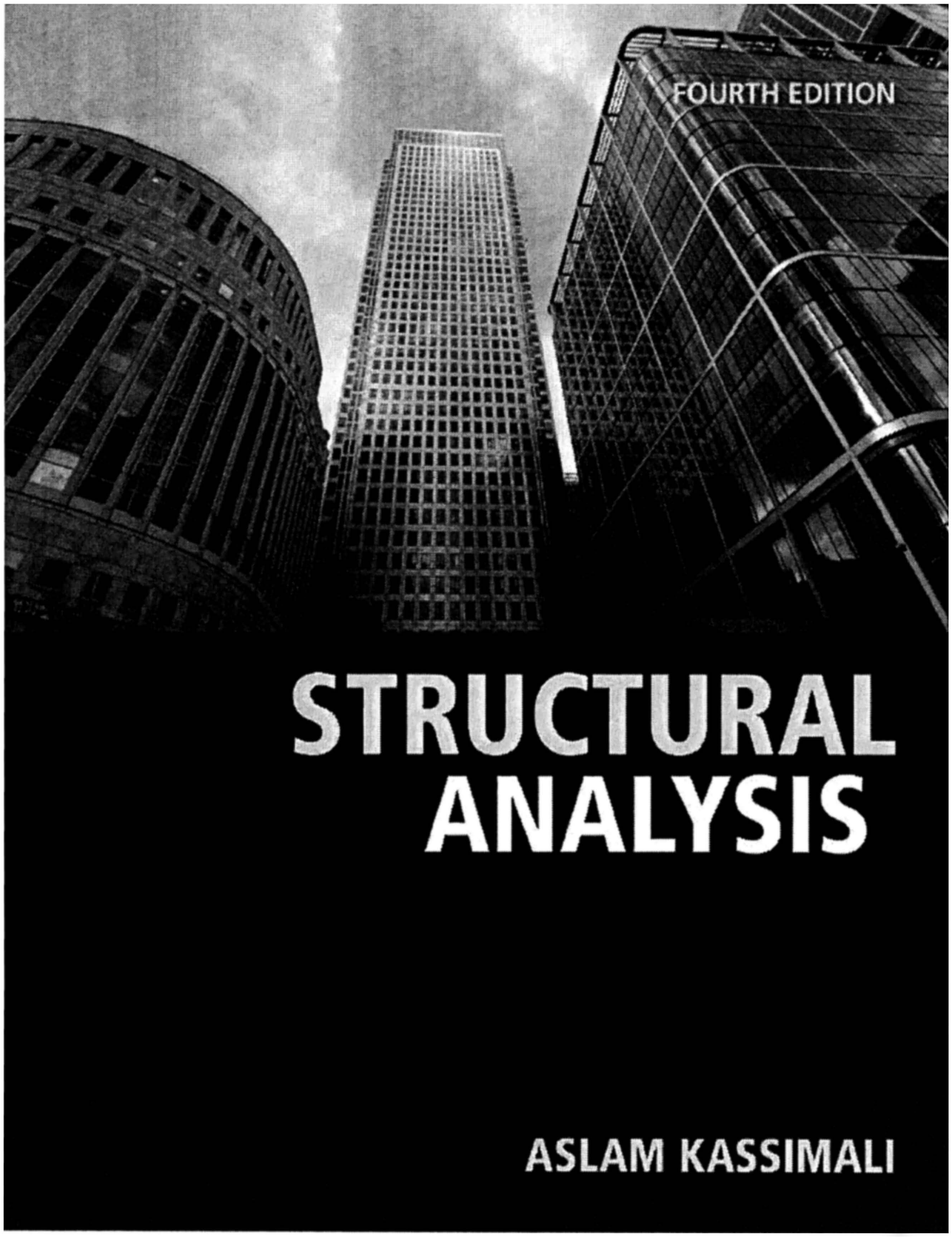
An Instructor's Solutions Manual to Accompany

Structural Analysis, 4th Edition

Aslam Kassimali



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An Instructor's Solutions Manual

For

Structural Analysis
Fourth Edition

Aslam Kassimali
Southern Illinois University Carbondale

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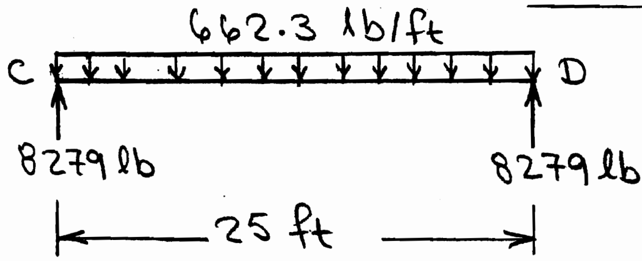
Chapter Two
Loads on Structures

CHAPTER 2

2.1 Beam CD

Uniformly distributed load $= 150(12)\left(\frac{4}{12}\right) + 490\left(\frac{18.3}{144}\right)$

$= \underline{662.3 \text{ lb/ft}}$



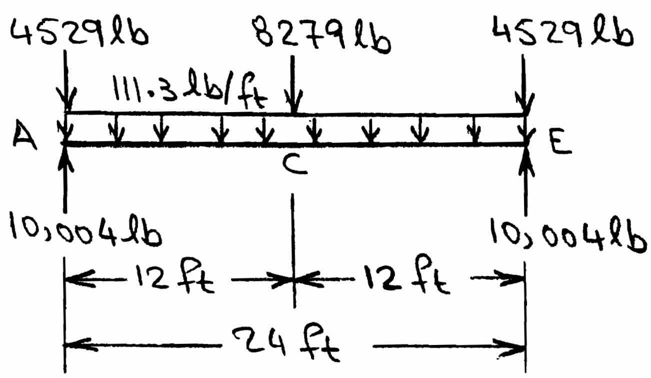
Girder AE

Uniformly distributed load $= 490\left(\frac{32.7}{144}\right) = \underline{111.3 \text{ lb/ft}}$

Concentrated load at C $= \underline{8279 \text{ lb}}$

Concentrated loads at A and E

$= \left[150(6)\left(\frac{4}{12}\right) + 490\left(\frac{18.3}{144}\right)\right]\left(\frac{25}{2}\right) = \underline{4529 \text{ lb}}$



2.2 See solution of Problem 2.1

Beam CD Uniformly distributed load

$= 662.3 + 120\left(\frac{6}{12}\right)(7) = 662.3 + 420 = \underline{1082.3 \text{ lb/ft}}$

Girder AE Uniformly distributed load $= \underline{111.3 \text{ lb/ft}}$

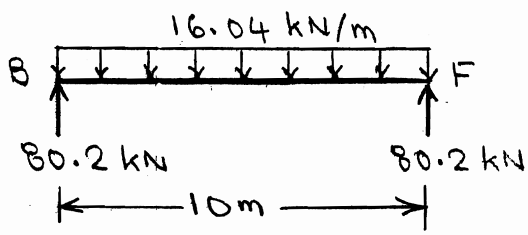
Concentrated load at C $= 8279 + 420\left(\frac{25}{2}\right) = \underline{13{,}529 \text{ lb}}$

Concentrated loads at A and E $= \underline{4529 \text{ lb}}$

2.3 Beam BF

Uniformly distributed load

$= 23.6(5)\left(\frac{130}{1000}\right) + 77\left(\frac{9100}{10^6}\right) = \underline{16.04 \text{ kN/m}}$



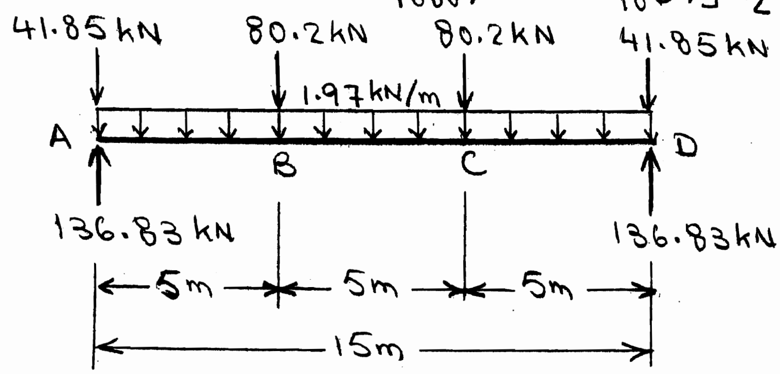
Girder AD

Uniformly distributed load $= 77\left(\frac{25600}{10^6}\right) = \underline{1.97 \text{ kN/m}}$

Concentrated loads at B and C $= \underline{80.2 \text{ kN}}$

Concentrated loads at A and D

$= \left[23.6(2.5)\left(\frac{130}{1000}\right) + 77\left(\frac{9100}{10^6}\right)\right]\frac{10}{2} = \underline{41.85 \text{ kN}}$



2.4

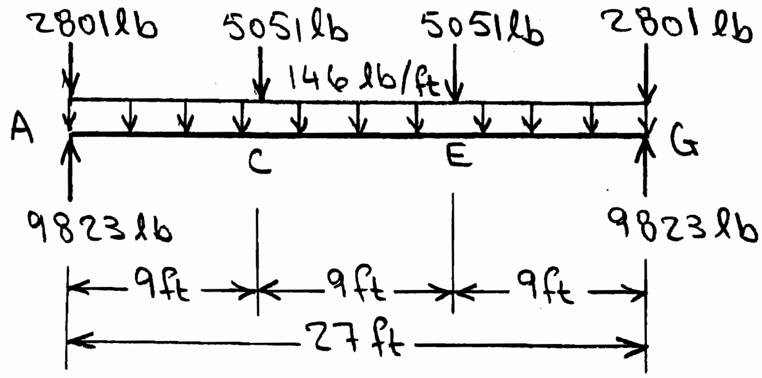
Uniformly distributed load $= 490\left(\frac{42.9}{144}\right) = \underline{146 \text{ lb/ft}}$

Concentrated loads at A and G

$$= \left[150(4.5)\left(\frac{4}{12}\right) + 490\left(\frac{16.2}{144}\right)\right]\left(\frac{20}{2}\right) = \underline{2801 \text{ lb}}$$

Concentrated loads at C and E

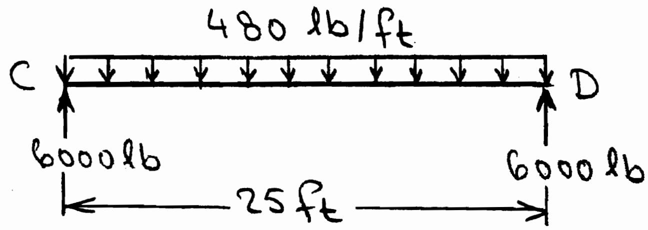
$$= \left[150(9)\left(\frac{4}{12}\right) + 490\left(\frac{16.2}{144}\right)\right]\left(\frac{20}{2}\right) = \underline{5051 \text{ lb}}$$



2.5 Live load = 40 psf

Beam CD

Uniformly distributed load = 40(12) = 480 lb/ft

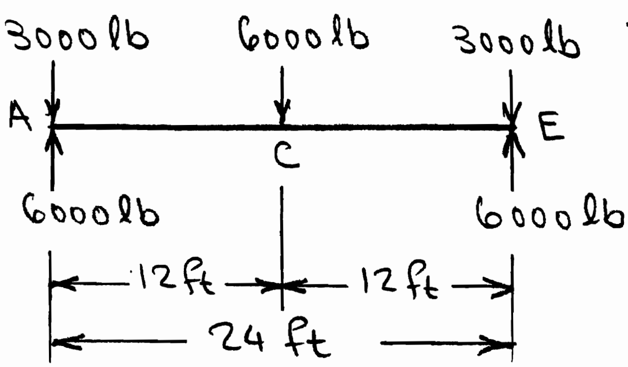


Girder AE

Concentrated load at C = 6000 lb

Concentrated loads at A and E = $[40(6)]\left(\frac{25}{2}\right)$

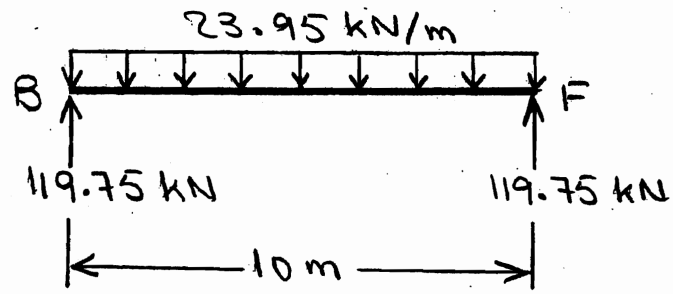
= 3000 lb



2.6 Live load = 4.79 kPa = 4.79 kN/m^2

Beam BF

Uniformly distributed load = 4.79(5) = 23.95 kN/m

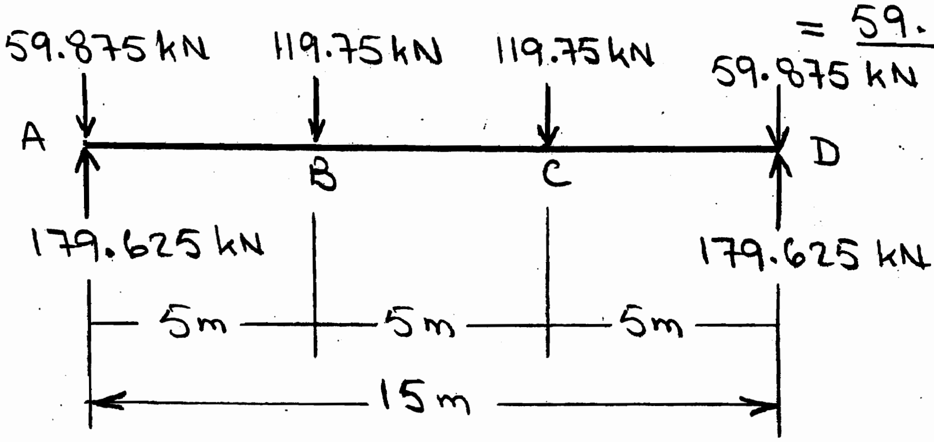


Girder AD

Concentrated loads at B and C = 119.75 kN

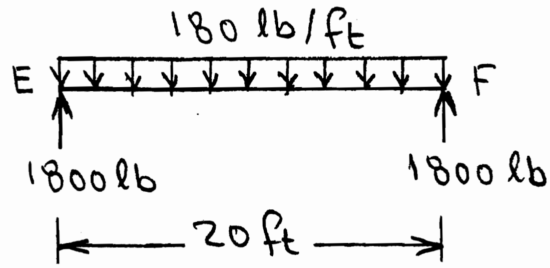
Concentrated loads at A and D = $[4.79(2.5)]\frac{10}{2}$

= 59.875 kN



2.7 Beam EF

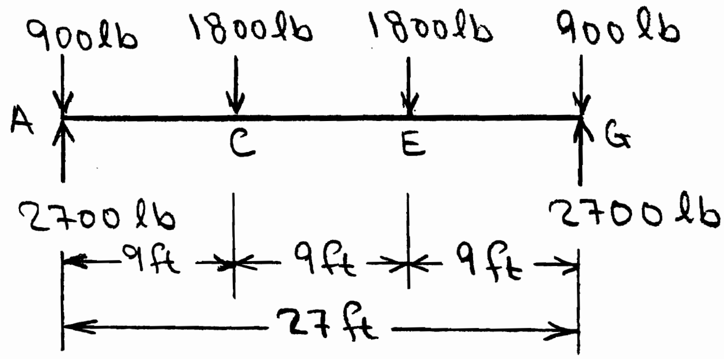
Uniformly distributed load = 20(9) = 180 lb/ft



Girder AG

Concentrated loads at C and E = 1800 lb

Concentrated loads at A and G = 1800/2 = 900 lb



Column A Concentrated load = 2700 lb

2.8 $V = 85$ mph, $h = 40 + (15/2) = 47.5$ ft,

$I = 1.0$, $z_g = 1200$ ft, $\alpha = 7.0$, $K_{zt} = 1$

and $K_d = 1$

$$K_h = 2.01\left(\frac{47.5}{1200}\right)^{2/7} = 0.8$$

$$q_h = 0.00256(0.8)(1)(1)(85)^2(1) = 14.8 \text{ psf}$$

$$G = 0.85$$

For $\theta = 45°$ and $h/L = 47.5/30 = 1.58$:

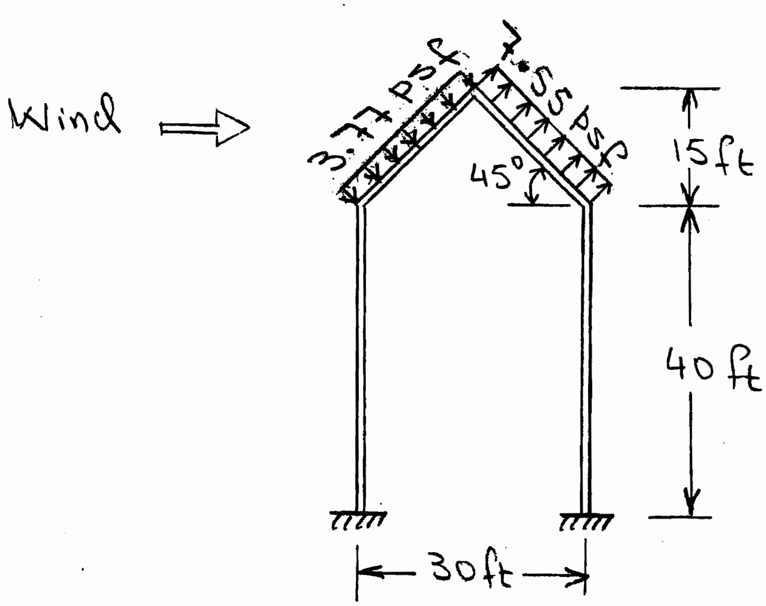
$C_p = 0.3$ for windward side

$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$p_h = 14.8(0.85)(0.3) = \underline{3.77 \text{ psf}}$ for windward side

$p_h = 14.8(0.85)(-0.6) = \underline{-7.55 \text{ psf}}$ for leeward side



2.9 $V = 40$ m/s, $h = 12 + \frac{5}{2} = 14.5$ m

$I = 1.15$, $z_g = 366$ m, $\alpha = 7.0$, $K_{zt} = 1$

and $K_d = 1$

$$K_h = 2.01\left(\frac{14.5}{366}\right)^{2/7} = 0.8$$

$$q_h = 0.613(0.8)(1)(1)(40)^2(1.15) = 902.34 \text{ N/m}^2$$

$G = 0.85$

Roof slope: $\theta = \tan^{-1}(5/6) = 39.8°$

$$\frac{h}{L} = \frac{14.5}{12} = 1.21$$

$C_p = -0.1$ and 0.25 for windward side

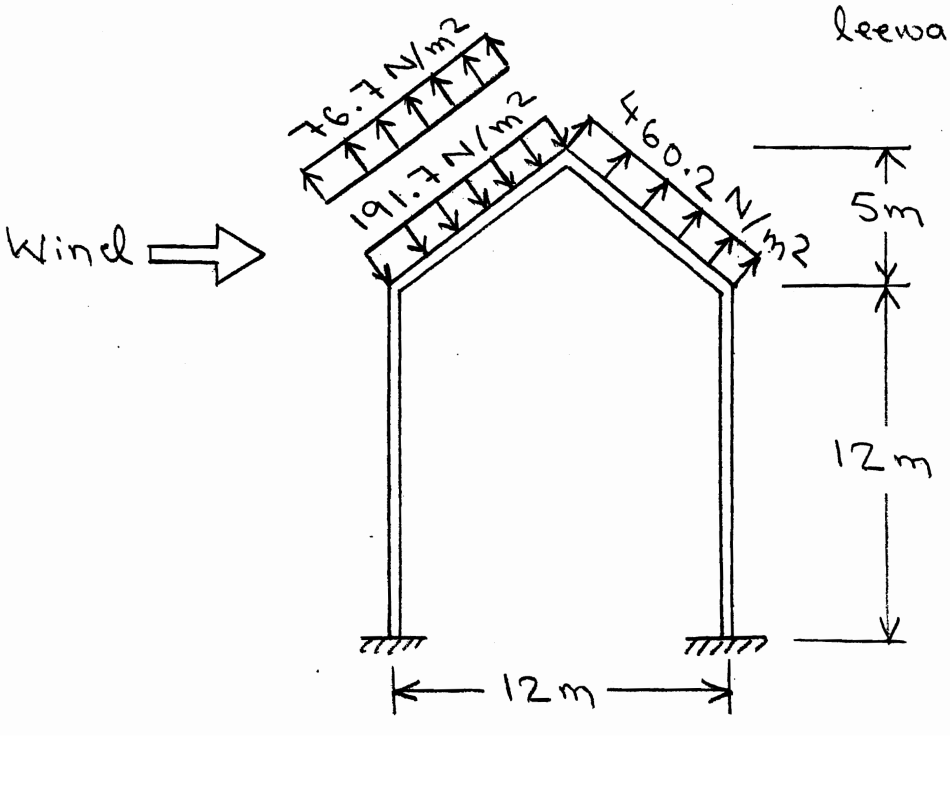
$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$p_h = (902.34)(0.85)(-0.1) = \underline{-76.7 \text{ N/m}^2}$ } for windward side

$p_h = (902.34)(0.85)(0.25) = \underline{191.7 \text{ N/m}^2}$ }

$p_h = (902.34)(0.85)(-0.6) = \underline{-460.2 \text{ N/m}^2}$ for leeward side



2.10

$V = 90$ mph, $h = 30 + \frac{11}{2} = 35.5$ ft

$I = 1.15$, $z_g = 900$ ft, $\alpha = 9.5$, $K_{zt} = 1$ and $K_d = 1$

$$K_h = 2.01\left(\frac{35.5}{900}\right)^{2/9.5} = 1.02$$

$$q_h = 0.00256(1.02)(1)(1)(90)^2(1.15) = 24.32 \text{ psf}$$

$G = 0.85$

Roof slope: $\theta = \tan^{-1}(11/20) = 28.8^\circ$

$$\frac{h}{L} = \frac{35.5}{40} = 0.89$$

$C_p = -0.3$ and 0.2 for windward side

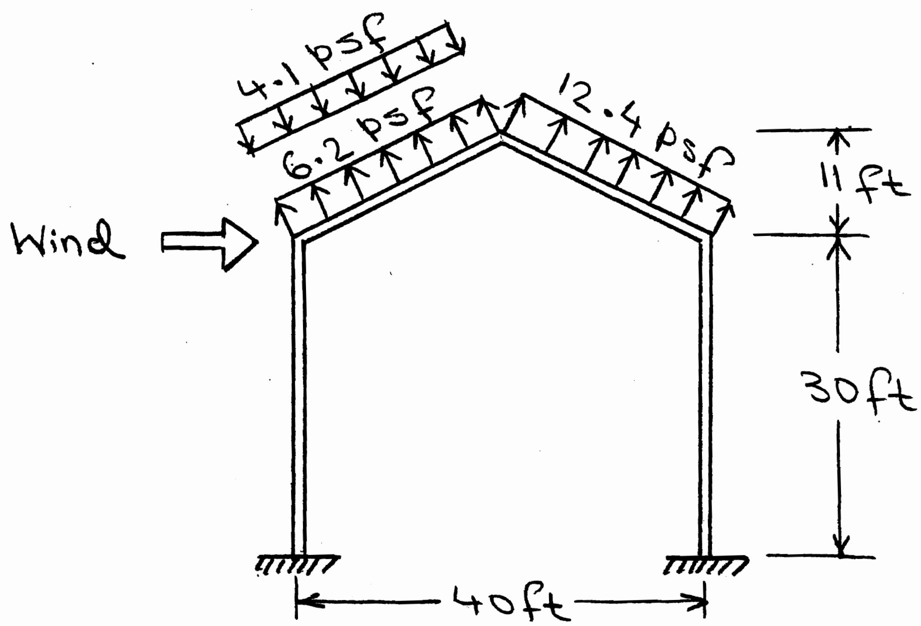
$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$p_h = 24.32(0.85)(-0.3) = \underline{-6.2 \text{ psf}}$ } for windward side

$p_h = 24.32(0.85)(0.2) = \underline{4.1 \text{ psf}}$ }

$p_h = 24.32(0.85)(-0.6) = \underline{-12.4 \text{ psf}}$ for leeward side



2.11 $V = 90$ mph, $I = 1.15$, $z_g = 900$ ft, $\alpha = 9.5$

From the solution of Problem 2.10:

$q_h = 24.32$ psf and $G = 0.85$

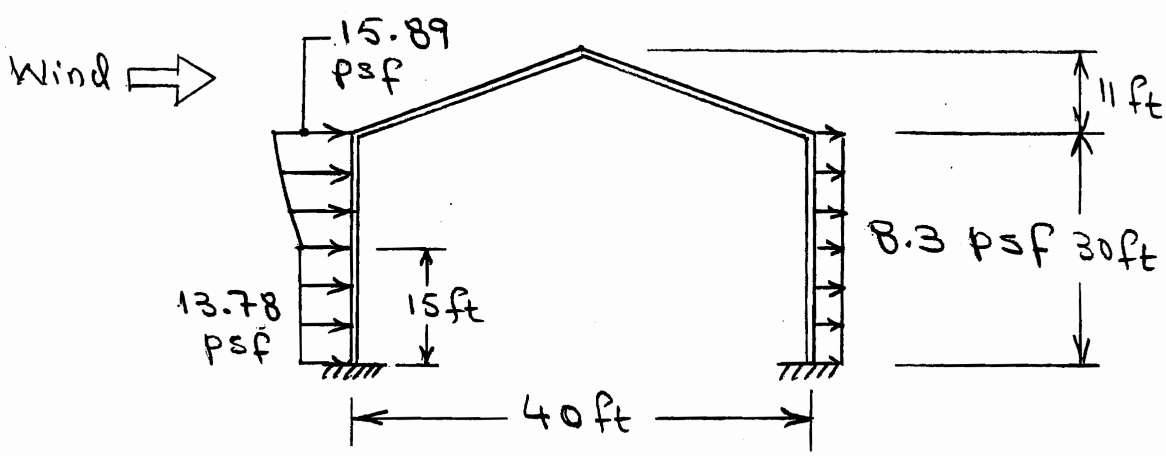
Leeward wall: For $L/B = 40/30 = 1.33$, $C_p = -0.4$

Thus, the wind pressure, $p_h = 24.32(0.85)(-0.4)$

$= \underline{-8.3 \text{ psf}}$

Windward wall: $C_p = 0.8$

z (ft)	K_z	q_z (psf)	p_z (psf)
30	0.98	23.37	15.89
25	0.95	22.65	15.4
20	0.90	21.46	14.59
15	0.85	20.27	13.78



2.12 $p_g = 20\,psf$, $C_e = 1$, $C_t = 1$, $I = 1.2$

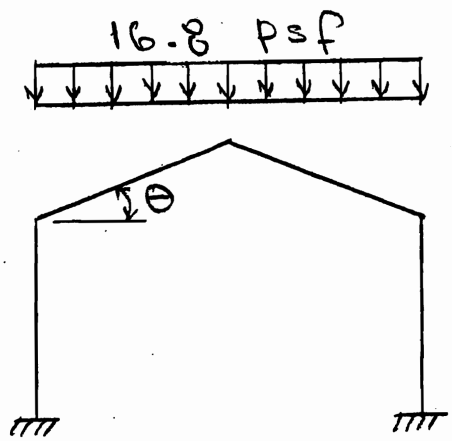
$$p_f = 0.7\,C_e\,C_t\,I\,p_g = 0.7(1)(1)(1.2)(20) = 16.8\,psf$$

$$\theta = \tan^{-1}(11/20) = 28.8^\circ,\quad \frac{70}{W} + 0.5 = \frac{70}{20} + 0.5 = 4^\circ$$

Therefore, the minimum values of p_f need not be considered.

$C_s = 1$

Balanced load $= p_s = C_s\,p_f = 1(16.8) = \underline{16.8\ psf}$



Balanced Snow Load

2.13 $p_g = 1.2\ kN/m^2$, $C_e = 1$, $C_t = 1$, $I = 1.1$

$$p_f = 0.7 C_e C_t I p_g = 0.7(1)(1)(1.1)(1.2) = 0.92\ kN/m^2$$

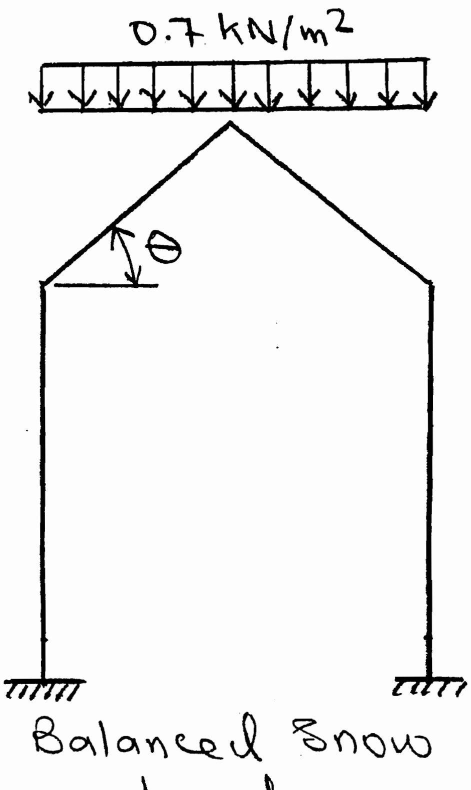
$\theta = \tan^{-1}(5/6) = 39.8^\circ$, $W = 6m = 19.7\ ft$

$$\frac{70}{W} + 0.5 = \frac{70}{19.7} + 0.5 = 4.1^\circ$$

Therefore, the minimum values of p_f need not be considered.

$$C_s = 1 - \frac{\theta - 30^\circ}{40^\circ} = 0.76$$

Balanced Load $= p_s = C_s p_f = 0.76(0.92) = \underline{0.7\ kN/m^2}$



Balanced Snow Load

Chapter Three
Equilibrium and Support Reactions

CHAPTER 3

3.1 (a) Internally stable, with $r=3$. It is statically determinate

(b) Internally stable, with $r=5$. It is statically indeterminate. $i_e = 5-3 = \underline{2}$

(c) Internally unstable, with $r=6$ and $e_c=2$. As $r > 3+e_c$, the structure is statically indeterminate. $i_e = 6-(3+2) = \underline{1}$

(d) Internally unstable with $r=3$ and $e_c=1$. As $r < 3+e_c$, the beam is statically unstable.

3.2 (a) Internally stable, with $r=5$. It is statically indeterminate. $i_e = 5-3 = \underline{2}$.

(b) Internally unstable, with $r=5$ and $e_c=2$. As $r=3+e_c$, the beam is statically determinate.

(c) Internally stable, with $r=6$. It is statically indeterminate. $i_e = 6-3 = \underline{3}$

(d) Internally unstable, with $r=6$ and $e_c=1$. As $r > 3+e_c$, the arch is statically indeterminate. $i_e = 6-(3+1) = \underline{2}$.

(e) Internally unstable, with $r=5$ and $e_c=2$. As $r=3+e_c$, the frame is statically determinate.

3.3 (a) Internally unstable, with $r = 5$ and $e_c = 3$.
As $r < 3 + e_c$, the beam is <u>statically unstable</u>.

(b) Internally unstable, with $r = 4$ and $e_c = 2$.
As $r < 3 + e_c$, the beam is <u>statically unstable</u>.

(c) Internally stable, with $r = 4$. It is <u>statically indeterminate</u>. $i_e = 4 - 3 = \underline{1}$.

(d) Internally unstable, with $r = 4$ and $e_c = 2$.
As $r < 3 + e_c$, the frame is <u>statically unstable</u>.

3.4 (a) Internally stable, with $r = 4$.

It is <u>statically indeterminate.</u>

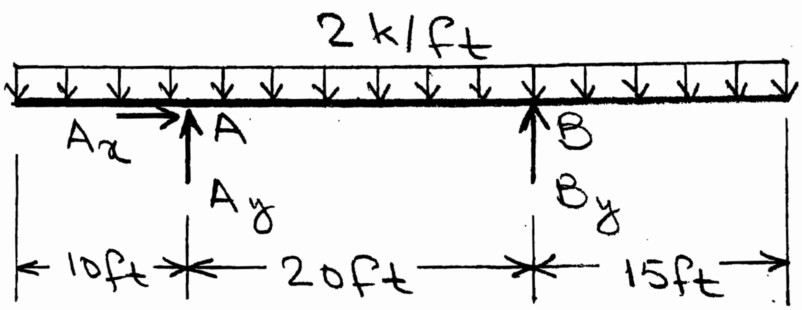
$i_e = 4 - 3 = \underline{1}.$

(b) Internally unstable, with $r = 4$ and $e_c = 2$. As $r < 3 + e_c$, the frame is <u>statically unstable.</u>

(c) Internally stable, with $r = 6$. It is <u>statically indeterminate.</u> $i_e = 6 - 3 = \underline{3}.$

(d) Internally unstable, with $r = 6$ and $e_c = 3$. As $r = 3 + e_c$, the frame is <u>statically determinate.</u>

3.5



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

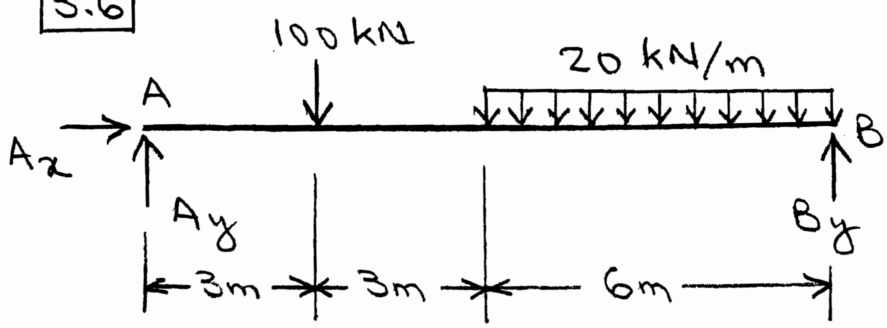
$+\circlearrowright \Sigma M_B = 0$ $\qquad$ $-A_y(20) + 2(45)(7.5) = 0$

$\underline{A_y = 33.75\text{ k}\uparrow}$

$+\uparrow \Sigma F_y = 0$ $\qquad$ $33.75 - 2(45) + B_y = 0$

$\underline{B_y = 56.25\text{ k}\uparrow}$

3.6



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

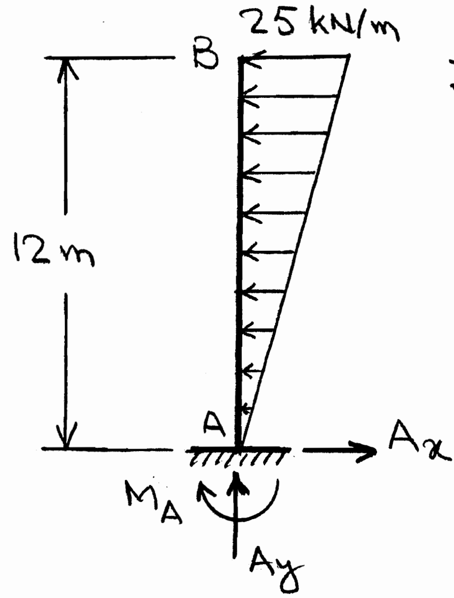
$+\circlearrowright \Sigma M_B = 0$ $\qquad$ $-A_y(12) + 100(9) + 20(6)3 = 0$

$\underline{A_y = 105\text{ kN}\uparrow}$

$+\uparrow \Sigma F_y = 0$ $\qquad$ $105 - 100 - 20(6) + B_y = 0$

$\underline{B_y = 115\text{ kN}\uparrow}$

3.7



$\xrightarrow{+} \Sigma F_x = 0$

$-\frac{1}{2}(25)(12) + A_x = 0$

$\underline{A_x = 150 \text{ kN} \rightarrow}$

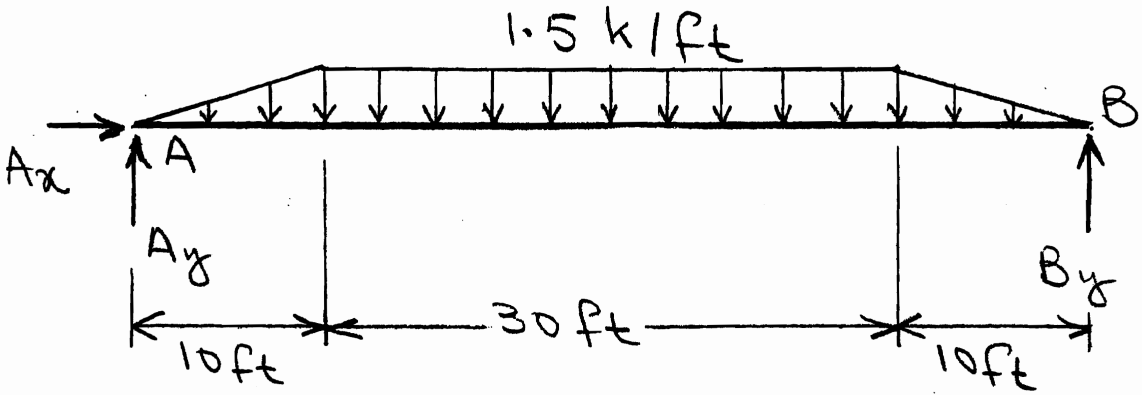
$\Sigma F_y = 0 \qquad \underline{A_y = 0}$

$+\circlearrowright \Sigma M_A = 0$

$-M_A + \frac{1}{2}(25)(12)(8) = 0$

$\underline{M_A = 1200 \text{ kN.m} \circlearrowright}$

3.8



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowleft \Sigma M_B = 0$

$$-A_y(50) + \frac{1}{2}(1.5)10\left(\frac{10}{3}+40\right) + 1.5(30)(25) + \frac{1}{2}(1.5)10\left(\frac{2}{3}\right)(10) = 0$$

