10.7: PROBLEM DEFINITION

Situation:

Water is flowing through a horizontal pipe (garden hose).
$D = 0.018\,\text{m}, \quad L = 20\,\text{m}.$
$f = 0.012$, $V = 1.5\,\text{m/s}$.

Find:

Pressure drop (Pa) for 20 m of hose.

Properties:

Water (15 °C), Table A.5, $\rho = 999\,\text{kg/m}^3$.

PLAN

1. Relate pressure drop to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Combine steps 1 & 2.

SOLUTION

1. Energy eqn. (location 1 upstream; location 2 is 20 m downstream)

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + 0 + 0 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + 0 + h_L \\
\text{since } V_1 &= V_2, \text{ KE terms cancel} \\
\Delta p &= \gamma h_L \qquad (1)
\end{aligned}$$

2. Darcy-Weisbach eqn.:

$$h_L = h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

3. Combine Eqs. (1) and (2):

$$\begin{aligned}
\Delta p &= \gamma \left(f \frac{L}{D} \frac{V^2}{2g} \right) = \frac{\rho V^2}{2} \left(f \frac{L}{D} \right) \\
&= \frac{(999\,\text{kg/m}^3)(1.5\,\text{m/s})^2}{2} \left(0.012 \times \frac{20\,\text{m}}{0.018\,\text{m}} \right)
\end{aligned}$$

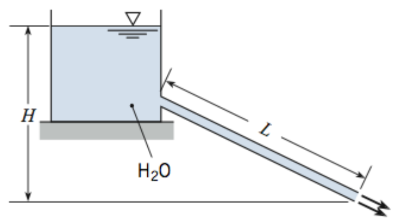
$$\boxed{\Delta p = 15.0\,\text{kPa}}$$

10.8: PROBLEM DEFINITION

Situation:

Water is flowing from a tank through a tube & then discharging to ambient.
$D = 0.008\,\text{m}, \; L = 6\,\text{m}.$
$H = 3\,\text{m}, \; f = 0.015.$

Sketch:



Find:

Exit velocity (m/s).
Discharge (L/s).
Sketch the HGL & EGL.

Assumptions:

The only head loss is in the tube.
Turbulent flow so $\alpha_2 = 1.0$.

Properties:

Water (15 °C), Table A.5, $\rho = 999\,\text{kg/m}^3 \quad \nu = 1.14 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Relate H to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Find V by combining steps 1 & 2.
4. Find Q by using the flow rate equation.

SOLUTION

1. Energy eqn. (location 1 at the free surface, location 2 at the pipe exit)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$0 + 0 + H + 0 = 0 + \frac{V_2^2}{2g} + 0 + h_L \qquad (1)$$

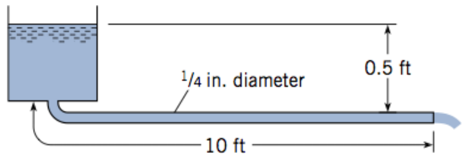
2. Darcy-Weisbach eqn.:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

10.18: PROBLEM DEFINITION

Situation:

Kerosene flows out a tank and through a tube.
$D = 0.25\,\text{in}$, $L = 10\,\text{ft}$.
$z_1 = 0.5\,\text{ft}$.



Find:

Mean velocity in the tube.
Discharge.

Assumptions:

Laminar flow so $\alpha = 2$.
Only head loss is in the tube.

Properties:

Kerosene (68 °F): $S = 0.8$.

PLAN

Apply the energy equation from the surface of the reservoir to the pipe outlet.

SOLUTION

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + 2V^2/2g + z_2 + 32\mu LV/(\gamma D^2) \\ 0 + 0 + 0.50 &= 0 + V^2/g + 32\mu LV/(\gamma D^2) \end{aligned}$$

Thus

$$\begin{aligned} V^2/g + 32\mu LV/(\gamma D^2) - 0.50 &= 0 \\ V^2/32.2 + 32(4\times10^{-5})(10)V/(0.80\times62.4\times(1/48)^2) - 0.50 &= 0 \\ V^2 + 19.0V - 16.1 &= 0 \end{aligned}$$

