(a) Draw the graph of $F(y)$.
(b) Compute the pf of Y.
(c) Compute $P(0 \le Y \le 2)$.

3.5 Consider the experiment of selecting one coin at random from an urn containing four pennies, three nickels, two dimes, and one quarter. Let X denote the monetary value of the coin that is selected. Compute the probability function of X.

3.6 The number of customers who make a reservation for a limousine service to an airport is a random variable X with probability function given by

$$f(x) = \frac{1}{21}(5 - |x - 3|), \quad x = 1, 2, 3, 4, 5, 6$$

The limousine has a capacity of five passengers and the cost is \$10 per passenger. Consequently, the revenue R per trip is also a random variable. Compute the probability function of R.

3.7 Consider a deck of five cards marked 1, 2, 3, 4, 5. Two of these cards are picked at random and without replacement; let W = sum of the numbers picked. Compute the pf of W.

3.8 Suppose a lot of 100 items contains 10 defective ones, and a sample of 5 items is selected at random from the lot. Let X denote the number of defective items in the sample; compute $P(X = x)$ for $x = 0, 1, 2, 3, 4, 5$. (*Hint:* This problem is similar to the computation of the probability function of the number of hearts in a five-card poker hand [Eq. (3.5)], with "defective" and "nondefective" substituted for "heart" and "nonheart.")

3.9 Suppose a lot of 50 items contains 4 defective ones, and a sample of 5 items is selected at random from the lot. Let X denote the number of defective items in the sample; compute the probability function of X.

3.10 Let X have the pf given by

$$f(x) = cx \quad \text{for } x = 1, 2, 3, 4, 5, 6; \qquad f(x) = 0 \quad \text{elsewhere}$$

(a) Compute c.
(b) Compute $F(x)$ for $x = 0, 1, 5, 7$.
(c) Compute $P(X \text{ is odd})$.
(d) Compute $P(X \text{ is even})$.
(e) Compute $P(X < 4)$.
(f) Compute $P(2 \le X \le 4)$.

3.11 Let X have the pf given by

$$f(x) = cx \quad \text{for } x = 1, 2, \ldots, n; \qquad f(x) = 0 \quad \text{elsewhere}$$

(a) Show that

$$c = \frac{2}{n(n+1)}$$

(b) Show that

$$F(x) = \frac{x(x+1)}{n(n+1)} \quad \text{for } x = 1, 2, \ldots, n$$

PROBLEMS

HW

3.13 The number of defects on a printed circuit board is a random variable X with pf given by

$$P(X = i) = c/(i+1), \quad i = 0, 1, \ldots, 4$$

(a) Compute the constant c.

(b) Compute the mean and variance of X.

C

3.14 The number of cells (out of 100) that exhibit chromosome aberrations is a random variable X with pf given by

$$P(X = i) = \frac{c(i+1)^2}{2^{i+1}}, \quad i = 0, 1, 2, 3, 4, 5$$

(a) Determine the value of the constant c.

(b) Compute the mean and variance of X.

PROBLEMS

3.29 The random variable X has a hypergeometric distribution with parameters $n = 4, N = 20, D = 6$.

(a) Compute $h(x), x = 0, 1, 2, 3, 4$, using Eq. (3.27).

(b) Compute $h(x), x = 0, 1, 2, 3, 4$, using the recurrence formula (3.30).

Which method is more efficient for computing the hypergeometric probabilities?

3.30 A lot of 10 components contains 2 that are defective. A random sample of size 4 is drawn. Let X denote the number of defective components drawn.

(a) Show that X has a hypergeometric distribution and identify the parameters N, D, n.

Use the formula from part (*a*) to compute the following probabilities:

(b) $P(X = 0)$ **(c)** $P(X = 1)$ **(d)** $P(X > 1)$

3.31 Suppose the sampling plan is to sample 10 percent of a lot of N items with the acceptance number $c = 0$. Compute the acceptance probability in each of the following cases:

(a) $N = 50, D = 1$ **(b)** $N = 100, D = 2$ **(c)** $N = 200, D = 4$

3.32 Suppose a sample of size 15 is taken from a lot of 50 items, and the lot is accepted if the number X of defective items in the sample is less than or equal to 1 (i.e., we accept the lot if $X \leq 1$). Compute P(acceptance) for the following values of D:

(a) $D = 1$ **(b)** $D = 3$ **(c)** $D = 6$

3.33 A group of 10 items contains 2 that are defective. Articles are taken one at a time from the lot and tested. Let Y denote the number of articles tested in order to find the first defective item. Compute the pf of Y.

3.34 An urn contains two nickels and three dimes. We select two coins at random, without replacement. Let X = monetary value of the coins in the sample; e.g., if the sample consists of a nickel and dime, $X = 15$ cents.