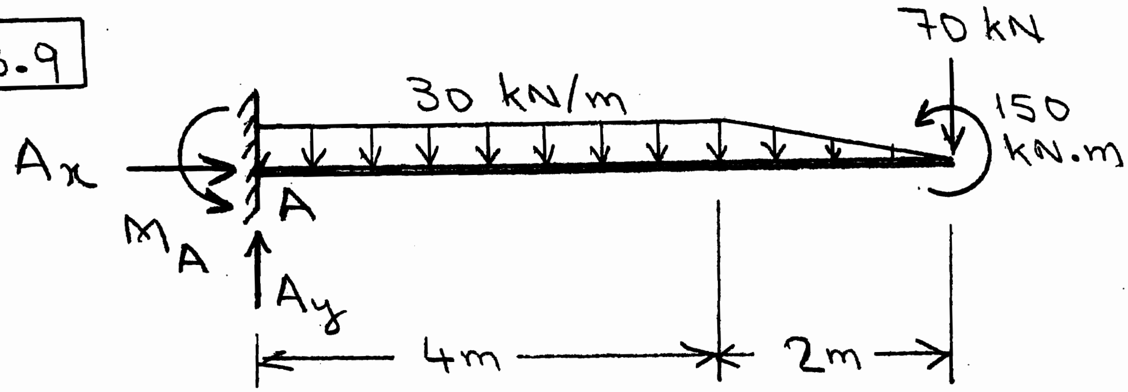
$\underline{A_y = 30\text{ k}\uparrow}$

$+\uparrow \Sigma F_y = 0$

$$30 - \frac{1}{2}(1.5)(10)(2) - 1.5(30) + B_y = 0$$

$\underline{B_y = 30\text{ k}\uparrow}$

3.9



$$\Sigma F_x = 0 \qquad \underline{A_x = 0}$$

$$+\uparrow \Sigma F_y = 0 \qquad A_y - 30(4) - \tfrac{1}{2}(30)(2) - 70 = 0$$

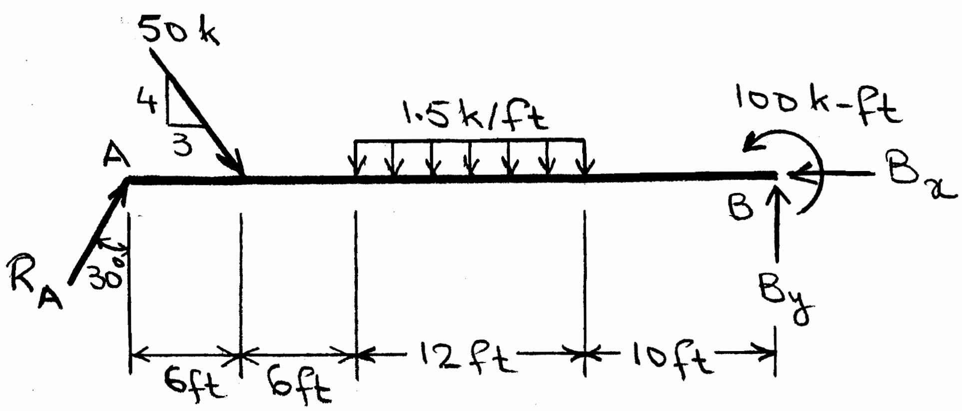
$$\underline{A_y = 220\ \text{kN}\uparrow}$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$M_A - 30(4)(2) - \tfrac{1}{2}(30)(2)\left(4 + \tfrac{2}{3}\right) - 70(6) + 150 = 0$$

$$\underline{M_A = 650\ \text{kN.m}\circlearrowleft}$$

3.10



$+\circlearrowleft \sum M_B = 0$

$$-R_A(\cos 30°)(34) + \frac{4}{5}(50)(28) + 1.5(12)(16) + 100 = 0$$

$R_A = 51.2\text{ k}$ ↗

$\overset{+}{\rightarrow} \sum F_x = 0$

$$51.2(\sin 30°) + \frac{3}{5}(50) - B_x = 0$$

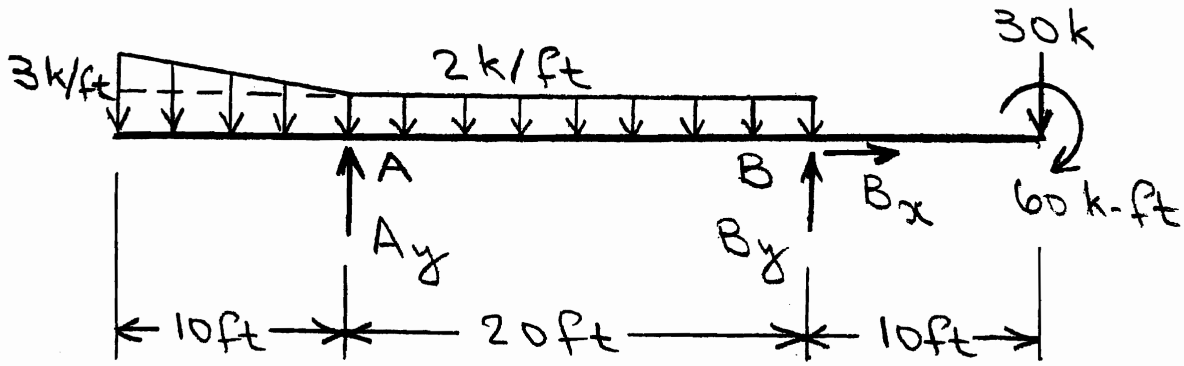
$B_x = 55.6\text{k} \leftarrow$

$+\uparrow \sum F_y = 0$

$$51.2(\cos 30°) - \frac{4}{5}(50) - 1.5(12) + B_y = 0$$

$B_y = 13.65\text{k} \uparrow$

3.11



$\Sigma F_x = 0$ $\quad\quad$ $\underline{B_x = 0}$

$+\circlearrowright \Sigma M_A = 0$

$\frac{1}{2}(1)(10)\left(\frac{20}{3}\right) - 2(30)(5) + B_y(20) - 30(30) - 60 = 0$

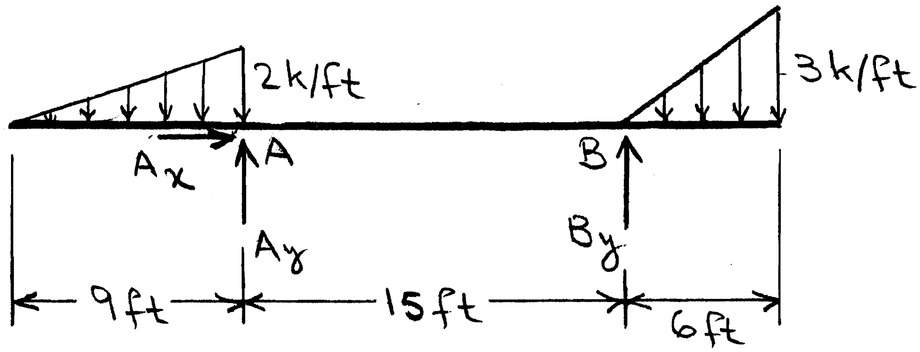
$\underline{B_y = 61.33\text{ k}\uparrow}$

$+\uparrow \Sigma F_y = 0$

$-\frac{1}{2}(1)(10) - 2(30) + A_y + 61.33 - 30 = 0$

$\underline{A_y = 33.67\text{ k}\uparrow}$

3.12



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowleft \Sigma M_B = 0$

$\frac{1}{2}(2)(9)\left[\frac{1}{3}(9)+15\right] - A_y(15) - \frac{1}{2}(3)(6)(4) = 0$

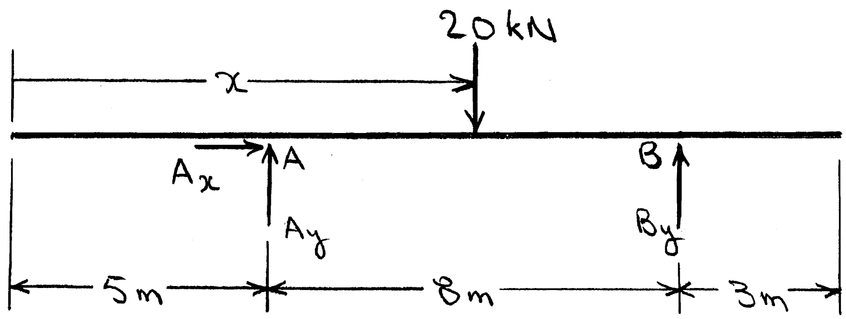
$\underline{A_y = 8.4\text{k}\uparrow}$

$+\uparrow \Sigma F_y = 0$

$-\frac{1}{2}(2)(9) + 8.4 + B_y - \frac{1}{2}(3)(6) = 0$

$\underline{B_y = 9.6\text{k}\uparrow}$

3.14



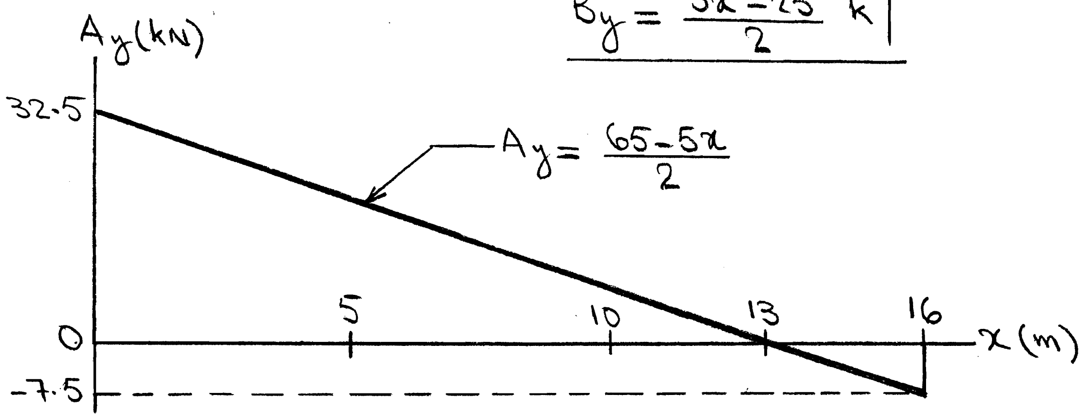
$\Sigma F_x = 0$ $\qquad\qquad$ $\underline{A_x = 0}$

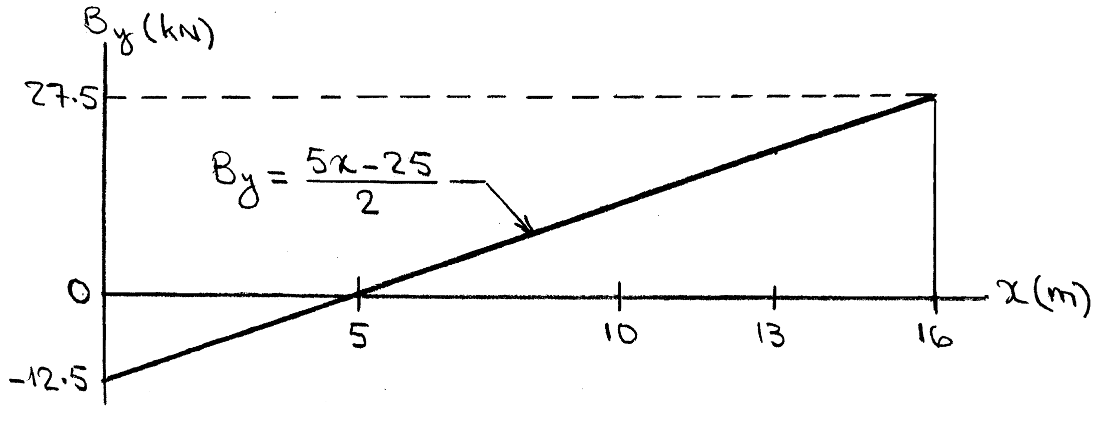
$+\circlearrowleft \Sigma M_B = 0 \qquad -A_y(8) + 20(13 - x) = 0$

$$\underline{A_y = \frac{65 - 5x}{2}\ \text{k}\uparrow}$$

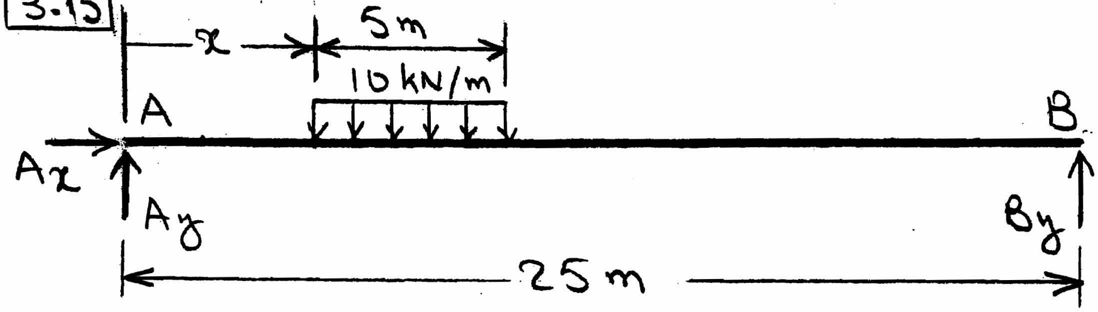
$+\uparrow \Sigma F_y = 0 \qquad \frac{65 - 5x}{2} - 20 + B_y = 0$

$$\underline{B_y = \frac{5x - 25}{2}\ \text{k}\uparrow}$$





3.15



$\Sigma F_x = 0 \qquad\qquad A_x = 0$

<u>$0 \leq x \leq 20\text{ m}$:</u>

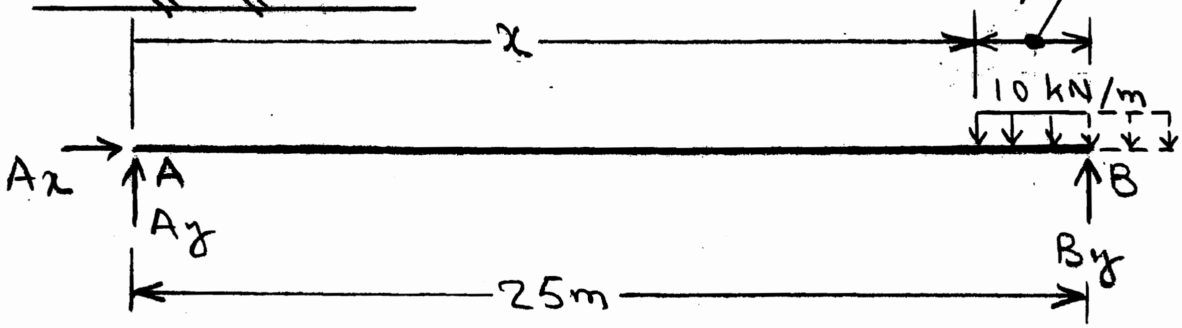
$+\circlearrowleft \Sigma M_B = 0 \qquad -A_y(25) + 10(5)[25 - (x+25)] = 0$

$A_y = 45 - 2x \text{ kN} \uparrow$

$+\uparrow \Sigma F_y = 0 \qquad (45-2x) - 10(5) + B_y = 0$

$B_y = 5 + 2x \text{ kN} \uparrow$

<u>$20\text{ m} \leq x \leq 25\text{ m}$:</u>



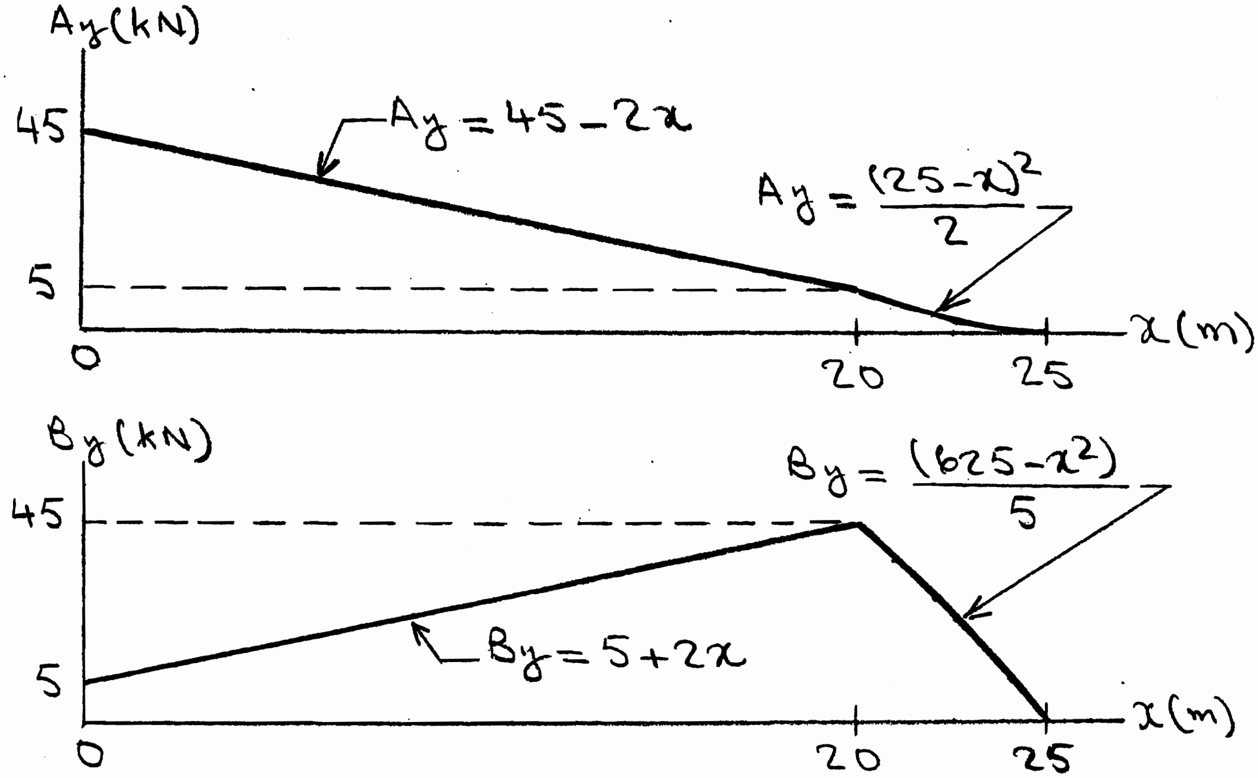
$+\circlearrowleft \Sigma M_B = 0$

$-A_y(25) + 10(25-x)\left(\frac{25-x}{2}\right) = 0 \qquad A_y = \frac{(25-x)^2}{5} \text{ kN} \uparrow$

$+\uparrow \Sigma F_y = 0 \qquad \frac{(25-x)^2}{5} - 10(25-x) + B_y = 0$

$B_y = \frac{(625 - x^2)}{5} \text{ kN} \uparrow$

13.15 (contd.)



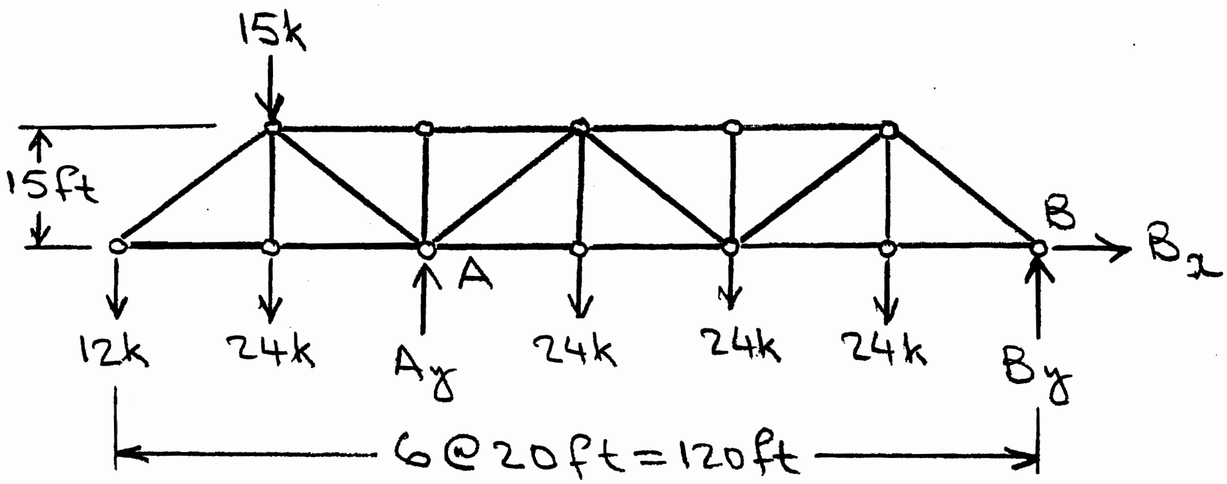
Ay(kN)
45
5
0
20
25
x(m)
Ay = 45 − 2x
Ay = (25 − x)^2 / 2
By(kN)
45
5
0
20
25
x(m)
By = 5 + 2x
By = (625 − x^2) / 5

3.16

$\underline{B_x = 0}$

$$A_y = B_y = \frac{2(50)+70}{2} = \underline{85\text{ kN}\uparrow}$$

3.17



$\sum F_x = 0$ $\qquad$ $\underline{B_x = 0}$

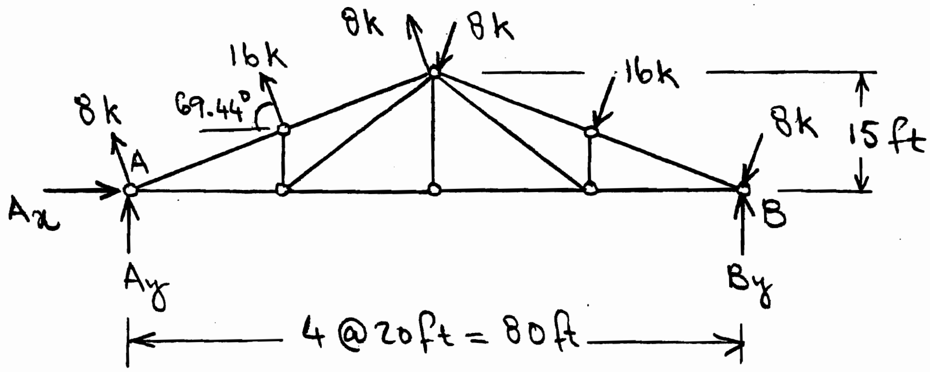
$$A_y = 12\left(\frac{6}{4}\right) + (24+15)\left(\frac{5}{4}\right) + 24\left(\frac{3}{4} + \frac{1}{2} + \frac{1}{4}\right)$$

$$\underline{A_y = 102.75 \text{ k} \uparrow}$$

$$B_y = 12 + 15 + 4(24) - 102.75$$

$$\underline{B_y = 20.25 \text{ k} \uparrow}$$

3.18



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad A_x - (8+16+8+8+16+8)\cos 69.44° = 0$

$\underline{A_x = 22.48\text{ k} \rightarrow}$

$+\circlearrowleft \Sigma M_B = 0 \qquad -A_y(80) + 2[16(7.5) + 8(15)]\cos 69.44°$

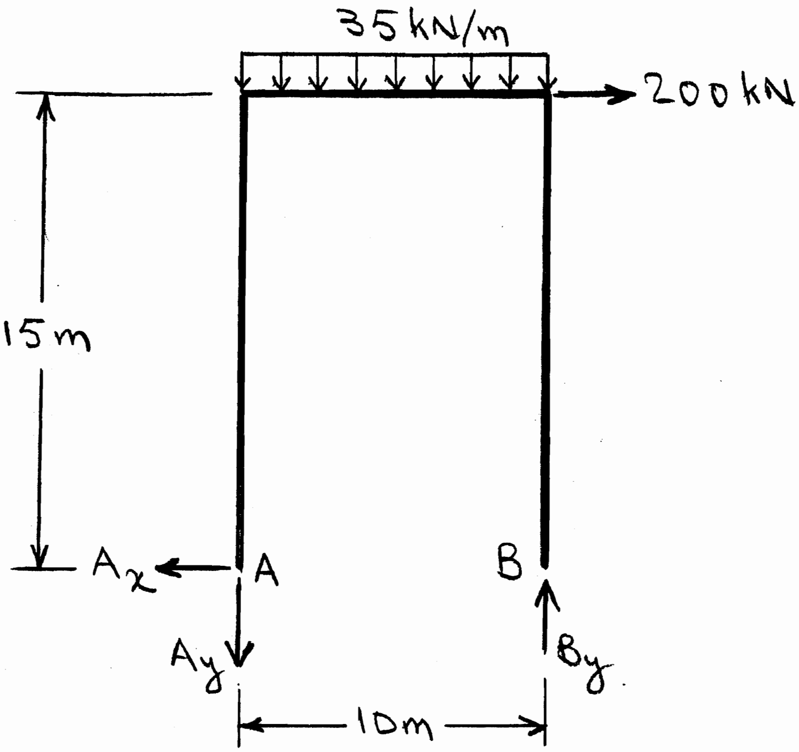
$+ [-8(80) - 16(60) + 16(20)]\sin 69.44° = 0$

$A_y = -12.87\text{ k} \qquad \underline{A_y = 12.87\text{ k} \downarrow}$

$+\uparrow \Sigma F_y = 0 \qquad -12.87 + (8+16+8-8-16-8)\sin 69.44° + B_y = 0$

$\underline{B_y = 12.87\text{ k} \uparrow}$

3.19



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$\underline{A_x = 200 \text{ kN} \leftarrow}$

$+\circlearrowright \Sigma M_B = 0$

$$A_y(10) + 35(10)(5) - 200(15) = 0$$

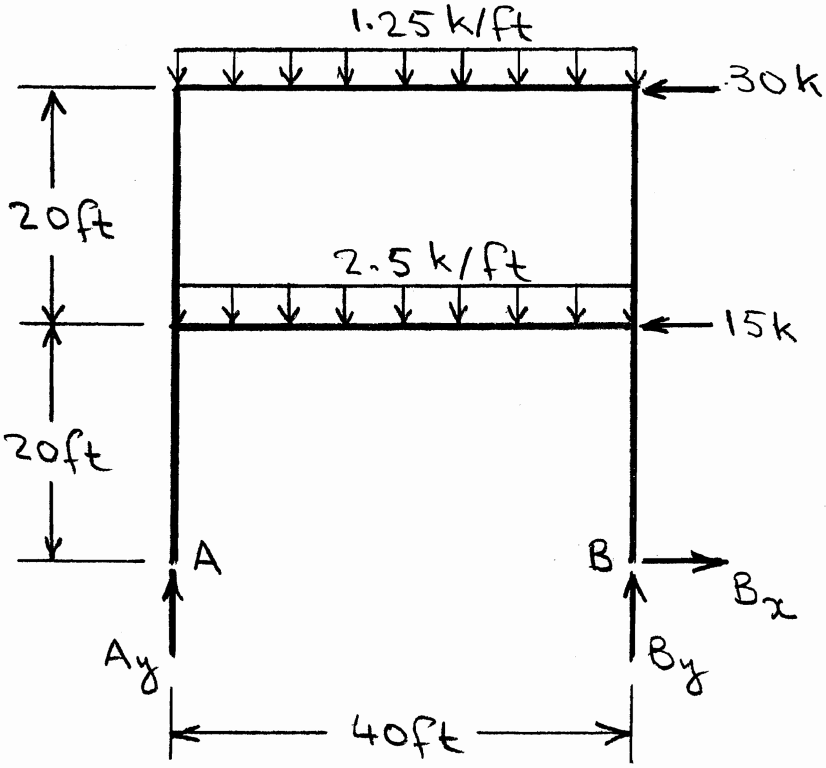
$\underline{A_y = 125 \text{ kN} \downarrow}$

$+\uparrow \Sigma F_y = 0$

$$-125 - 35(10) + B_y = 0$$

$\underline{B_y = 475 \text{ kN} \uparrow}$

3.20



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad B_x - 15 - 30 = 0 \qquad \underline{B_x = 45\text{ k} \rightarrow}$

$+\circlearrowleft \Sigma M_B = 0$

$$-A_y(40) + 2.5(40)(20) + 15(20) + 1.25(40)(20) + 30(40) = 0$$

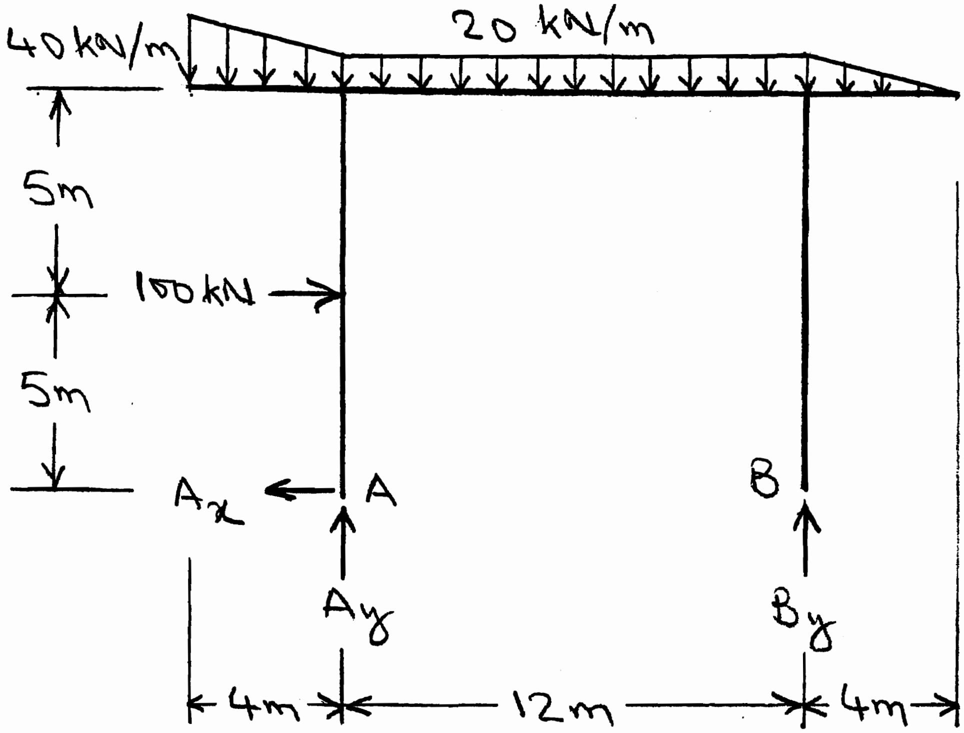
$\underline{A_y = 112.5\text{ k} \uparrow}$

$+\uparrow \Sigma F_y = 0$

$$112.5 - 2.5(40) - 1.25(40) + B_y = 0$$

$\underline{B_y = 37.5\text{ k} \uparrow}$

3.21



$\overset{+}{\rightarrow} \sum F_x = 0 \qquad 100 - A_x = 0$

$\underline{A_x = 100 \text{ kN} \leftarrow}$

$+\circlearrowleft \sum M_A = 0$

$-100(5) + \frac{1}{2}(20)(4)\left(\frac{2}{3}\right)4 - 20(16)4 + B_y(12)$

$-\frac{1}{2}(20)4\left(12 + \frac{4}{3}\right) = 0$

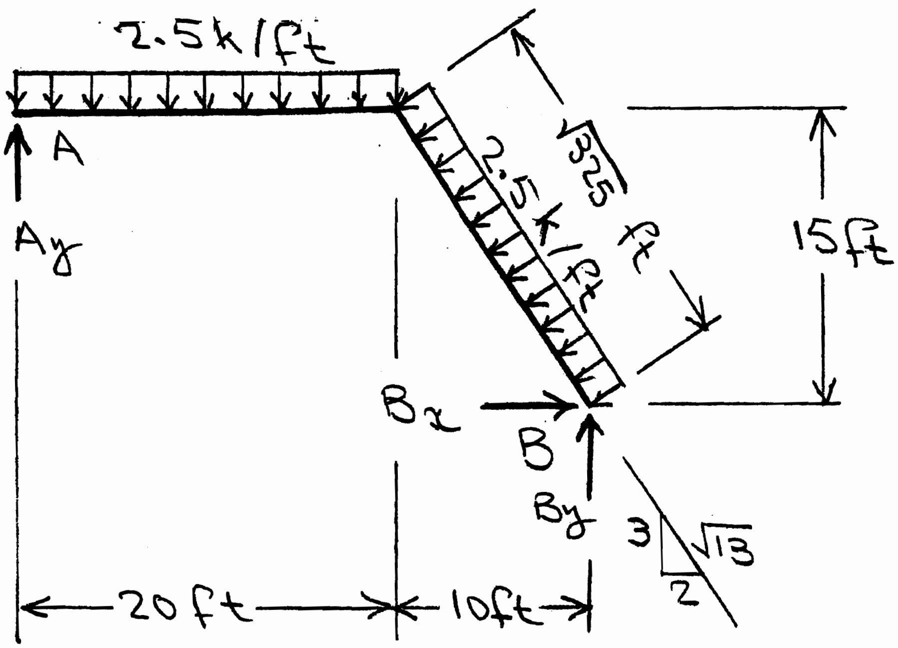
$\underline{B_y = 183.89 \text{ kN} \uparrow}$

$+\uparrow \sum F_y = 0$

$A_y - \frac{1}{2}(20)4 - 20(16) - \frac{1}{2}(20)4 + 183.89 = 0$

$\underline{A_y = 216.11 \text{ kN} \uparrow}$

3.22



$\xrightarrow{+} \Sigma F_x = 0 \qquad B_x - 2.5(\sqrt{325})\left(\frac{3}{\sqrt{13}}\right) = 0$

$$\underline{B_x = 37.5\text{ k} \rightarrow}$$

$+\circlearrowright \Sigma M_B = 0$

$$-A_y(30) + 2.5(20)(20) + 2.5(\sqrt{325})\left(\frac{\sqrt{325}}{2}\right) = 0$$

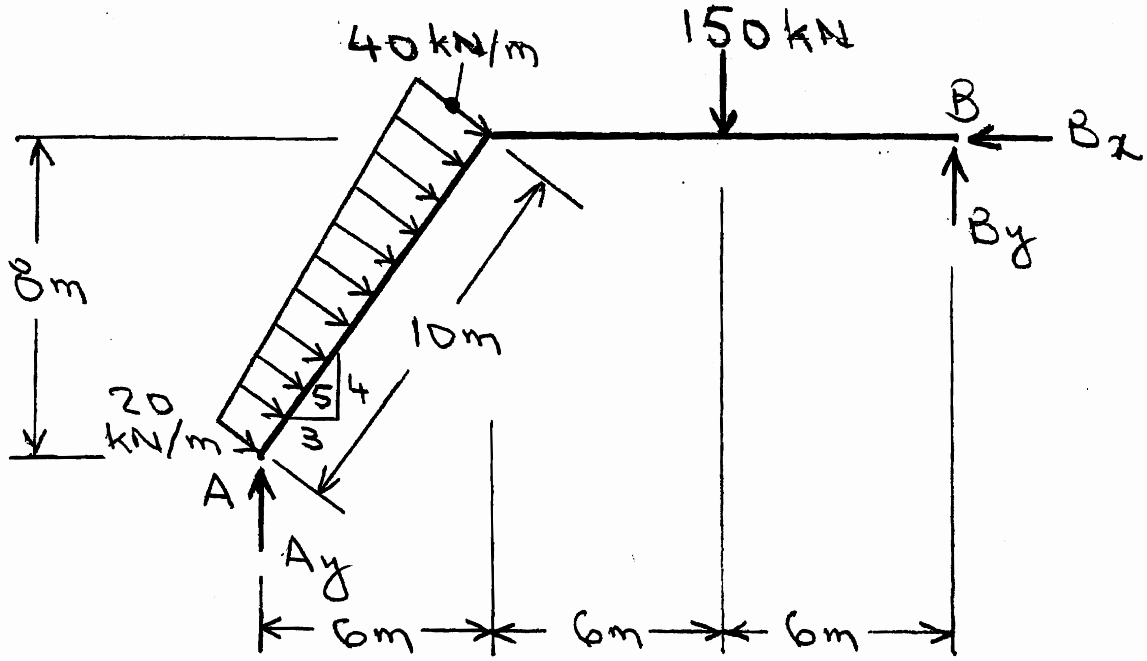
$$\underline{A_y = 46.875\text{ k} \uparrow}$$

$+\uparrow \Sigma F_y = 0$

$$46.875 - 2.5(20) - 2.5(\sqrt{325})\left(\frac{2}{\sqrt{13}}\right) + B_y = 0$$

$$\underline{B_y = 28.125\text{ k} \uparrow}$$

3.23



$$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad \left(\frac{20+40}{2}\right)10\left(\frac{4}{5}\right) - B_x = 0$$

$$\underline{B_x = 240 \text{ kN} \leftarrow}$$

$$+\circlearrowright \Sigma M_A = 0$$

$$-20(10)5 - \frac{1}{2}(20)10\left(\frac{20}{3}\right) - 150(12) + 240(8) + B_y(18) = 0$$

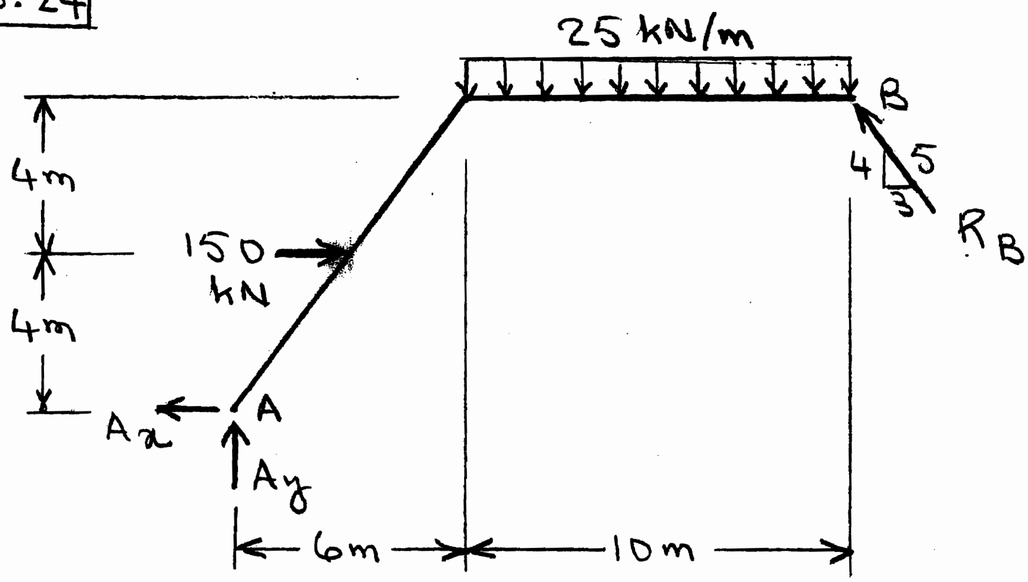
$$\underline{B_y = 85.93 \text{ kN} \uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$A_y - \left(\frac{20+40}{2}\right)10\left(\frac{3}{5}\right) - 150 + 85.93 = 0$$

$$\underline{A_y = 244.07 \text{ kN} \uparrow}$$

3.24



$+\circlearrowright \Sigma M_A = 0$

$-150(4) - 25(10)(11) + \frac{3}{5}R_B(8) + \frac{4}{5}R_B(16) = 0$

$R_B = 190.3$ kN ↖

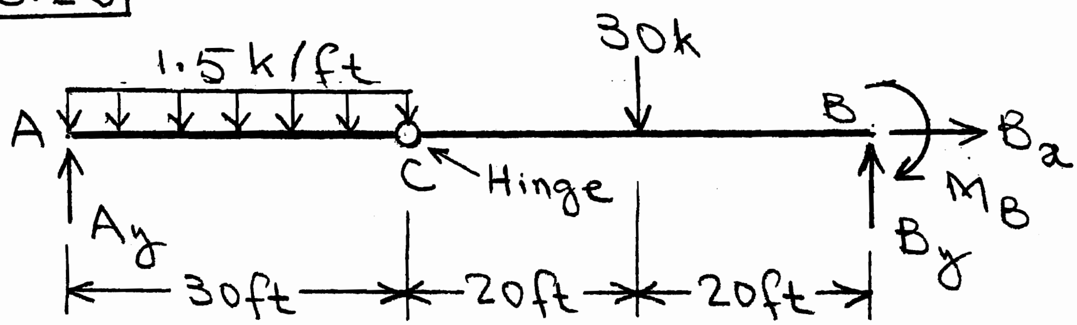
$\xrightarrow{+} \Sigma F_x = 0 \qquad -A_x + 150 - \frac{3}{5}(190.3) = 0$

$A_x = 35.8$ kN ←

$+\uparrow \Sigma F_y = 0 \qquad A_y - 25(10) + \frac{4}{5}(190.3) = 0$

$A_y = 97.7$ kN ↑

3.25



$\sum F_x = 0$ $\qquad$ $\underline{B_x = 0}$

$+\circlearrowright \sum M_C^{AC} = 0$ $\qquad -A_y(30) + 1.5(30)15 = 0$

$\underline{A_y = 22.5\text{ k}\uparrow}$

$+\uparrow \sum F_y = 0$ $\qquad 22.5 - 1.5(30) - 30 + B_y = 0$

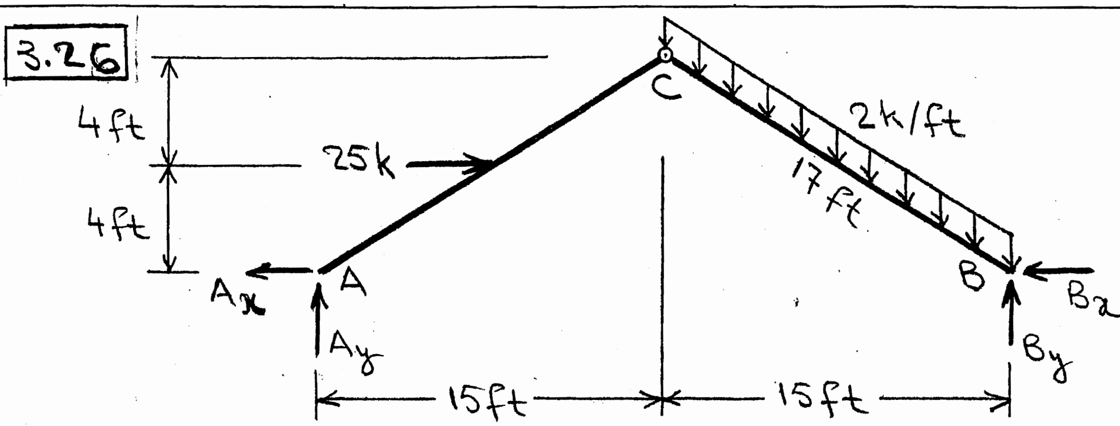
$\underline{B_y = 52.5\text{ k}\uparrow}$

