ENME 3770 Engineering Thermodynamics
Homework # 4, Summer 2009

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Pr 1. Air enters an uninsulated nozzle operating at steady state at 760°R with negligible velocity and exits the nozzle at 520°R with a velocity of 1500ft/s. Assuming ideal gas behavior and neglecting PE effects, determine the heat transfer per unit mass of air flowing, in Btu/lb. (10 points)

Sol. Using energy rate balance for states 1 and 2,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \tfrac{1}{2}V_1^2 + gZ_1\right) - \dot{m}_2\left(h_2 + \tfrac{1}{2}V_2^2 + gZ_2\right)$$

Here, $\dot{m}_1 = \dot{m}_2 = \dot{m}$ (conservation of mass)

$V_1 \approx 0$, $V_2 = 1500$ft/s, $Z_1 \approx 0$, $Z_1 \approx 0$, $\dot{W}_{cv} = 0$

State 1: $T_1 = 760°R$, so from Table A-22E, $h_1 = 182.08$ Btu/lb

State 2: $T_2 = 520°R$, so from Table A-22E, $h_2 = 124.27$ Btu/lb

Energy rate balance becomes after the above-mentioned assumptions,

$$0 = \dot{Q}_{cv} + \dot{m}(h_1) - \dot{m}\left(h_2 + \tfrac{1}{2}V_2^2\right)$$

$$\Rightarrow \quad \frac{\dot{Q}_{cv}}{\dot{m}} = h_2 + \tfrac{1}{2}V_2^2 - h_1$$

$$\Rightarrow \quad \frac{\dot{Q}_{cv}}{\dot{m}} = 124.27\frac{\text{Btu}}{\text{lbm}} + \frac{\tfrac{1}{2}\times 1500^2\,\frac{\text{ft}^2}{\text{s}^2}}{778\frac{\text{ft.lbf}}{\text{Btu}}\times 32.2\frac{\text{ft}}{\text{s}^2}} - 182.08\frac{\text{Btu}}{\text{lbm}} = -12.9\frac{\text{Btu}}{\text{lbm}}$$

Pr 2. Air expands through a turbine from 10 bar, 900K to 1 bar, 500K. The inlet velocity is small compared to the exit velocity of 100m/s. The turbine operates at steady state and develops a power output of 3200kW. Heat transfer between the turbine and its surroundings and PE effects are negligible. Calculate the mass flow rate, in kg/s, and the exit area. (10 points)

Sol. Using energy rate balance for states 1 and 2,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \tfrac{1}{2}V_1^2 + gZ_1\right) - \dot{m}_2\left(h_2 + \tfrac{1}{2}V_2^2 + gZ_2\right)$$

Here, $\dot{m}_1 = \dot{m}_2 = \dot{m}$ (conservation of mass)

$V_1 \approx 0$, $V_2 = 100$m/s, $Z_1 \approx 0$, $Z_1 \approx 0$, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 3200\text{kW} = 3200\text{kJ/s}$

State 1: $P_1 = 10$ bar, $T_1 = 900$K, so from Table A-22, $h_1 = 932.93$ kJ/kg

State 2: $P_2 = 1$ bar, $T_2 = 500$K, so from Table A-22, $h_2 = 503.02$ kJ/kg

Using ideal gas law, $v_2 = \dfrac{RT_2}{P_2} = \dfrac{\left(287\frac{\text{J}}{\text{kg.K}}\right)\times 500\text{K}}{1\times 10^5\frac{\text{N}}{\text{m}^2}} = 1.435\ \text{m}^3/\text{kg}$

