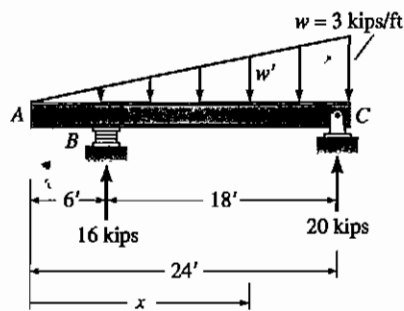


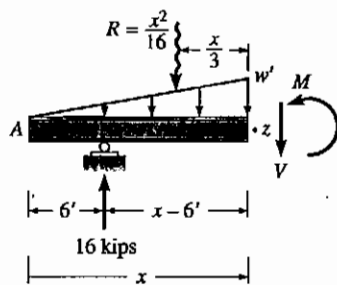
# HOMEWORK 7

## EXAMPLE 5.4

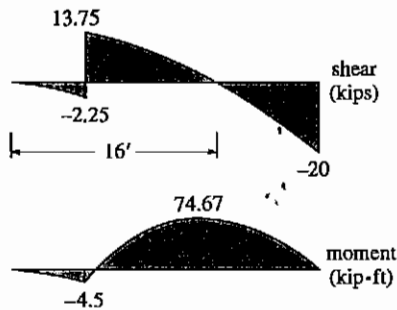
1



(a)



(b)



(c)

- Write the equations for shear and moment on a vertical section between supports  $B$  and  $C$  for the beam in Figure 5.10a.
- Using the equation for shear in part (a), determine the point where the shear is zero (the point of maximum moment).
- Plot the variation of the shear and moment between  $B$  and  $C$ .

### Solution

- Cut the free body shown in Figure 5.10b by passing a section through the beam a distance  $x$  from point  $A$  at the left end. Using similar triangles, express  $w'$ , the ordinate of the triangular load at the cut (consider the triangular load on the free body and on the beam), in terms of  $x$  and the ordinate of the load curve at support  $C$ .

$$\frac{w'}{x} = \frac{3}{24} \quad \text{therefore} \quad w' = \frac{x}{8}$$

Compute the resultant of the triangular load on the free body in Figure 5.10b.

$$R = \frac{1}{2} x w' = \frac{1}{2} (x) \left( \frac{x}{8} \right) = \frac{x^2}{16}$$

Compute  $V$  by summing forces in the vertical direction.

$$\uparrow \Sigma F_y = 0$$

$$0 = 16 - \frac{x^2}{16} - V$$

$$V = 16 - \frac{x^2}{16} \tag{1}$$

Compute  $M$  by summing moments about the cut.

$$\circlearrowleft \Sigma M_z = 0$$

$$0 = 16(x - 6) - \frac{x^2}{16} \left( \frac{x}{3} \right) - M$$

$$M = -96 + 16x - \frac{x^3}{48} \tag{2}$$

- Set  $V = 0$  and solve Equation 1 for  $x$ :

$$0 = 16 - \frac{x^2}{16} \quad \text{and} \quad x = 16 \text{ ft}$$

- See Figure 5.10c for a plot of  $V$  and  $M$ .

Figure 5.10

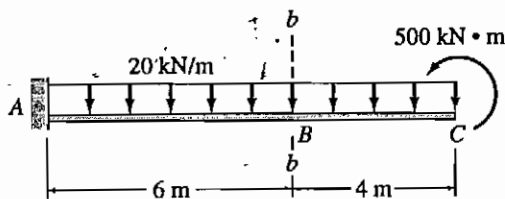


Fig. 5.4

*Shear* Considering the external forces acting downward as positive, we write

$$S = +20(4) = +80 \text{ kN} \qquad S = 80 \text{ kN} \quad \text{Ans.}$$

Note that the 500 kN-m couple does not have any effect on shear.

