HW #5 - 4.22 d, e, f, g, h, i ; 4.24 d, e, f ;
4.37 ; 4.38 ; 4.42

Problems 4.22–4.40

Problem 4.22

(a) $P(Z < 1) = 0.8413$	*(b)* $P(Z < -1) = 0.1587$	*(c)* $P(\lvert Z\rvert < 1) = 0.6826$
(d) $P(Z < -1.64) = 0.0505$	*(e)* $P(Z > 1.64) = 0.0505$	*(f)* $P(\lvert Z\rvert < 1.64) = 0.8990$
(g) $P(Z > 2) = 0.0228$	*(h)* $P(\lvert Z\rvert > 2) = 0.0456$	*(i)* $P(\lvert Z\rvert < 1.96) = 0.95$

Problem 4.24

(a) $P(Z < c = -0.67) = 0.25$	*(b)* $P(Z < c = 0.67) = 0.75$
(c) $P(\lvert Z\rvert < c = 0.67) = 0.5$	*(d)* $P(Z < c = 1.04) = 0.85$
(e) $P(Z > c = 1.96) = 0.025$	*(f)* $P(\lvert Z\rvert < c = 1.96) = 0.95$

Problem 4.37 *The random variable Y is binomial with parameters* $n = 100$, $p = 0.7$, *so* $npq = 21$, $\sqrt{npq} = 4.58$.
We use the normal approximation without the continuity correction.

(a) $P(Y = y) = b(y; 100; 0.7)$, $(y = 0, 1, \ldots, 100)$.
(b) $P(Y \geq 75) \approx 0.1379$.
(c) $P(Y \geq 80) \approx 0.0146$.
(d) $P(Y \leq 65) \approx 0.1379$.

Problem 4.38

(a) If the drug is worthless then the number of recoveries is a binomial random variable with parameters $n = 200$, $p = 0.40$. *We use the normal approximation to the binomial (with* $np = 80$, $npq = 48$*) to conclude that*

$$P(X \geq 100) = P\left(\frac{X - 80}{\sqrt{48}} \geq 2.88\right) = 0.002.$$

(b) If the drug is really 80% effective then the number of recoveries is a binomial random variable with parameters $n = 200$, $p = 0.80$. *We use the normal approximation to the binomial (with* $np = 160$, $npq = 32$*) to conclude that*

$$P(X \geq 100) = P\left(\frac{X - 1600}{\sqrt{32}} \geq -10.61\right) = 1.00.$$

Problem 4.42

(a) We obtain the median by solving the equation

$$P(T < t) = 1 - \exp((-t/26710)^{1.053}) = 0.5$$

for t. *This leads to the following sequence of calculations:*

$$\begin{aligned}
\exp((-t/26710)^{1.053}) &= 0.5; \text{ taking logarithms of both sides yields} \\
(-t/26710)^{1.053}) &= \ln 0.5 = -0.6931, \text{ so} \\
1.053 \ln\left(\frac{t}{26710}\right) &= \ln 0.6931. \text{ Therefore} \\
\ln\left(\frac{t}{26710}\right) &= \frac{\ln 0.6931}{1.053} = -0.3481 \\
\text{Thus, the median} &= 26710 \times \exp(-0.3481) = 1885\text{[illegible] hrs.}
\end{aligned}$$

(b) The proportion of engine fans that fail on an 8000 hour warranty is given by $1 - \exp(-(8000/26710)^{1.053}) = 0.245$.