Solving the above quadratic equation for V yields:

$$\boxed{V = 0.81 \text{ ft/s}}$$

Check Reynolds number to see if flow is laminar

$$\begin{aligned} \text{Re} &= VD\rho/\mu \\ &= 0.81\times(1/48)(1.94\times0.8)/(4\times10^{-5}) \\ \text{Re} &= 654.8 \text{ (laminar)} \\ Q &= VA \\ &= 0.81\times(\pi/4)(1/48)^2 = 2.76\times10^{-4} \text{ cfs} \end{aligned}$$

$$\boxed{Q = 2.76 \times 10^{-4} \text{ cfs}}$$

10.43: PROBLEM DEFINITION

Situation:

Water flows with a through a horizontal run of PVC pipe

$V = 2\,\mathrm{m/s}$, $L = 50\,\mathrm{m}$.

Nominal diameter 2.5" Schedule 40. $D = 2.45\,\mathrm{in.} = 0.0622\,\mathrm{m}$.

Find:

(a) Pressure drop in kPa.

(b) Head loss in meters.

(c) Power in watts needed to overcome the head loss.

Assumptions:

1.) Assume $k_s = 0$.

2.) Assume $\alpha_1 = \alpha_2$, where subscripts 1 and 2 denote the inlet and exit of the pipe.

Properties:

Water (10 °C), Table A.5:

$\rho = 1000\,\mathrm{kg/m^3}$, $\gamma = 9810\,\mathrm{N/m^3}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/s}$.

PLAN

To establish laminar or turbulent flow, calculate the Reynolds number. Then find the appropriate friction factor (f) and apply the Darcy-Weisbach equation to find the head loss. Next, find the pressure drop using the energy equation. Lastly, find power using $P = \dot{m} g h_f$.

SOLUTION

Reynolds number

$$\begin{aligned}
\mathrm{Re} &= \frac{VD}{\nu} \\
&= \frac{(2\,\mathrm{m/s})(0.0622\,\mathrm{m})}{(1.31 \times 10^{-6}\,\mathrm{m^2/s})} \\
&= 94,960
\end{aligned}$$

Thus, flow is turbulent.

Friction factor (f) (Swamee-Jain equation)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{94{,}960^{0.9}}\right)\right]^2} \\
&= 0.0181
\end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.0181\left(\frac{50\,\mathrm{m}}{0.0622\,\mathrm{m}}\right)\frac{(2\,\mathrm{m/\,s})^2}{2\times 9.81\,\mathrm{m/\,s^2}} \\ &= 2.966\,\mathrm{m} \end{aligned}$$

$$\boxed{h_f = 2.97\,\mathrm{m}\quad \text{(part b)}}$$

Energy equation

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_t+h_L$$

Select a control volume surrounding the pipe. After analysis of each term, the energy equation simplifies to

$$\begin{aligned} \frac{p_1}{\gamma} &= \frac{p_2}{\gamma}+h_f \\ \text{or}\quad \Delta p &= \gamma h_f \\ &= \left(9810\,\mathrm{N/\,m^3}\right)(2.966\,\mathrm{m}) \\ &= 29,096\,\mathrm{kPa} \end{aligned}$$

$$\boxed{\Delta p = 29.1\,\mathrm{kPa}\quad \text{(part a)}}$$

Flow rate equation

$$\begin{aligned} \dot m &= \rho A V \\ &= \left(1000\,\mathrm{kg/\,m^3}\right)\left(\frac{\pi\,(0.0622\,\mathrm{m})^2}{4}\right)(2\,\mathrm{m/\,s}) \\ &= 6.077\,\mathrm{kg/\,s} \end{aligned}$$

Power equation

$$\begin{aligned} \dot W &= \dot m g h_f \\ &= (6.077\,\mathrm{kg/\,s})\left(9.81\,\mathrm{m/\,s^2}\right)(2.966\,\mathrm{m}) \\ &= 176.8\,\mathrm{W} \end{aligned}$$

$$\boxed{\text{Power to overcome head loss} = 177\,\mathrm{W}\quad \text{(part c)}}$$

REVIEW

1. The pressure drop (29 kPa) is about 1/3 of an atmosphere This value could be decreased by increasing the pipe diameter to lower the speed of the water.

