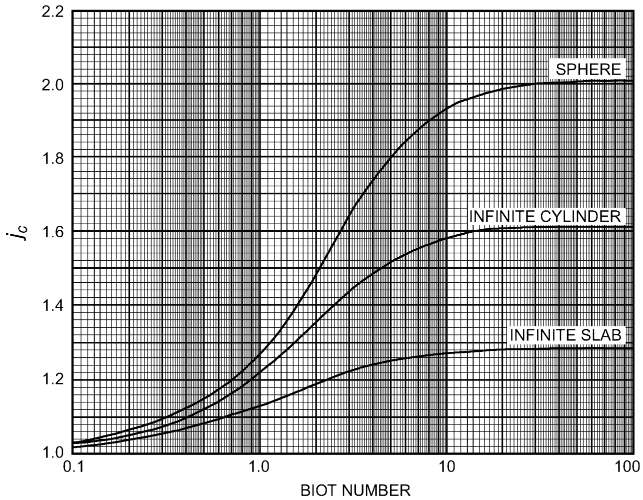

Fig. 3 Relationship Between j_c Value for Thermal Center Temperature and Biot Number for Various Shapes

formulas that model this cooling behavior incorporate two factors, *f* and *j*, which represent the slope and intercept, respectively, of the temperature history curve. The *j* factor is a measure of lag between the onset of cooling and the exponential decrease in the temperature of the food. The *f* factor represents the time required for a 90% reduction in the nondimensional temperature difference. Graphically, the *f* factor corresponds to the time required for the linear portion of the temperature history curve to pass through one log cycle. The *f* factor is a function of the Biot number, and the *j* factor is a function of the Biot number and the location within the food.

The general form of the cooling time model is

$$Y = \frac{T_m - T}{T_m - T_i} = je^{-2.303\theta/f} \tag{3}$$

where θ is the cooling time. This equation can be rearranged to give cooling time explicitly as

$$\theta = \frac{-f}{2.303}\ln\left(\frac{Y}{j}\right) \tag{4}$$

Determination of *f* and *j* Factors for Slabs, Cylinders, and Spheres

From analytical solutions, Pflug et al. (1965) developed charts for determining *f* and *j* factors for foods shaped either as infinite slabs, infinite cylinders, or spheres. They assumed uniform initial temperature distribution in the food, constant surrounding medium temperature, convective heat exchange at the surface, and constant thermophysical properties. Figure 2 can be used to determine *f* values and Figures 3 to 5 can be used to determine *j* values. Because the *j* factor is a function of location within the food, Pflug et al. presented charts for determining *j* factors for center, mass average, and surface temperatures.

As an alternative to Figures 2 to 5, Lacroix and Castaigne (1987a) presented expressions for estimating *f* and j_c factors for the thermal center temperature of infinite slabs, infinite cylinders, and spheres. These expressions, which depend on geometry and Biot number, are summarized in Tables 1 to 3. In these expressions, α is the thermal diffusivity of the food (see Chapter 9) and *L* is the characteristic dimension, defined as the shortest distance from the thermal center of the food to its surface. For an infinite slab, *L* is the half thickness. For an infinite cylinder or a sphere, *L* is the radius.

By using various combinations of infinite slabs and infinite cylinders, the *f* and *j* factors for infinite rectangular rods, finite cylinders, and rectangular bricks may be estimated. Each of these shapes can be generated by intersecting infinite slabs and infinite cylinders: two infinite slabs of proper thickness for the infinite rectangular rod, one infinite slab and one infinite cylinder for the finite cylinder, or three infinite orthogonal slabs of proper thickness for the rectangular brick. The *f* and *j* factors of these composite bodies can be estimated by

$$\frac{1}{f_{comp}} = \sum_i \left(\frac{1}{f_i}\right) \tag{5}$$

$$j_{comp} = \prod_i j_i \tag{6}$$

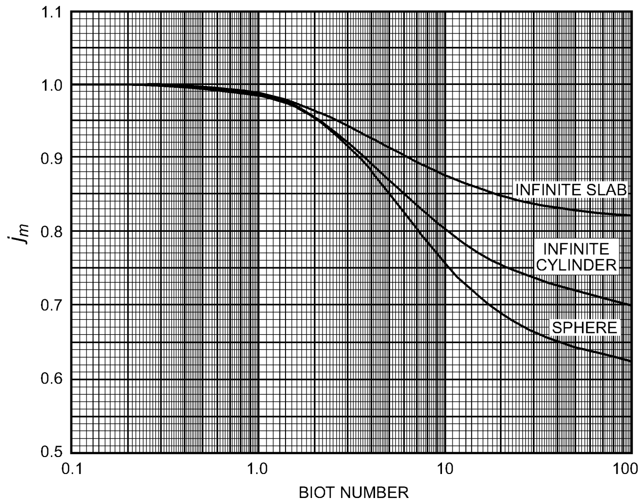

Fig. 4 Relationship Between j_m Value for Mass Average Temperature and Biot Number for Various Shapes

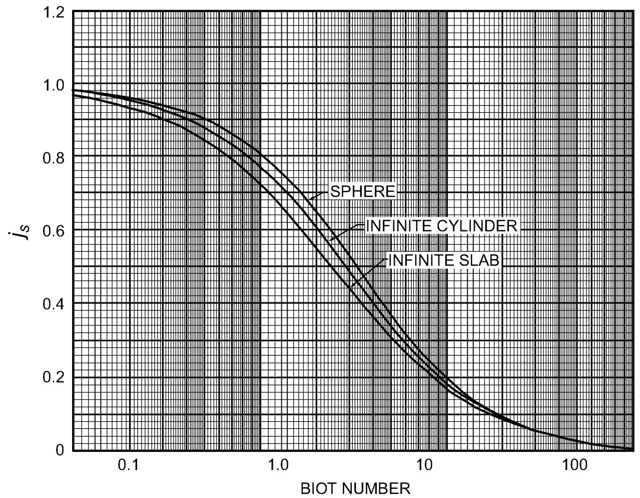

Fig. 5 Relationship Between j_s Value for Surface Temperature and Biot Number for Various Shapes

where the subscript i represents the appropriate infinite slab(s) or infinite cylinder. To evaluate the f_i and j_i of Equations (5) and (6), the Biot number must be defined, corresponding to the appropriate infinite slab(s) or infinite cylinder.

Determination of *f* and *j* Factors for Irregular Shapes

Smith et al. (1968) developed, for the case of irregularly shaped foods and Biot number approaching infinity, a shape factor called the geometry index G, which is obtained as follows:

$$G = 0.25 + \frac{3}{8B_1^2} + \frac{3}{8B_2^2} \quad (7)$$

where B_1 and B_2 are related to the cross-sectional areas of the food:

$$B_1 = \frac{A_1}{\pi L^2} \qquad B_2 = \frac{A_2}{\pi L^2} \quad (8)$$

where L is the shortest distance between the thermal center of the food and its surface, A_1 is the minimum cross-sectional area containing L, and A_2 is the cross-sectional area containing L that is orthogonal to A_1.

G is used in conjunction with the inverse of the Biot number m and a nomograph (shown in Figure 6) to obtain the characteristic value M_1^2. Smith et al. showed that the characteristic value M_1^2 can be related to the f factor by

$$f = \frac{2.303L^2}{M_1^2\alpha} \quad (9)$$

where α is the thermal diffusivity of the food. In addition, an expression for estimating a j_m factor used to determine the mass average temperature is given as

$$j_m = 0.892e^{-0.0388M_1^2} \quad (10)$$

Table 1 Expressions for Estimating *f* and j_c Factors for Thermal Center Temperature of Infinite Slabs

Biot Number Range	Equations for *f* and *j* factors
Bi ≤ 0.1	$\frac{f\alpha}{L^2} = \frac{\ln 10}{\text{Bi}}$ $j_c = 1.0$
0.1 < Bi ≤ 100	$\frac{f\alpha}{L^2} = \frac{\ln 10}{u^2}$ $j_c = \frac{2\sin u}{u + \sin u \cos u}$ *where* $u = 0.860972 + 0.312133\ln(\text{Bi}) + 0.007986[\ln(\text{Bi})]^2 - 0.016192[\ln(\text{Bi})]^3 - 0.001190[\ln(\text{Bi})]^4 + 0.000581[\ln(\text{Bi})]^5$
Bi > 100	$\frac{f\alpha}{L^2} = 0.9332$ $j_c = 1.273$

Source: Lacroix and Castaigne (1987a)

Table 2 Expressions for Estimating *f* and j_c Factors for Thermal Center Temperature of Infinite Cylinders

Biot Number Range	Equations for *f* and *j* factors
Bi ≤ 0.1	$\frac{f\alpha}{L^2} = \frac{\ln 10}{2\ \text{Bi}}$ $j_c = 1.0$
0.1 < Bi ≤ 100	$\frac{f\alpha}{L^2} = \frac{\ln 10}{v^2}$ $j_c = \frac{2J_1(v)}{v[J_0^2(v) - J_1^2(v)]}$ *where* $v = 1.257493 + 0.487941\ln(\text{Bi}) + 0.025322[\ln(\text{Bi})]^2 - 0.026568[\ln(\text{Bi})]^3 - 0.002888[\ln(\text{Bi})]^4 + 0.001078[\ln(\text{Bi})]^5$ and $J_0(v)$ and $J_1(v)$ are zero and first-order Bessel functions, respectively.
Bi > 100	$\frac{f\alpha}{L^2} = 0.3982$ $j_c = 1.6015$

Source: Lacroix and Castaigne (1987a)

Table 3 Expressions for Estimating f and j_c Factors for Thermal Center Temperature of Spheres

Biot Number Range	Equations for f and j factors
Bi ≤ 0.1	$\frac{f\alpha}{L^2} = \frac{\ln 10}{3\ \text{Bi}}$ $j_c = 1.0$
0.1 < Bi ≤ 100	$\frac{f\alpha}{L^2} = \frac{\ln 10}{w^2}$ $j_c = \frac{2(\sin w - w\cos w)}{w - \sin w \cos w}$ *where* $w = 1.573729 + 0.642906\ln(\text{Bi}) + 0.047859[\ln(\text{Bi})]^2 - 0.03553[\ln(\text{Bi})]^3 - 0.004907[\ln(\text{Bi})]^4 + 0.001563[\ln(\text{Bi})]^5$
Bi > 100	$\frac{f\alpha}{L^2} = 0.2333$ $j_c = 2.0$

Source: Lacroix and Castaigne (1987a)

