

Section 5.5 Case 2: Study Period ¹ Useful Life

Up until now, study periods and useful lives have been the same length. The study period is frequently taken to be a common multiple of the alternatives' lives when study period \neq useful life.

Repeatability Assumption (page 211)

Conditions:

1. Study period is either indefinitely long or equal to a common multiple of the lives of the alternative.
2. The cash flows associated with an alternative's initial life span are representative of what will happen in succeeding life spans.

Example – Problem 5-24a (pp. 236-237)

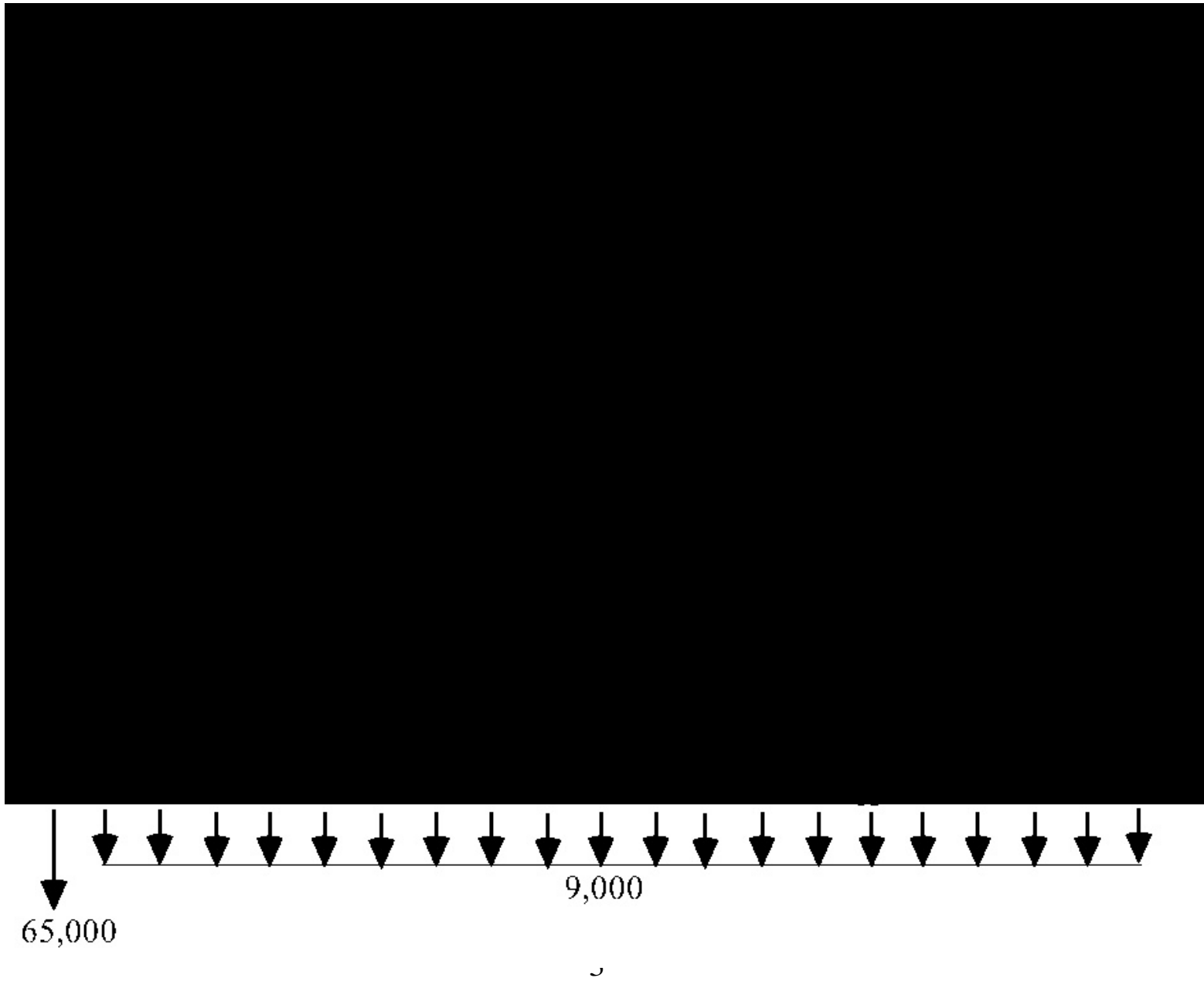
Cost Alternatives; Study Period > Useful Life; MARR = 15%