$+\circlearrowright \sum M_A = 0$

$-1.5(30)15 - 30(50) + 52.5(70) - M_B = 0$

$\underline{M_B = 1500\text{ k-ft} \circlearrowright}$

3.26



$+\circlearrowleft \Sigma M_B = 0$

$-A_y(30) - 25(4) + 2(17)\left(\frac{15}{2}\right) = 0$ $\qquad$ $\underline{A_y = 5.17\ k \uparrow}$

$+\circlearrowleft \Sigma M_C^{AC} = 0$

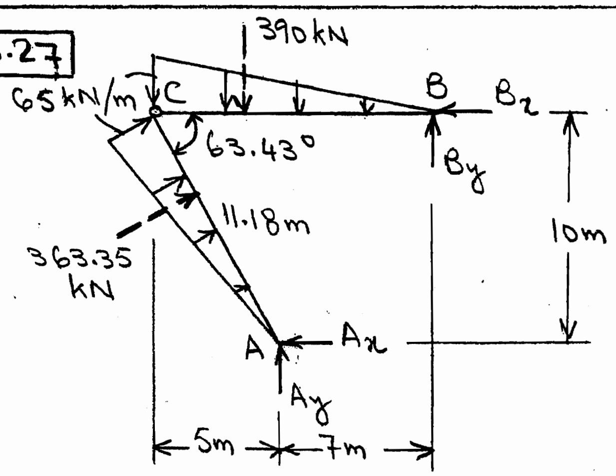
$-5.17(15) - A_x(8) + 25(4) = 0$ $\qquad$ $\underline{A_x = 2.81\ k \leftarrow}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$-2.81 + 25 - B_x = 0$ $\qquad$ $\underline{B_x = 22.19\ k \leftarrow}$

$+\uparrow \Sigma F_y = 0 \qquad 5.17 - 2(17) + B_y = 0$ $\qquad$ $\underline{B_y = 28.83\ k \uparrow}$

3.27



$+\circlearrowleft \Sigma M_C^{BC} = 0$

$-390\left(\frac{12}{3}\right) + B_y(12) = 0$

$\underline{B_y = 130\ kN \uparrow}$

$+\circlearrowleft \Sigma M_A = 0$

$B_x(10) + 130(7) + 390(1)$

$-363.35\left(\frac{2}{3}\right)11.18 = 0$

$\underline{B_x = 140.82\ kN \leftarrow}$

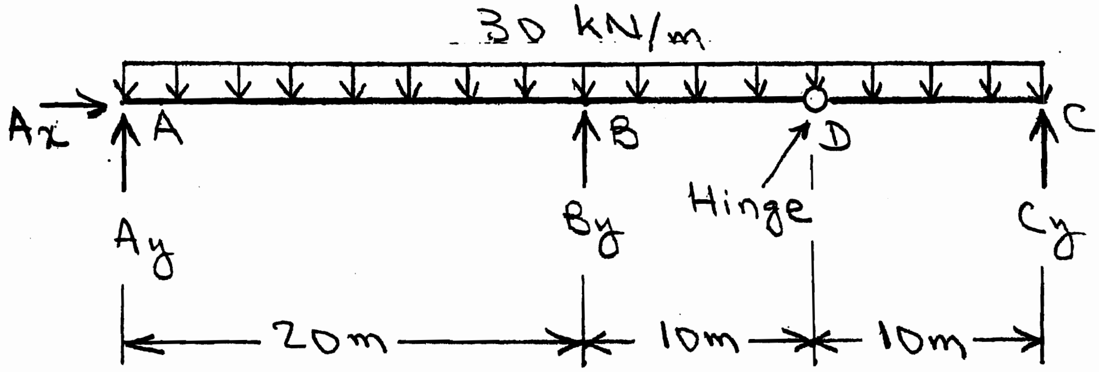
$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 363.35 \sin 63.43^\circ - A_x - 140.82 = 0$

$\underline{A_x = 184.16\ kN \leftarrow}$

$+\uparrow \Sigma F_y = 0 \qquad A_y + 363.35 \cos 63.43^\circ - 390 + 130 = 0$

$\underline{A_y = 97.48\ kN \uparrow}$

3.28



$\Sigma F_x = 0$ $\qquad\qquad$ $\underline{A_x = 0}$

$+\circlearrowright \Sigma M_D^{CD} = 0$ $\qquad$ $C_y(10) - 30(10)5 = 0$

$$\underline{C_y = 150 \text{ kN}\uparrow}$$

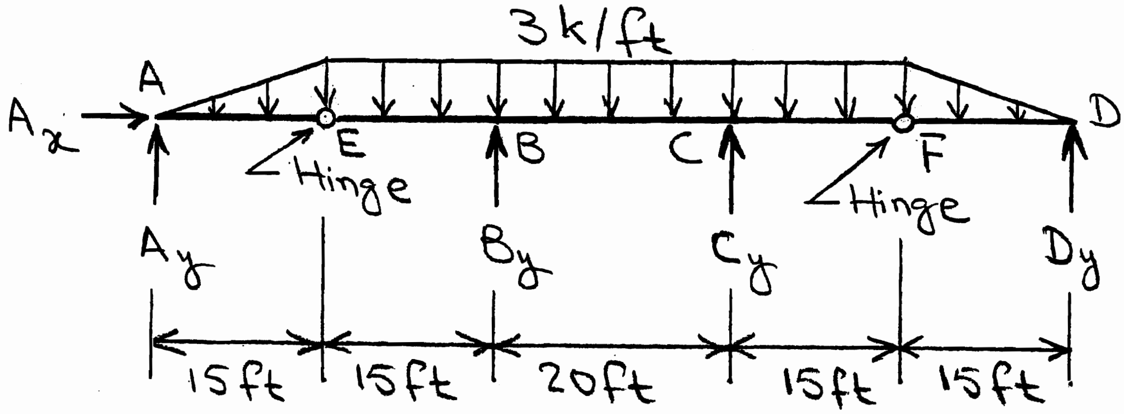
$+\circlearrowright \Sigma M_A = 0$ $\qquad$ $-30(40)20 + B_y(20) + 150(40) = 0$

$$\underline{B_y = 900 \text{ kN}\uparrow}$$

$+\uparrow \Sigma F_y = 0$ $\qquad$ $A_y - 30(40) + 900 + 150 = 0$

$$\underline{A_y = 150 \text{ kN}\uparrow}$$

3.29



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowleft \Sigma M_E^{AE} = 0 \qquad -A_y(15) + \frac{1}{2}(3)15(5) = 0$

$\underline{A_y = 7.5\text{ k}\uparrow}$

$+\circlearrowleft \Sigma M_F^{DF} = 0 \qquad D_y(15) - \frac{1}{2}(3)15(5) = 0$

$\underline{D_y = 7.5\text{ k}\uparrow}$

$+\circlearrowleft \Sigma M_C = 0$

$-7.5(50) + \frac{1}{2}(3)15(40) + 3(50)10 - B_y(20)$

$-\frac{1}{2}(3)15(20) + 7.5(30) = 0$

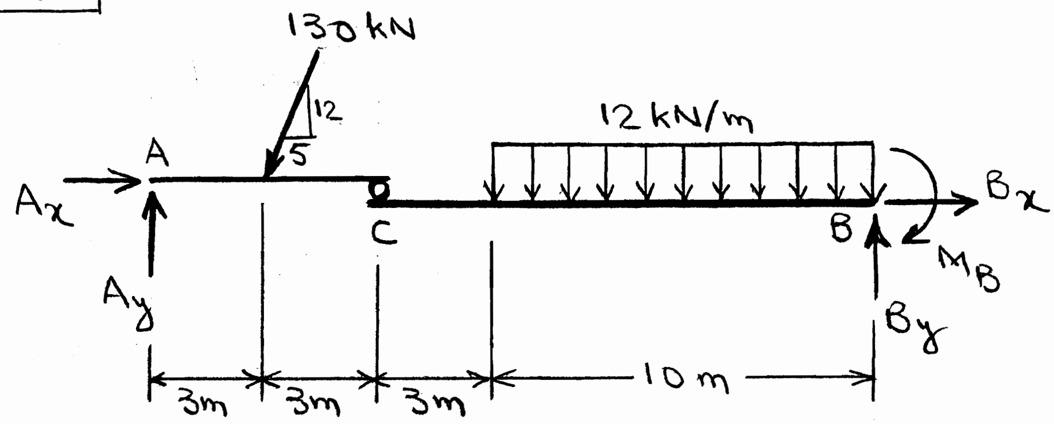
$\underline{B_y = 90\text{ k}\uparrow}$

$+\uparrow \Sigma F_y = 0$

$7.5 - \frac{1}{2}(3)15 - 3(50) - \frac{1}{2}(3)15 + 90 + C_y + 7.5 = 0$

$\underline{C_y = 90\text{ k}\uparrow}$

3.30



$\overset{+}{\rightarrow} \Sigma F_x^{AC} = 0 \qquad A_x - \frac{5}{13}(130) = 0 \qquad \underline{A_x = 50 \text{ kN} \rightarrow}$

$+\circlearrowright \Sigma M_C^{AC} = 0 \qquad -A_y(6) + \frac{12}{13}(130)(3) = 0 \qquad \underline{A_y = 60 \text{ kN} \uparrow}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 50 - \frac{5}{13}(130) + B_x = 0 \qquad \underline{B_x = 0}$

$+\uparrow \Sigma F_y = 0 \qquad 60 - \frac{12}{13}(130) - 12(10) + B_y = 0$

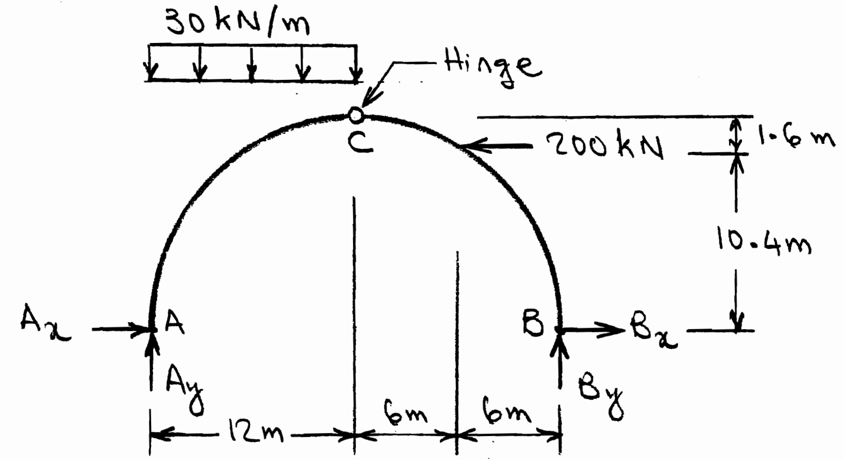
$\underline{B_y = 180 \text{ kN} \uparrow}$

$+\circlearrowright \Sigma M_A = 0$

$$-\frac{12}{13}(130)(3) - 12(10)(9+5) + 180(19) - M_B = 0$$

$\underline{M_B = 1380 \text{ kN}\cdot\text{m} \circlearrowright}$

3.31



$+\circlearrowleft \Sigma M_B = 0 \qquad -A_y(24) + 30(12)(18) + 200(10.4) = 0$

$\underline{A_y = 356.67 \text{ kN} \uparrow}$

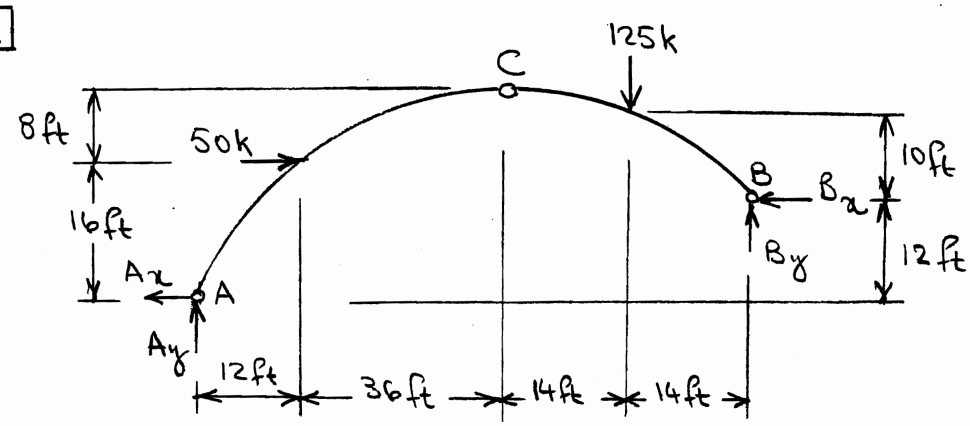
$+\circlearrowleft \Sigma M_C^{AC} = 0 \qquad A_x(12) - 356.67(12) + 30(12)(6) = 0$

$\underline{A_x = 176.67 \text{ kN} \rightarrow}$

$\Sigma F_x = 0 \qquad 176.67 - 200 + B_x = 0 \qquad \underline{B_x = 23.33 \text{ kN} \rightarrow}$

$\Sigma F_y = 0 \qquad 356.67 - 30(12) + B_y = 0 \qquad \underline{B_y = 3.33 \text{ kN} \uparrow}$

3.32



$+\circlearrowleft \Sigma M_C^{AC} = 0 \qquad -A_x(24) - A_y(48) + 50(8) = 0$

$3A_x + 6A_y = 50 \quad \text{——} \quad (1)$

$+\circlearrowleft \Sigma M_B = 0 \qquad -A_x(12) - A_y(76) - 50(4) + 125(14) = 0$

$6A_x + 38A_y = 775 \quad \text{——} \quad (2)$

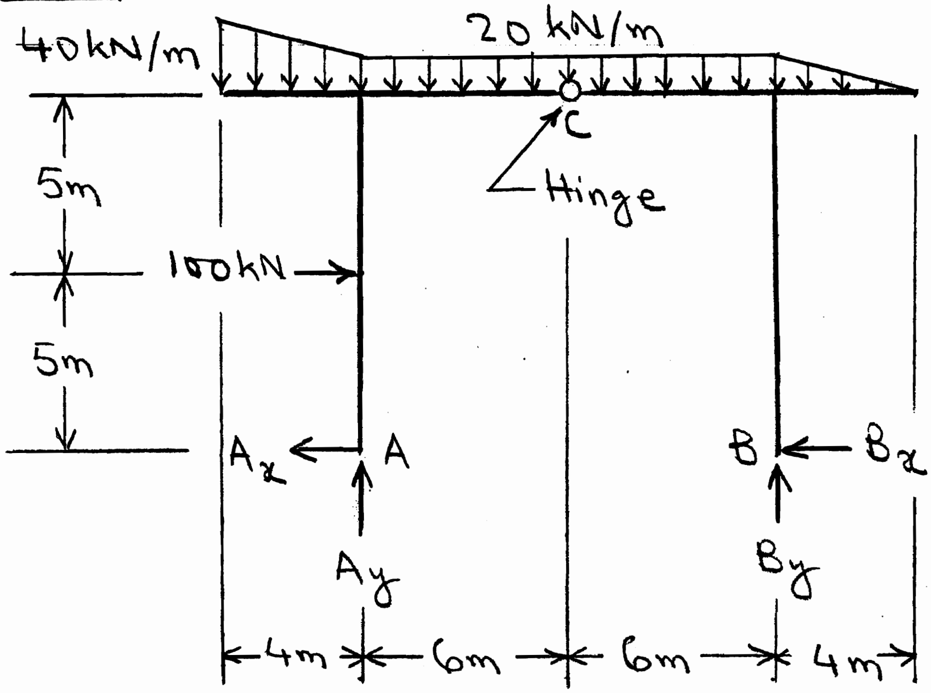
Solving (1) and (2) simultaneously, we obtain

$A_x = -35.25\text{k} = \underline{35.25\text{k} \rightarrow} \qquad \underline{A_y = 25.96 \text{ k} \uparrow}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 35.25 + 50 - B_x = 0 \qquad \underline{B_x = 85.25 \text{ k} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \qquad 25.96 - 125 + B_y = 0 \qquad \underline{B_y = 99.04 \text{ k} \uparrow}$

3.33



$$+\circlearrowleft \Sigma M_A = 0$$

$$-100(5) + \frac{1}{2}(20)(4)\left(\frac{8}{3}\right) - 20(16)4 - \frac{1}{2}(20)4\left(12+\frac{4}{3}\right) + B_y(12) = 0$$

$$\underline{B_y = 183.89 \text{ kN} \uparrow}$$

$$+\circlearrowleft \Sigma M_C^{BC} = 0$$

$$-20(6)3 - \frac{1}{2}(20)4\left(6+\frac{4}{3}\right) + 183.89(6) - B_x(10) = 0$$

$$\underline{B_x = 45 \text{ kN} \leftarrow}$$

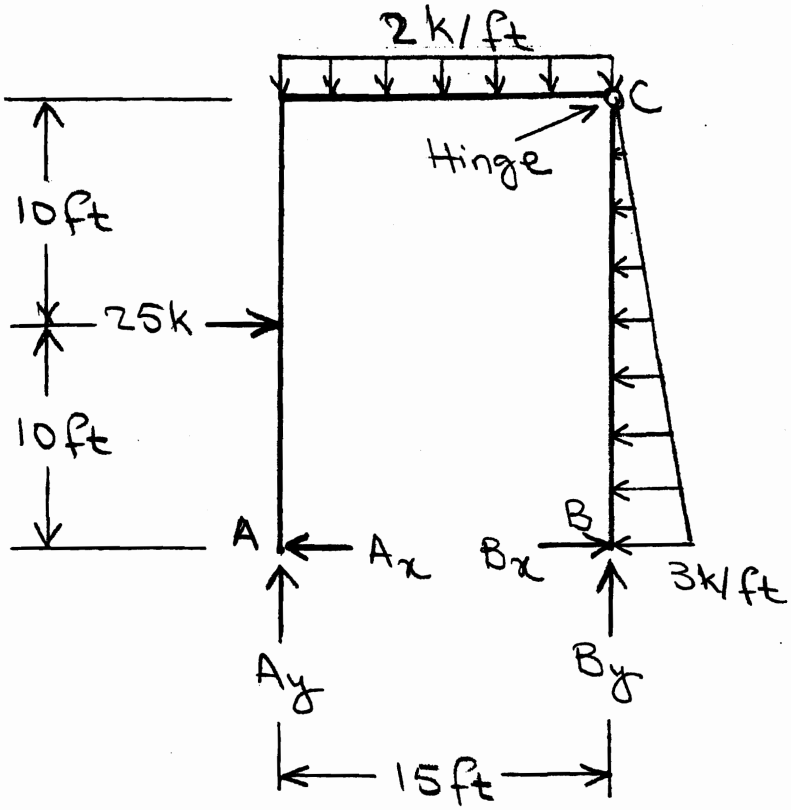
$$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 100 - A_x - 45 = 0$$

$$\underline{A_x = 55 \text{ kN} \leftarrow}$$

$$+\uparrow \Sigma F_y = 0 \qquad A_y - \frac{1}{2}(20)4 - 20(16) - \frac{1}{2}(20)4 + 183.89 = 0$$

$$\underline{A_y = 216.11 \text{ kN} \uparrow}$$

3.34



$+\circlearrowright \sum M_A = 0$

$$-25(10) - 2(15)7.5 + \tfrac{1}{2}(3)20\left(\tfrac{20}{3}\right) + B_y(15) = 0$$

$$\underline{B_y = 18.33\ k \uparrow}$$

$+\sum M_C^{BC} = 0 \qquad -\tfrac{1}{2}(3)20\left(\tfrac{40}{3}\right) + B_x(20) = 0$

$$\underline{B_x = 20\ k \rightarrow}$$

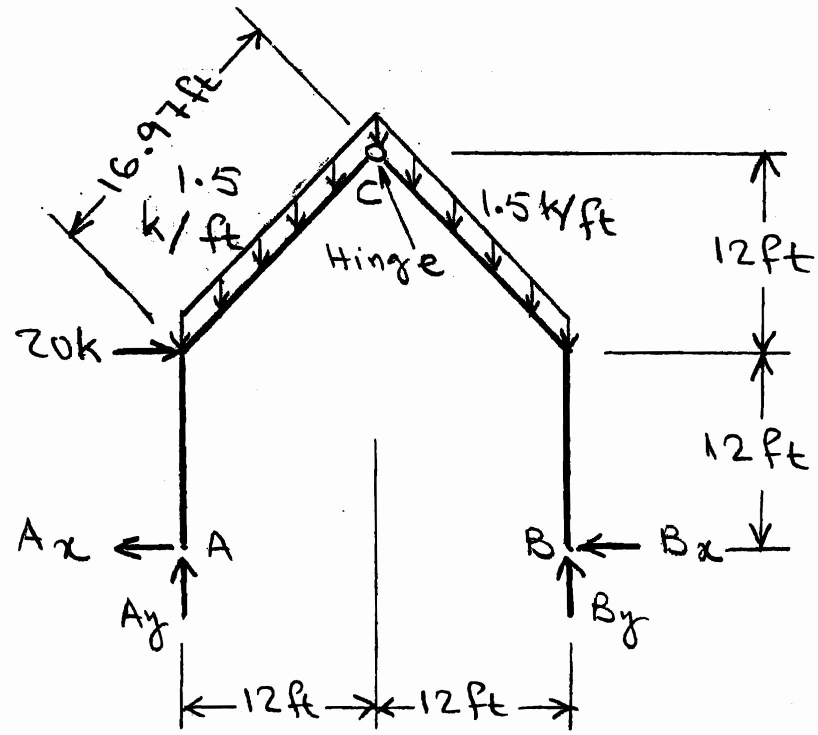
$\overset{+}{\rightarrow} \sum F_x = 0 \qquad -A_x + 25 - \tfrac{1}{2}(3)20 + 20 = 0$

$$\underline{A_x = 15\ k \leftarrow}$$

$+\uparrow \sum F_y = 0 \qquad A_y - 2(15) + 18.33 = 0$

$$\underline{A_y = 11.67\ k \uparrow}$$

3.35



$+\circlearrowleft \Sigma M_B = 0$

$$-A_y(24) - 20(12) + 1.5(16.97)(18) + 1.5(16.97)(6) = 0$$

$$\underline{A_y = 15.46 \text{ k} \uparrow}$$

$+\circlearrowleft \Sigma M_C^{AC} = 0$

$$-A_x(24) - 15.46(12) + 20(12) + 1.5(16.97)(6) = 0$$

$$\underline{A_x = 8.63 \text{ k} \leftarrow}$$

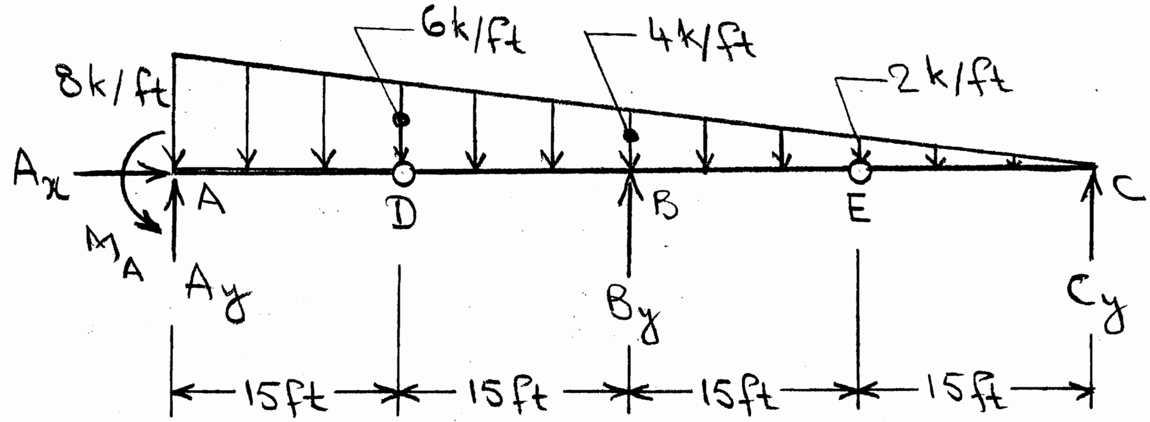
$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad -8.63 + 20 - B_x = 0$

$$\underline{B_x = 11.37 \text{ k} \leftarrow}$$

$+\uparrow \Sigma F_y = 0 \qquad 15.46 - 2(1.5)(16.97) + B_y = 0$

$$\underline{B_y = 35.45 \text{ k} \uparrow}$$

3.36



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowright \Sigma M_E^{CE} = 0 \qquad C_y(15) - \frac{1}{2}(2)(15)(5) = 0$ $\qquad$ $\underline{C_y = 5\text{ k}\uparrow}$

$+\circlearrowright \Sigma M_D^{CD} = 0 \qquad 5(45) - \frac{1}{2}(6)(45)(15) + B_y(15) = 0$

$\underline{B_y = 120\text{ k}\uparrow}$

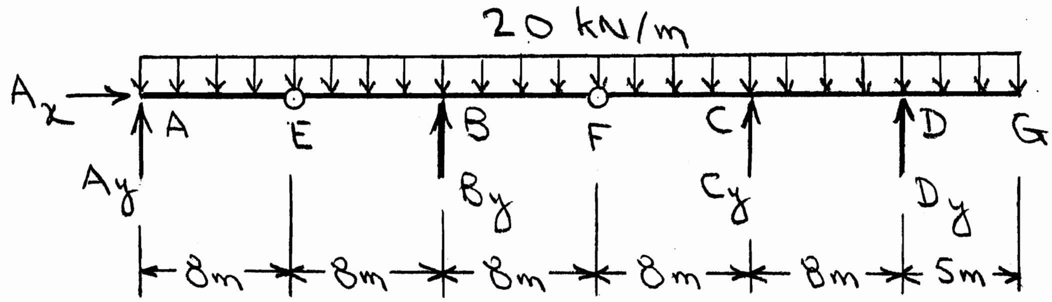
$+\circlearrowright \Sigma M_A = 0 \qquad M_A - \frac{1}{2}(8)(60)(20) + 120(30) + 5(60) = 0$

$\underline{M_A = 900\text{ k-ft}\circlearrowright}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - \frac{1}{2}(8)(60) + 120 + 5 = 0$

$\underline{A_y = 115\text{ k}\uparrow}$

3.37



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowleft \Sigma M_E^{AE} = 0 \qquad -A_y(8) + 20(8)(4) = 0$ $\qquad$ $\underline{A_y = 80 \text{ kN}\uparrow}$

$+\circlearrowleft \Sigma M_F^{AF} = 0 \qquad -80(24) + 20(24)(12) - B_y(8) = 0$

$\underline{B_y = 480 \text{ kN}\uparrow}$

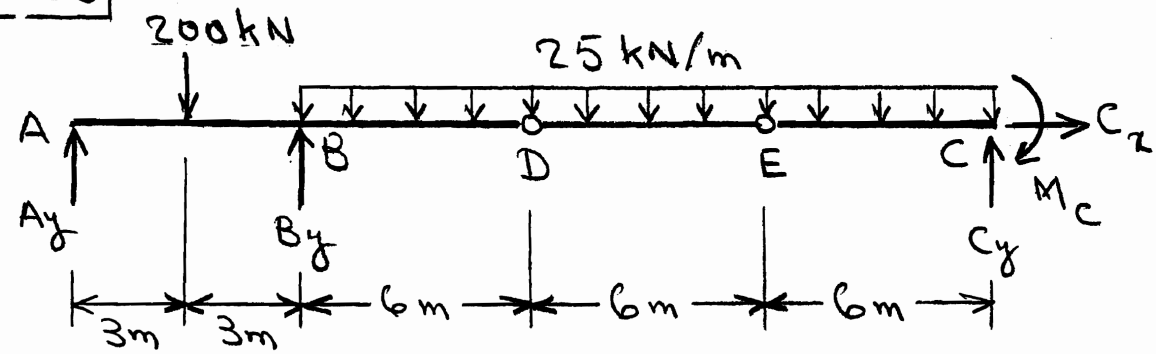
$+\circlearrowleft \Sigma M_D = 0 \qquad -80(40) - 480(24) - C_y(8) + 20(45)(17.5) = 0$

$\underline{C_y = 128.75 \text{ kN}\uparrow}$

$+\uparrow \Sigma F_y = 0 \qquad 80 + 480 + 128.75 + D_y - 20(45) = 0$

$\underline{D_y = 211.25 \text{ kN}\uparrow}$

3.38



$\Sigma F_x = 0$ $\qquad$ $\underline{C_x = 0}$

$+\circlearrowleft \Sigma M_D^{AD} = 0 \quad -A_y(12) + 200(9) - B_y(6) + 25(6)(3) = 0$

$$2A_y + B_y = 375 \qquad (1)$$

$+\circlearrowleft \Sigma M_E^{AE} = 0 \quad -A_y(18) + 200(15) - B_y(12) + 25(12)(6) = 0$

$$1.5A_y + B_y = 400 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

$$A_y = -50 \text{ kN} = \underline{50 \text{ kN} \downarrow}$$

$$\underline{B_y = 475 \text{ kN} \uparrow}$$

$+\uparrow \Sigma F_y = 0 \quad -50 - 200 + 475 - 25(18) + C_y = 0$

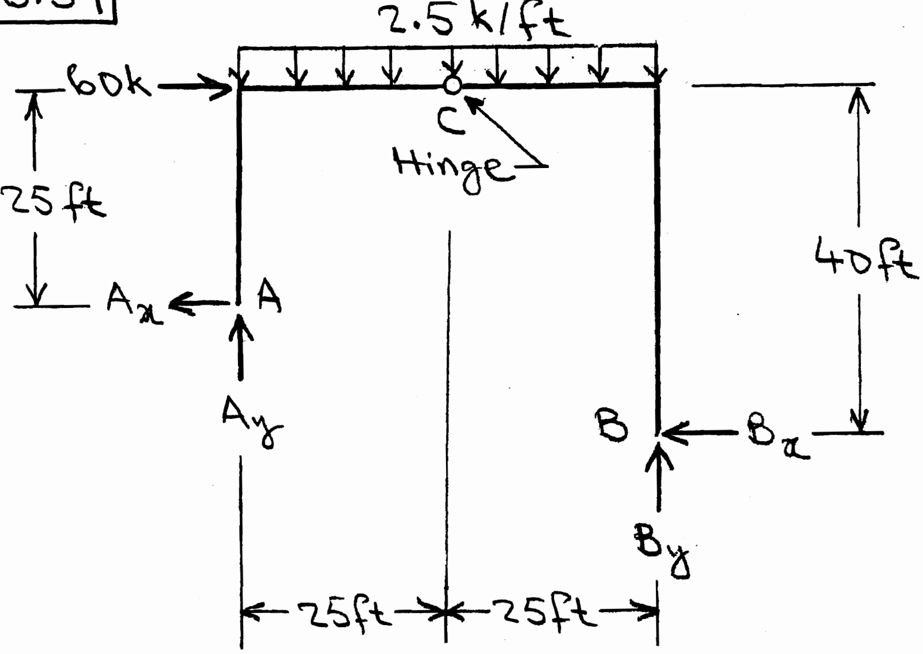
$$\underline{C_y = 225 \text{ kN} \uparrow}$$

$+\circlearrowleft \Sigma M_C = 0$

$$50(24) + 200(21) - 475(18) + 25(18)(9) - M_C = 0$$

$$\underline{M_C = 900 \text{ kN.m} \circlearrowright}$$

3.39



$+\circlearrowleft \sum M_A = 0$

$-60(25) - 2.5(50)25 - B_x(15) + B_y(50) = 0$

$3B_x - 10B_y = -925 \qquad (1)$

$+\circlearrowleft \sum M_C^{BC} = 0$

$-2.5(25)(12.5) - B_x(40) + B_y(25) = 0$

$8B_x - 5B_y = -156.25 \qquad (2)$

Solving Eqs. (1) and (2) simultaneously, we obtain

$B_x = 47.12 \text{ k} \leftarrow$

$B_y = 106.63 \text{ k} \uparrow$

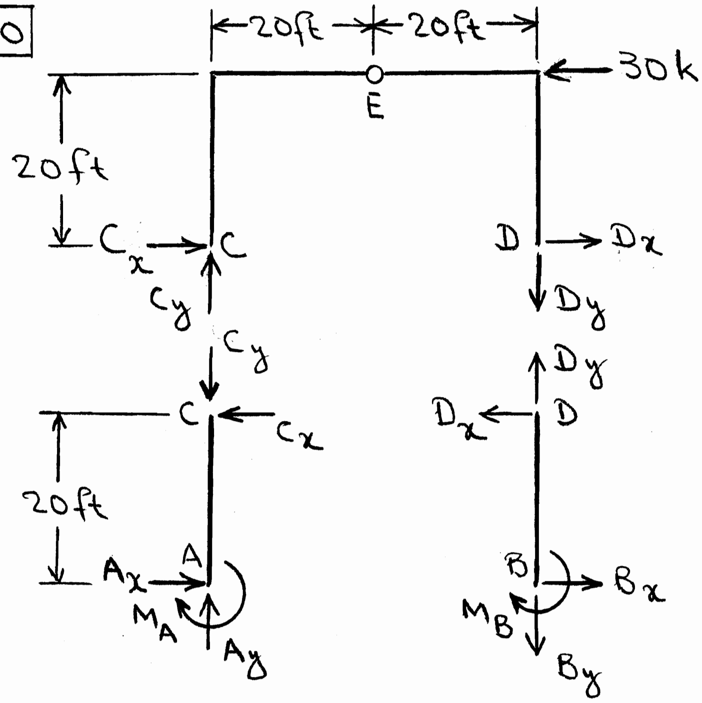
$\overset{+}{\rightarrow} \sum F_x = 0 \qquad -A_x + 60 - 47.12 = 0$

$A_x = 12.88 \text{ k} \leftarrow$

$+\uparrow \sum F_y = 0 \qquad A_y - 2.5(50) + 106.63 = 0$

$A_y = 18.37 \text{ k} \uparrow$

3.40



Considering the free body of portion CED, we determine the forces at the internal hinges C and D to be

$+\circlearrowleft \Sigma M_D = 0 \quad -C_y(40) + 30(20) = 0 \quad C_y = 15\text{ k}$

$+\circlearrowleft \Sigma M_E^{CE} = 0 \quad C_x(20) - 15(20) = 0 \quad C_x = 15\text{k}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad 15 - 30 + D_x = 0 \quad D_x = 15\text{k}$

$+\uparrow \Sigma F_y = 0 \quad 15 - D_y = 0 \quad D_y = 15\text{k}$

Considering the equilibrium of portion AC, we compute the reactions at support A to be

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad A_x - 15 = 0 \quad \underline{A_x = 15\text{ k} \rightarrow}$

$+\uparrow \Sigma F_y = 0 \quad A_y - 15 = 0 \quad \underline{A_y = 15\text{k} \uparrow}$

$+\circlearrowleft \Sigma M_A = 0 \quad 15(20) - M_A = 0 \quad \underline{M_A = 300\text{ k-ft} \circlearrowright}$

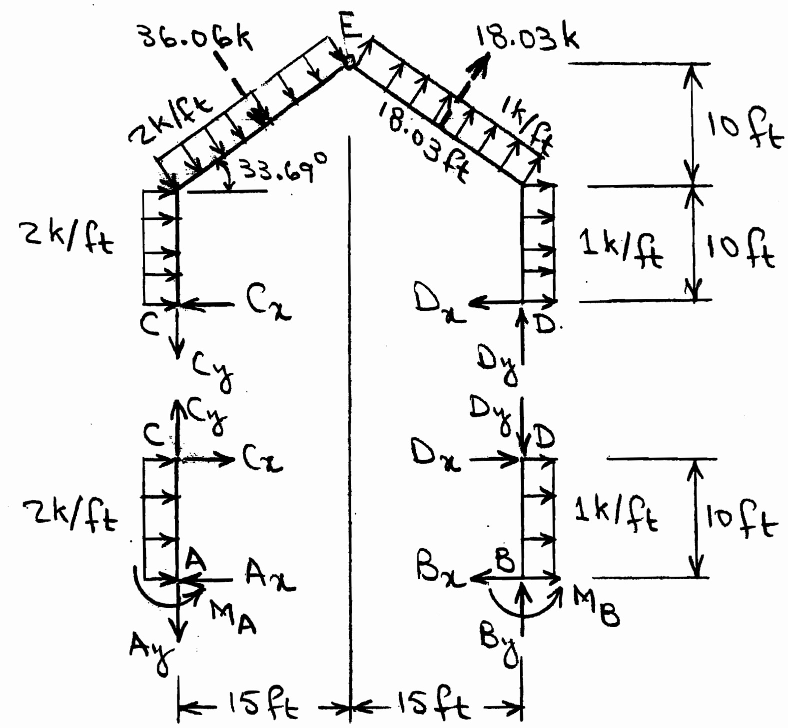
Similarly, considering the equilibrium of portion BD:

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad B_x - 15 = 0 \quad \underline{B_x = 15\text{k} \rightarrow}$

$+\uparrow \Sigma F_y = 0 \quad -B_y + 15 = 0 \quad \underline{B_y = 15\text{k} \downarrow}$

$+\circlearrowleft \Sigma M_B = 0 \quad 15(20) - M_B = 0 \quad \underline{M_B = 300\text{k-ft} \circlearrowright}$

3.41



Considering the free body of portion CED:

$+\circlearrowleft \Sigma M_D = 0 \quad C_y(30) - (2+1)(10)5 - (36.06+18.03)\sin 33.69°(15) + 36.06\cos 33.69°(22.5) - 18.03\cos 33.69°(7.5) = 0$

$C_y = 1.25\text{ k}$

$+\circlearrowleft \Sigma M_E^{CE} = 0 \quad -C_x(20) + 1.25(15) + 2(10)(15) + 36.06\left(\frac{18.03}{2}\right) = 0$

$C_x = 32.19\text{ k}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad -32.19 + (2+1)10 + (36.06+18.03)\sin 33.69° - D_x = 0$

$D_x = 27.81\text{ k}$

$+\uparrow \Sigma F_y = 0 \quad -1.25 - 36.06\cos 33.69° + 18.03\cos 33.69° + D_y = 0$

$D_y = 16.25\text{ k}$

Free body of portion AC:

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad 32.19 + 2(10) - A_x = 0 \qquad \underline{A_x = 52.19\text{ k} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \quad -A_y + 1.25 = 0 \qquad \underline{A_y = 1.25\text{ k} \downarrow}$

$+\circlearrowleft \Sigma M_A = 0 \quad M_A - 32.19(10) - 2(10)5 = 0 \qquad \underline{M_A = 421.9\text{ k-ft} \circlearrowleft}$

Free body of portion BD:

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad 27.81 + 1(10) - B_x = 0 \qquad \underline{B_x = 37.81\text{ k} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \quad B_y - 16.25 = 0 \qquad \underline{B_y = 16.25\text{ k} \uparrow}$

$+\circlearrowleft \Sigma M_B = 0 \quad M_B - 27.81(10) - 1(10)5 = 0 \qquad \underline{M_B = 328.1\text{ k-ft} \circlearrowleft}$

Chapter Four
Plane and Space Trusses

CHAPTER 4

4.1 (a) $m = 2,\ r = 3,\ j = 3;\ m + r < 2j;$ Unstable

(b) $m = 3,\ r = 3,\ j = 3;\ m + r = 2j$

Statically determinate

(c) $m = 2,\ r = 4,\ j = 3;\ m + r = 2j$

Statically determinate

(d) $m = 3,\ r = 4,\ j = 4;\ m + r < 2j;$ Unstable

4.2 (a) $m = 6,\ r = 3,\ j = 4;\ m + r > 2j$

Statically indeterminate; $i = (6 + 3) - 2(4) = \underline{1}$

(b) $m = 9,\ r = 3,\ j = 6;\ m + r = 2j$

Statically determinate

(c) $m = 9,\ r = 3,\ j = 6;\ m + r = 2j$

Unstable, because two rigid portions are connected by an internal hinge and three reactions are not sufficient to prevent relative rotation of one rigid part with respect to the other.

