CHAPTER 5
SOLUTIONS

5-1 Theoretical oxygen demand of glutamic acid

Given: 63 mg/L of glutamic acid and oxidation reactions

Solution:

a. Compute the gram molecular weights of glutamic acid and oxygen consumed

GMW of $C_5H_9O_4N$ = 147

GMW of oxygen (4.5 O_2 + 2 O_2) = 208

b. Calculate the ThOD

$$\text{ThOD} = (63\,\text{mg/L})\frac{208}{147} = 89.15\,\text{mg/L}$$

5-2 Theoretical oxygen demand of bacterial cells

Given: 30 mg/L of bacterial cells and oxidation reactions

Solution:

a. Compute gram molecular weights

GMW of $C_5H_7NO_2$ = 113

GMW of oxygen (5 O_2 + 2 O_2) = 224

b. Calculate the ThOD

$$\text{ThOD} = (30\,\text{mg/L})\frac{224}{113} = 59.47\,\text{mg/L}$$

5-3 Theoretical oxygen demand of acetic acid

Given: 300 mg/L of acetic acid and oxidation reaction

Solution:

a. Compute gram molecular weights

GMW of CH_3COOH = 60

GMW of oxygen = 64

b. Calculate ThOD

$$\text{ThOD} = (300\,\text{mg/L})\frac{64}{60} = 320\,\text{mg/L}$$

5-4 Rate constant

Given: BOD_5 = 220.0; L = 320.0 mg/L

Solution:

a. Setup Eqn. 5-5

$220.0 = 320.0\ (1-10^{-K(5)})$

$0.6875 = 1 - 10^{-K(5)}$

$-0.3125 = -10^{-K(5)}$

b. Divide through by -1 and take log of both sides

$\log(0.3125) = \log(10^{-K(5)})$

$-0.5051 = -K(5)$

c. Solve for K

K = 0.1010 or 0.101 d^{-1}

5-5 Rate constant

Given: 7 day BOD = 60.0; L = 85.0 mg/L

Solution:

a. Setup Eqn. 5-5

$60.0 = 85.0\ (1 - 10^{-K(7)})$

$0.7059 = 1 - 10^{-K(7)}$

$-0.2941 = -10^{-K(7)}$

b. Divide through by -1 and take log of both sides

$\log(0.2941) = \log(10^{-K(7)})$

$-0.5315 = -K(7)$

$K = 0.0759\ d^{-1}$

5-6 Rate constant BOD_6

Given: 6 day BOD = 213 mg/L; L = 318.4 mg/L

Solution:

a. Set up Eqn. 5-5

$213 = 318.4(1 - e^{-k(6)})$

$0.6690 = 1 – e^{-k(6)}$

$-0.3310 = -e^{-k(6)}$

b. Divide through by -1 and take natural logarithm of both sides

$\ln(0.3310) = \ln(e^{-k(6)})$

$-1.1055 = -k(6)$

$k = 0.1843\ d^{-1}$

5-7 Converting K from base 10 to base e

Given: $K = 0.101\ d^{-1}$ from Problem 5-1

Solution:

$k = (0.101)(\ln 10)$

$k = (0.101)(2.303) = 0.233\ d^{-1}$

5-8 Converting K from base 10 to base e

Given: $K = 0.0759\ d^{-1}$ from Problem 5-2

Solution:

$$k = (0.0759)(\ln 10)$$

$$k = (0.0759)(2.303) = 0.175\ d^{-1}$$

5-9 Convert k from base e to base 10

Given: k = 0/1843 from Problem 5-6

Solution:

$$K = \frac{k}{\ln 10} = \frac{0.1843}{2.303} = 0.0800 d^{-1}$$

5-10 Converting to 15 ^{0}C

Given: $K = 0.101\ d^{-1}$ at 20 ^{0}C

Solution:

a. Setup Eqn. 5-6 and note that the temperature is between 4 and 20 ^{0}C so θ is 1.135:

$$K = (0.101)(1.135)^{15 - 20}$$

$$K = (0.101)(0.531) = 0.0536\ d^{-1}$$

5-11 Converting from 25 ^{0}C to 16 ^{0}C

Given: $K = 0.0759\ d^{-1}$ at 25 ^{0}C from Problem 5-5

Solution:

a. Must convert K at 25 ^{0}C to K at 20 ^{0}C first

$$0.0759 = K_{20}\ (1.056)^{25 - 20}$$

$$K_{20} = \frac{0.0759}{1.3132} = 0.0578$$

b Then convert to 16 ^{0}C

$$K = (0.0578)(1.135)^{16 - 20}$$

$$K = (0.0578)(0.6026) = 0.0348 \text{ d}^{-1}$$

5-12 BOD_5 ?

Given: Oxygen consumption = 2.00 mg/L; 1.00% sample

Solution:

a. This an application of Eqn. 5-19 with the dilution factor defined by Eqn. 5-18

$$BOD_5 = (2.00)\frac{100\%}{1.00\%} = 200 \text{ mg/L}$$

5-13 Sample size

Given: BOD_5 = 350.0 mg/L; oxygen consumption = 4.00 mg/L

Solution:

a. Setup Eqn. 5-19 with X = sample size in percent

$$350.0 = (4.00)\frac{100\%}{X}$$

$$X = \frac{(4.00)(100\%)}{350.0} = 1.14\% \text{ sample size}$$

5-14 Sample size

Given: BOD_5 = 327 mg/L; oxygen consumption = 4.8 mg/L

Solution:

a. Set up Eqn. 5-19 with X = sample size

$$327 = (4.8)\frac{100\%}{X}$$

$$X = \frac{(4.8)(100\%)}{327} = 1.47\% \text{ sample size}$$

5-15 BOD_5 for different K's

Given: L = 280.0 mg/L; K = 0.0800 d^{-1} or K = 0.120 d^{-1}

Solution:

a. For K = 0.0800 d^{-1}

$BOD_5 = 280.0\ (1 - 10^{-(0.08)(5)}) = 280.0\ (1 - 0.3981)$

$BOD_5 = 280.0\ (0.6019) = 168.53$ or 169 mg/L

b. For K = 0.120 d^{-1}

$BOD_5 = 280.0\ (1 - 10^{-(0.12)(5)}) = 280.0\ (1 - .2512)$

$BOD_5 = 280.0\ (0.7488) = 209.67$ or 210 mg/L

5-16 Ultimate BOD's for different K's

Given: BOD_5 = 280.0; K = 0.0800 d^{-1} or K = 0.120 d^{-1}

Solution:

a. For K = 0.0800 d^{-1}

$280.0 = L\ (1 - 10^{-(0.08)(5)})$

$$L = \frac{280.0}{(1 - 0.3981)}$$

L = 465.199 or 465.2 mg/L

b. For K = 0.120 d^{-1}

$280 = L\ (1-10^{-(0.12)(5)})$

$$L = \frac{280.0}{(1 - 0.2512)}$$

L = 373.93 or 373.9 mg/L

5-17 Plot BOD curves and find when L occurs

Given: Data in 5-15

Solution:

Day	BOD(t) for K=0.0800 d^{-1}	BOD(t) for K=0.120 d^{-1}
	0.08	0.12
2	86.3	118.9
5	168.5	209.7
10	235.6	262.3
20	273.0	278.9
35	279.6	280.0

Ultimate BOD occurs at 34.5 d $\pm$ 0.5 d

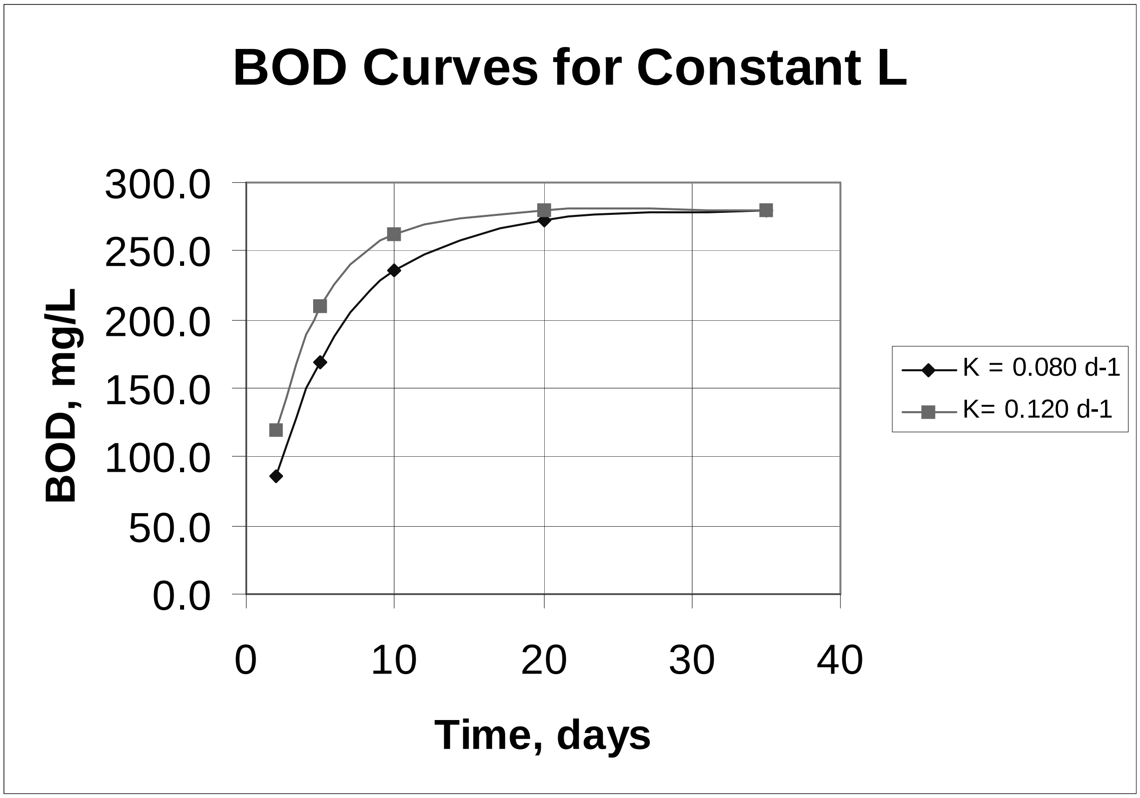


Figure S-5-17: BOD curves for constant L

5-18 Plot BOD curves and find when L occurs

Given: Data in 5-16

Solution:

Day	BOD(t) for K=0.0800 d^{-1}	BOD(t) for K=0.120 d^{-1}
	0.08	0.12
1	78.3	90.3
2	143.4	158.8
5	280.0	280.0
10	391.5	350.3
20	453.5	372.4
35	464.5	373.9