*Bending Moment* Considering the counterclockwise moments as positive, we write

$$M = 500 - 20(4)(2) = 340 \text{ kN} \cdot \text{m} \qquad M = 340 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

The reader may check the results by summing forces and moments on portion AB of the beam after computing the reactions at support A.

## 5.2 SHEAR AND BENDING MOMENT DIAGRAMS

Shear and bending moment diagrams depict the variations of these quantities along the length of the member. Such diagrams can be constructed by using the method of sections described in the preceding section. Proceeding from one end of the member to the other (usually from left to right), sections are passed, after each successive change in loading, along the length of the member to determine the equations expressing the shear and bending moment in terms of the distance of the section from a fixed origin. The values of shear and bending moments determined from these equations are then plotted as ordinates against the position with respect to a member end as abscissa to obtain the shear and bending moment diagrams. This procedure is illustrated by the following examples.

(2)

### EXAMPLE 5.3

Draw the shear and bending moment diagrams for the beam shown in Fig. 5.5(a).

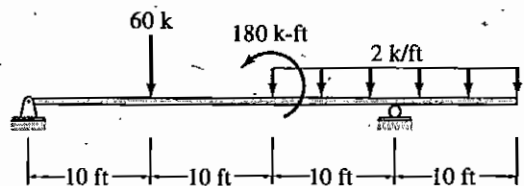
**SOLUTION** *Reactions* See Fig. 5.5(b).

$$+ \rightarrow \sum F_x = 0 \qquad A_x = 0$$

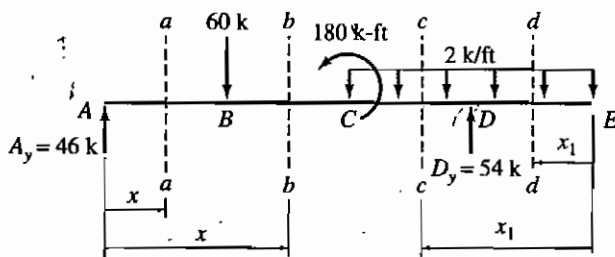
$$+ \zeta \sum M_D = 0$$

$$- A_y(30) + 60(20) + 180 + 2(20)(0) = 0$$

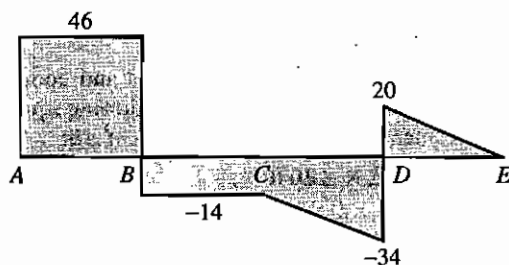
$$A_y = 46 \text{ k} \uparrow$$



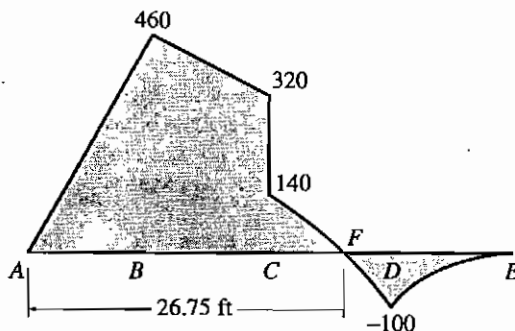
(a)



(b)



(c) Shear Diagram (k)



(d) Bending Moment Diagram (k-ft)

Fig. 5.5

$$+\uparrow \sum F_y = 0$$

$$46 - 60 - 2(20) + D_y = 0$$

$$D_y = 54 \text{ k} \uparrow$$

**Shear Diagram** To determine the equation for shear in segment  $AB$  of the beam, we pass a section  $aa$  at a distance  $x$  from support  $A$ , as shown in Fig. 5.5(b). Considering the free body to the left of this section, we obtain

$$S = 46 \text{ k} \quad \text{for} \quad 0 < x < 10 \text{ ft}$$

As this equation indicates, the shear is constant at 46 k from an infinitesimal distance to the right of point  $A$  to an infinitesimal distance to the left of point  $B$ . At point  $A$ , the shear increases abruptly from 0 to 46 k, so a vertical line is drawn from 0 to 46 on the shear diagram (Fig. 5.5(c)) at  $A$  to indicate this change. This is followed by a horizontal line from  $A$  to  $B$  to indicate that the shear remains constant in this segment.