(d) $m = 7,\ r = 4,\ j = 5;\ m + r > 2j$

Statically indeterminate; $i = (7 + 4) - 2(5) = \underline{1}$

4.3 (a) $m = 17,\ r = 3,\ j = 9;\quad m + r > 2j$

Statically indeterminate; $i = (17 + 3) + 2(9) = \underline{2}$

(b) $m = 12,\ r = 3,\ j = 7;\quad m + r > 2j$

Statically indeterminate; $i = (12 + 3) - 2(7) = \underline{1}$

(c) $m = 25,\ r = 4,\ j = 14;\quad m + r > 2j$

Statically indeterminate; $i = (25 + 4) - 2(14) = \underline{1}$

(d) $m = 24,\ r = 6,\ j = 15;\quad m + r = 2j$

Statically determinate

4.4 (a) $m = 22,\ r = 4,\ j = 13;\quad m + r = 2j$

Statically determinate

(b) $m = 14,\ r = 3,\ j = 9;\quad m + r < 2j$; Unstable

(c) $m = 11,\ r = 3,\ j = 7;\quad m + r = 2j$

Statically determinate

(d) $m = 23,\ r = 4,\ j = 13;\quad m + r > 2j$

Statically indeterminate; $i = (23 + 4) - 2(13) = \underline{1}$

4.5 (a) $m = 14,\ r = 4,\ j = 9;\quad m + r = 2j$

Unstable, because two rigid portions are connected by a hinge.

(b) $m = 24,\ r = 3,\ j = 14;\quad m + r < 2j$; Unstable

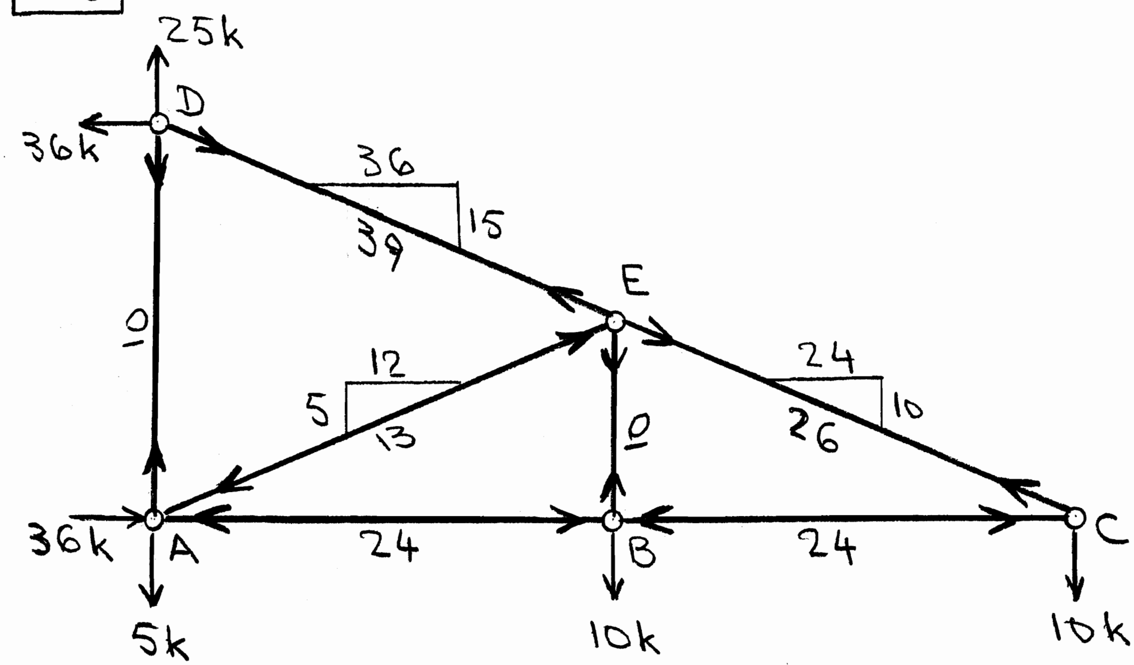
(c) $m = 9,\ r = 3,\ j = 6;\quad m + r = 2j$

Statically determinate

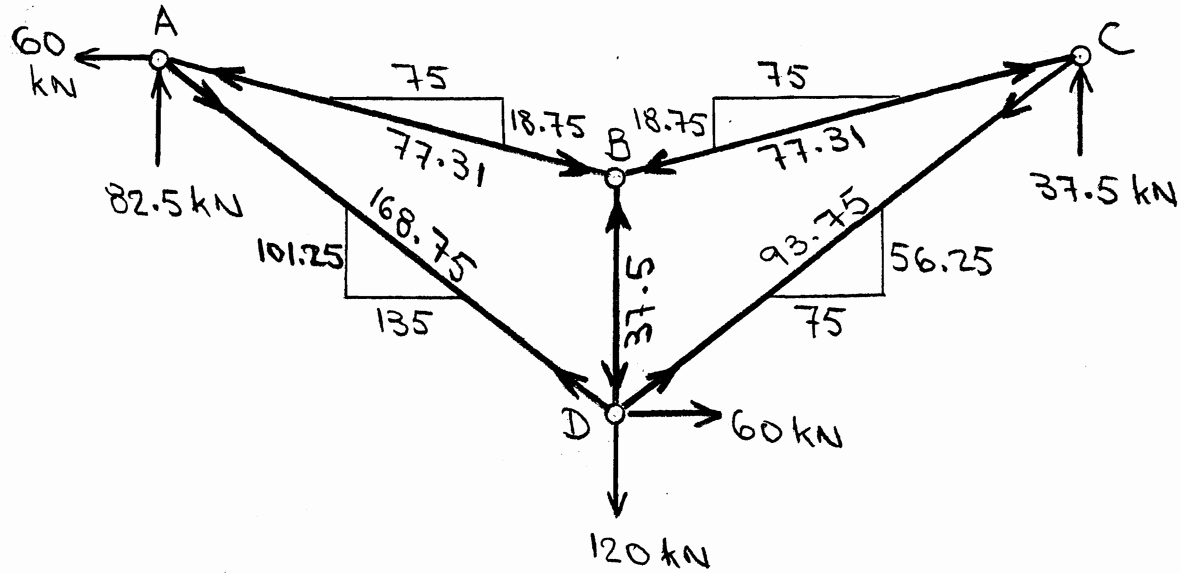
(d) $m = 25,\ r = 3,\ j = 14;\quad m + r = 2j$

Unstable, because two rigid portions are connected by three parallel members

4.6

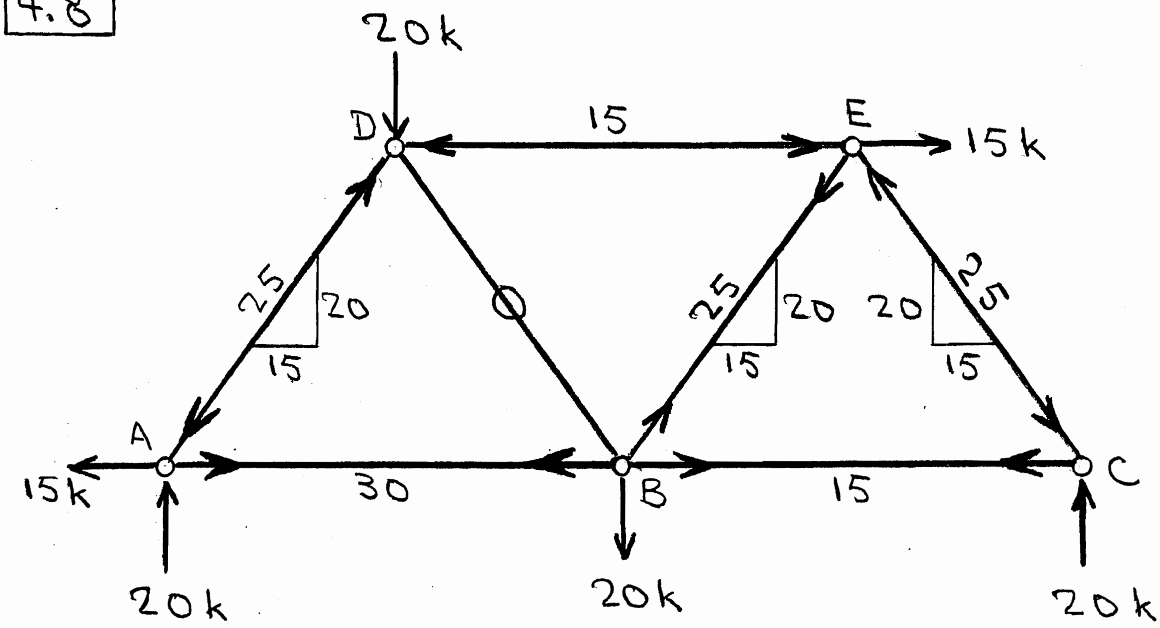


4.7

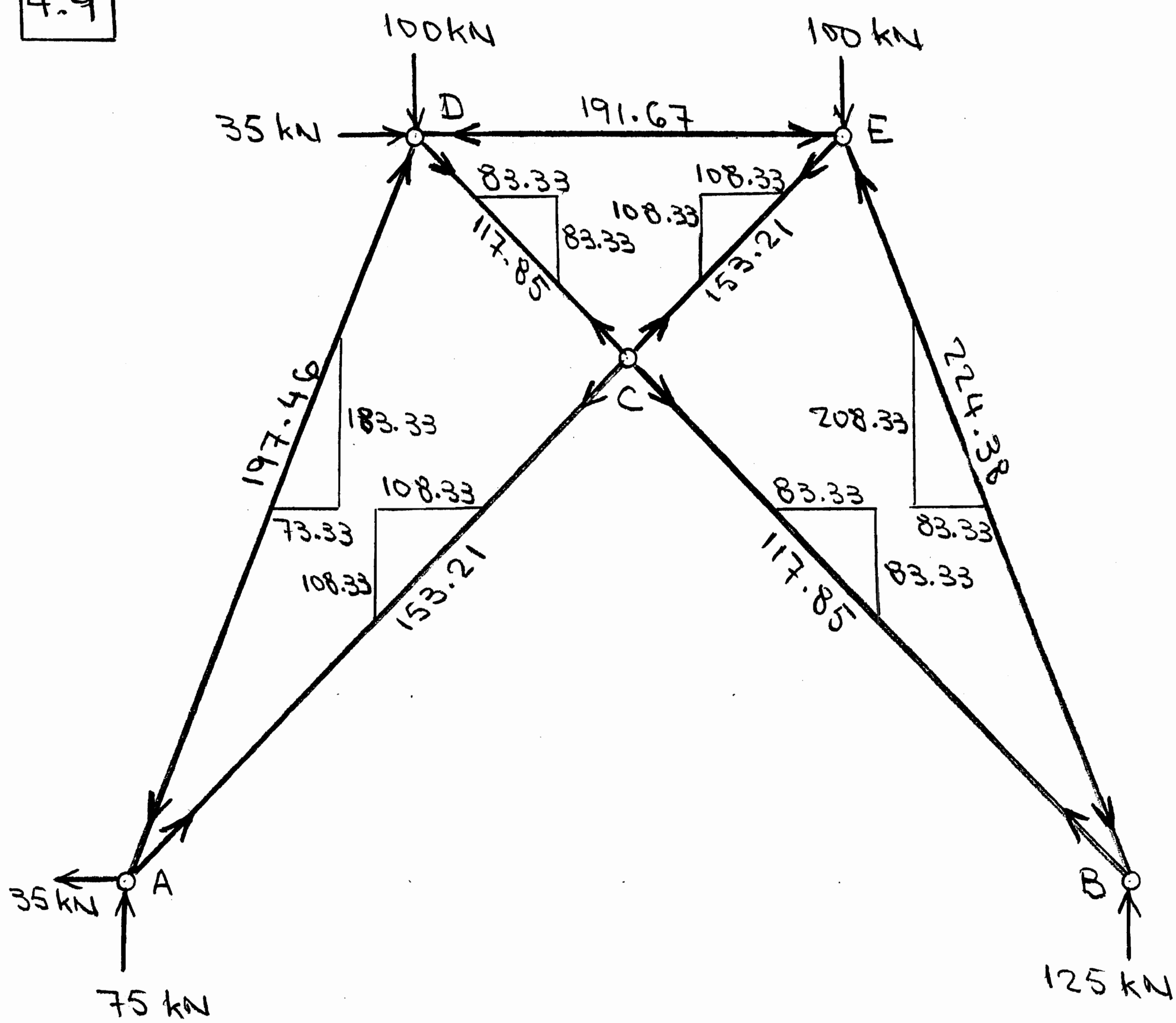


A
B
C
D
60 kN
82.5 kN
37.5 kN
60 kN
120 kN
75
18.75
77.31
75
18.75
77.31
168.75
101.25
135
37.5
93.75
56.25
75

4.8

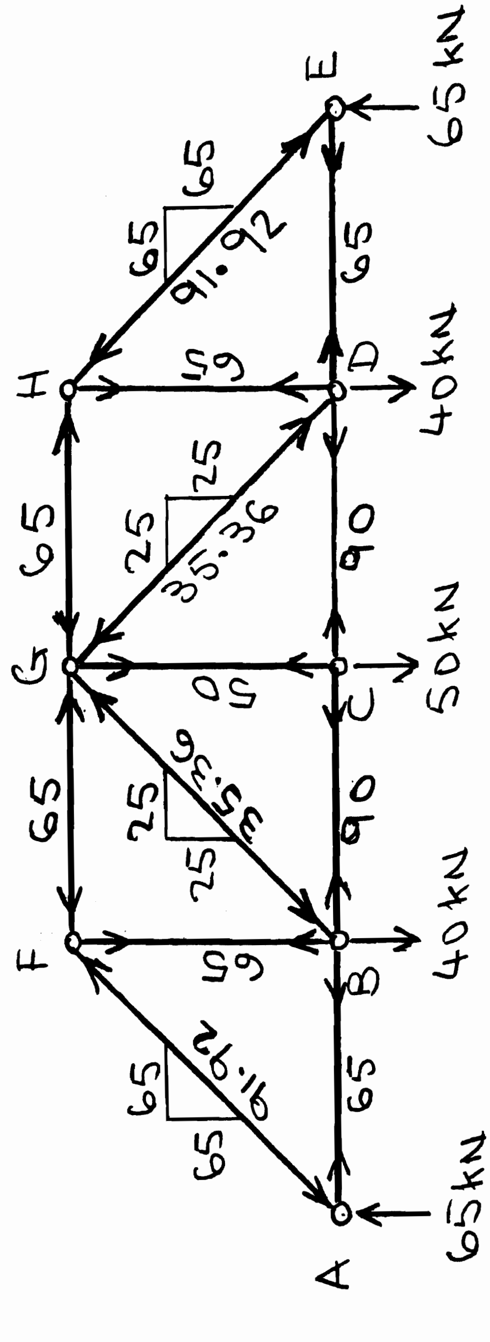


4.9

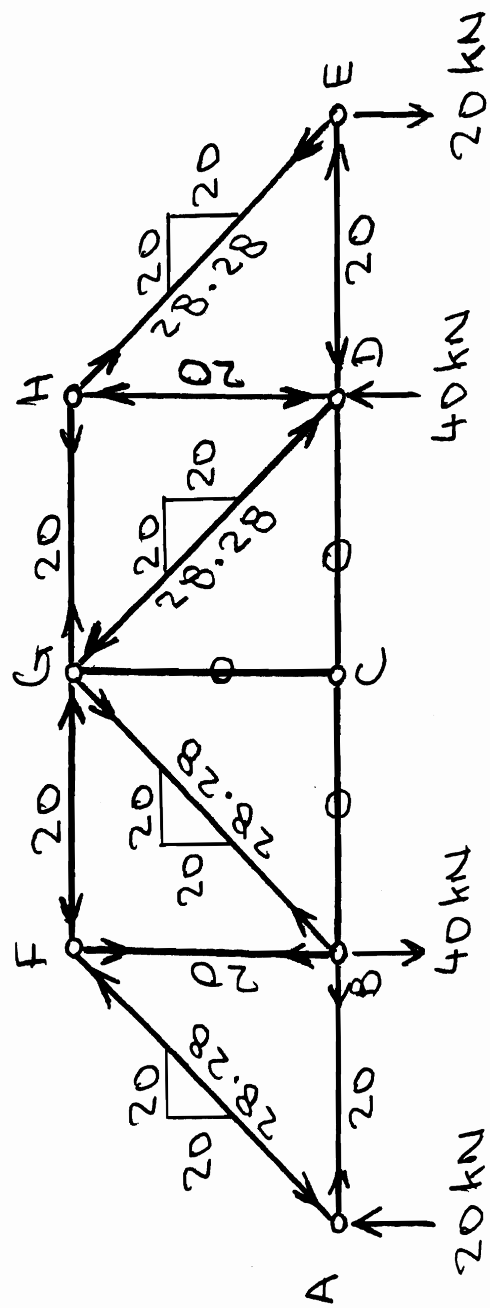


100kN
100 kN
35 kN
D
191.67
E
83.33
108.33
108.33
83.33
117.85
153.21
C
197.46
183.33
208.33
224.38
108.33
83.33
73.33
83.33
108.33
153.21
117.85
83.33
35 kN
A
B
75 kN
125 kN

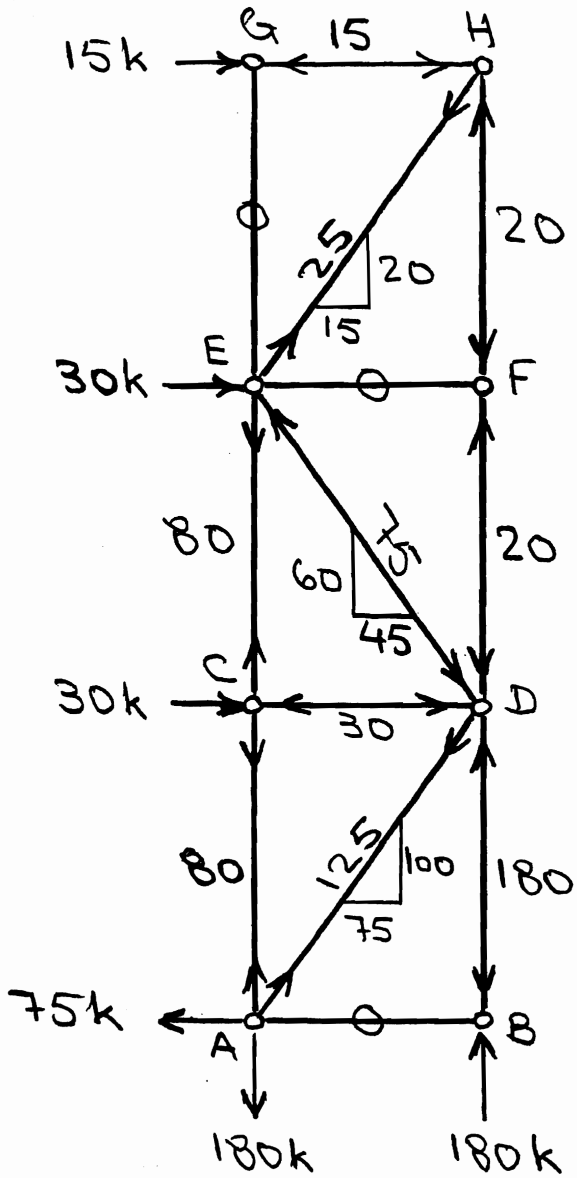
4.10


A
B
C
D
E
F
G
H
65 kN
40 kN
50 kN
40 kN
65 kN
65
91.92
65
65
65
25
25
35.36
50
25
25
35.36
65
65
90
90
65
65
65
91.92
65

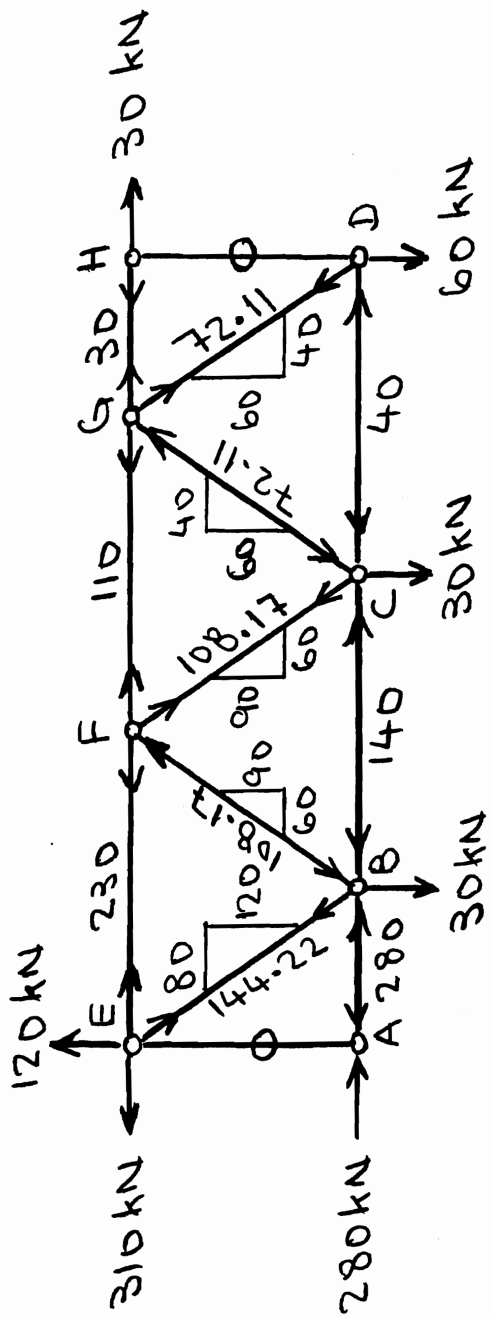
4.11



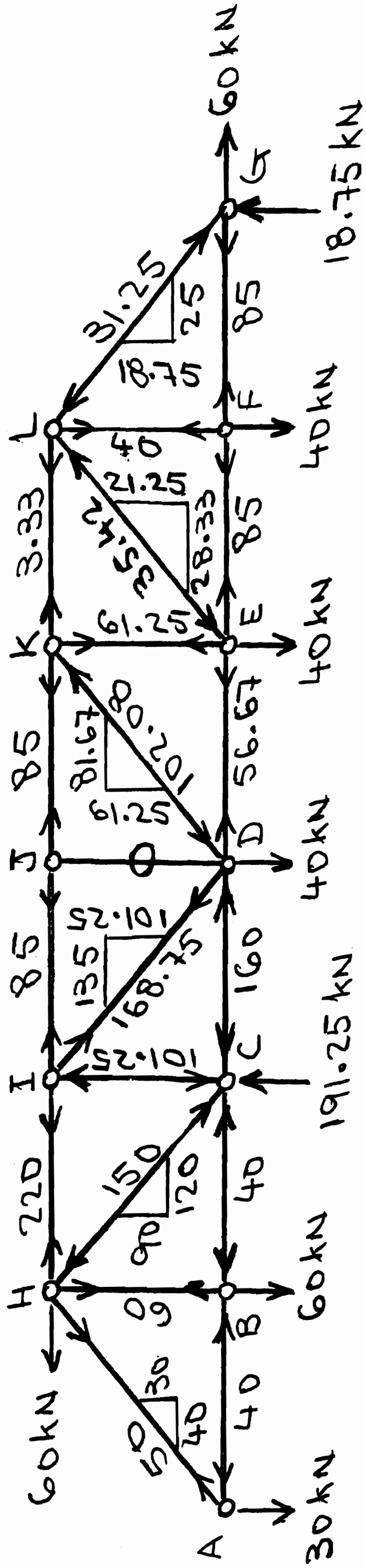
4.12


G
15
H
15k
0
20
25
20
15
E
30k
0
F
80
60
75
45
20
C
30k
30
D
80
125
100
75
180
75k
A
0
B
180k
180k

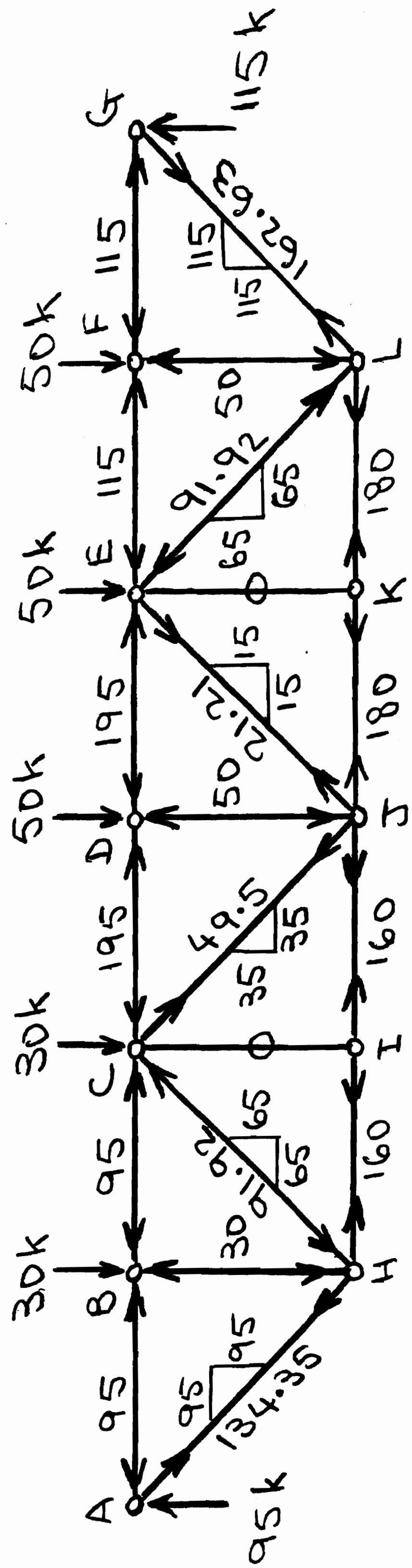
4.13


30 kN
H
0
D
60 kN
30
72.11
40
60
G
72.11
40
60
40
110
30 kN
C
108.17
60
90
F
108.17
90
60
140
230
B
30 kN
120.80
80
144.22
120 kN
E
280
A
0
310 kN
280 kN

4.14

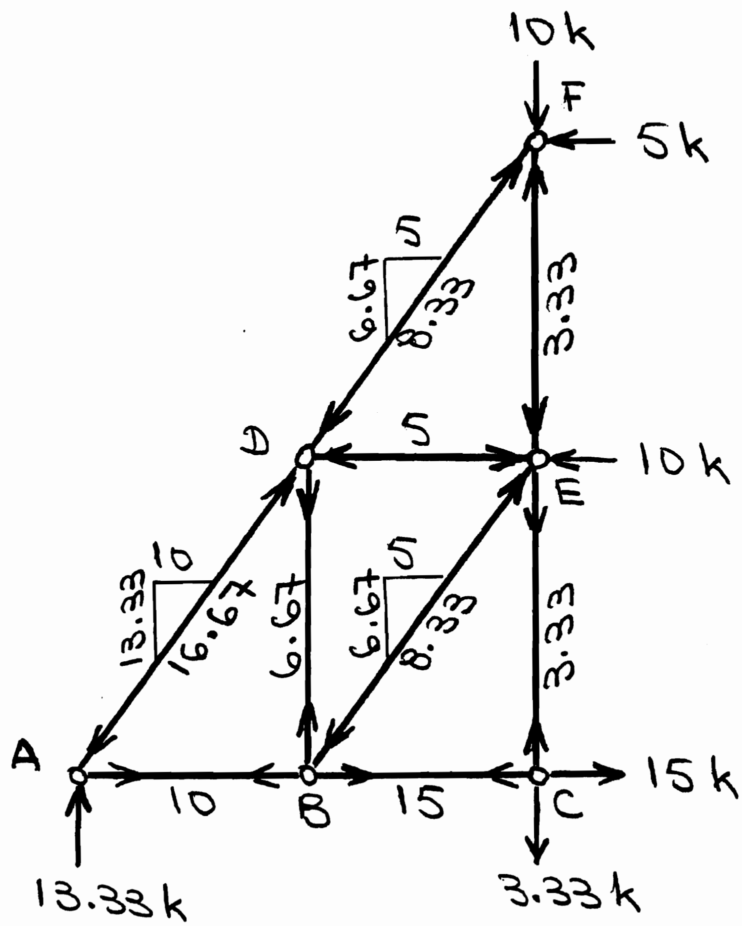

A
B
C
D
E
F
G
H
I
J
K
L
30 kN
60 kN
60 kN
191.25 kN
40 kN
40 kN
40 kN
18.75 kN
60 kN
50
30
40
40
60
150
90
120
220
40
101.25
135
168.75
101.25
85
160
0
85
61.25
81.67
102.08
56.67
61.25
35.42
21.25
28.33
3.33
85
40
31.25
18.75
25
85

4.15

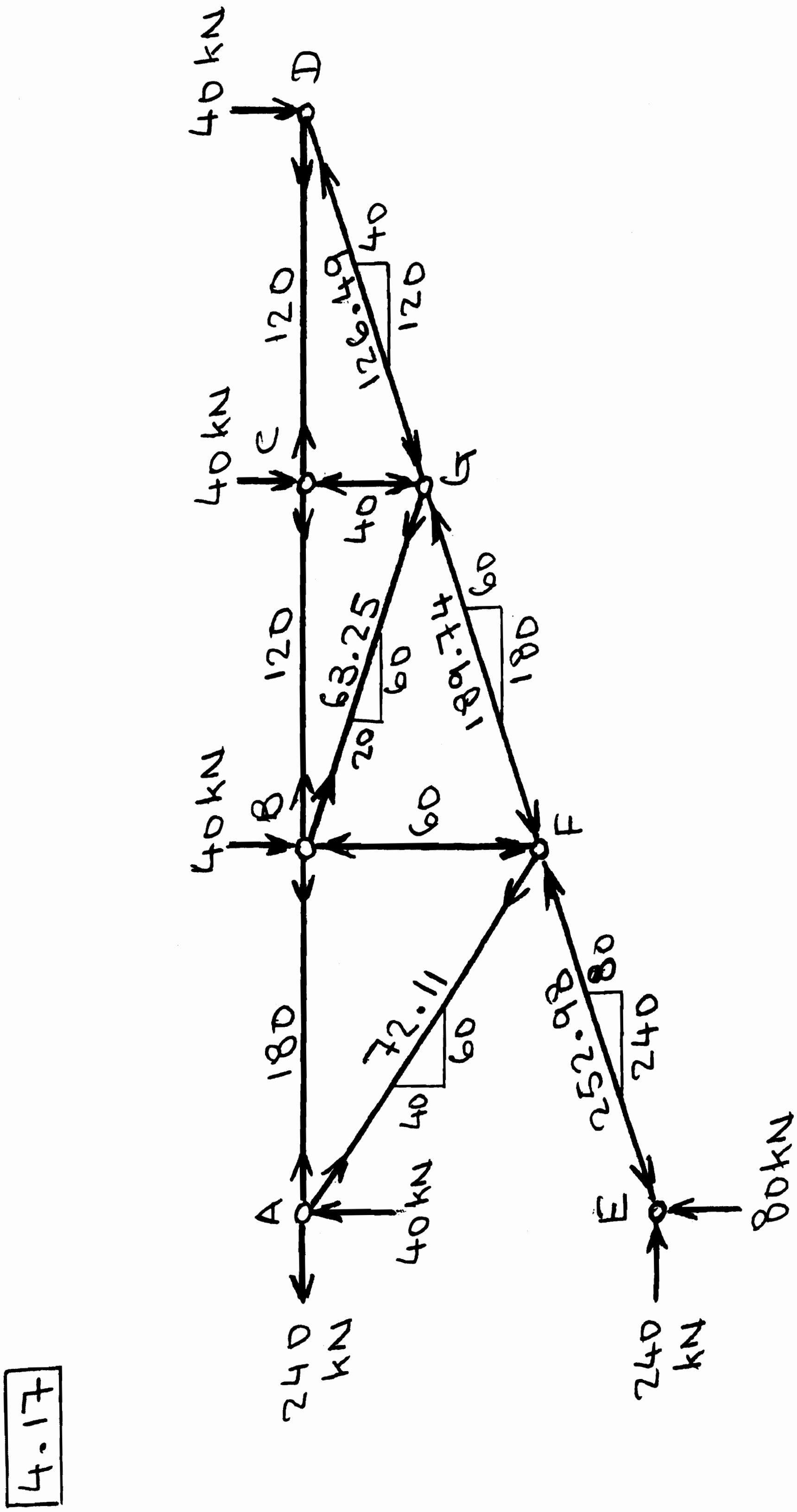


A
B
C
D
E
F
G
H
I
J
K
L
95 k
115 k
30k
30k
50k
50k
50k
95
95
195
195
115
115
134.35
91.92
49.5
21.21
91.92
162.63
30
0
50
0
50
160
160
180
180

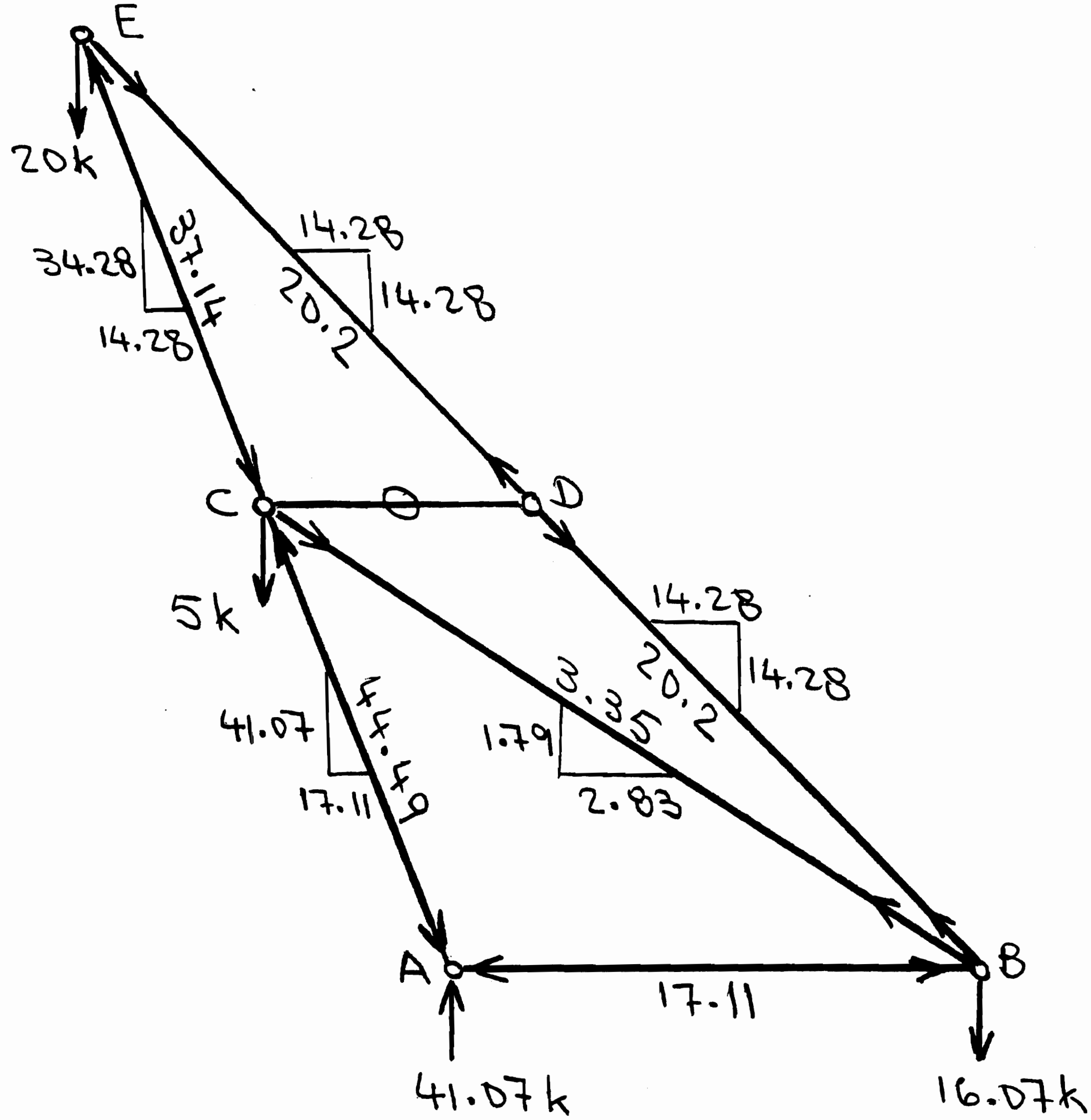
4.16



10 k
F
5 k
5
6.67
8.33
3.33
D
5
10 k
E
10
13.33
16.67
6.67
5
6.67
8.33
3.33
A
10
B
15
C
15 k
13.33 k
3.33 k

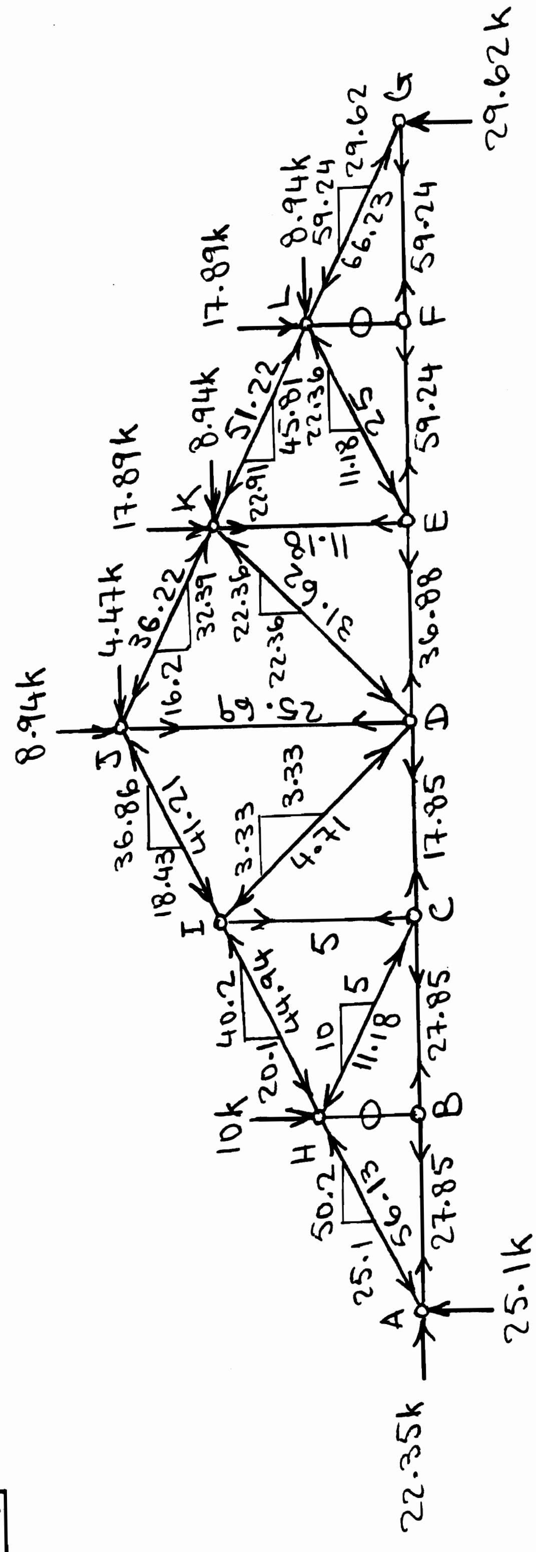

4.17
40 kN
D
120
126.49
40
120
40 kN
C
40
G
120
63.25
60
20
189.74
60
180
40 kN
B
60
F
180
72.11
60
40
252.98
80
240
A
40 kN
E
80 kN
240 kN
240 kN