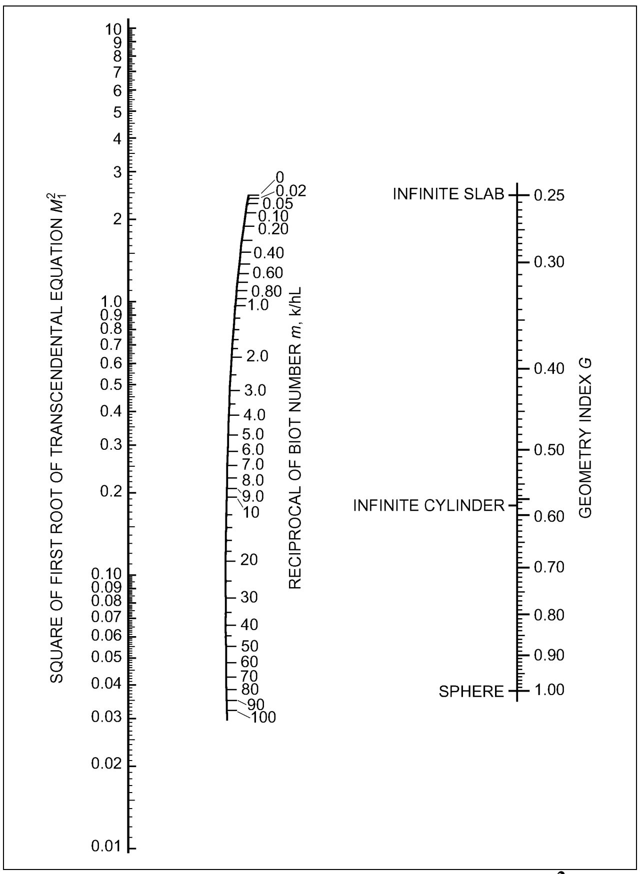

Fig. 6 Nomograph for Estimating Value of M_1^2 from Reciprocal of Biot Number and Smith's (1966) Geometry Index

As an alternative to estimating M_1^2 from the nomograph developed by Smith et al. (1968), Hayakawa and Villalobos (1989) obtained regression formulas for estimating M_1^2. For Biot numbers approaching infinity, their regression formula is

$$\ln\left(M_1^2\right) = 2.2893825 + 0.35330539X_g - 3.8044156X_g^2 - 9.6821811X_g^3 - 12.0321827X_g^4 - 7.1542411X_g^5 - 1.6301018X_g^6 \qquad (11)$$

where $X_g = \ln(G)$. Equation (11) is applicable for $0.25 \le G \le 1.0$. For finite Biot numbers, Hayakawa and Villalobos (1989) gave the following:

$$\ln\left(M_1^2\right) = 0.92083090 + 0.83409615X_g - 0.78765739X_b - 0.04821784X_g X_b - 0.04088987X_g^2 - 0.10045526X_b^2 + 0.01521388X_g^3 + 0.00119941X_g X_b^3 + 0.00129982X_b^4 \qquad (12)$$

where $X_g = \ln(G)$ and $X_b = \ln(1/\text{Bi})$. Equation (12) is applicable for $0.25 \le G \le 1.0$ and $0.01 \le 1/\text{Bi} \le 100$.

Cooling Time Estimation Methods Based on Equivalent Heat Transfer Dimensionality

Product geometry can also be considered using a shape factor called the **equivalent heat transfer dimensionality** (Cleland and Earle 1982a), which compares total heat transfer to heat transfer through the shortest dimension. Cleland and Earle developed an expression for estimating the equivalent heat transfer dimensionality of irregularly shaped foods as a function of Biot number. This overcomes the limitation of the geometry index G, which was derived for the case of Biot number approaching infinity. However, the cooling time estimation method developed by Cleland and Earle requires the use of a nomograph. Lin et al. (1993, 1996a, 1996b) expanded on this method to eliminate the need for a nomograph.

In the method of Lin et al., the cooling time of a food or beverage is estimated by a first term approximation to the analytical solution for convective cooling of a sphere:

$$\theta = \frac{3\rho c L^2}{\omega^2 k E}\ln\left(\frac{j}{Y}\right) \qquad (13)$$

Equation (13) is applicable for center temperature if $Y_c < 0.7$ and for mass average temperature if $Y_m < 0.55$, where Y_c is the fractional unaccomplished temperature difference based on final center temperature and Y_m is the fractional unaccomplished temperature difference based on final mass average temperature. In Equation (13), θ is cooling time, ρ is the food's density, c is the food's specific heat, L is the food's radius or half-thickness, k is the food's thermal conductivity, j is the lag factor, E is the equivalent heat transfer dimensionality, and ω is the first root (in radians) of the following transcendental function:

$$\omega\cot\omega + \text{Bi} - 1 = 0 \qquad (14)$$

In Equation (13), the equivalent heat transfer dimensionality E is given as a function of Biot number:

$$E = \frac{\text{Bi}^{4/3} + 1.85}{\frac{\text{Bi}^{4/3}}{E_\infty} + \frac{1.85}{E_0}} \qquad (15)$$

E_0 and E are the equivalent heat transfer dimensionalities for the limiting cases of Bi = 0 and Bi $\to \infty$, respectively. The definitions of E_0 and E use the dimensional ratios β_1 and β_2:

$$\beta_1 = \frac{\text{Second shortest dimension of food}}{\text{Shortest dimension of food}} \quad (16)$$

$$\beta_2 = \frac{\text{Longest dimension of food}}{\text{Shortest dimension of food}} \quad (17)$$

For two-dimensional, irregularly shaped foods, E_0 (the equivalent heat transfer dimensionality for Bi = 0) is given by

$$E_0 = \left(1 + \frac{1}{\beta_1}\right)\left[1 + \left(\frac{\beta_1 - 1}{2\beta_1 + 2}\right)^2\right] \quad (18)$$

For three-dimensional, irregularly shaped foods, E_0 is

$$E_0 = 1.5\frac{\beta_1 + \beta_2 + \beta_1^2(1+\beta_2) + \beta_2^2(1+\beta_1)}{\beta_1\beta_2(1+\beta_1+\beta_2)} - \frac{[(\beta_1 - \beta_2)^2]^{0.4}}{15} \quad (19)$$

For finite cylinders, bricks, and infinite rectangular rods, E_0 may be determined as follows:

$$E_0 = 1 + \frac{1}{\beta_1} + \frac{1}{\beta_2} \quad (20)$$

For spheres, infinite cylinders, and infinite slabs, E_0 = 3, 2, and 1, respectively.

For both two-dimensional and three-dimensional food items, the general form for E at Bi $\to \infty$ is given as

$$E_\infty = 0.75 + p_1 f(\beta_1) + p_2 f(\beta_2) \quad (21)$$

where

$$f(\beta) = \frac{1}{\beta^2} + 0.01 p_3 \exp\left[\beta - \frac{\beta^2}{6}\right] \quad (22)$$

with β_1 and β_2 as previously defined. The geometric parameters p_1, p_2, and p_3 are given in Table 4 for various geometries.

Lin et al. (1993, 1996a, 1996b) also developed an expression for the lag factor j_c applicable to the thermal center of a food as

$$j_c = \frac{\mathrm{Bi}^{1.35} + \frac{1}{\lambda}}{\frac{\mathrm{Bi}^{1.35}}{j_\infty} + \frac{1}{\lambda}} \quad (23)$$

where j_∞ is as follows:

$$j_\infty = 1.271 + 0.305\exp(0.172\gamma_1 - 0.115\gamma_1^2) + 0.425\exp(0.09\gamma_2 - 0.128\gamma_2^2) \quad (24)$$

and the geometric parameters λ, γ_1, and γ_2 are given in Table 4.

For the mass average temperature, Lin et al. gave the lag factor j_m as follows:

$$j_m = \mu j_c \quad (25)$$

where

$$\mu = \left(\frac{1.5 + 0.69\,\mathrm{Bi}}{1.5 + \mathrm{Bi}}\right)^N \quad (26)$$

and N is the number of dimensions of a food in which heat transfer is significant (see Table 4).

Algorithms for Estimating Cooling Time

The following suggested algorithm for estimating cooling time of foods and beverages is based on the equivalent heat transfer dimensionality method by Lin et al. (1993, 1996a, 1996b).

Table 4 Geometric Parameters

Shape	N	p_1	p_2	p_3	γ_1	γ_2	λ
Infinite slab ($\beta_1 = \beta_2 = \infty$)	1	0	0	0	∞	∞	1
Infinite rectangular rod ($\beta_1 \ge 1$, $\beta_2 = \infty$)	2	0.75	0	−1	$4\beta_1/\pi$	∞	γ_1
Brick ($\beta_1 \ge 1$, $\beta_2 \ge \beta_1$)	3	0.75	0.75	−1	$4\beta_1/\pi$	$1.5\beta_2$	γ_1
Infinite cylinder ($\beta_1 = 1$, $\beta_2 = \infty$)	2	1.01	0	0	1	∞	1
Infinite ellipse ($\beta_1 > 1$, $\beta_2 = \infty$)	2	1.01	0	1	β_1	∞	γ_1
Squat cylinder ($\beta_1 = \beta_2$, $\beta_1 \ge 1$)	3	1.01	0.75	−1	$1.225\beta_1$	$1.225\beta_2$	γ_1
Short cylinder ($\beta_1 = 1$, $\beta_2 \ge 1$)	3	1.01	0.75	−1	β_1	$1.5\beta_2$	γ_1
Sphere ($\beta_1 = \beta_2 = 1$)	3	1.01	1.24	0	1	1	1
Ellipsoid ($\beta_1 \ge 1$, $\beta_2 \ge \beta_1$)	3	1.01	1.24	1	β_1	β_2	γ_1

Source: Lin et al. (1996b)

1. Determine thermal properties of the food (see Chapter 9).
2. Determine surface heat transfer coefficient for cooling (see Chapter 9).
3. Determine characteristic dimension L and dimensional ratios β_1 and β_2 using Equations (16) and (17).
4. Calculate Biot number using Equation (1).
5. Calculate equivalent heat transfer dimensionality E for food geometry using Equation (15). This calculation requires evaluation of E_0 and E using Equations (18) to (22).
6. Calculate lag factor corresponding to thermal center and/or mass average of food using Equations (23) to (26).
7. Calculate root of transcendental equation given in Equation (14).
8. Calculate cooling time using Equation (13).

The following alternative algorithm for estimating the cooling time of foods and beverages is based on the use of f and j factors.

1. Determine thermal properties of food (see Chapter 9).
2. Determine surface heat transfer coefficient for cooling process (see Chapter 9).
3. Determine characteristic dimension L of food.
4. Calculate Biot number using Equation (1).
5. Calculate f and j factors by one of the following methods:
   (a) Method of Pflug et al. (1965): Figures 2 to 5.
   (b) Method of Lacroix and Castaigne (1987a): Tables 1, 2, and 3.
   (c) Method of Smith et al. (1968): Equations (7) to (10) and Figure 6.
   (d) Method of Hayakawa and Villalobos (1989): Equations (11) and (12) in conjunction with Equations (7) to (10).
6. Calculate cooling time using Equation (4).

SAMPLE PROBLEMS FOR ESTIMATING COOLING TIME

Example 1. A piece of ham, initially at 160°F, is to be cooled in a blast freezer. The air temperature within the freezer is 30°F and the surface heat transfer coefficient is estimated to be 8.5 Btu/h·ft²·°F. The overall dimension of the ham is 4 by 6.5 by 11 in. Estimate the time required for the mass average temperature of the ham to reach 50°F. Thermophysical properties for ham are given as follows:

c = 0.89 Btu/lb·°F
k = 0.22 Btu/h·ft·°F
ρ = 67.5 lb/ft³

Solution: Use the algorithm based on the method of Lin et al. (1993, 1996a, 1996b).

Step 1: Determine the ham's thermal properties (c, k, ρ).
These were given in the problem statement.

Step 2: Determine the heat transfer coefficient h.
The heat transfer coefficient is given as h = 8.5 Btu/h·ft²·°F.

Step 3: Determine the characteristic dimension L and dimensional ratios β_1 and β_2.