	A	B
Investment cost	\$14,000	\$65,000
Annual costs	14,000	9,000
Useful life	5	20
Market value @ end of useful life	8,000	13,000

a. If the study period = 20 years, which alternative is preferred?

Hint: First draw a cash flow diagram.

Problem 5-24a (continued)



Problem 5-24a (continued)

$$\begin{aligned}PW_A &= -14,000 - 6,000(P|F, 15, 5) - 6,000(P|F, 15, 10) \\ &\quad - 6,000(P|F, 15, 15) + 8,000(P|F, 15, 20) - 14,000(P|A, 15, 20) \\ &= -\$106,345\end{aligned}$$

$$\begin{aligned}PW_B &= -65,000 - 9,000(P|A, 15, 20) + 13,000(P|F, 15, 20) \\ &= -\$120,539\end{aligned}$$

Select A to minimize costs

Also,

$$AW_A = -\$106,345(A|P, 15, 20) = -\$16,990$$

$$AW_B = -\$120,539(A|P, 15, 20) = -\$19,262$$

Consider the AW over the useful life of Alternative A:

$$\begin{aligned}AW_A &= -14,000(A|P, 15, 5) - 14,000 + 8,000(A|F, 15, 5) \\ &= -16,990\end{aligned}$$

$$\text{Life 1: } AW_{1-5} = -16,990$$

$$\text{Life 2: } AW_{6-10} = -14,000(A|P, 15, 5) - 14,000 + 8,000(A|F, 15, 5) = -16,990$$

$$\text{Life 3: } AW_{11-15} = -16,990$$

$$\text{Life 4: } AW_{16-20} = -16,990$$

Shortcut: If the study period equals a common multiple of the alternatives' lives, simply compare AW computed over the respective useful lives (assuming repeatability is valid).

Case 2: Study Period > Useful Life

What if the study period is not a common multiple of the alternatives' lives or repeatability is not applicable?

Use the Cotermination Assumption (page 211).

Procedure: The cash flows of the alternatives need to be adjusted to terminate at the end of the study period.

- **Cost alternatives:** Assuming repeatability, repeat part of the useful life of the original alternative, and then use an estimated MV to truncate it at the end of the study period. Without repeatability, we must purchase/lease the service/asset for the remaining years.
- **Investment alternatives:** Assume all cash flows will be reinvested at the MARR to the end of the study period (i.e., calculate FW at end of useful life and move this to the end of the study period using the MARR).

Case 2: Study Period < Useful Life

When the study period is explicitly stated to be shorter than the useful life, use the cotermination assumption.

Procedure: The cash flows of the alternatives need to be adjusted to terminate at the end of the study period.

- Truncate the alternative at the end of the study period using an estimated Market Value.

Example of Cotermination (Case 2)

Suppose the study period had been stated to be 20 years. Which boiler would you recommend?

	Boiler A	Boiler B
Investment cost	\$50,000	\$120,000
Useful life	20 yrs.	40 yrs.
Salvage value @ end of useful life	10,000	20,000
Annual costs	9,000	6,000

Useful life of A = 20 years = study period

Useful life of B = 40 years > study period

Assume $MV_B @ \text{EOY } 20 = \$50,000$

The MARR is 10% per year.

Example Continued (Cotermination)

$$AW_A(10\%) = -14,700$$

$$\begin{aligned} AW_B(10\%) &= -6,000 - [20,000(A|P, 10, 20) - 50,000(A|F, 10, 20)] \\ &= -19,225 \end{aligned}$$

Still select Boiler A to minimize costs.

