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NCE 3300
COMPUTATIONAL METHODS IN CIVIL ENGINEERING

TEST 2

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1. The tensile strength S (measured in grams per square centimeter) of a fiber thread has a pdf given by

$$f(x) = (3/4)\, x^2\, (2 - x), \qquad 0 < x < 2$$
$$f(x) = 0, \qquad \text{elsewhere.}$$

Find E(X) and V(X).

$0 < x < 2$

means everything in between

$E(x) \int_a^b x f x \Rightarrow \int_0^2 \left(\frac{3}{4}\right) x^2 (2-x)(x) = 6/5$ ✓

$\int_a^b (x-4)^2 f(x)$

$V(x) = E(x^2) - [E(x)]^2$

$3/4 \cdot \int_0^2 \left(x^2 \cdot x^2 \cdot (2-x)\right) dx$

$= 8/5$

$\frac{8}{5} - \left(\frac{6}{5}\right)^2 = \boxed{.16}$ ✓

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2. **(Binomial Distribution)** A large industrial firm allows a discount on any invoice that is paid within 90 days. Of all invoices, 10% receive the discount. In a company audit, 12 invoices are sampled at random. What is the probability that more than 4 of the 12 sampled invoices receive the discount?

$1 - P(x \le 4) = 1 - p(x=0) - p(x=1) - P(x=2) - p(x=3) - p(x=4)$

Let $p = .1$
$n = 12$

$p(x=0) = \binom{12}{0} .1^0 (1-.1)^{12-0} = \frac{12!}{0!(12-0)!}(0.1^0)(1-0.1)^{12-0} =$

$(1)(1) = .2824$

$p(y=1) = \binom{12}{1}(.1^1)(1-.1)^{12-1} = .3765$

$p(x=2) = \binom{12}{2}(.1^2)(1-.1)^{12-2} = .2301$

$p(x=3) = \binom{12}{3}(.1^3)(1-0.1)^9 = .0852$

$p(x=4) = \frac{12!}{4!(12-4)!}(.1^4)(1-0.1)^8 = 495(.1)^4(.4304$
$.5721)$

$= .021308$

$1 - .3765 - .2301 - .0852 - .021308$
$- .2824$

$= .43\%$

$= .0043$

3. Assume automobile arrivals at a gasoline station are **POISSON DISTRIBUTED** and occur at an average rate of 30 per hour (30 per 60 minutes). The station has only one gasoline pump. If all cars are assumed to require one minute to obtain fuel, what is the probability that a waiting line will occur at the pump? (P(X>1) = ?)

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given

30 per 60 min

1 per 2 min

$\frac{1}{2}$

T=1 min

Prob of waiting in line =

$1 - F_X(1) - F_X(0) = 1 - e^{-1/2}\left[\frac{1}{2} + 1\right] =$

$= 1 - .6065306597\left[3/2\right]$

$= 1 - .9097959896$

$= .090204$

$\boxed{9.02\%}$ ✓

~~90.97%~~

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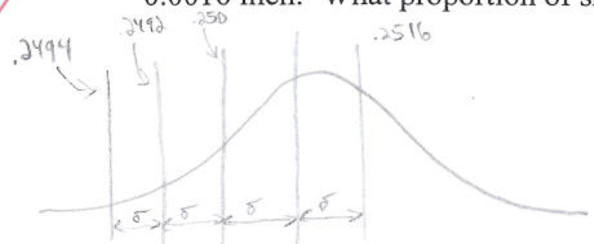
4. The diameter of a shaft is **normally distributed** with mean 0.2508 inch and standard deviation 0.0008 inch. The specifications on the shaft are 0.2500 $\pm$ 0.0016 inch. What proportion of shafts conforms to specifications?



$M = .2508$

$\sigma = .0008$

$.2500 \pm .0016$

$.2486$ to $.2516$

$P(.2484 \leq x \leq .2516)$

$P(\quad \leq \frac{x-M}{\sigma} \leq \quad)$

$\frac{.2516 - .2508}{.0008} = 1 = Z$

$\frac{.2484 - .2508}{.0008} = -3 = Z$

$-3 \leq 1$

Appendix A (p.548 + 549)

$.0013 + .8413$

$\boxed{.8423}$

$\boxed{84.23\%}$

this one	mine
4000	2000
180	20
25	.5
5%	10%

5. An elevator in the Engineering building has a weight capacity of 4000 pounds. Assume that the weight X of a randomly selected person is **normally distributed** with mean 180 pounds and standard deviation 25 pounds. What is the maximum number N of people allowed on the elevator such that the probability of their combined weight exceeding 4000 pound load limit is less than 5%?

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$$\frac{T - n \times 180}{25\sqrt{n}} \sim N(0,1)$$

overload occurs if $T > 4000$

$$\frac{T - n \times 180}{25\sqrt{n}} > \frac{4000 - n \times 180}{25\sqrt{n}} = 1.645 \leftarrow \boxed{\text{Table}_{.05}^{\infty} \text{ A.4}}$$

$$\frac{4000 - n \times 180}{25\sqrt{n}} = 1.645$$

$$41.125\sqrt{n} = 4000 - n \times 180$$

$$41.125\sqrt{n} + 180n = 4000$$

$$41.125\sqrt{n} + 180n - 4000 = 0$$

let $x = \sqrt{n}$

$$41.125x + 180x^2 - 4000 = 0$$

$$x(41.125 + 180x) - 4000 = 0$$

$$+180x^2 + 41.125x - 4000 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-41.125 \pm \sqrt{41.125^2 - 4(180)(-4000)}}{2(180)}$$

$$\frac{-41.125 \pm \sqrt{1691.2656 + 2880000}}{2(180)} = \frac{-41.125 \pm 1697.554}{360} = 4.601 = x = n^2 = 21.1709$$

$\boxed{21 = \text{max \# of passengers}}$