For cooling time problems, the characteristic dimension is the shortest distance from the thermal center of a food to its surface. Assuming that the thermal center of the ham coincides with its geometric center, the characteristic dimension becomes

$$L = (4/12 \text{ ft})/2 = 0.1667 \text{ ft}$$

The dimensional ratios then become [Equations (16) and (17)]

$$\beta_1 = \frac{6.5}{4} = 1.625$$

$$\beta_2 = \frac{11}{4} = 2.75$$

Step 4: Calculate the Biot number.

$$\text{Bi} = hL/k = (8.5)(0.1667)/0.22 = 6.44$$

Step 5: Calculate the heat transfer dimensionality.
Using Equation (19), E_0 becomes

$$E_0 = 1.5\frac{1.625 + 2.75 + 1.625^2(1 + 2.75) + 2.75^2(1 + 1.625)}{(1.625)(2.75)(1 + 1.625 + 2.75)} - \frac{[(1.625 - 2.75)^2]^{0.4}}{15} = 2.06$$

Assuming the ham to be ellipsoidal, the geometric factors can be obtained from Table 4:

$$p_1 = 1.01 \qquad p_2 = 1.24 \qquad p_3 = 1$$

From Equation (22),

$$f(\beta_1) = \frac{1}{1.625^2} + (0.01)(1)\exp\left(1.625 - \frac{1.625^2}{6}\right) = 0.4114$$

$$f(\beta_2) = \frac{1}{2.75^2} + (0.01)(1)\exp\left(2.75 - \frac{2.75^2}{6}\right) = 0.1766$$

From Equation (21),

$$E_\infty = 0.75 + 1.01 \times 0.4114 + 1.24 \times 0.1766 = 1.38$$

Thus, using Equation (15), the equivalent heat transfer dimensionality becomes

$$E = \frac{6.44^{4/3} + 1.85}{\dfrac{6.44^{4/3}}{1.38} + \dfrac{1.85}{2.06}} = 1.44$$

Step 6: Calculate the lag factor applicable to the mass average temperature.

From Table 4, $\lambda = \beta_1$, $\gamma_1 = \beta_1$, and $\gamma_2 = \beta_2$. Using Equation (24), j_∞ becomes

$$j_\infty = 1.271 + 0.305\exp[(0.172)(1.625) - (0.115)(1.625)^2] + 0.425\exp[(0.09)(2.75) - (0.128)(2.75)^2] = 1.78$$

Using Equation (23), the lag factor applicable to the center temperature becomes

$$j_c = \frac{6.44^{1.35} + \dfrac{1}{1.625}}{\dfrac{6.44^{1.35}}{1.78} + \dfrac{1}{1.625}} = 1.72$$

Using Equations (25) and (26), the lag factor for the mass average temperature becomes

$$j_m = \left[\frac{1.5 + (0.69)(6.44)}{1.5 + 6.44}\right]^3(1.72) = 0.721$$

Step 7: Find the root of transcendental Equation (14):

$$\omega\cot\omega + \text{Bi} - 1 = 0$$

$$\omega\cot\omega + 6.44 - 1 = 0$$

$$\omega = 2.68$$

Step 8: Calculate cooling time.
The unaccomplished temperature difference is

$$Y = \frac{T_m - T}{T_m - T_i} = \frac{30 - 50}{30 - 160} = 0.1538$$

Using Equation (13), the cooling time becomes

$$\theta = \frac{3 \times 67.5 \times 0.89(0.1667)^2}{(2.68)^2 \times 0.22 \times 1.44}\ln\left(\frac{0.721}{0.1538}\right) = 3.40 \text{ h}$$

Example 2. Repeat the cooling time calculation of Example 1, but use Hayakawa and Villalobos' (1989) estimation algorithm based on the use of f and j factors.

Solution.

Step 1: Determine the thermal properties of the ham.
The thermal properties of ham are given in Example 1.

Step 2: Determine the heat transfer coefficient.
From Example 1, h = 8.5 Btu/h·ft²·°F.

Step 3: Determine the characteristic dimension L and the dimensional ratios β_1 and β_2.
From Example 1, L = 0.1667 ft, β_1 = 1.625, β_2 = 2.75.

Step 4: Calculate the Biot number.
From Example 1, Bi = 6.44.

Step 5: Calculate the f and j factors using the method of Hayakawa and Villalobos (1989).

For simplicity, assume the cross sections of the ham to be ellipsoidal. The area of an ellipse is the product of π times half the minor axis times half the major axis, or

$$A_1 = \pi L^2\beta_1 \qquad A_2 = \pi L^2\beta_2$$

Using Equations (7) and (8), calculate the geometry index G:

$$B_1 = \frac{A_1}{\pi L^2} = \frac{\pi L^2\beta_1}{\pi L^2} = \beta_1 = 1.625$$

$$B_2 = \frac{A_2}{\pi L^2} = \frac{\pi L^2\beta_2}{\pi L^2} = \beta_2 = 2.75$$

$$G = 0.25 + \frac{3}{(8)(1.625)^2} + \frac{3}{(8)(2.75)^2} = 0.442$$

Using Equation (12), determine the characteristic value M_1^2:

$$X_g = \ln(G) = \ln(0.442) = -0.816$$

$$X_b = \ln(1/\text{Bi}) = \ln(1/6.44) = -1.86$$

$$\begin{aligned}\ln(M_1^2) = {} & 0.92083090 + 0.83409615(-0.816) - 0.78765739(-1.86) \\ & - 0.04821784(-0.816)(-1.86) - 0.04088987(-0.816)^2 \\ & - 0.10045526(-1.86)^2 + 0.01521388(-0.816)^3 \\ & + 0.00119941(-0.816)(-1.86)^3 + 0.00129982(-1.86)^4 \\ = {} & 1.27\end{aligned}$$

$$M_1^2 = 3.56$$

From Equation (9), the f factor becomes

$$f = \frac{2.303L^2}{M_1^2\alpha} = \frac{2.303L^2\rho c}{M_1^2 k}$$

$$f = \frac{(2.303)(0.1667)^2(67.5)(0.89)}{(3.56)(0.22)} = 4.91 \text{ h}$$

From Equation (10), the j factor becomes

$$j_m = 0.892e^{(-0.0388)(3.56)} = 0.777$$

Step 6: Calculate cooling time.

From Example 1, the unaccomplished temperature difference was found to be $Y = 0.1538$. Using Equation (4), the cooling time becomes

$$\theta = -\frac{4.91}{2.303}\ln\left(\frac{0.1538}{0.777}\right) = 3.45 \text{ h}$$

FREEZING TIMES OF FOODS AND BEVERAGES

As discussed at the beginning of this chapter, freezing of foods and beverages is not an isothermal process but rather occurs over a range of temperatures. This section discusses Plank's basic freezing time estimation method and its modifications; methods that calculate freezing time as the sum of the precooling, phase change, and subcooling times; and methods for irregularly shaped foods. These methods are divided into three subgroups: (1) equivalent heat transfer dimensionality, (2) mean conducting path, and (3) equivalent sphere diameter. All of these freezing time estimation methods use thermal properties of foods (Chapter 9).

Plank's Equation

One of the most widely known simple methods for estimating freezing times of foods and beverages was developed by Plank (1913, 1941). Convective heat transfer is assumed to occur between the food and the surrounding cooling medium. The temperature of the food is assumed to be at its initial freezing temperature, which is constant throughout the freezing process. Furthermore, constant thermal conductivity for the frozen region is assumed. Plank's freezing time estimation is as follows:

$$\theta = \frac{L_f}{T_f - T_m}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right) \tag{27}$$

where L_f is the volumetric latent heat of fusion (see Chapter 9), T_f is the initial freezing temperature of the food, T_m is the freezing medium temperature, D is the thickness of the slab or diameter of the sphere or infinite cylinder, h is the convective heat transfer coefficient, k_s is the thermal conductivity of the fully frozen food, and P and R are geometric factors. For an infinite slab, $P = 1/2$ and $R = 1/8$. For a sphere, $P = 1/6$ and $R = 1/24$; for an infinite cylinder, $P = 1/4$ and $R = 1/16$.

Plank's geometric factors indicate that an infinite slab of thickness D, an infinite cylinder of diameter D, and a sphere of diameter D, if exposed to the same conditions, would have freezing times in the ratio of 6:3:2. Hence, a cylinder freezes in half the time of a slab and a sphere freezes in one-third the time of a slab.

Modifications to Plank's Equation

Various researchers have noted that Plank's method does not accurately predict freezing times of foods and beverages. This is because, in part, Plank's method assumes that foods freeze at a constant temperature, and not over a range of temperatures as is the case in actual food freezing processes. In addition, the frozen food's thermal conductivity is assumed to be constant; in reality, thermal conductivity varies greatly during freezing. Another limitation of Plank's equation is that it neglects precooling and subcooling, the removal of sensible heat above and below the freezing point. Consequently, researchers have developed improved semianalytical/empirical cooling and freezing time estimation methods that account for these factors.

Cleland and Earle (1977, 1979a, 1979b) incorporated corrections to account for removal of sensible heat both above and below the food's initial freezing point as well as temperature variation during freezing. Regression equations were developed to estimate the geometric parameters P and R for infinite slabs, infinite cylinders, spheres, and rectangular bricks. In these regression equations, the effects of surface heat transfer, precooling, and final subcooling are accounted for by the Biot, Plank, and Stefan numbers, respectively.

In this section, the Biot number is defined as

$$\text{Bi} = \frac{hD}{k_s} \tag{28}$$

where h is the convective heat transfer coefficient, D is the characteristic dimension, and k_s is the thermal conductivity of the fully frozen food. In freezing time calculations, the characteristic dimension D is defined to be twice the shortest distance from the thermal center of a food to its surface: the thickness of a slab or the diameter of a cylinder or a sphere.

In general, the Plank number is defined as follows:

$$\text{Pk} = \frac{C_l(T_i - T_f)}{\Delta H} \tag{29}$$

where C_l is the volumetric specific heat of the unfrozen phase and ΔH is the food's volumetric enthalpy change between T_f and the final food temperature (see Chapter 9). The Stefan number is similarly defined as

$$\text{Ste} = \frac{C_s(T_f - T_m)}{\Delta H} \tag{30}$$

where C_s is the volumetric specific heat of the frozen phase.

In Cleland and Earle's method, Plank's original geometric factors P and R are replaced with the modified values given in Table 5, and the latent heat L_f is replaced with the volumetric enthalpy change of the food ΔH_{14} between the freezing temperature T_f and the final center temperature, assumed to be 14°F. As shown in Table 5, P and R are functions of the Plank and Stefan numbers. Both parameters should be evaluated using the enthalpy change ΔH_{14}. Thus, the modified Plank equation takes the form

$$\theta = \frac{\Delta H_{14}}{T_f - T_m}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right) \tag{31}$$

where k_s is the thermal conductivity of the fully frozen food.

Equation (31) is based on curve-fitting of experimental data in which the product final center temperature was 14°F. Cleland and Earle (1984) noted that this prediction formula does not perform as well in situations with final center temperatures other than 14°F. Cleland and Earle proposed the following modified form of Equation (31) to account for different final center temperatures:

$$\theta = \frac{\Delta H_{14}}{T_f - T_m}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right)\left[1 - \frac{1.65\,\text{Ste}}{k_s}\ln\left(\frac{T_c - T_m}{T_{ref} - T_m}\right)\right] \tag{32}$$

where T_{ref} is 14°F, T_c is the final product center temperature, and ΔH_{14} is the volumetric enthalpy difference between the initial freezing temperature T_f and 14°F. The values of P, R, Pk, and Ste should be evaluated using ΔH_{14}, as previously discussed.

