$\bar{y}$ = yaverage

100

ENCE 3300
COMPUTATIONAL METHODS IN CIVIL ENGINEERING

Best Fit Line

TEST 4

plug these x's in $a_0 + a_1(x)$

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1. Find the least-squares line approximation to the data listed. ✓
Also find: the standard error of the estimate; the cofficient of determination; and the correlation coefficient.

$S_r = \Sigma e_i = (y_i - (a_0 + a_i x_i))^2$

$A_1 = \dfrac{n \Sigma x_i y_i - (\Sigma x_i \cdot \Sigma y_i)}{n \Sigma x_i^2 - (\Sigma x_i)^2}$

$A_0 = \bar{y} - A_1 \bar{x}$

$S_{y/x} = \sqrt{\dfrac{S_r}{n-2}}$ $S_r = \Sigma e_i$

$R^2 = \dfrac{S_T - S_R}{S_T}$

$S_T = \Sigma (y - \bar{y})^2$

$x_i \cdot y_i = x \cdot y$ $x^2 = x_i^2$ $(y - \text{yaverage})^2$

	X	Y	$x_i \cdot y_i$	x_i^2	e_i	$(y-\bar{y})^2$
	2	102.6	205.2	4	0.0139669	226.63948
	4	107.0	428	16	2.2011769	113.51944
	6	108.4	650.4	36	0.0227736	85.646696
	8	110.0	880	64	2.5136661	58.592136
	10	114.0	1140	100	0.3844	13.355736
	12	117.6	1411.2	144	0.0029752	0.0029757
	14	120.2	1682.8	196	0.2392099	6.4793157
	16	125.0	2000	256	1.6291041	53.955636
	18	126.0	2268	324	0.5748397	69.646536
	20	131.2	2624	400	1.9804165	183.47922
	22	132.2	2908.4	484	0.3934711	211.57012
Σ	132	1294.2	16198	2024	9.956	1022.8873
AVG.	12	117.6545455				

✓

$A_1 = \dfrac{(11)(16198) - (132 \cdot 1294.2)}{(11)(2024) - 17424} = \dfrac{7343.6}{4840} = 1.517272727$ ✓

$A_0 = 117.6545455 - (1.57272727)(12) = 99.447273$ ✓

yaverage − A_1 (xaverage) (xaverage)

$S_{y/x} = \sqrt{\dfrac{9.956}{11-2}} = 1.05177$ ✓

Σe_i

11#'s

$R^2 = \dfrac{(1022.8873 - 9.956)}{1022.8873} = 0.9902667674$ ✓

$\Sigma(y-\bar{y})^2 - \Sigma e_i$

$\Sigma(y-\bar{y})^2$

$R = \sqrt{r^2} = 0.9951214837$

✓

random work

~~102.6 − (99.44 + 1.517276(x))~~

~~y~~ − ($a_0 + A_1 x_2$) = ?

102.6 − (99.45 + 1.52 × 2) = ?

$?^2 = \Sigma e_i = S_r$

$?^2$

EAST SQUARES LINE APPROX. = $y_i - ((99.447273) + 1.5172727(x_i))$

TANDARD ERROR OF ESTIMATE = 1.051771

OEFFICIENT OF DETERMINATION = 0.9902667674

CORRELATION COEFFICIENT = 0.9951214837

In Base 10

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2. What transform must be used on the equations below to allow the use of linear least squares?

$y = \overset{\downarrow}{a}\, e^{bx}$

a. $y = ae^{bx}$ $\Rightarrow$ TRANSFORM $=$ $\ln y = \ln a_1 - bx$ ✓

$\ln e = 1$

$\ln e = 1$

$\underset{\downarrow}{y} = \underset{\downarrow}{a}x^{b}$

b. $y = ax^{b}$ $\Rightarrow$ TRANSFORM $\Rightarrow$ $\ln y = \ln a + b \ln x$ ✓

c. $y = ax/(b + x)$ $\Rightarrow$ TRANSFORM $\Rightarrow$ $\frac{1}{y} = \frac{b}{a} \cdot \frac{1}{x} + \frac{1}{a} \Rightarrow \beta = a_1\alpha + a_0$ ✓