4.18

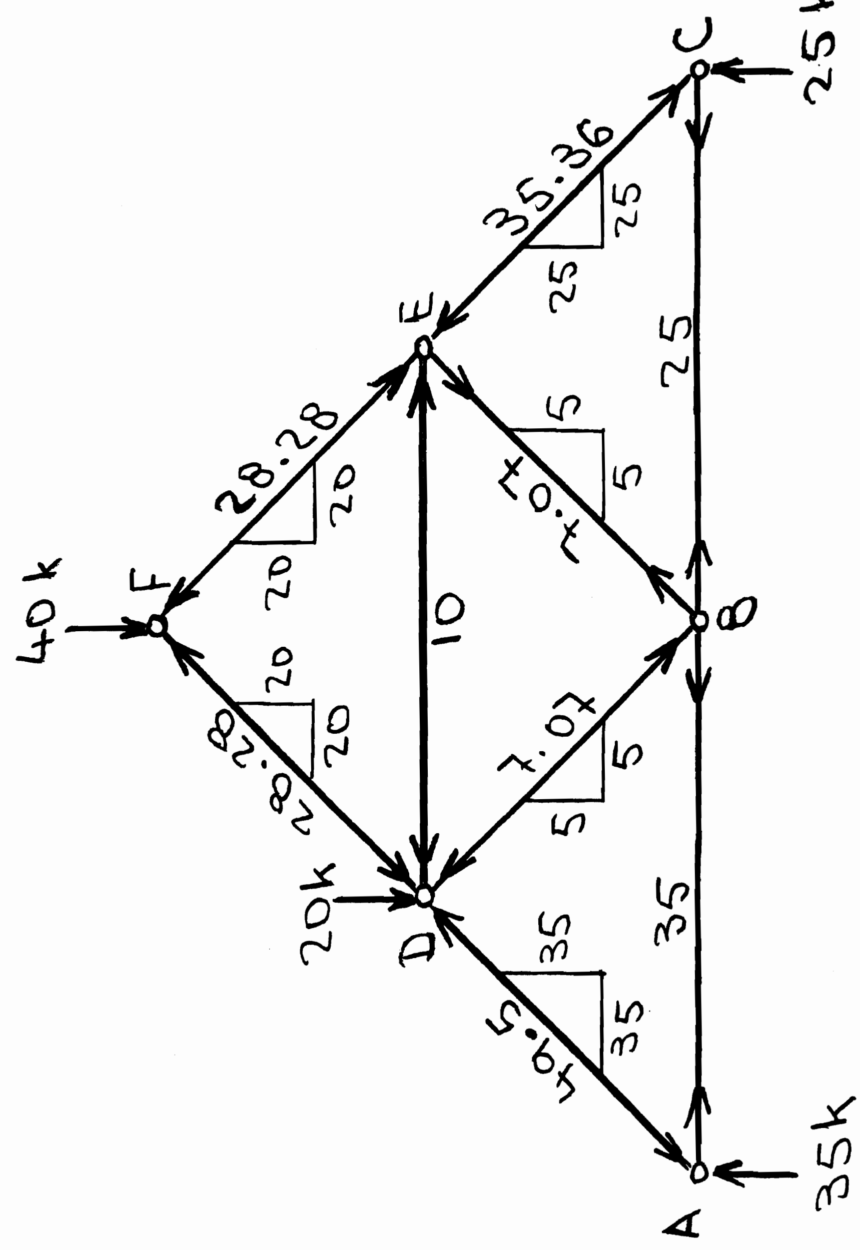


E
20k
34.28
37.14
14.28
14.28
20.2
14.28
C
0
D
5k
14.28
20.2
14.28
41.07
44.49
17.11
1.79
3.35
2.83
A
B
17.11
41.07k
16.07k

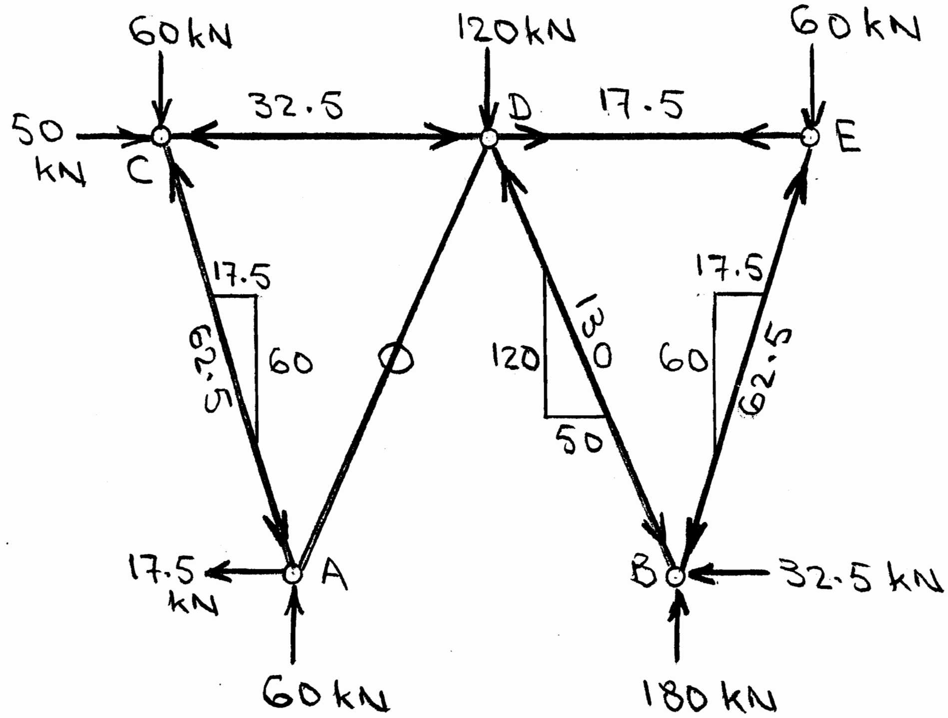
4.19



4.20

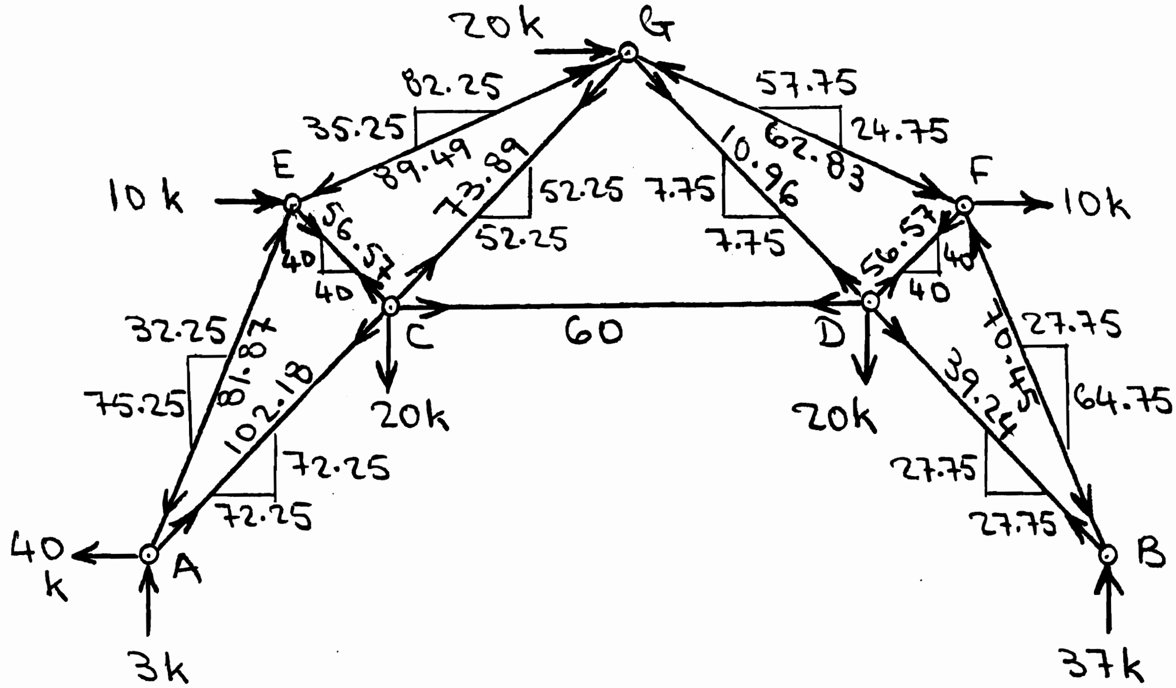


4.21

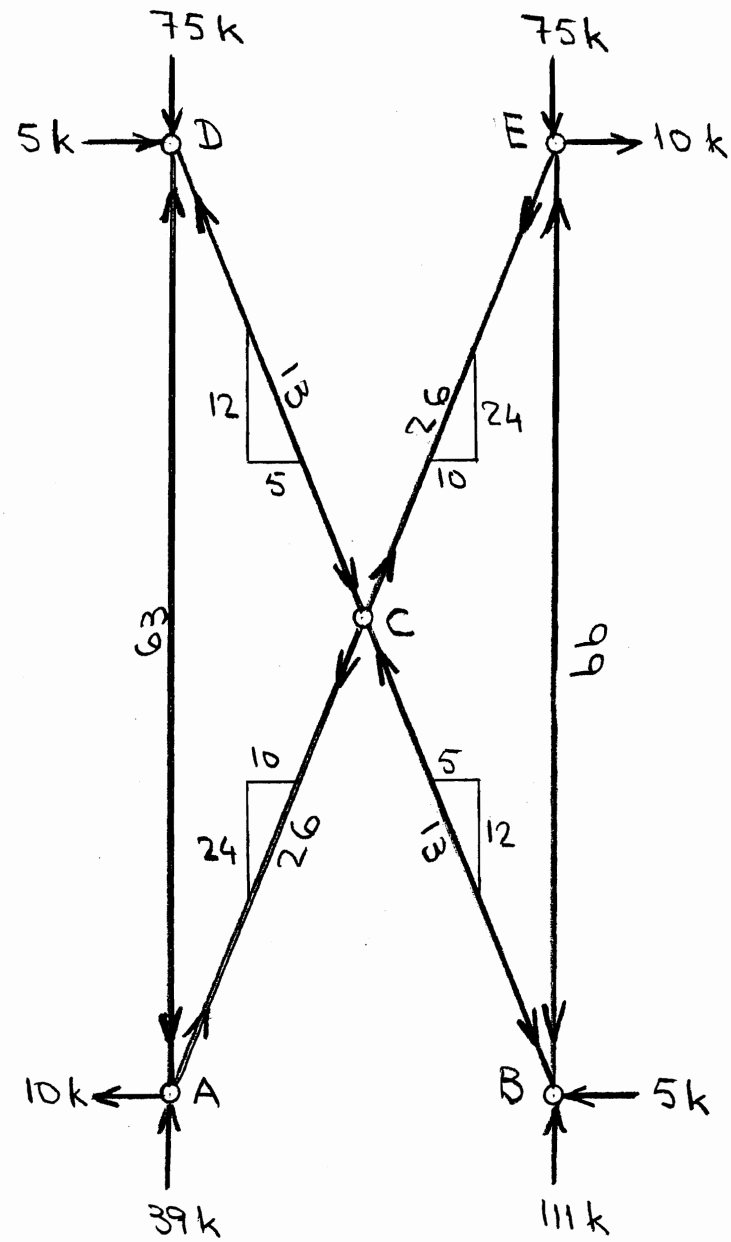


60 kN
120 kN
60 kN
50 kN
C
32.5
D
17.5
E
17.5
62.5
60
0
120
130
50
17.5
60
62.5
17.5 kN
A
B
32.5 kN
60 kN
180 kN

4.22

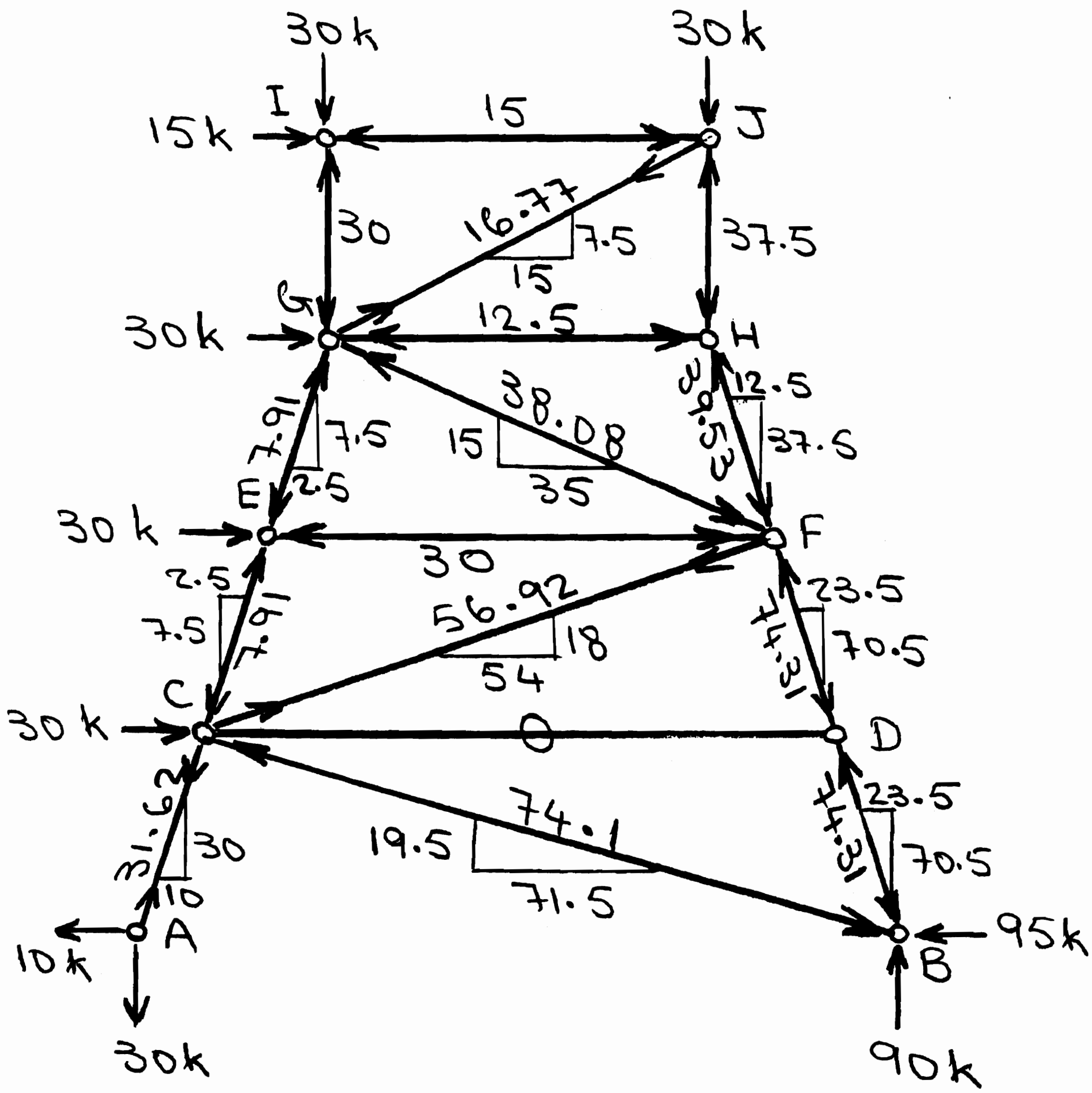

20k
G
82.25
35.25
57.75
24.75
62.83
89.49
73.89
52.25
7.75
10.96
E
F
10 k
10k
56.57
40
52.25
7.75
56.57
40
40
40
C
60
D
32.25
27.75
81.87
70.45
75.25
102.18
39.24
64.75
20k
20k
72.25
27.75
72.25
27.75
40 k
A
B
3k
37k

4.23



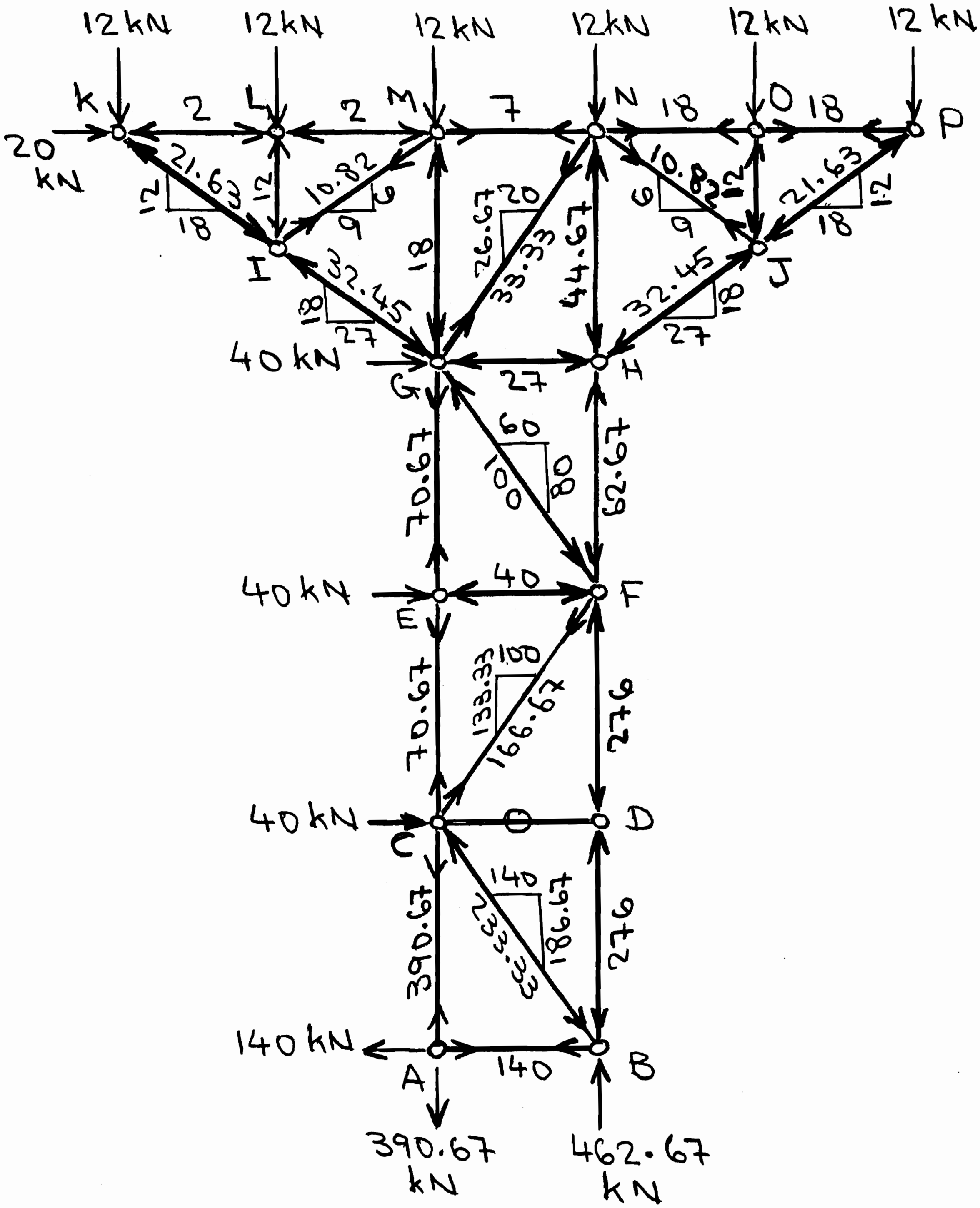
75 k
75 k
5 k
D
E
10 k
12
13
5
26
24
10
63
C
99
10
24
26
5
13
12
10 k
A
B
5 k
39 k
111 k

4.24

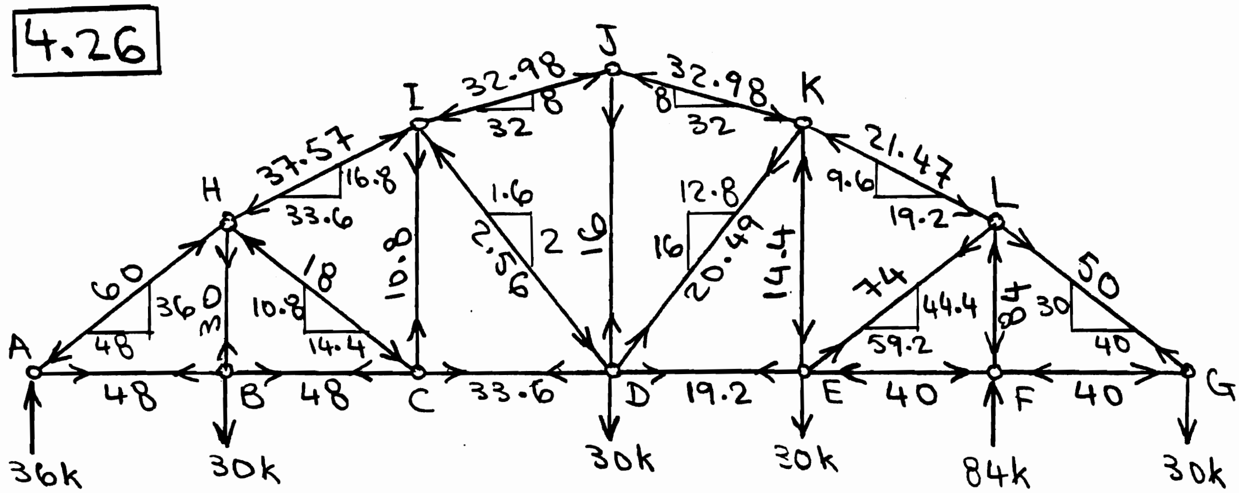


30k
30k
I
J
15k
15
16.77
7.5
15
30
37.5
G
H
30k
12.5
7.91
7.5
2.5
15
38.08
35
9.53
12.5
37.5
E
F
30 k
30
2.5
7.5
7.91
56.92
18
54
74.31
23.5
70.5
C
D
30 k
0
31.62
30
10
19.5
74.1
71.5
74.31
23.5
70.5
A
B
10k
30k
95k
90k

4.25



4.26



32.98
J
32.98
I
K
8
32
8
32
37.57
21.47
H
16.8
9.6
L
33.6
19.2
1.6
12.8
60
36
18
10.8
2.56
2
16
20.49
14.4
74
44.4
50
30
10.8
14.4
59.2
40
A
48
B
48
C
33.6
D
19.2
E
40
F
40
G
36k
30k
30k
30k
84k
30k

4.27 Reactions:

$\overset{+}{\rightarrow}\Sigma F_x = 0 \quad A_x + 50 = 0 \quad A_x = -50 \quad \underline{A_x = 50\text{ kN}\leftarrow}$

$+\circlearrowleft \Sigma M_C^{AC} = 0 \quad -A_y(8) + 120(4) = 0 \quad \underline{A_y = 60\text{ kN}\uparrow}$

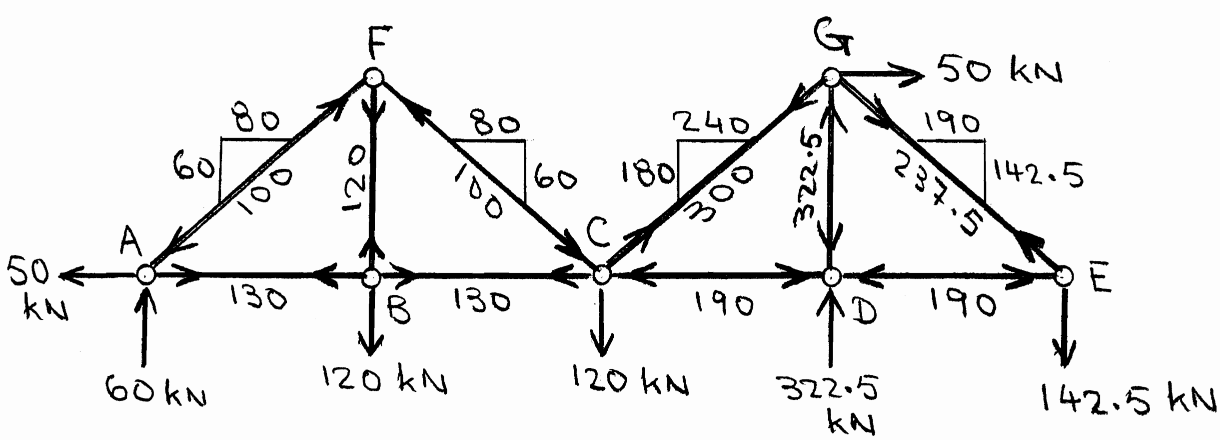
$+\circlearrowleft \Sigma M_E = 0 \quad -60(16) + 120(12) + 120(8) - 50(3) - D_y(4) = 0$

$\underline{D_y = 322.5\text{ kN}\uparrow}$

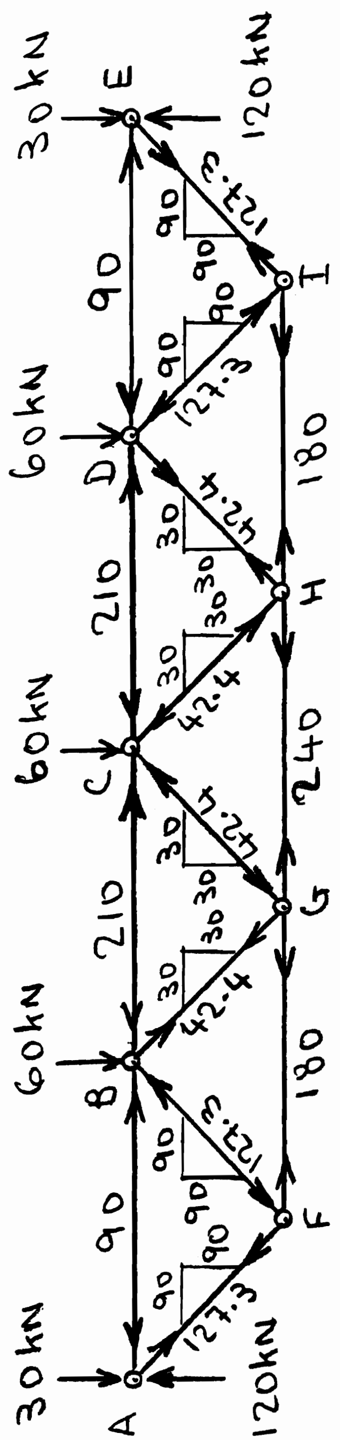
$+\uparrow \Sigma F_y = 0 \quad 60 - 120 - 120 + 322.5 + E_y = 0$

$E_y = -142.5\text{ kN} \quad \underline{E_y = 142.5\text{ kN}\downarrow}$

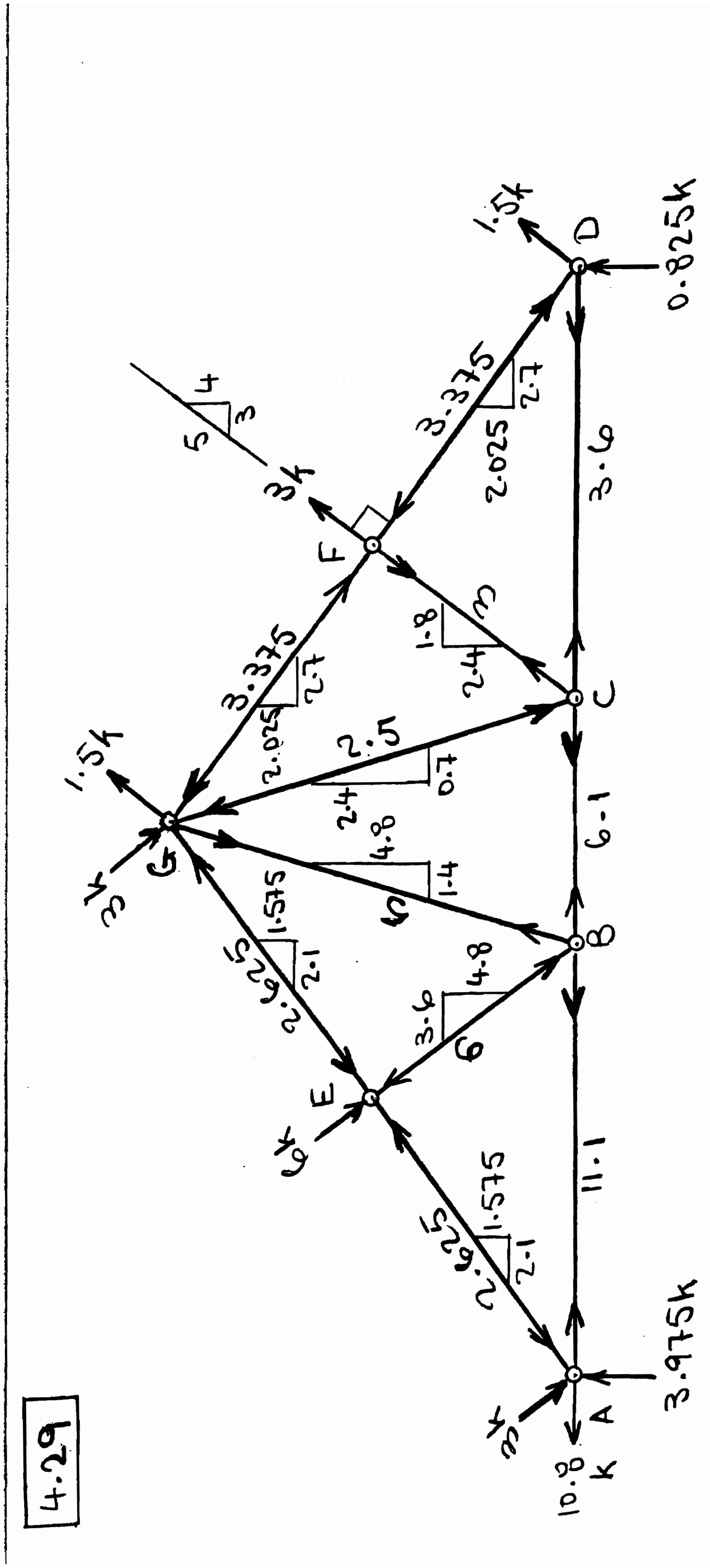
Member Forces:



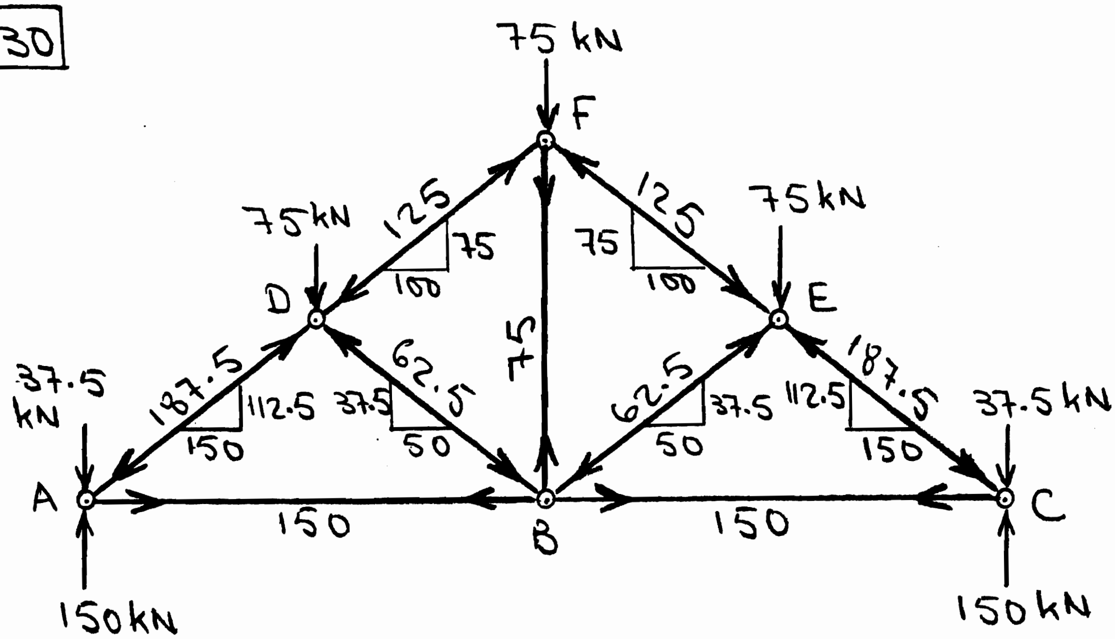
4.28



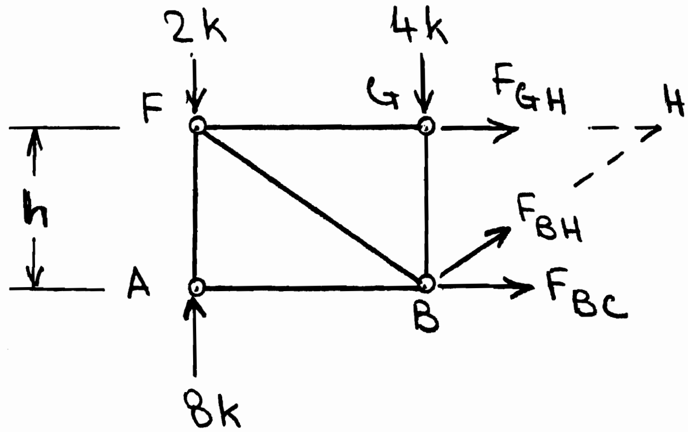
A
B
C
D
E
F
G
H
I
30 kN
60 kN
60 kN
60 kN
30 kN
120 kN
120 kN
90
210
210
90
127.3
42.4
42.4
42.4
42.4
127.3
127.3
127.3
180
240
180


4.29
A
B
C
D
E
F
G
K
3.975k
0.825k
1.5k
3k
6k
10.8
11.1
6.1
3.6
2.625
3.375
2.5
1.575
2.025
2.7
2.1
4.8
2.4
1.8
0.7
1.4
5
4
3

4.30


75 kN
F
75kN
125
75
100
125
75
100
75kN
D
E
75
37.5
kN
187.5
112.5
150
37.5
62.5
50
62.5
50
37.5
112.5
187.5
150
37.5 kN
A
150
B
150
C
150kN
150kN

4.31 Section through members BC, BH and GH:



$+\circlearrowleft \Sigma M_B = 0 \qquad -8(6) + 2(6) - F_{GH}(h) = 0$

$$F_{GH} = -\frac{36}{h} \qquad (1)$$

$+\circlearrowleft \Sigma M_H = 0 \qquad -8(12) + 2(12) + 4(6) + F_{BC}(h) = 0$

$$F_{BC} = \frac{48}{h} \qquad (2)$$

Equations (1) and (2) indicate that the magnitudes of F_{GH} and F_{BC} are inversely proportional to the truss height h.

