Chapter 3: Fluid Statics

After Paterson, Pennsylvania State University

Pressure

- **Pressure** is defined as a *normal force exerted by a fluid per unit area*.
- Units of pressure are N/m^2, which is called a **pascal** (Pa).
- More common: use multiples of Pa
  - *Kilopascal,* 1 kPa = 10^3 Pa
  - *Megapascal,* 1 MPa = 10^6 Pa
- Other units:
  - 1 *bar = 100 kPa*
  - 1 *atm = 101.325 kPa*

Absolute, gage, and vacuum pressures

- Actual pressure at a given point is called the **absolute pressure**.
- Pressure-measuring devices will read zero in the atmosphere
  - **Gage pressure**, $P_{gage} = P_{abs} - P_{atm}$.
- Pressure below atmospheric pressure are called **vacuum pressure**, $P_{vac} = P_{atm} - P_{abs}$.

Absolute, gage, and vacuum pressures

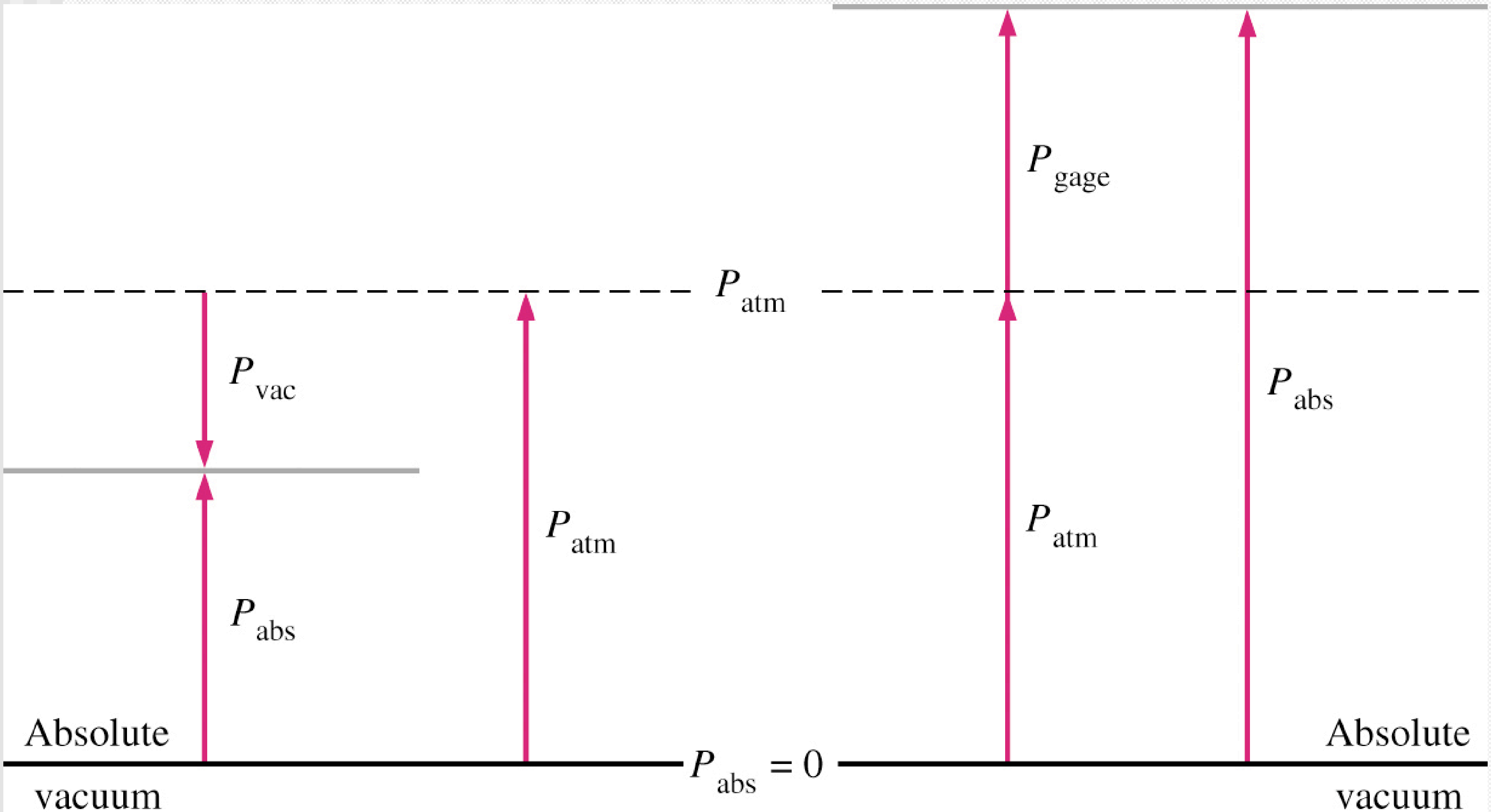
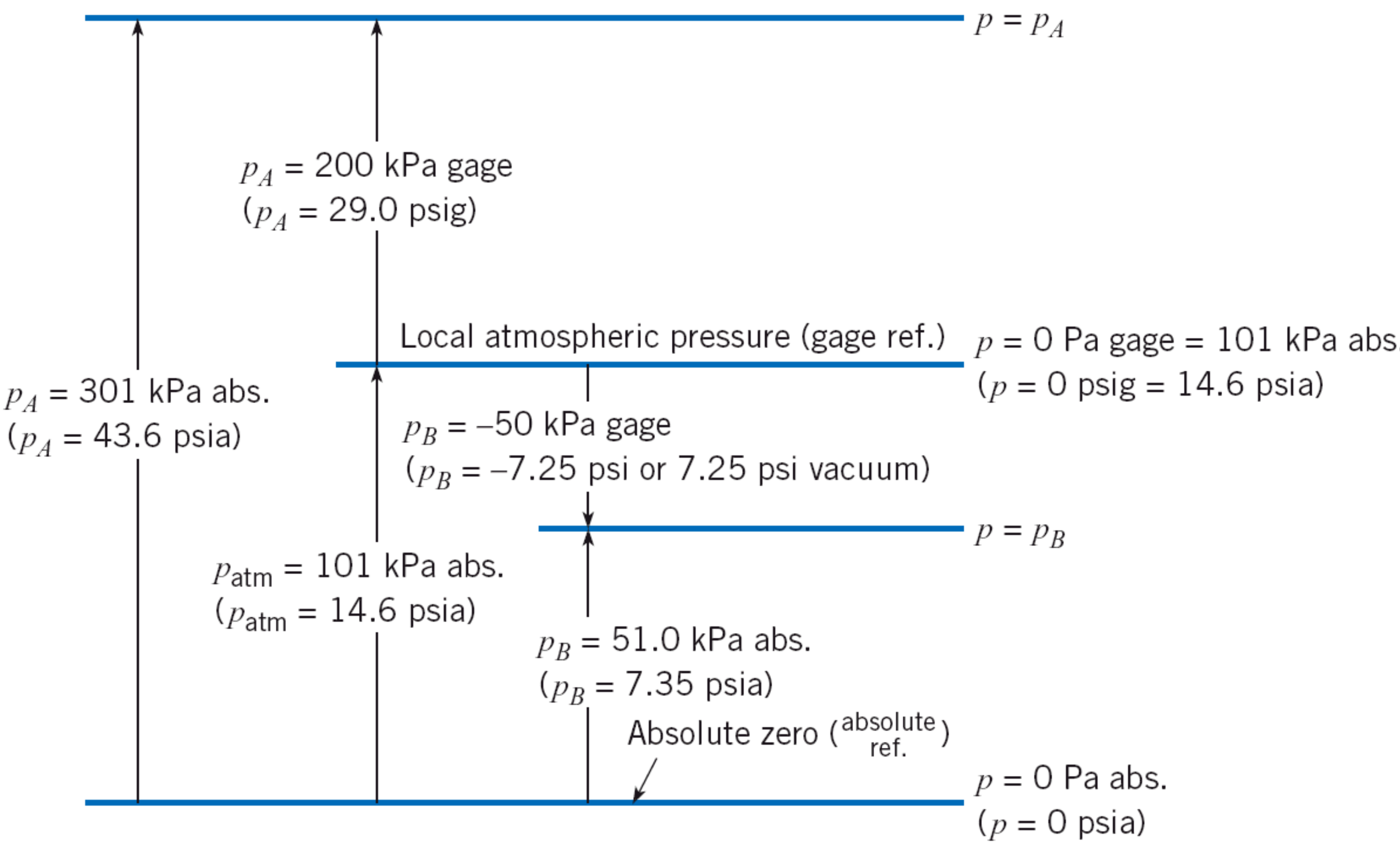


Figure 3.3 (p. 34)

Example of pressure relations.



