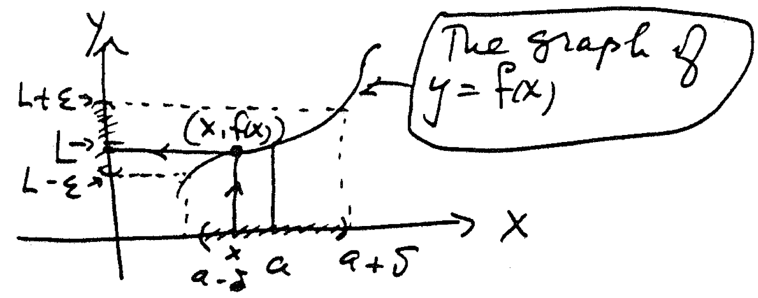
Math 2107 **Partial Study Guide For Exam #1**

All Except #(8) and #(10.b.ii)

① By definition, $\lim_{x \to a} f(x) = L$ if & only if for any given error bound $\varepsilon > 0$, there exists a deviation bound $\delta > 0$ so that

$$0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon$$

Picture:



② By definition f is continuous at a in case

(i) f is defined at a (i.e., $f(a)$ exists),

(ii) $\lim_{x \to a} f(x)$ exists, and

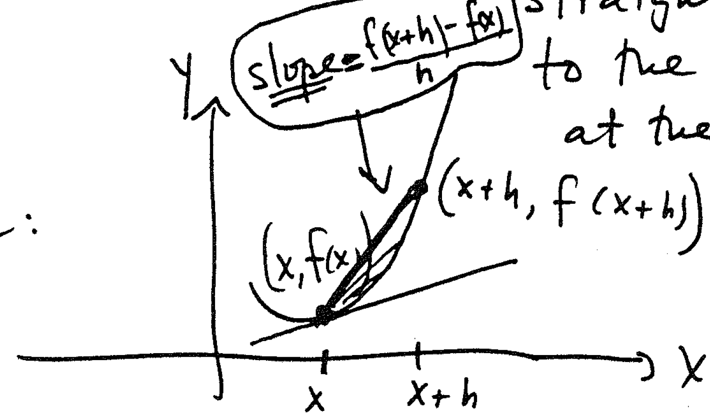
(iii) $\lim_{x \to a} f(x) = f(a)$

③ By definition, the derivative of f at x, denoted $f'(x)$, is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Geometrically $f'(x)$ = the slope of the straight line tangent to the graph of f at the point $(x, f(x))$.

Picture:



④ <u>Theorem</u>: If $f'(a)$ exists, f is continuous at a

<u>Proof</u>:

$$\lim_{h\to 0}\{f(a+h)\} = \lim_{h\to 0}\left\{h\cdot\left[\frac{f(a+h)-f(a)}{h}\right]+f(a)\right\}$$

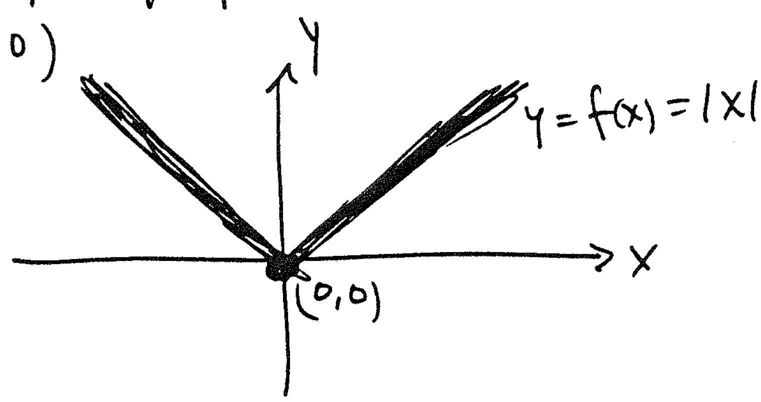
$$= \lim_{h\to 0}\{h\}\cdot\lim_{h\to 0}\left[\frac{f(a+h)-f(a)}{h}\right]+\lim_{h\to 0}[f(a)]$$

$$= \quad 0 \quad \cdot f'(a) \; + f(a) = f(a)$$

⑤ If $f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$, then

f is everywhere continuous because the graph of f is unbroken, but $f'(0)$ <u>does</u> <u>not</u> <u>exist</u> since the graph of f has a corner at $(0, f(0)) = (0,0)$

<u>Picture</u>:



⑥ $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$$

$$e = e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} + \cdots$$

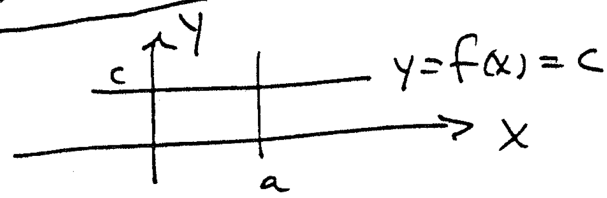
$$\approx 2.718281828459045\ldots\ldots$$

