2

11|12|07

98

ENCE 3300
COMPUTATIONAL METHODS IN CIVIL ENGINEERING

TEST 3

σ μ

1. The tensile strength of paper is modeled by a normal distribution with a mean of 35 pounds per square inch and a standard deviation of 2 pounds per square inch.

Normal Dist.

25

   (a) What is the probability that the strength of a sample is less than 39 lb/in^2?
   (b) If the specifications require the tensile strength to exceed 29 lb/in^2, what proportion of the sample is scrapped?

(A.) $P(x<39)$

$\sigma = 2$ Standard Deviation

$\mu = 35$ mean

$z = \frac{x-\mu}{\sigma}$ $z = \frac{\text{quantity} - \text{mean}}{\text{standard deviation}}$

$= \frac{39-35}{2}$

$z = 2$

$\boxed{P(z<2) = 0.9772 = 97.72\%}$

p.549

(B.) $P(x>29)$

$\sigma = 2$

$\mu = 35$

$z = \frac{29-35}{2}$

$z = -3$

$P(z<-3) =$ scrapped

$\boxed{P(z<-3) = 0.0013 = 0.13\%}$

Not Scrapped:

$P(z>-3) = 1 - P(z<-3)$

$= 1 - 0.0013$

$= 0.9987$

normal distribution probability

24 pts

2. The weight of a certain box of floor tile is modeled by a normal distribution with a mean of 20 pounds and a standard deviation of 0.5 pounds. You want to transport the tile in a mini-van with a cargo load capacity of 2000 lb. Determine the maximum number of boxes allowed in the mini-van so that the total cargo load exceeds the 2000 lb weight limit with a probability less than 10%.

23

X follows a normal dist

$X \sim N(\text{mean}, \text{variance}^2)$

$\sigma = 0.5$ lbs

$\mu = 20$ lbs

$X \sim N(20, 0.25)$

Cargo Capacity = 2000 lbs.

boxes allowed = ? such that prob. cargo > 2000 = 0.01

(a)

$$P(W_N > 2000) = P\left(\frac{(W_n - N \times 20)}{(\sqrt{N \times 0.25})} > \underbrace{\frac{(2000 - N \times 20)}{(\sqrt{N \times 0.25})}}_{a}\right) = 0.1$$

10%

FROM TABLE

$P(x > a) = 0.1 \quad a = -1.285$

So...

$$\frac{2000 - N \times 20}{0.5\sqrt{N}} = 1.285$$

$2000 - 20N = 0.6425\sqrt{N}$

$0.6425\sqrt{N} + 20N - 2000 = 0$

SUBSTITUTE $x = \sqrt{N}$

$(0.6425x) + (20x^2) - 2000 = 0$

$\left[-0.6425 \pm \sqrt{0.6425^2 - 4(20)(-2000)}\right] / 2(20) \Rightarrow \dfrac{-0.6425 \pm 400}{40} = 9.98 = X$

(only + integer)

$N = X^2$

$\boxed{N = 99.68 \text{ boxes}}$

-2

cannot have 0.68 boxes

$\boxed{N = 100}$ boxes

99

Definition prob

3. The tensile strength S (measured in grams per square centimeter) of a fiber thread has a pdf given by

25

$$f(x) = (3/4)\,x^2\,(2 - x), \qquad 0 < x < 2$$
$$f(x) = 0, \qquad \text{elsewhere.}$$

Find E(X) and V(X).

$$E(x) = \int_0^2 x\left(\frac{3}{4}\right)x^2(2-x)\,dx$$

$$= \int_0^2 x\left(\frac{3}{4}\right)(2x^2 - x^3)\,dx = \frac{3}{4}\int_0^2 2x^3 - x^4\,dx$$

$$\Rightarrow \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]\Bigg|_0^2$$

$$= \frac{3}{4}\left[8 - \frac{32}{5}\right] - 0$$

$$= 1.2 - 0$$

$$\boxed{E(x) = 1.2}$$

$$E(x^2) = \int_0^2 x^2\left(\frac{3}{4}\right)x^2(2-x)\,dx = \frac{3}{4}\int_0^2 2x^4 - x^5\,dx$$

$$= \frac{3}{4}\left[\frac{2x^5}{5} - \frac{x^6}{6}\right]\Bigg|_0^2 = \frac{3}{4}\left[\frac{64}{5} - \frac{64}{6}\right] - 0 \Rightarrow E(x^2) = 1.6$$

E(X) V(X)
PDF
Tensile Strength
derivative/integral

$$V(x) = E(x^2) - [E(x)]^2 = 1.6 - 1.44 = \boxed{0.16 = V(x)}$$

$N(175, 400)$

4. A ski lift has a weight capacity of 500 lb. Assume that the weight X of a randomly selected person is N(175,400) distributed. What is the maximum number N of people allowed on the ski lift such that the probability of their combined weight exceeding 500 lb load limit is less than 5%?

25

$$P(W_N > 500) = P\left(\frac{W_N - N \times 175}{\sqrt{N \times 400}} > \overbrace{\frac{500 - N \times 175}{\sqrt{N \times 400}}}^{a}\right) = 0.05$$

$P(x > a) = 0.05$

$a = 1.645$ ← p. 550 A.4

$$\rightarrow \frac{500 - 175N}{20\sqrt{N}} = 1.645$$

$1.645(20)\sqrt{N} + 175N - 500 = 0$

$x = \sqrt{N}$ (sub in x for $\sqrt{N}$)

$$\underset{B}{32.9x} + \underset{A}{175x^2} \underset{C}{- 500} = 0$$

$$\frac{-32.9 \pm \sqrt{32.9^2 - 4(175)(-500)}}{2(175)}$$

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\frac{-32.9 \pm 592.52}{2(175)}$$

$x = 1.5989$

$N = x^2 \Rightarrow \boxed{N = 2.556 \text{ people}}$

! cannot have 0.556 of a person

$\Rightarrow \boxed{N = 2 \text{ people}}$

Max #