REMEMBER: Atmospheric pressure is

1 atm = 14.696 psia
= 101.325 kPa
= 1.01 bar

0 psig = 14.696 psia

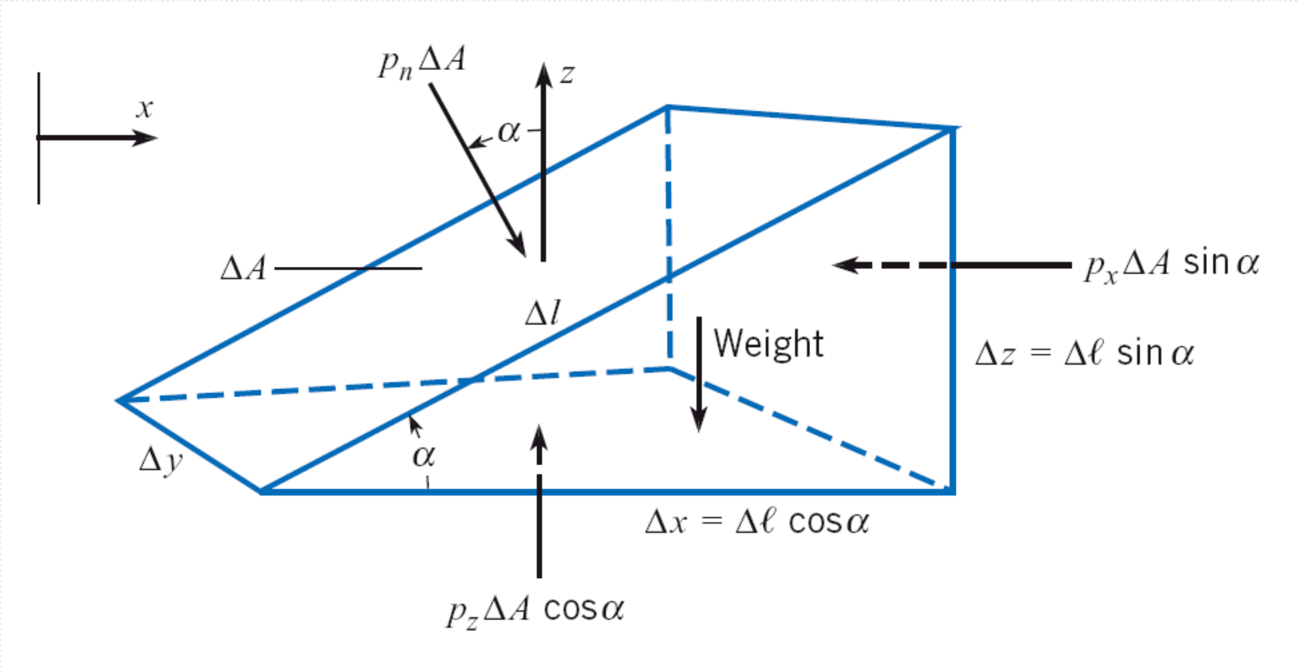
Absolute pressure (P_{abs}) =
gage pressure (psig) +
atmospheric pressure (P_{atm})

Pressure at a Point

- Pressure at a any point in a fluid is the same in all directions.
- Pressure has a magnitude, but not a specific direction, and thus it is a scalar quantity.
- Proof:

Figure 3.1 (p. 31)

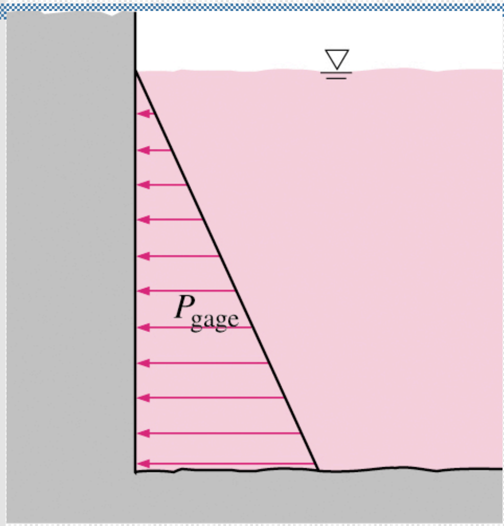
Pressure forces on a fluid element in equilibrium.

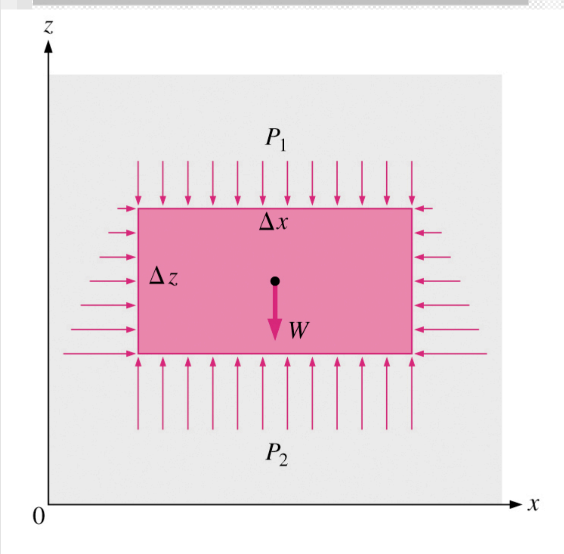


$$(p_n \Delta y \Delta l)\sin\alpha - p_x\left(\Delta y \Delta l \sin\alpha\right) = 0$$

$$p_n = p_x$$

Variation of Pressure with Depth





- In the presence of a gravitational field, pressure increases with depth because more fluid rests on deeper layers.
- To obtain a relation for the variation of pressure with depth, consider rectangular element
  - Force balance in *z*-direction gives