For K = 0.0800 d^{-1} L occurs at day 40 $\pm$ 5

For K = 0.120 d^{-1} L occurs at day 30 ± 5

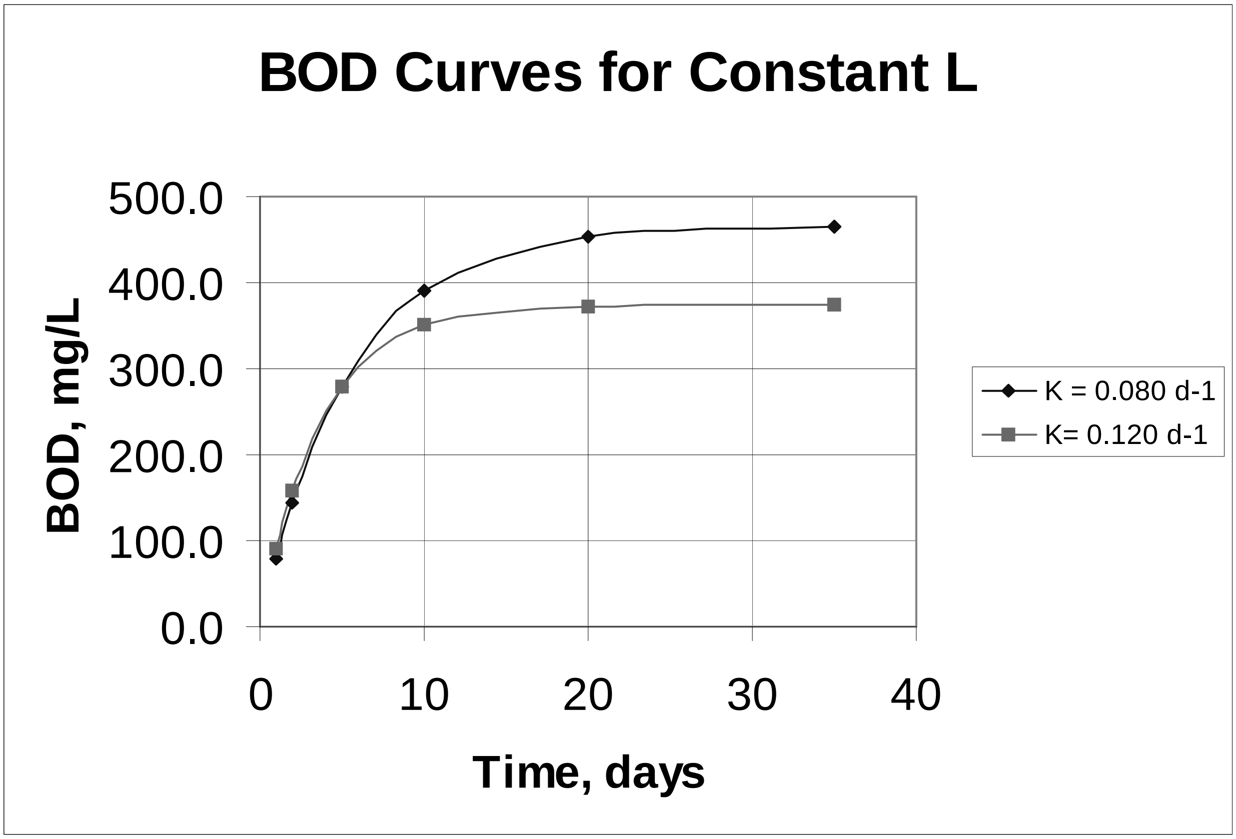


Figure S-5-18: BOD curves for constant L

5-19 Thomas' graphical method of k

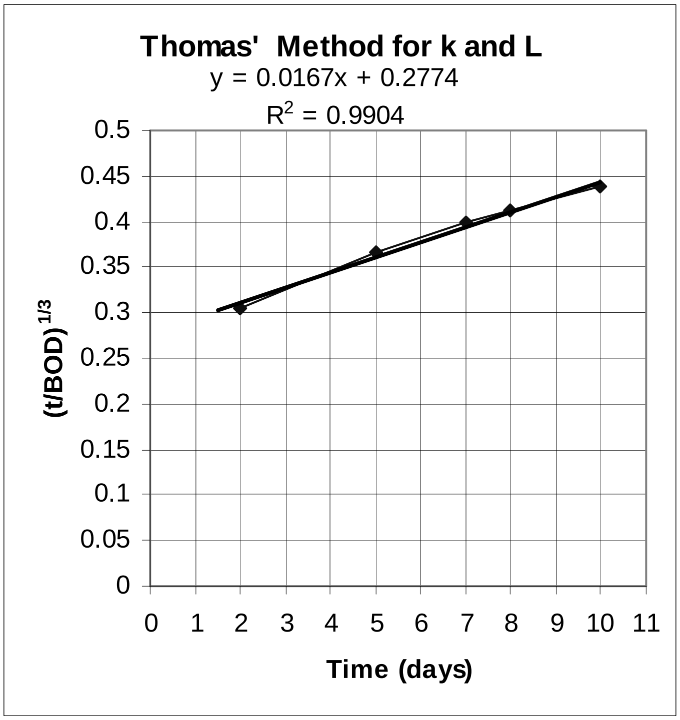
Given: Tabular data for BOD at different times

Solution:

a. Compute values of $(t/BOD_t)^{1/3}$

Day	BOD	Day	$(t/BOD)^{(1/3)}$
2	70	2	0.3057111
5	102.4	5	0.3655026
7	111	7	0.3980388
8	114	8	0.4124729
10	118.8	10	0.4382563

b. Plot the data and determine the slope and intercept



The intercept is A = 0.2774

The slope is B = 0.0167

c. Calculate the rate constant (base e)

$$k = \frac{6B}{A} = \frac{(6)(0.0167)}{0.2774} = 0.36 d^{-1}$$

5-20 Thomas' graphical method for k

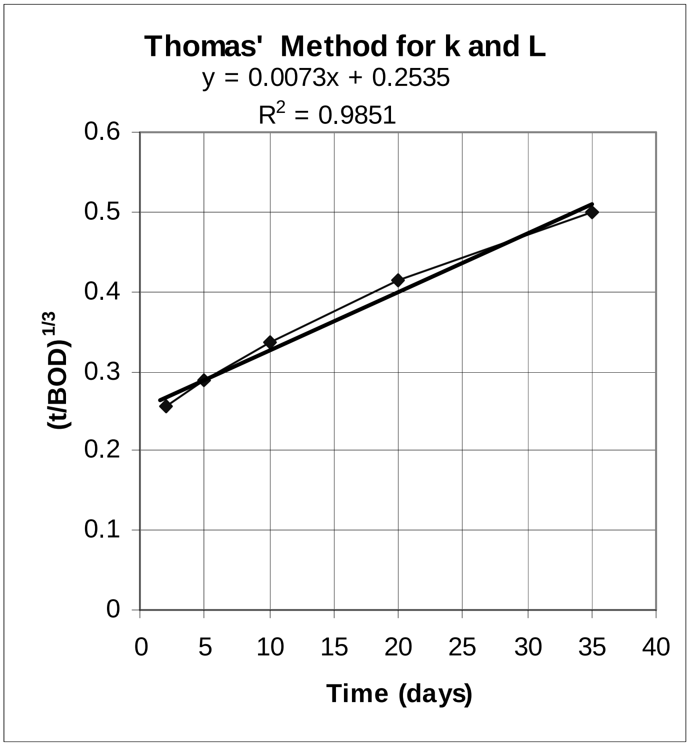
Given: Tabular data for BOD at different times

Solution:

a. Begin by computing values of $(t/BOD_t)^{1/3}$

Day	BOD	Day	$(t/BOD)^{(1/3)}$
2	119	2	0.25615
5	210	5	0.287685
10	262	10	0.336692
20	279	20	0.415409
35	279.98	35	0.500012

b. From the following find the intercept and slope



Intercept A = 0.2535

Slope B = 0.0073

c. Rate constant is

$$k = \frac{6B}{A} = \frac{(6)(0.0073)}{0.2535} = 0.17d^{-1}$$

5-21 Thomas' graphical method for k

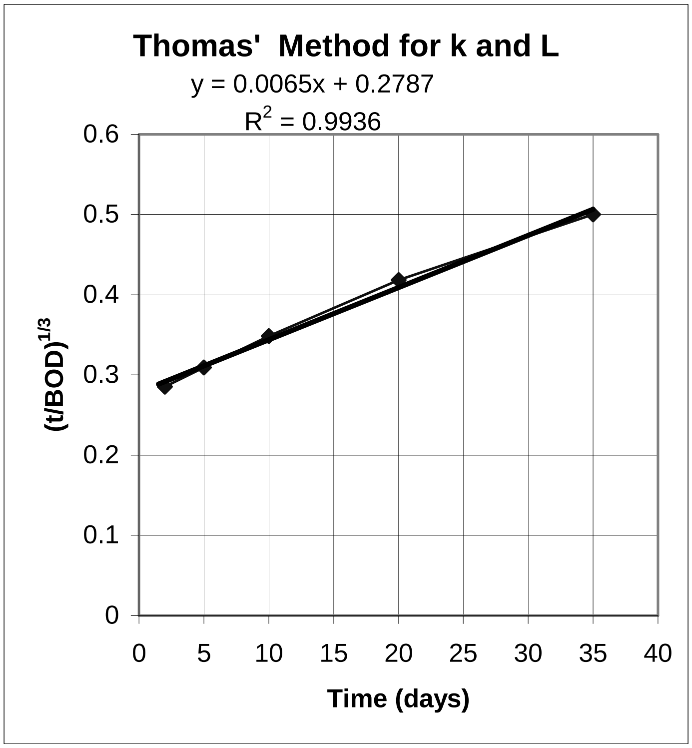
Given: Tabular data for BOD at different times

Solution:

a. Begin by computing values of $(t/BOD_t)^{1/3}$

Day	BOD	Day	$(t/BOD)^{(1/3)}$
2	86	2	0.285438
5	169	5	0.309287
10	236	10	0.348629
20	273	20	0.41843
35	279.55	35	0.500268

b. From the following plot find the intercept and slope



Intercept A = 0.2787

Slope B = 0.0065

c. Rate constant is

$$k = \frac{6B}{A} = \frac{(6)(0.0065)}{0.2787} = 0.14d^{-1}$$

5-22 Theoretical NBOD of glutamic acid

Given: Problem 5-1, 63 mg/L of glutamic acid

Solution:

a. Calculate the amount of nitrogen that is oxidized.

$$(63\,\text{mg/L})\frac{14\text{gN}}{147\text{gC}_5\text{H}_9\text{O}_4\text{N}} = 6.00\,\text{mgN/L}$$

b. Using the relationship from Eqn 5-20

Theo. NBOD = (6.00 mg N/L)(4.57 mg O_2/mg N) = 27.42 or 27 mg O_2/L

5-23 Theoretical NBOD of bacterial cells

Given: Problem 5-2, 30 mg/L of bacterial cells. (NOTE: the concentration is not specified in the first printing of the 3rd edition.)