2. The power to overcome the frictional head loss is small, about 1/4 of a horsepower.

3. Combine Eqs. (1) and (2):

$$\begin{aligned} H &= \frac{V_2^2}{2g}\left(1 + f\frac{L}{D}\right) \\ V_2 &= \sqrt{\frac{2gH}{1 + f\frac{L}{D}}} \\ &= \sqrt{\frac{2\,(9.81\,\mathrm{m/s^2})\,(3\,\mathrm{m})}{1 + 0.015\frac{(6\,\mathrm{m})}{(0.008\,\mathrm{m})}}} \end{aligned}$$

$$\boxed{V = 2.19\,\mathrm{m/s}}$$

4. Flow rate equation:

$$Q = VA = V\frac{\pi D^2}{4} = (2.192\,\mathrm{m/s})\,\frac{\pi\,(0.008\,\mathrm{m})^2}{4}$$

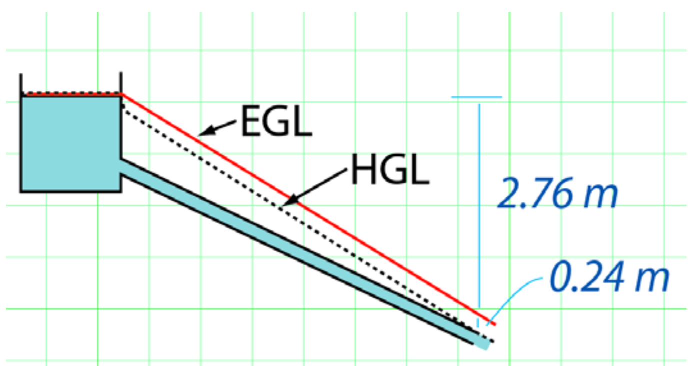
$$\boxed{Q = 0.110\ \mathrm{L/s}}$$

5. Sketch HGL & EGL

- Locate the EGL & HGL on free surface of tank.
- Velocity head and head loss:

$$\begin{aligned} \frac{V^2}{2g} &= \frac{(2.192\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} = 0.24\,\mathrm{m} \\ h_f &= f\frac{L}{D}\frac{V^2}{2g} = 0.015\frac{(6\,\mathrm{m})}{(0.008\,\mathrm{m})}\frac{(2.192\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} = 2.76\,\mathrm{m} \end{aligned}$$

- Locate the EGL and HGL at the end of the pipe. Sketch lines.



REVIEW Check the turbulent flow assumption.

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\nu} = \frac{(2.192\,\mathrm{m/s})\,(0.008\,\mathrm{m})}{(1.14\times10^{-6}\,\mathrm{m^2/s})} \\ \mathrm{Re} &= 15400 > 3000 \end{aligned}$$

Thus, the assumption of turbulent flow is valid.

10.47: PROBLEM DEFINITION

Situation:

Water is pumped through a vertical steel pipe to an elevated tank.

$D = 10\,\text{cm}$, $p_1 = 1.6\,\text{MPa}$.

$L = 80\,\text{m}$, $Q = 0.02\,\text{m}^3/\,\text{s}$.

Find:

Pressure at point 80 m above pump.

Properties:

Water (20 °C), Table A.5: $\gamma = 9790\,\text{N}/\,\text{m}^3$.

SOLUTION

$$\begin{aligned}
\text{Re} &= \frac{4Q}{\pi D \nu} \\
&= \frac{4 \times 0.02\,\text{m}^3/\,\text{s}}{\pi \times 0.10\,\text{m} \times 10^{-6}\,\text{m}^2/\,\text{s}} = 2.55 \times 10^5 \\
\frac{k_s}{D} &= \frac{4.6 \times 10^{-2}}{100} = 4.6 \times 10^{-4}
\end{aligned}$$

Resistance coefficient

$$f = 0.0185$$

Then

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

where

$$\begin{aligned}
V &= \frac{0.02\,\text{m}^3/\,\text{s}}{\pi/4) \times (0.1\,\text{m})^2} = 2.546 \text{ m/s} \\
h_f &= 0.0185 \times \frac{80\,\text{m}}{0.10\,\text{m}} \times \frac{(2.546\,\text{m}/\,\text{s})^2}{2 \times 9.81\,\text{m}/\,\text{s}^2} = 4.89 \text{ m}
\end{aligned}$$

Energy equation (from pump to location 80 m higher)

