1. (a) State <u>Rolle's Theorem</u>.

ANSWER: **Rolle's Theorem**:

Hypotheses:

[6 points]

① The function $f: [a, b] \to \mathbb{R}$ is defined and continuous on the Closed interval $[a, b]$

② The fundion $f: (a, b) \to \mathbb{R}$ is differentiable on the open interval (a, b)

Conclusion:

[3 points]

③ $f(a) = f(b)$

Conclusion: There exists a least one point $c \in \mathbb{R}$ with $a < c < b$ for which $f'(c) = 0$ the slope of the straight Line tangent to the graph of f at $(c, f(c))$

(b) State <u>The Mean Value Theorem</u>.

ANSWER: **The Mean Value Theorem**:

Hypotheses:

[4 points]

① $f: [a, b] \to \mathbb{R}$ is defined & continuous on the closed interval $[a, b]$

② $f: [$

1 & 2 Same as Rolles

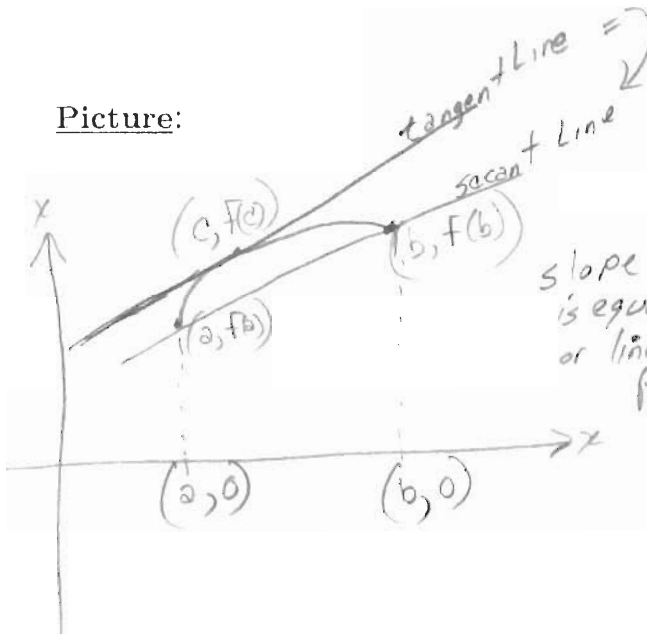
Conclusion:

[5 points]

There exist at least one point $c \in \mathbb{R}$ with $a < c < b$ for which $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Picture:

[7 points]



slope of tangent line is equal to secant line or line connecting the end points

2. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function with the property that $f'(x)$ exists for all $x \in \mathbb{R}$ and is given by the formula

$4x^2 \cdot (x-3)$
$4(x-0)\cdot(x-0)\cdot(x-3)$
x^2 times 4

$$f'(x) = 4x^2 \cdot (x - 3) .$$

$f(x) = (x+2)\ (3-x)(x-4)$

a. Draw the sign graph for $f'(x)$.

$[x-(-2)]\ (-1)(x-3)\ (x-4)$

[11 points]

ANSWER:

another one =

$f'(x) = (x-3)(x+1)\ (2+x)\ (4-x)\ (x-5)(6-x)$

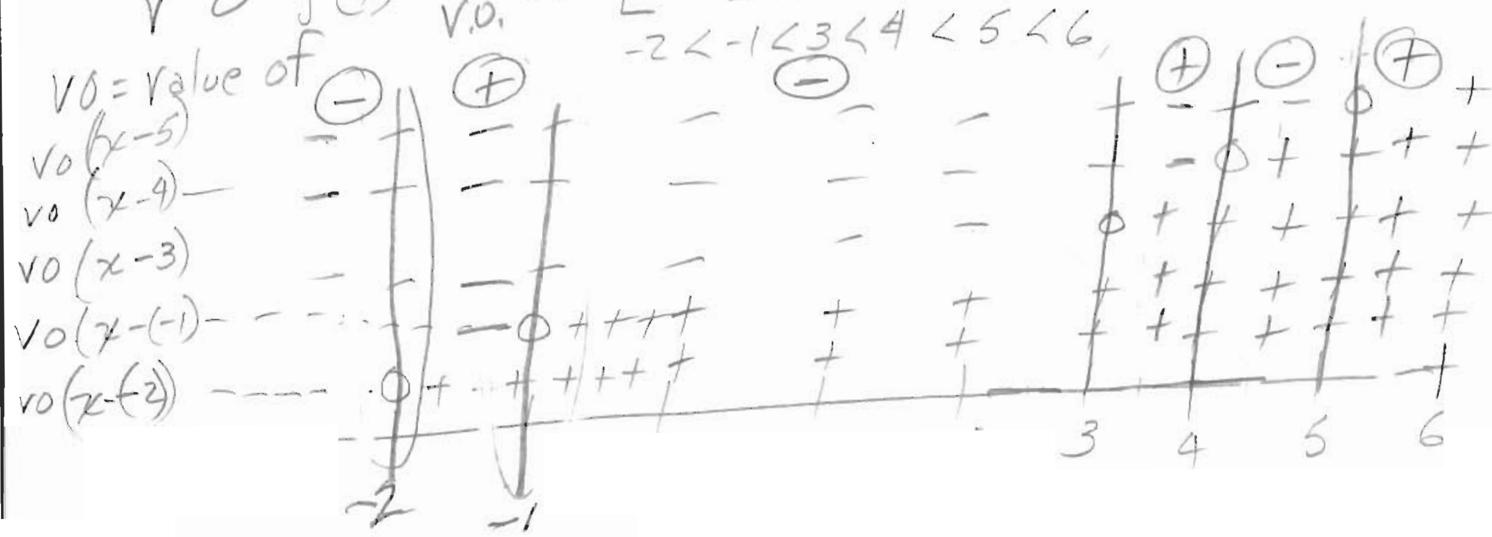
$= (x-3)\ (x-(-1))\ (x-(-2))(-1)\cdot(x-4)\ (x-5)(-1)(x-6)$