Solution:

a. Calculate the amount of nitrogen that is oxidized.

$$(30\,\text{mg/L})\left(\frac{14\text{gN}}{113\text{gC}_5\text{H}_7\text{NO}_2}\right) = 3.72\,\text{mgN/L}$$

b. Using the relationship from Eqn 5-20

Theo. NBOD = (3.72 mg N/L)(4.57 mg O_2/mg N) = 16.99 or 17 mg O_2/L

5-24 Theoretical NBOD of casein

Given: 200 mg/L of casein ($C_8H_{12}O_3N_2$) and reactions

Solution:

a. Calculate amount of nitrogen oxidized

$$(200\,\text{mg/L})\left(\frac{28\text{gN}}{184\text{gC}_8\text{H}_{12}\text{O}_3\text{N}_2}\right) = 30.43\,\text{mgN/L}$$

b. Using the relationship from Eqn. 5-20

Theo. NBOD = (30.43 mg N/L)(4.57 mg O_2/mg N) = 139.07 or 140 mg O_2/L

5-25 Derive an expression for temperature of mixed river & waste

Given: Assume specific heat and density of river and wastewater are same

Solution:

a. Begin with Eqn. 5-27

Heat lost by wastewater = Heat gained by river

b. Using the change in enthalpy we can write

$(m_{ww})(C_p)(\Delta T) = (m_{river})(C_p)(\Delta T)$
where m_{ww} = mass of the wastewater
m_{river} = mass of the river water

c. Use the assumption that the specific heats are the same and cancel them from "b." Expand the ΔT expressions to show the final temperature (T_s) and initial temperatures.

$(m_{ww})(T_w - T_s) = (m_{river})(T_s - T_r)$

d. Multiplying through and collecting terms

$m_{ww}T_w - m_{ww}T_s = m_{river}T_s - m_{river}T_r$

$m_{ww}T_w + m_{river}T_r = m_{river}T_s + m_{ww}T_s$

$m_{ww}T_w + m_{river}T_r = (T_s)(m_{river} + m_{ww})$

e. Solve for T_s

$$T_s = \frac{m_{WW}T_{WW} + m_{river}T_r}{m_{river} + m_{WW}}$$

f. Recognizing that

$m_{ww} = (Q_w)$(density of ww)
$m_{river} = (Q_r)$(density of river water)

g. And using the assumption that the densities are equal, then

$$T_s \frac{Q_W T_W + Q_r T_r}{Q_W + Q_r}$$

5-26 Tannery initial ultimate BOD after mixing

Given: Tannery Q_w = 0.011 m^3/s, BOD_5 = 590 mg/L, Creek Q_r = 1.7 m^3/s, BOD_5 upstream of tannery = 0.6 mg/L. Tannery k = 0.115 d^{-1}, Creek k = 3.7 d^{-1}

Solution:

a. Calculate the ultimate BOD of tannery wastewater using Eqn 4-2. NOTE: k's are lower case, therefore in base e.

$$L_0 = \frac{590\,mg/L}{1-\exp[(-0.115)(5)]} = \frac{590}{1-0.56} = 1349.2\,mg/L$$

Thus, L_w (Eqn 5-26) = 1349.2 mg/L

b. Calculate the ultimate BOD of Cattaraugus Creek

$$L_0 = \frac{0.6\,mg/L}{1-\exp[(-3.7)(5)]} = \frac{0.6}{1-9.24\times10^{-9}} = 0.6\,mg/L$$

Thus, L_r (Eqn 5-26) = 0.6 mg/L

c. Calculate the initial ultimate BOD using Eqn 5-26

$$L_a = \frac{(0.011\,m^3/s)(1349.2\,mg/L)+(1.7\,m^3/s)(0.6\,mg/L)}{(0.011\,m^3/s)+(1.7\,m^3/s)} = \frac{14.84+1.02}{1.711}$$

L_a = 9.27 or 9 mg/L

5-27 Initial ultimate BOD after mixing – Cherry Creek

Given: Pittsburgh Q_W = 0.126 m^3/s; BOD_5 = 34 mg/L;
Cherry Creek Q_r = 0.126 m^3/s; BOD_5 = 1.2 mg/L;
Pittsburgh k = 0.222 d^{-1}; Cherry Creek k = 0.090 d^{-1}; Temperatures = 20 ^oC for each

Solution:

a. Calculate ultimate BOD of Pittsburgh wastewater using Eqn. 5-4. Note k is lowercase, therefore in base e.

$$L_0 = \frac{34\,\text{mg/L}}{1-\exp[(-0.222)(5)]} = \frac{34}{1-0.3296} = 50.71\,\text{mg/L}$$

Thus, L_W = 50.71 mg/L

b. Calculate ultimate BOD of Cherry Creek

$$L_0 = \frac{1.2\,\text{mg/L}}{1-\exp[(-0.090)(5)]} = \frac{1.2}{1-0.6376} = 3.31\,\text{mg/L}$$

Thus, L_r = 3.31 mg/L

c. Calculate initial ultimate BOD using Eqn. 5-26

$$L_a = \frac{(0.126\,\text{m}^3/\text{s})(50.71\,\text{mg/L}) + (0.126\,\text{m}^3/\text{s})(3.31\,\text{mg/L})}{(0.126\,\text{m}^3/\text{s}) + (0.126\,\text{m}^3/\text{s})} = \frac{6.3895 + 0.4171}{0.252}$$

L_a = 27.01 or 27 mg/L

5-28 Mixed ultimate BOD

Given: Problem 5-26; data from glue factory and municipal WWTP

Solution:

a. The mixed ultimate BOD is the weighted average of the three wastewater streams and the creek

$$L_a = \frac{(0.011)(1349.2) + (1.7)(0.6) + (0.13)(255) + (0.02)(75)}{0.011 + 1.7 + 0.13 + 0.02} = \frac{50.51}{1.86}$$

L_a = 27.14 or 30 mg/L

5-29 Initial ultimate BOD of Ambrosia River

Given: Flows and ultimate BOD's for 3 creeks

Solution:

$$L_a = \frac{(0.252)(27) + (0.13)(8) + (0.02)(16)}{0.252 + 0.13 + 0.02} = \frac{8.164}{0.402}$$

L_a = 20.31 or 20 mg/L

5-30 Deoxygenation and reaeration rate constants

Given: Table of data

Solution:

a. Calculate the deoxygenation rate constant using Eqn 5-43

$$k_d = 0.20 + \frac{0.5}{1.0}(0.4) = 0.40 d^{-1}$$

b. Calculate the reaeration rate constant using Eqn 5-44

$$k_r = \frac{3.9(0.5)^{0.5}}{(1.0)^{1.5}} = \frac{3.9(0.71)}{1.0} = 2.76 d^{-1}$$

5-31 k_d and k_r for flood conditions

Given: Wastewater k = 0.20 d^{-1} at 20 ^oC and stream flood conditions

Solution:

a. Calculate temperature of mixture of stream and wastewater using Eqn. 5-30.

$$T_f = \frac{(0.126\, m^3/s)(293K) + (0.252\, m^3/s)(283K)}{0.126 + 0.252} = \frac{108.23}{0.3780} = 286.33K \text{ or } 13.33\ ^oC$$

b. Calculate the deoxygenation rate constant at 20 ^oC

$$k_d = 0.20 + \frac{2.5}{4.0}(0.6) = 0.58 d^{-1}$$

c. Correct the deoxygenation rate constant to 13.33 ^oC using Eqn. 5-6

$$k_d \text{at} 13.33°C = (0.58)(1.135)^{13.33-20} = 0.25 d^{-1}$$

d. Calculate the rearating rate constant at 20 ^oC

$$k_r = \frac{3.9(2.5)^{0.5}}{(4.0)^{1.5}} = 0.77 d^{-1}$$

e. Correct the reaeration rate to 13.33 ^oC using Eqn. 5-6. Note that θ = 1.024 as discussed below Eqn. 5-44.

$$k_r \text{at} 13.33°C = (0.77)(1.024)^{13.33-20} = 0.66 d^{-1}$$

5-32 Critical point and critical DO

Given: L_a = 50 mg/L; DO = saturation; river temp = 10 °C; k_d = 0.30 d^{-1}; k_r = 0.30 d^{-1}

Solution:

a. Since the DO in the river is at saturation after the wastewater and river have mixed, the initial deficit (D_a) is 0.0 mg/L.

b.Note that this is a special case of k_d = k_r so Eqn 5-46 applies.

$$t_c = \frac{1}{0.30}\left(1 - \frac{0.0}{50}\right) = 3.33\text{d}$$

c. The critical deficit is found using Eqn 5-42 with t = t_c

$$D = [(0.30)(3.33)(50) + 0]\exp[(-0.30)(3.33)] = 18.41\,\text{mg/L}$$

d. The critical DO is found by solving Eqn 5-31 for DO with DO_s = 11.33 mg/L (Table A-2 at 10 °C)

DO = DO_s - D

DO = 11.33 -18.41 = -7.08 mg/L

Since the Do cannot be negative, the critical DO = 0.0 mg/L.

5-33 Critical point and critical DO

Given: Problem 5-32 and new temperatures

Solution:

a. Calculate the new k_d using Eqn 5-6 with θ = 1.135

Calculate k_d at 20 °C

$$k_{20} = \frac{k_{10}}{(\theta)^{10-20}} = \frac{0.30}{(1.135)^{-10}} = \frac{0.30}{0.2819} = 1.0643\text{d}^{-1}$$

Calculate k_d at 15 °C

$$k_d = k_{15} = 1.0643(1.135)^{15-20} = (1.0643)(0.5309) = 0.5651\text{d}^{-1}$$

b. Calculate new k_r using Eqn 4-5 with θ = 1.024

Calculate k_r at 20 °C

$$k_{20} = \frac{k_{10}}{(\theta)^{10-20}} = \frac{0.30}{(1.024)^{-10}} = \frac{0.30}{0.7889} = 0.3803\text{d}^{-1}$$

Calculate k_r at 15 °C

$k_r = k_{15} = 0.3803(1.024)^{15 - 20} = (0.3803)(0.8882) = 0.3378 \text{ d}^{-1}$

c. Calculate the critical time using Eqn 5-45. Note: since the DO is at saturation, D_a = 0.0 mg/L

$$t_c = \frac{1}{0.3378 - 0.5651} \ln\left[\frac{0.3378}{0.5651}\left(1 - (0)\frac{0.3378 - 0.5651}{(0.5651)(50)}\right)\right]$$

t_c = (-4.399)ln[0.5978] = 2.264

d. The critical deficit is found using Eqn 5-41 with t = 2.264

$$D_c = \frac{(0.5651)(50)}{(0.3378 - 0.5651)}\left[\exp((-0.5651)(2.264)) - \exp((-0.3378)(2.264))\right] + 0$$

D_c = 23.27 mg/L

e. The critical DO is found by solving Eqn 5-31 for DO with DO_s = 10.15 mg/L Appendix A-2 at 15 °C

DO = 10.15 - 23.27 = - 13.12 or 0.0 mg/L since DO cannot be negative

5-34 Comparison of reaeration equations

Given: Churchill, Elemore and Buckingham equation; Eqn. 5-44; depth = 1 m; velocity values