$a_0 = \frac{1}{a} \Rightarrow a = \frac{1}{a_0}$

$a_1 = \frac{b}{a} \Rightarrow b = a_1 a$

$\frac{1}{y} = \ln y$

$\frac{ax}{b+x} = \frac{a}{b+x} \cdot x$

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3. For the falling parachutist problem, Example problem 1.2, assume that the first jumper has a mass of 68.1 kg and a drag coefficient of 12.5 kg/s. If a second jumper has a mass of 75 kg and a drag coefficient of 14 kg/s, how long will it take the second jumper to attain the same velocity reached by the first jumper in 10 s?

9.81 × 68.1

e = 2.71

Jumper (1) Time in 10 sec = 44.87 = $\overset{kg/s}{\text{Vel}}$

Jumper (2) goal = 44.87

$$\frac{gm}{c}\left(1 - e \wedge - c/m\,(t)\right)$$

$$44.87\,(\text{Vel.}) = \frac{(9.8)(75\,kg)}{14}\left(1 - e^{-(14/75)t}\right)$$

$$44.87 = 52.5 - 52.5e^{-0.18666t}$$

$$\frac{44.87 - 52.5}{52.5} = e^{-0.18666T}$$

e = yellow

$$\ln 0.14533 = \ln\left(e^{-0.18666T}\right)$$

$$\boxed{= T = 10.33\ \text{sec}}$$

✓

Sat

4. A terrorist decides to poison all the people in an airtight building by pumping a mixture of gas that contains 10% toxic gas into the 100,000 ft^3 building (which holds 6000 lb$_m$ of air initially) at a rate of 6 lb$_m$/min. When the concentration of the toxic gas reaches 4%, people will start to die. The terrorist assumes that the mass of gas in the building will remain constant and calculated how long it will take to kill the people. You realize that the building is airtight and that the mass of gas in the building will increase with time. Will it take more or less time than the terrorist originally calculated to accumulate a toxic level of gas into the building? (Make sure you do both the terrorist's calculation and the corrected calculation.)

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TERRORIST CALCULATION

A(T) = #m of gas at time (t) min after process

$\frac{da(t)}{dt}$ = # lbm of gas - gas out

gas in = (6 lb) x (0.1) = 0.6 lbm/min gas OUT = $\frac{A(t)}{6000} \times \frac{6 lb}{min} = \frac{A(T)}{1000}$

$\frac{dA}{dt} = 0.6 - \frac{A}{1000} \Rightarrow \frac{600-A}{1000}$

$d(t) = \frac{1000}{600-A}\,da \Rightarrow -1000 \ln|600-A| = t + C$

$\Rightarrow \ln|600-A| = -T/1000 + C$

$\Rightarrow 600 - A = Ce^{-t/1000}$

$\Rightarrow A = 600 - Ce^{-t/1000}$

$C_{toxic} = \frac{(6 lb/min)(.10)(t)}{6000 + 6t}$

6000 lbm x .04 ~~0.04~~ = 240 lb TOXIC Air needed to kill population

$(6 lb/min)(\overset{(10\%)}{.15})(t) = [6000 + 6t].4$

$\frac{240 \text{ lb}}{0.6 \frac{lb}{min}} = \boxed{400 \text{ min}}$ ~ 6.7 hrs = length of time needed to kill

.36t = 240

t = .667 min

10% of 6

	min	mass (lbs)	% gas	%gas / mass
6x	1	6006	0.6 6x.10	= 9.99 E-5
6x	25	6150	15 150x.10	= .0024 15/6150
6x	50	6300	30 300x.1	= 0.0047
6x	150	6900	90 900x.1	= 0.130
6x	400	8400	240	= 0.28
6x	500	9000	300	0.03
6x	600	9600	310	= 0.0375

67

APPROX:

70 min