What would the market value of Boiler B @ EOY 20 have to be in order to select Boiler B instead of A?

Example Continued (Cotermination)

Market Value of Boiler B to reverse the decision = ?

Set $AW_A = AW_B$, let X be the unknown market value

$$-14,700 = -6,000 - \{120,000(A|P, 10, 20) - X(A|F, 10, 20)\}$$

$$-8,700 = -\{14,100 - 0.0175X\}$$

$$5,400 = 0.0175X$$

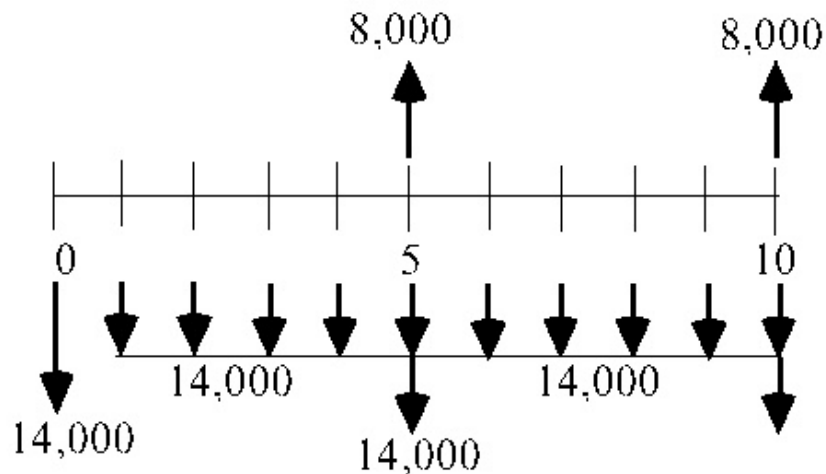
$X = \$308,571$ therefore, $MV_B > \$308,571$ to favor B

Such a value is very unlikely because X is more than the initial cost of Boiler B.

Problem 5-24a Revisited with Cotermination

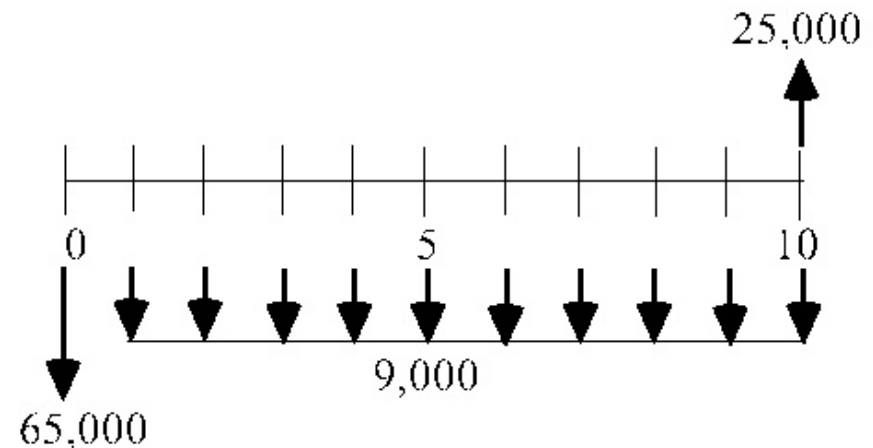
If the study period = 10 years and the estimated market value for alternate B = \$25,000 @ EOY 10, which alternative is preferred?

Alternative A:



$$AW_A = -\$16,990$$

Alternative B:



$$\begin{aligned} AW_B &= -65,000(A|P, 15\%, 10) - 9,000 \\ &\quad + 25,000(A|F, 15\%, 10) \\ &= -\$20,722 \end{aligned}$$

Alternative A is still preferred.