For h = 3 ft: $F_{GH} = -\frac{36}{3} = -12\text{ k} = \underline{12\text{ k (C)}}$

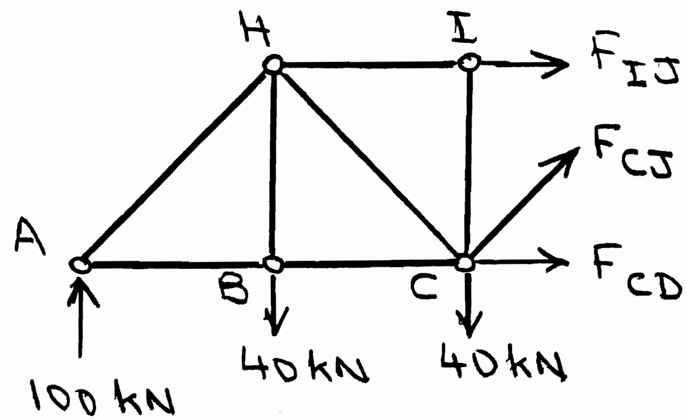
$F_{BC} = \frac{48}{3} = \underline{16\text{ k (T)}}$

For h = 6 ft: $F_{GH} = -\frac{36}{6} = -6\text{ k} = \underline{6\text{ k (C)}}$

$F_{BC} = \frac{48}{6} = \underline{8\text{ k (T)}}$

4.32

Section through members CD, CJ and IJ:



$+\uparrow \Sigma F_y = 0$

$100 - 2(40) + \left(\frac{1}{\sqrt{2}}\right) F_{CJ} = 0$

$F_{CJ} = -28.28 \text{ kN}$

$\underline{F_{CJ} = 28.28 \text{ kN (C)}}$

$+\circlearrowleft \Sigma M_C = 0$

$-100(10) + 40(5) - F_{IJ}(5) = 0$

$F_{IJ} = -160 \text{ kN}$

$\underline{F_{IJ} = 160 \text{ kN (C)}}$

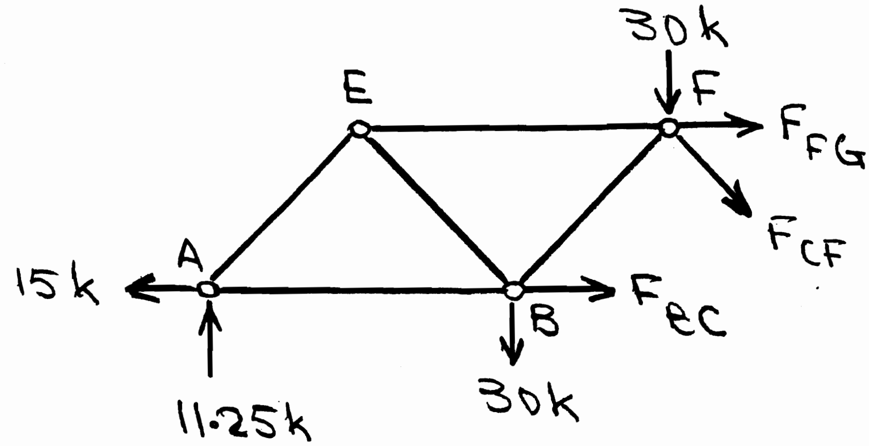
$\overset{+}{\rightarrow} \Sigma F_x = 0$

$F_{CD} - \left(\frac{1}{\sqrt{2}}\right) 28.28 - 160 = 0$

$\underline{F_{CD} = 180 \text{ kN (T)}}$

4.33

Section through members BC, CF and FG:



$+\uparrow \Sigma F_y = 0$

$$11.25 - 30 - 30 - \left(\frac{1}{\sqrt{2}}\right) F_{CF} = 0$$

$$F_{CF} = -68.94\text{ k}$$

$$\underline{F_{CF} = 68.94\text{ k (C)}}$$

$+\circlearrowright \Sigma M_F = 0$

$$-15(10) - 11.25(30) + 30(10) + F_{BC}(10) = 0$$

$$\underline{F_{BC} = 18.75\text{ k (T)}}$$

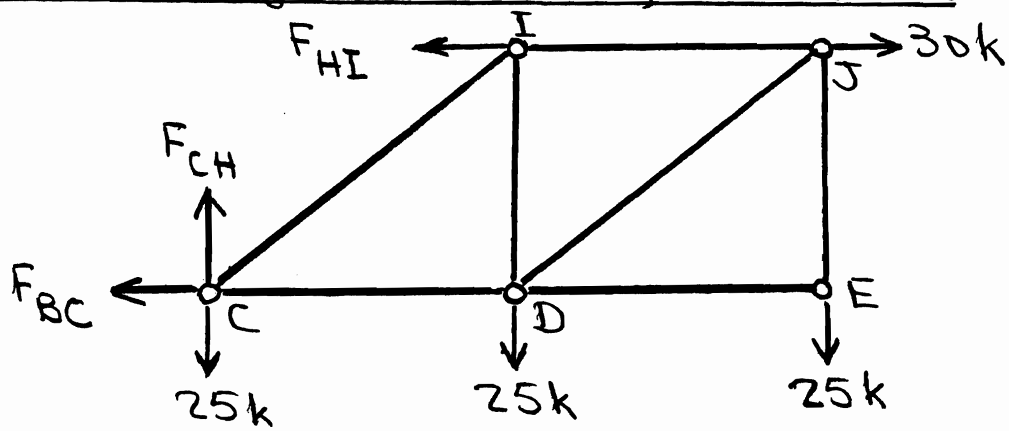
$\overset{+}{\rightarrow} \Sigma F_x = 0$

$$-15 + 18.75 - \left(\frac{1}{\sqrt{2}}\right) 68.94 + F_{FG} = 0$$

$$\underline{F_{FG} = 45\text{ k (T)}}$$

4.34

Section through members BC, CH and HI:



$+\uparrow \Sigma F_y = 0 \qquad F_{CH} - 3(25) = 0$

$F_{CH} = 75\text{ k (T)}$

$+\circlearrowleft \Sigma M_C = 0$

$F_{HI}(15) - 30(15) - 25(20) - 25(40) = 0$

$F_{HI} = 130\text{ k (T)}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

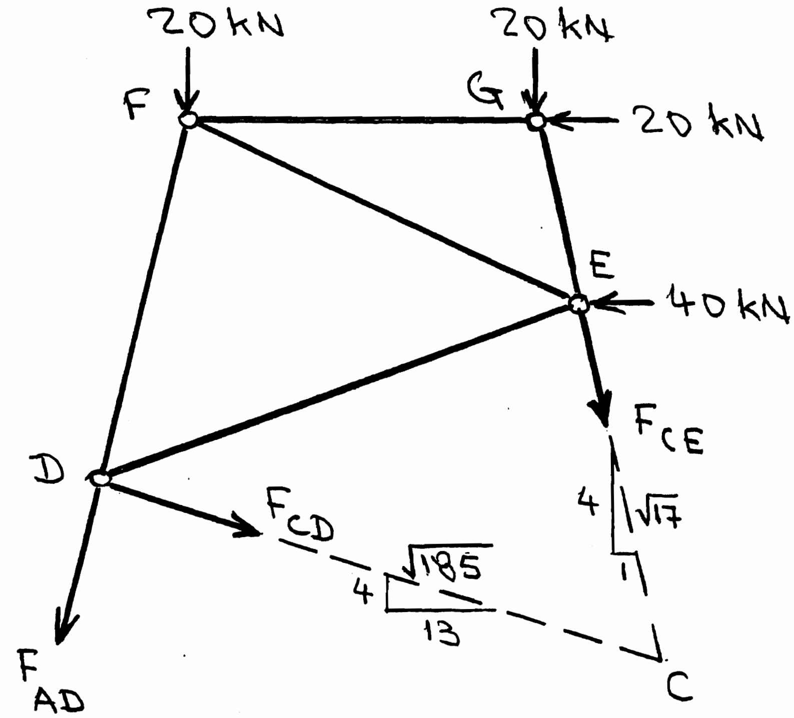
$-F_{BC} - 130 + 30 = 0$

$F_{BC} = -100\text{ k}$

$F_{BC} = 100\text{ k (C)}$

4.35

Section through members AD, CD and CE:



$+\circlearrowleft \Sigma M_D = 0$

$$-20(1.5) - 20(7.5) + 20(6) + 40(3) - \frac{1}{\sqrt{17}} F_{CE}(3) - \frac{4}{\sqrt{17}} F_{CE}(8.25) = 0$$

$F_{CE} = 6.87$ kN (T)

$+\circlearrowleft \Sigma M_C = 0$

$$20(8.25) + 20(2.25) + 20(9) + 40(6) + \frac{1}{\sqrt{17}} F_{AD}(3) + \frac{4}{\sqrt{17}} F_{AD}(9.75) = 0$$

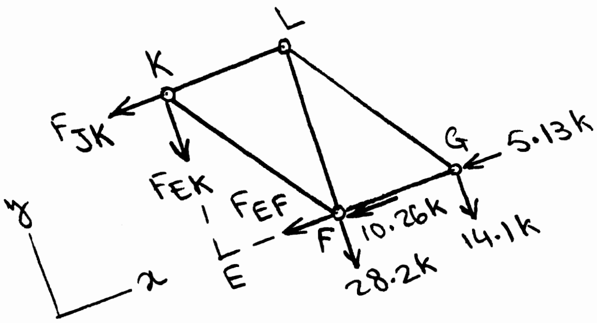
$F_{AD} = -61.85$ kN $\quad F_{AD} = 61.85$ kN (C)

$+\uparrow \Sigma F_y = 0$

$$\frac{4}{\sqrt{17}}(61.85) - 20 - 20 - \frac{4}{\sqrt{17}}(6.87) - \frac{4}{\sqrt{185}} F_{CD} = 0$$

$F_{CD} = 45.34$ kN (T)

4.36 Section through members EF, EK and JK:



$\Sigma F_y = 0 \quad -F_{EK} - 28.2 - 14.1 = 0$

$F_{EK} = -42.3\text{k} = \underline{42.3\text{k (C)}}$

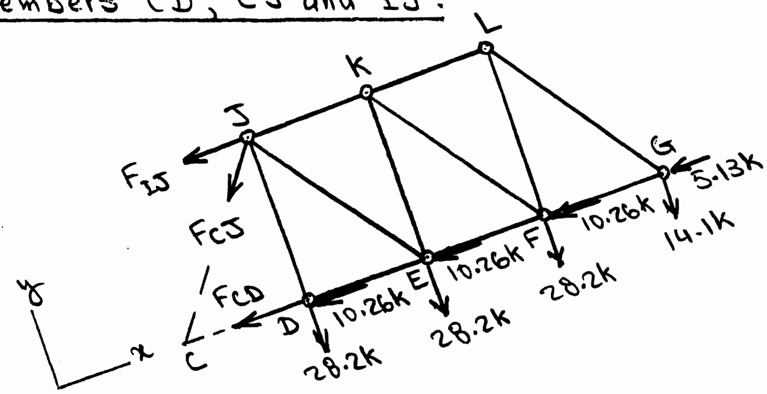
$+\circlearrowleft \Sigma M_K = 0 \quad -F_{EF}(20) - 28.2(15) - 14.1(30)$
$- (5.13 + 10.26)(20) = 0$

$F_{EF} = -57.69\text{k} = \underline{57.69\text{k (C)}}$

$+\circlearrowleft \Sigma M_E = 0 \quad F_{JK}(20) - 28.2(15) - 14.1(30) = 0$

$\underline{F_{JK} = 42.3\text{k (T)}}$

Section through members CD, CJ and IJ:



$\Sigma F_y = 0 \quad -\left(\frac{4}{5}\right)F_{CJ} - 3(28.2) - 14.1 = 0$

$F_{CJ} = -123.38\text{k} = \underline{123.38\text{k (C)}}$

$+\circlearrowleft \Sigma M_J = 0 \quad -F_{CD}(20) - [3(10.26) + 5.13](20) - 28.2(15 + 30)$
$- 14.1(45) = 0$

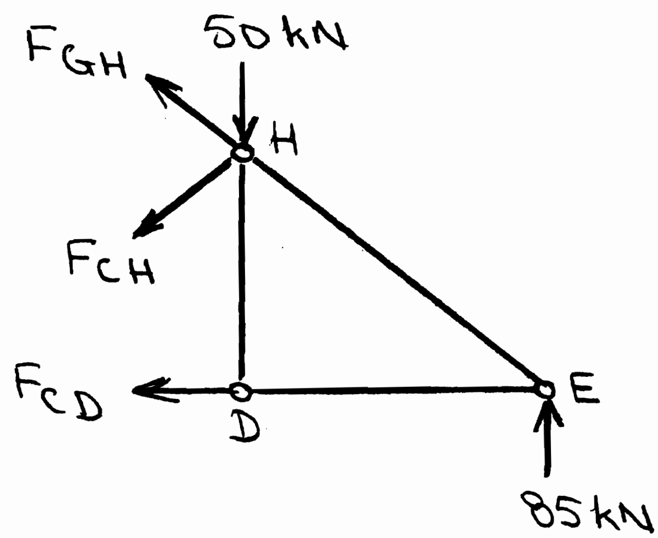
$F_{CD} = -131.09\text{k} = \underline{131.09\text{k (C)}}$

$+\circlearrowleft \Sigma M_C = 0 \quad F_{IJ}(20) - 28.2(15 + 30 + 45) - 14.1(60) = 0$

$\underline{F_{IJ} = 169.2\text{k (T)}}$

4.37

Section through members CD, CH and GH:



$+\circlearrowleft \Sigma M_E = 0$

$50(4) + \frac{4}{5} F_{CH}(3) + \frac{3}{5} F_{CH}(4) = 0$

$F_{CH} = -41.67 \text{ kN}$ $\quad$ $\underline{F_{CH} = 41.67 \text{ kN (C)}}$

$+\uparrow \Sigma F_y = 0$

$\frac{3}{5} F_{GH} - 50 + \frac{3}{5}(41.67) + 85 = 0$

$F_{GH} = -100 \text{ kN}$ $\quad$ $\underline{F_{GH} = 100 \text{ kN (C)}}$

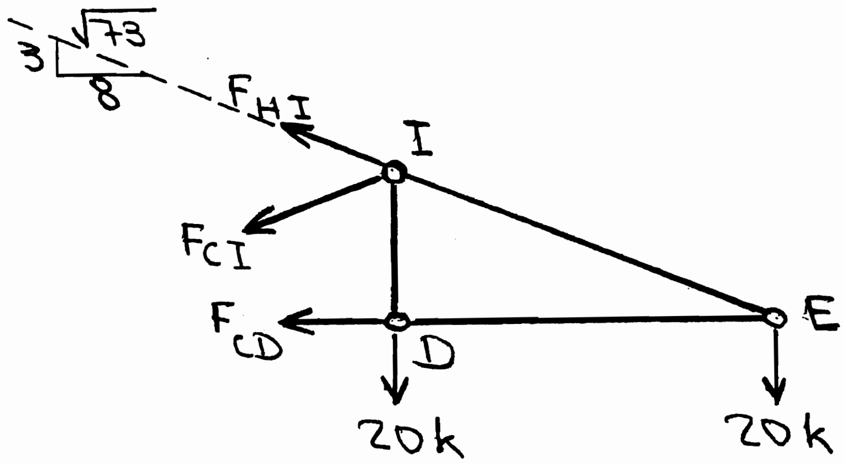
$+\circlearrowleft \Sigma M_H = 0$

$-F_{CD}(3) + 85(4) = 0$

$\underline{F_{CD} = 113.33 \text{ kN (T)}}$

4.38

Section through members CD, CI and HI:



$+\circlearrowright \Sigma M_I = 0 \qquad -F_{CD}(3.75) - 20(10) = 0$

$F_{CD} = -53.33\text{ k} \qquad \underline{F_{CD} = 53.33\text{ k (C)}}$

$+\circlearrowright \Sigma M_E = 0$

$20(10) + \frac{8}{\sqrt{73}} F_{CI}(3.75) + \frac{3}{\sqrt{73}} F_{CI}(10) = 0$

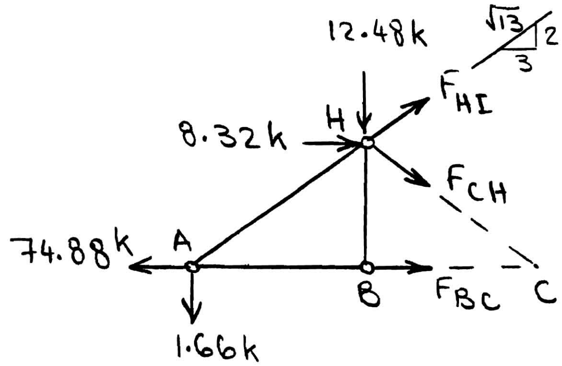
$F_{CI} = -28.48\text{ k} \qquad \underline{F_{CI} = 28.48\text{ k (C)}}$

$+\uparrow \Sigma F_y = 0$

$\frac{3}{\sqrt{73}} F_{HI} + \frac{3}{\sqrt{73}}(28.48) - 20 - 20 = 0$

$\underline{F_{HI} = 85.44\text{ k (T)}}$

4.39 Section through members BC, CH and HI:



$$+\circlearrowleft \Sigma M_H = 0 \quad -74.88\left(\frac{40}{3}\right) + 1.66(20) + F_{BC}\left(\frac{40}{3}\right) = 0$$

$$\underline{F_{BC} = 72.39 \text{ k (T)}}$$

$$+\circlearrowleft \Sigma M_C = 0 \quad 1.66(40) + 12.48(20) - 8.32\left(\frac{40}{3}\right)$$

$$-\frac{3}{\sqrt{13}}F_{HI}\left(\frac{40}{3}\right) - \frac{2}{\sqrt{13}}F_{HI}(20) = 0$$

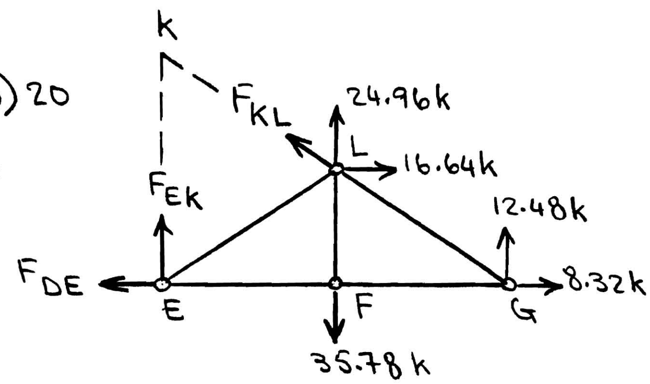
$$\underline{F_{HI} = 9.24 \text{ k (T)}}$$

$$+\circlearrowleft \Sigma M_A = 0 \quad -8.32\left(\frac{40}{3}\right) - 12.48(20) - \frac{3}{\sqrt{13}}F_{CH}\left(\frac{40}{3}\right)$$

$$-\frac{2}{\sqrt{13}}F_{CH}(20) = 0$$

$$F_{CH} = -16.25 \text{ k} = \underline{16.25 \text{ k (C)}}$$

Section through members DE, EK and KL:



$$+\circlearrowleft \Sigma M_K = 0$$

$$-F_{DE}\left(\frac{80}{3}\right) + (24.96 - 35.78)20$$

$$+ 12.48(40) + 16.64\left(\frac{40}{3}\right)$$

$$+ 8.32\left(\frac{80}{3}\right) = 0$$

$$\underline{F_{DE} = 27.25 \text{ k (T)}}$$

$$+\circlearrowleft \Sigma M_G = 0$$

$$-F_{EK}(40) + (35.78 - 24.96)20 - 16.64\left(\frac{40}{3}\right) = 0$$

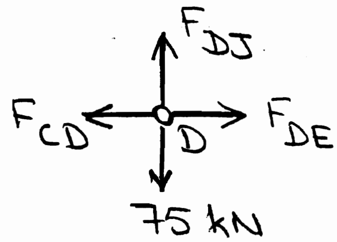
$$F_{EK} = -0.14 \text{ k} = \underline{0.14 \text{ k (C)}}$$

$$+\circlearrowleft \Sigma M_E = 0 \quad (24.96 - 35.78)20 + 12.48(40) - 16.64\left(\frac{40}{3}\right)$$

$$+\frac{3}{\sqrt{13}}F_{KL}\left(\frac{40}{3}\right) + \frac{2}{\sqrt{13}}F_{KL}(20) = 0$$

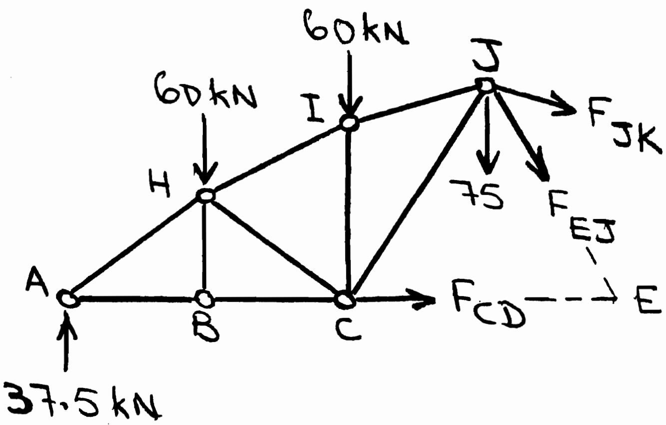
$$F_{KL} = -2.75 \text{ k} = \underline{2.75 \text{ k (C)}}$$

4.40 Joint D:



$\Sigma F_y = 0 \qquad F_{DJ} = 75\text{ kN (T)}$

Section through members CD, DJ, EJ and JK:



$+\circlearrowleft \Sigma M_J = 0$

$-37.5(12) + 60(8) + 60(4) + F_{CD}(6) = 0$

$F_{CD} = -45\text{ kN} \qquad \underline{F_{CD} = 45\text{ kN (C)}}$

$+\circlearrowleft \Sigma M_E = 0$

$-37.5(16) + 60(12) + 60(8) + 75(4)$

$-\frac{4}{\sqrt{17}} F_{JK}(6) + \frac{1}{\sqrt{17}} F_{JK}(4) = 0$

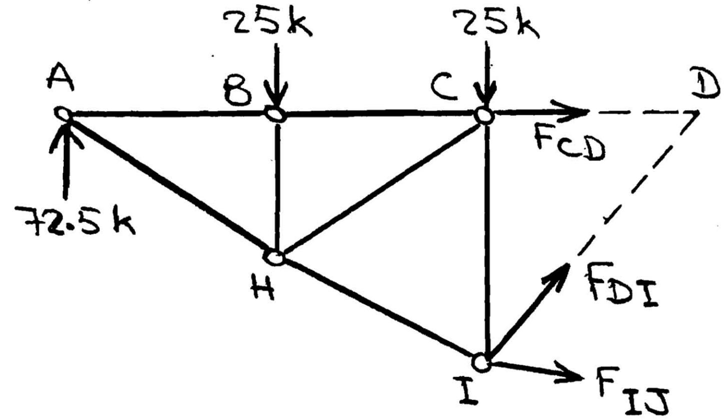
$\underline{F_{JK} = 185.54\text{ kN (T)}}$

$+\uparrow \Sigma F_y = 0$

$37.5 - 60 - 60 - 75 - \frac{3}{\sqrt{13}} F_{EJ} - \frac{1}{\sqrt{17}}(185.54) = 0$

$F_{EJ} = -243.37\text{ kN} \quad F_{EJ} = 243.37\text{ kN (C)}$

4.41 Section through members CD, DI and IJ:



$+\circlearrowleft \Sigma M_I = 0 \qquad -72.5(60) + 25(30) - F_{CD}(35) = 0$

$F_{CD} = -102.86\text{ k} \qquad \underline{F_{CD} = 102.86\text{ k (C)}}$

$+\circlearrowleft \Sigma M_D = 0$

$-72.5(90) + 25(60) + 25(30) + \frac{6}{\sqrt{37}} F_{IJ}(35)$

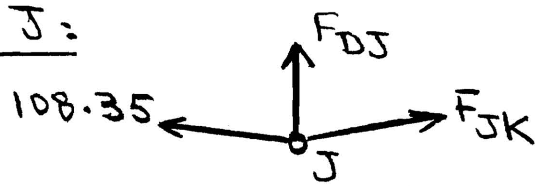
$+ \frac{1}{\sqrt{37}} F_{IJ}(30) = 0 \qquad F_{IJ} = 108.35\text{ k (T)}$

$+\uparrow \Sigma F_y = 0$

$72.5 - 25 - 25 - \frac{1}{\sqrt{37}}(108.35) + \frac{7}{\sqrt{85}} F_{DI} = 0$

$F_{DI} = -6.17\text{ k} \qquad \underline{F_{DI} = 6.17\text{ k (C)}}$

Joint J:



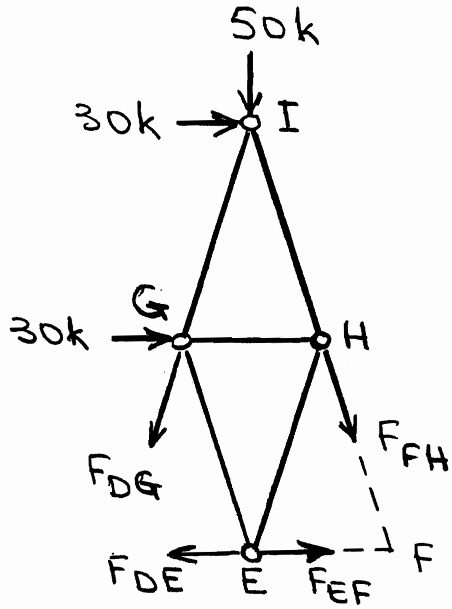
$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad -\frac{6}{\sqrt{37}}(108.35) + \frac{6}{\sqrt{37}} F_{JK} = 0$

$F_{JK} = 108.35\text{ k (T)}$

$+\uparrow \Sigma F_y = 0 \qquad 2\left(\frac{1}{\sqrt{37}}\right)108.35 + F_{DJ} = 0$

$F_{DJ} = -35.63\text{ k} \qquad \underline{F_{DJ} = 35.63\text{ k (C)}}$

4.42 Section through members DG, DE, EF and FH:



$+\circlearrowleft \Sigma M_F = 0$

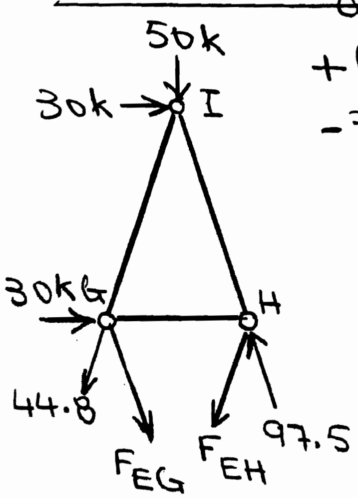
$$50(6) - 30(18) - 30(9) + \frac{1}{\sqrt{10}} F_{DG}(9) + \frac{3}{\sqrt{10}} F_{DG}(9) = 0$$

$$\underline{F_{DG} = 44.8 \text{ k (T)}}$$

$+\uparrow \Sigma F_y = 0 \qquad -\frac{3}{\sqrt{10}}(44.8) - 50 - \frac{3}{\sqrt{10}} F_{FH} = 0$

$$F_{FH} = -97.5 \text{ k} = 97.5 \text{ k (C)}$$

Section through members DG, EG, EH and FH:



$+\circlearrowleft \Sigma M_G = 0$

$$-30(9) - 50(3) + \frac{3}{\sqrt{10}}(97.5)(6) - \frac{3}{\sqrt{10}} F_{EH}(6) = 0$$

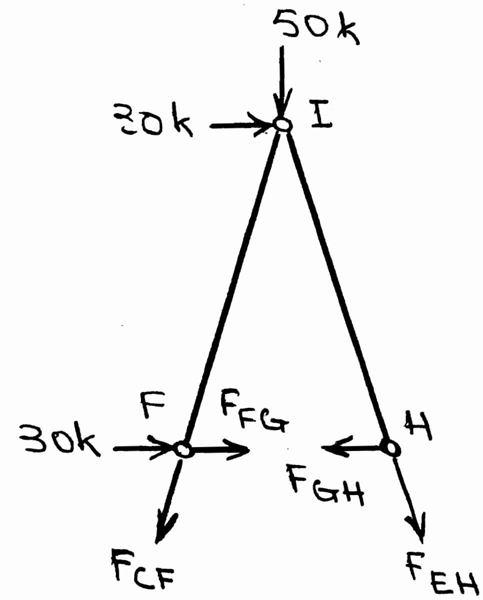
$$\underline{F_{EH} = 23.72 \text{ k (T)}}$$

$+\circlearrowleft \Sigma M_H = 0$

$$50(3) - 30(9) + \frac{3}{\sqrt{10}}(44.8)(6) + \frac{3}{\sqrt{10}} F_{EG}(6) = 0$$

$F_{EG} = -23.72 \text{ k} \qquad \underline{F_{EG} = 23.72 \text{ k (C)}}$

4.43 Section through members CF, FG, GH and EH:



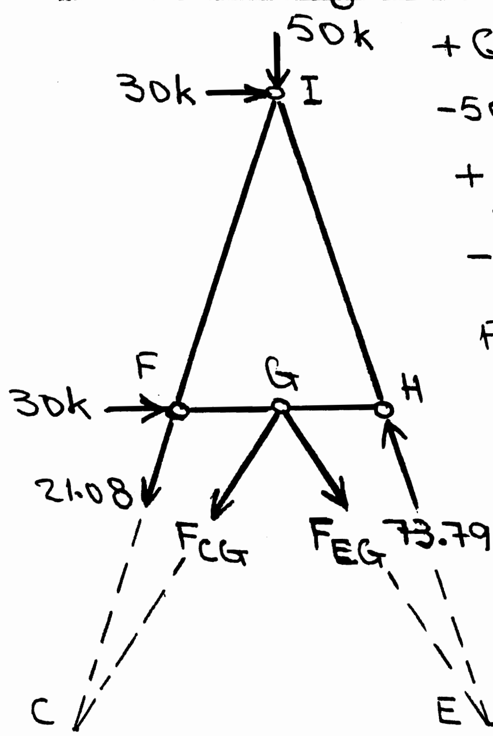
$+\circlearrowleft \sum M_H = 0 \qquad 50(3) - 30(9) + \frac{3}{\sqrt{10}} F_{CF}(6) = 0$

$F_{CF} = 21.08 \text{ k (T)}$

$+\circlearrowleft \sum M_F = 0 \qquad -50(3) - 30(9) - \frac{3}{\sqrt{10}} F_{EH}(6) = 0$

$F_{EH} = -73.79 \text{ k} = 73.79 \text{ k (C)}$

Section through members CF, CG, EG and EH:



$+\circlearrowleft \sum M_C = 0$

$-50(6) - 30(18) - 30(9) + \frac{1}{\sqrt{10}}(73.79)(9)$
$+ \frac{3}{\sqrt{10}}(73.79)(9) - \frac{2}{\sqrt{13}} F_{EG}(9)$
$- \frac{3}{\sqrt{13}} F_{EG}(6) = 0$

$F_{EG} = -27.04 \text{ k}$

$F_{EG} = 27.04 \text{ k (C)}$

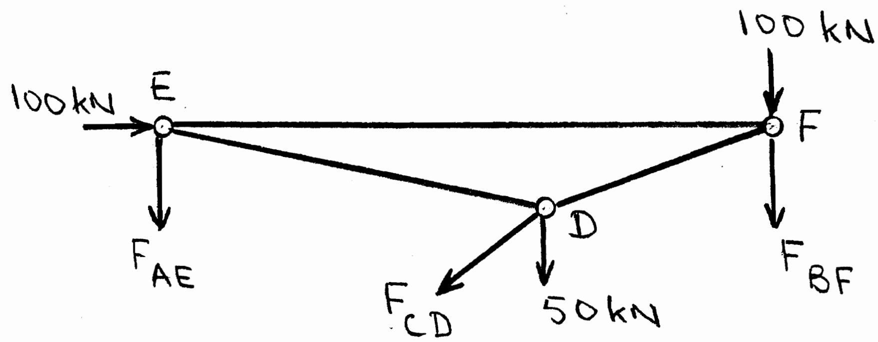
$+\circlearrowleft \sum M_E = 0$

$50(6) - 30(18) - 30(9)$
$+ \frac{1}{\sqrt{10}}(21.08)(9) + \frac{3}{\sqrt{10}}(21.08)(9)$
$+ \frac{2}{\sqrt{13}} F_{CG}(9) + \frac{3}{\sqrt{13}} F_{CG}(6) = 0$

$F_{CG} = 27.04 \text{ k (T)}$

4.44

Section through members AE, CD and BF:



$\xrightarrow{+} \Sigma F_x = 0 \qquad 100 - \left(\frac{4}{5}\right) F_{CD} = 0 \qquad \underline{F_{CD} = 125 \text{ kN (T)}}$

$+\circlearrowleft \Sigma M_F = 0 \qquad F_{AE}(16) - \frac{4}{5}(125)(2) + \frac{3}{5}(125)(6) + 50(6) = 0$

$F_{AE} = -34.38 \text{ kN} = \underline{34.38 \text{ kN (C)}}$

$+\uparrow \Sigma F_y = 0 \qquad 34.38 - \frac{3}{5}(125) - 50 - 100 - F_{BF} = 0$

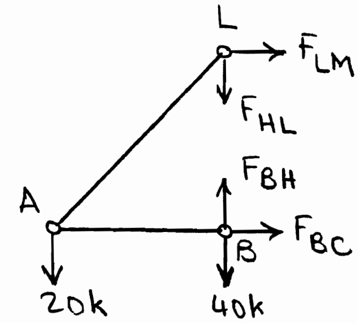
$F_{BF} = -190.62 \text{ kN} = \underline{190.62 \text{ kN (C)}}$

4.45 Section through members BC, BH, HL and LM:

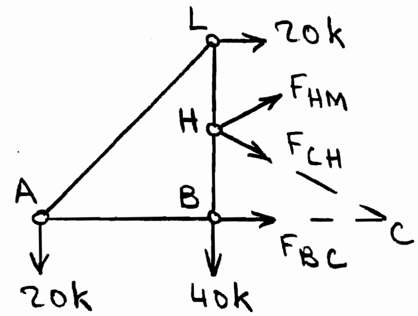
$+\circlearrowleft \Sigma M_B = 0$

$20(30) - F_{LM}(30) = 0$

$F_{LM} = 20\text{ k (T)}$



Section through members BC, CH, HM and LM:



$+\circlearrowleft \Sigma M_C = 0$

$20(60) + 40(30) - 20(30)$

$- \frac{2}{\sqrt{5}} F_{HM}(15) - \frac{1}{\sqrt{5}} F_{HM}(30) = 0$

$F_{HM} = 67.08\text{ k (T)}$

$+\uparrow \Sigma F_y = 0 \qquad -20 - 40 + \frac{1}{\sqrt{5}}(67.08) - \frac{1}{\sqrt{5}} F_{CH} = 0$

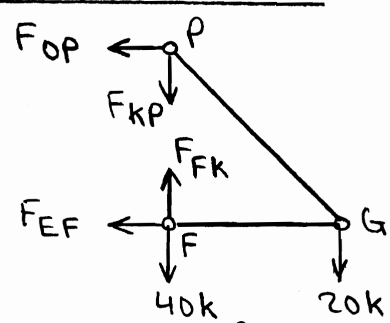
$F_{CH} = -67.08\text{ k} = 67.08\text{ k (C)}$

Section through members EF, FK, KP, and OP:

$+\circlearrowleft \Sigma M_P = 0$

$- F_{EF}(30) - 20(30) = 0$

$F_{EF} = -20\text{ k} = 20\text{ k (C)}$



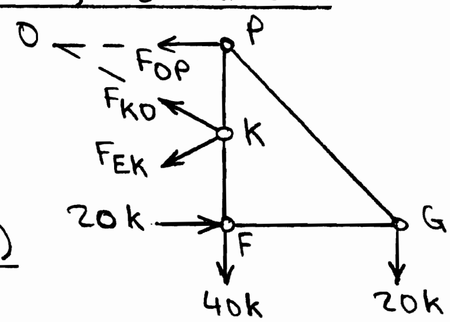
Section through members EF, EK, KO and OP:

$+\circlearrowleft \Sigma M_O = 0$

$-20(60) - 40(30) + 20(30)$

$- \frac{2}{\sqrt{5}} F_{EK}(15) - \frac{1}{\sqrt{5}} F_{EK}(30) = 0$

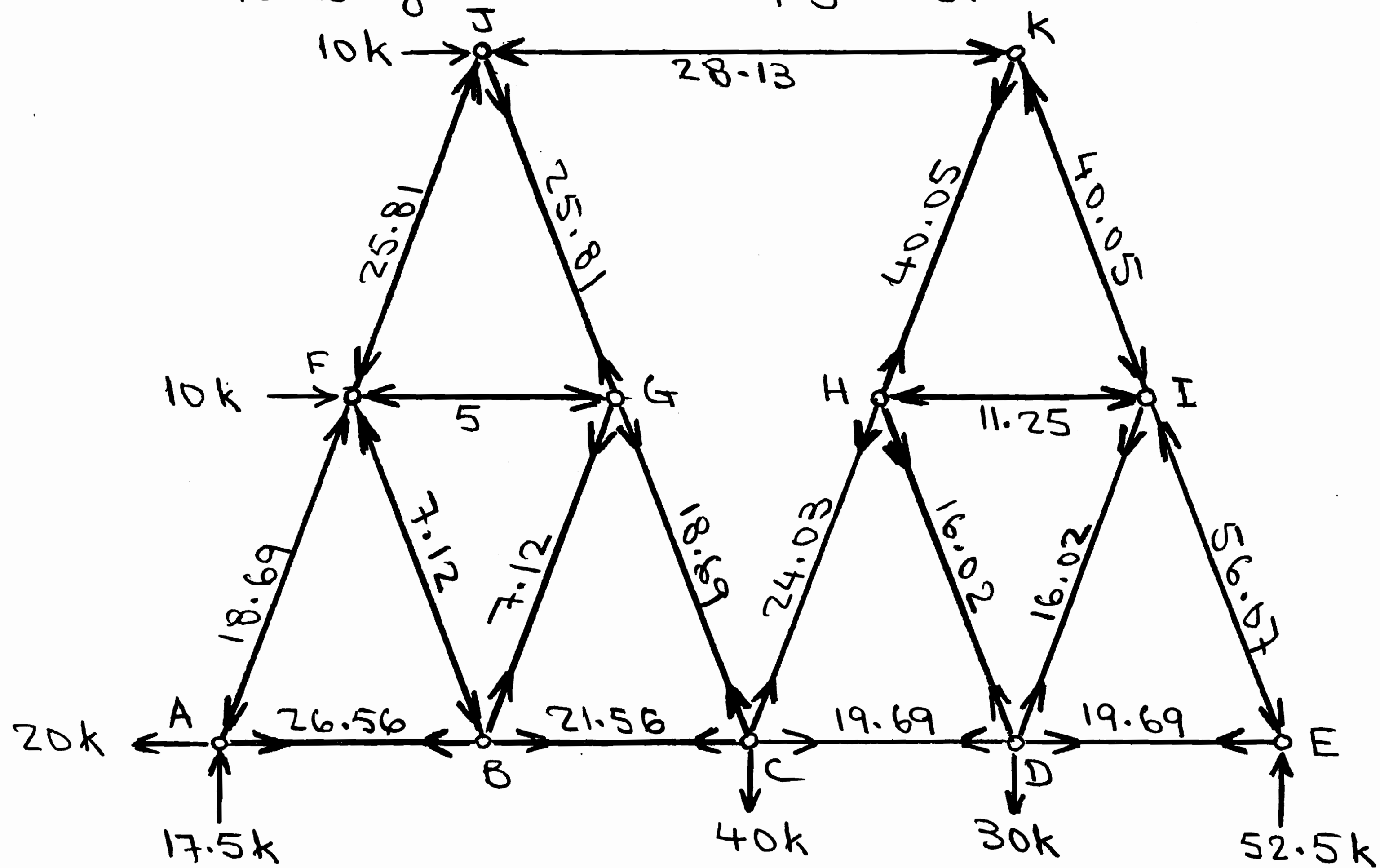
$F_{EK} = -67.08\text{ k} = 67.08\text{ k (C)}$



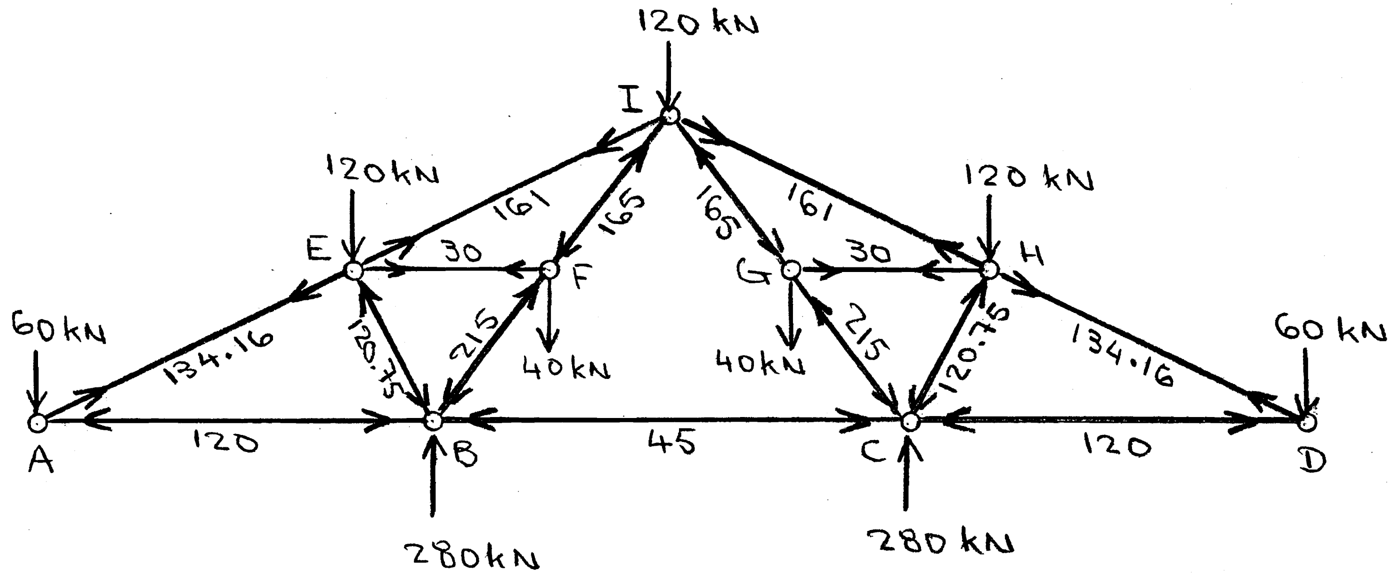
$+\uparrow \Sigma F_y = 0$

$\frac{1}{\sqrt{5}}(67.08) + \frac{1}{\sqrt{5}} F_{KO} - 40 - 20 = 0 \qquad F_{KO} = 67.08\text{ k (T)}$

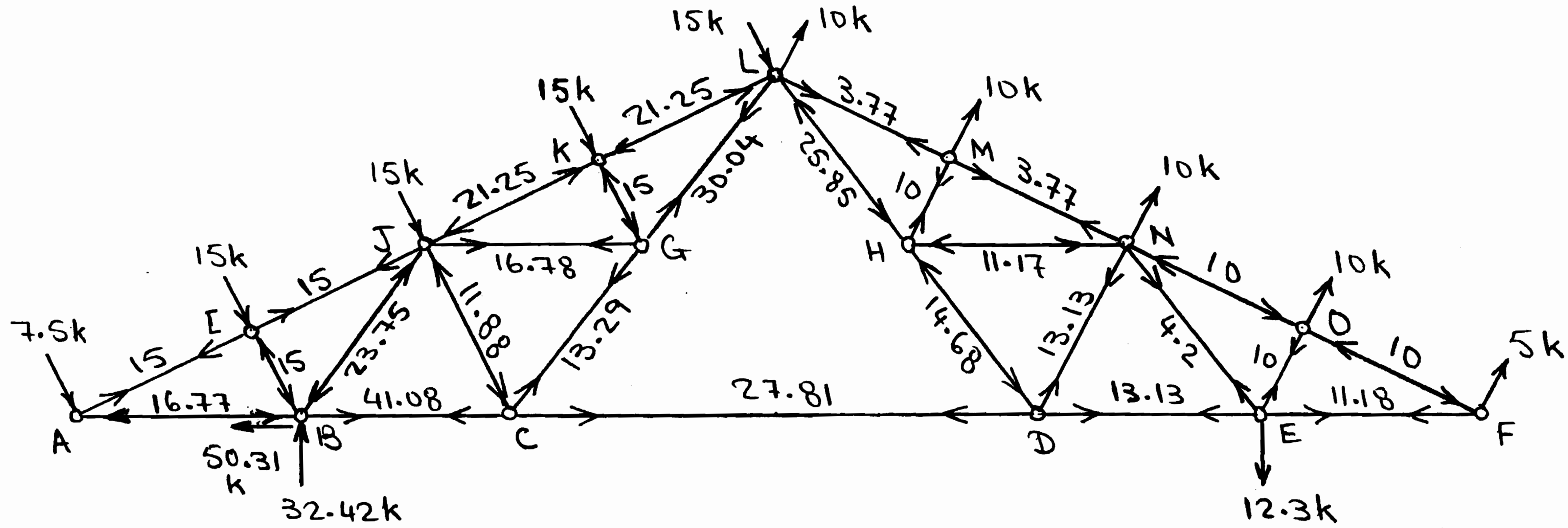
4.46 Considering a section through members BC, CG and JK, compute the force in member JK: $F_{JK} = 28.13\text{k (C)}$. Then, determine the remaining member forces by the method of joints.