Solution:

a. Calculate k_r using Eqn. 5-44 and the equation given in problem for the given velocities

v (m/s)	k_r (d^{-1})	k_r (d^{-1})
0.05	0.872067	0.2615
0.1	1.233288	0.523
0.2	1.744133	1.046
0.4	2.466577	2.092

b. Plot k_r values versus speed

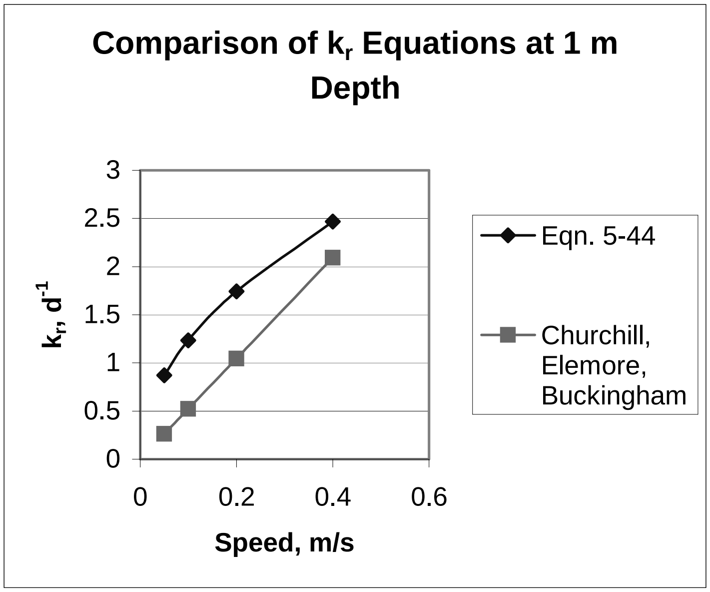


Figure S-5-34: Comparison of k_r equations

c. Discuss possible reasons for the difference in results

O’Connor and Dobbins equation is based on molecular diffusion. The Churchill, Elemore and Buckingham equation is based on field measurements.

5-35 DO if L_w is reduced 50%

Given: DO at critical point is 4.0 mg/L; L_r = 0.0 mg/L; D_a = 0.0 mg/L; no change in Q_w or Q_r ; DO_s = 10.83 mg/L; L_w is reduced 50%.

Solution:

a. Calculate D_c for the case with DO = 4.0 mg/L from Eqn 5-31

$$D_c = DO_s - DO$$

$$D_c = 10.83 - 4.0 = 6.83 \text{ mg/L}$$

b. From Eqn 5-26, with L_r = 0.0 mg/L, find

$$L_a = \left(\frac{Q_W}{Q_W + Q_r}\right) L_W$$

Therefore, if L_w is reduced 50%, then L_a is reduced 50%.

c. By observation of Eqn 5-45, with D_a = 0.0 mg/L, the value for t_c does not change with change in L_a because D_a = 0.0 mg/L.

d. The value of D_c from Eqn 5-41 is directly proportional to L_w, i.e. if L_w is 0.50 of the initial value, L_a is 0.50 of the initial value and D_c is 0.50 of the initial value. Then

D_c =(0.50)(6.83) = 3.415 mg/L

e. The DO in the river is then

DO = 10.83 - 3.415 = 7.415 mg/L

5-36 DO at Alittlebit and at critical point

Given: Table of data for wastewater and river; Alittlebit = 5.79 km downstream; standard = 5.00 mg/L; NOTE: K_d and K_r are given in base 10.

Solution:

Part I - DO at Alittlebit

a. Calculate initial ultimate BOD with Eqn. 5-26; Q_w = 0.280 m^3/s; L_w = 6.44 mg/L; Q_r = 0.877 m^3/s; L_r = 7.00 mg/L

$$L_a = \frac{(0.280)(6.44) + (0.877)(7.00)}{0.280 + 0.877} = 6.8645 \text{ mg/L}$$

b. From Appendix A, Table A-2 find DO_s = 7.92 mg/L at 28 ^{0}C

c. Calculate initial deficit with Eqn. 5-32; Q_w = 0.280 m^3/s; DO_w = 1.00 mg/L; Q_r = 0.877 m^3/s; DO_r = 6.00 mg/L

$$D_a = 7.92 - \left(\frac{(0.280)(1.00) + (0.877)(6.00)}{0.280 + 0.877}\right) = 3.13 \text{ mg/L}$$

d. Calculate travel time from Big Bear to Alittlebit

$$t = \frac{(5.79\,\text{km})(1000\,\text{m/km})}{(0.650\,\text{m/s})(86400\,\text{s/d})} = 0.1031\text{d}$$

e. Calculate deficit (base 10 because K's are in base 10)

$$D = \frac{(0.199)(6.8645)}{0.370 - 0.199}\left(10^{-(0.199)(0.1031)} - 10^{-(0.370)(0.1031)}\right) + 3.13\left(10^{-(0.370)(0.1031)}\right)$$

$$D = \frac{1.3660}{0.171}(0.9539 - 0.9159) + 3.13(0.9159) = 7.9885(0.03795) + 2.8668 = 3.17$$

f. Calculate DO with Eqn. 5-31 and DO_s = 7.92 mg/L

DO = 7.92 - 3.17 = 4.75 mg/L at Alittlebit

Part II - Critical DO

a. Critical time (t_c)

$$t_c = \frac{1}{0.370 - 0.199}\log\left[\frac{0.370}{0.199}\left(1 - 3.13\left(\frac{0.370 - 0.199}{(0.199)(6.8645)}\right)\right)\right]$$

$$t_c = \frac{1}{0.171}\log[1.8593(1 - 3.13(0.125))] = 0.3141\text{d}$$

b. Calculate critical deficit and DO

$$D = 7.9885\left(10^{-(0.199)(0.3141)} - 10^{-(0.370)(0.3141)}\right) + 3.13\left(10^{-(0.370)(0.3141)}\right)$$

D_c = 7.9885 (0.86595 - 0.76521) + 3.13 (0.76521)

D_c = 7.9885 (0.10074) + 2.395 = 3.19988 or 3.20

DO = 7.92 - 3.20 = 4.72 mg/L

DO is reduced below standard of 5.00 mg/L

5-37 DO at Alittlebit if river temperature drops to 12 ^{0}C

Given: Data in Problem 5-36 and new river temperature

Solution:

a. The initial ultimate BOD and travel time are the same as in Problem 5-36: L_a =6.8645; t = 0.1031 d.

b. The initial deficit changes because the saturation temperature changes. From Appendix A, Table A-2 find DO_s = 10.83 mg/L at 12 ^{0}C

c. Calculate initial deficit with Eqn. 5-32; Q_w = 0.280 m^3/s; DO_w = 1.00 mg/L; Q_r = 0.877 m^3/s; DO_r = 6.00 mg/L

$$D_a = 10.83 - \left(\frac{(0.280)(1.00)+(0.877)(6.00)}{0.280+0.877}\right) = 10.83 - 4.790 = 6.04\,\text{mg/L}$$

d. Calculate temperature of mixed river water and wastewater.

$$T_f = \frac{(28)(0.280)+(12)(0.877)}{0.280+0.877} = 15.87°\text{C}$$

e. Correct K_d and K_r using Eqn. 5-6

For K_d

$0.199 = K_{20}(1.056)^{28 - 20}$

$0.199 = K_{20}(1.5464)$

$K_{20} = 0.128689$

At 15.87 ^{0}C

$K_d = 0.128689(1.135)^{15.87 - 20}$

$K_d = 0.128689(0.5927) = 0.07628\ d^{-1}$

For K_r

$0.370 = K_r(1.024)^{28 - 15.87}$

$0.370 = K_r(1.3333)$

$K_r = 0.2775\ d^{-1}$

f. Calculate deficit at Alittlebit (base 10 because K's are in base 10)

$$D = \frac{(0.07628)(6.8645)}{0.2775 - 0.07628}\left(10^{-(0.07628)(0.1031)} - 10^{-(0.2775)(0.1031)}\right) + 6.04\left(10^{-(0.2775)(0.1031)}\right)$$

$$D = \frac{0.5236}{0.2012}(0.9821 - 0.9362) + 6.04(0.9362)$$

D = 2.6024(0.045809) + 5.6546 = 5.774 or 5.77 mg/L

g. Calculate DO with Eqn. 5-31 and DO_s = 10.83 mg/L

DO = 10.83 - 5.77 = 5.06 mg/L at Alittlebit

5-38 DO with ultimate BOD given in kg/d

Given: Tabular data with L_a = 1100 kg/d; NOTE: rate constants are in base e NOT base 10

Solution:

a. Calculate travel time

$$t = \frac{(1.609\text{km})(1000\,\text{m/km})}{(0.100\,\text{m/s})(86400\,\text{s/d})} = 0.186\text{d}$$

b. Convert kg/d to mg/L (See Example 5-6.)

$$L_a = \frac{(1100\,\text{kg/d})(10^6\ \text{mg/kg})}{(2.40\,\text{m}^3/\text{s})(86400\,\text{s/d})} = 5.31\text{mg/L}$$

c. Calculate deficit

$$D = \frac{(1.911)(5.31)}{4.49 - 1.911}\left(e^{-(1.199)(0.186)} - e^{-(4.49)(0.186)}\right) + 0$$

D = 3.93(0.7009 - 0.4338) = 1.046 mg/L

d. Calculate DO

From Table in Appendix at 17 ^{0}C find DO_s = 9.74 mg/L

DO = 9.74 - 1.046 = 8.69 mg/L

5-39 DO at 2.880 km from waste discharge

Given: Table of data

Solution:

a. Calculate travel time

$$t = \frac{(2.880\text{km})(1000\,\text{m/km})}{(0.100\,\text{m/s})(86400\,\text{s/d})} = 0.333\text{d}$$

b. Calculate L_a in mg/L

$$L_a = \frac{(1125\,\text{kg/d})(10^6\ \text{mg/kg})}{(0.300\,\text{m}^3/\text{s})(86400\,\text{s/d})(1000\,\text{L/m}^3)} = 43.40\,\text{mg/L}$$

c. Calculate D. Note capital K, thus base 10.

$$D = \frac{(1.839)(43.40)}{2.030 - 1.830}\left(10^{-(1.830)(0.333)} - 10^{-(2.030)(0.333)}\right) + 0 = 13.94\,\text{mg/L}$$

d. Calculate DO. From Table A-2, DO_S at 18°C = 9.54 mg/L

DO = 9.54 – 13.94 = -4.40 mg/L

e. Cannot have a negative DO. Thus,

DO = 0.00 mg/L

5-40 DO at Avepitaeonmi

Given: Table of data; Avepitaeonmi is 15.55 km downstream; standard = 5.00 mg/L

Solution:

Part I. DO at Avepitaeonmi

a. Calculate t

$$t = \frac{(15.55\text{km})(1000\,\text{m/km})}{(0.390\,\text{m/s})(86400\,\text{s/d})} = 0.462\text{d}$$

b. Calculate k_d (base e)

$$k_d = 0.4375 + \frac{0.390}{2.80}(0.200)$$

$k_d = 0.4375 + 0.02785 = 0.4654$ at 20 ^{0}C

At 16 ^{0}C

$k_d = 0.4654\ (1.135)^{16 - 20}$

$k_d = 0.4654\ (0.6026) = 0.2805\ d^{-1}$

c. Calculate k_r

$$k_r = \frac{3.9(0.390)^{0.5}}{(2.80)^{1.5}} = 0.5212 \text{ at } 20\ ^0\text{C}$$

At 16 ^{0}C

$k_r = 0.5212\ (1.024)^{16 - 20}$

$k_r = 0.5212\ (0.9095) = 0.474\ d^{-1}$

d. Calculate D_a

From Table A-2 @ 16 ^{0}C $DO_s = 9.95$ mg/L

$$D_a = 9.95 - \left(\frac{(0.1507)(1.00) + (1.08)(7.95)}{0.1507 + 1.08}\right) = 2.85\,\text{mg/L}$$

e. Calculate L_w

$$L_W = \frac{BOD_t}{\left(1 - e^{-(k)(t)}\right)} = \frac{128}{\left(1 - e^{-(0.4375)(5)}\right)} = 144.17\,\text{mg/L}$$

f. Calculate L_a from Eqn. 5-26

$$L_a = \frac{(0.1507)(144.17) + (1.08)(11.40)}{0.1507 + 1.08} = 27.65\,\text{mg/L}$$

g. Calculate Deficit

$$D = \frac{(0.2805)(27.65)}{0.474 - 0.2805}\left(e^{-(0.2805)(0.462)} - e^{-(0.474)(0.462)}\right) + 2.85\left(e^{-(0.474)(0.462)}\right)$$

$$D = \frac{7.7558}{0.1935}(0.8785 - 0.8033) + 2.85(0.8033) = 5.30\,\text{mg/L}$$

g. Calculate DO with Eqn. 5-31 and DO_s = 9.95 mg/L

DO = 9.95 - 5.30 = 4.65 mg/L at Avepitaeonmi

Part II. Critical DO

a. Calculate t_c

$$t_c = \frac{1}{0.474 - 0.2805}\ln\left[\frac{0.474}{0.2805}\left(1 - 2.85\left(\frac{0.474 - 0.2805}{(0.2805)(27.65)}\right)\right)\right]$$

t_c = 5.168 ln [1.6898 (0.9288)] = 2.33 d

b. Calculate D_c

$$D_c = \frac{(0.2805)(27.65)}{0.474 - 0.2805}\left(e^{-(0.2805)(2.33)} - e^{-(0.474)(2.33)}\right) + 2.85\left(e^{-(0.474)(2.33)}\right)$$

$$D_c = \frac{7.7558}{0.1935}(0.520188 - 0.3314) + 2.85(0.3314) = 8.51\,\text{mg/L}$$

c. Calculate DO with Eqn. 5-31 and DO_s = 9.95 mg/L

DO = 9.95 - 8.51 = 1.44 mg/L at critical point

d. Distance to Critical Point is the product of the speed of the stream and the travel time

x = (v)(t) = (0.390 m/s)(86,400 s/d)(2.33 d)(1.0 x 10^{-3} km/m)

x = 78.5 km downstream from Watapitae

e. The assimilative capacity is restricted.

5-41 DO at Avepitaeonmi if wastewater discharge is 30 mg/L

Given: Data in Problem 5-40 but BOD_5 of Watapitae wastewater is reduced to 30.00 mg/L

Solution:

Part I. DO at Avepitaeonmi

a. From Problem 4-29 the following remain the same:
$t = 0.462$ d; $k_d = 0.2805\ d^{-1}$; $k_r = 0.474\ d^{-1}$; $D_a = 2.85$ mg/L

b. Recalculate L_w

$$L_W = \frac{BOD_t}{\left(1 - e^{-(k)(t)}\right)} = \frac{30.00}{\left(1 - e^{-(0.4375)(5)}\right)} = 33.79\ mg/L$$

c. Calculate L_a from Eqn. 5-26

$$L_a = \frac{(0.1507)(33.79) + (1.08)(11.40)}{0.1507 + 1.08} = 14.14\ mg/L$$

d. Calculate Deficit

$$D = \frac{(0.2805)(14.14)}{0.474 - 0.2805}\left(e^{-(0.2805)(0.462)} - e^{-(0.474)(0.462)}\right) + 2.85\left(e^{-(0.474)(0.462)}\right)$$

$$D = \frac{3.966}{0.1935}(0.8785 - 0.8033) + 2.85(0.8033) = 3.83\ mg/L$$

e. Calculate DO with Eqn. 5-31 and $DO_s = 9.95$ mg/L

DO = 9.95 - 3.83 = 6.12 mg/L at Avepitaeonmi

Part II. Critical DO

a. Calculate t_c

$$t_c = \frac{1}{0.474 - 0.2805}\ln\left[\frac{0.474}{0.2805}\left(1 - 2.85\left(\frac{0.474 - 0.2805}{(0.2805)(14.14)}\right)\right)\right]$$

$t_c = 5.168 \ln [1.6898 (0.8610)] = 1.937$ d

b. Calculate D_c

$$D = \frac{(0.2805)(14.14)}{0.474 - 0.2805}\left(e^{-(0.2805)(1.937)} - e^{-(0.474)(1.937)}\right) + 2.85\left(e^{-(0.474)(1.937)}\right)$$

$$D = \frac{3.966}{0.1935}(0.580732 - 0.399169) + 2.85(0.399169) = 4.86\,\text{mg/L}$$

c. Calculate DO with Eqn. 5-31 and DO_s = 9.95 mg/L

DO = 9.95 - 4.86 = 5.09 mg/L at critical point

d. Distance to Critical Point

x = (v)(t) = (0.390 m/s)(86 400 s/d)(1.937 d)(1.0 x 10^{-3} km/m)

x = 65.27 km downstream from Watapitae

e. The assimilative capacity is restricted but much less so at Avepitaeonmi (6.12 - 5.00 = 1.12 mg/L above standard).

5-42 Blue Ox DO

Given: Table of data; DNR river DO = 5.00 mg/L

Solution:

a. Calculate L_W

$$L_W = \frac{BOD_t}{1 - e^{-kt}} = \frac{90}{1 - e^{-(0.3685)(5)}} = 106.94\,\text{mg/L}$$

Note: L_W is calculated at 20°C, not at stream temperature.

b. Calculate L_a from Eqn. 5-26

$$L_a = \frac{Q_W L_W + Q_r L_r}{Q_W + Q_r} = \frac{(1.148)(106.94) + (7.222)(7.66)}{1.148 + 7.222} = 21.28\,\text{mg/L}$$

c. Calculate D_a using Table A-2

$D_a = DO_{sat} - DO$

$$DO = \frac{DO_W Q_W + DO_r Q_r}{Q_W + Q_r} = \frac{(1)(1.148) + (6)(7.222)}{1.148 + 7.222} = 5.31$$

DO_{sat} = 10.15mg/L (from Table A-2)

Da = 10.15 – 5.31 = 4.84 mg/L

d. Calculate k_d

$$k_d(20°C) = 0.3685 + \frac{0.3}{2.92}(0.1) = 0.3788 d^{-1}$$

$$k_d(15°C) = 0.3788(1.135)^{15-20} = 0.2011 d^{-1}$$

e. Calculate k_r

$$k_r(20°C) = \frac{3.9(0.3)^{0.5}}{(2.92)^{1.5}} = 0.4281 d^{-1}$$

$$k_r(15°C) = 0.4281(1.024)^{15-20} = 0.3802 d^{-1}$$

f. Calculate t_c

$$t_c = \frac{1}{0.3802 - 0.2011} \ln\left[\frac{0.3802}{0.2011}\left(1 - 4.84\left(\frac{0.3802 - 0.2011}{(0.2011)(21.28)}\right)\right)\right] = 2.293 d$$

g. Calculate D_c

$$D_c = \frac{(0.2011)(21.28)}{0.3802 - 0.2011}\left(e^{-(0.2011)(2.293)} - e^{-(0.3802)(2.293)}\right) + 4.84\left(e^{-(0.3802)(2.293)}\right)$$

D_c = 7.096 mg/L

h. Calculate DO_c

$DO_c = DO_{sat} - D_c$ = 10.15 – 7.096 = 3.054 mg/L

i. Calculate distance to critical point

x = (v)(t) = (0.30 m/s)(86400 s/d)(2.293 d)(1.0 x 10^{-3} km/m) = 59.44 km

j. See example spreadsheet on following page

k. Plot DO vs. Time

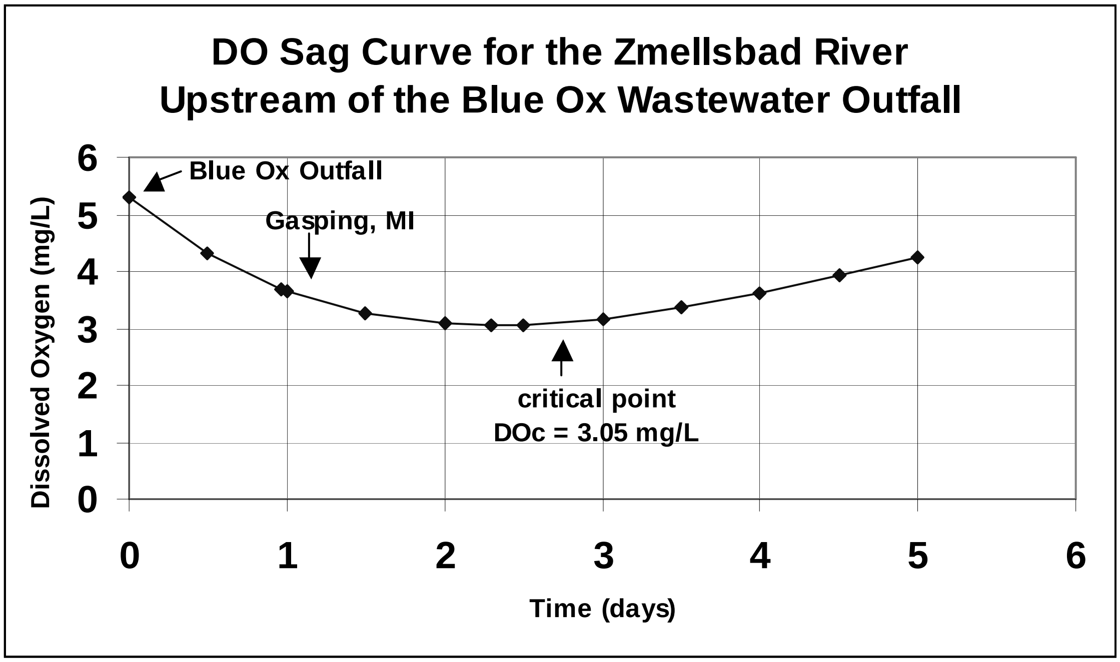


Figure S-5-42: DO sag curve for Zmellsbad River

Input Data

BOD_5 of Waste	90	mg/L
Rate Const (k)	0.3685	d^{-1} at 20 ^oC
Q_w	1.148	m^3/s
DO_w	1	mg/L
L_r	7.66	mg/L
Q_r	7.222	m^3/s
DO_r	6	mg/L
T_r	15	^oC
DO_{sat}	10.15	mg/L
River Speed	0.3	m/s
River Depth	2.92	m
η	0.1	
Distance	25.11	km