$$\sum F_z = ma_z = 0$$

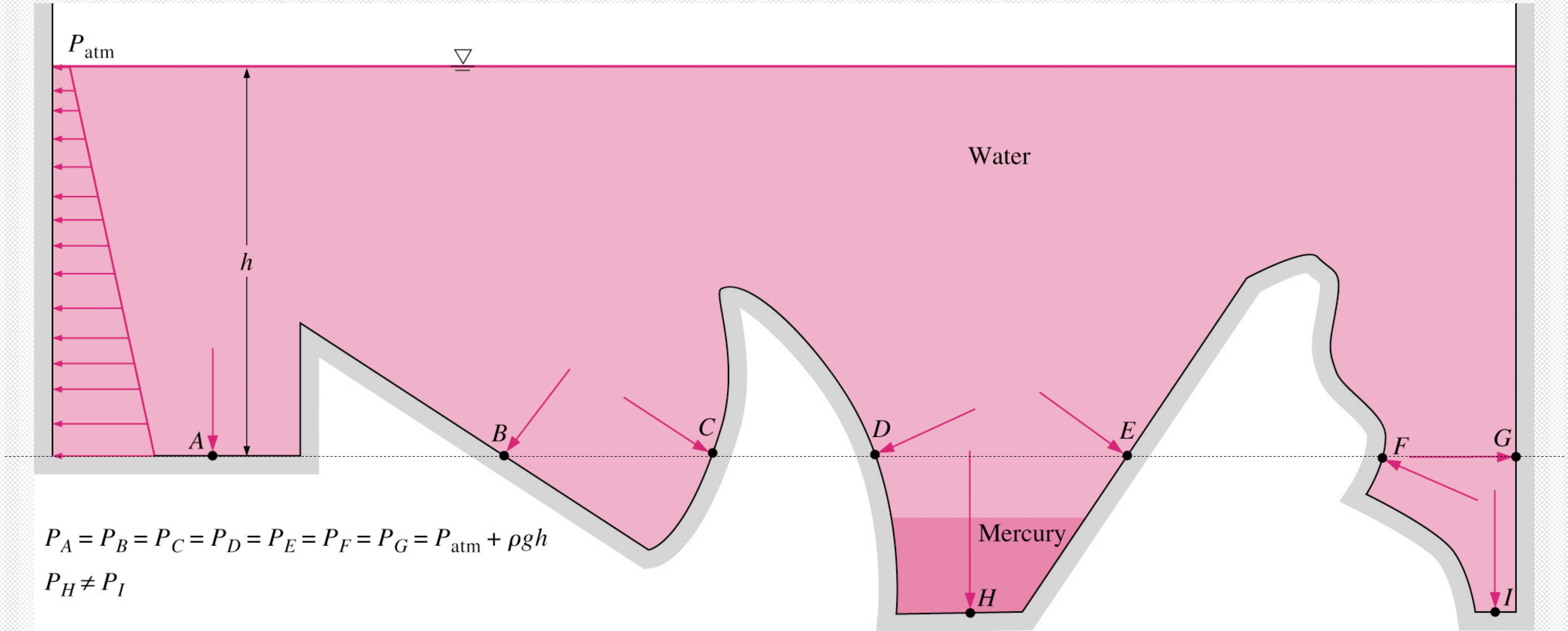
$$P_2 \Delta x \Delta y - P_1 \Delta x \Delta y - \rho g \Delta x \Delta z \Delta y = 0$$

  - Dividing by $\Delta x \Delta y$ and rearranging gives

$$\Delta P = P_2 - P_1 = \rho g \Delta z = \gamma_s \Delta z$$

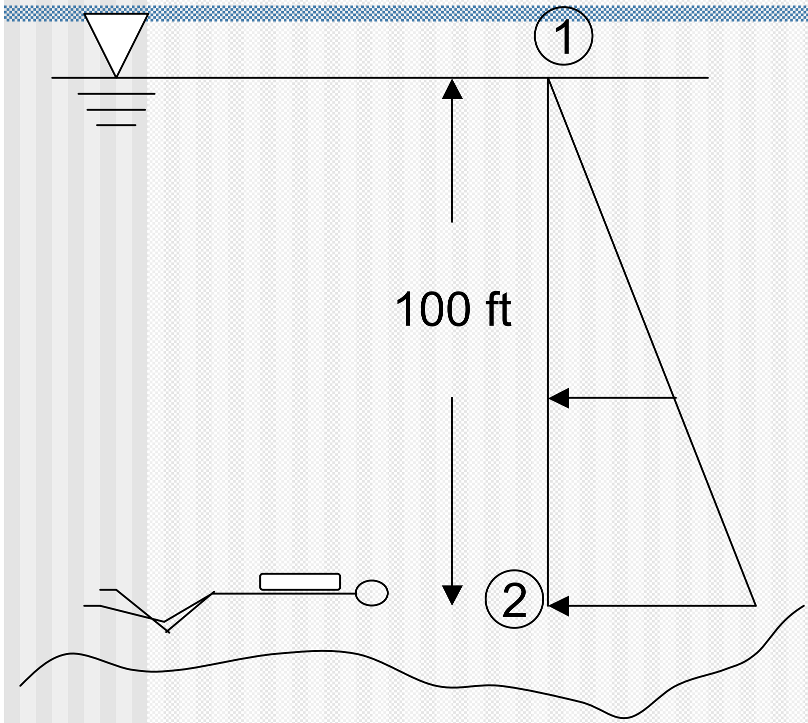
Variation of Pressure with Depth

- Pressure in a fluid at rest is independent of the shape of the container.
- Pressure is the same at all points on a horizontal plane in a given fluid.



Scuba Diving and Hydrostatic Pressure

Scuba Diving and Hydrostatic Pressure



- Pressure on diver at 30.5 m?

$$P_{gage,2} = \rho g z = \left(998 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(30.5m\right) = 298.5kPa$$

$$= 298.5kPa\left(\frac{1atm}{101.325kPa}\right) = 2.95atm$$

$$P_{abs,2} = P_{gage,2} + P_{atm} = 2.95atm + 1atm = 3.95atm$$

- Danger of emergency ascent?

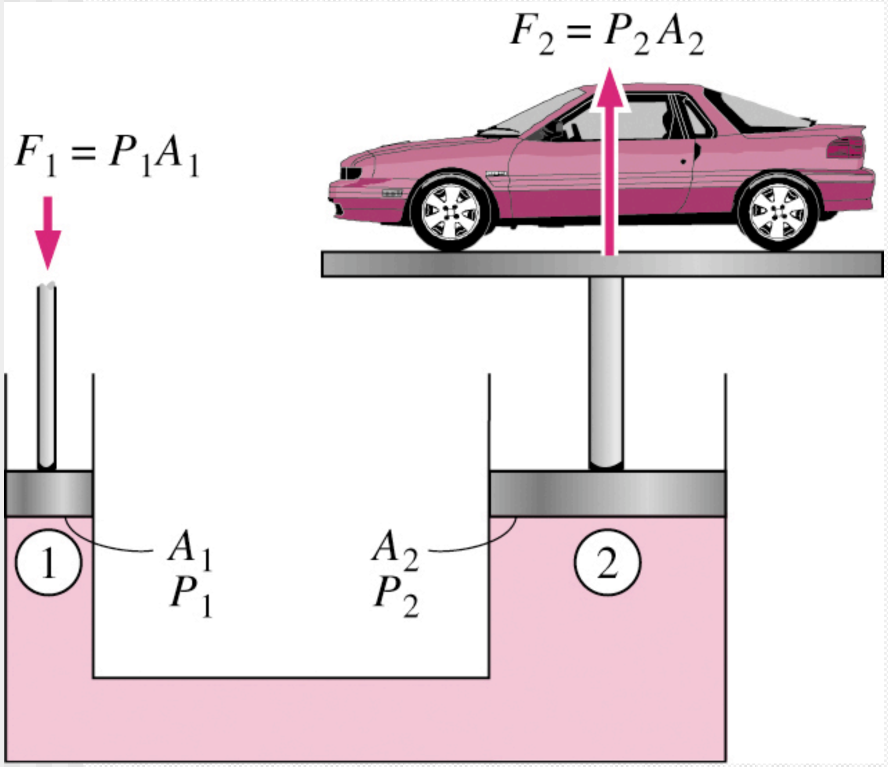
Boyle's law

$$P_1V_1 = P_2V_2$$

$$\frac{V_1}{V_2} = \frac{P_2}{P_1} = \frac{3.95atm}{1atm} \approx 4$$

If you hold your breath on ascent, your lung volume would increase by a factor of 4, which would result in embolism and/or death.