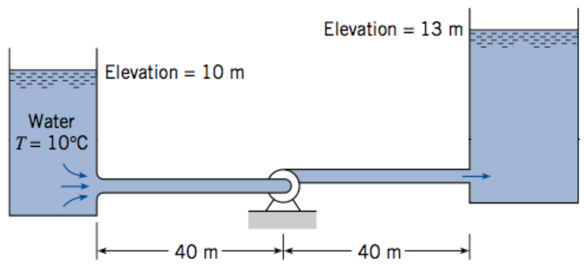
$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f \\
\frac{1.6 \times 10^6\,\text{Pa}}{9,790\,\text{N}/\,\text{m}^3} + \frac{V_1^2}{2g} &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + 80 + 4.89 \\
V_1 &= V_2 \\
& \boxed{p_2 = 769\,\text{kPa}}
\end{aligned}$$

10.60: PROBLEM DEFINITION

Situation:

Water is pumped between reservoirs through a steel pipe.
$Q = 0.1\,\mathrm{m^3/s}$, $D = 15\,\mathrm{cm}$.

Sketch:



Find:

Power that is supplied to the system by the pump.

Properties:

From Table 10.4: $k_s = 0.046$ mm.
Water (10 °C), Table A.5: $\upsilon = 1.3 \times 10^{-6}\,\mathrm{m^2/s}$, $\gamma = 9810\,\mathrm{N/m^3}$.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.1\,\mathrm{m^3/s}}{(\pi/4) \times (0.15\,\mathrm{m})^2} \\ &= 5.66\ \mathrm{m/s} \\ \frac{V^2}{2g} &= 1.63\ \mathrm{m} \\ \frac{k_s}{D} &= \frac{0.0046}{15} = 0.0003 \end{aligned}$$

Reynolds number

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\upsilon} = \frac{5.66\,\mathrm{m/s} \times 0.15\,\mathrm{m}}{1.3 \times 10^{-6}\,\mathrm{m^2/s}} \\ &= 6.4 \times 10^5 \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.8)

$$f = 0.016$$

Energy equation (between the reservoir surfaces)

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\
h_p &= z_2 - z_1 + \frac{V^2}{2g}(K_e + f\frac{L}{D} + K_{\mathrm{E}}) \\
&= 13\,\mathrm{m} - 10\,\mathrm{m} + 1.63\,\mathrm{m} \times (0.1 + 0.016 \times \frac{80\,\mathrm{m}}{0.15\,\mathrm{m}} + 1) \\
&= 3 + 15.7 = 18.7\ \mathrm{m}
\end{aligned}$$

Power equation

$$\begin{aligned}
P &= Q\gamma h_p \\
&= 0.10\,\mathrm{m}^3/\,\mathrm{s} \times 9810\,\mathrm{N}/\,\mathrm{m}^3 \times 18.7\,\mathrm{m} \\
&= 18,345\ \mathrm{W}
\end{aligned}$$

$$\boxed{P = 18.3\,\mathrm{kW}}$$

10.65: PROBLEM DEFINITION

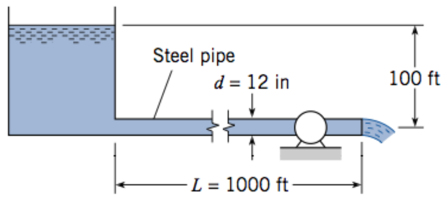
Situation:

Water flows out of reservoir, through a steel pipe and a turbine.

$Q = 5\,\text{ft}^3/\,\text{s}$, $\eta = 0.8$, $\Delta z = 100\,\text{ft}$.

$D = 12\,\text{in}$, $L = 1000\,\text{ft}$.

Sketch:



Find:

Power delivered by turbine.

Assumptions:

Turbulent flow, so $\alpha_2 \approx 1$.

Properties:

Water (70°F), Table A.5: $\nu = 1.06 \times 10^{-5}\ \text{ft}^2/\text{s}$

PLAN

Apply the energy equation from the reservoir water surface to the jet at the end of the pipe.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_T + \sum h_L \\
0 + 0 + z_1 &= 0 + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_T + \left(K_e + f\frac{L}{D}\right)\frac{V^2}{2g} \\
z_1 - z_2 &= h_T + (1 + 0.5 + f\frac{L}{D})\frac{V^2}{2g} \\
100\ \text{ft} &= h_T + \left(1.5 + f\frac{L}{D}\right)\frac{V^2}{2g}
\end{aligned}$$

