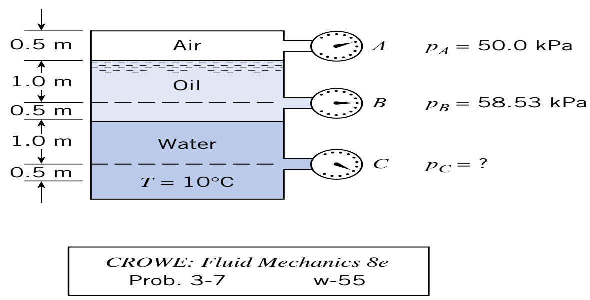
3.11: PROBLEM DEFINITION

Situation:
A closed tank contains air, oil, and water.

Find:
Specific gravity of oil.
Pressure at C (kPa-gage).

Sketch:



Properties:
Water (10 °C), Table A.5, $\gamma = 9810\,\text{N/m}^3$.

PLAN

1. Find the oil specific gravity by applying the hydrostatic equation from A to B.
2. Apply the hydrostatic equation to the water.
3. Apply the hydrostatic equation to the oil.
4. Find the pressure at C by combining results for steps 2 and 3.

SOLUTION

1. Hydrostatic equation (from oil surface to elevation B):

$$\begin{aligned} p_A + \gamma z_A &= p_B + \gamma z_B \\ 50{,}000\ \text{N/m}^2 + \gamma_{\text{oil}}\,(1\ \text{m}) &= 58{,}530\ \text{N/m}^2 + \gamma_{\text{oil}}\,(0\ \text{m}) \\ \gamma_{\text{oil}} &= 8530\ \text{N/m}^3 \end{aligned}$$

Specific gravity:

$$S = \frac{\gamma_{\text{oil}}}{\gamma_{\text{water}}} = \frac{8530\ \text{N/m}^3}{9810\ \text{N/m}^3}$$

$$\boxed{S_{\text{oil}} = 0.87}$$

2. Hydrostatic equation (in water):

$$p_c = (p_{\text{btm of oil}}) + \gamma_{\text{water}}\,(1\,\text{m})$$

3. Hydrostatic equation (in oil):

$$p_{\text{btm of oil}} = (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m})$$

4. Combine equations:

$$\begin{aligned} p_c &= (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m}) + \gamma_{\text{water}}\,(1\,\text{m}) \\ &= \left(58,530\,\text{Pa} + 8530\,\text{N/m}^3 \times 0.5\,\text{m}\right) + 9810\,\text{N/m}^3\,(1\,\text{m}) \\ &= 72,605\ \text{N/m}^2 \end{aligned}$$

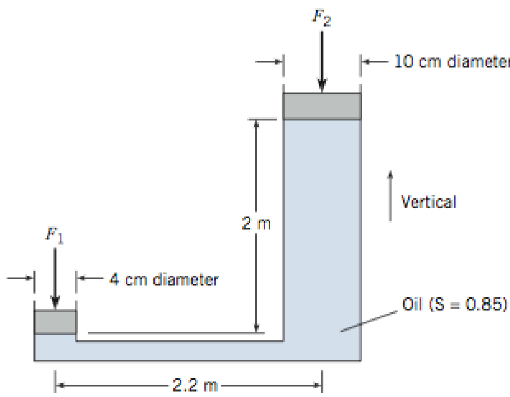
$$\boxed{p_c = 72.6\ \text{kPa-gage}}$$

3.13: PROBLEM DEFINITION

Situation:

A force is applied to a piston.

$F_1 = 200\,\text{N}$, $d_1 = 4\,\text{cm}$, $d_2 = 10\,\text{cm}$.



Find:

Force resisted by piston.

Assumptions:

Neglect piston weight.

PLAN

Apply the hydrostatic equation and equilibrium.