Energy rate balance becomes after the above-mentioned assumptions,

$$0 = \dot{W}_{cv} + \dot{m}(h_1) - \dot{m}\left(h_2 + \tfrac{1}{2}V_2^2\right)$$

$$\Rightarrow \quad 0 = 3200\times 10^3\frac{\text{J}}{\text{s}} + \dot{m}\left(932.93\times 10^3\frac{\text{J}}{\text{kg}}\right) - \dot{m}\left(503.02\times 10^3\frac{\text{J}}{\text{kg}} + \tfrac{1}{2}\times 100^2\frac{\text{m}^2}{\text{s}^2}\times 1\frac{\text{J}}{\text{kg}}\frac{\text{s}^2}{\text{m}^2}\right)$$

$$\Rightarrow \quad 0 = 3200\times 10^3\frac{\text{J}}{\text{s}} + \dot{m}\left(424.91\times 10^3\frac{\text{J}}{\text{kg}}\right)$$

$$\Rightarrow \quad \dot{m} = 7.531\frac{\text{kg}}{\text{s}}$$

Volume flow rate, $A_2V_2 = \dot{m}v_2 \Rightarrow A_2\times 100 = 7.531\times 1.435 \Rightarrow A_2\times 100 = 0.108\text{m}^2$

4.57 At steady state, a well-insulated compressor takes in air at 60°F, 14.2 lbf/in², with a volumetric flow rate of 1200 ft³/min and compresses it to 500°F, 120 lbf/in². KE and PE changes from inlet to exit can be neglected. Determine the compressor power, in hp, and the volumetric flow rate at the exit, in ft³/min. (10 points)

Sol. Using energy rate balance for states 1 and 2,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \tfrac{1}{2}V_1^2 + gZ_1\right) - \dot{m}_2\left(h_2 + \tfrac{1}{2}V_2^2 + gZ_2\right)$$

Here, $\dot{m}_1 = \dot{m}_2 = \dot{m}$ (conservation of mass)

$V_1 \approx 0$, $V_2 = 0$, $Z_1 \approx 0$, $Z_1 \approx 0$, $\dot{Q}_{cv} = 0$, volume for rate, $q_1 = 1200\text{ft}^3/\text{min} = 20\text{ft}^3/\text{s}$

State 1: $P_1 = 14.2$ lbf/in², $T_1 = 60°F = 520°R$, so from Table A-22E, $h_1 = 124.27$ Btu/lb

State 2: $P_2 = 120$ lbf/in², $T_2 = 500°F = 960°R$, so from Table A-22E, $h_2 = 231.06$ Btu/lb

Using ideal gas law,

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{1.99}{28.965}\frac{\text{Btu}}{\text{lb.°R}}\times 778.2\frac{\text{ft.lb}}{\text{Btu}}\right)\times 520°\text{R}}{14.2\frac{\text{lb}}{\text{in}^2}\times 144\frac{\text{in}^2}{\text{ft}^2}} = 13.5964\ \text{ft}^3/\text{lb}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{\left(\frac{1.99}{28.965}\frac{\text{Btu}}{\text{lb.°R}}\times 778.2\frac{\text{ft.lb}}{\text{Btu}}\right)\times 960°\text{R}}{120\frac{\text{lb}}{\text{in}^2}\times 144\frac{\text{in}^2}{\text{ft}^2}} = 2.97\ \text{ft}^3/\text{lb}$$

For conservation of mass, $\dot{m} = \dfrac{q_1}{v_1} = \dfrac{q_2}{v_2} \Rightarrow \dfrac{1200}{13.5964} = \dfrac{q_2}{2.97} \Rightarrow q_2 = 262.12\text{ft}^3/\text{min}$

Mass flow rate, $\dot{m} = \dfrac{q_1}{v_1} = \dfrac{20}{13.5964} = 1.47\text{lb/s}$

Energy rate balance becomes after the above-mentioned assumptions,

$$0 = -\dot{W}_{cv} + \dot{m}h_1 - \dot{m}h_2$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

$$\Rightarrow \quad \dot{W}_{cv} = 1.47\frac{\text{lb}}{\text{s}}(124.27 - 231.06)\frac{\text{Btu}}{\text{lb}} = -156.9813\ \text{Btu/s} = -222\text{hp}$$