Next, by using section  $bb$  (Fig. 5.5(b)), we determine the equation for shear in segment  $BC$  as

$$S = 46 - 60 = -14 \text{ k} \quad \text{for} \quad 10 \text{ ft} < x \leq 20 \text{ ft}$$

The abrupt change in shear from 46 k at an infinitesimal distance to the left of  $B$  to  $-14$  k at an infinitesimal distance to the right of  $B$  is shown on the shear diagram (Fig. 5.5(c)) by a vertical line from  $+46$  to  $-14$ . A horizontal line at  $-14$

is then drawn from  $B$  to  $C$  to indicate that the shear remains constant at this value throughout this segment.

To determine the equations for shear in the right half of the beam, it is convenient to use another coordinate,  $x_1$ , directed to the left from the end  $E$  of the beam, as shown in Fig. 5.5(b). The equations for shear in segments  $ED$  and  $DC$  are obtained by considering the free bodies to the right of sections  $dd$  and  $cc$ , respectively. Thus

$$S = 2x_1 \quad \text{for} \quad 0 \leq x_1 < 10 \text{ ft}$$

and

$$S = 2x_1 - 54 \quad \text{for} \quad 10 \text{ ft} < x_1 \leq 20 \text{ ft}$$

These equations indicate that the shear increases linearly from zero at  $E$  to  $+20$  k at an infinitesimal distance to the right of  $D$ ; it then drops abruptly to  $-34$  k at an infinitesimal distance to the left of  $D$ ; and from there it increases linearly to  $-14$  k at  $C$ . This information is plotted on the shear diagram, as shown in Fig. 5.5(c).

Ans.

**Bending Moment Diagram** Using the same sections and coordinates employed previously for computing shear, we determine the following equations for bending moment in the four segments of the beam. For segment  $AB$ :

$$M = 46x \quad \text{for} \quad 0 \leq x \leq 10 \text{ ft}$$

For segment  $BC$ :

$$M = 46x - 60(x - 10) = -14x + 600 \quad \text{for} \quad 10 \text{ ft} \leq x < 20 \text{ ft}$$

For segment  $ED$ :

$$M = -2x_1 \left( \frac{x_1}{2} \right) = -x_1^2 \quad \text{for} \quad 0 \leq x_1 \leq 10 \text{ ft}$$

For segment  $DC$ :

$$M = -x_1^2 + 54(x_1 - 10) = -x_1^2 + 54x_1 - 540 \quad \text{for} \quad 10 \text{ ft} \leq x_1 < 20 \text{ ft}$$

The first two equations, for the left half of the beam, indicate that the bending moment increases linearly from 0 at  $A$  to 460 k-ft at  $B$ ; it then decreases linearly to 320 k-ft at  $C$ , as shown on the bending moment diagram in Fig. 5.5(d). The last two equations for the right half of the beam are quadratic in  $x_1$ . The values of  $M$  computed from these equations are plotted on the bending moment diagram shown in Fig. 5.5(d). It can be seen that  $M$  decreases from 0 at  $E$  to  $-100$  k-ft at  $D$ , and it then increases to  $+140$  k-ft at an infinitesimal distance to the right of  $C$ . Note that at  $C$ , the bending moment drops abruptly by an amount  $320 - 140 = 180$  k-ft, which is equal to the magnitude of the moment of the counterclockwise external couple acting at this point.