Calculation of Ultimate BOD of Waste	L_W =	106.9418	mg/L
Mixed Ultimate BOD	L_a=	21.27715	mg/L
DO after mixing	DO=	5.314217	mg/L
Initial Mixed Deficit	D_a=	4.835783	mg/L
Deoxygenation rate constant at 20 °C	k_d=	0.378774	d^{-1} at 20 °C
Deoxygenation rate constant at 15 °C	k_d (at 15°C)=	0.201095	d^{-1} at 15 °C
Rearation rate constant at 20 °C	k_r (at 20°C)=	0.428106	d^{-1} at 20 °C
Rearation rate constant at 15 °C	k_r (at 15°C)=	0.380234	d^{-1} at 15 °C

	Time (d)	kr-kd	k_d*L_a/(k_r-k_d)	kd*t	kr*t	D (mg/L)	DO (mg/L)
	0.96875	0.1791	23.8849	0.1948	0.3684	6.4775	3.6725
Blue Ox Outfall →	0	0.1791	23.8849	0.0000	0.0000	4.8358	5.3142
	0.5	0.1791	23.8849	0.1005	0.1901	5.8491	4.3009
Gasping →	0.96875	0.1791	23.8849	0.1948	0.3684	6.4775	3.6725
	1	0.1791	23.8849	0.2011	0.3802	6.5100	3.6400
	1.5	0.1791	23.8849	0.3016	0.5704	6.8964	3.2536
	2	0.1791	23.8849	0.4022	0.7605	7.0710	3.0790
critical point →	2.2931	0.1791	23.8849	0.4611	0.8719	7.0957	3.0543
	2.5	0.1791	23.8849	0.5027	0.9506	7.0846	3.0654
	3	0.1791	23.8849	0.6033	1.1407	6.9773	3.1727
X_c (km) = 59.44	3.5	0.1791	23.8849	0.7038	1.3308	6.7816	3.3684
	4	0.1791	23.8849	0.8044	1.5209	6.5229	3.6271
	4.5	0.1791	23.8849	0.9049	1.7111	6.2214	3.9286
	5	0.1791	23.8849	1.0055	1.9012	5.8930	4.2570

5-43 Blue Ox ultimate BOD

Given: Problem 4-42; DEQ criteria of critical DO must be above 5.00 mg/L; hint in problem statement

Solution:

a. Set-up spreadsheet (continued on following page)

Input Data

BOD_5 of Waste	34.44	mg/L
Rate Const (k)	0.3685	d^{-1} at 20 ^oC
Q_w	1.148	m^3/s
DO_w	1	mg/L
L_r	7.66	mg/L
Q_r	7.222	m^3/s
DO_r	6	mg/L
T_r	15	^oC
DO_{sat}	10.15	mg/L
River Speed	0.3	m/s
River Depth	2.92	m
η	0.1	
Distance	25.11	km

Calculation of Ultimate BOD of Waste	L_W =	40.92307	mg/L	=	4059	kg/d
Mixed Ultimate BOD	L_a=	12.22225	mg/L			
DO after mixing	DO=	5.314217	mg/L			
Initial Mixed Deficit	D_a=	4.835783	mg/L			
Deoxygenation rate constant at 20 ^oC	k_d=	0.378774	d^{-1} at 20 ^oC			
Deoxygenation rate constant at 15 ^oC	k_d (at 15^{o}C)=	0.201095	d^{-1} at 15 ^oC			
Rearation rate constant at 20 ^oC	k_r (at 20^{o}C)=	0.428106	d^{-1} at 20 ^oC			
Rearation rate constant at 15 ^oC	k_r (at 15^{o}C)=	0.380234	d^{-1} at 15 ^oC			

Time (d)	kr-kd	k_d*L_a/(k_r-k_d)	kd*t	kr*t	D	DO
0.9688	0.1791	13.7202	0.1948	0.3684	5.1447	5.0053
0	0.1791	13.7202	0.0000	0.0000	4.8358	5.3142
0.5	0.1791	13.7202	0.1005	0.1901	5.0616	5.0884
0.96875	0.1791	13.7202	0.1948	0.3684	5.1447	5.0053
1	0.1791	13.7202	0.2011	0.3802	5.1466	5.0034
critical point → 1.1301	0.1791	13.7202	0.2273	0.4297	5.1500	5.0000
1.5	0.1791	13.7202	0.3016	0.5704	5.1249	5.0251
2	0.1791	13.7202	0.4022	0.7605	5.0238	5.1262
2.5	0.1791	13.7202	0.5027	0.9506	4.8650	5.2850
3	0.1791	13.7202	0.6033	1.1407	4.6657	5.4843
3.5	0.1791	13.7202	0.7038	1.3308	4.4394	5.7106
4	0.1791	13.7202	0.8044	1.5209	4.1966	5.9534
4.5	0.1791	13.7202	0.9049	1.7111	3.9456	6.2044
5	0.1791	13.7202	1.0055	1.9012	3.6926	6.4574

b. Answer: L_W = 40.92 mg/L as seen in above spreadsheet

c. Convert to kg/d

$$L_W = 40.92\,\text{mg/L}\left(\frac{\text{kg}}{10^6\,\text{mg}}\right)\left(\frac{1000\text{L}}{\text{m}^3}\right)(1.148\,\text{m}^3/\text{s})(86400\,\text{s/d}) = 4059\,\text{kg/d}$$

4059 kg/d of BOD may be discharged to keep the DO above the DEQ water quality criteria of 5.00 mg/L at the critical point.

5-44 Population growth at Watapitae

Given: Data from Problems 5-40 and 5-41; 5% per year population growth with corresponding increase in wastewater discharge; BOD_5 = 30.00 mg/L

Solution:

a. The critical DO of 5.09 mg/L (See solution to Problem 5-41) leaves little room for growth, i.e. 5.09 - 5.00 = 0.09 mg/L. Because the deficit is a function of t_c which, in turn, is a function of L_a and, in turn, a function of Q_w, the analytical solution is not trivial. This problem was solved by trial and error using a spreadsheet program.

By trial and error Q_{wmax} = 0.164 m^3/s

Then $Q_{wmax} = Q_w(1 + i)^n$

where i = growth rate and n = years

Solving for n

$0.164 = 0.1507(1 + 0.05)^n$

$1.088 = (1.05)^n$

log(1.088) = n(log(1.05)
0.0367 = n(0.0211)

n = 1.733 year's growth until secondary treatment becomes inadequate to maintain DO above 5.00 mg/L at critical point.

b. A variation of this problem is to define the constraint at Avepitaeonmi, i.e. 6.12 - 5.00 = 1.12 mg/L capacity available. This allows for an analytical solution since the travel time is fixed. This problem was solved by trial and error using a spreadsheet program.

By trial and error $Q_{wmax} = 0.383\ m^3/s$

Then $Q_{wmax} = Q_w(1 + i)^n$

where i = growth rate and n = years

Solving for n

$0.383 = 0.1507(1 + 0.05)^n$

$2.5415 = (1.05)^n$

log(2.5415) = n(log(1.05)

0.40509 = n(0.0211)

n = 19.12 year's growth

until secondary treatment becomes inadequate to maintain DO above 5.00 mg/L at Avepitaeonmi.

5-45 Rework 5-40 with ice cover

Given: Problem 5-40; reaeration = 0; $t_r = 2\ ^0C$

Solution:

Part I. DO at Avepitaeonmi

a. Calculate t

$$t = \frac{(15.55\text{km})(1000\,\text{m/km})}{(0.390\,\text{m/s})(86400\,\text{s/d})} = 0.462\text{d}$$

b. Calculate temperature of mixed wastewater and river water

$$T_f = \frac{(0.1507)(2) + (1.08)(16)}{0.1507 + 1.08} = 3.7°\text{C}$$

c. Calculate k_d (base e)

$$k_d(20°\text{C}) = 0.4375 + \frac{0.390}{2.80}(0.200) = 0.4654\text{d}^{-1}$$

$$k_d(3.7°\text{C}) = 0.4654(1.135)^{3.7-20} = 0.0590\text{d}^{-1}$$

d. Calculate k_r

$k_r = 0.0$

e. Calculate D_a

From Table A-2 @ 3.7 ^{0}C DO_s by interpolation = 13.24 mg/L

$$D_a = 13.24 - \frac{(0.1507)(1.00) + (1.08)(7.95)}{0.1507 + 1.08} = 6.14\,\text{mg/L}$$

f. Calculate L_w

$$L_W = \frac{BOD_t}{(1 - e^{-kt})} = \frac{128}{(1 - e^{-(0.4375)(5)})} = 144.17\,\text{mg/L}$$

g. Calculate L_a from Eqn. 5-26

$$L_a = \frac{(0.1507)(144.17) + (1.08)(11.40)}{0.1507 + 1.08} = 27.65\,\text{mg/L}$$

h. Calculate Deficit

Two methods of solution are possible. One is to apply Eqn. 4-40 directly. Alternatively, we note that without reaeration, the deficit is directly proportional to the rate of BOD exertion (Eqn. 4-34) so that:

$$\frac{dD}{dt} = \frac{d(BOD)}{dt} = \frac{dL_t}{dt}$$

And we know from Eqn. 5-2 that

$$\frac{dL_t}{dt} = -kL_t$$

with k = deoxygenation rate = k_d, so that the deficit is equal to BOD exerted, i.e $D = L_a - L_t$. Using the integrated form as in the derivation for Eqn. 5-2

$L_t = L_a\exp(-k_d t)$

$L_t = 27.65 \exp\{(-0.0590)(0.462)\} = 26.91$ mg/L

And the deficit would be

$D = L_a - L_t + D_a$

D = 27.65 - 26.91 + 6.14 = 6.88 mg/L

i. The DO would be

DO = 13.24 - 6.88 = 6.36 mg/L

Part II. DO at Critical Point

Because there is no competition between aeration and deoxygenation, there is no critical point. However, there are two other points of interest: (1) the point where DO = 5.00 mg/L and (2) the point where DO = 0.0 mg/L.

a. Solving for point where DO = 5.00 mg/L

$DO = 5.00 = DO_s - D = 13.24 - D$
D = 13.24 - 5.00 = 8.24 mg/L

and from above

$D = L_a - L_t + D_a$

$8.24 = 27.65 - L_t + 6.14$
$L_t = 25.55$

Using Eqn. 5-2 to find t

$L_t = L_a \exp(-k_d t)$

$25.55 = 27.65 \exp\{(-0.0590)(t)\}$

$0.924 = \exp\{(-0.0590)(t)\}$

$\ln(0.924) = \ln(\exp\{(-0.0590)(t)\})$

$- 0.078988 = - 0.0590(t)$

$t = 1.33879$ d

The distance to this point is then the product of the stream speed and the travel time