V O $f'(x) = (-1)\cdot(-1)\ [x-(-2)]\ [x-(-1)]\ [x-3]\ [x-4]\ [x-5]\ [x-6]$

V.O.

$-2 < -1 < 3 < 4 < 5 < 6$

VO = Value of



Behavior of F

THE SIGN GRAPH OF:

$$f'(x) = 4x^2 \cdot (x - 3) .$$

b. **DETERMINE**: (1) those <u>intervals</u> on which f is <u>strictly increasing</u> :

[2 points]

$f \nearrow$ on $(3, +\infty)$ $0 < x < \infty$

AND

(2) those <u>intervals</u> on which f is <u>strictly decreasing</u> :

[2 points]

$f \searrow$ on $(-\infty, 0) \cup (0, 3)$

3. Make a **careful sketch of the graph** of the function $f : \mathbb{R} \to \mathbb{R}$ defined for all $x \in \mathbb{R}$ by the equation

$f''(x) = 12x^2 - 24x = 12x(x-2)$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

$$y = f(x) = x^4 - 4x^3. = x^3 \cdot (x-4)$$

Where is it (+) (-)

Be sure to label the axes, the curve, the x-intercepts [if any], the y-intercepts [if any], the local maximum points [if any], the local minimum points [if any], the inflection points [if any], and, finally, the regions [if any] on which the curve is *concave-up* or *concave-down* .

CAUTION: **No credit will be given for just a bunch of plotted points**! Your picture must be accompanied by a supporting mathematical analysis that is **thorough**. [This means that **you must justify**, for example, **why** a local maximum is a local maximum, or why an inflection point is an inflection point, or why the curve is concave-up or concave-down.]

[20 points] Sign graph of f'

You may, if necessary, continue the answer to this problem on the next page.

Problem 3 continued (if necessary) :

4. Find the dimensions of the *open-topped* box of *greatest volume* which can be made from a *rectangular* piece of cardboard 9 inches *wide* by 24 inches *long* by cutting a *square* from *each corner* and bending the *sides* up.

NOTE: Your solution must employ the techniques of calculus; and your answer must be supported by a *thorough mathematical analysis* (i.e., you must justify why a *maximum* occurs!).

[20 points]

SOLUTION:

You may, if necessary, continue the answer to this problem on the next page.

Problem 4 continued

5. Use differentials to approximate

$$\sqrt[3]{128} \quad .$$

[NOTE: No credit for any other method: DO NOT USE A CALCULATOR!]

Write your answer in the form "a.bc" .

SOLUTION:

[10 points]

6. **Evaluate the following indefinite integrals:**

(a) $\displaystyle\int \frac{x^{15} + x^{\frac{7}{3}}}{x^3}\, dx =$

[5 points]

(b) $\displaystyle\int_0^9 \sqrt{x}\, dx =$

[5 points]

EXTRA-CREDIT: [10 points: But, any score over 100 will be truncated to 100.]

NOTE: You may do just one of the following two problems. If you do more than one problem be sure to cross out what you do not want counted; otherwise I will count only the first problem not crossed out .

(1) The sum of one number and two times a second number is 32 . What numbers should be selected so that the product of the two numbers is as large as possible ?

NOTE: Your solution must employ the techniques of calculus.

SOLUTION:

[10 points]

The Second Possible Extra-Credit Problem

(2) Evaluate the following indefinite integral:

$$\int x \cdot \sqrt{x - 2}\ dx =$$

[10 points]