

5.44: PROBLEM DEFINITION

Situation:

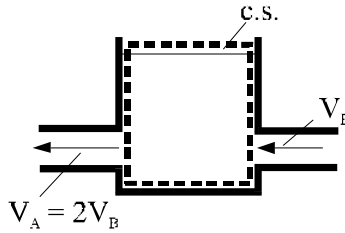
The level in the tank is influenced by the motion of pistons A and B moving left.

$$V_A = 2V_B, D_A = 3 \text{ in}, D_B = 6 \text{ in}.$$

Find:

Determine whether the water level is rising, falling or staying the same.

Sketch:



PLAN

Apply the continuity equation. Select a control volume as shown above. Assume it is coincident with and moves with the water surface.

SOLUTION

Continuity equation

$$\begin{aligned}\dot{m}_o - \dot{m}_i &= -\frac{d}{dt} \int_{cv} \rho dV \\ \rho 2V_B A_A - \rho V_B A_B &= -\rho \frac{d}{dt} \int_{cv} dV\end{aligned}$$

where $A_A = (\pi/4)3^2$; $A_B = (\pi/4)6^2$ and $A_A = (1/4)A_B$. Then

$$\begin{aligned}2V_B(1/4)A_B - V_B A_B &= -\frac{d}{dt} \int_{CV} dV \\ V_B A_B ((1/2) - 1) &= -\frac{d}{dt} \int_{CV} dV \\ \frac{d}{dt} \int_{CV} dV &= (1/2)V_B A_B \\ \frac{d}{dt}(Ah) &= (1/2)V_B A_B \\ A \frac{dh}{dt} &= (1/2)V_B A_B\end{aligned}$$

Because $(1/2)V_B A_B$ is positive dh/dt is positive; therefore, one concludes that the water surface is **rising**.

5.46: PROBLEM DEFINITION**Situation:**

Pipe flows A and B merge into a single pipe.

$$Q_A = 0.02t \text{ m}^3/\text{s}, Q_B = 0.008t^2 \text{ m}^3/\text{s}.$$

$$A_{\text{exit}} = 0.01 \text{ m}^2, t = 1 \text{ s}.$$

Find:

Velocity at the exit, V_{exit} .

Acceleration at the exit, a_{exit} .

Assumptions:

Incompressible flow.

PLAN

Apply the continuity equation.

SOLUTION

Since the flow is incompressible, the unsteady term is zero. Continuity equation

$$\begin{aligned} Q_{\text{exit}} &= Q_A + Q_B \\ V_{\text{exit}} &= \left(\frac{1}{A_{\text{exit}}} \right) (Q_A + Q_B) \\ &= \left(\frac{1}{0.01 \text{ m}^2} \right) (.02t \text{ m}^3/\text{s} + 0.008t^2 \text{ m}^3/\text{s}) \\ &= 2t \text{ m/s} + 0.8t^2 \text{ m/s} \end{aligned}$$

Then at $t = 1$ sec,

$$V_{\text{exit}} = 2.8 \text{ m/s}$$

The acceleration along a pathline at the ($s \rightarrow x$) exit is

$$a_{\text{exit}} = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x}$$

Since V varies with time, but not with position, there is no convective acceleration so

$$a_{\text{exit}} = \frac{\partial V}{\partial t} = 2 + 1.6t \text{ m/s}$$

Then at $t = 1$ sec

$$a_{\text{exit}} = 3.6 \text{ m/s}^2$$

5.52: PROBLEM DEFINITION

Situation:

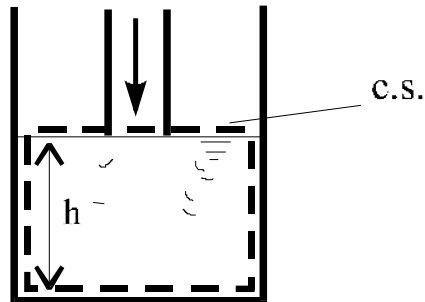
A round tank is being filled with water.

$$V_p = 10 \text{ ft/s}, D_T = 4 \text{ ft}, D_p = 1 \text{ ft}.$$

Find:

Rate at which the water surface is rising.

Sketch:



PLAN

Apply the continuity equation and let the c.s. move up with the water surface in the tank.

SOLUTION

Continuity equation

$$0 = \frac{d}{dt} \int_{CV} \rho dV + \dot{m}_o - \dot{m}_i$$
$$0 = \frac{d}{dt} (hA_T) - ((10 + V_R)A_p)$$

where A_T = tank area, V_R = rise velocity and A_p = pipe area.

$$0 = A_T \frac{dh}{dt} - 10A_p - V_RA_p$$

but $dh/dt = V_R$ so

$$0 = A_TV_R - 10A_p - V_RA_p$$
$$V_R = \frac{10A_p}{A_T - A_p} = \frac{(10 \text{ ft/s}) (\pi/4)(1 \text{ ft})^2}{[(\pi/4)(4 \text{ ft})^2 - (\pi/4)(1 \text{ ft})^2]}$$

$$V_R = (2/3) \text{ ft/s}$$

5.61: PROBLEM DEFINITION**Situation:**

A tank has one inflow and two outflows.

$$D_{4in} = 4 \text{ in} = 0.333 \text{ ft}, V_{4in} = 10 \text{ ft/s.}$$

$$D_{6in} = 6 \text{ in} = 0.5 \text{ ft}, V_{6in} = 7 \text{ ft/s.}$$

$$D_{3in} = 3 \text{ in} = 0.25 \text{ ft}, V_{3in} = 4 \text{ ft/s.}$$

$$D_{\text{tank}} = 3 \text{ ft.}$$

Find:

Is the tank filling or emptying.

Rate at which the tank level is changing: $\frac{dh}{dt}$

SOLUTION

$$\text{Inflow} = (10 \text{ ft/s}) \left(\frac{\pi}{4}\right) \left(\frac{4}{12} \text{ ft}\right)^2 = 0.8727 \text{ cfs}$$

$$\text{Outflow} = (7 \text{ ft/s}) \left(\frac{\pi}{4}\right) (0.5 \text{ ft})^2 + (4 \text{ ft/s}) \left(\frac{\pi}{4}\right) (0.25 \text{ ft})^2 = 1.571 \text{ cfs}$$

Outflow > Inflow, Thus,

tank is emptying

$$\begin{aligned} \frac{dh}{dt} &= -\frac{Q}{A} \\ &= -\frac{(1.571 - 0.8727) \text{ ft}^3/\text{s}}{\pi (3 \text{ ft})^2} \end{aligned}$$

$\frac{dh}{dt} = -0.0247 \text{ ft/s}$