MATH 2107

Take-Home Quiz 1 ANSWER KEY

PART A: State whether the following statements are *true* (T) or *false* (F).

1. F . In fact: $\lim_{x\to 0} 2 = 2$.

2. F . In fact: $\lim_{x\to -1} \frac{x^2+x}{x^2-1} = \frac{1}{2}$.

3. T . The $\lim_{x\to 1} \frac{3x^2-6x+3}{x-1} = 0$.

4. F . In fact: $\lim_{x\to 2} \frac{3-\sqrt{x+7}}{2-x} = \frac{1}{6}$.

5. T . The $\lim_{x\to a^+} (mx+b) = ma+b$.

6. T . The $\lim_{x\to \infty} \frac{x}{\sqrt{x^2-4}} = 1$.

7. F . In fact: $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2-4}} = -1$.

8. T . The $\lim_{x\to 2^+} \sqrt{x-2} = 0$.

9. F . In fact: $\lim_{x\to 2} \sqrt{x-2}$ does *not* exist.

10. T . The $\lim_{x\to 2^-} \sqrt{x-2}$ does *not* exist.

11. T . The $\lim_{x\to \infty} \frac{x^3}{x^2+1} = \infty$.

12. F . It is False that: To *prove* that $\lim_{x\to 2} (3x-2) = 4$, we must show that , for *some* neighborhood $N = (4-\epsilon, 4+\epsilon)$ of 4, there is a deleted neighborhood $D = (2-\delta, 2) \cup (2, 2+\delta)$ of 2 such that for all $x$ in $D$, $3x-2$ is in $N$.

13. T . It is True that: To *prove* that $\lim_{x\to 2} (3x-2) = 4$, we must show that , for *any given* neighborhood $N = (4-\epsilon, 4+\epsilon)$ of 4, there is a deleted neighborhood $D = (2-\delta, 2) \cup (2, 2+\delta)$ of 2 such that for all $x$ in $D$, $3x-2$ is in $N$ .

14. F . It is False that: The function $f(x) = \frac{2}{x-3}$ can be made *continuous* at $x = 3$.

15. T . It is True that: The function $f(x) = \frac{2x^2-12x+18}{x-3}$ can be made *continuous* at $x = 3$.

PART B: *Circle* the *one* correct choice.

1. $\lim_{x\to 3^+} \frac{|x-3|}{x-3}$ is
   b) 1

2. $\lim_{x\to 3^-} \frac{|x-3|}{x-3}$ is
   d) -1

3. $\lim_{x\to 3} \frac{|x-3|}{x-3}$ is
   e) none of the above

4. $\lim_{x\to +\infty} \frac{|x-3|}{x-3}$ is
   b) 1

5. $\lim_{x\to -\infty} \frac{|x-3|}{x-3}$ is
   c) -1

6. $\lim_{x\to -2} \frac{x^2-x-6}{x^2+3x+2}$
   b) is 5

7. $\lim_{x\to a} \frac{x^2-a}{x-a^2}$ , for $a \neq 0$ or $a \neq 1$,
   d) is -1

8. $\lim_{x\to a} a$ is
   b) a

9. If $f(x) = |x-3|$, then $\lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$ is
   a) 1

10. If $f(x) = |x-3|$, then $\lim_{h\to 0^-} \frac{f(3+h)-f(3)}{h}$ is
   c) -1

MATH 2107

Take-Home Quiz 1

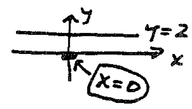
NAME (Print): KEY  Student I.D.:

NAME (Signature):

PART A: State whether the following statements are *true* (T) or *false* (F).

1. F $\lim_{x\to 0} 2 = 0$.

$\lim_{x\to 0} 2 = \boxed{2}$




2. F $\lim_{x\to -1} \frac{x^2+x}{x^2-1} = -\frac{1}{2}$.

$$\frac{x^2+x}{x^2-1} = \frac{x\cdot(x+1)}{(x-1)\cdot(x+1)} = \frac{x}{x-1} \mapsto \frac{-1}{-1-1} = \frac{-1}{-2} = \boxed{\frac{1}{2}}$$

3. T $\lim_{x\to 1} \frac{3x^2-6x+3}{x-1} = 0$.

$$\frac{3(x^2-2x+1)}{x-1} = \frac{3(x-1)(x-1)}{x-1} = 3(x-1) \mapsto 3(1-1) = \boxed{0}$$

$\boxed{\text{Note}}$: $1 \,|\, 3 \;\; -6 \;\; 3$; $\;3 \;\; -3$; $\;3 \;\; -3 \;\; \boxed{0}$ $\Longrightarrow$ $3x^2-6x+3 = (x-1)\cdot(3x-3) = (x-1)\cdot 3\cdot(x-1)$