7. Basic Properties of Limits

(a) If $f(x) = c$ for all x, $\lim_{x \to a} f(x) = c$

i.e., $\boxed{\lim_{x \to a} c = c}$ ← Constant Function Rule

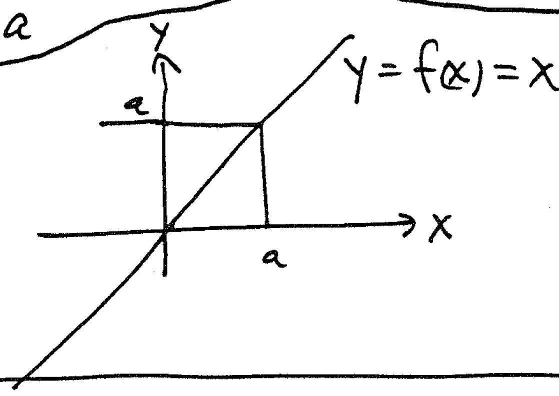
Picture:



(b) If $f(x) = x$ for all x, $\lim_{x \to a} f(x) = a$

i.e., $\boxed{\lim_{x \to a} x = a}$ ← Identity Function Rule

Picture:



Assume now that $\lim_{x \to a} f(x)$ & $\lim_{x \to a} g(x)$ exist

THEN

(c) **Sum Rule**

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a}[f(x)] + \lim_{x \to a}[g(x)]$$

(d) **Product Rule**

$$\lim_{x \to a} [f(x) \cdot g(x)] = \left(\lim_{x \to a}[f(x)]\right) \cdot \left(\lim_{x \to a}[g(x)]\right)$$

(e) **Reciprocal Rule**

$$\lim_{x \to a} \left[\frac{1}{f(x)}\right] = \frac{1}{\lim_{x \to a}[f(x)]} \quad \text{if } \lim_{x \to a} f(x) \neq 0$$

(f) **Replacement Rule**

If the functions f & g agree for all x near a (but not necessarily including a) then $\lim_{x \to a}[f(x)] = \lim_{x \to a}[g(x)]$

(g) $$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} [f(x)]}{\lim_{x \to a} [g(x)]}, \text{ if } \lim_{x \to a} [g(x)] \neq 0$$

Quotient Rule

(h) If

$f(x) \leq h(x) \leq g(x)$ for all x

in some neighborhood of a

(with the possible exception of a itself)

then

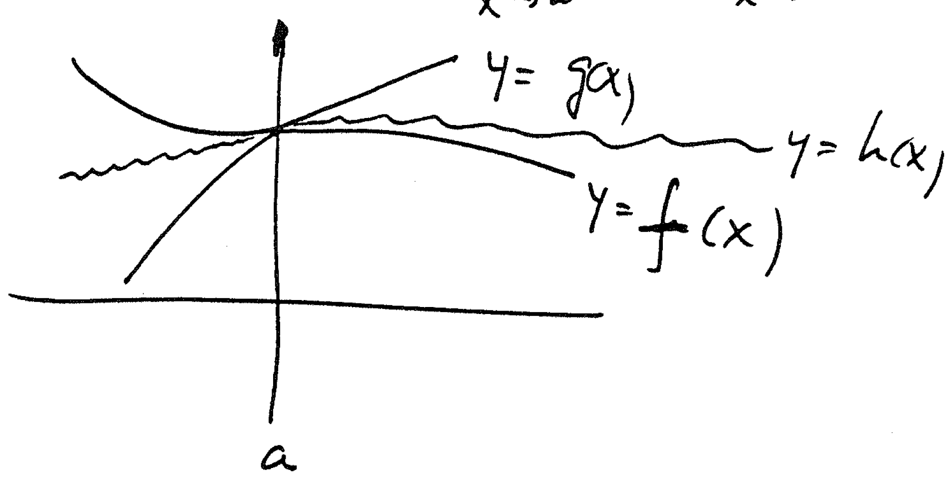
$$\lim_{x \to a} [f(x)] = \lim_{x \to a} [g(x)] \implies \lim_{x \to a} h(x) \text{ exists}$$

and

$$\lim_{x \to a} [h(x)] = \lim_{x \to a} [f(x)]$$

Squeeze Play Rule

Picture



(i) $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(j) $$\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$

For Exam #2

8. **Basic Derivative Rules**

ⓐ $(f+g)' = f' + g'$ ⓑ $(c\cdot f)' = c\cdot f'$, c = constant

ⓒ $(f\cdot g)' = f\cdot g' + g\cdot f'$ ⓓ $\left(\frac{f}{g}\right)' = \frac{g\cdot f' - f\cdot g'}{g^2}$

ⓔ $f(x) = x^c$, where c = constant $\Rightarrow f'(x) = c\,x^{c-1}$

For Exam #1

9. Find the equation of the straight line that is ⓐ <u>tangent to</u> <u>and</u> ⓑ <u>normal to</u> the graph of the function

$$f(x) = 5x^2 + 3x - 2$$

at the point $(1,6)$

10. ⓐ Be able to compute <u>limits</u> using the rules.