Hung and Thompson (1983) also improved on Plank's equation to develop an alternative freezing time estimation method for infinite slabs. Their equation incorporates the volumetric change in enthalpy ΔH_0 for freezing as well as a weighted average temperature difference between the food's initial temperature and the freezing medium temperature. This weighted average temperature difference ΔT is given as follows:

$$\Delta T = (T_f - T_m) + \frac{(T_i - T_f)^2 \frac{C_l}{2} - (T_f - T_c)^2 \frac{C_s}{2}}{\Delta H_0} \quad (33)$$

where T_c is the food's final center temperature and ΔH_0 is its enthalpy change between initial and final center temperatures; the latter is assumed to be 0°F. Empirical equations were developed to estimate P and R for infinite slabs as follows:

$$P = 0.7306 - 1.083\,\mathrm{Pk} + \mathrm{Ste}\left(15.40U - 15.43 + 0.01329\frac{\mathrm{Ste}}{\mathrm{Bi}}\right) \quad (34)$$

$$R = 0.2079 - 0.2656U(\mathrm{Ste}) \quad (35)$$

where $U = \Delta T/(T_f - T_m)$. In these expressions, Pk and Ste should be evaluated using the enthalpy change ΔH_0. The freezing time prediction model is

$$\theta = \frac{\Delta H_0}{\Delta T}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right) \quad (36)$$

Cleland and Earle (1984) applied a correction factor to the Hung and Thompson model [Equation (36)] and improved the prediction accuracy of the model for final temperatures other than 0°F. The correction to Equation (36) is as follows:

$$\theta = \frac{\Delta H_0}{\Delta T}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right)\left[1 - \frac{1.65\,\mathrm{Ste}}{k_s}\ln\left(\frac{T_c - T_m}{T_{ref} - T_m}\right)\right] \quad (37)$$

where T_{ref} is 0°F, T_c is the product final center temperature and ΔH_0 is the volumetric enthalpy change between the initial temperature T_i and 0°F. The weighted average temperature difference ΔT, Pk, and Ste should be evaluated using ΔH_0.

Precooling, Phase Change, and Subcooling Time Calculations

Total freezing time θ is as follows:

$$\theta = \theta_1 + \theta_2 + \theta_3 \quad (38)$$

where θ_1, θ_2, and θ_3 are the precooling, phase change, and subcooling times, respectively.

DeMichelis and Calvelo (1983) suggested using Cleland and Earle's (1982a) equivalent heat transfer dimensionality method, discussed in the Cooling Times of Foods and Beverages section of this chapter, to estimate precooling and subcooling times. They also suggested that the phase change time be calculated with Plank's equation, but with the thermal conductivity of the frozen food evaluated at temperature $(T_f + T_m)/2$, where T_f is the food's initial freezing temperature and T_m is the temperature of the cooling medium.

Lacroix and Castaigne (1987a, 1987b, 1988) suggested the use of f and j factors to determine precooling and subcooling times of foods and beverages. They presented equations (see Tables 1 to 3) for estimating the values of f and j for infinite slabs, infinite cylinders, and spheres. Note that Lacroix and Castaigne based the Biot number on the shortest distance between the thermal center of the food and its surface, not twice that distance.

Lacroix and Castaigne (1987a, 1987b, 1988) gave the following expression for estimating precooling time θ_1:

$$\theta_1 = f_1 \log\left(j_1 \frac{T_m - T_i}{T_m - T_f}\right) \quad (39)$$

where T_m is the coolant temperature, T_i is the food's initial temperature, and T_f is the initial freezing point of the food. The f_1 and j_1 factors are determined from a Biot number calculated using an average thermal conductivity, which is based on the frozen and unfrozen food's thermal conductivity evaluated at $(T_f + T_m)/2$. See Chapter 9 for the evaluation of food thermal properties.

The expression for estimating subcooling time θ_3 is

$$\theta_3 = f_3 \log\left(j_3 \frac{T_m - T_f}{T_m - T_c}\right) \quad (40)$$

where T_c is the final temperature at the center of the food. The f_3 and j_3 factors are determined from a Biot number calculated using the thermal conductivity of the frozen food evaluated at the temperature $(T_f + T_m)/2$.

Lacroix and Castaigne model the phase change time θ_2 with Plank's equation:

$$\theta_2 = \frac{L_f D^2}{(T_f - T_m)k_c}\left(\frac{P}{2\mathrm{Bi}_c} + R\right) \quad (41)$$

where L_f is the food's volumetric latent heat of fusion, P and R are the original Plank geometric shape factors, k_c is the frozen food's thermal conductivity at $(T_f + T_m)/2$, and Bi_c is the Biot number for the subcooling period ($\mathrm{Bi}_c = hL/k_c$).

Lacroix and Castaigne (1987a, 1987b) adjusted P and R to obtain better agreement between predicted freezing times and experimental data. Using regression analysis, Lacroix and Castaigne suggested the following geometric factors:

For **infinite slabs**

$$P = 0.51233 \quad (42)$$

$$R = 0.15396 \quad (43)$$

For **infinite cylinders**

$$P = 0.27553 \quad (44)$$

$$R = 0.07212 \quad (45)$$

For **spheres**

$$P = 0.19665 \quad (46)$$

$$R = 0.03939 \quad (47)$$

For **rectangular bricks**

$$P = P'\left(-0.02175\frac{1}{\mathrm{Bi}_c} - 0.01956\frac{1}{\mathrm{Ste}} - 1.69657\right) \quad (48)$$

$$R = R'\left(5.57519\frac{1}{\mathrm{Bi}_c} + 0.02932\frac{1}{\mathrm{Ste}} + 1.58247\right) \quad (49)$$

For rectangular bricks, P' and R' are calculated using the expressions given in Table 5 for the P and R of bricks.

Pham (1984) also devised a freezing time estimation method, similar to Plank's equation, in which sensible heat effects were considered by calculating precooling, phase-change, and subcooling times separately. In addition, Pham suggested using a mean freezing point, assumed to be 3°F below the initial freezing point of the food, to account for freezing that takes place over a range of temperatures. Pham's freezing time estimation method is stated in terms of the

Table 5 Expressions for *P* and *R*

Shape	*P* and *R* Expressions	Applicability
Infinite slab	$P = 0.5072 + 0.2018\,\text{Pk} + \text{Ste}\left(0.3224\,\text{Pk} + \frac{0.0105}{\text{Bi}} + 0.0681\right)$ $R = 0.1684 + \text{Ste}(0.2740\,\text{Pk} - 0.0135)$	$2 \le h \le 88$ Btu/h·ft²·°F $0 \le D \le 4.7$ in. $T_i \le 104$°F $-49 \le T_m \le 5$°F
Infinite cylinder	$P = 0.3751 + 0.0999\,\text{Pk} + \text{Ste}\left(0.4008\,\text{Pk} + \frac{0.0710}{\text{Bi}} - 0.5865\right)$ $R = 0.0133 + \text{Ste}(0.0415\,\text{Pk} + 0.3957)$	$0.155 \le \text{Ste} \le 0.345$ $0.5 \le \text{Bi} \le 4.5$ $0 \le \text{Pk} \le 0.55$
Sphere	$P = 0.1084 + 0.0924\,\text{Pk} + \text{Ste}\left(0.231\,\text{Pk} - \frac{0.3114}{\text{Bi}} + 0.6739\right)$ $R = 0.0784 + \text{Ste}(0.0386\,\text{Pk} - 0.1694)$	$0.155 \le \text{Ste} \le 0.345$ $0.5 \le \text{Bi} \le 4.5$ $0 \le \text{Pk} \le 0.55$
Brick	$P = P_2 + P_1[0.1136 + \text{Ste}(5.766P_1 - 1.242)]$ $R = R_2 + R_1[0.7344 + \text{Ste}(49.89R_1 - 2.900)]$ where $P_2 = P_1\left[1.026 + 0.5808\,\text{Pk} + \text{Ste}\left(0.2296\,\text{Pk} + \frac{0.0182}{\text{Bi}} + 0.1050\right)\right]$ $R_2 = R_1[1.202 + \text{Ste}(3.410\,\text{Pk} + 0.7336)]$ and $P_1 = \frac{\beta_1\beta_2}{2(\beta_1\beta_2 + \beta_1 + \beta_2)}$ $R_1 = \frac{Q}{2}\left[(r-1)(\beta_1 - r)(\beta_2 - r)\ln\left(\frac{r}{r-1}\right) - (s-1)(\beta_1 - s)(\beta_2 - s)\ln\left(\frac{s}{s-1}\right)\right] + \frac{1}{72}(2\beta_1 + 2\beta_2 - 1)$ in which $\frac{1}{Q} = 4\left[(\beta_1 - \beta_2)(\beta_1 - 1) + (\beta_2 - 1)^2\right]^{1/2}$ $r = \frac{1}{3}\left\{\beta_1 + \beta_2 + 1 + [(\beta_1 - \beta_2)(\beta_1 - 1) + (\beta_2 - 1)^2]^{1/2}\right\}$ $s = \frac{1}{3}\left\{\beta_1 + \beta_2 + 1 - [(\beta_1 - \beta_2)(\beta_1 - 1) + (\beta_2 - 1)^2]^{1/2}\right\}$ and $\beta_1 = \frac{\text{Second shortest dimension of food}}{\text{Shortest dimension of food}}$ $\beta_2 = \frac{\text{Longest dimension of food}}{\text{Shortest dimension of food}}$	$0.155 \le \text{Ste} \le 0.345$ $0 \le \text{Pk} \le 0.55$ $0 \le \text{Bi} \le 22$ $1 \le \beta_1 \le 4$ $1 \le \beta_2 \le 4$

Source: Cleland and Earle (1977, 1979a, 1979b)

volume and surface area of the food and is, therefore, applicable to foods of any shape. This method is given as

$$\theta_i = \frac{Q_i}{hA_s\Delta T_{mi}}\left(1 + \frac{\text{Bi}_i}{k_i}\right) \qquad i = 1, 2, 3 \tag{50}$$

where θ_1 is the precooling time, θ_2 is the phase change time, θ_3 is the subcooling time, and the remaining variables are defined as shown in Table 6.

Pham (1986a) significantly simplified the previous freezing time estimation method to yield

$$\theta = \frac{V}{hA_s}\left(\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right)\left(1 + \frac{\text{Bi}_s}{4}\right) \tag{51}$$

in which

$$\begin{aligned}\Delta H_1 &= C_l(T_i - T_{fm})\\ \Delta H_2 &= L_f + C_s(T_{fm} - T_c)\end{aligned} \tag{52}$$

$$\begin{aligned}\Delta T_1 &= \frac{T_i + T_{fm}}{2} - T_m\\ \Delta T_2 &= T_{fm} - T_m\end{aligned} \tag{53}$$

where C_l and C_s are volumetric specific heats above and below freezing, respectively, T_i is the initial food temperature, L_f is the volumetric latent heat of freezing, and V is the volume of the food.