A point at which the bending moment is zero is termed the *point of inflection*. To determine the location of the point of inflection  $F$  (Fig. 5.5(d)), we set  $M = 0$  in the equation for bending moment in segment  $DC$  to obtain

$$M = -x_1^2 + 54x_1 - 540 = 0$$

from which  $x_1 = 13.25$  ft; that is, point  $F$  is located at a distance of 13.25 ft from end  $E$ , or  $40 - 13.25 = 26.75$  ft from support  $A$  of the beam, as shown in Fig. 5.5(d).

Ans.

③

EXAMPLE 5.4

Draw the shear and bending moment diagrams for the beam shown in Fig. 5.6(a).

SOLUTION

Reactions See Fig. 5.6(b).

$$+ \rightarrow \sum F_x = 0$$

$$B_x = 0$$

$$+ \zeta \sum M_c = 0$$

$$\left(\frac{1}{2}\right)(9)(27)\left(\frac{9}{3}\right) - B_y(6) = 0$$

$$B_y = 60.75 \text{ kN} \uparrow$$

$$+ \uparrow \sum F_y = 0$$

$$-\left(\frac{1}{2}\right)(9)(27) + 60.75 + C_y = 0$$

$$C_y = 60.75 \text{ kN} \uparrow$$

**Shear Diagram** To determine the equations for shear in segments  $AB$  and  $BC$  of the beam, we pass sections  $aa$  and  $bb$  through the beam, as shown in Fig. 5.5(b). Considering the free bodies to the left of these sections and realizing that the load intensity,  $w(x)$ , at a point at a distance  $x$  from end  $A$  is  $w(x) = \left(\frac{27}{9}\right)x = 3x \text{ kN/m}$ , we obtain the following equations for shear in segments  $AB$  and  $BC$ , respectively:

$$S = -\left(\frac{1}{2}\right)(x)(3x) = -\frac{3x^2}{2} \quad \text{for} \quad 0 \leq x < 3 \text{ m}$$

$$S = -\left(\frac{3x^2}{2}\right) + 60.75 \quad \text{for} \quad 3 \text{ m} < x < 9 \text{ m}$$

The values of  $S$  computed from these equations are plotted to obtain the shear diagram shown in Fig. 5.6(c). The point  $D$  at which the shear is zero is obtained from the equation

$$S = -\left(\frac{3x^2}{2}\right) + 60.75 = 0$$

from which  $x = 6.36 \text{ m}$ .

Ans.

**Bending Moment Diagram** Using the same sections employed previously for computing shear, we determine the following equations for bending moment in segments  $AB$  and  $BC$ , respectively:

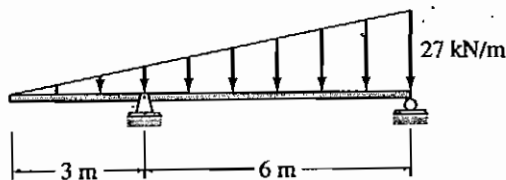
$$M = -\left(\frac{1}{2}\right)(x)(3x)\left(\frac{x}{3}\right) = -\frac{x^3}{2} \quad \text{for} \quad 0 \leq x \leq 3 \text{ m}$$

$$M = -\left(\frac{x^3}{2}\right) + 60.75(x - 3) \quad \text{for} \quad 3 \text{ m} \leq x \leq 9 \text{ m}$$

