HW#6 – 6.3, 6.4, 6.7, 6.9, 6.10, & 6.20

Problem 6.3 *(a)* $P(X < 60) = P((X-65)/10 < (60-65)/10) = P(Z < -0.5) = 0.3085$.

(b) $T = X_1 + \ldots X_8 \sim N(8 \times 65; 8 \times 100) = N(520; 800)$

Eight hours equals 480 *minutes, so*

$P(T < 480) = P((T-520)/\sqrt{800} < (480-520)/\sqrt{800}) = P(Z < -1.414) = 0.0793$.

Problem 6.4

(a) $\overline{X} \sim N(2; 4/100) = N(2; 0.04)$ *so*

$$\begin{aligned} P(1.9 < \overline{X} < 2.1) &= P\left(\frac{-0.1}{0.2} < \frac{\overline{X}-2}{0.2} < \frac{0.1}{0.2}\right) \\ &= P(-0.5 < Z < 0.5) = 0.383. \end{aligned}$$

(b) Solve the following equation for n :

$$\begin{aligned} P(1.9 < \overline{X} < 2.1) &= P\left(\frac{-0.1}{2/\sqrt{n}} < \frac{\overline{X}-2}{2/\sqrt{n}} < \frac{0.1}{2/\sqrt{n}}\right) = 0.9 \textit{ so} \\ \frac{0.1}{2/\sqrt{n}} &= z(0.05) = 1.645; \textit{ consequently,} \\ n &= (2 \times 16.45)^2 = 1082.41 \textit{ or } n = 1083. \end{aligned}$$

Problem 6.7

(a) $\sigma(\overline{X}-\overline{Y}) = \sqrt{\sigma^2(\overline{X}) + \sigma^2(\overline{Y})} = 1347.22$. *Therefore,* $\overline{X}-\overline{Y} \sim N(0; (1347.22)^2)$.

$$\begin{aligned} \textit{(b) } P(|\overline{X}-\overline{Y}| > 2500) &= P\left(\frac{\overline{X}-\overline{Y}}{1347.22} > \frac{2500}{1347.22}\right) \\ &= P(|Z| > 1.86) = 0.0628. \\ \textit{(c) } P(\overline{X}-\overline{Y} < -2500) &= P\left(\frac{\overline{X}-\overline{Y}}{1347.22} < \frac{-2500}{1347.22}\right) \\ &= P(Z < -1.86) = 0.0314 \end{aligned}$$

Problem 6.9

(a) $E(W_n) = n \times 175$ *and* $V(W_n) = n \times 400$.
(b) $W_n \sim N(n \times 175; n \times 400)$.
(c)

$$\begin{aligned} P(W_{18} > 3000) &= P\left(\frac{W_{18} - 18 \times 175}{\sqrt{18 \times 400}} > \frac{-150}{84.85}\right) \\ &= P(Z > -1.77) = 0.9616. \end{aligned}$$

d) The maximum number N *of persons allowed on the elevator satisfies the following (quadratic) equation in* $\sqrt{N}$.

$$\begin{aligned} P(W_N > 3000) &= P\left(\frac{W_N - N \times 175}{\sqrt{N \times 400}} > \frac{3000 - N \times 175}{\sqrt{N \times 400}}\right) \\ &= 0.05. \text{ Therefore,} \end{aligned}$$

$$\frac{3000 - N \times 175}{\sqrt{N \times 400}} = 1.645$$

The largest integer $N \geq 16.4$ *is* $N = 16$.

Problem 6.10

(a) $T \approx N(48 \times 10, 48 \times 0.5) = N(480, 24)$.
(b) We convert 31 *lbs to* 496 *oz. Thus*

$$\begin{aligned} P(T > 31 \text{ lbs}) &= P(T > 496) \\ &= P\left(\frac{T - 480}{\sqrt{24}} > \frac{16}{\sqrt{24}}\right) \\ &= P(Z > 3.26) = 0.0006. \end{aligned}$$

Problem 6.20

$$\begin{aligned} P(1.5 \leq s \leq 2.9) &= P\left(\frac{15 \times (1.5)^2}{5} \leq \frac{15s^2}{5} \leq \frac{15 \times (2.9)^2}{5}\right) \\ &= P(6.75 \leq \chi^2_{15} \leq 25.23) \approx 0.95 - 0.05 = 0.90. \end{aligned}$$