HW #3 – 3.6, 3.13, 3.31, 3.45, 3.57

Problem 3.6

r	10	20	30	40	50
$f(r)$	3/21	4/21	5/21	4/21	5/21

Note: The van takes 5 passengers when $X \geq 5$, so $P(X \geq 5) = P(X = 5) + P(X = 6)$.

Problem 3.13

$$(a)\ c^{-1} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 2.283. \quad c = 0.438$$

So $c = 0.438$.

(b) $E(X) = 1.1899$, $E(X^2) = 3.1898$, $V(X) = 1.7739$.

Problem 3.31

$$P(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

$$(a)\ n = 5, N = 50, D = 1$$

$$P(X = 0) = \frac{\binom{49}{5}}{\binom{50}{5}} = 0.90$$

(b) $N = 100, D = 2, n = 10$ $\quad P(x = 0) = 0.81$

(c) $N = 200, D = 4, n = 20$ $\quad P(x = 0) = 0.65$

Problem 3.45 *The number of patients who recover is given by the binomially distributed random variable X with parameters $n = 20$ and $p = 0.4$ (if the drug is worthless) and $p = 0.8$ if the recovery rate for patients given the drug is indeed 80%.*

$$(a)\ P(X \geq 12) = 1 - B(11; 20; 0.4) = 1 - 0.943 = 0.057$$
$$(b)\ P(X \geq 12) = 1 - B(11; 20; 0.8) = B(8; 20; 0.20) = 0.99.$$

Problem 3.57 *Let X denote the number of unvaccinated students. It has a binomial distribution with parameters $n = 50$, $p = 0.01$. We approximate it with the Poisson distribution with parameter $\lambda = 50 \times 0.01 = 0.5$.*

$$(a)\ P(X = 0) = p(0; 0.5) = 0.6065$$
$$(b)\ P(X = 1) = p(1; 0.5) = 0.3033$$
$$(c)\ P(X \geq 2) = 1 - P(X \leq 1) = 0.0902.$$