$x = (v)(t) = (0.390 \text{ m/s})(1.338789 \text{ d})(86.400 \text{ s/d})(1 \times 10^{-3} \text{ km/m}) = 45.11 \text{ km}$

b. Solving for point where DO = 0.00 mg/L

$DO = 0.00 = DO_s - D = 13.24 - D$

$D = 13.24 - 0.00 = 13.24$
and from above

$D = L_a - L_t + D_a$

$13.24 = 27.65 - L_t + 6.14$

$L_t = 20.55$

Using Eqn. 5-2 to find t

$L_t = L_a \exp(-k_d t)$

$20.55 = 27.65 \exp\{(-0.0590)(t)\}$

$0.743 = \exp\{(-0.0590)(t)\}$

$\ln(0.743) = \ln(\exp\{(-0.0590)(t)\})$

$- 0.296765 = - 0.0590(t)$

$t = 5.0299$ d

The distance to this point is then the product of the stream speed and the travel time

$x = (v)(t) = (0.390 \text{ m/s})(5.0299 \text{ d})(86{,}400 \text{ s/d})(1 \times 10^{-3} \text{ km/m}) = 169.49$ km

5-46 Improving the Watertown wastewater

Given: Problem 5-36; DO cheaper than BOD removal;

Solution:

An assumption was made that either BOD reduction or an increase in DO_w was used but not a combination. Because there is some latitude in assigning costs and the final critical DO other answers may be acceptable. A spreadsheet program was used to select a DO_w = 2.7 mg/L

Check

$$D_a = 7.92 - \frac{(0.280)(2.7) + (0.877)(6.00)}{0.280 + 0.877} = 2.72 \text{ mg/L}$$

Since the K's are in base 10

$t_c = 5.848 \log [1.8593(1-2.72(0.125))]$

$t_c = 5.848 \log (1.2178) = (5.848)(0.08889)$

$t_c = 0.5198$ d

$$D_c = 7.9885\left(10^{-(0.199)(0.5198)} - 10^{-(0.370)(0.5198)}\right) + 2.76\left(10^{-(0.379)(0.5198)}\right)$$

$D_c = 1.1652 + 1.7467 = 2.91$ mg/L

$DO_c = 7.92 - 2.91 = 5.01$ mg/L

5-47 Allowable ultimate BOD from Watapitae

Given: Data from Prob. 5-23; allow 1.50 mg/L above DNR regulations at Avepitaeonmi

Solution:

a. Allowable deficit

$$D = 9.95 - (1.50 + 5.00) = 3.45 \text{ mg/L}$$

b. Allowable initial ultimate BOD

$$L_a = \frac{(D - D_a \exp[(-k_r)(t)])(k_r - k_d)}{k_d(\exp[(-k_d)(t)] - \exp[(-k_r)(t)])}$$

$$L_a = \frac{(3.45 - 2.85\exp[(-0.474)(0.462)])(0.474 - 0.2805)}{0.2805(\exp[(-0.2805)(0.462)] - \exp[(-0.474)(0.462)])} = 10.66 \text{ mg/L}$$

c. Allowable ultimate BOD in discharge (solve Eqn. 5-26 for L_w)

$$L_W = \frac{L_a(Q_W + Q_r) - Q_r L_r}{Q_W}$$

$$L_W = \frac{10.66(0.1507 + 1.08) - (1.08)(11.40)}{0.1507} = 5.35 \text{ mg/L}$$

d. Allowable Mass Discharge in kg/d (Note: mg/L = g/m^3)

Q_wL_w = (0.1507 m^3/s)(5.35 mg/L)(86,400 s/d)(1 x 10^{-3} kg/g)

Q_wL_w = 69.67 or 70 kg/d

5-48 Allowable mass discharge

Given: Assume D_a = 0.0; L_r = 0.0; DO must be kept > 4.00 mg/ at 8.05 km; K_d = 1.80; K_r = 2.20; T = 12 ^{0}C; Q_r = 5.95 m^3/s;
v = 0.300 m/s; Q_w = 0.0130 m^3/s

Solution:

a. From Table A-2 at T = 12 ^{0}C find DO_s = 10.83 mg/L

b. Allowable deficit

$D = DO_s - DO_{allowable}$

D = 10.83 - 4.00 = 6.83 mg/L

c. Travel time to 8.05 km downstream

$$t = \frac{(8.05 km)(1000 m/km)}{(0.300 m/s)(86400 s/d)} = 0.3106 d$$

d. Solve DO Sag Eqn. for L_a (NOTE: in base 10)

$$L_a = D\left(\frac{K_r - K_d}{K_d}\right)\left(\frac{1}{10^{-(K_d)(t)} - 10^{-(K_r)(t)}}\right) + 0$$

$$L_a = 6.83\left(\frac{2.20 - 1.80}{1.80}\right)\left(\frac{1}{10^{-(1.80)(0.3106)} - 10^{-(2.20)(0.3106)}}\right) + 0 = 22.07 mg/L$$

e. Solve Eqn. 5-26 for Q_wL_w (NOTE: mg/L = g/m^3)

$Q_wL_w = L_a(Q_w + Q_r) - Q_rL_r$

Q_wL_w = [22.0707 g/m^3(0.0130 + 5.95)] - [(5.95)(0.00)] = 131.6076 g/s

Q_wL_w = (131.6076 g/s)(10^{-3} kg/g)(86,400 s/d) = 1.1371 x 10^4 or 1.14 x 10^4 kg/d

5-49 Nitrogenous BOD

Given: Problem 5-48 and 3.00 mg/L ammonia-N.

Solution:

a. Calculated values of DO_s, D and t same as in 5-48

b. Solve Eqn. 5-47 for L_a

$$L_a = \left(\frac{K_r - K_d}{K_d}\right)\left(\frac{1}{10^{-(K_d)(t)} - 10^{-(K_r)(t)}}\right)\left[D - \frac{K_n L_n}{K_r - K_n}\left(10^{-(K_n)(t)} - 10^{-(K_r)(t)} - D_a\left(10^{-(K_r)(t)}\right)\right)\right]$$

c. Note from Problem 5-48 D_a = 0.0

d. Compute L_n

L_n = NBOD = (4.57 g/g of N)(3.00 mg/L) = 13.71 mg/L

where 4.57 is theoretical ultimate oxygen demand Eqn. 5-20.

e. Calculate L_a

$$L_a = \left(\frac{2.20 - 1.80}{1.80}\right)\left(\frac{1}{10^{-(0.900)(0.3106)} - 10^{-(2.20)(0.3106)}}\right)\left[6.83 - \frac{(0.900)(13.71)}{2.20 - 0.900}\left(10^{-(0.900)(0.3106)} - 10^{-(2.20)(0.3106)}\right)\right]$$

L_a = (3.14)(3.81) = 11.98 mg/L

f. Find L_w (NOTE:from 5-48 L_r = 0)

$$L_W = \frac{L_a(Q_W + Q_r) - Q_r L_r}{Q_W}$$

$$L_W = \frac{11.98(0.0130 + 5.95) - (5.95)(0.0)}{0.0130} = 5495\,\text{mg/L}$$

g. Allowable mass discharge in kg/d

Q_wL_w = (0.0130 m^3/s)(10^3 L/m^3)(86,400 s/d)(5495 mg/L)

Q_wL_w = 6.172 x 10^9 mg/d or 6.2 x 10^3 kg/d

5-50 DO sag curve for Big Head River

Given: Table of data

Solution:

a. See Problem 5-42 for example spreadsheet calculations

b. Calculate DO at critical point during the summer as in the following example spreadsheet

Input Data **Summer**

BOD_5 of Waste	100	mg/L
Rate Const (k)	0.3685	d^{-1} at 20 ^oC
Q_w	0.2	m^3/s
DO_w	0	mg/L
L_r	7.66	mg/L
Q_r	0.241	m^3/s
DO_r	8	mg/L
T_w	28	^oC
T_r	28	^oC
DO_{sat}	8.38	mg/L
River Speed	0.15	m/s
River Depth	1	m
η	0.3	

Calculation of Ultimate BOD of Waste	L_W =	118.8242	mg/L
Mixed Ultimate BOD	L_a=	58.07462	mg/L
DO after mixing	DO=	4.371882	mg/L
Initial Mixed Deficit	D_a=	4.008118	mg/L
Deoxygenation rate constant at 20 ^oC	k_d=	0.4135	d^{-1} at 20 ^oC
Deoxygenation rate constant at 28 ^oC	k_d (at 28^{o}C)=	0.639421	d^{-1} at 15 ^oC
Rearation rate constant at 20 ^oC	k_r (at 20^{o}C)=	1.510464	d^{-1} at 20 ^oC
Rearation rate constant at 28 ^oC	k_r (at 28^{o}C)=	1.826038	d^{-1} at 15 ^oC

Summer continued

		Time (d)	kr-kd	$k_d*L_a/(k_r-k_d)$	kd*t	kr*t	D (mg/L)	DO (mg/L)	
		0.96875	1.1866	31.2941	0.6194	1.769	12.1915	-3.8115	Cannot be negative, so = 0.0
µBrew Outfall →		0	1.1866	31.2941	0	0	4.0081	4.3719	
		0.5	1.1866	31.2941	0.3197	0.913	11.7806	-3.4006	Cannot be negative, so = 0.0
critical point →		0.7688	1.1866	31.2941	0.4916	1.4039	12.4385	-4.0585	Cannot be negative, so = 0.0
		0.96875	1.1866	31.2941	0.6194	1.769	12.1915	-3.8115	Cannot be negative, so = 0.0
		1	1.1866	31.2941	0.6394	1.826	12.1163	-3.7363	Cannot be negative, so = 0.0
		1.5	1.1866	31.2941	0.9591	2.7391	10.2292	-1.8492	Cannot be negative, so = 0.0
		2	1.1866	31.2941	1.2788	3.6521	8.0033	0.3767	
		2.5	1.1866	31.2941	1.5986	4.5651	6.0433	2.3367	
		3	1.1866	31.2941	1.9183	5.4781	4.4819	3.8981	
X_c (km) =	9.96	3.5	1.1866	31.2941	2.238	6.3911	3.2925	5.0875	
		4	1.1866	31.2941	2.5577	7.3042	2.4064	5.9736	
		4.5	1.1866	31.2941	2.8774	8.2172	1.7539	6.6261	
		5	1.1866	31.2941	3.1971	9.1302	1.2764	7.1036	