Review of Chapter 5

Comparison of Mutually Exclusive Alternatives (MEAs)

- Investment Alternatives (usually includes DN)
- Cost Alternatives (must do)

Equivalent Worth Methods of Comparing MEAs

- Maximize worth or Minimize costs
- Use the MARR as the interest rate

Comparing MEAs using the IRR

- Never simply choose the MEA that maximizes IRR
- Compare MEAs by looking at the IRR on the incremental investment/cost required to choose higher FC MEAs

Study Period \neq Useful Life

- Repeatability Assumption
- Cotermination Assumption

Problem 5-21 (Useful Life ¹ Study Period)

Given two MEAs, which one (if either) should be implemented?

MARR = 15% per year

Machine:	A	B
Investment cost	-\$20,000	-\$30,000
Annual Revenues	10,000	14,000
Annual Expenses	-4,400	-8,600
Market Value	4,000	0
Useful Life (yrs)	5	10

(a) Use the AW method

Assume Repeatability, study period = 10 years

$$\begin{aligned}AW_A &= -20,000(A|P, 15\%, 5) + 5,600 + 4,000(A|F, 15\%, 5) \\ &= \$227\end{aligned}$$

$$\begin{aligned}AW_B &= -30,000(A|P, 15\%, 10) + 5,400 \\ &= -\$579\end{aligned}$$

Choose Alternative A

Problem 5-21 Continued

(b) Use the PW method (explicitly compare alternatives over ten years)

$$\begin{aligned}PW_A &= -20,000 + 5,600(P|A, 15\%, 10) - 16,000(P|F, 15\%, 5) \\ &\quad + 4,000(P|F, 15\%, 10) \\ &= \$1,138.88\end{aligned}$$

$$\begin{aligned}PW_B &= -30,000 + 5,400(P|A, 15\%, 10) \\ &= -\$2,899\end{aligned}$$

We confirm our decision in part (a) to choose Alternative A.

Problem 5-21b continued

What if we could not assume repeatability?

Use the FW method, Study period = 10 years

Assume the FW at the end of the useful life of Alternative A will be reinvested at the MARR until the end of the study period.

$$\begin{aligned}FW_A &= -20,000(F|P, 15\%, 10) + 5,600(F|A, 15\%, 5)(F|P, 15\%, 5) \\ &\quad + 4,000(F|P, 15\%, 5) \\ &= \$3079\end{aligned}$$

$$\begin{aligned}FW_B &= -30,000(F|P, 15\%, 10) + 5,400(F|A, 15\%, 10) \\ &= -\$11,728\end{aligned}$$

Chapter 5 Review Problem – Hokie Manufacturing

Two mutually exclusive engineering projects, A and B, are being considered by Hokie Manufacturing. Both projects have a life of 10 years. Estimated cash flows are shown below. The MARR is 15%. You have been hired to determine which of the two projects, if either, Hokie Manufacturing should undertake.

Perform your analysis using an incremental procedure.

	<u>A</u>	<u>B</u>
Initial Investment	\$45,000	\$55,000
Annual Revenues	12,500	14,000
Annual Costs	2,500	2,500
Salvage Value	1,500	4,000

Hokie Manufacturing Continued

Solve by incremental present worth

- DN \rightarrow A \rightarrow B
- Compare DN \rightarrow A

$$PW_{DN}(15\%) = 0$$

$$PW_{DN \rightarrow A}(15\%) = PW_A(15\%) =$$

$$= -45,000 + 10,000(P/A, 15\%, 10) + 1500(P/F, 15\%, 10)$$

$$= \$5,558.80$$

Since $PW_{DN \rightarrow A} = \$5,558.80$, we know that the incremental investment is worthwhile at a MARR of 15%. Keep A, Drop DN.

Hokie Manufacturing Continued

- Compare A \rightarrow B

$$PW_{A \rightarrow B} (15\%)$$

$$= -10,000 + 1,500(P/A, 15\%, 10) + 2,500(P/F, 15\%, 10)$$

$$= -1,853.8$$

Since $PW_{A \rightarrow B} = -\$1,853.8$, we know that the incremental investment is NOT profitable at a MARR of 15%. Keep A, Drop B.

We also know when the $PW_{A \rightarrow B} (\text{MARR}) < 0$,
then the $IRR_{A \rightarrow B} < \text{MARR}$