4. F $\lim_{x\to 2} \frac{3-\sqrt{x+7}}{2-x} = 1$.

$$\frac{3-\sqrt{x+7}}{2-x}\cdot\frac{3+\sqrt{x+7}}{3+\sqrt{x+7}} = \frac{9-(x+7)}{2-x}\cdot\frac{1}{3+\sqrt{x+7}}$$

$$= \frac{9-7-x}{2-x}\cdot\frac{1}{3+\sqrt{x+7}} \mapsto \frac{1}{3+\sqrt{9}} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

5. T $\lim_{x\to a^+} (mx+b) = ma+b$.

6. T $\lim_{x\to\infty} \frac{x}{\sqrt{x^2-4}} = 1$.

$$\frac{x}{\sqrt{x^2\cdot(1-\frac{4}{x^2})}} = \frac{x}{\sqrt{x^2}\sqrt{1-\frac{4}{x^2}}} = \frac{x}{|x|\sqrt{1-\frac{4}{x^2}}}$$

$$= \frac{x}{x}\cdot\frac{1}{\sqrt{1-\frac{4}{x^2}}} \mapsto \frac{1}{\sqrt{1-0}} = \frac{1}{1} = \boxed{1}$$

For $x>0$

7. F $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2-4}} = 1$.

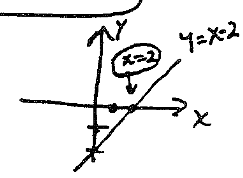
For $x<0$

$$\frac{x}{\sqrt{x^2-4}} = \frac{x}{|x|\sqrt{1-\frac{4}{x^2}}} = \frac{x}{-x\sqrt{1-\frac{4}{x^2}}}$$

$$= \frac{-1}{\sqrt{1-\frac{4}{x^2}}} \mapsto \frac{-1}{\sqrt{1-0}} = \boxed{-1}$$

8. T $\lim_{x\to 2^+} \sqrt{x-2} = 0$.

$x>2 \Rightarrow x-2>0$




9. F $\lim_{x\to 2} \sqrt{x-2} = 0$. Does not exist

$\because$ the $\lim_{x\to 2}(-)$ exists $\Updownarrow$ $\lim_{x\to 2^-}(-) = \lim_{x\to 2^+}(-)$

10. T $\lim_{x\to 2^-} \sqrt{x-2}$ does *not* exist.

$x<2 \Rightarrow x-2<0$ so $\sqrt{x-2} \notin \mathbb{R}$.

i.e. $\sqrt{\text{negative number}} \neq$ real number

11. T $\lim_{x\to\infty} \frac{x^3}{x^2+1} = \infty$.

$$\frac{x^3}{x^2+1} = \frac{x^3}{x^2+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{x}{1+\frac{1}{x^2}}$$

$$\Downarrow$$

$$\frac{\infty}{1+0} = \infty$$

12. F To *prove* that $\lim_{x \to 2}(3x-2)=4$, we must show that , for *some* neighborhood $N=(4-\epsilon, 4+\epsilon)$ of 4, there is a deleted neighborhood $D=(2-\delta,2)\cup(2,2+\delta)$ of 2 such that for all $x$ in $D$, $3x-2$ is in $N$.

13. T To *prove* that $\lim_{x \to 2}(3x-2)=4$, we must show that , for *any given* neighborhood $N=(4-\epsilon, 4+\epsilon)$ of 4, there is a deleted neighborhood $D=(2-\delta,2)\cup(2,2+\delta)$ of 2 such that for all $x$ in $D$, $3x-2$ is in $N$ .

14. F The function $f(x)=\frac{2}{x-3}$ can be made *continuous* at $x=3$.

Sign of $x-3$   $- - - - 0 + + + +$   $x=3$

$\therefore \lim_{x \to 3^+} \frac{2}{x-3} = +\infty$

while $\lim_{x \to 3^-} \frac{2}{x-3} = -\infty$, so $\lim_{x \to 3} f(x)$ does not exist.

15. T The function $f(x)=\frac{2x^2-12x+18}{x-3}$ can be made *continuous* at $x=3$.

$$\frac{2(x^2-6x+9)}{x-3} = \frac{2(x-3)(x-3)}{x-3} = 2(x-3) \longmapsto 2(3-3)=0$$

PART B: *Circle* the *one* correct choice.