Pham suggested that the mean freezing temperature T_{fm} used in Equations (52) and (53) mainly depended on the cooling medium temperature T_m and product center temperature T_c. By curve fitting to existing experimental data, Pham (1986a) proposed the following equation to determine the mean freezing temperature for use in Equations (52) and (53):

$$T_{fm} = 23.46 + 0.263T_c + 0.105T_m \tag{54}$$

where all temperatures are in °F.

Geometric Considerations

Equivalent Heat Transfer Dimensionality. Similar to their work involving cooling times of foods, Cleland and Earle (1982b) also introduced a geometric correction factor, called the **equivalent heat transfer dimensionality *E***, to calculate the freezing times of irregularly shaped foods. The freezing time of an irregularly shaped object θ_{shape} was related to the freezing time of an infinite slab θ_{slab} using the equivalent heat transfer dimensionality:

$$\theta_{shape} = \theta_{slab}/E \tag{55}$$

Table 6 Definition of Variables for Freezing Time Estimation Method

Process	Variables
Precooling	$i = 1$ $k_1 = 6$ $Q_1 = C_l(T_i - T_{fm})V$ $\text{Bi}_1 = (\text{Bi}_l + \text{Bi}_s)/2$ $\Delta T_{m1} = \frac{(T_i - T_m) - (T_{fm} - T_m)}{\ln\left(\frac{T_i - T_m}{T_{fm} - T_m}\right)}$
Phase change	$i = 2$ $k_2 = 4$ $Q_2 = L_f V$ $\text{Bi}_2 = \text{Bi}_s$ $\Delta T_{m2} = T_{fm} - T_m$
Subcooling	$i = 3$ $k_3 = 6$ $Q_3 = C_s(T_{fm} - T_c)V$ $\text{Bi}_3 = \text{Bi}_s$ $\Delta T_{m3} = \frac{(T_{fm} - T_m) - (T_o - T_m)}{\ln\left(\frac{T_{fm} - T_m}{T_o - T_m}\right)}$

Source: Pham (1984)

Notes: A_s = area through which heat is transferred
Bi_l = Biot number for unfrozen phase
Bi_s = Biot number for frozen phase
Q_1, Q_2, Q_3 = heats of precooling, phase change, and subcooling, respectively
ΔT_{m1}, ΔT_{m2}, ΔT_{m3} = corresponding log-mean temperature driving forces
T_c = final thermal center temperature
T_{fm} = mean freezing point, assumed 3°F below initial freezing point
T_o = mean final temperature
V = volume of food

Freezing time of the infinite slab is then calculated from one of the many suitable freezing time estimation methods.

Using data collected from a large number of freezing experiments, Cleland and Earle (1982b) developed empirical correlations for the equivalent heat transfer dimensionality applicable to rectangular bricks and finite cylinders. For rectangular brick shapes with dimensions D by $\beta_1 D$ by $\beta_2 D$, the equivalent heat transfer dimensionality was given as follows:

$$E = 1 + W_1 + W_2 \tag{56}$$

where

$$W_1 = \left(\frac{\text{Bi}}{\text{Bi} + 2}\right)\frac{5}{8\beta_1^3} + \left(\frac{2}{\text{Bi} + 2}\right)\frac{2}{\beta_1(\beta_1 + 1)} \tag{57}$$

and

$$W_2 = \left(\frac{\text{Bi}}{\text{Bi} + 2}\right)\frac{5}{8\beta_2^3} + \left(\frac{2}{\text{Bi} + 2}\right)\frac{2}{\beta_2(\beta_2 + 1)} \tag{58}$$

For finite cylinders where the diameter is smaller than the height, the equivalent heat transfer dimensionality was given as

$$E = 2.0 + W_2 \tag{59}$$

In addition, Cleland et al. (1987a, 1987b) developed expressions for determining the equivalent heat transfer dimensionality of infinite slabs, infinite and finite cylinders, rectangular bricks, spheres, and two- and three-dimensional irregular shapes. Numerical methods were used to calculate the freezing or thawing times for these shapes. A nonlinear regression analysis of the resulting numerical data yielded the following form for the equivalent heat transfer dimensionality:

Table 7 Geometric Constants

Shape	G_1	G_2	G_3
Infinite slab	1	0	0
Infinite cylinder	2	0	0
Sphere	3	0	0
Finite cylinder (diameter > height)	1	2	0
Finite cylinder (height > diameter)	2	0	1
Infinite rod	1	1	0
Rectangular brick	1	1	1
Two-dimensional irregular shape	1	1	0
Three-dimensional irregular shape	1	1	1

Source: Cleland et al. (1987a)

$$E = G_1 + G_2E_1 + G_3E_2 \tag{60}$$

where

$$E_1 = X\left(2.32/\beta_1^{1.77}\right)\frac{1}{\beta_1} + \left[1 - X\left(2.32/\beta_1^{1.77}\right)\right]\frac{0.73}{\beta_1^{2.50}} \tag{61}$$

$$E_2 = X\left(2.32/\beta_1^{1.77}\right)\frac{1}{\beta_2} + \left[1 - X\left(2.32/\beta_2^{1.77}\right)\right]\frac{0.50}{\beta_2^{3.69}} \tag{62}$$

and G_1, G_2, and G_3 are given in Table 7. In Equations (61) and (62), the function X with argument ϕ is defined as

$$X(\phi) = \phi/\left(\text{Bi}^{1.34} + \phi\right) \tag{63}$$

Using the freezing time prediction methods for infinite slabs and various multidimensional shapes developed by McNabb et al. (1990), Hossain et al. (1992a) derived infinite series expressions for E of infinite rectangular rods, finite cylinders, and rectangular bricks. For most practical freezing situations, only the first term of these series expressions is significant. The resulting expressions for E are given in Table 8.

Hossain et al. (1992b) also presented a semianalytically derived expression for the equivalent heat transfer dimensionality of two-dimensional, irregularly shaped foods. An equivalent "pseudoelliptical" infinite cylinder was used to replace the actual two-dimensional, irregular shape in the calculations. A pseudoellipse is a shape that depends on the Biot number. As the Biot number approaches infinity, the shape closely resembles an ellipse. As the Biot number approaches zero, the pseudoelliptical infinite cylinder approaches an infinite rectangular rod. Hossain et al. (1992b) stated that, for practical Biot numbers, the pseudoellipse is very similar to a true ellipse. This model pseudoelliptical infinite cylinder has the same volume per unit length and characteristic dimension as the actual food. The resulting expression for E is as follows:

$$E = 1 + \frac{1 + \frac{2}{\text{Bi}}}{\beta_2^2 + \frac{2\beta_2}{\text{Bi}}} \tag{64}$$

In Equation (64), the Biot number is based on the shortest distance from the thermal center to the food's surface, not twice that distance. Using this expression for E, the freezing time θ_{shape} of two-dimensional, irregularly shaped foods can be calculated with Equation (55).

Hossain et al. (1992c) extended this analysis to predicting freezing times of three-dimensional, irregularly shaped foods. In this work, the irregularly shaped food was replaced with a model ellipsoid shape having the same volume, characteristic dimension, and smallest cross-sectional area orthogonal to the characteristic dimension, as the actual food item. An expression was presented for E of a pseudoellipsoid as follows:

Table 8 Expressions for Equivalent Heat Transfer Dimensionality

Shape	Expressions for Equivalent Heat Transfer Dimensionality E
Infinite rectangular rod ($2L$ by $2\beta_1 L$)	$E = \left(1+\frac{2}{\mathrm{Bi}}\right)\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 4\sum_{n=1}^{\infty}\left[\frac{(\sin z_n)}{z_n^3\left(1+\frac{\sin^2 z_n}{\mathrm{Bi}}\right)\left(\frac{z_n}{\mathrm{Bi}}\sinh(z_n\beta_1)+\cosh(z_n\beta_1)\right)}\right]\right\}^{-1}$ where z_n are roots of $\mathrm{Bi} = z_n \tan(z_n)$ and $\mathrm{Bi} = hL/k$, where L is the shortest distance from the center of the rectangular rod to the surface.
Finite cylinder, height exceeds diameter (radius L and height $2\beta_1 L$)	$E = \left(2+\frac{4}{\mathrm{Bi}}\right)\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 8\sum_{n=1}^{\infty}\left[y_n^3 J_1(y_n)\left(1+\frac{y_n^2}{\mathrm{Bi}^2}\right)\left(\cosh(\beta_1 y_n)+\frac{y_n}{\mathrm{Bi}}\sinh(\beta_1 y_n)\right)\right]^{-1}\right\}^{-1}$ where y_n are roots of $y_n J_1(y_n) - \mathrm{Bi}\, J_0(y_n) = 0$; J_0 and J_1 are Bessel functions of the first kind, order zero and one, respectively; and $\mathrm{Bi} = hL/k$, where L is the radius of the cylinder.
Finite cylinder, diameter exceeds height (radius $\beta_1 L$ and height $2L$)	$E = \left(1+\frac{2}{\mathrm{Bi}}\right)\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 4\sum_{n=1}^{\infty}\frac{\sin z_n}{z_n^2(z_n+\cos z_n \sin z_n)\left(I_0(z_n\beta_1)+\frac{z_n}{\mathrm{Bi}}I_1(z_n\beta_1)\right)}\right\}^{-1}$ where z_n are roots of $\mathrm{Bi} = z_n \tan(z_n)$; I_0 and I_1 are Bessel function of the second kind, order zero and one, respectively; and $\mathrm{Bi} = hL/k$, where L is the radius of the cylinder.
Rectangular brick ($2L$ by $2\beta_1 L$ by $2\beta_2 L$)	$E = \left(1+\frac{2}{\mathrm{Bi}}\right)\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 4\sum_{n=1}^{\infty}\left[\frac{\sin z_n}{z_n^3\left(1+\frac{\sin^2 z_n}{\mathrm{Bi}}\right)\left[\frac{z_n}{\mathrm{Bi}}\sinh(z_n\beta_1)+\cosh(z_n\beta_1)\right]}\right]\right.$ $- 8\beta_2^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\sin z_n \sin z_m\left[\left(\cosh(z_{nm})+\frac{z_{nm}}{\mathrm{Bi}\beta_2}\sinh(z_{nm})\right)\right.\right.$ $\left.\left.\left.z_n z_m z_{nm}^2\left(1+\frac{1}{\mathrm{Bi}}\sin^2 z_n\right)\left(1+\frac{1}{\mathrm{Bi}\beta_1}\sin^2 z_m\right)\right]^{-1}\right]\right\}^{-1}$ where z_n are roots of $\mathrm{Bi} = z_n \tan(z_n)$; z_m are the roots of $\mathrm{Bi}\beta_1 = z_m \tan(z_m)$; $\mathrm{Bi} = hL/k$, where L is the shortest distance from the thermal center of the rectangular brick to the surface; and z_{nm} is given as $z_{nm}^2 = z_n^2\beta_2^2 + z_m^2\left(\frac{\beta_2}{\beta_1}\right)^2$

Source: Hossain et al. (1992a)

Table 9 Summary of Methods for Determining Equivalent Heat Transfer Dimensionality

Slab		Cleland et al. (1987a, 1987b) Equations (60) to (63)	
Infinite cylinder		Cleland et al. (1987a, 1987b) Equations (60) to (63)	
Sphere		Cleland et al. (1987a, 1987b) Equations (60) to (63)	
Finite cylinder (diameter > height)		Cleland et al. (1987a, 1987b) Equations (60) to (63)	Hossain et al. (1992a) Table 8
Finite cylinder (height > diameter)	Cleland and Earle (1982a, 1982b) Equations (58) and (59)	Cleland et al. (1987a, 1987b) Equations (60) to (63)	Hossain et al. (1992a) Table 8
Infinite rod		Cleland et al. (1987a, 1987b) Equations (60) to (63)	Hossain et al. (1992a) Table 8
Rectangular brick	Cleland and Earle (1982a, 1982b) Equations (56) to (58)	Cleland et al. (1987a, 1987b) Equations (60) to (63)	Hossain et al. (1992a) Table 8
2-D irregular shape (infinite ellipse)		Cleland et al. (1987a, 1987b) Equations (60) to (63)	Hossain et al. (1992b) Equation (64)
3-D irregular shape (ellipsoid)		Cleland et al. (1987a, 1987b) Equations (60) to (63)	Hossain et al. (1992b) Equation (65)

$$E = 1 + \frac{1 + \frac{2}{\text{Bi}}}{\beta_1^2 + \frac{2\beta_1}{\text{Bi}}} + \frac{1 + \frac{2}{\text{Bi}}}{\beta_2^2 + \frac{2\beta_2}{\text{Bi}}} \quad (65)$$

In Equation (65), the Biot number is based on the shortest distance from the thermal center to the surface of the food, not twice that distance. With this expression for E, freezing times θ_{shape} of three-dimensional, irregularly shaped foods may be calculated using Equation (55).

