

1. (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function and let $a \in \mathbb{R}$. Prove that f is continuous at a whenever $f'(a)$ exists.

[10 points]

- (b) Give an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ and a point $a \in \mathbb{R}$ such that f is continuous at a but $f'(a)$ does not exist.

[To get credit, you must give at least some indication of why f is continuous at a and why $f'(a)$ does not exist.]

[5 points]

2. (a) State the Chain Rule. Be sure to state the hypotheses as well as the conclusion.

ANSWER:

The Chain Rule:

Hypotheses:

[1 point]

Conclusions:

[4 points]

- (b) Complete the following:

[5 points: 1 each]

$$(F \cdot S)' =$$

$$\left(\frac{T}{B}\right)' =$$

$$(c \cdot F)' =$$

$$(F + G)' =$$

$$(F^n)' =$$

3. Assume that the equation

$$y^4 - xy^3 + x^3 = 1$$

defines y as a differentiable function of x . Use implicit differentiation

to find $\frac{dy}{dx}$. Then find the equation of the straight line tangent to

the graph of $y^4 - xy^3 + x^3 = 1$ at the point $(1, 1)$.

[15 points]

SOLUTION:

4. For each of the following, use the various rules that we have developed [not the definition] to compute $f'(x)$ when $f(x)$ is given.

Then **SIMPLIFY** your answers as much as possible.

NOTE: DO NOT COMPUTE $f'(x)$ DIRECTLY FROM THE DEFINITION!

[20 points: 5 each]

a. $f(x) = x + \frac{x}{x-4}$

b. $f(x) = \left(\frac{x^8 - 1}{x^8 + 1}\right)^6$

c. $f(x) = \frac{\sin x}{1 - \cos x}$

d. $f(x) = -\ln[\cos^2 x]$

5. For each of the following, use the various rules that we have developed [not the definition] to compute $f'(x)$ when $f(x)$ is given.

Do **NOT** simplify your answers.

NOTE: DO NOT COMPUTE $f'(x)$ DIRECTLY FROM THE DEFINITION!

[40 points: 5 each]

a. $f(x) = \sec(6x^8)$

b. $f(x) = e^{\sin[\ln(x^4+8)]}$

c. $f(x) = \cosh[\tan^{-1}(x^{15})]$

d. $f(x) = \sinh[\sin^{-1}(e^{x^2})]$

5. (Continued):

e. $f(x) = [\cot(5x^4)]^6$

f. $f(x) = \cos[\sin(e^{4x})]$

g. $f(x) = x^6 \cdot \tan\left(\frac{1}{x^5}\right)$

h. $f(x) = \frac{(x^2 + 6) \cdot \sqrt[3]{5x + 6}}{x^4 + 2x^2}$ [Use logarithmic differentiation.]

EXTRA-CREDIT: Circle the *one* correct choice.

[12 points : But, any exam score over 100 will be truncated to 100.]

For each of the following, use the various rules that we have developed

[not the definition] to compute $f'(x)$ when $f(x)$ is given.

Do **NOT** simplify your answers.

NOTE: DO NOT COMPUTE $f'(x)$ DIRECTLY FROM THE DEFINITION!

a. $f(x) = [\tan^6 x]^{\sin x}$

[8 points]

b. $f(x) = 6^{\log_6(6^{x^6})} + \log_6(x)$

[4 points]