4.47 Considering a section through members BC, FI and EI, compute the force in member BC: $F_{BC} = 45\text{ kN (C)}$. Then, determine the remaining member forces by the method of joints.

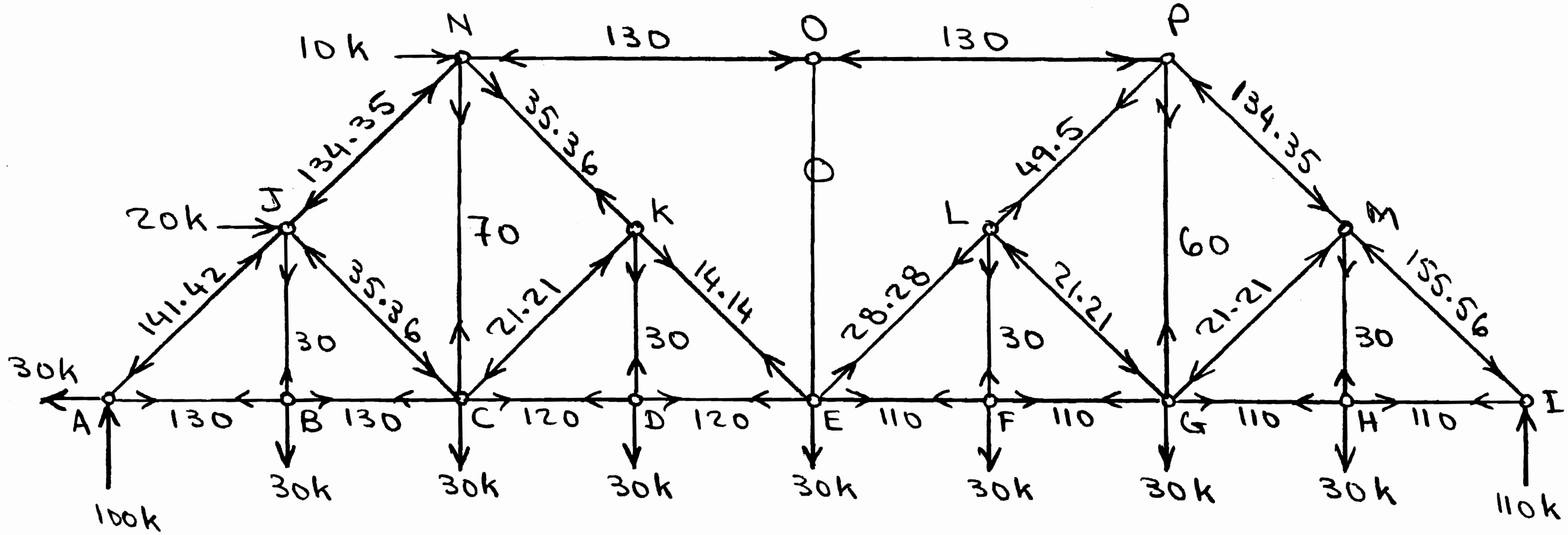


4.48 Considering a section through members CD, GL and KL, compute the force in member CD: $F_{CD} = 27.81\,k\ (T)$. Then, determine the remaining member forces by the method of joints.

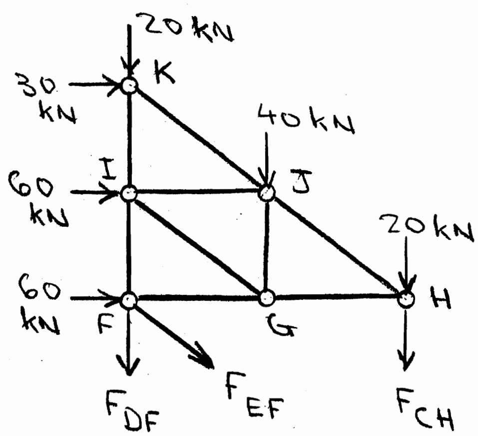


4.49 Considering a section through members DE, EK and NO, compute the force in member NO: $\underline{F_{NO} = 130\text{k (C)}}$.

Then, determine the remaining member forces by the method of joints.



4.50 Section through members DF, EF and CH:



$+\circlearrowleft \Sigma M_F = 0 \quad -30(6) - 60(3) - 40(4) - 20(8) - F_{CH}(8) = 0$

$F_{CH} = -85 \text{ kN} = \underline{85 \text{ kN (C)}}$

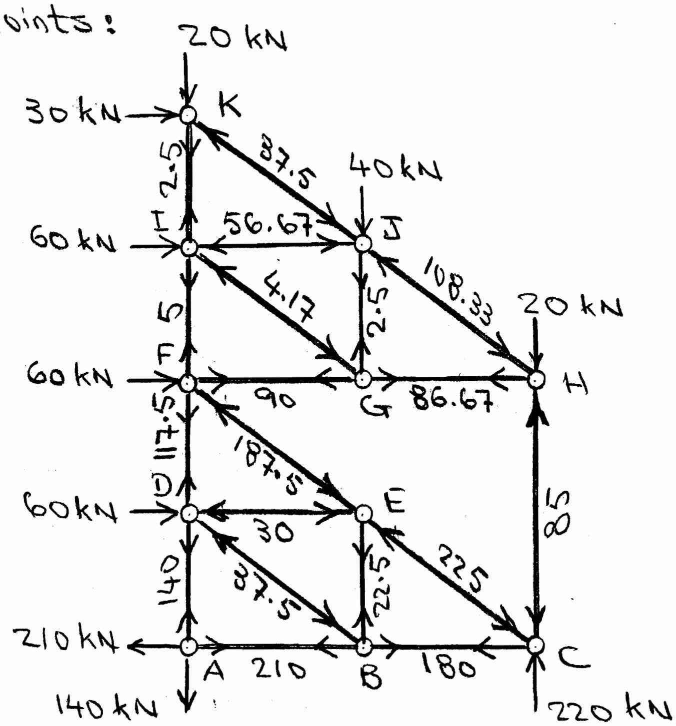
$\xrightarrow{+} \Sigma F_x = 0 \quad 30 + 60 + 60 + \left(\frac{4}{5}\right) F_{EF} = 0$

$F_{EF} = -187.5 \text{ kN} = \underline{187.5 \text{ kN (C)}}$

$+\uparrow \Sigma F_y = 0 \quad -20 - 40 - 20 - F_{DF} + 85 + \left(\frac{3}{5}\right)(187.5) = 0$

$\underline{F_{DF} = 117.5 \text{ (T)}}$

The remaining member forces can now be determined by the method of joints:



4.51

Member	Projection x (ft)	Projection y (ft)	Projection z (ft)	Length (ft)
AD	3	10	5	11.58
BD	8	10	7	14.59
CD	2	10	4	10.95

Joint D:

$$\Sigma F_x = 0 \quad -\left(\frac{3}{11.58}\right)F_{AD} + \left(\frac{8}{14.59}\right)F_{BD} + \left(\frac{2}{10.95}\right)F_{CD} + 6 = 0$$

$$\Sigma F_y = 0 \quad -\left(\frac{10}{11.58}\right)F_{AD} - \left(\frac{10}{14.59}\right)F_{BD} - \left(\frac{10}{10.95}\right)F_{CD} - 12 = 0$$

$$\Sigma F_z = 0 \quad \left(\frac{5}{11.58}\right)F_{AD} + \left(\frac{7}{14.59}\right)F_{BD} - \left(\frac{4}{10.95}\right)F_{CD} = 0$$

Solving these simultaneous equations, we obtain:

$F_{AD} = 1.12 \text{ k (T)}$

$F_{BD} = -7.56 \text{ k} = 7.56 \text{ k (C)}$

$F_{CD} = -8.54 \text{ k} = 8.54 \text{ k (C)}$

4.52

Member	Projection			Length (m)
	x (m)	y (m)	z (m)	
AB	0	0	3	3
AC	4	0	3	5
BC	4	0	0	4
AD	0	7	3	7.62
BD	0	7	0	7
CD	4	7	0	8.06

Joint D:

$\Sigma F_x = 0 \qquad \left(\frac{4}{8.06}\right) F_{CD} = 0 \qquad \underline{F_{CD} = 0}$

$\Sigma F_z = 0 \qquad \left(\frac{3}{7.62}\right) F_{AD} + 40 = 0 \qquad F_{AD} = -101.6 \text{ kN}$

$\underline{F_{AD} = 101.6 \text{ kN (C)}}$

$\Sigma F_y = 0 \qquad \left(\frac{7}{7.62}\right) 101.6 - F_{BD} - 60 = 0$

$\underline{F_{BD} = 33.33 \text{ kN (T)}}$

Joint B:

$\Sigma F_x = 0 \qquad \underline{F_{BC} = 0}$

$\Sigma F_z = 0 \qquad \underline{F_{AB} = 0}$

Joint C:

$\Sigma F_z = 0 \qquad \left(\frac{3}{5}\right) F_{AC} = 0 \qquad \underline{F_{AC} = 0}$

4.53

Member	Projection			Length (ft)
	x (ft)	y (ft)	z (ft)	
AB	20	0	0	20
AC	14	0	10	17.2
BC	6	0	10	11.66
AD	10	20	5	22.91
BD	10	20	5	22.91
CD	4	20	5	21

Joint D:

$$\Sigma F_x = 0 \qquad -\left(\frac{10}{22.91}\right)F_{AD} + \left(\frac{10}{22.91}\right)F_{BD} + \left(\frac{4}{21}\right)F_{CD} + 25 = 0$$

$$\Sigma F_y = 0 \qquad -\left(\frac{20}{22.91}\right)F_{AD} - \left(\frac{20}{22.91}\right)F_{BD} - \left(\frac{20}{21}\right)F_{CD} - 30 = 0$$

$$\Sigma F_z = 0 \qquad \left(\frac{5}{22.91}\right)F_{AD} + \left(\frac{5}{22.91}\right)F_{BD} - \left(\frac{5}{21}\right)F_{CD} = 0$$

Solving these equations, we obtain:

$F_{AD} = 16.63\text{ k (T)}$ $\qquad F_{BD} = -33.84\text{ k} = 33.84\text{ k (C)}$

$F_{CD} = -15.74\text{ k} = 15.74\text{ k (C)}$

Joint A:

$$\Sigma F_z = 0 \qquad -\left(\frac{5}{22.91}\right)16.63 - \left(\frac{10}{17.2}\right)F_{AC} = 0$$

$$F_{AC} = -6.24\text{ k} = 6.24\text{ k (C)}$$

$$\Sigma F_x = 0 \qquad \left(\frac{10}{22.91}\right)16.63 - \left(\frac{14}{17.2}\right)6.24 + F_{AB} = 0$$

$$F_{AB} = -2.18\text{ k} = 2.18\text{ k (C)}$$

Joint B:

$$\Sigma F_z = 0 \qquad \left(\frac{5}{22.91}\right)33.84 - \left(\frac{10}{11.66}\right)F_{BC} = 0$$

$$F_{BC} = 8.61\text{ k (T)}$$

4.54

Member	Projection			Length (m)
	x (m)	y (m)	z (m)	
AB	8	0	0	8
CD	8	0	0	8
BC	0	0	4	4
AD	0	0	4	4
BD	8	0	4	8.94
AE	4	8	2	9.17
BE	4	8	2	9.17
CE	4	8	2	9.17
DE	4	8	2	9.17

Joint C: $\Sigma F_y = 0 \qquad \left(\frac{8}{9.17}\right) F_{CE} = 0 \qquad \underline{F_{CE} = 0}$

$\Sigma F_z = 0 \qquad \underline{F_{BC} = 0}$

Joint E: $\Sigma F_x = 0 \qquad \frac{4}{9.17}\left(-F_{AE} + F_{BE} - F_{DE}\right) + 30 = 0$

$\Sigma F_y = 0 \qquad -\frac{8}{9.17}\left(F_{AE} + F_{BE} + F_{DE}\right) - 60 = 0$

$\Sigma F_z = 0 \qquad \frac{2}{9.17}\left(F_{AE} + F_{BE} - F_{DE}\right) + 40 = 0$

Solving these equations, we obtain:

$F_{AE} = -57.31 \text{ kN} = \underline{57.31 \text{ kN (C)}}$;

$F_{BE} = -68.78 \text{ kN} = \underline{68.78 \text{ kN (C)}}$ and $\underline{F_{DE} = 57.31 \text{ kN (T)}}$

Joint A:

$\Sigma F_x = 0 \qquad -\left(\frac{4}{9.17}\right) 57.31 + F_{AB} = 0 \qquad \underline{F_{AB} = 25 \text{ kN (T)}}$

$\Sigma F_z = 0 \qquad \left(\frac{2}{9.17}\right) 57.31 - F_{AD} = 0 \qquad \underline{F_{AD} = 12.5 \text{ kN (T)}}$

Joint D:

$\Sigma F_z = 0 \qquad \left(\frac{2}{9.17}\right) 57.31 + 12.5 + \left(\frac{4}{8.94}\right) F_{BD} = 0$

$F_{BD} = -55.87 \text{ kN} = \underline{55.87 \text{ kN (C)}}$

$\Sigma F_x = 0 \qquad \left(\frac{4}{9.17}\right) 57.31 - \left(\frac{8}{8.94}\right) 55.87 + F_{CD} = 0$

$\underline{F_{CD} = 25 \text{ kN (T)}}$

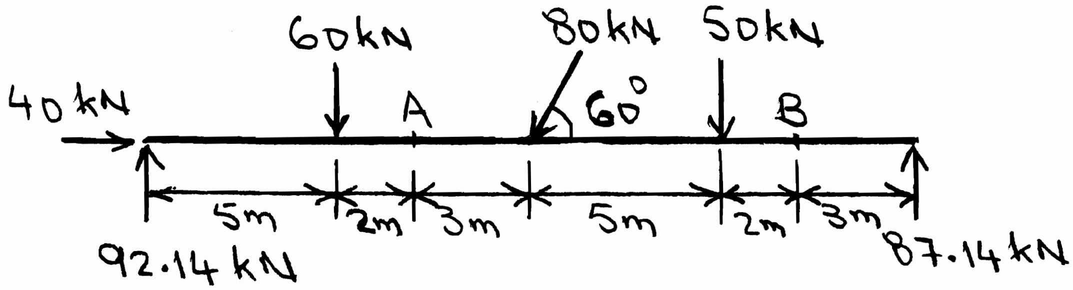
4.55

Member	Force (k)
AB	29.17 (T)
CD	15.83 (C)
BC	7.22 (T)
AD	0
AC	0.5 (T)
EF	28.33 (T)
GH	10.0 (C)
FG	5.55 (C)
EH	5.55 (C)
FH	2.0 (T)
AE	4.12 (T)
BF	8.25 (C)
CG	80.41 (C)
DH	39.17 (C)
BE	52.25 (C)
CF	8.83 (T)
DG	45.12 (T)
AH	2.21 (C)

Chapter Five
Beams and Frames: Shear and Bending Movement

CHAPTER 5

5.1



$Q_A = \underline{-40 \text{ kN}}$

$S_A = 92.14 - 60 = \underline{32.14 \text{ kN}}$

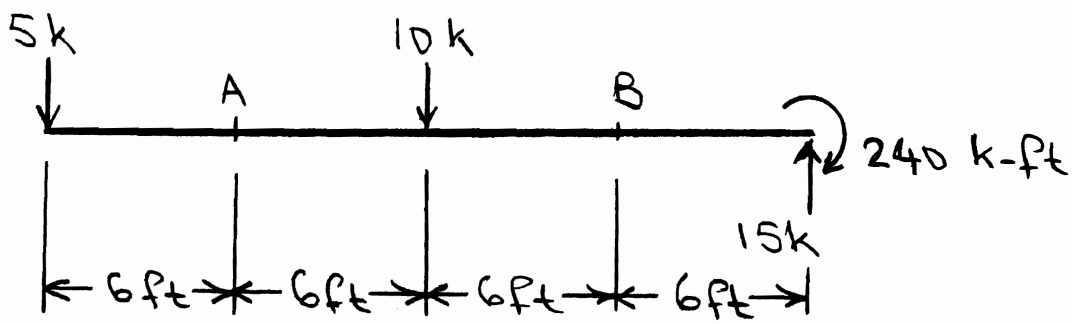
$M_A = 92.14(7) - 60(2) = \underline{524.98 \text{ kN.m}}$

$Q_B = \underline{0}$

$S_B = \underline{-87.14 \text{ kN}}$

$M_B = 87.14(3) = \underline{261.42 \text{ kN.m}}$

5.2



$Q_A = \underline{0}$

$S_A = \underline{-5\ k}$

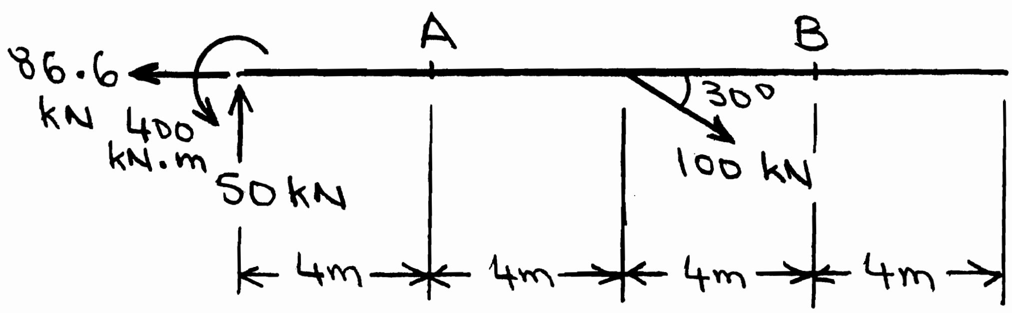
$M_A = -5(6) = \underline{-30\ k\text{-}ft}$

$Q_B = \underline{0}$

$S_B = -5 - 10 = \underline{-15\ k}$

$M_B = -5(18) - 10(6) = \underline{-150\ k\text{-}ft}$

5-3



$Q_A = 100\cos 30^\circ = \underline{86.6\ kN}$

$S_A = 100\sin 30^\circ = \underline{50\ kN}$

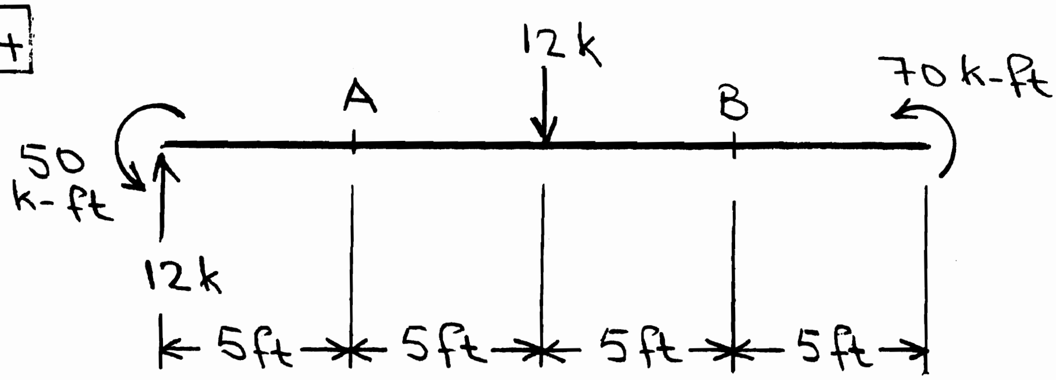
$M_A = -100\sin 30^\circ (4) = \underline{-200\ kN.m}$

$Q_B = \underline{0}$

$S_B = \underline{0}$

$M_B = \underline{0}$

5.4



$Q_A = \underline{0}$

$S_A = \underline{12\ k}$

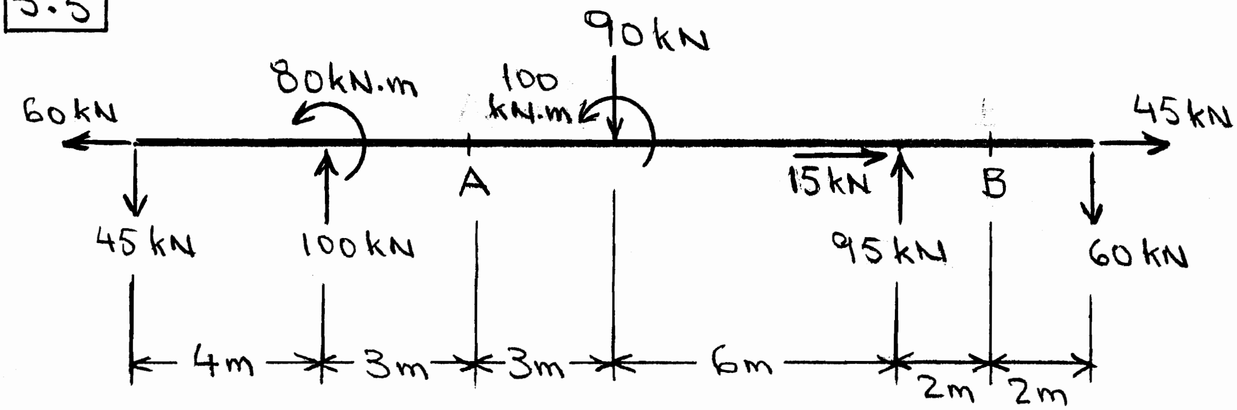
$M_A = 12(5) - 50 = \underline{10\ k\text{-}ft}$

$Q_B = \underline{0}$

$S_B = \underline{0}$

$M_B = \underline{70\ k\text{-}ft}$

5.5



$Q_A = \underline{60 \text{ kN}}$

$S_A = -45 + 100 = \underline{55 \text{ kN}}$

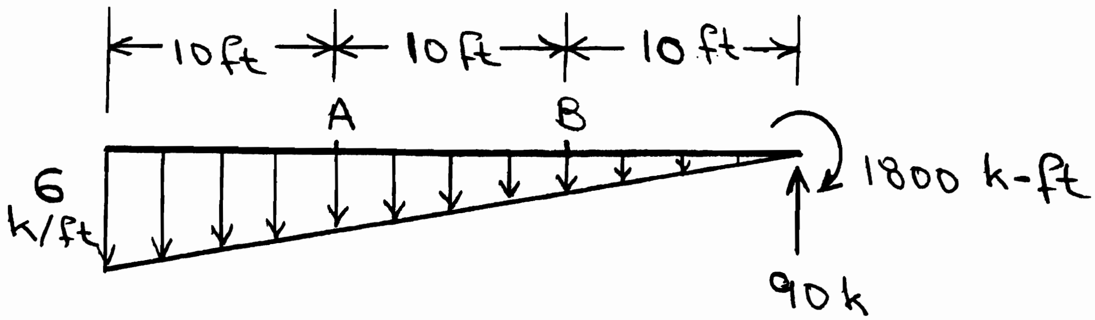
$M_A = -45(7) - 80 + 100(3) = \underline{-95 \text{ kN.m}}$

$Q_B = \underline{45 \text{ kN}}$

$S_B = \underline{60 \text{ kN}}$

$M_B = -60(2) = \underline{-120 \text{ kN.m}}$

5.6



$Q_A = \underline{0}$

$S_A = -\frac{(6+4)}{2}(10) = \underline{-50\ k}$

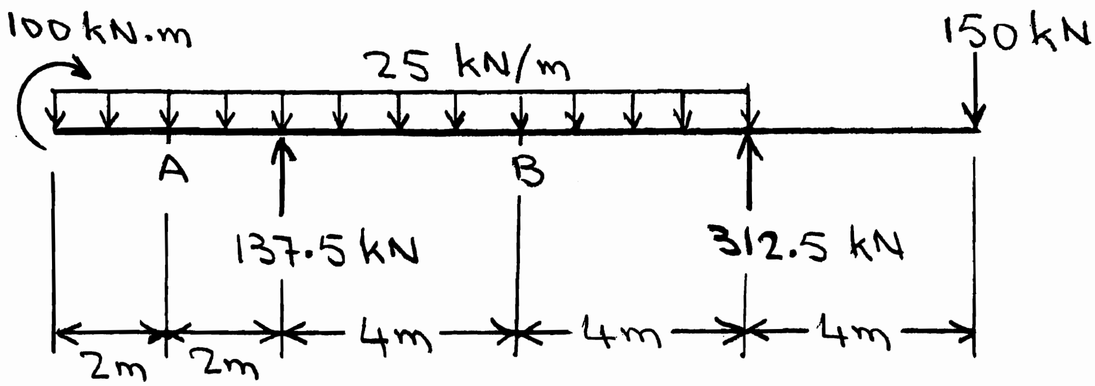
$M_A = -4(10)5 - \frac{1}{2}(2)(10)\left(\frac{20}{3}\right) = \underline{-266.67\ k\text{-}ft}$

$Q_B = \underline{0}$

$S_B = -\frac{(6+2)}{2}(20) = \underline{-80\ k}$

$M_B = -2(20)10 - \frac{1}{2}(4)(20)\left(\frac{40}{3}\right) = \underline{-933.33\ k\text{-}ft}$

5.7



$Q_A = \underline{0}$

$S_A = -25(2) = \underline{-50 \text{ kN}}$

$M_A = 100 - 25(2)(1) = \underline{50 \text{ kN.m}}$

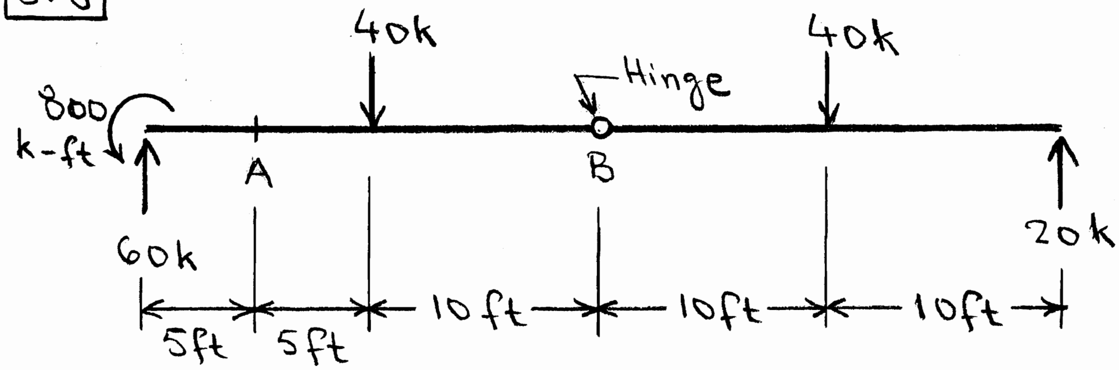
$Q_B = \underline{0}$

$S_B = 150 - 312.5 + 25(4) = \underline{-62.5 \text{ kN}}$

$M_B = -150(8) + 312.5(4) - 25(4)(2)$

$M_B = \underline{-150 \text{ kN.m}}$

5.8



$Q_A = \underline{0}$

$S_A = \underline{60\text{ k}}$

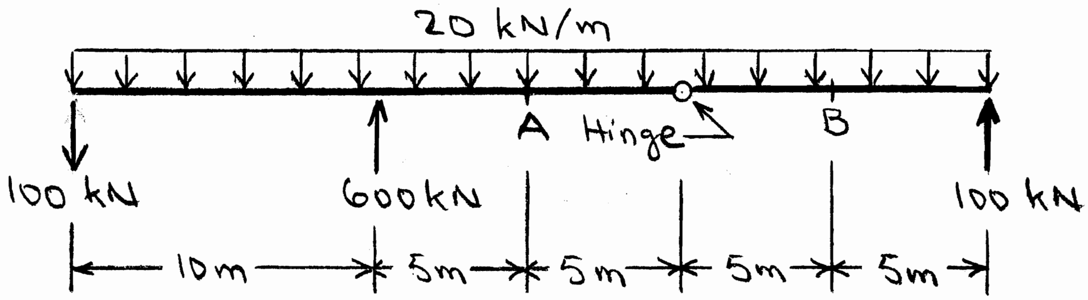
$M_A = -800 + 60(5) = \underline{-500\text{ k-ft}}$

$Q_B = \underline{0}$

$S_B = -20 + 40 = \underline{20\text{ k}}$

$M_B = 20(20) - 40(10) = \underline{0}$

5.9



$Q_A = \underline{0}$

$S_A = -100 - 20(15) + 600 = \underline{200\ kN}$

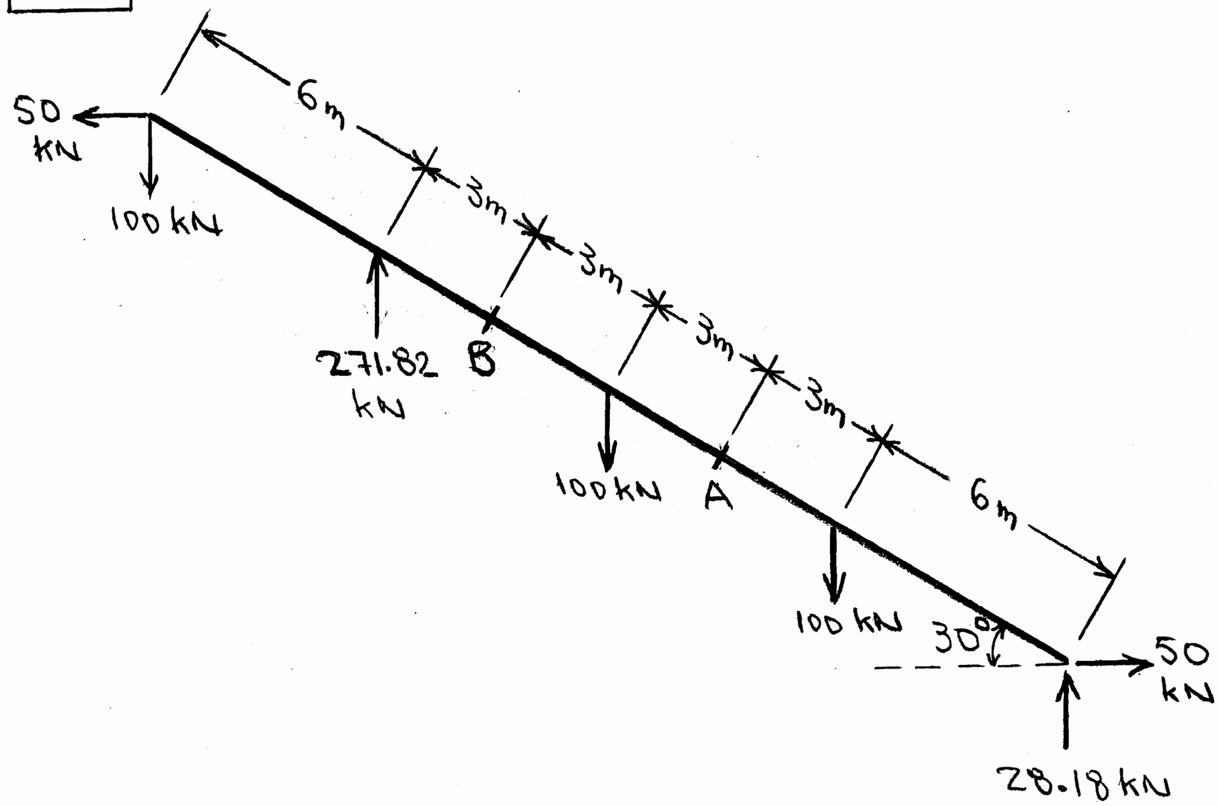
$M_A = -100(15) - 20(15)(7.5) + 600(5) = \underline{-750\ kN \cdot m}$

$Q_B = \underline{0}$

$S_B = -100 + 20(5) = \underline{0}$

$M_B = 100(5) - 20(5)(2.5) = \underline{250\ kN \cdot m}$

5.10



$Q_A = 50(\cos 30°) - 28.18(\sin 30°) + 100(\sin 30°) = \underline{79.2 \text{ kN}}$

$S_A = -50(\sin 30°) - 28.18(\cos 30°) + 100(\cos 30°) = \underline{37.2 \text{ kN}}$

$M_A = 50(\sin 30°)(9) + 28.18(\cos 30°)(9) - 100(\cos 30°)(3)$

$= \underline{184.8 \text{ kN.m}}$

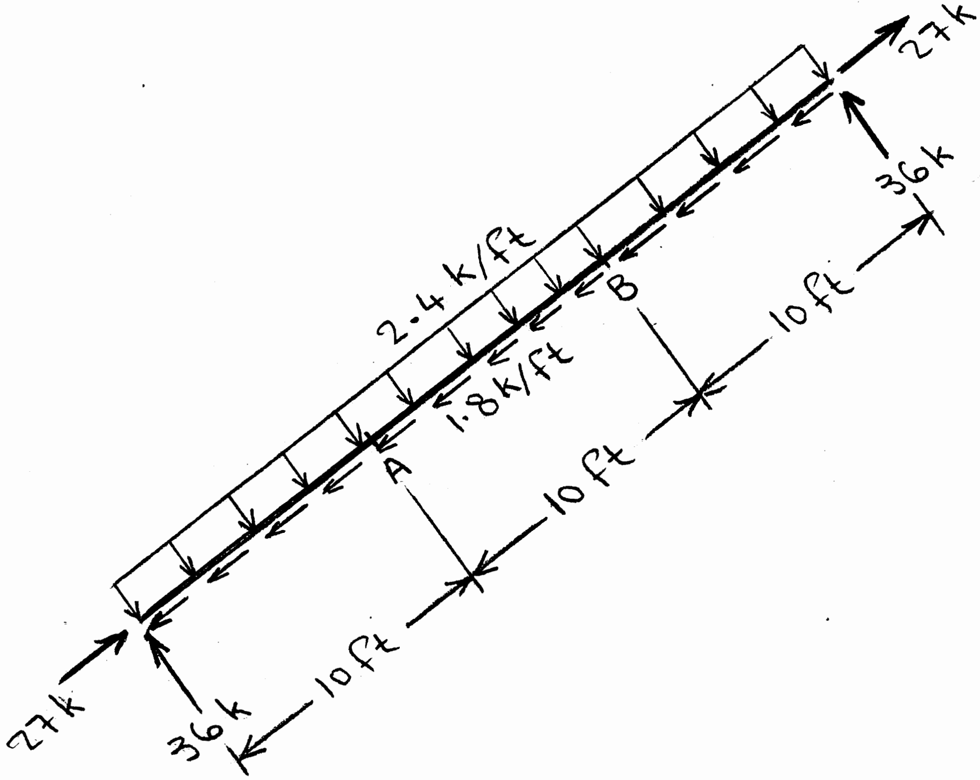
$Q_B = 50(\cos 30°) - 100(\sin 30°) + 271.82(\sin 30°) = \underline{129.2 \text{ kN}}$

$S_B = -50(\sin 30°) - 100(\cos 30°) + 271.82(\cos 30°) = \underline{123.8 \text{ kN}}$

$M_B = -50(\sin 30°)(9) - 100(\cos 30°)(9) + 271.82(\cos 30°)(3)$

$= \underline{-298.2 \text{ kN.m}}$

5.11



$Q_A = -27 + 1.8(10) = \underline{-9\ k}$

$S_A = 36 - 2.4(10) = \underline{12\ k}$

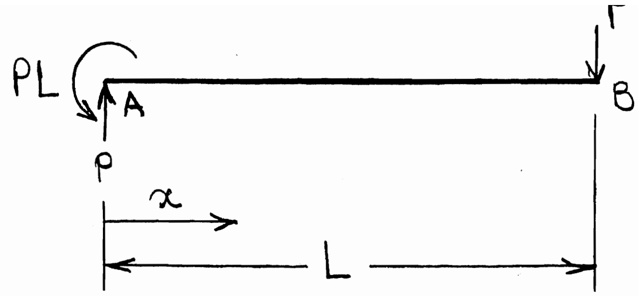
$M_A = 36(10) - 2.4(10)(5) = \underline{240\ k\text{-}ft}$

$Q_B = 27 - 1.8(10) = \underline{9\ k}$

$S_B = -36 + 2.4(10) = \underline{-12\ k}$

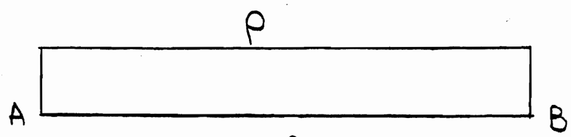
$M_B = 36(10) - 2.4(10)(5) = \underline{240\ k\text{-}ft}$

5.12

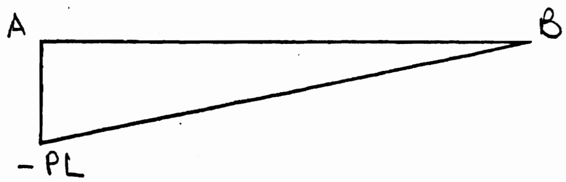


$S = \underline{P}$ for $0 < x < L$

$M = -PL + Px = \underline{P(x - L)}$ for $0 < x \leq L$

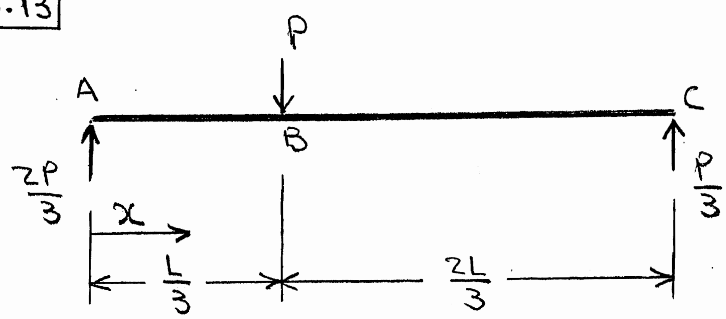


Shear Diagram



Bending Moment Diagram

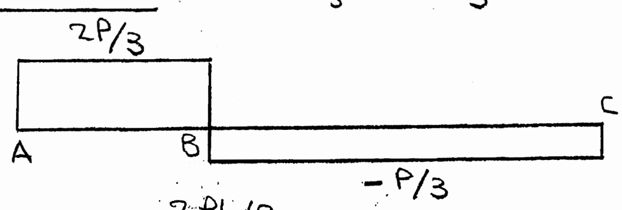
5.13



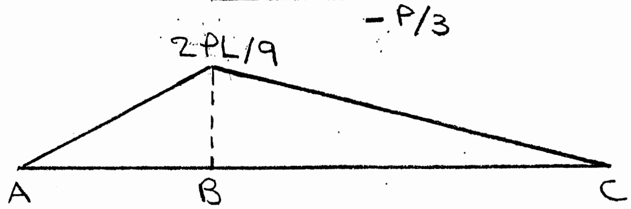
$\underline{0 < x < \frac{L}{3}:}$ $\quad S = \frac{2P}{3}$ $\quad M = \frac{2P}{3}x$