Table 9 summarizes the methods that have been discussed for determining the equivalent heat transfer dimensionality of various geometries. These methods can be used with Equation (55) to calculate freezing times.

Mean Conducting Path. Pham's freezing time formulas, given in Equations (50) and (51), require knowledge of the Biot number. To calculate the Biot number of a food, its characteristic dimension must be known. Because it is difficult to determine the characteristic dimension of an irregularly shaped food, Pham (1985) introduced the concept of the **mean conducting path**, which is the mean heat transfer length from the surface of the food to its thermal center, or $D_m/2$. Thus, the Biot number becomes

$$\text{Bi} = \frac{hD_m}{k} \quad (66)$$

where D_m is twice the mean conducting path.

For rectangular blocks of food, Pham (1985) found that the mean conducting path was proportional to the geometric mean of the block's two shorter dimensions. Based on this result, Pham (1985) presented an equation to calculate the Biot number for rectangular blocks of food:

$$\frac{\text{Bi}}{\text{Bi}_o} = 1 + \left\{\left[1.5\sqrt{\beta_1} - 1\right]^{-4} + \left[\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right)\left(1 + \frac{4}{\text{Bi}_o}\right)\right]^{-4}\right\}^{-0.25} \quad (67)$$

where Bi_o is the Biot number based on the shortest dimension of the block D_1, or $\text{Bi}_o = hD_1/k$. The Biot number can then be substituted into a freezing time estimation method to calculate the freezing time for rectangular blocks.

Pham (1985) noted that, for squat-shaped foods, the mean conducting path $D_m/2$ could be reasonably estimated as the arithmetic mean of the longest and shortest distances from the surface of the food to its thermal center.

Equivalent Sphere Diameter. Ilicali and Engez (1990) and Ilicali and Hocalar (1990) introduced the **equivalent sphere diameter** concept to calculate the freezing time of irregularly shaped foods. In this method, a sphere diameter is calculated based on the volume and the volume-to-surface-area ratio of the irregularly shaped food. This equivalent sphere is then used to calculate the freezing time of the food item.

Considering an irregularly shaped food item where the shortest and longest distances from the surface to the thermal center were designated as D_1 and D_2, respectively, Ilicali and Engez (1990) and Ilicali and Hocalar (1990) defined the volume-surface diameter D_{vs} as the diameter of a sphere having the same volume-to-surface-area ratio as the irregular shape:

$$D_{vs} = 6V/A_s \quad (68)$$

where V is the volume of the irregular shape and A_s is its surface area. In addition, the volume diameter D_v is defined as the diameter of a sphere having the same volume as the irregular shape:

$$D_v = (6V/\pi)^{1/3} \quad (69)$$

Because a sphere is the solid geometry with minimum surface area per unit volume, the equivalent sphere diameter $D_{eq,s}$ must be greater than D_{vs} and smaller than D_v. In addition, the contribution of the volume diameter D_v has to decrease as the ratio of the longest to the shortest dimensions D_2/D_1 increases, because the object will be essentially two-dimensional if $D_2/D_1 \gg 1$. Therefore, the equivalent sphere diameter $D_{eq,s}$ is defined as follows:

$$D_{eq,s} = \frac{1}{\beta_2 + 1}D_v + \frac{\beta_2}{\beta_2 + 1}D_{vs} \quad (70)$$

Thus, predicting the freezing time of the irregularly shaped food is reduced to predicting the freezing time of a spherical food with diameter $D_{eq,s}$. Any of the previously discussed freezing time methods for spheres may then be used to calculate this freezing time.

Evaluation of Freezing Time Estimation Methods

As noted previously, selecting an appropriate estimation method from the plethora of available methods can be challenging for the designer. Thus, Becker and Fricke (1999a, 1999b, 1999c, 2000a, 2000b) quantitatively evaluated selected semianalytical/empirical food freezing time estimation methods for regularly and irregularly shaped foods. Each method's performance was quantified by comparing its numerical results to a comprehensive experimental freezing time data set compiled from the literature. The best-performing methods for each shape are listed in Table 10.

Algorithms for Freezing Time Estimation

The following suggested algorithm for estimating the freezing time of foods and beverages is based on the modified Plank equation presented by Cleland and Earle (1977, 1979a, 1979b). This algorithm is applicable to simple food geometries, including infinite slabs, infinite cylinders, spheres, and three-dimensional rectangular bricks.

1. Determine thermal properties of food (see Chapter 9).
2. Determine surface heat transfer coefficient for the freezing process (see Chapter 9).
3. Determine characteristic dimension D and dimensional ratios β_1 and β_2 using Equations (16) and (17).
4. Calculate Biot, Plank, and Stefan numbers using Equations (28), (29), and (30), respectively.
5. Determine geometric parameters P and R given in Table 5.
6. Calculate freezing time using Equation (31) or (32), depending on the final temperature of the frozen food.

The following algorithm for estimating freezing times of foods and beverages is based on the method of equivalent heat transfer dimensionality. It is applicable to many food geometries, including infinite rectangular rods, finite cylinders, three-dimensional rectangular bricks, and two- and three-dimensional irregular shapes.

1. Determine thermal properties of the food (see Chapter 9).

Table 10 Estimation Methods of Freezing Time of Regularly and Irregularly Shaped Foods

Shape	Methods
Infinite slab	Cleland and Earle (1977), Hung and Thompson (1983), Pham (1984, 1986a)
Infinite cylinder	Cleland and Earle (1979a), Pham (1986a), Lacroix and Castaigne (1987a)
Short cylinder	Cleland et al. (1987a, 1987b), Hossain et al. (1992a), equivalent sphere diameter technique
Rectangular brick	Cleland and Earle (1982b), Cleland et al. (1987a, 1987b), Hossain et al. (1992a)
Two-dimensional irregular shape	Hossain et al. (1992b)
Three-dimensional irregular shape	Hossain et al. (1992c), equivalent sphere diameter technique

2. Determine surface heat transfer coefficient for the freezing process (see Chapter 9).
3. Determine characteristic dimension D and dimensional ratios β_1 and β_2 using Equations (16) and (17).
4. Calculate Biot, Plank, and Stefan numbers using Equations (28), (29), and (30), respectively.
5. Calculate freezing time of an infinite slab using a suitable method. Suitable methods include
   (a) Equation (31) or (32) in conjunction with the geometric parameters P and R given in Table 5.
   (b) Equation (36) or (37) in conjunction with Equations (33), (34), and (35).
6. Calculate the food's equivalent heat transfer dimensionality. Refer to Table 9 to determine which equivalent heat transfer dimensionality method is applicable to the particular food geometry.
7. Calculate the freezing time of the food using Equation (55).

SAMPLE PROBLEMS FOR ESTIMATING FREEZING TIME

Example 3. A rectangular brick-shaped package of beef (lean sirloin) measuring 1.5 by 4.5 by 6 in. is to be frozen in a blast freezer. The beef's initial temperature is 50°F, and the freezer air temperature is –22°F. The surface heat transfer coefficient is estimated to be 7.4 Btu/h·ft^2·°F. Calculate the time required for the thermal center of the beef to reach 14°F.

Solution: Because the food is a rectangular brick, the algorithm based on the modified Plank equation by Cleland and Earle (1977, 1979a, 1979b) is used.

Step 1: Determine the thermal properties of lean sirloin.

As described in Chapter 9, the thermal properties can be calculated as follows:

Property	At –40°F (Fully Frozen)	At 14°F (Final Temp.)	At 28.9°F (Initial Freezing Point)	At 50°F (Initial Temp.)
Density, lb/ft^3	$\rho_s = 63.6$	$\rho_s = 63.6$	$\rho_l = 67.2$	$\rho_l = 67.2$
Enthalpy, Btu/lb	—	$H_s = 35.81$	$H_l = 117.8$	—
Specific heat, Btu/lb·°F	$c_s = 0.504$	—	—	$c_l = 0.840$
Thermal conductivity, Btu/(h·ft·°F)	$k_s = 0.96$	—	—	—

Volumetric enthalpy difference between the initial freezing point and 14°F:

$$\Delta H_{14} = \rho_l H_l - \rho_s H_s$$

$$\Delta H_{14} = (67.2)(117.8) - (63.6)(35.8) = 5640 \text{ Btu/ft}^3$$

Volumetric specific heats:

$$C_s = \rho_s c_s = (63.6)(0.504) = 32.05 \text{ Btu/(ft}^3\cdot°\text{F)}$$

$$C_l = \rho_l c_l = (67.2)(0.84) = 56.45 \text{ Btu/(ft}^3\cdot°\text{F)}$$

Step 2: Determine the surface heat transfer coefficient.

The surface heat transfer coefficient is estimated to be 7.4 Btu/h·ft^2·°F.

Step 3: Determine the characteristic dimension D and the dimensional ratios β_1 and β_2.

For freezing time problems, the characteristic dimension D is twice the shortest distance from the thermal center of the food to its surface. For this example, $D = 1.5/12 = 0.125$ ft.

Using Equations (16) and (17), the dimensional ratios then become

$$\beta_1 = 4.5/1.5 = 3$$

$$\beta_2 = 6.0/1.5 = 4$$

Step 4: Using Equations (28) to (30), calculate the Biot, Plank, and Stefan numbers.

$$\text{Bi} = \frac{hD}{k_s} = \frac{(7.4)(0.125)}{0.96} = 0.964$$

$$\text{Pk} = \frac{C_l(T_i - T_f)}{\Delta H_{10}} = \frac{56.45(50 - 28.9)}{5640} = 0.211$$

$$\text{Ste} = \frac{C_s(T_f - T_m)}{\Delta H_{10}} = \frac{32.05[28.9 - (-22)]}{5640} = 0.289$$

Step 5: Determine the geometric parameters P and R for the rectangular brick.