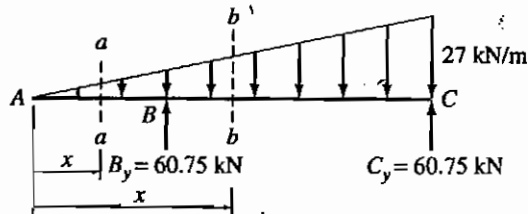
The values of  $M$  computed from these equations are plotted to obtain the bending moment diagram shown in Fig. 5.6(d). To locate the point at which the bending moment is maximum, we differentiate the equation for  $M$  in segment  $BC$  with respect to  $x$  and set the derivative  $dM/dx$  equal to zero; that is,

$$\frac{dM}{dx} = \left(-\frac{3x^2}{2}\right) + 60.75 = 0$$

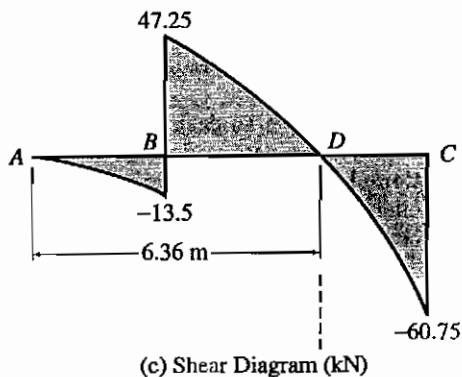
from which  $x = 6.36 \text{ m}$ . This indicates that the maximum bending moment occurs at the same point at which the shear is zero. Also, a comparison of the expressions for  $dM/dx$  and  $S$  in segment  $BC$  indicates that the two equations are identical; that



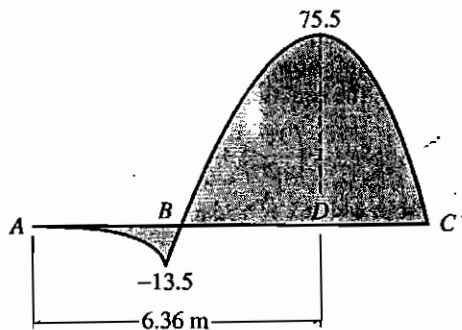
(a)



(b)



(c) Shear Diagram (kN)



(d) Bending Moment Diagram (kN · m)

Fig. 5.6

is, the slope of the bending moment diagram at a point is equal to the shear at that point. (This relationship, which is generally valid, is discussed in detail in a subsequent section.)

Finally, the magnitude of the maximum moment is determined by substituting  $x = 6.36$  m into the equation for  $M$  in segment  $BC$ :

$$M_{\max} = -\left[\frac{(6.36)^3}{2}\right] + 60.75(6.36 - 3) = 75.5 \text{ kN} \cdot \text{m}$$

Ans.

(clockwise) couple acts at  $B$ , so the bending moment increases abruptly at this point by an amount equal to the magnitude of the moment of the couple. The largest value (global maximum) of the bending moment over the entire length of the beam occurs at just to the right of  $B$ . (Note that no abrupt change, or discontinuity, occurs in the shear diagram at this point.) Finally, as the shear in segments  $BC$  and  $CD$  is constant and negative, the bending moment diagram in these segments consists of straight lines with negative slopes.

Ans.

*Qualitative Deflected Shape* See Fig. 5.10(e).

Ans.

## EXAMPLE 5.7

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.11(a).

### SOLUTION

*Reactions* (See Fig. 5.11(b)).

$$+ \rightarrow \sum F_x = 0$$

$$B_x = 0$$

$$+ \zeta \sum M_C = 0$$

$$\frac{1}{2}(3)(12)(24) - B_y(20) + 3(20)(10) - \frac{1}{2}(3)(6)(2) = 0$$

$$B_y = 50.7 \text{ k}$$

$$B_y = 50.7 \text{ k} \uparrow$$

$$+ \uparrow \sum F_y = 0$$

$$-\frac{1}{2}(3)(12) + 50.7 - 3(20) - \frac{1}{2}(3)(6) + C_y = 0$$

$$C_y = 36.3 \text{ k}$$

$$C_y = 36.3 \text{ k} \uparrow$$

*Shear Diagram*

*Point A*  $S_A = 0$

$$\text{Point B } S_{B,L} = 0 - \frac{1}{2}(3)(12) = -18 \text{ k}$$

$$S_{B,R} = -18 + 50.7 = 32.7 \text{ k}$$

$$\text{Point C } S_{C,L} = 32.7 - 3(20) = -27.3 \text{ k}$$

$$S_{C,R} = -27.3 + 36.3 = 9 \text{ k}$$

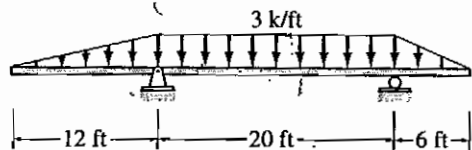
$$\text{Point D } S_D = 9 - \frac{1}{2}(3)(6) = 0$$