(a) Compute the pf of X. **(b)** Compute $E(X)$. **(c)** Compute $V(X)$.

3.35 An urn contains two white balls and four red balls. Balls are drawn one by one without replacement until two white balls have appeared. Denote by U the number of balls that must be drawn until both white balls have appeared. Compute the pf of U.

(b) Compute the acceptance probabilities for $D = 50$ and acceptance number $c = 1$.
(c) Compute the acceptance probabilities for $D = 50$ and acceptance number $c = 0$.

3.45 It is known that 40 percent of patients naturally recover from a certain disease. A drug is claimed to cure 80 percent of all patients with the disease. An experimental group of 20 patients are given the drug. What is the probability that at least 12 recover if:
(a) The drug is worthless and hence the true recovery rate 40 percent? p = .4
(b) The drug is really 80 percent effective? p = .8

3.46 (Continuation of Prob. 3.45) Suppose 200 patients are given the drug. Let R denote the number of patients who recover. Compute $E(R)$ and $V(R)$ for each of the following cases:
(a) The drug is worthless.
(b) The drug is really 80 percent effective as claimed.

3.47 Use Chebyshev's inequality to estimate the probability that the sample proportion obtained from a poll of voters with a sample size of $n = 2500$ differs from the true proportion by an amount less than or equal to 0.04.

3.48 Let X denote the number of heads in n tosses of a fair coin. Use Chebyshev's inequality to obtain a lower bound on the following probabilities:
(a) $P(225 \le X \le 275), n = 500$
(b) $P(80 \le X \le 120), n = 200$
(c) $P(45 \le X \le 55), n = 100$

3.49 R. Wolf (1882) threw a die 20,000 times and recorded the number of times each of the six faces appeared. The results follow.

Face	1	2	3	4	5	6
Frequency	3407	3631	3176	2916	3448	3422

Source: D. J. Hand et al., *Small Data Sets*, London, Chapman & Hall, 1994.

(a) Compute an upper bound on the probability that the number of times the number 5 appeared is greater than or equal to the observed value, 3448. Assume that the dice are fair, and use Chebyshev's inequality to obtain the upper bound.
(b) Compute an upper bound on the probability that the number of times the number 4 appeared is less than or equal to the observed value, 2916.

3.50 Can Chebyshev's inequality be improved? Consider the random variable X with the following probability function:

$$P(X = \pm a) = p, \qquad P(X = 0) = 1 - 2p, \quad 0 < p < 0.5$$

Show that

$$P(|X| \ge a) = \frac{V(X)}{a^2}$$

3.55 **(a)** Compute the Poisson probabilities $p(x; 1.5)$, $x = 0, 1, 2, 3$, using Eq. (3.47).

(b) Compute the Poisson probabilities $p(x; 1.5)$, $x = 0, 1, 2, 3$, using the formula

$$p(x; \lambda) = P(x; \lambda) - P(x - 1; \lambda)$$

where the values of the distribution function

$$P(x; \lambda) = \sum_{0 \le y \le x} p(y; \lambda)$$

are provided in Table A.2 in the appendix.

3.56 A lot of 10,000 articles has 150 defective items. A random sample of 100 articles is selected. Let X denote the number of defective items in the sample. Give the formula for:

(a) The *exact* pf $f_X(x)$ of X.

(b) The binomial approximation of $f_X(x)$.

(c) The Poisson approximation of the binomial distribution of part (*b*).

3.57 If 99 percent of the students entering a junior high school are vaccinated against the measles, what is the probability that a group of 50 students includes:

(a) No unvaccinated students?

(b) Exactly one unvaccinated student?

(c) Two or more unvaccinated students?

3.58 A batch of dough yields 500 cookies. How many chocolate chips should be mixed into the dough so that the probability of a cookie having no chocolate chips is ≤ 0.01?

3.59 A taxi company has a limousine with a seating capacity of N. The cost of each seat is \$5, and the price per seat is \$10. The number of reservations X is Poisson distributed with parameter $\lambda = 5$, i.e., $P(X = x) = e^{-5}5^x/x!$, $x = 0, 1, \ldots$. Compute the expected net revenue when:

(a) $N = 4$

(b) $N = 5$

(c) Which is more profitable to operate, a four- or five-seat limousine?

3.60 A sampling plan has $n = 100$, $c = 2$, and the lot size $N = 10{,}000$. Compute the approximate acceptance probabilities for the following values of D:

(a) $D = 50$ **(b)** $D = 100$ **(c)** $D = 200$

(*Hint:* Use the Poisson approximation to the binomial.)

3.7 MATHEMATICAL DETAILS AND DERIVATIONS