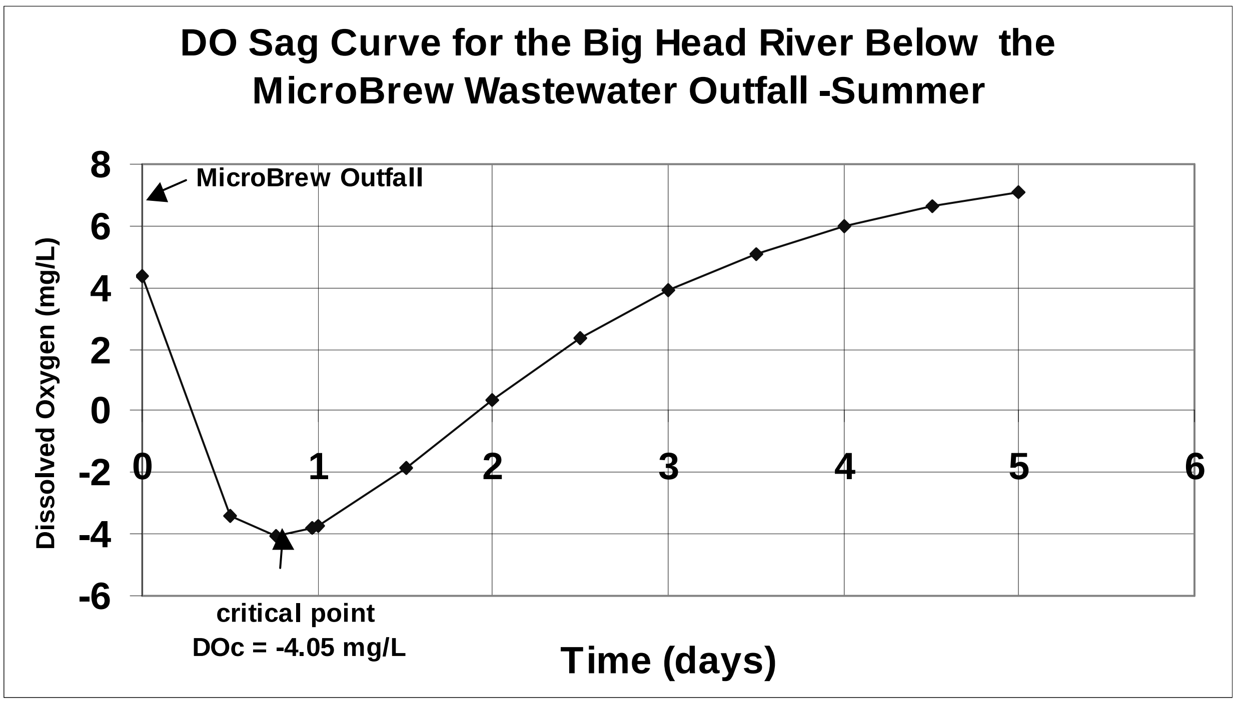


Figure S-5-50a: DO sag curve for Big Head River in the summer

b. Calculate DO at critical point during the winter as in the example spreadsheet on the following page

Input Data	Winter	
BOD_5 of Waste	100	mg/L
Rate Const (k)	0.3685	d^{-1} at 20 °C
Q_w	0.2	m^3/s
DO_w	0	mg/L
L_r	7.66	mg/L
Q_r	0.483	m^3/s
DO_r	8	mg/L
T_w	28	°C
T_r	4	°C
DO_{sat}	13.13	mg/L
River Speed	0.15	m/s
River Depth	2	m
η	0.3	

Calculation of Ultimate BOD of Waste	L_W =	118.8242	mg/L
Mixed Ultimate BOD	L_a=	40.21175	mg/L
DO after mixing	DO=	5.657394	mg/L
Initial Mixed Deficit	D_a=	7.472606	mg/L
Deoxygenation rate constant at 20 °C	k_d=	0.391	d^{-1} at 20 °C
Deoxygenation rate constant at 5.3 °C	k_d (at 5.3°C)=	0.125529	d^{-1} at 15 °C
Rearation rate constant at 20 °C	k_r (at 20°C)=	0.534029	d^{-1} at 20 °C
Rearation rate constant at 5.3 °C	k_r (at 5.3°C)=	0.43167	d^{-1} at 15 °C

Winter Continued		Time (d)	kr-kd	$k_d*L_a/(k_r-k_d)$	kd*t	kr*t	D (mg/L)	DO (mg/L)
		0.96875	0.3061	16.4883	0.1216	0.4182	8.6658	4.4642
µBrew Outfall →		0	0.3061	16.4883	0	0	7.4726	5.6574
		0.5	0.3061	16.4883	0.0628	0.2158	8.2198	4.9102
		0.96875	0.3061	16.4883	0.1216	0.4182	8.6658	4.4642
		1	0.3061	16.4883	0.1255	0.4317	8.6882	4.4418
		1.5	0.3061	16.4883	0.1883	0.6475	8.9401	4.1899
		2	0.3061	16.4883	0.2511	0.8633	9.0251	4.1049
critical point →		2.0626	0.3061	16.4883	0.2589	0.8904	9.0261	4.1039
		2.5	0.3061	16.4883	0.3138	1.0792	8.9829	4.1471
		3	0.3061	16.4883	0.3766	1.295	8.8449	4.2851
X_c (km) =	26.73	3.5	0.3061	16.4883	0.4394	1.5108	8.636	4.494
		4	0.3061	16.4883	0.5021	1.7267	8.3759	4.7541
		4.5	0.3061	16.4883	0.5649	1.9425	8.0801	5.0499
		5	0.3061	16.4883	0.6276	2.1584	7.7608	5.3692

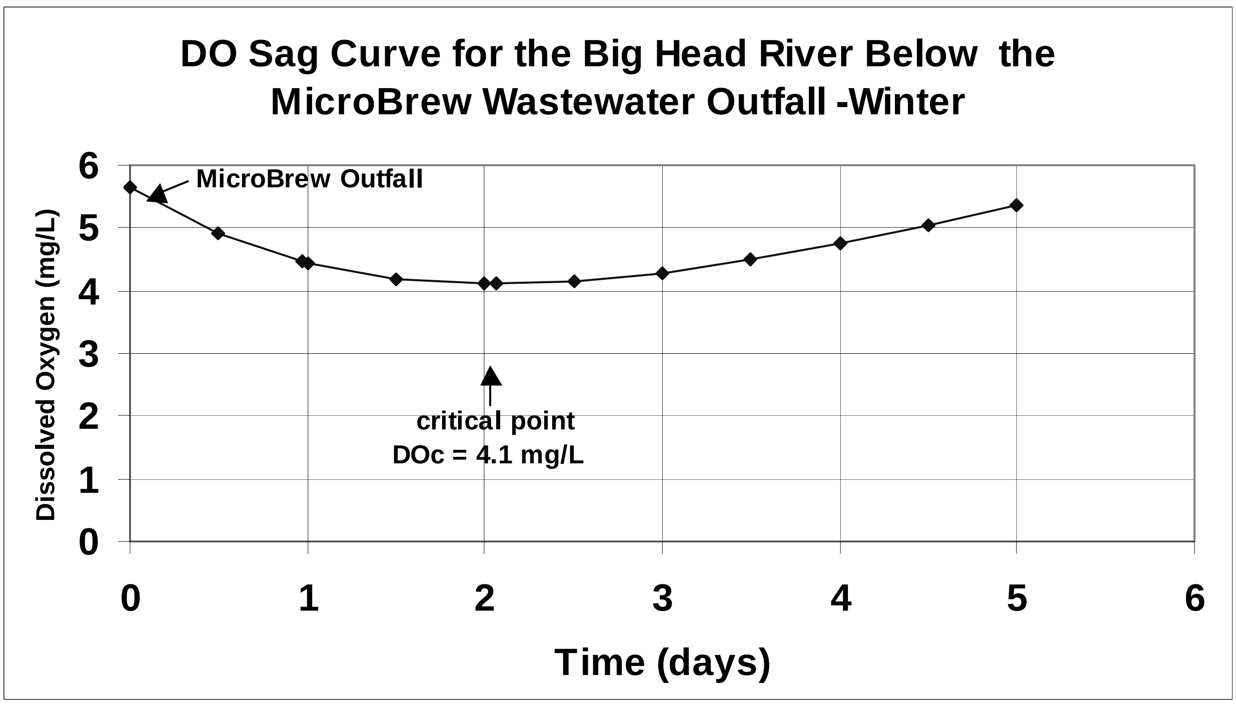


Figure S-5-50b: DO sag curve for the Big Head River in the winter

5-51 Salt water intrusion

Given: $z \neq 40h$

Solution:

a. Begin with Eqn. 5-50

$$z = \frac{\rho_f}{\rho_s - \rho_f} h$$

b. Substitute densities of fresh water and salt water given in text

$$z = \frac{1.00}{1.025 - 1.00} h = \frac{1.00}{0.025} h = 40.0h$$

5-52 Salt water intrusion – maximum pumping

Given: Hydraulic conductivity = 4.63 x 10^{-5} m/s; well screen 30 m above interface; upcoming must be less than 10 m

Solution:

a. Solve Eqn. 5-51 for Q

$$Q = \frac{(z_\infty)(2\pi)(\rho_s - \rho_f)KL}{\rho_f}$$

b. Convert K into compatible units

K = (4.63 x 10^{-5} m/s)(86400 s/d) = 4.00 m/d

c. Calculate flow rate

$$Q = \frac{(10\text{m})(2\pi)(1.025\,\text{g/m}^3 - 1.000\,\text{g/m}^3)(4.00\,\text{m/d})(30\text{m})}{1.000\,\text{g/m}^3} = 188.5\,\text{m}^3/\text{d}$$

DISCUSSION QUESTIONS

5-1 Effect of treatment on values of k

Given:Students determined rate constants (k) for treated and untreated sewage

Solution:

Because easily degraded organics are more completely removed than less readily degraded organics, the raw sewage rate constants will be higher.

5-2 Setting BOD standards

Given: Select BOD_5 or ultimate BOD as standard

Solution:

Ultimate BOD would be chosen to limit the total impact on the stream. However, if the rate constant was very high, BOD_5 might be more appropriate because the rate of deaeration would be excessive.

5-3 Location of temperature measurements

Given: Three measurements: 33 °C, 18 °C, 21 °C and measurement locations of 1 m above, 1 m deep and 10 m deep

Solution:

Because the lake is deep and probably stratified in July, the best guess is:

1 m above = 33 °C
1 m below = 21 °C
10 m below = 18 °C

5-4 Location of critical point

Given: Critical point is 18 km downstream for untreated wastewater

Solution:

Treated wastewater will have a lower rate constant (k). This in turn will reduce the value of k_d. The critical point will be displaced downstream. Although the effect of decreasing L_w is to make L_a smaller and hence to make t_c smaller, the decrease in k_d is controlling.

5-5 Lake productivity

Given: Remote lake in Canada with no waste sources but highly turbid and very low DO in hypolimnion

Solution:

This would be classified as a eutrophic lake. High turbidity eliminates oligotrophic productivity. Large mats of floating algae point to a eutrophic lake. Assuming summer stratification, the hypolimnion DO of 1.0 mg/L is close to anaerobic which would eliminate mesotrophic productivity.

5-6 Lack of lake acidification

Given: Lakes in Illinois, Indiana, and Kentucky and rainwater pH of 4.4

Solution:

The calcareous bedrock geology appears to supply enough alkalinity to buffer the pH to keep the lakes from being acidified.