Checks

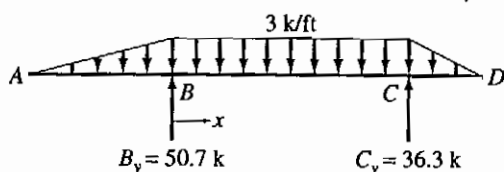
The shear diagram is shown in Fig. 5.11(c). The shape of the diagram between the ordinates just computed is obtained by applying the condition that the slope of the shear diagram at any point is equal to the load intensity at that point. For example, as the load intensity at  $A$  is zero, so is the slope of the shear diagram at  $A$ . Between  $A$  and  $B$ , the load intensity is negative and it decreases linearly from zero at  $A$  to  $-3 \text{ k/ft}$  at  $B$ . Thus the slope of the shear diagram is negative in this segment, and it decreases (becomes more steep) continuously from  $A$  to just to the left of  $B$ . The rest of the shear diagram is constructed by using similar reasoning.

Ans.

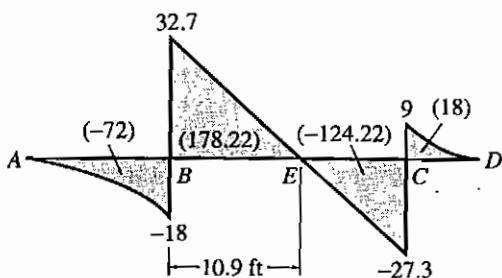
The point of zero shear,  $E$ , is located by using the similar triangles forming the shear diagram between  $B$  and  $C$ .



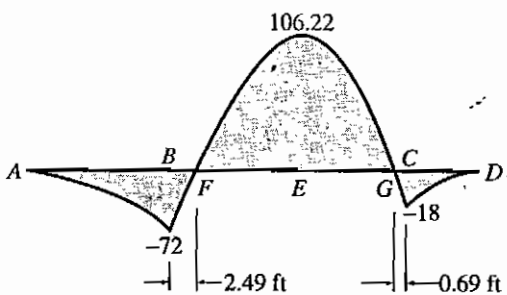
(a)



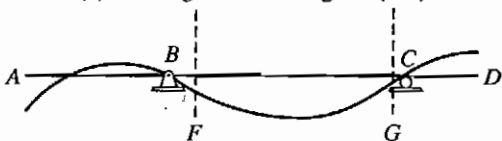
(b)



(c) Shear Diagram (k)



(d) Bending Moment Diagram (k-ft)



(e) Qualitative Deflected Shape

Fig. 5.11

To facilitate the construction of the bending moment diagram, the areas of the various segments of the shear diagram have been computed and are shown in parentheses on the shear diagram (Fig. 5.11(c)). It should be noted that the areas of the parabolic spandrels,  $AB$  and  $CD$ , can be obtained by using the formula for the area of this shape given in Appendix A.

**Bending Moment Diagram**

Point A  $M_A = 0$

Point B  $M_B = 0 - 72 = -72 \text{ k}\cdot\text{ft}$

Point E  $M_E = -72 + 178.22 = 106.22 \text{ k}\cdot\text{ft}$

Point C  $M_C = 106.22 - 124.22 = -18 \text{ k}\cdot\text{ft}$

Point D  $M_D = -18 + 18 = 0$

Checks

The shape of the bending moment diagram between these ordinates is obtained by using the condition that the slope of the bending moment diagram at any point is equal to the shear at that point. The bending moment diagram thus constructed is shown in Fig. 5.11(d).