SOLUTION

1. Equilibrium (piston 1)

$$\begin{aligned} F_1 &= p_1 A_1 \\ p_1 &= \frac{F_1}{A_1} \\ &= \frac{4 \times 200\,\text{N}}{\pi \cdot (0.04\,\text{m})^2\ \text{m}^2} \\ &= 1.592 \times 10^5\,\text{Pa} \end{aligned}$$

2. Hydrostatic equation

$$\begin{aligned} p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\ p_2 &= p_1 + (S\gamma_{\text{water}})(z_1 - z_2) \\ &= 1.592 \times 10^5\,\text{Pa} + \left(0.85 \times 9810\,\text{N}/\,\text{m}^3\right)(-2\,\text{m}) \\ &= 1.425 \times 10^5\,\text{Pa} \end{aligned}$$

3. Equilibrium (piston 2)

$$\begin{aligned} F_2 &= p_2 A_2 \\ &= \left(1.425 \times 10^5 \,\mathrm{N/\,m^2}\right)\left(\frac{\pi\,(0.1\,\mathrm{m})^2}{4}\right) \\ &= 1119\,\mathrm{N} \end{aligned}$$

$$\boxed{F_2 = 1120 \text{ N}}$$

3.21: PROBLEM DEFINITION

Situation:

An odd tank contains water, air and a liquid.

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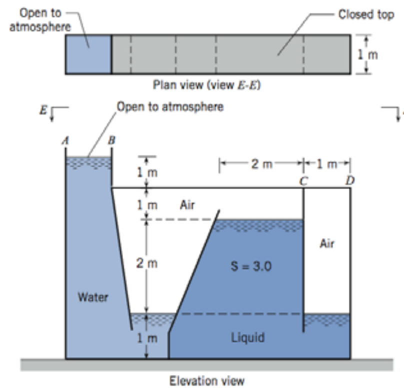


Find:

Maximum gage pressure (kPa).
Where will maximum pressure occur.
Hydrostatic force (in kN) on top of the last chamber, surface CD.

Properties:

$\gamma_{\text{water}} = 9810 \text{ N/m}^3$.

PLAN

1. To find the maximum pressure, apply the manometer equation.
2. To find the hydrostatic force, multiply pressure times area.

SOLUTION

1. Manometer eqn. (start at surface AB; neglect pressure changes in the air; end at the bottom of the liquid reservoir)

$$
\begin{aligned}
0 + 4 \times \gamma_{\mathrm{H_2O}} + 3 \times 3\gamma_{\mathrm{H_2O}} &= p_{\max} \\
p_{\max} &= 13\,\mathrm{m} \times 9,810\,\mathrm{N/\,m^3} \\
&= 127,530 \text{ N/m}^2
\end{aligned}
$$

$$
\boxed{p_{\max} = 127.5 \text{ kPa}}
$$

Answer ⇒ Maximum pressure will be at the bottom of the liquid that has a specific gravity of $S = 3$.

2. Hydrostatic force

$$
\begin{aligned}
F_{CD} &= pA \\
&= (127,530\,\mathrm{N/\,m^2} - 1\,\mathrm{m} \times 3 \times 9810\,\mathrm{N/\,m^3}) \times 1 \text{ m}^2 \\
&\boxed{F_{CD} = 98.1 \text{ kN}}
\end{aligned}
$$

3.22: PROBLEM DEFINITION

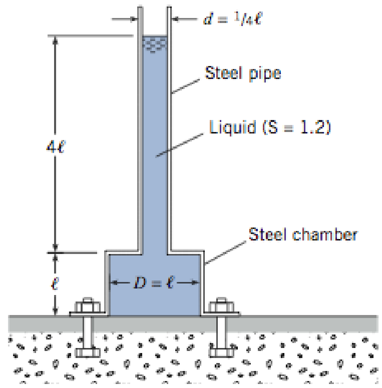
Situation:

A steel pipe is connected to a steel chamber.

$\ell = 2.5\,\text{ft}$, $W = 600\,\text{lbf}$.

$D_1 = 0.25\ell$, $z_1 = 5\ell$.

$D_2 = \ell$, $S = 1.2$.



Find:

Force exerted on chamber by bolts (lbf).

Properties:

$\gamma_{\text{water}} = 62.4\,\text{lbf}/\,\text{ft}^3$.

PLAN

Apply equilibrium and the hydrostatic equation.

