Math 2107

I. Calculate the following limits
(use $+\infty$ or $-\infty$ if appropriate)

a. $\lim_{x \to a} \dfrac{x^3 - a^3}{x - a}$

b. $\lim_{x \to 0} \dfrac{\tan 7x}{3x}$

c. $\lim_{x \to 0} \dfrac{\sec x - 1}{x}$

d. $\lim_{x \to -\infty} \dfrac{x+1}{\sqrt{x^2+1}}$

e. $\lim_{x \to 2} \dfrac{x^2+x-6}{x^2-3x+2}$

f. $\lim_{x \to 0} \dfrac{x}{\sin(2x)}$

g. $\lim_{x \to \infty} \dfrac{3x-5}{\sqrt{4x^2+7x-5}}$

h. $\lim_{x \to 2^+} \dfrac{1}{2-x}$

i. $\lim_{x \to 3} \sqrt{\dfrac{x^2-9}{x-3}}$

j. $\lim_{x \to 0} \dfrac{\tan 5x}{x}$

I. (Continued) - Calculate the following limits. (Use $+\infty$ or $-\infty$ if appropriate)

k. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+1}}{x}$

l. $\lim\limits_{x \to 2^+} \dfrac{x-3}{x-2}$

m. $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4}$

n. $\lim\limits_{x \to 0} \dfrac{1-\cos^2 x}{x}$

o. $\lim\limits_{x \to \infty} \dfrac{4-3x-12x^2}{1+5x+3x^2}$

p. $\lim\limits_{x \to 1} \dfrac{2-\sqrt{x+3}}{x-1}$

II. Use the <u>definition</u> of the derivative to calculate $f'(x)$ if:

a. $f(x) = \sqrt{x}$

b. $f(x) = 2x^2-3x+1$

c. $f(x) = \sqrt{x-4}$

d. $f(x) = 3x^2-10x$

Note: No credit for any other method.

III. Use the definition of the derivative to calculate $f'(4)$ if $f(x) = \sqrt{x}$.
(No credit for any other method.)

IV. What is the slope of the curve
$$y = x^3 - 6x + 2$$
where it crosses the y-axis?

V. Given $f(x) = 2x^2 + 4x - 4$, find the equations of the lines described below:

(1) The tangent line at

a (0, -4) b (1, 2) c (-1, -6)

(2) The normal line at

d (1, 2) e (-1, -6)

VI. What is the equation of the straight line having slope 4 & tangent to the graph of $f(x) = 3x^2 + 1$?

VII. At what point is the line $12x - y = 16$ tangent to the graph of $y = x^3$?

VIII. State whether the following statements are true (T) or false (F).

a. ___ $f(x) = x^{1/3}$ is differentiable for all values of x.

b. ___ Let $f(x) = \begin{cases} 2x + 5, & \text{if } x \neq 3 \\ -4, & \text{if } x = 3 \end{cases}$. Then f is continuous at $x = 3$.

c. ___ If $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and $k \in \mathbb{R}$ with $f(a) < k < f(b)$, then there is at least one $c \in \mathbb{R}$ with $a < c < b$ such that $k = f(c)$.