It can be seen from this figure that the maximum negative bending moment occurs at point B, whereas the maximum positive bending moment, which has the largest absolute value over the entire length of the beam, occurs at point E. **Ans.**

To locate the points of inflection, F and G, we set equal to zero the equation for bending moment in segment BC, in terms of the distance x from the left support point B (Fig. 5.11(b)):

$$M = -\left(\frac{1}{2}\right)(3)(12)(4+x) + 50.7x - 3(x)\left(\frac{x}{2}\right) = 0$$

or

$$-1.5x^2 + 32.7x - 72 = 0$$

from which  $x = 2.49 \text{ ft}$  and  $x = 19.31 \text{ ft}$  from B.

**Qualitative Deflected Shape** A qualitative deflected shape of the beam is shown in Fig. 5.11(e). The bending moment is positive in segment FG, so the beam is bent concave upward in this region. Conversely, since the bending moment is negative in segments AF and GD, the beam is bent concave downward in these segments.

**Ans.**

5

**EXAMPLE 5.8**

Draw the shear and bending moment diagrams and the qualitative deflected shape for the beam shown in Fig. 5.12(a).

**SOLUTION** Reactions (See Fig. 5.12(b).)

$$+\zeta \sum M_B^{BD} = 0$$

$$-20(10)(5) + C_y(10) - 100(15) = 0$$

$$C_y = 250 \text{ kN}$$

$$C_y = 250 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$A_y - 20(10) + 250 - 100 = 0$$

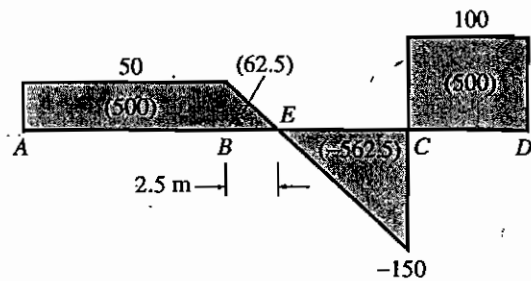
$$A_y = 50 \text{ kN}$$

$$A_y = 50 \text{ kN} \uparrow$$

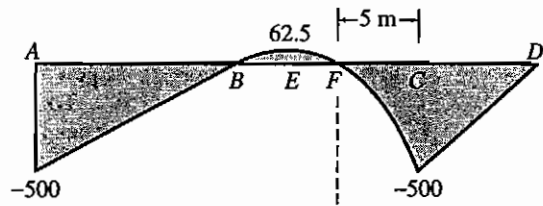
$$+\zeta \sum M_A = 0$$

$$M_A - 20(10)(15) + 250(20) - 100(25) = 0$$

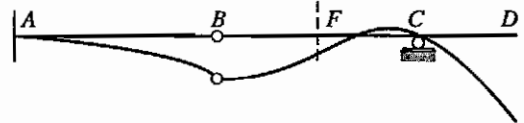
$$M_A = 500 \text{ kN}\cdot\text{m} \quad M_A = 500 \text{ kN}\cdot\text{m} \curvearrowright$$



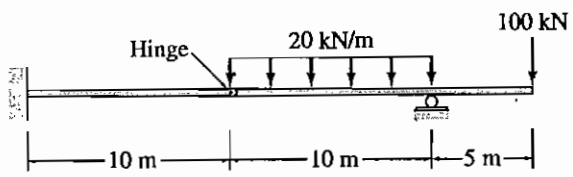
(c) Shear Diagram (kN)