Pr 4. A feedwater heater operates at steady state with liquid water entering at inlet 1 at 7 bar, 42°C, and a mass flow rate of 70 kg/s. A separate steam of water enters at inlet 2 as a two-phase liquid-vapor mixture at 7 bar with a quality of 98%. Saturated liquid at 7 bar exits the feedwater heater at 3. Ignoring heat transfer with the surroundings and neglecting KE and PE effects, determine the mass flow rate, in kg/s, at inlet 2. (10 points)

Sol. For conservation of mass, $\dot{m}_{in} = \dot{m}_{out} \Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$

For conservation of energy, $H_{in} = H_{out} \Rightarrow H_1 + H_2 = H_3 \Rightarrow \dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_3h_3$

Here $\dot{m}_1 = 70\text{kg/s}$, so, $\dot{m}_3 = 70 + \dot{m}_2$

State 1, $P_1 = 7$ bar, $T_1 = 42°C$, from Table A-5, $h_1 = 176.5481$ kJ/kg

State 2, $P_2 = 7$ bar, x = 0.98, from Table A-2, $h_2 = 2722.174$ kJ/kg

State 3, $P_3 = 7$ bar (Saturated), from Table A-2, $h_3 = 697.22$ kJ/kg

So, satisfying the energy equation,

$$70\times 176.5481 + \dot{m}_2\times 2722.174 = (\dot{m}_2 + 70)\times 697.22 \quad \Rightarrow \dot{m}_2 = 17.99\text{kg/s}$$

Pr 5. Ammonia enters the expansion valve of a refrigeration system at a pressure of 1.4 MPa and a temperature of 32°C and exits at 0.08 MPa. If the refrigerant undergoes a throttling process, what is the quality of the refrigerant exiting the expansion valve? (10 points)

Sol. $h(T, P) \approx h_f(T) + v_f(T)[P - P_{sat}(T)]$ or $h(T, P) \approx h_f(T)$

State 1: $P_1 = 1.4$ MPa, $T_1 = 32°C$, so from Table A-13

$h_f(T) = h_f(32°C) = 332.17$ kJ/kg, $v_f(T) = 0.0016887$ m³/g, $P_{sat}(T) = 12.38$ bar

So, $h_1 = 332170 + 0.\ 0016887 \times [1.4 \times 10^6 - 12.38 \times 10^5] = 332444$ J = 332.444 kJ

Or, $h_1' = 332.17$ kJ

As this is a throttling process, $h_2 = h_1$.

So, $h_2 = 332.444$ kJ or $h_2' = 332.17$ kJ

State 2: From Table A-13, for $P_2 = 0.8$ bar, $h_f = 9.04$ kJ/kg, $h_g = 1391.78$ kJ/kg

So, for $h_2 = 332.444$ kJ, x = 0.233866, or for $h_2 = 332.17$ kJ, x = 0.233688.

This insignificant difference reflects the incompressibility of liquid ammonia.

Pr 6. A rigid tank of volume 0.75m³ is initially evacuated. A hole develops in the wall, and air form the surroundings at 1 bar, 25°C flows until the pressure in the tank reaches 1 bar. Heat transfer between the contents of the tank and the surroundings is negligible. Determine the final temperature in the tank, in °C. (10 points)

Sol. The energy rate balance reduces with negligible KE, PE and Heat transfer,

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_i\frac{dm_{cv}}{dt}$$

Integrating, $\Delta U_{cv} = -W_{cv} + h_i\,\Delta m_{cv}$,

Since the tank is initially evacuated, the terms, ΔU_{cv} and Δm_{cv} reduce to the internal energy and mass within the tank at the end of the process and no work is done as the tank is leaked to air,

$\Delta U_{cv} = m_2u_2$

and $\Delta m_{cv} = m_2 - 0$

$W_{cv} = 0$

So, $m_2u_2 = m_2\,h_1 \Rightarrow u_2 = h_1$

At state 1, $P_1 = 1$ bar, $T_1 = 25°C$, from Table A-22, $h_1 = 298.18$ kJ/kg

So, for $u_2 = h_1 = 298.18$ kJ/kg, from Table A-22, $T_2 = 416.54K = 143.54°C$