DUE WED

Duplicate in Bay if needed

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ENCE 3300
COMPUTATIONAL METHODS IN CIVIL ENGINEERING

TEST 2

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1. The random variable X has the pdf defined by

   $f(x) = (6 - x)/k$; $x = -2, -1, 0, 1, 2$

   $f(x) = 0$; elsewhere

   a. Find k;

   b. Find E(X);

   c. Find V(X).

(a) $f(-2) = \frac{8}{k} = \frac{8}{30}$

$f(-1) = \frac{7}{k} = \frac{7}{30}$

$f(0) = \frac{6}{k} = \frac{6}{30}$

$f(1) = \frac{5}{k} = \frac{5}{30}$

$f(2) = \frac{4}{k} = \frac{4}{30}$

$\frac{30}{k} = 1$

$\frac{30}{k} = \frac{30}{30}$

$\frac{30}{k} = 1$

$\therefore$ $k = 30$

(b) $E(X) = \Sigma x f(x)$

$= (-2)(\frac{8}{30}) + (-1)(\frac{7}{30}) + (0)(\frac{6}{30}) + (1)(\frac{5}{30}) + (2)(\frac{4}{30})$

$E(X) = -\frac{1}{3} = -.3333$

(c) $V(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \Sigma x^2 f(x)$

$= (-2)^2 \frac{8}{30} + (-1)^2 \frac{7}{30} + (0)^2 \frac{6}{30} + (1)^2 \frac{5}{30} + (2)^2 \frac{4}{30}$

$E(X^2) = 2$

$V(X) = 2 - (-\frac{1}{3})^2 = 1.889$

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2. It is known that screws produced by a certain company will have 10 defective screws in a lot of 100. The company sells screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace? (What is the probability that a package will have 2 or more defective screws?) (Use the hypergeometric distribution.) $P(x \geq 2)$

$P(x \geq 2) = 1 - P(x \leq 2)$

$P(x \geq 2) = 1 - [P(x=0) + P(x=1) + P(x=2)]$

D = # deff

N = total number in group

n = sample

$$h(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

$$h(0) = \frac{\binom{10}{0}\binom{100-10}{10-0}}{\binom{100}{10}} = \frac{(1)\binom{90}{10}}{\binom{100}{10}} = .330$$

$$h(1) = \frac{\binom{10}{1}\binom{100-10}{10-1}}{\binom{100}{10}} = \frac{\binom{10}{1}\binom{90}{9}}{\binom{100}{10}} = .40799$$

$$\cancel{h(2) = \frac{\binom{10}{2}\binom{100-10}{10-2}}{\binom{100}{10}} = \frac{\binom{10}{2}\binom{90}{8}}{\binom{100}{10}} = .2015}$$

$$P(x \geq 2) = 1 - [P(x=0) + P(x=1)]$$

$$= 1 - [.330 + .408]$$

$$= 1 - .738 = 0.262$$

✓

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3. If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine (Use the binominal distribution) the probability that out of 2000 individuals,
   (a) exactly 3 individuals will suffer a bad reaction. $b(x; n, p)$ $b(x=3)$
   (b) More than 2 will suffer a bad reaction. $b(x>2)$

np

$p = 0.001$

$P = 1 - x=0, x=1, x=2$

p = prob = frequency dist. = sample proportion

$n = 2000$ $p = .001$

$1 - b(x=0) - b(x=1) - b(x=2)$

(a) $P(X=3)$

$$b(x) = \binom{n}{x} p^x [1-p]^{n-x}$$

$$b(3) = \binom{2000}{3} (.001)^3 [1-0.001]^{2000-3}$$

$$= \binom{2000}{3} (.001)^3 (.999)^{1997}$$

$$b(3) = .181$$

(b) $P(X>2)$?

$$P(X>2) = 1 - P(X \leq 2) \Rightarrow 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$P(X=0) = \binom{2000}{0} (.001)^0 (.999)^{2000} = .135$$

$$P(X=1) = \binom{2000}{1} (.001)^1 (.999)^{1999} = .271$$

$$P(X=2) = \binom{2000}{2} (.001)^2 (.999)^{1998} = .271$$

$$P(X>2) = 1 - [.135 + .271 + .271]$$

$$= 1 - .677 = .323 \text{ or } 32.3\%$$

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4. Assume automobile arrivals at a gasoline station are Poisson distributed and occur at an average rate of 40 per hour (40 per 60 minutes). The station has only one gasoline pump. If all cars are assumed to require one minute to obtain fuel, what is the probability that a waiting line will occur at the pump? (Probability that 2 or more automobiles arrive in a minute.)

$P(x \geq 2)$

$\lambda = \frac{40}{60} = \frac{2}{3}$

$P(x \geq 2) = 1 - P(x=0) + P(x=1) + P(x=2)$

$P(x=0) = \frac{e^{-2/3}\left(\frac{2}{3}\right)^0}{0!} = .513$

$P(x=1) = \frac{e^{-2/3}\left(\frac{2}{3}\right)^1}{1!} = .342$

$P(x=2) = \frac{e^{-2/3}\left(\frac{2}{3}\right)^2}{2!} = .114$

$P(x \geq 2) = .513 + .342 + .114 = .969$

$1 - [.513 + .342]$

$\boxed{1 - .855 = 0.145}$