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# ENCE 3300
# COMPUTATIONAL METHODS IN CIVIL ENGINEERING

## TEST 2

see Carter T-2 #3 for full in[illegible] form

1. A certain type of component is packaged in lots of four. Let X represent the number of properly functioning components in a randomly chosen lot. Assume that the probability that exactly x components function is proportional to x; in other words, assume that the probability density function of X is given by:

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$$f(x) = c\,x, \quad x = 1, 2, 3, \text{ or } 4$$
$$f(x) = 0, \quad \text{elsewhere.}$$

where c is a constant.

a) Find the value of the constant c so that f(x) is a probability density function.

b) Find P(X = 2).

c) Find E(X).

d) Find V(X).

a) $c(1) + c(2) + c(3) + c(4) = 1$

$c(1+2+3+4) = 1$

$\boxed{c = 0.1}$ ✓

c) $E(x) = \sum_x x f(x)$

$= [1(0.1) + 2(0.2) + 3(0.3) + 4(0.4)]$

$\boxed{E(x) = 3}$ ✓

b) $P(x=2) = cx$

$P(x=2) = 0.1(2)$

$\boxed{P(x=2) = 0.2}$ ✓

| x | f(x) |
|---|---|
| 1 | 0.1 |
| 2 | 0.2 |
| 3 | 0.3 |
| 4 | 0.4 |

= 1 total

d) $V(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \sum_x x^2 f(x)$

$= [1^2(0.1) + 2^2(0.2) + 3^2(0.3) + 4^2(0.4)]$

$E(x^2) = 10$

$V(x) = 10 - 3^2$

$\boxed{V(x) = 1}$ ✓

2. (Hypergeometric Distribution) Of 50 buildings in an industrial park, 12 have electrical code violations. If 10 buildings are selected at random for inspection, what is the probability that exactly 3 of the 10 have code violations?

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$N = 50$
$D = 12$
$n = 10$
$x = 3$

$$h(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

$$P(\text{acceptance}) = P(x=3) = \frac{\binom{12}{3}\binom{38}{7}}{\binom{50}{10}}$$

$$\frac{\left(\frac{\text{total violation}}{\text{violations w/ the desired}}\right)\left(\frac{\text{total clean}}{\text{total desired clean}}\right)}{\left(\frac{\text{total buildings}}{\text{selected buildings}}\right)}$$

$$h(3) = \frac{\left(\frac{12!}{3!(12-3)!}\right)\left(\frac{(50-12)!}{(10-3)!(50-12-10+3)!}\right)}{\frac{50!}{10!(50-10)!}}$$

$$h(3) = \frac{\left(\frac{12!}{3!(7!)}\right)\left(\frac{38!}{7!(25!)}\right)}{\frac{50!}{10!(40!)}} = 0.2703$$

$$\boxed{= 27.03\%}$$

$\frac{30!}{(50-10)!}$ $\frac{10 \cdot 41 \cdot 40!}{40!}$

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3. (Binomial Distribution) A large industrial firm allows a discount on any invoice that is paid within 30 days. Of all invoices, 10% receive the discount. In a company audit, 12 invoices are sampled at random. What is the probability that fewer than 4 of the 12 sampled invoices receive the discount?

a) $P(X<4)$ $p=0.1$

$n=12$

$B(x;n,p)=\sum_{0\le y\le x} b(y;n,p)$

$B(x;n,1-p)=1-B(n-1-x;n,p)$

$P(x<4)=1-B(0;12;0.1)$

$P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$

$P(X=0)=\binom{12}{0}0.1^0(1-0.1)^{12-0}$

$\frac{12!}{0!(12-0)!}(0.1^0)(1-0.1)^{12}=0.2824$

$P(X=1)=\binom{12}{1}(0.1^1)(1-0.1)^{12-1}=0.3765$

$P(X=2)=\binom{12}{2}(0.1^2)(1-0.1)^{12-2}=$

$\frac{12!}{2!(12-2)!}(0.1^2)(0.9)^{10}=0.2301$

$P(X=3)=\binom{12}{3}(0.1^3)(1-0.1)^9=0.0852$

$\sum(P>4)=0.2824+0.3765+0.2301+0.0852$

$\boxed{=0.9742}$ ✓

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4. Assume that the number of hits on a certain website during a fixed time interval follows a Poisson distribution. Assume that the mean rate of hits is 5 per minute. Find the probability that there will be exactly 17 hits in the next three minutes.

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

$$P(X=17) = e^{-15} \frac{15^{17}}{17!}$$

$$\boxed{P(X=17) = 8.04\%}$$

$e^x$

2nd on graph calc