Pascal's Law



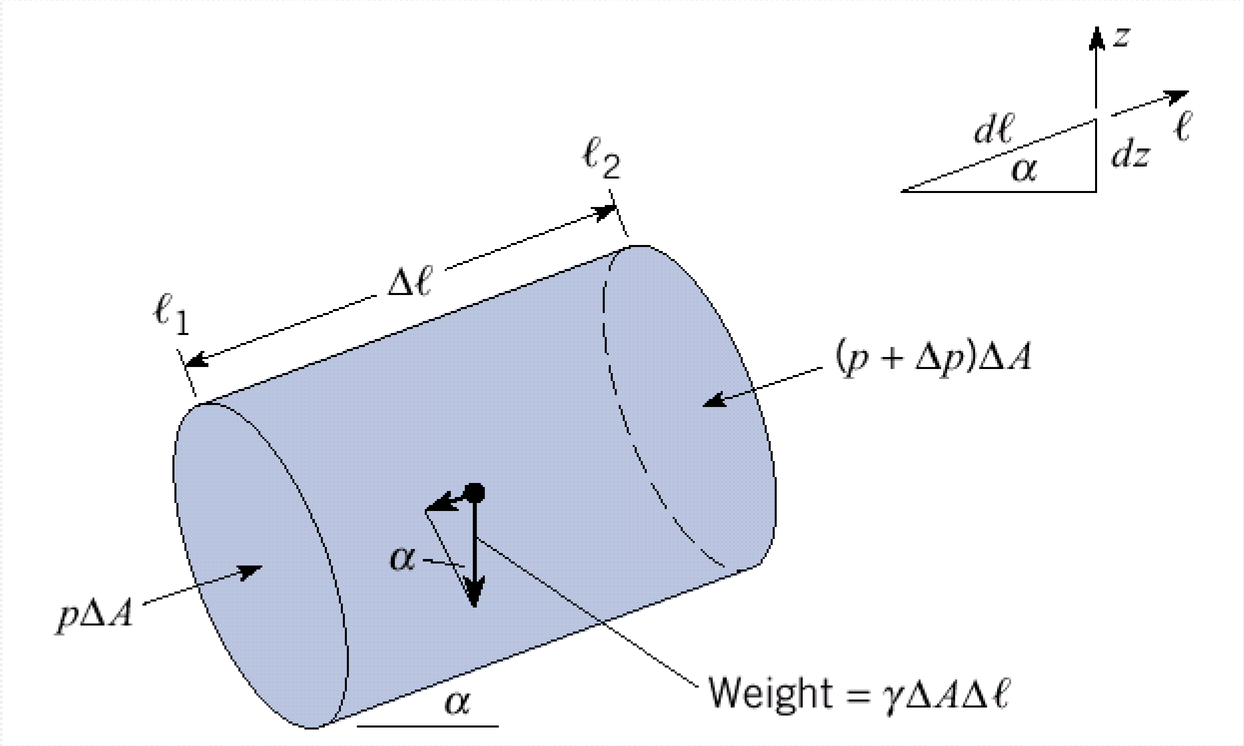
- Pressure applied to a confined fluid increases the pressure throughout by the same amount.
- In picture, pistons are at same height:

$$p_1 = p_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

- Ratio A_2/A_1 is called *ideal mechanical advantage*

Pressure Variation with Elevation

- For a static fluid, pressure varies only with the elevation within the fluid.



Basic equation for hydrostatic pressure variation with elevation

Write force balance in volume element, in the *l* direction:

$$p\Delta A - (p + \Delta p)\Delta A - \gamma \Delta A \Delta l \sin\alpha = 0$$

Dividing by $\Delta l \Delta A$ we get:

$$\frac{\Delta p}{\Delta l} = -\gamma \sin\alpha; \quad or, \frac{dp}{dl} = -\gamma \sin\alpha = -\gamma \frac{dz}{dl}$$

$$\frac{dp}{dl} = -\gamma \frac{dz}{dl} \quad (3.5)$$

$$\frac{dp}{dz} = -\gamma \quad (3.6)$$

Pressure Variation for a Uniform-Density Fluid

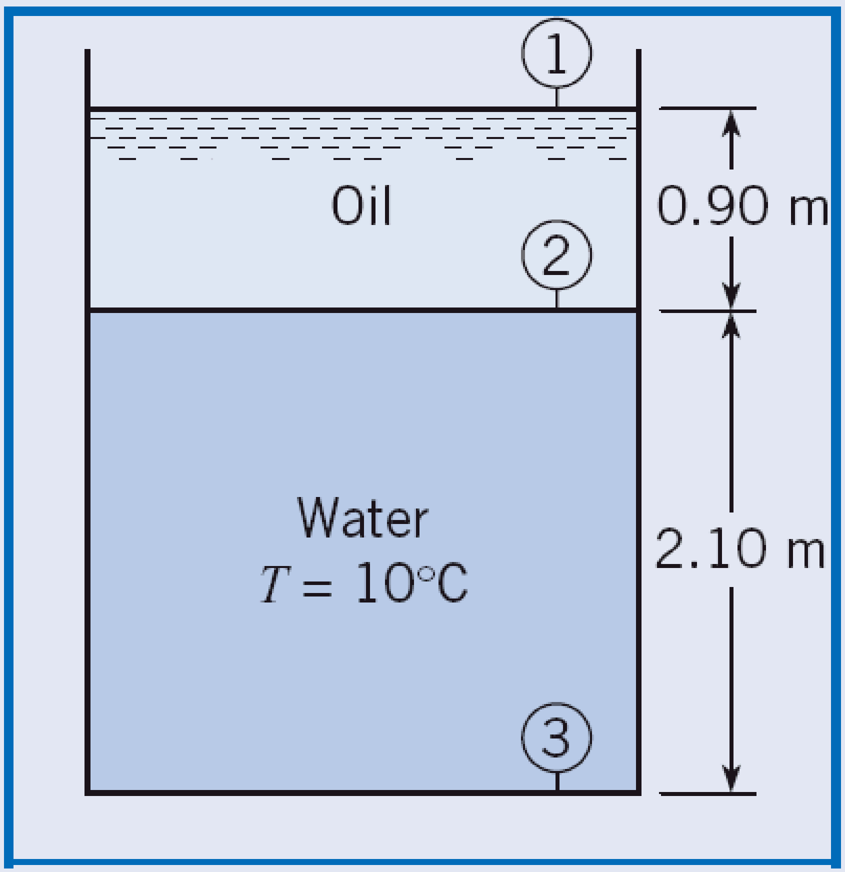
Integrating Equation 3.6:

$$p = -\gamma \ z + \text{constant}$$

$$\left(\frac{p}{\gamma} + z \right) = \text{constant}$$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

Example 3.4 (p. 38)



