2-6 (a) $p = 1{,}000 - 0.2D$
$C_T = 1{,}000 + 2D^2$

Profit(loss) $= \text{Total Revenue} - \text{Total Cost}$
$= (1{,}000 - 0.2D)D - (1{,}000 + 2D^2)$
$= 1{,}000D - 2.2D^2 - 1{,}000$

$$\frac{d\,\text{Profit}}{dD} = 1{,}000 - 4.4D^* = 0$$

$D^* = \underline{227.27}$ units per year

(b) $$\frac{d^2(\text{Profit})}{dD^2} = -4.4 < 0$$

Since the second derivative is negative, profit has been maximized at D^*.

2-8 $D = 780 - 10p$ units per month; $p = 78 - 0.1D$

$$D^* = \frac{a - c_v}{2b} = \frac{78 - 30}{2(0.1)} = \underline{240 \text{ units per month}} \qquad \text{(Eqn 2. -10)}$$

Profit (Loss) $= \text{Total revenue} - \text{Total cost}$
$= 78D - 0.1D^2 - (800 + 30D)$
$= 78(240) - 0.1(240)^2 - [800 + 30(240)]$
$= \underline{\$4{,}960}$ per month (profit)

$$D' = \frac{-(78-30) \pm \left[(78-30)^2 - 4(-0.1)(-800)\right]^{0.5}}{2(-0.1)}$$

$$= \frac{-48 \pm 44.54}{-0.2}$$

$$D_1' = \frac{-48 + 44.54}{-0.2} = 17.3, \text{ or } 18 \text{ units / month}$$

$$D_2' = \frac{-48 - 44.54}{-0.2} = 462.7, \text{ or } 463 \text{ units / month}$$

2-10 (a) $p = 700 - 0.05D$; $C_F = \$1,000,000$/month; $c_v = \$131.50$ per unit

The unit demand, D, is one thousand board feet.

$$D^* = \frac{a - c_v}{2b} = \frac{700 - 131.50}{2(0.05)} = \underline{5,685 \text{ units / month}} \qquad \text{(Eqn. 2-10)}$$

$$\begin{aligned} \text{Profit (loss)} &= 700D - 0.05D^2 - (1,000,000 + 131.50D) \\ &= [\,700(5,685) - 0.05(5,685)^2] - [\$1,000,000 + \$131.50(5,685)] \\ &= \$615,961.25 \text{ / month} \qquad \text{(maximum profit)} \end{aligned}$$

(b) $$D' = \frac{568.5 \pm \sqrt{(568.5)^2 - 4(0.05)(1,000,000)}}{2(0.05)}$$

$$D_1' = \frac{568.5 - 351}{0.1} = 2,175 \text{ units / month}$$

$$D_2' = \frac{568.5 + 351}{0.1} = 9,195 \text{ units / month}$$

Range of profitable demand is 2,175 units to 9,195 units per month.

2-12 $p = 6 - 0.001D$

$$D^* = \frac{6 - 4}{2(0.001)} = \underline{1,000 \text{ units per month}} \qquad \text{(Equation 2-10)}$$

$$\begin{aligned} \text{Profit (loss)} &= \text{Total Revenue - Total Cost} \\ &= (aD - bD^2) - (C_F + c_vD) \\ &= [6(1,000) - 0.001(1,000)^2] - [1,000 + 4(1,000)] \\ &= 5,000 - 5,000 = \underline{0} \qquad \text{(maximum profit)} \end{aligned}$$

Thus, the company, at best, can break even.

2-14 C_F = \$504,000 per month; c_v = \$166 per unit; p = \$328 per unit

$$D' = \frac{C_F}{p - c_v} = \frac{\$504{,}000}{(\$328 - \$166)/\text{unit}} = \underline{3{,}112 \text{ pumps per month}}$$

Reduced Costs: C_F at (-18%) = \$504,000 (1 - 0.18) = \$413,280

c_v at (- 6%) = \$166 (1 - 0.06) = \$156.04

$$D' = \frac{\$413{,}280}{(\$328 - \$156.04)/\text{unit}} = \underline{2{,}404 \text{ pumps per month}} \qquad \text{(Eqn. 2-13)}$$

$$\frac{3{,}112 - 2{,}404}{3{,}112} = 0.2275, \text{ or a 22.75\% reduction in the breakeven point.}$$