Determine P from Table 5.

$$P_1 = \frac{(3)(4)}{2[(3)(4) + 3 + 4]} = 0.316$$

$$P_2 = 0.316\left\{1.026 + (0.5808)(0.211) + 0.289\left[(0.2296)(0.211) + \frac{0.0182}{0.964} + 0.1050\right]\right\} = 0.379$$

$$P = 0.379 + 0.316\{0.1136 + 0.289[(5.766)(0.316) - 1.242]\} = 0.468$$

Determine R from Table 5.

$$\frac{1}{Q} = 4[(3-4)(3-1) + (4-1)^2]^{1/2} = 10.6$$

$$r = \frac{1}{3}\left\{3 + 4 + 1 + \left[(3-4)(3-1) + (4-1)^2\right]^{1/2}\right\} = 3.55$$

$$s = \frac{1}{3}\left\{3 + 4 + 1 - \left[(3-4)(3-1) + (4-1)^2\right]^{1/2}\right\} = 1.78$$

$$R_1 = \frac{1}{(10.6)(2)}\left\{(3.55-1)(3-3.55)(4-3.55)\ln\left[\frac{3.55}{3.55-1}\right] - (1.78-1)(3-1.78)(4-1.78)\ln\left[\frac{1.78}{1.78-1}\right]\right\} + \frac{1}{72}[(2)(3) + (2)(4) - 1] = 0.0885$$

$$R_2 = 0.0885\{1.202 + 0.289[(3.410)(0.211) + 0.7336]\} = 0.144$$

$$R = 0.144 + 0.0885\{0.7344 + 0.289[(49.89)(0.0885) - 2.900]\} = 0.248$$

Step 6: Calculate the beef's freezing time.

Because the final temperature at the thermal center of the beef is given to be 14°F, use Equation (31) to calculate the freezing time:

$$\theta = \frac{5640}{28.9 - (-22)}\left[\frac{(0.468)(0.125)}{7.4} + \frac{(0.248)(0.125)^2}{0.96}\right] = 1.32 \text{ h}$$

Example 4. Orange juice in a cylindrical container, 1.0 ft diameter by 1.5 ft tall, is to be frozen in a blast freezer. The initial temperature of the juice is 41°F and the freezer air temperature is –31°F. The surface heat transfer coefficient is estimated to be 5.3 Btu/h·ft^2·°F. Calculate the time required for the thermal center of the juice to reach 0°F.

Solution: Because the food is a finite cylinder, the algorithm based on the method of equivalent heat transfer dimensionality (Cleland et al. 1987a, 1987b) is used. This method requires calculation of the freezing time of an infinite slab, which is determined using the method of Hung and Thompson (1983).

Step 1: Determine the thermal properties of orange juice.

Using the methods described in Chapter 9, the thermal properties of orange juice are calculated as follows:

Property	At −40°F (Fully Frozen)	At 0°F (Final Temp.)	At 41°F (Initial Temp.)
Density, lb/ft^3	$\rho_s = 60.6$	$\rho_s = 60.6$	$\rho_l = 64.9$
Enthalpy, Btu/lb	—	$H_s = 17.5$	$H_l = 164$
Specific heat, Btu/lb·°F	$c_s = 0.420$	—	$c_l = 0.933$
Thermal cond., Btu/lb·ft·°F	$k_s = 1.29$	—	—
Initial freezing temperature: $T_f = 31.3$°F			

Volumetric enthalpy difference between $T_i = 41$°F, and 0°F:

$$\Delta H_0 = \rho_l H_l - \rho_s H_s$$

$$\Delta H_0 = (64.9)(164.0) - (60.6)(17.5) = 9580 \text{ Btu/ft}^3$$

Volumetric specific heats:

$$C_s = \rho_s c_s = (60.6)(0.420) = 25.45 \text{ Btu/ft}^3\cdot°\text{F}$$

$$C_l = \rho_l c_l = (64.9)(0.933) = 60.55 \text{ Btu/ft}^3\cdot°\text{F}$$

Step 2: Determine the surface heat transfer coefficient.

The surface heat transfer coefficient is estimated to be 5.3 Btu/h·ft^2·°F.

Step 3: Determine the characteristic dimension D and the dimensional ratios β_1 and β_2.

For freezing time problems, the characteristic dimension is twice the shortest distance from the thermal center of the food item to its surface. For the cylindrical sample of orange juice, the characteristic dimension is equal to the diameter of the cylinder:

$$D = 1.0 \text{ ft}$$

Using Equations (16) and (17), the dimensional ratios then become

$$\beta_1 = \beta_2 = \frac{1.5}{1.0} = 1.5$$

Step 4: Using Equations (28) to (30), calculate the Biot, Plank, and Stefan numbers.

$$\text{Bi} = \frac{hD}{k_s} = \frac{(5.3)(1.0)}{1.29} = 4.11$$

$$\text{Pk} = \frac{C_l(T_i - T_f)}{\Delta H_0} = \frac{(60.55)(41 - 31.3)}{9580} = 0.0613$$

$$\text{Ste} = \frac{C_s(T_f - T_m)}{\Delta H_0} = \frac{(25.45)[31.3 - (-31)]}{9580} = 0.166$$

Step 5: Calculate the freezing time of an infinite slab.

Use the method of Hung and Thompson (1983). First, find the weighted average temperature difference given by Equation (33).

$$\Delta T = [31.3 - (-31)] + \frac{(41 - 31.3)^2(60.55/2) - (31.3 - 0)^2(25.45/2)}{9580} = 61.3 \text{ °F}$$

Determine the parameter U:

$$U = \frac{61.3}{31.3 - (-31)} = 0.984$$

Determine the geometric parameters P and R for an infinite slab using Equations (34) and (35):

$$P = 0.7306 - (1.083)(0.0613) + (0.166)\left[(15.40)(0.984) - 15.43 + \frac{(0.01329)(0.166)}{4.11}\right] = 0.616$$

$$R = 0.2079 - (0.2656)(0.984)(0.166) = 0.165$$

Determine the freezing time of the slab using Equation (36):

$$\theta = \frac{9580}{61.3}\left[\frac{(0.616)(1.0)}{5.3} + \frac{0.165(1.0)^2}{1.29}\right] = 38.2 \text{ h}$$

Step 6: Calculate the equivalent heat transfer dimensionality for a finite cylinder.

Use the method presented by Cleland et al. (1987a, 1987b), Equations (60) to (63), to calculate the equivalent heat transfer dimensionality. From Table 7, the geometric constants for a cylinder are

$$G_1 = 2 \qquad G_2 = 0 \qquad G_3 = 1$$

Calculate E_2:

$$\phi = \frac{2.32}{\beta_2^{1.77}} = \frac{2.32}{1.5^{1.77}} = 1.132$$

$$X(1.132) = \frac{1.132}{4.11^{1.34} + 1.132} = 0.146$$

$$E_2 = \frac{0.146}{1.5} + (1 - 0.146)\frac{0.50}{1.5^{3.69}} = 0.193$$

Thus, the equivalent heat transfer dimensionality E becomes

$$E = G_1 + G_2E_1 + G_3E_2$$

$$E = 2 + (0)(E_1) + (1)(0.193) = 2.193$$

Step 7: Calculate freezing time of the orange juice using Equation (55):

$$\theta_{shape} = \theta_{slab}/E = 38.2/2.193 = 17.4 \text{ h}$$

SYMBOLS

- A_1 = cross-sectional area in Equation (8), ft^2
- A_2 = cross-sectional area in Equation (8), ft^2
- A_s = surface area of food, ft^2
- B_1 = parameter in Equation (7)
- B_2 = parameter in Equation (7)
- Bi = Biot number
- Bi_1 = Biot number for precooling = $(\text{Bi}_l + \text{Bi}_s)/2$
- Bi_2 = Biot number for phase change = Bi_s
- Bi_3 = Biot number for subcooling = Bi_s
- Bi_c = Biot number evaluated at $k_c = hD/k_c$
- Bi_l = Biot number for unfrozen food = hD/k_l
- Bi_o = Biot number based on shortest dimension = hD_1/k
- Bi_s = Biot number for fully frozen food = hD/k_s
- c = specific heat of food, Btu/lb·°F
- C_l = volumetric specific heat of unfrozen food, Btu/ft^3·°F
- C_s = volumetric specific heat of fully frozen food, Btu/ft^3·°F
- D = slab thickness or cylinder/sphere diameter, ft
- D_1 = shortest dimension, ft
- D_2 = longest dimension, ft
- $D_{eq,s}$ = equivalent sphere diameter, ft
- D_m = twice the mean conducting path, ft
- D_v = volume diameter, ft
- D_{vs} = volume-surface diameter, ft
- E = equivalent heat transfer dimensionality
- E_0 = equivalent heat transfer dimensionality at Bi = 0
- E_1 = parameter given by Equation (61)
- E_2 = parameter given by Equation (62)
- E_∞ = equivalent heat transfer dimensionality at Bi $\rightarrow \infty$
- f = cooling time parameter
- f_1 = cooling time parameter for precooling
- f_3 = cooling time parameter for subcooling
- f_{comp} = cooling parameter for a composite shape
- G = geometry index
- G_1 = geometric constant in Equation (60)
- G_2 = geometric constant in Equation (60)
- G_3 = geometric constant in Equation (60)
- h = heat transfer coefficient, Btu/h·ft^2·°F
- $I_0(x)$ = Bessel function of second kind, order zero
- $I_1(x)$ = Bessel function of second kind, order one
- j = cooling time parameter

j_1 = cooling time parameter for precooling
j_3 = cooling time parameter for subcooling
j_c = cooling time parameter applicable to thermal center
j_{comp} = cooling time parameter for a composite shape
j_m = cooling time parameter applicable to mass average
j_s = cooling time parameter applicable to surface temperature
$J_0(x)$ = Bessel function of first kind, order zero
$J_1(x)$ = Bessel function of first kind, order one
j_∞ = lag factor parameter given by Equation (24)
k = thermal conductivity of food, Btu/h·ft·°F
k_c = thermal conductivity of food evaluated at $(T_f + T_m)/2$, Btu/h·ft·°F
k_l = thermal conductivity of unfrozen food, Btu/h·ft·°F
k_s = thermal conductivity of fully frozen food, Btu/h·ft·°F
L = half thickness of slab or radius of cylinder/sphere, ft
L_f = volumetric latent heat of fusion, Btu/ft^3
m = inverse of Biot number
M_1^2 = characteristic value of Smith et al. (1968)
N = number of dimensions
p_1 = geometric parameter given in Table 4
p_2 = geometric parameter given in Table 4
p_3 = geometric parameter given in Table 4
P = Plank's geometry factor
P' = geometric factor for rectangular bricks calculated using method in Table 5
P_1 = intermediate value of Plank's geometric factor
P_2 = intermediate value of Plank's geometric factor
Pk = Plank number = $C_l(T_i - T_f)/\Delta H$
Q = parameter in Table 5
Q_1 = volumetric heat of precooling, Btu/ft^3
Q_2 = volumetric heat of phase change, Btu/ft^3
Q_3 = volumetric heat of subcooling, Btu/ft^3
r = parameter given in Table 5
R = Plank's geometry factor
R' = geometric factor for rectangular bricks calculated using method in Table 5
R_1 = intermediate value of Plank's geometric factor
R_2 = intermediate value of Plank's geometric factor
s = parameter given in Table 5
Ste = Stefan number = $C_s(T_f - T_m)/\Delta H$
T = product temperature, °F
T_c = final center temperature of food, °F
T_f = initial freezing temperature of food, °F
T_{fm} = mean freezing temperature, °F
T_i = initial temperature of food, °F
T_m = cooling or freezing medium temperature, °F
T_o = mean final temperature, °F
T_{ref} = reference temperature for freezing time correction factor, °F
u = parameter given in Table 1
U = parameter in Equations (34) and (35) = $\Delta T/(T_f - T_m)$
v = parameter given in Table 2
V = volume of food, ft^3
w = parameter given in Table 3
W_1 = parameter given by Equation (57)
W_2 = parameter given by Equation (58)
x = coordinate direction
$X(\phi)$ = function given by Equation (63)
X_b = parameter in Equation (12)
X_g = parameter in Equations (11) and (12)
y = coordinate direction
Y = fractional unaccomplished temperature difference
Y_c = fractional unaccomplished temperature difference based on final center temperature
Y_m = fractional unaccomplished temperature difference based on final mass average temperature
y_n = roots of transcendental equation; $y_n J_1(y_n) - \text{Bi}\, J_0(y_n) = 0$
z = coordinate direction
z_m = roots of transcendental equation; $\text{Bi}\,\beta_1 = z_m \tan(z_m)$
z_n = roots of transcendental equation; $\text{Bi} = z_n \tan(z_n)$
z_{nm} = parameter given in Table 8