Find gage pressure at bottom

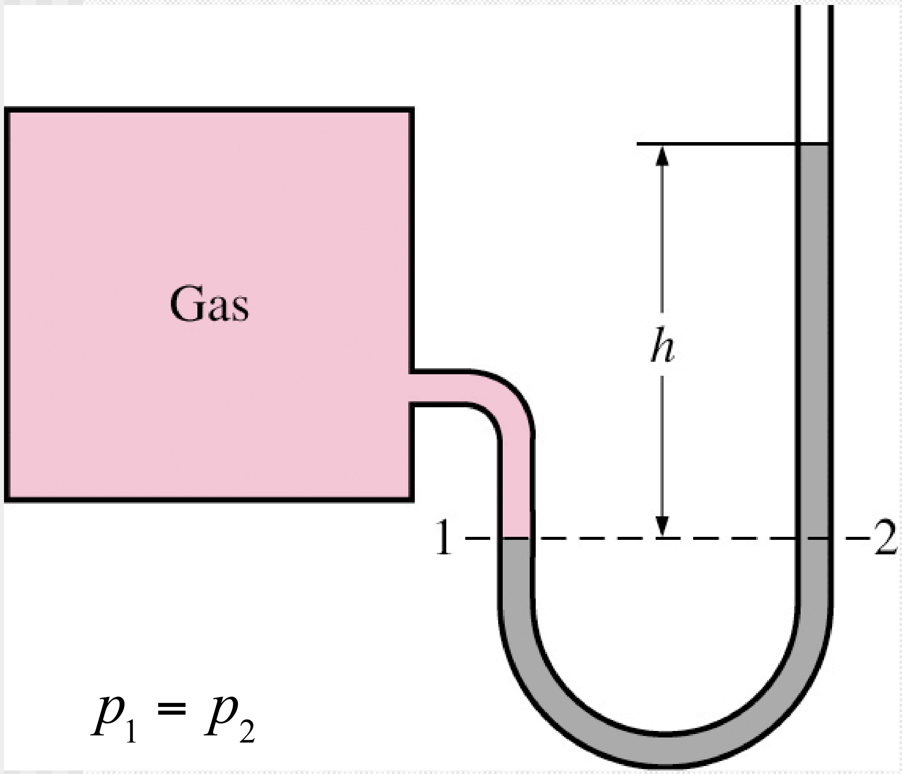
$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$$p_2 = 0.9m \times 0.8 \times 9810 \frac{N}{m^3} = 7.06 \text{ kPa gage}$$

$$\frac{p_2}{\gamma} + z_2 = \frac{p_3}{\gamma} + z_3$$

$$p_3 = 9810\left(\frac{7060}{9810} + 2.1\right) = 27.7 \text{ kPa gage}$$

The Manometer

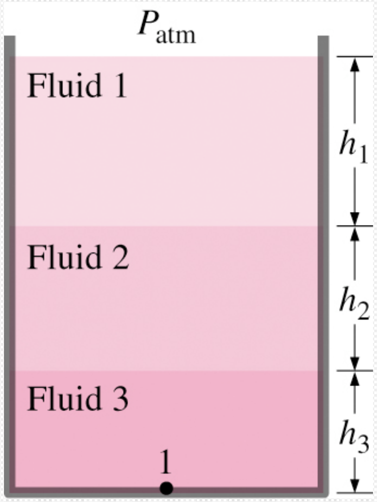


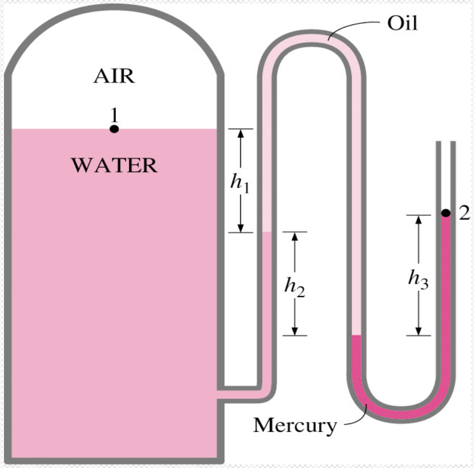
$$p_1 = p_2$$

$$p_2 = p_{atm} + \rho g h$$

- An elevation change of Δh in a fluid at rest corresponds to $\Delta p/\rho g$.
- A device based on this is called a **manometer.**
- Manometer: U-tube containing one or more fluids such as mercury, water, alcohol, or oil.
- Heavy fluids such as mercury are used if large pressure differences are anticipated.

Multifluid Manometer

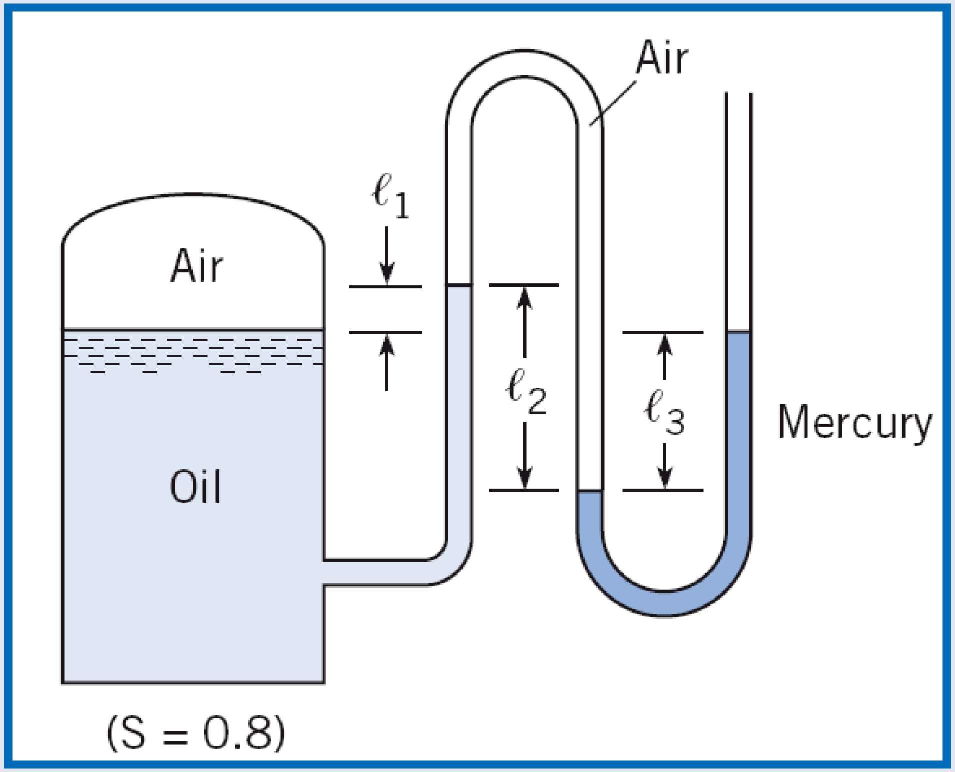




- For multi-fluid systems
  - Pressure change across a fluid column of height *h* is $\Delta p = \rho g h$.
  - Pressure increases downward, and decreases upward.
  - Two points at the same elevation in a continuous fluid are at the same pressure.
  - Pressure can be determined by adding and subtracting $\rho g h$ terms.

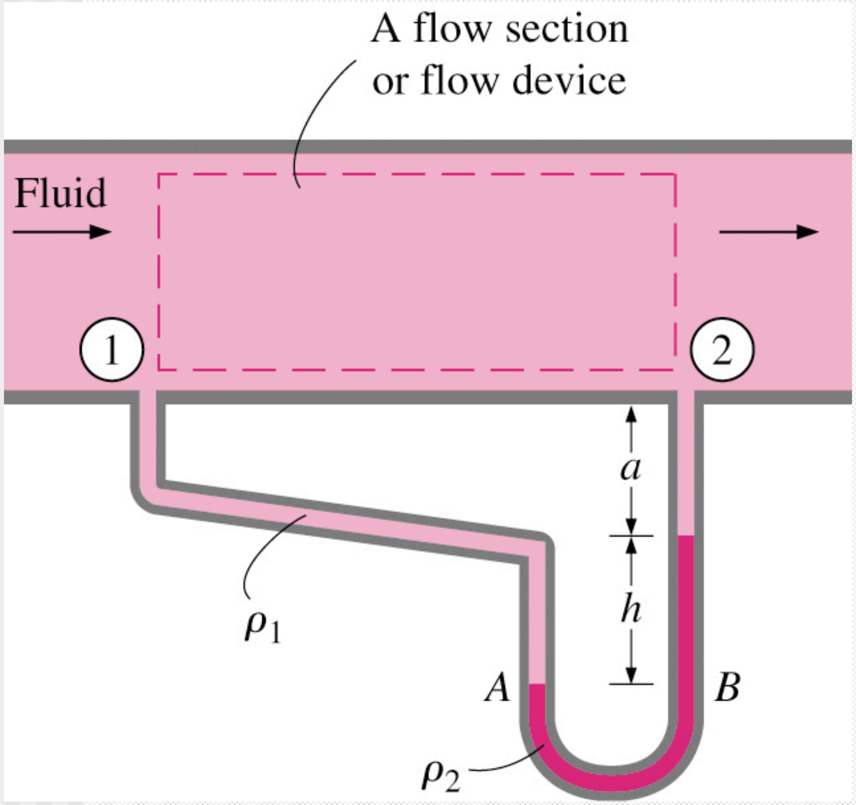
$$p_2 = p_1 + \sum_{down} \gamma_i h_i - \sum_{uo} \gamma_i h_i \qquad (3.17)$$