SOLUTION

1. Equilibrium. (system is the steel structure plus the liquid within)

$$\text{(Weight of the steel)} = \begin{array}{l}\text{(Force exerted by bolts)} + \text{(Weight of the liquid)} + \\ \text{(Pressure force acting on the bottom of the free body)}\end{array}$$

$$F_B + W_{\text{liquid}} + W_s = p_2 A_2 \tag{1}$$

2. Hydrostatic equation (location 1 is on surface; location 2 at the bottom)

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma_{\text{liquid}}} + z_2 \\ 0 + 5\ell &= \frac{p_2}{1.2\gamma_{\text{water}}} + 0 \\ p_2 &= 1.2\gamma_{\text{water}} 5\ell \\ &= 1.2 \times 62.4 \times 5 \times 2.5 \\ &= 936 \text{ psfg} \end{aligned}$$

3. Area

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi \ell^2}{4} = \frac{\pi \times (2.5\,\text{ft})^2}{4} = 4.909\,\text{ft}^2$$

4. Weight of liquid

$$\begin{aligned} W_{\text{liquid}} &= \left(A_2\ell + \frac{\pi d^2}{4}4\ell\right)\gamma_{\text{liquid}} = \left(A_2\ell + \frac{\pi\ell^3}{16}\right)(1.2)\,\gamma_{\text{water}} \\ &= \left(\left(4.909\,\text{ft}^2\right)(2.5\,\text{ft}) + \frac{\pi\,(2.5\,\text{ft})^3}{16}\right)(1.2)\left(62.4\frac{\text{lbf}}{\text{ft}^3}\right) \\ &= 1148.7\,\text{lbf} \end{aligned}$$

5. Substitute numbers into Eq. (1)

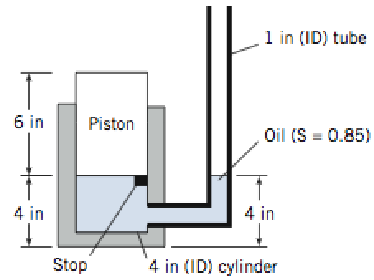
$$\begin{aligned} F_B + (1148.7\,\text{lbf}) + (600\,\text{lbf}) &= \left(936\,\text{lbf}/\,\text{ft}^2\right)\left(4.909\,\text{ft}^2\right) \\ F_B &= 2846\,\text{lbf} \end{aligned}$$

$$\boxed{F_B = 2850\ \text{lbf}}$$

3.25: PROBLEM DEFINITION

Situation:

Oil is added to the tube so the piston rises 1 inch.
$W_{\text{piston}} = 10\,\text{lbf}$, $S = 0.85$.
$D_p = 4\,\text{in}$, $D_{\text{tube}} = 1\,\text{in}$.

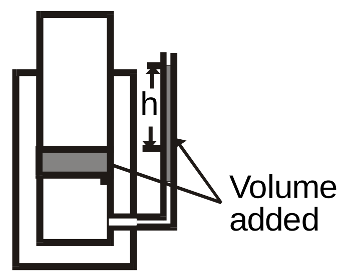


Find:

Volume of oil (in^3) that is added.

SOLUTION

Notice that the oil fills the apparatus as shown below.



Pressure acting on the bottom of the piston

$$\begin{aligned} p_p A_p &= 10 \text{ lbf} \\ p_p &= \frac{10\,\text{lbf}}{A_p} = \frac{10\,\text{lbf}}{\pi\,(4\,\text{in})^2/4} \\ &= 0.796 \text{ psig } = 114.6 \text{ psfg} \end{aligned}$$

Hydrostatic equation (apply to liquid in the tube)

$$\begin{aligned} \gamma_{\text{oil}} h &= 114.6 \text{ psfg} \\ h &= 114.6/(62.4 \times 0.85) = 2.161 \text{ ft} = 25.9 \text{ in} \end{aligned}$$