1. $\lim_{x \to 3+} \frac{|x-3|}{x-3}$ is
   a) 3
   b) 1
   c) 0
   d) -1
   e) none of the above

$\forall x>3$, $x-3>0$, so $\forall x>3$

$$\frac{|x-3|}{x-3} = \frac{x-3}{x-3} = \boxed{1}$$

2. $\lim_{x \to 3-} \frac{|x-3|}{x-3}$ is
   a) 3
   b) 1
   c) 0
   d) -1
   e) none of the above

$x<3 \Leftrightarrow x-3<0$, So for $x<3$

$$\frac{|x-3|}{x-3} = \frac{-(x-3)}{x-3} = -1 \longmapsto \boxed{-1}$$

3. $\lim_{x \to 3} \frac{|x-3|}{x-3}$ is   Does not exist!
   a) 3
   b) 1
   c) 0
   d) -1
   e) none of the above

$$\therefore \left[\exists \lim_{x\to 3}\right] \Leftrightarrow \left[\lim_{x \to 3^+} = \lim_{x \to 3^-}\right]$$

4. $\lim_{x \to +\infty} \frac{|x-3|}{x-3}$ is
   a) $+\infty$
   b) 1
   c) 0
   d) -1
   e) none of the above

$\forall x>3$, $x-3>0$, So

$$\forall x>3,\ \frac{|x-3|}{x-3} = \frac{x-3}{x-3} = \boxed{1}$$

sign of $x-3$ : 3

5. $\lim_{x\to-\infty}\frac{|x-3|}{x-3}$ is

a) 1

b) 0

c) -1

d) $-\infty$

e) none of the above

$\forall\, x<3,\ x-3<0$, so

$\frac{|x-3|}{x-3} = \frac{-(x-3)}{x-3} = \boxed{-1}$

9. If $f(x)=|x-3|$, then $\lim_{x\to3^+}\frac{f(x)-f(3)}{x-3}$ is

a) 1

b) 0

c) -1

d) 3

e) none of the above

$x>3 \Updownarrow x-3>0$

for $x>3$

$\forall x>3$, $\frac{f(x)-f(3)}{x-3} = \frac{|x-3|-|3-3|}{x-3} = \frac{|x-3|}{x-3} = \frac{x-3}{x-3}$

$\boxed{1}$

6. $\lim_{x\to-2}\frac{x^2-x-6}{x^2+3x+2}$

a) does not exist

b) is 5

c) is 1

d) is 0

e) is -5

$x^2-x-6=[x-(-2)](x-3)$

$$\begin{array}{r|rrr} -2 & 1 & -1 & -6 \\ & & -2 & 6 \\ \hline & 1 & -3 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrr} -2 & 1 & 3 & 2 \\ & & -2 & -2 \\ \hline & 1 & 1 & \boxed{0} \end{array}$$

$\Rightarrow x^2+3x+2=[x-(-2)][x+1]$

$\frac{x^2-x-6}{x^2+3x+2} = \frac{(x-3)\cdot(x+2)}{(x+1)(x+2)} = \frac{x-3}{x+1} \mapsto \frac{-2-3}{-2+1} = \frac{-5}{-1} = \boxed{5}$

10. If $f(x)=|x-3|$, then $\lim_{h\to0^-}\frac{f(3+h)-f(3)}{h}$ is

a) 1

b) 0

c) -1

d) 3

e) none of the above

$\forall\, h<0$

$\frac{f(3+h)-f(3)}{h} = \frac{|3+h-3|-|3-3|}{h}$

$= \frac{|h|}{h} = \frac{-h}{h}$

$= \boxed{-1}$

7. $\lim_{x\to a}\frac{x^2-a}{x-a^2}$, for $a\neq0$ or $a\neq1$,

a) does not exist

b) is 0

c) is 1

d) is -1

e) is 2

$a-a^2=a(1-a)=0 \Leftrightarrow a=0$ or $a=1$

$\therefore$ if $a\neq0$ or $a\neq1$

$\lim_{x\to a}\frac{x^2-a}{x-a} = \frac{a^2-a}{a-a^2} = \frac{-(a-a^2)}{a-a^2} = \boxed{-1}$

8. $\lim_{x\to a} a$ is

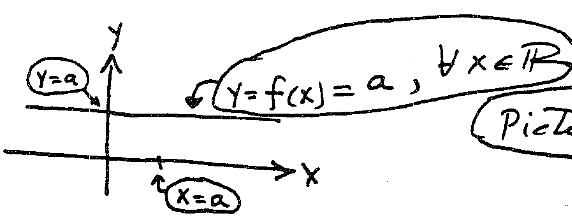
a) $a^2$

b) a

c) 0

d) 1

e) none of the above




Picture for $a>0$