Greek

α = thermal diffusivity of food, ft^2/h
β_1 = ratio of second shortest dimension to shortest dimension, Equation (16)
β_2 = ratio of longest dimension to shortest dimension, Equation (17)
γ_1 = geometric parameter from Lin et al. (1996b)
γ_2 = geometric parameter from Lin et al. (1996b)
ΔH = volumetric enthalpy difference, Btu/ft^3
ΔH_1 = volumetric enthalpy difference = $C_l(T_i - T_{fm})$, Btu/ft^3
ΔH_2 = volumetric enthalpy difference = $L_f + C_s(T_{fm} - T_c)$, Btu/ft^3
ΔH_{14} = volumetric enthalpy difference between initial freezing temperature T_f and 14°F, Btu/ft^3
ΔH_0 = volumetric enthalpy difference between initial temperature T_i and 0°F, Btu/ft^3
ΔT = weighted average temperature difference in Equation (33), °F
ΔT_1 = temperature difference = $(T_i + T_{fm})/2 - T_m$, °F
ΔT_2 = temperature difference = $T_{fm} - T_m$, °F
ΔT_{m1} = temperature difference for precooling, °F
ΔT_{m2} = temperature difference for phase change, °F
ΔT_{m3} = temperature difference for subcooling, °F
θ = cooling or freezing time, h
θ_1 = precooling time, h
θ_2 = phase change time, h
θ_3 = tempering time, h
θ_{shape} = freezing time of an irregularly shaped food, h
θ_{slab} = freezing time of an infinite slab-shaped food, h
λ = geometric parameter from Lin et al. (1996b)
μ = parameter given by Equation (26)
ρ = density of food, lb/ft^3
ϕ = argument of function X, Equation (63)
ω = first root of Equation (14)

REFERENCES

Becker, B.R. and B.A. Fricke. 1999a. Evaluation of semi-analytical/empirical freezing time estimation methods, part I: Regularly shaped food items. *International Journal of HVAC&R Research* (now *HVAC&R Research*) 5(2):151-169.

Becker, B.R. and B.A. Fricke. 1999b. Evaluation of semi-analytical/empirical freezing time estimation methods, part II: Irregularly shaped food items. *International Journal of HVAC&R Research* (now *HVAC&R Research*) 5(2):171-187.

Becker, B.R. and B.A. Fricke. 1999c. Freezing times of regularly shaped food items. *International Communications in Heat and Mass Transfer* 26(5):617-626.

Becker, B.R. and B.A. Fricke. 2000a. Evaluation of semi-analytical/empirical freezing time estimation methods, part I: Regularly shaped food items (RP-888). *Technical Paper* 4352, presented at the ASHRAE Winter Meeting, Dallas.

Becker, B.R. and B.A. Fricke. 2000b. Evaluation of semi-analytical/empirical freezing time estimation methods, part II: Irregularly shaped food items (RP-888). *Technical Paper* 4353, presented at the ASHRAE Winter Meeting, Dallas.

Cleland, A.C. 1990. *Food refrigeration processes: Analysis, design and simulation*. Elsevier Science, London.

Cleland, A.C. and R.L. Earle. 1977. A comparison of analytical and numerical methods of predicting the freezing times of foods. *Journal of Food Science* 42(5):1390-1395.

Cleland, A.C. and R.L. Earle. 1979a. A comparison of methods for predicting the freezing times of cylindrical and spherical foodstuffs. *Journal of Food Science* 44(4):958-963, 970.

Cleland, A.C. and R.L. Earle. 1979b. Prediction of freezing times for foods in rectangular packages. *Journal of Food Science* 44(4):964-970.

Cleland, A.C. and R.L. Earle. 1982a. A simple method for prediction of heating and cooling rates in solids of various shapes. *International Journal of Refrigeration* 5(2):98-106.

Cleland, A.C. and R.L. Earle. 1982b. Freezing time prediction for foods—A simplified procedure. *International Journal of Refrigeration* 5(3): 134-140.

Cleland, A.C. and R.L. Earle. 1984. Freezing time predictions for different final product temperatures. *Journal of Food Science* 49(4):1230-1232.

Cleland, D.J., A.C. Cleland, and R.L. Earle. 1987a. Prediction of freezing and thawing times for multi-dimensional shapes by simple formulae—Part 1: Regular shapes. *International Journal of Refrigeration* 10(3): 156-164.

Cleland, D.J., A.C. Cleland, and R.L. Earle. 1987b. Prediction of freezing and thawing times for multi-dimensional shapes by simple formulae—Part 2: Irregular shapes. *International Journal of Refrigeration* 10(4): 234-240.

DeMichelis, A. and A. Calvelo. 1983. Freezing time predictions for brick and cylindrical-shaped foods. *Journal of Food Science* 48: 909-913, 934.

Hayakawa, K. and G. Villalobos. 1989. Formulas for estimating Smith et al. parameters to determine the mass average temperature of irregularly shaped bodies. *Journal of Food Process Engineering* 11(4):237-256.

Heldman, D.R. 1975. *Food process engineering*. AVI, Westport, CT.

Hossain, M.M., D.J. Cleland, and A.C. Cleland. 1992a. Prediction of freezing and thawing times for foods of regular multi-dimensional shape by using an analytically derived geometric factor. *International Journal of Refrigeration* 15(4):227-234.

Hossain, M.M., D.J. Cleland, and A.C. Cleland. 1992b. Prediction of freezing and thawing times for foods of two-dimensional irregular shape by using a semi-analytical geometric factor. *International Journal of Refrigeration* 15(4):235-240.

Hossain, M.M., D.J. Cleland, and A.C. Cleland. 1992c. Prediction of freezing and thawing times for foods of three-dimensional irregular shape by using a semi-analytical geometric factor. *International Journal of Refrigeration* 15(4):241-246.

Hung, Y.C. and D.R. Thompson. 1983. Freezing time prediction for slab shape foodstuffs by an improved analytical method. *Journal of Food Science* 48(2):555-560.

Ilicali, C. and S.T. Engez. 1990. A simplified approach for predicting the freezing or thawing times of foods having brick or finite cylinder shape. In *Engineering and food*, vol. 2, pp. 442-456. W.E.L. Speiss and H. Schubert, eds. Elsevier Applied Science, London.

Ilicali, C. and M. Hocalar. 1990. A simplified approach for predicting the freezing times of foodstuffs of anomalous shape. In *Engineering and food*, vol. 2, pp. 418-425. W.E.L. Speiss and H. Schubert, eds. Elsevier Applied Science, London.

Lacroix, C. and F. Castaigne. 1987a. Simple method for freezing time calculations for infinite flat slabs, infinite cylinders and spheres. *Canadian Institute of Food Science and Technology Journal* 20(4):252-259.

Lacroix, C. and F. Castaigne. 1987b. Simple method for freezing time calculations for brick and cylindrical shaped food products. *Canadian Institute of Food Science and Technology Journal* 20(5):342-349.

Lacroix, C. and F. Castaigne. 1988. Freezing time calculation for products with simple geometrical shapes. *Journal of Food Process Engineering* 10(2):81-104.

Lin, Z., A.C. Cleland, G.F. Serrallach, and D.J. Cleland. 1993. Prediction of chilling times for objects of regular multi-dimensional shapes using a general geometric factor. *Refrigeration Science and Technology* 1993-3:259-267.

Lin, Z., A.C. Cleland, D.J. Cleland, and G.F. Serrallach. 1996a. A simple method for prediction of chilling times for objects of two-dimensional irregular shape. *International Journal of Refrigeration* 19(2):95-106.

Lin, Z., A.C. Cleland, D.J. Cleland, and G.F. Serrallach. 1996b. A simple method for prediction of chilling times: Extension to three-dimensional irregular shaped. *International Journal of Refrigeration* 19(2):107-114.

McNabb, A., G.C. Wake, and M.M. Hossain. 1990. Transition times between steady states for heat conduction: Part I—General theory and some exact results. *Occasional Publications in Mathematics and Statistics* 20, Massey University, New Zealand.

Pflug, I.J., J.L. Blaisdell, and J. Kopelman. 1965. Developing temperature-time curves for objects that can be approximated by a sphere, infinite plate, or infinite cylinder. *ASHRAE Transactions* 71(1):238-248.

Pham, Q.T. 1984. An extension to Plank's equation for predicting freezing times for foodstuffs of simple shapes. *International Journal of Refrigeration* 7:377-383.

Pham, Q.T. 1985. Analytical method for predicting freezing times of rectangular blocks of foodstuffs. *International Journal of Refrigeration* 8(1): 43-47.

Pham, Q.T. 1986a. Simplified equation for predicting the freezing time of foodstuffs. *Journal of Food Technology* 21(2):209-219.

Pham, Q.T. 1986b. Freezing of foodstuffs with variations in environmental conditions. *International Journal of Refrigeration* 9(5):290-295.

Pham, Q.T. 1987. A converging-front model for the asymmetric freezing of slab-shaped food. *Journal of Food Science* 52(3):795-800.

Pham, Q.T. 1991. Shape factors for the freezing time of ellipses and ellipsoids. *Journal of Food Engineering* 13:159-170.

Plank, R. 1913. Die Gefrierdauer von Eisblocken. *Zeitschrift für die gesamte Kälte Industrie* 20(6):109-114.

Plank, R. 1941. Beitrage zur Berechnung und Bewertung der Gefriergeschwindigkeit von Lebensmitteln. *Zeitschrift für die gesamte Kälte Industrie* 3(10):1-24.

Smith, R.E. 1966. *Analysis of transient heat transfer from anomalous shape with heterogeneous properties*. Ph.D. dissertation, Oklahoma State University, Stillwater.

Smith, R.E., G.L. Nelson, and R.L. Henrickson. 1968. Applications of geometry analysis of anomalous shapes to problems in transient heat transfer. *Transactions of the ASAE* 11(2):296-302.

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