But

$$\begin{aligned} V &= \frac{Q}{A} = \frac{5\,\mathrm{ft}^3/\,\mathrm{s}}{(\pi/4)\,(1\,\mathrm{ft})^2} = 6.37\ \mathrm{ft/s} \\ \frac{V^2}{2g} &= 0.6293\ \mathrm{ft} \\ \mathrm{Re} &= \frac{VD}{v} = 6.0 \times 10^5 \end{aligned}$$

From Fig. 10.8 $f = 0.015$ for $k_s/D = 0.000167$. Then

$$\begin{aligned} 100\ \mathrm{ft} &= h_T + \left(1.5 + 0.0150 \times \frac{1000\,\mathrm{ft}}{1\,\mathrm{ft}}\right)(0.6293\,\mathrm{ft}) \\ h_T &= (100 - 10.35)\,\mathrm{ft} = 89.65\,\mathrm{ft} \end{aligned}$$

Power equation

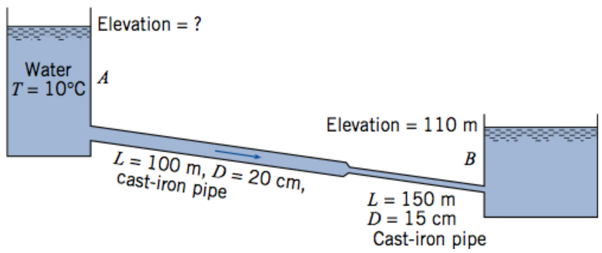
$$\begin{aligned} P &= Q\gamma h_T \eta \\ &= \left(5\,\mathrm{ft}^3/\,\mathrm{s}\right)\left(62.4\,\mathrm{lbf}/\,\mathrm{ft}^3\right)(89.6\,\mathrm{ft})\,(0.80) \\ &= 22,364\ \mathrm{ft}\cdot\mathrm{lbf/s} \\ &\boxed{P = 40.7\ \text{horsepower}} \end{aligned}$$

10.82: PROBLEM DEFINITION

Situation:

Two reservoirs are connected by a cast-iron pipe of varying diameter.
$z_2 = 110\,\text{m}$, $Q = 0.3\,\text{m}^3/\text{s}$.
$D_1 = 20\,\text{cm}$, $L_1 = 100\,\text{m}$.
$D_2 = 15\,\text{cm}$, $L_2 = 150\,\text{m}$.

Sketch:



Find:

Water surface elevation in reservoir A.

Properties:

From Table 10.4: $k_s = 0.26$ mm.
Water (10 °C), Table A.5: $\nu = 1.3 \times 10^{-6}$ m²/s.

SOLUTION

$$\frac{k_s}{D_{20}} = \frac{0.26}{200} = 0.0013$$

$$\frac{k_s}{D_{15}} = 0.0017$$

$$V_{20} = \frac{Q}{A_{20}} = \frac{0.03\,\text{m}^3/\text{s}}{\pi/4 \times (0.20\,\text{m})^2} = 0.955 \text{ m/s}$$

$$\frac{Q}{A_{15}} = 1.697 \text{ m/s}$$

$$\text{Re}_{20} = \frac{VD}{v} = \frac{0.955\,\text{m/s} \times 0.2\,\text{m}}{1.3 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.5 \times 10^5$$

$$\text{Re}_{15} = \frac{1.697\,\text{m/s} \times 0.15\,\text{m}}{1.3 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.9 \times 10^5$$

From Fig. 10-14: $f_{20} = 0.022$; $f_{15} = 0.024$

$$\begin{aligned}
z_1 &= z_2 + \sum h_L \\
z_1 &= 110 + \frac{V_{20}^2}{2g}\left(0.5 + 0.022 \times \frac{100\,\mathrm{m}}{0.2\,\mathrm{m}} + 0.19\right) \\
&\quad + \frac{V_{15}^2}{2g}\left[\left(0.024 \times \frac{150\,\mathrm{m}}{0.15\,\mathrm{m}}\right) + 1.0 + 0.19\right] \\
&= 110\,\mathrm{m} + 0.0465\,\mathrm{m}(11.7) + 0.1468\,\mathrm{m}(25.19) \\
&= 110 + 0.535 + 3.70 = 114.2\,\mathrm{m}
\end{aligned}$$

$$\boxed{z_1 = 114\,\mathrm{m}}$$