Example 3.9 (p. 44)



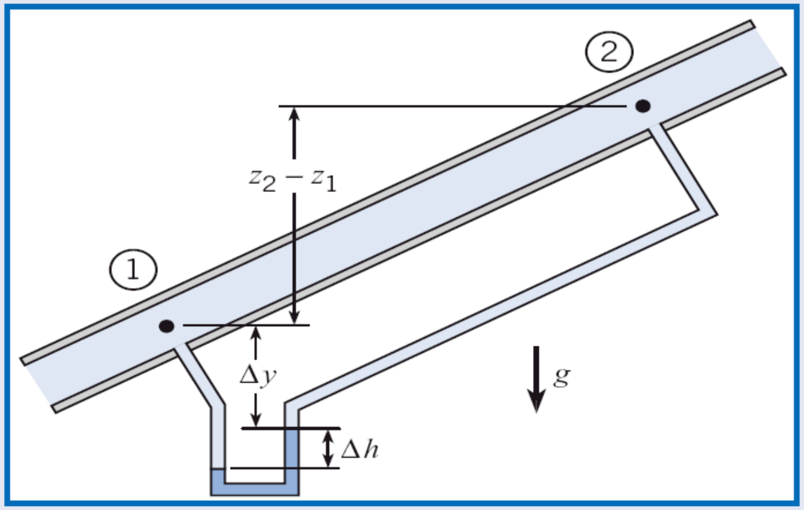
What is the pressure of the air in the tank if ℓ_1 = 40 cm, ℓ_2 = 100 cm, and ℓ_3 = 80 cm?

Measuring Pressure Drops



- Manometers are well-suited to measure pressure drops across valves, pipes, heat exchangers, etc.
- Relation for pressure drop p_1-p_2 is obtained by starting at point 1 and adding or subtracting ρgh terms until we reach point 2.
- If fluid in pipe is a gas, $\rho_2 >> \rho_1$ and $p_1 - p_2 = \rho gh$

Example 3.10, p. 45. Find the difference in piezometric pressure and piezometric head between point 1 and 2.



$$p_2 = p_1 + \sum_{down} \gamma_i h_i - \sum_{uo} \gamma_i h_i \qquad (3.17)$$

$$p_2 = p_1 + \gamma_w (\Delta y + \Delta h) - \gamma_{Hg} \Delta h - \gamma_w (\Delta y + z_2 - z_1)$$

$$(p_2 + \gamma_w z_2) - (p_1 + \gamma_w z_1) = \Delta h (\gamma_w - \gamma_{Hg})$$

$$(p_i + \gamma_w z_i) = \text{piezometric pressure}$$

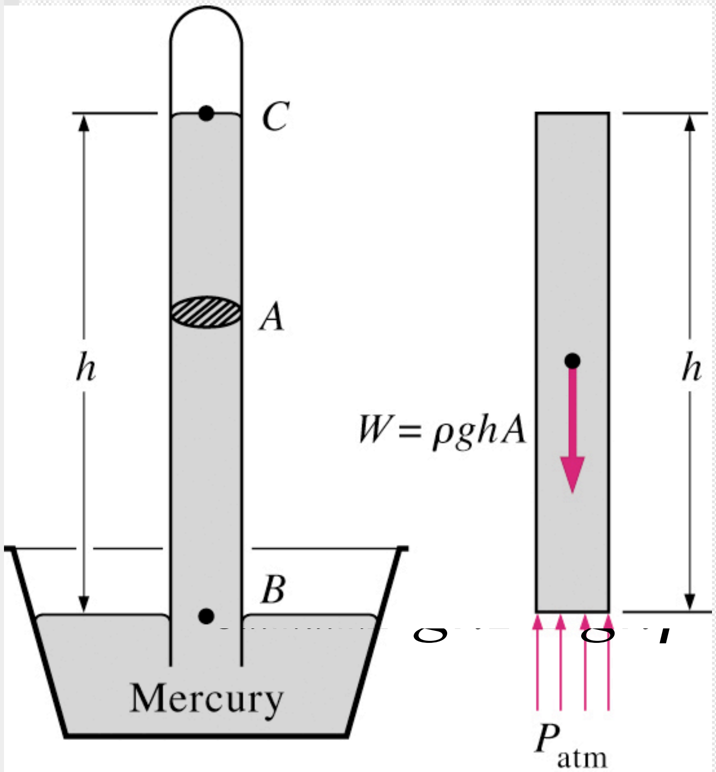
$$p_{z_2} - p_{z_1} = \Delta h (\gamma_w - \gamma_{Hg})$$

$$\text{Piezometric head, h} = \frac{\text{Piezometric pressure}}{\gamma_w}$$

$$h_2 - h_1 = \frac{p_{z_2} - p_{z_1}}{\gamma_w} = \frac{\Delta h (\gamma_w - \gamma_{Hg})}{\gamma_w}$$

$$h_2 - h_1 = \Delta h (1 - S_{Hg})$$

The Barometer

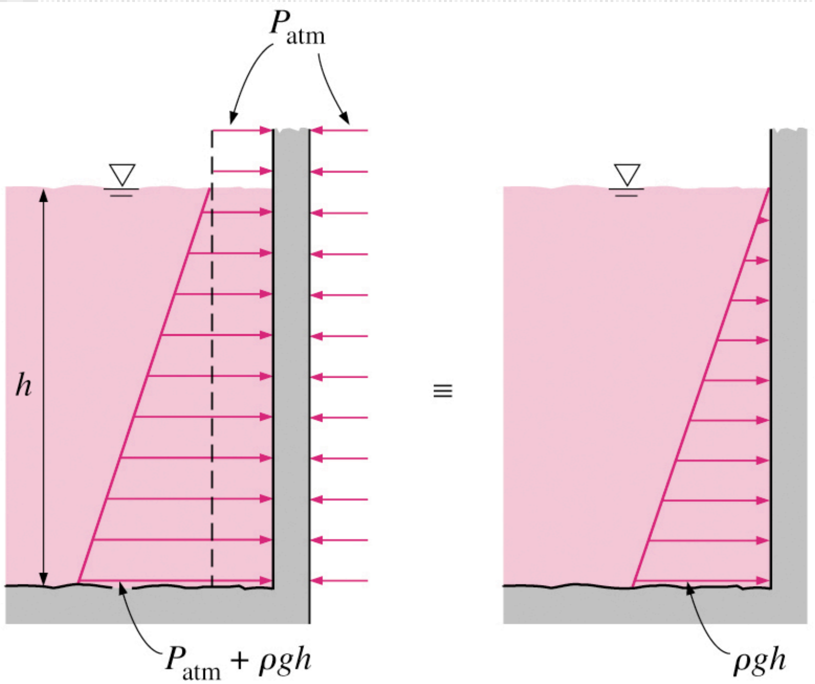


- Atmospheric pressure is measured by a deviced called a **barometer**; thus, atmospheric pressure is often referred to as the *barometric pressure*.
- P_C can be taken to be zero since there is only Hg vapor above point C, and it is very low relative to P_{atm}.
- Change in atmospheric pressure due to elevation has many effects: Cooking, nose bleeds, engine performance, aircraft performance.