Calculate volume

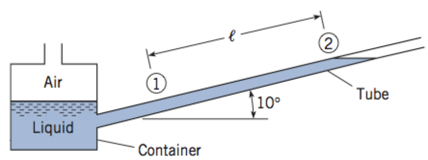
$$\begin{aligned} V\!\!\!/_{\text{added}} &= V\!\!\!/_{\text{left}} + V\!\!\!/_{\text{right}} \\ &= \frac{\pi\,(4\,\text{in})^2\,(1\,\text{in})}{4} + \frac{\pi\,(1\,\text{in})^2\,(1\,\text{in} + 25.9\,\text{in})}{4} \end{aligned}$$

$$\boxed{V\!\!\!/_{\text{added}} = 33.7 \text{ in.}^3}$$

3.35: PROBLEM DEFINITION

Situation:

State 1: air at p_{atm}, liquid in tube at elevation 1.
State 2: air is pressurized; liquid at elevation 2.
$D_{\text{container}} = 10D_{\text{tube}}$, $\ell = 3\,\text{ft}$.



Find:

Pressure in the air within the container (psfg).

Properties:

liquid, $\gamma = 50\,\text{lbf/ft}^3$.

PLAN

1. Find the decrease in liquid level in the container by using conservation of mass.
2. Find the pressure in the container by apply the manometer equation.

SOLUTION

1. Conservation of mass (applied to liquid)

$$\begin{aligned}
\text{Gain in mass of liq. in tube} &= \text{Loss of mass of liq. in container} \\
(\text{Volume change in tube})\,\rho_{\text{liquid}} &= (\text{Volume change in container })\,\rho_{\text{liquid}} \\
\forall_{\text{tube}} &= \forall_{\text{container}}
\end{aligned}$$

$$\begin{aligned}
(\pi/4)D_{\text{tube}}^2 \times \ell &= (\pi/4)D_{\text{container}}^2 \times (\Delta h)_{\text{container}} \\
(\Delta h)_{\text{container}} &= \left(\frac{D_{\text{tube}}}{D_{\text{container}}}\right)^2 \ell \\
(\Delta h)_{\text{container}} &= \left(\frac{1}{10}\right)^2 \times 3\,\text{ft} \\
&= 0.03\,\text{ft}
\end{aligned}$$

2. Manometer equation (point 1 = free surface of liquid in the tube; point 2 = free surface of liquid in the container)

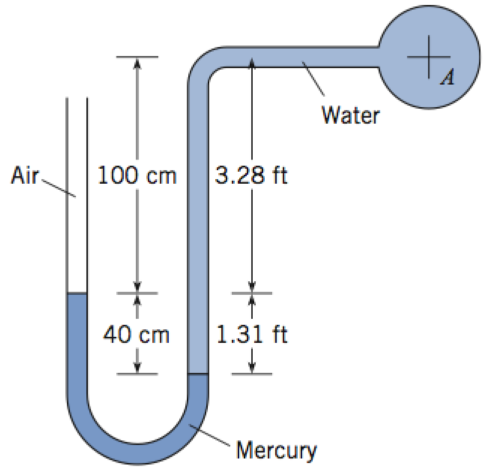
$$\begin{aligned}
p_{\text{container}} &= (\ell \sin 10^\circ + \Delta h)\gamma \\
&= (3 \sin 10^\circ + .03)\,\text{ft} \times 50\,\text{lbf/ft}^3 \\
&= 27.548\,\text{lbf/ft}^2
\end{aligned}$$

$$\boxed{p_{\text{container}} = 27.5 \text{ psfg}}$$

3.36: PROBLEM DEFINITION

Situation:

A pipe system has a manometer attached to it.



Find:

Gage pressure at center of pipe A (psi, kPa).

Properties:

Mercury, Table A.4: $\gamma = 1.33 \times 10^5\,\text{N/m}^3$.

Water, Table A.5: $\gamma = 9810\,\text{N/m}^3$.

PLAN

Apply the manometer equation.