ⓑ Be able to compute derivatives
 (i) from the definition
 (ii) by using the rules.

11. Suppose that $f:\mathbb{R}\to\mathbb{R}$ is a function. What characteristic of the graph of f enables you to tell at a glance that

ⓐ f is everywhere continuous?

ⓑ f is everywhere differentiable?

12. If A and B are subsets of the set $\mathbb{R}$ of all real numbers, be able to look at the graph of a function $f: A \to B$ and tell at a glance whether for a given point $p \in \mathbb{R}$

ⓐ f is <u>defined</u> at p.

ⓑ $\lim_{x\to p} f(x)$ exists

ⓒ f is <u>continuous</u> at p

ⓓ f is <u>differentiable</u> at p

Math 2107

Calculate the following limits. [No work, no credit]
(Use $+\infty$ or $-\infty$ where appropriate): $\underset{x\to(-)}{\mathrm{L}} f(x) := \underset{x\to(-)}{\mathrm{Lim}} f(x)$

1. $\underset{x\to a}{\mathrm{L}} \dfrac{x^3-a^3}{x-a} =$

2. $\underset{x\to -1}{\mathrm{L}} \dfrac{x^4-1}{x+1} =$

3. $\underset{x\to 0}{\mathrm{L}} \dfrac{1+\sin x}{\cos x} =$

4. If $f(x) = |x-2|$, then $\underset{h\to 0^+}{\mathrm{L}} \dfrac{f(2+h)-f(2)}{h} =$

5. $\underset{x\to 0}{\mathrm{L}} \dfrac{\tan 6x}{2x} =$

6. $\underset{x\to 0}{\mathrm{L}} \dfrac{\sec x - 1}{x} =$

7. $\underset{h\to 0}{\mathrm{L}} \dfrac{\sqrt{4+h}-2}{h} =$

8. $\underset{x\to +\infty}{\mathrm{L}} \dfrac{x}{\sqrt{x^2-x}} =$

9. $\underset{x\to -\infty}{\mathrm{L}} \dfrac{x}{\sqrt{x^2-x}} =$

10. $\underset{x\to +\infty}{\mathrm{L}} \dfrac{2x}{x-1} =$

11. $\underset{x\to 1^-}{\mathrm{L}} \dfrac{2x}{x-1} =$

12. $\underset{x\to 3}{\mathrm{L}} \dfrac{x-3}{x^2-4x+3} =$

13. $\underset{x\to 1}{\mathrm{L}} \dfrac{\sqrt{x}-1}{x-1} =$

14. $\underset{x\to 0}{\mathrm{L}}\ x\cdot\cot(2x) =$

15. $\underset{x\to 0}{\mathrm{L}}\ \tan x\cdot\cot x =$

16. $\underset{x\to 0^-}{\mathrm{L}} \left(1+\dfrac{1}{x}\right) =$

17. $\underset{x\to 0}{\mathrm{L}} \dfrac{\sin 4x}{5x} =$

18. $\underset{x\to +\infty}{\mathrm{L}} \dfrac{x}{\sqrt{4x^2-1}} =$

19. $\underset{x\to \frac{\pi}{2}}{\mathrm{L}} \dfrac{\sec x}{\tan x} =$

20. If
$$f(x) = \begin{cases} x^2+1, & \text{if } x<-2 \\ x+7, & \text{if } x\geq -2 \end{cases}$$
then $\underset{x\to -2}{\mathrm{L}} f(x) = ?$

21. $\underset{x\to 0}{\mathrm{Lim}} \dfrac{\tan 7x}{3x} =$

22. $\underset{x\to -\infty}{\mathrm{L}} \dfrac{x+1}{\sqrt{x^2+1}} =$

23. $\underset{x\to 2}{\mathrm{L}} \dfrac{x^2+x-6}{x^2-3x+2} =$

24. $\underset{x\to 0}{\mathrm{L}} \dfrac{x}{\sin(2x)} =$

25. $\underset{x\to \infty}{\mathrm{L}} \dfrac{3x-5}{\sqrt{4x^2+7x-5}} =$

26. $\underset{x\to 2^+}{\mathrm{L}} \dfrac{1}{2-x} =$

27. $\underset{x\to 3}{\mathrm{L}} \sqrt{\dfrac{x^2-9}{x-3}} =$

28. $\underset{x\to 0}{\mathrm{L}} \dfrac{\tan 5x}{x} =$