Hoover Dam

Hoover Dam

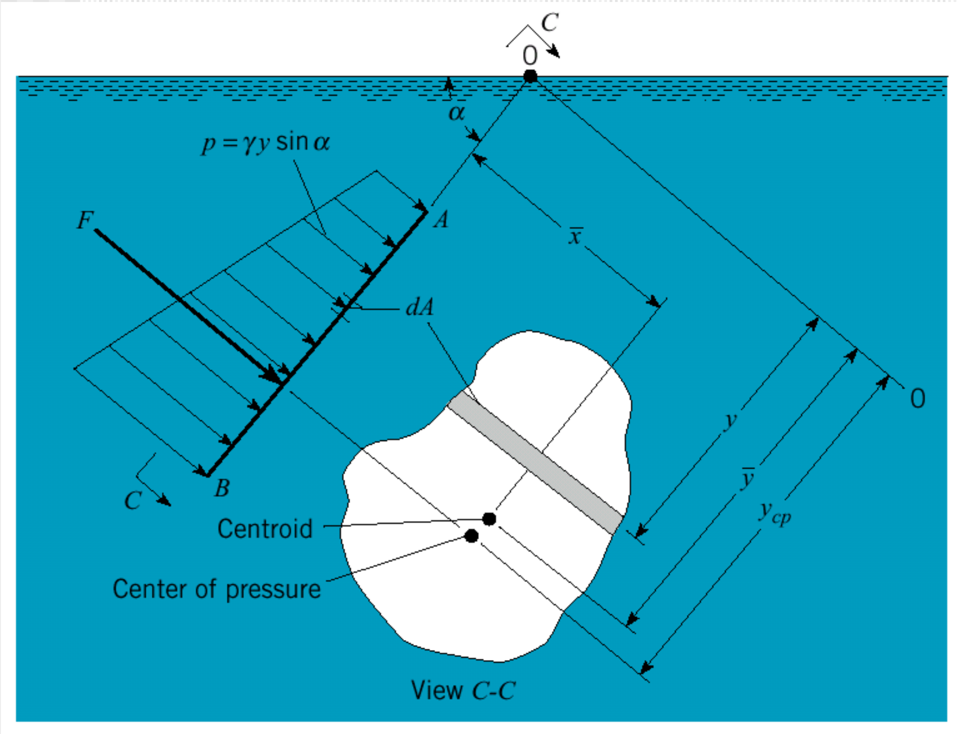
Hydrostatic Forces on Plane Surfaces



(*a*) P_{atm} considered (*b*) P_{atm} subtracted

- On a *plane* surface, the hydrostatic forces form a system of parallel forces
- For many applications, magnitude and location of application, which is called **center of pressure**, must be determined.
- Atmospheric pressure p_{atm} can be neglected when it acts on both sides of the surface.

Forces on plane areas



Need to find:

- Magnitude of resultant *F*
- Centroid of the area
- Location of the center of pressure: point of application of resultant force

Magnitude of Resultant Force

Pressure on differential strip: $p = \gamma h = \dfrac{dF}{dA}$

But $h = y \sin\alpha$; therefore: $dF = \gamma y \sin\alpha dA$

Total force on plane: $F = \int \gamma y \sin\alpha dA = \gamma \sin\alpha \int y dA$

Definition of centroid: $\bar{y} = \dfrac{1}{A}\int y dA$. Therefore:

$F = \gamma \sin\alpha \cdot A\bar{y}$

$\boxed{F = \gamma \bar{h} A}$ where $\bar{h} = \bar{y} \sin\alpha$

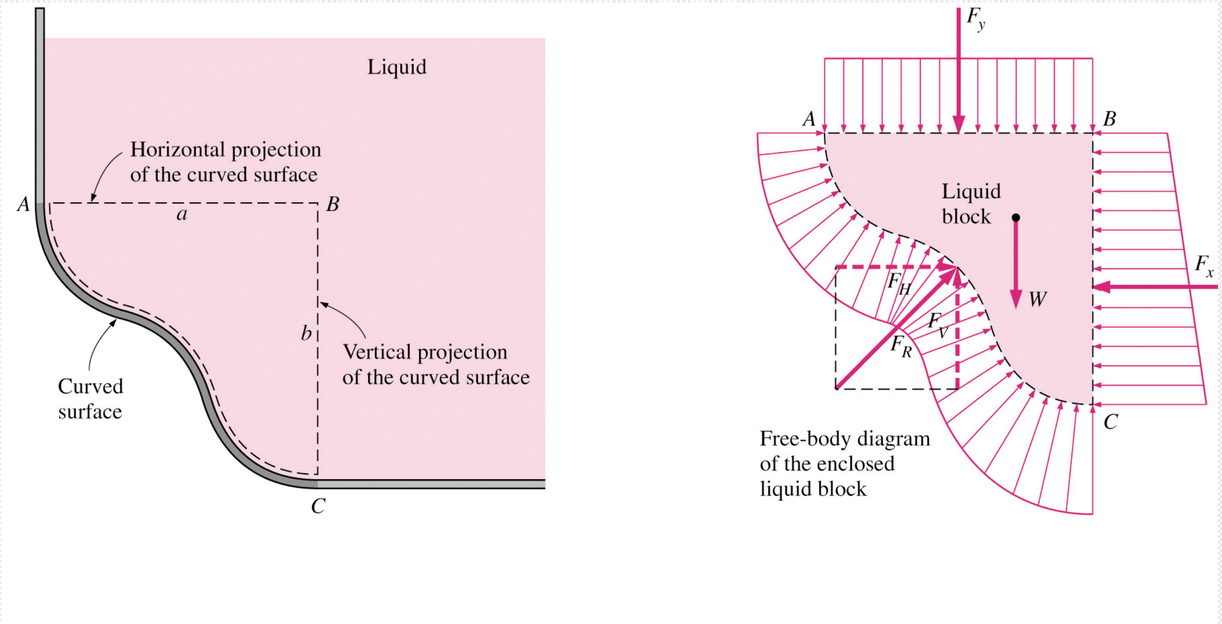
Position of the center of pressure

- Line of action of resultant force *F* does not pass through the centroid of the surface. In general, it lies underneath where the pressure is higher.
- Vertical location of **Center of Pressure** is determined by equation the moment of the resultant force to the moment of the distributed pressure force.

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A} \qquad (3.25)$$

- The moment of inertia is tabulated for simple geometries.

Hydrostatic Forces on Curved Surfaces



- F_R on a curved surface is more involved since it requires integration of the pressure forces that change direction along the surface.
- Easiest approach: determine horizontal and vertical components F_H and F_V separately.