SOLUTION

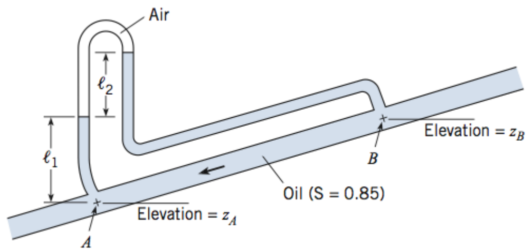
Manometer equation

$$
\begin{aligned}
p_A &= 1.31\,\text{ft} \times 847\,\text{lbf/ft}^3 - 4.59\,\text{ft} \times 62.4\,\text{lbf/ft}^3 \\
&= 823.2 \text{ psf} \\
& \boxed{p_A = 5.72 \text{ psig}} \\
p_A &= 0.4\,\text{m} \times 1.33 \times 10^5\,\text{N/m}^3 - 1.4\,\text{m} \times 9810\,\text{N/m}^3 \\
& \boxed{p_A = 39.5 \text{ kPa gage}}
\end{aligned}
$$

3.40: PROBLEM DEFINITION

Situation:

A system is described in the problem statement.
$\ell_1 = 1\,\text{m}$, $\ell_2 = 0.5\,\text{m}$.
$z_A = 10\,\text{m}$, $z_B = 11\,\text{m}$.



Find:

(a) Difference in pressure between points A and B (kPa).
(b) Difference in piezometric head between points A and B (m).

Properties:

$\gamma = 9810\,\text{N/m}^3$, $S = 0.85$.

PLAN

Apply the manometer equation.

SOLUTION

Manometer equation (apply from A to B)

$$\begin{aligned} p_A - (1\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) + (0.5\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) &= p_B \\ p_A - p_B &= 4169\,\text{Pa} \end{aligned}$$

$$\boxed{p_A - p_B = 4.17 \text{ kPa}}$$

Piezometric head

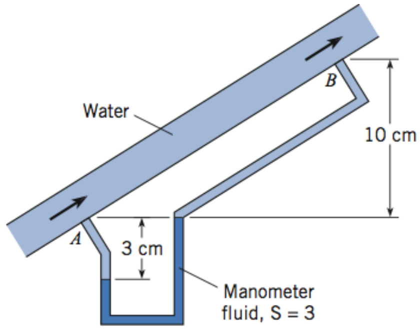
$$\begin{aligned} h_A - h_B &= \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right) \\ &= \frac{p_A - p_B}{\gamma} + (z_A - z_B) \\ &= \frac{4169\,\text{N/m}^2}{0.85 \times 9810\,\text{N/m}^3} - 1\,\text{m} \\ &= -0.5 \text{ m} \end{aligned}$$

$$\boxed{h_A - h_B = -0.50 \text{ m}}$$

3.44: PROBLEM DEFINITION

Situation:

Manometer–measuring pressure difference in a pipe.



Find:

(a) Pressure difference $(p_A - p_B)$ in kPa.
(b) Piezometric pressure difference $(p_{zA} - p_{zB})$ in kPa.

Properties:

$S = 3.0$.

PLAN

Apply the manometer equation. Use the definition of piezometric pressure.

SOLUTION

Manometer equation (apply between points A & B)

$$p_B = p_A + 0.03\gamma_f - 0.03\gamma_m - 0.1\gamma_f$$

or

$$p_A - p_B = -0.03(\gamma_f - \gamma_m) + 0.1\gamma_f$$

Substitute in values

$$p_A - p_B = -0.03\,\mathrm{m}(9810\,\mathrm{N/m^3} - 3 \times 9810\,\mathrm{N/m^3}) + 0.1 \times 9810\,\mathrm{N/m^3}$$

$$\boxed{p_A - p_B = 1.57\,\mathrm{kPa}}$$

Definition of piezometric pressure

$$p_z \equiv p + \gamma z$$

Thus

$$\begin{aligned} p_{zA} - p_{zB} &= \left(p_A + \gamma_{H2O}\, z_A\right) - \left(p_B + \gamma_{H2O}\, z_B\right) \\ &= (p_A - p_B) + \gamma_{H2O}\,(z_A - z_B) \\ &= 1.57\,\mathrm{kPa} + \left(9.81\,\mathrm{kN/m^3}\right)(-0.1\,\mathrm{m}) = 0.589\ \mathrm{kPa} \end{aligned}$$

$$\boxed{p_{zA} - p_{zB} = 0.589\ \mathrm{kPa}}$$