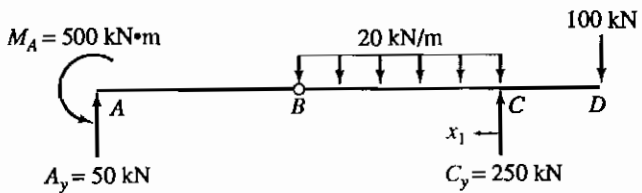
(d) Bending Moment Diagram (kN·m)



(e) Qualitative Deflected Shape



(a)



(b)

Fig. 5.12

*Shear Diagram*

Point A  $S_{A,R} = 50$  kN

Point B  $S_B = 50 + 0 = 50$  kN

Point C  $S_{C,L} = 50 - 20(10) = -150$  kN

$S_{C,R} = -150 + 250 = 100$  kN

Point D  $S_D = 100 - 100 = 0$

The shear diagram is shown in Fig. 5.12(c).

Checks

Ans.

*Bending Moment Diagram*

Point A  $M_{A,R} = -500$  kN·m

Point B  $M_B = -500 + 500 = 0$

Point E  $M_E = 0 + 62.5 = 62.5$  kN·m

Point C  $M_C = 62.5 - 562.5 = -500$  kN·m

Point D  $M_D = -500 + 500 = 0$

Checks

The bending moment diagram is shown in Fig. 5.12(d). The point of inflection *F* can be located by setting equal to zero the equation for bending moment in segment *BC*, in terms of the distance  $x_1$  from the right support point *C*

6

### EXAMPLE 5.11

Draw the shear and moment curves and sketch the deflected shape of the continuous beam in Figure 5.19a. The support reactions are given.

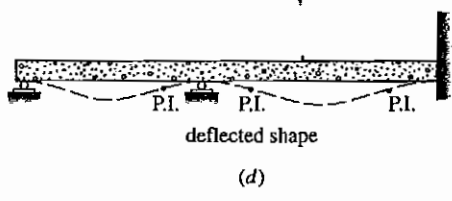
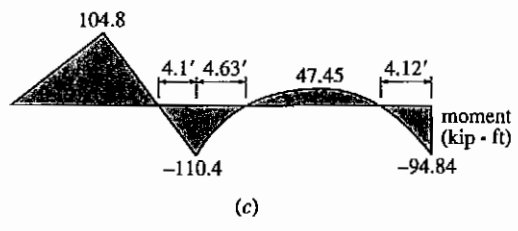
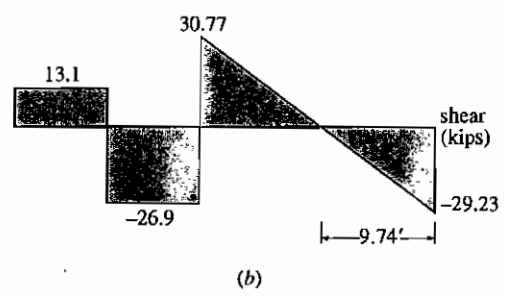
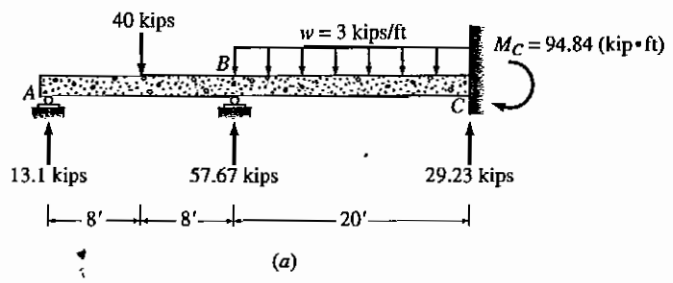


Figure 5.19

### Solution

Because the beam is indeterminate to the second degree, the reactions must be determined by one of the methods of indeterminate analysis covered in Chapters 11 through 13. Once the reactions are established, the procedure to draw the shear and moment curves is identical to that used in Examples 5.6 to 5.10. Figure 5.19d shows the deflected shape of the structure. Points of inflection are indicated by small black dots.