Hydrostatic Forces on Curved Surfaces

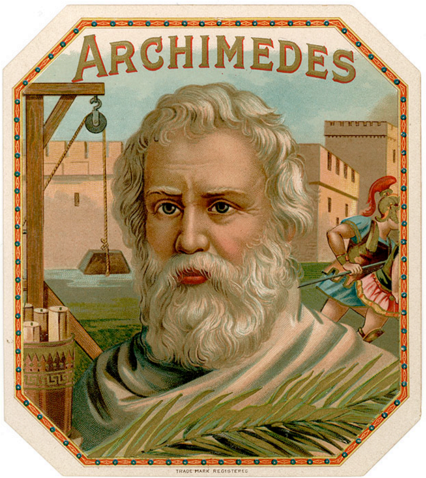
- Horizontal component of total hydrostatic pressure on any surface = total pressure ***on the projection*** of that surface on the vertical plane.
- Vertical component of total hydrostatic pressure on any surface = weight of volume of liquid extending vertically from the surface to the free liquid surface.
- Magnitude of force $F_R=(F_H^2+F_V^2)^{1/2}$
- Angle of force is $\alpha = tan^{-1}(F_V/F_H)$

Buoyancy and Stability

- Buoyancy is due to the fluid displaced by a body. $F_B = \gamma_f Vol.$
- **Archimedes principle** : The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

Examples of Archimedes Principle

The Golden Crown of Hiero II, King of Syracuse

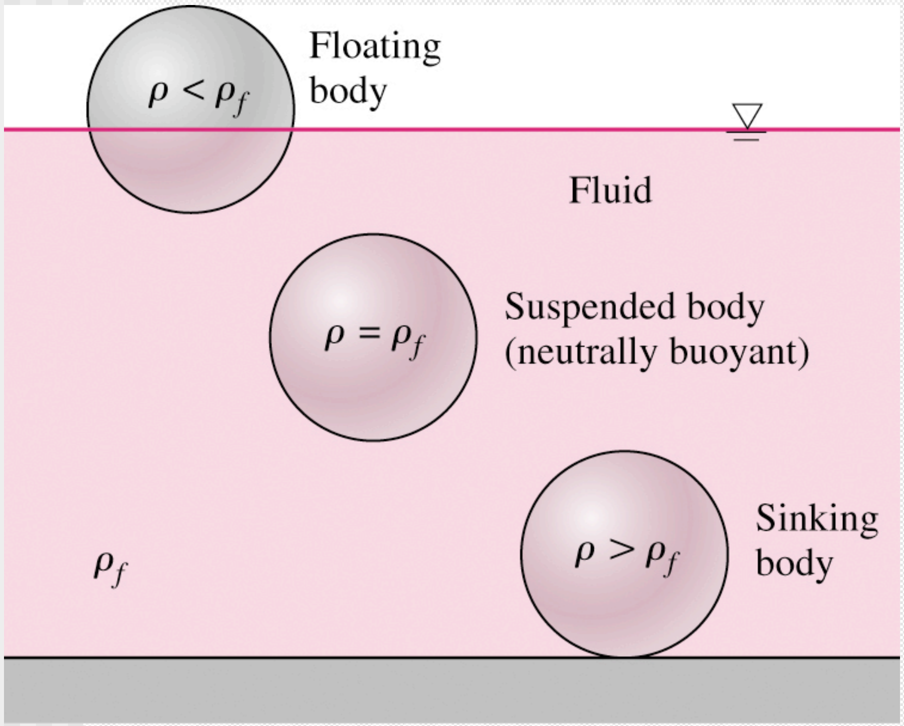


- Archimedes, 287-212 B.C.
- Hiero, 306-215 B.C.
- Hiero learned of a rumor where the goldsmith replaced some of the gold in his crown with silver. Hiero asked Archimedes to determine whether the crown was pure gold.
- Archimedes had to develop a nondestructive testing method

The Golden Crown of Hiero II, King of Syracuse

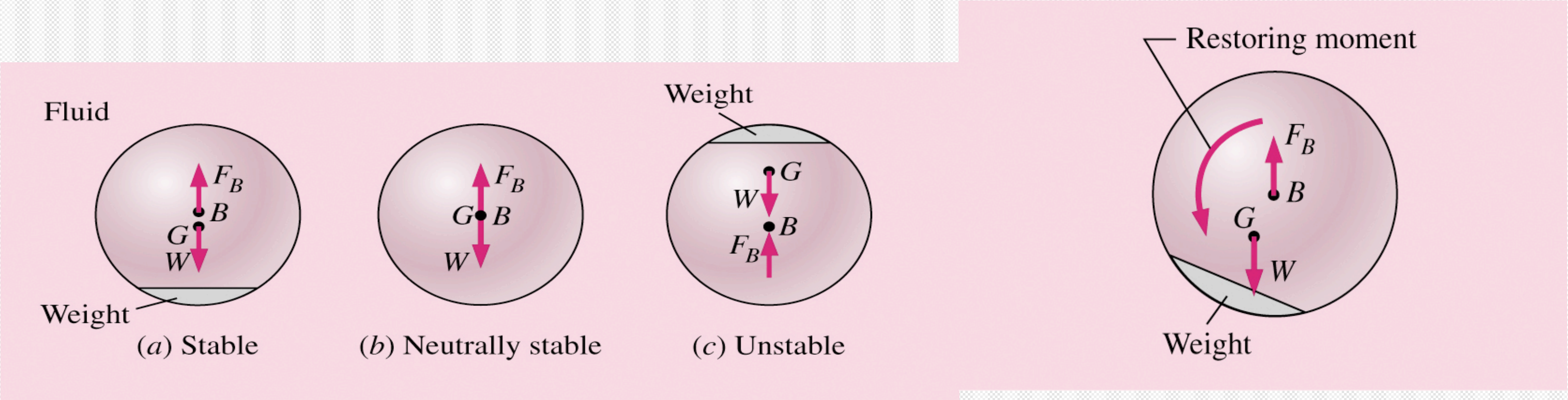
- The weight of the crown and nugget are the same in air: $W_c = \rho_c V_c = W_n = \rho_n V_n$.
- If the crown is pure gold, $\rho_c = \rho_n$ which means that the volumes must be the same, $V_c = V_n$.
- In water, the buoyancy force is $B = \rho_{H2O} V$.
- If the scale becomes unbalanced, this implies that the $V_c \neq V_n$, which in turn means that the $\rho_c \neq \rho_n$
- Goldsmith was shown to be a fraud!

Buoyancy and Stability



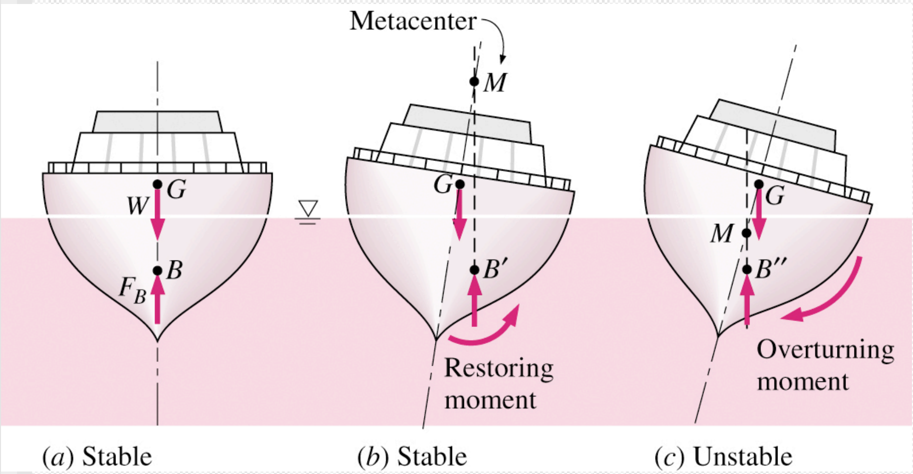
- Buoyancy force F_B is equal only to the displaced volume $\rho_f g V_{displaced}$.
- Three scenarios possible
  1. $\rho_{body} < \rho_{fluid}$: Floating body
  2. $\rho_{body} = \rho_{fluid}$: Neutrally buoyant
  3. $\rho_{body} > \rho_{fluid}$: Sinking body

Stability of Immersed Bodies



- Rotational stability of immersed bodies depends upon relative location of *center of gravity G* and *center of buoyancy B*.
  - *G* below *B*: stable
  - *G* above *B*: unstable
  - *G* coincides with *B*: neutrally stable.

Stability of Floating Bodies



- If body is bottom heavy (*G* lower than *B*), it is always stable.
- Floating bodies can be stable when *G* is higher than *B* due to shift in location of center buoyancy and creation of restoring moment.
- Measure of stability is the metacentric height G*M*. If *GM*>1, ship is stable.