$\underline{\frac{L}{3} < x < L:}$ $\quad S = \frac{2P}{3} - P = -\frac{P}{3}$ $\quad M = \frac{2P}{3}x - P\left(x - \frac{L}{3}\right)$

$$= \frac{P}{3}(L - x)$$

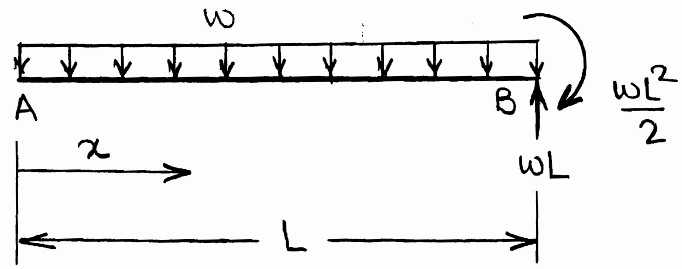


Shear Diagram



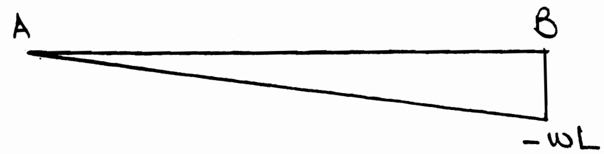
Bending Moment Diagram

5.14

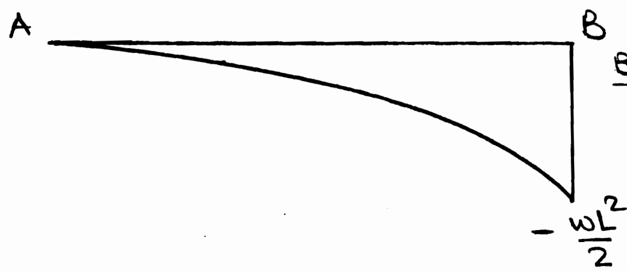


$S = \underline{-wx}$ for $0 \leqslant x < L$

$M = \underline{-\frac{wx^2}{2}}$ for $0 \leqslant x \leqslant L$

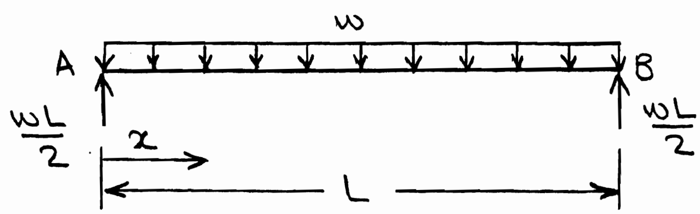


Shear Diagram



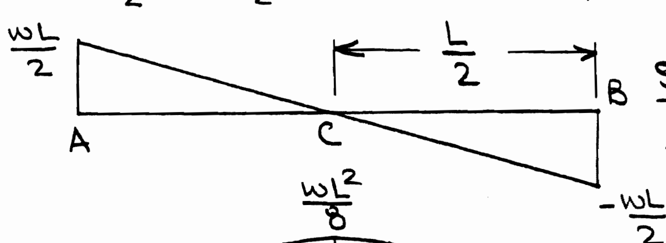
Bending Moment Diagram

5.15

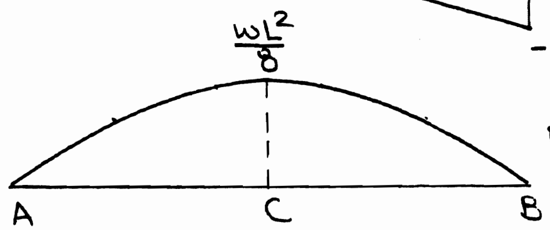


$S = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$ for $0 < x < L$

$M = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{wx}{2}(L - x)$ for $0 \leqslant x \leqslant L$

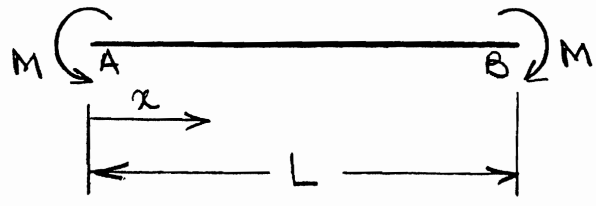


Shear Diagram



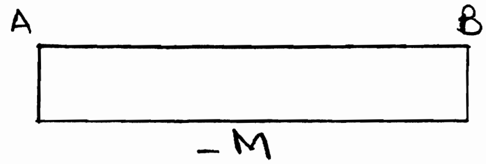
Bending Moment Diagram

5.16



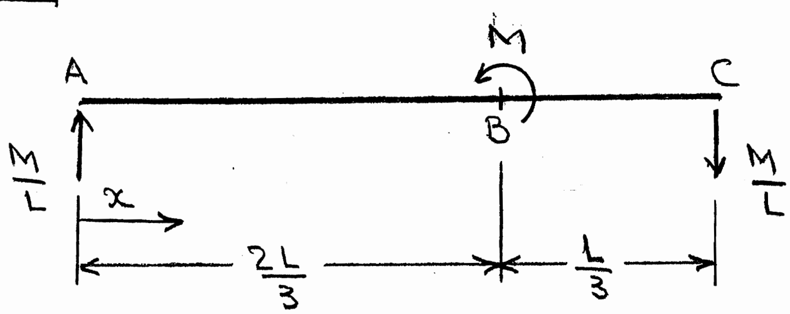
$S = \underline{0}$

$M_x = \underline{-M}$ for $0 < x < L$



Bending Moment Diagram

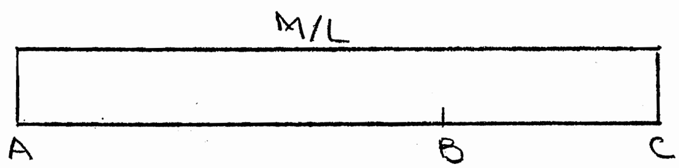
5.17



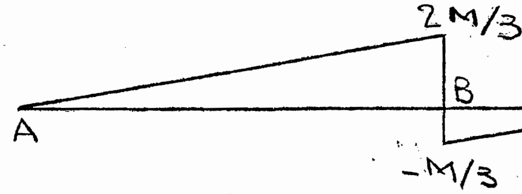
$S = \frac{M}{L}$ for $0 < x < L$

$B.M. = \frac{Mx}{L}$ for $0 \leq x < \frac{2L}{3}$

$B.M. = \frac{Mx}{L} - M = \frac{M}{L}(x - L)$ for $\frac{2L}{3} < x \leq L$

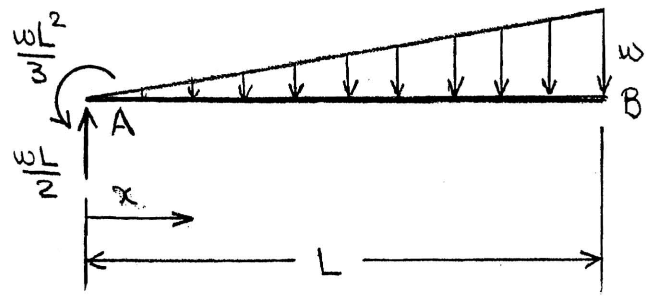


Shear Diagram



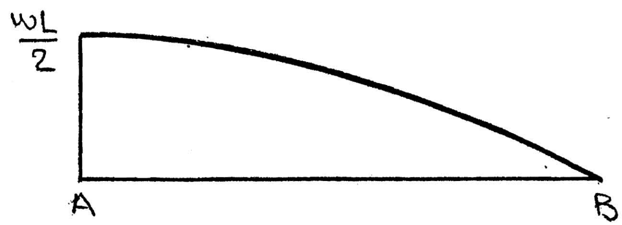
Bending Moment Diagram

5.18

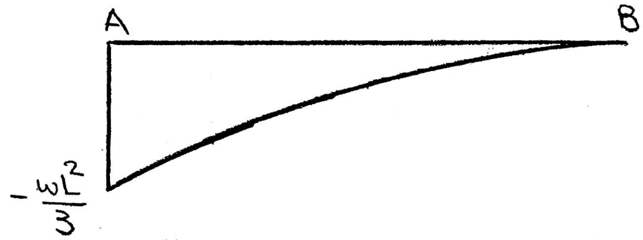


$$S = \frac{wL}{2} - \frac{1}{2}(x)\left(\frac{wx}{L}\right) = \frac{wL}{2}\left[1 - \left(\frac{x}{L}\right)^2\right] \qquad 0 < x \leq L$$

$$M = -\frac{wL^2}{3} + \left(\frac{wL}{2}\right)x - \left(\frac{wx^2}{2L}\right)\left(\frac{x}{3}\right) = -\frac{wL^2}{3}\left[1 - \frac{3}{2}\left(\frac{x}{L}\right) + \frac{1}{2}\left(\frac{x}{L}\right)^3\right] \qquad 0 < x \leq L$$

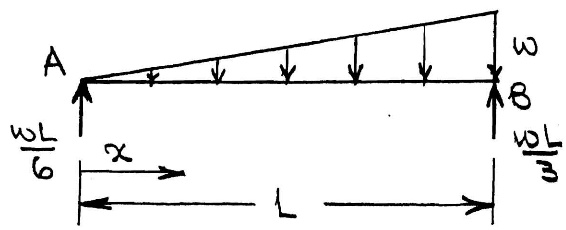


Shear Diagram



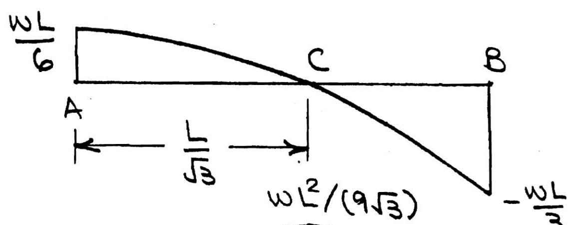
Bending Moment Diagram

5.19

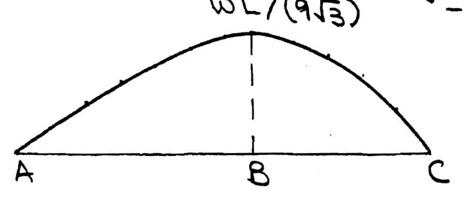


$$S = \frac{wL}{6} - \frac{1}{2}\left(\frac{wx}{L}\right)x = \frac{w}{6L}(L^2 - 3x^2) \quad \text{for } 0 < x < L$$

$$M = \frac{wL}{6}x - \frac{wx^2}{2L}\left(\frac{x}{3}\right) = \frac{wx}{6L}(L^2 - x^2) \quad \text{for } 0 \leq x \leq L$$

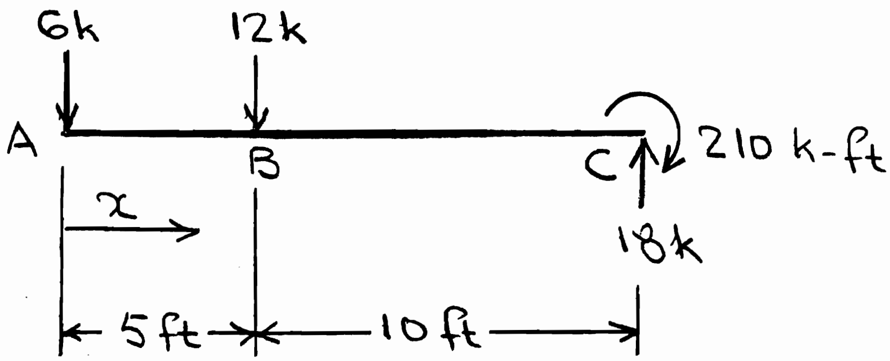


Shear Diagram



Bending Moment Diagram

5.20

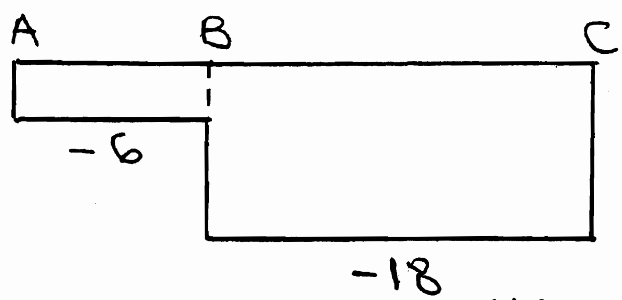


$0 < x < 5'$: $S = -6$

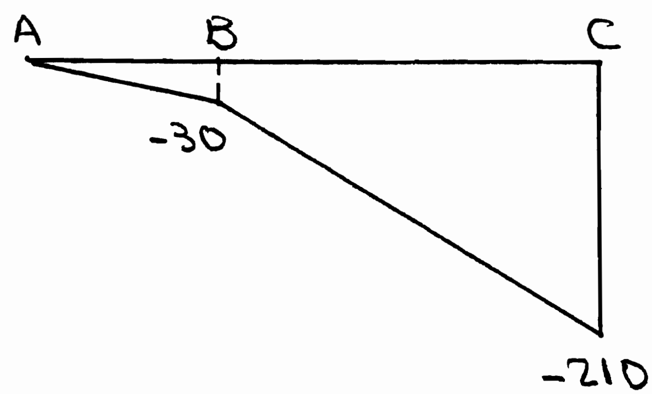
$M = -6x$

$5' < x < 15'$: $S = -6 - 12 = -18$

$M = -6x - 12(x-5) = 60 - 18x$

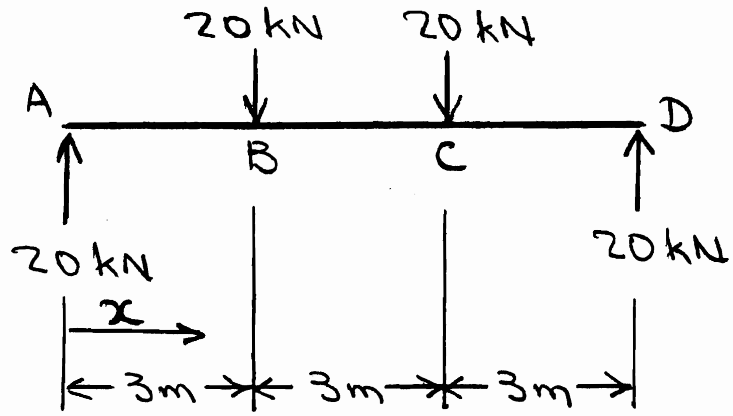


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.21



$0 < x < 3\text{m}$:

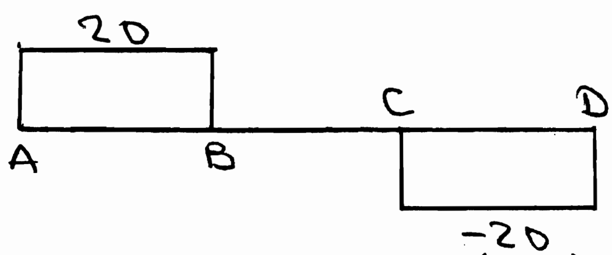
$$S = 20$$
$$M = 20x$$

$3\text{m} < x < 6\text{m}$:

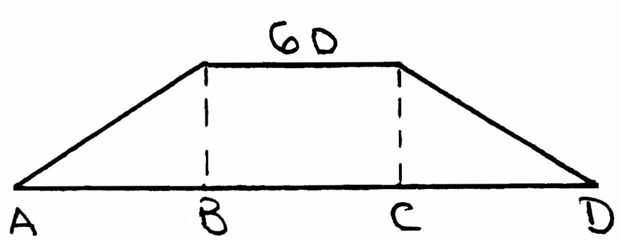
$$S = 20 - 20 = 0$$
$$M = 20x - 20(x-3) = 60$$

$6\text{m} < x < 9\text{m}$:

$$S = 20 - 20 - 20 = -20$$
$$M = 20x - 20(x-3) - 20(x-6)$$
$$= 180 - 20x$$

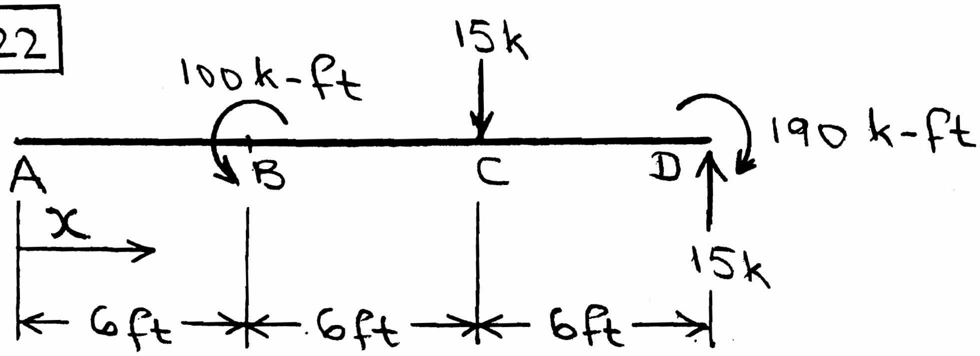


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

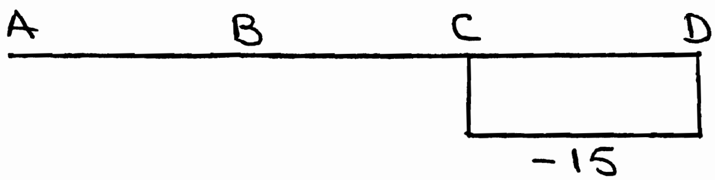
5.22



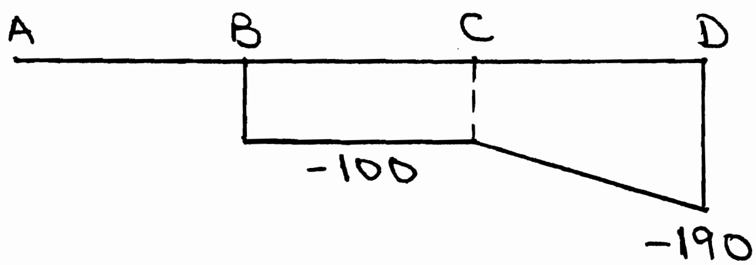
$0 \leq x < 6'$: $S = 0$, $M = 0$

$6' < x < 12'$: $S = 0$, $M = -100$

$12' < x < 18'$: $S = -15$, $M = -100 - 15(x-12) = 80 - 15x$

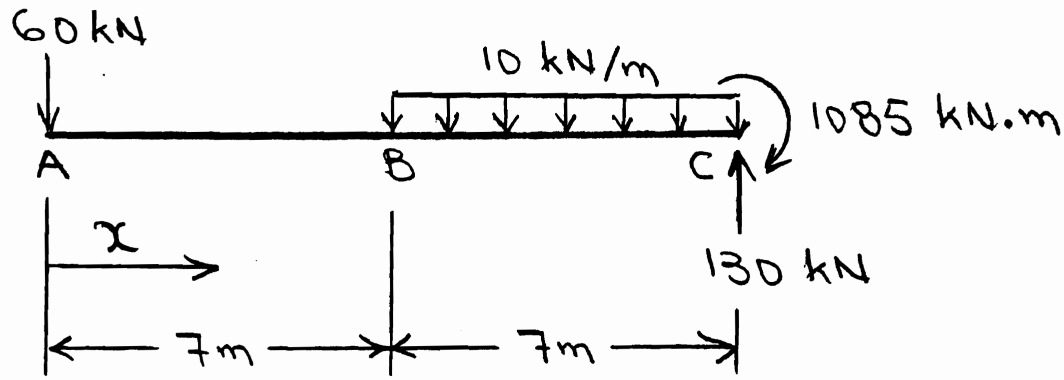


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.23



$0 < x \leqslant 7\text{m}$:

$$S = -60$$

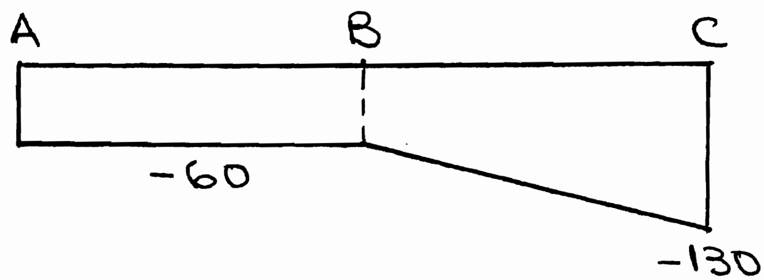
$$M = -60x$$

$7\text{m} \leqslant x < 14\text{m}$:

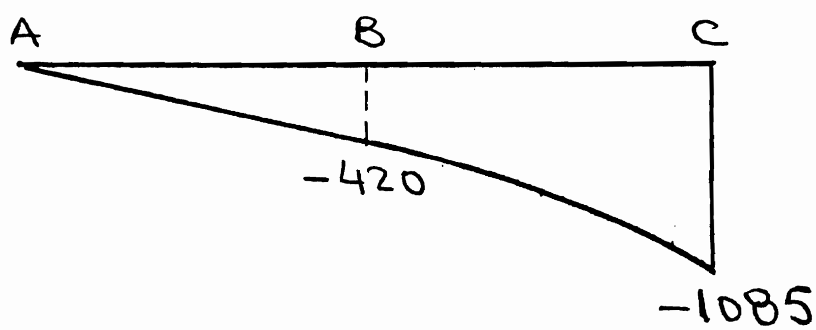
$$S = -60 - 10(x-7) = 10(1-x)$$

$$M = -60x - \frac{10(x-7)^2}{2}$$

$$= -5x^2 + 10x - 245$$

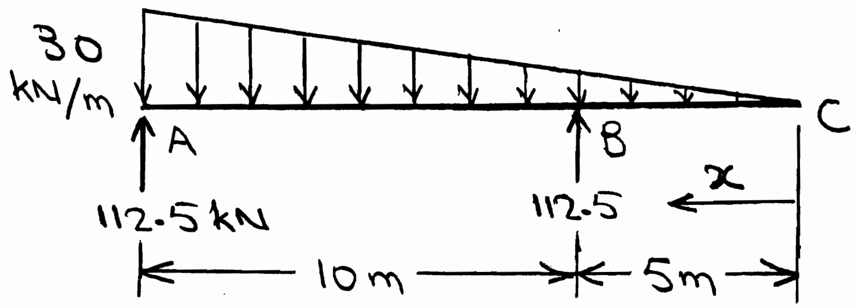


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.24



$0 \leq x < 5\text{m}$:

$$S = \frac{1}{2}(2x)x = x^2$$

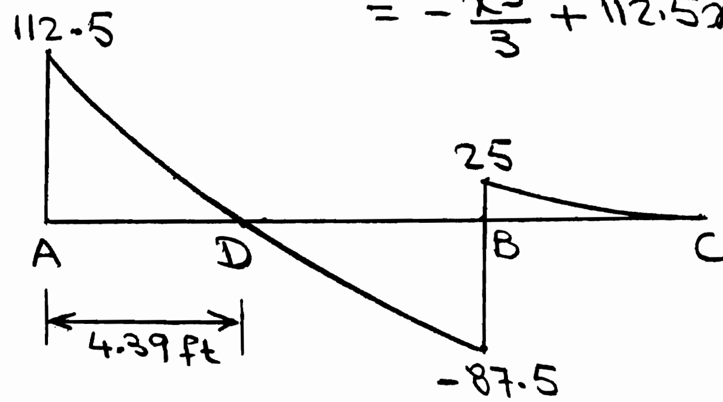
$$M = -\frac{1}{2}(2x)x\left(\frac{x}{3}\right) = -\frac{x^3}{3}$$

$5\text{m} < x < 10\text{m}$:

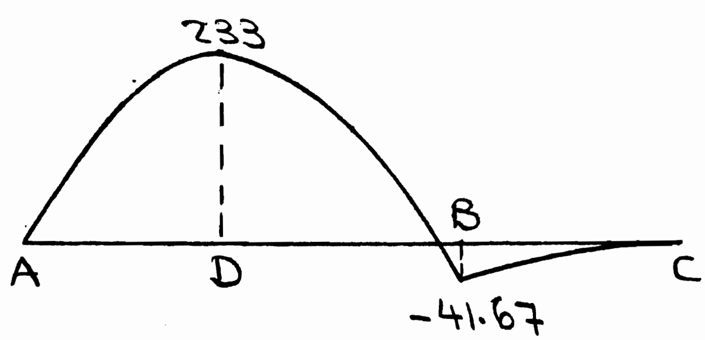
$$S = x^2 - 112.5$$

$$M = -\frac{x^3}{3} + 112.5(x-5)$$

$$= -\frac{x^3}{3} + 112.5x - 562.5$$

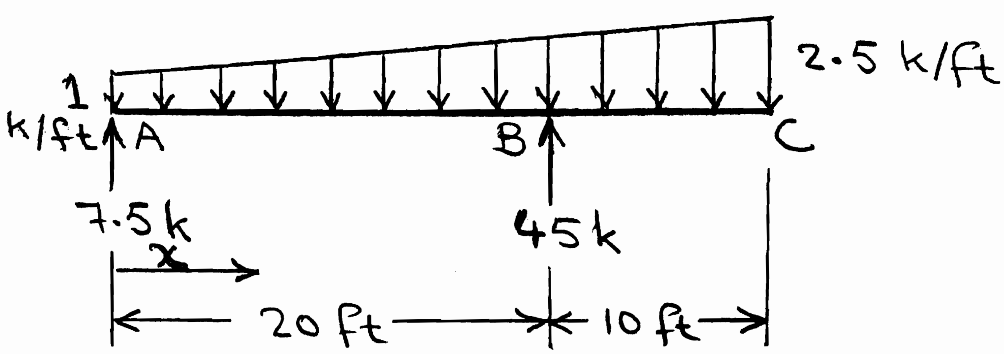


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.25

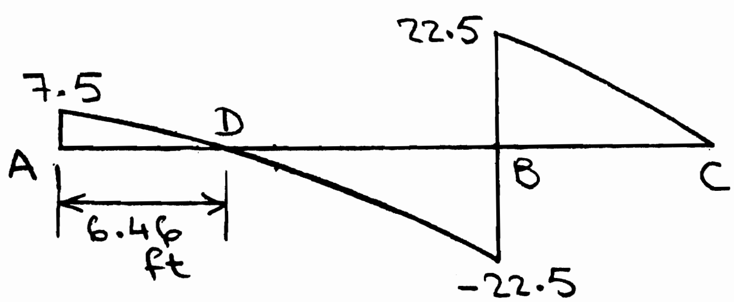


$0 < x < 20'$:

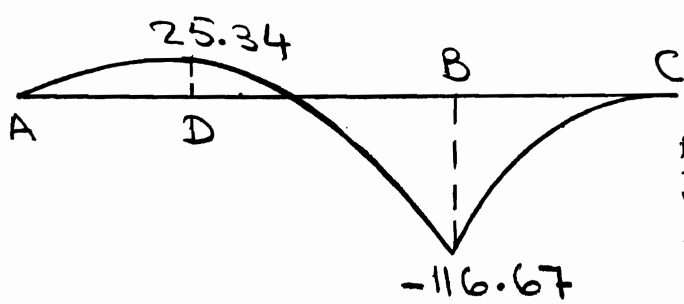
$$S = 7.5 - 1(x) - \frac{1}{2}\left(\frac{x}{20}\right)x$$
$$= -\frac{x^2}{40} - x + 7.5$$
$$M = 7.5x - 1\left(\frac{x^2}{2}\right) - \frac{1}{2}\left(\frac{x}{20}\right)x\left(\frac{x}{3}\right)$$
$$= -\frac{x^3}{120} - \frac{x^2}{2} + 7.5x$$

$20' < x < 30'$:

$$S = 7.5 - 1(x) - \frac{1}{2}\left(\frac{x}{20}\right)x + 45$$
$$= -\frac{x^2}{40} - x + 52.5$$
$$M = 7.5x - 1\left(\frac{x^2}{2}\right) - \frac{x^3}{120} + 45(x-20)$$
$$= -\frac{x^3}{120} - \frac{x^2}{2} + 52.5x - 900$$

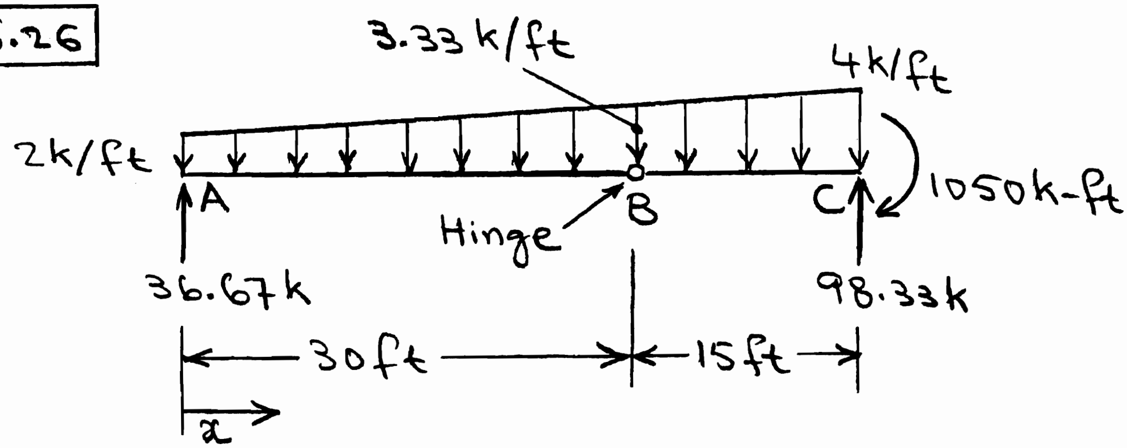


Shear Diagram (k)



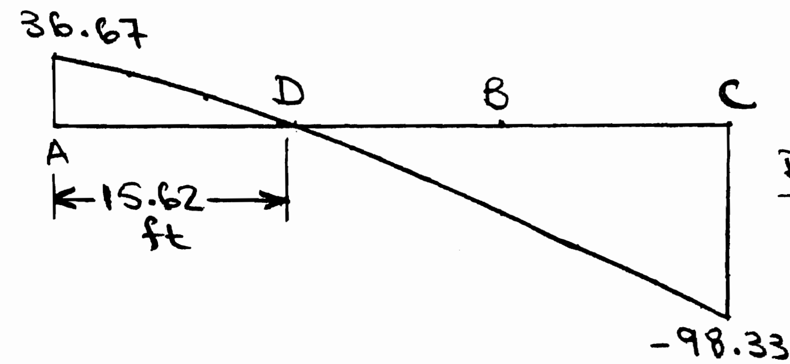
Bending Moment Diagram (k-ft)

5.26

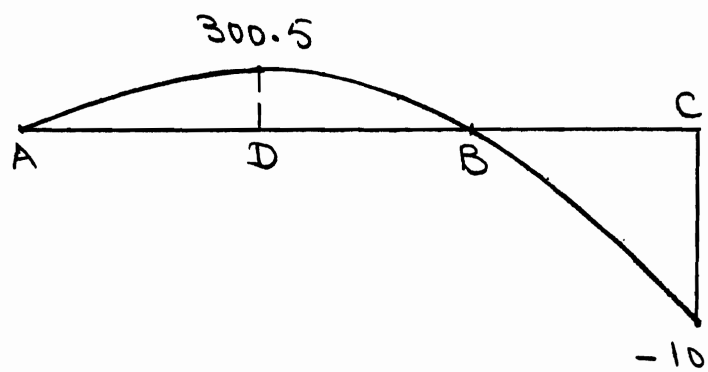


$0 \leqslant x \leqslant 45'$:

$$S = 36.67 - 2x - \frac{1}{2}\left(\frac{2}{45}x\right)x$$
$$= 36.67 - 2x - \frac{x^2}{45}$$
$$M = 36.67x - 2(x)\frac{x}{2} - \frac{1}{2}\left(\frac{2}{45}x\right)x\left(\frac{x}{3}\right)$$
$$= 36.67x - x^2 - \frac{x^3}{135}$$

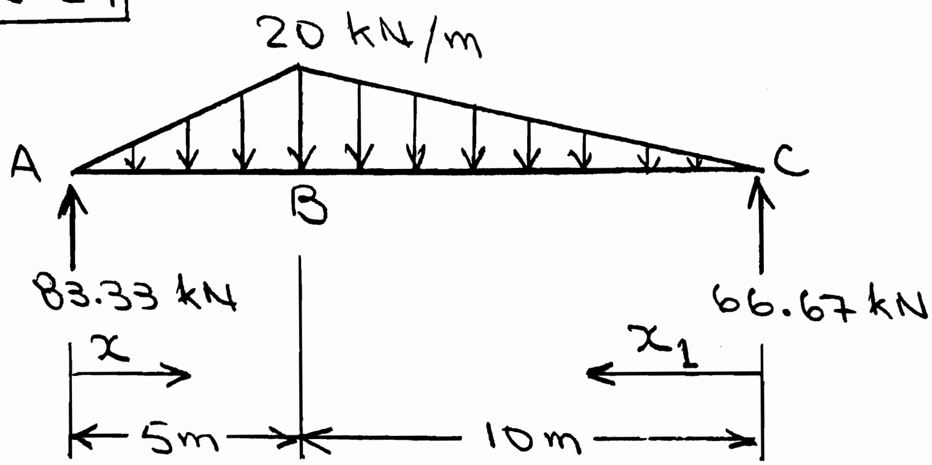


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.27



$0 < x \leq 5\text{m}$:

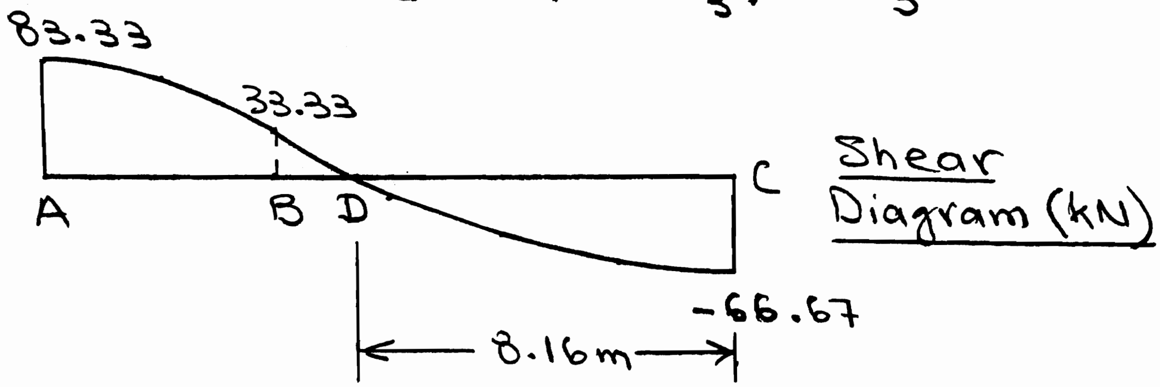
$$S = 83.33 - \frac{1}{2}\left(\frac{20x}{5}\right)x = -2x^2 + 83.33$$

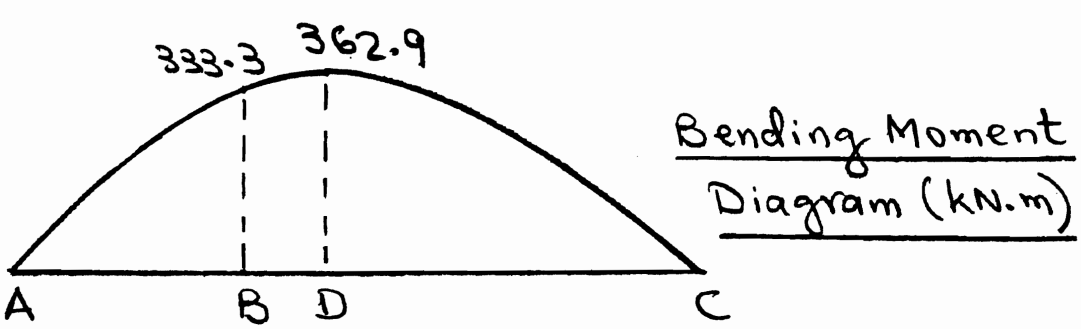
$$M = 83.33x - \frac{1}{2}\left(\frac{20x}{5}\right)x\left(\frac{x}{3}\right) = -\frac{2x^3}{3} + 83.33x$$

$0 < x_1 \leq 10\text{ m}$:

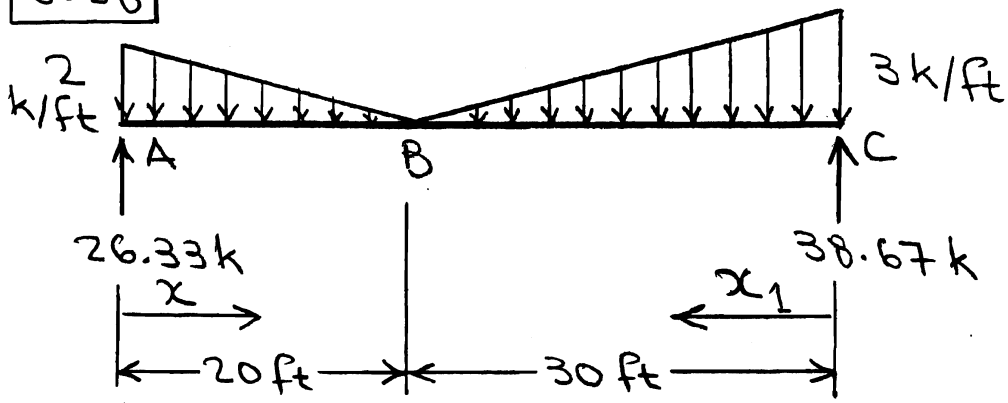
$$S = -66.67 + \frac{1}{2}(2x_1)x_1 = x_1^2 - 66.67$$

$$M = 66.67x_1 - \frac{1}{2}(2x_1)x_1\left(\frac{x_1}{3}\right) = -\frac{x_1^3}{3} + 66.67x_1$$





5.28



$0 < x \leq 20'$:

$$S = 26.33 - 2x + \frac{1}{2}\left(\frac{x}{10}\right)x = \frac{x^2}{20} - 2x + 26.33$$

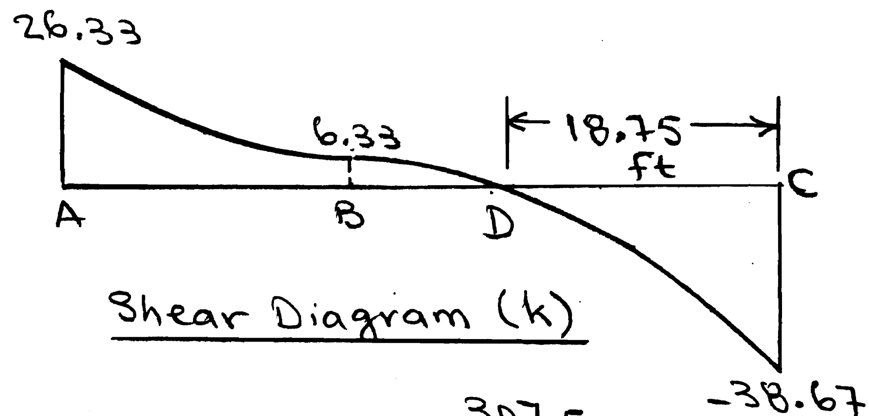
$$M = 26.33x - x^2 + \frac{1}{2}\left(\frac{x}{10}\right)x\left(\frac{x}{3}\right) = \frac{x^3}{60} - x^2 + 26.33x$$

$0 < x_1 \leq 30'$:

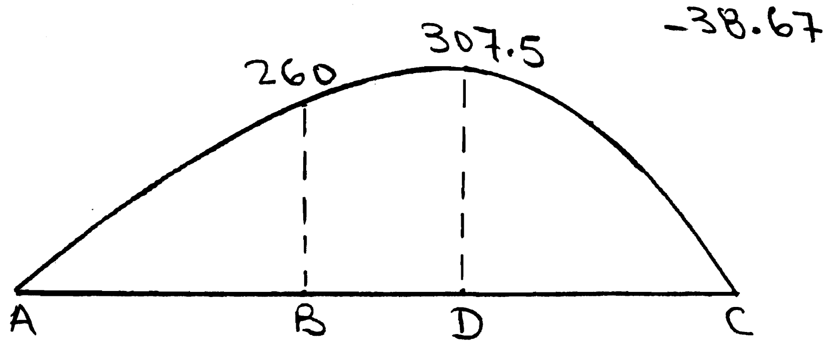
$$S = -38.67 + 3x_1 - \frac{1}{2}\left(\frac{x_1}{10}\right)x_1 = -\frac{x_1^2}{20} + 3x_1 - 38.67$$

$$M = 38.67x_1 - 3\frac{x_1^2}{2} + \frac{1}{2}\left(\frac{x_1}{10}\right)x_1\left(\frac{x_1}{3}\right)$$

$$= \frac{x_1^3}{60} - \frac{3x_1^2}{2} + 38.67x_1$$

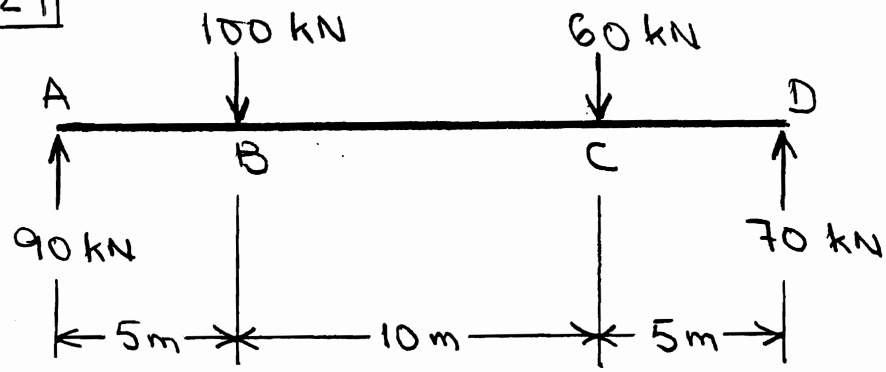


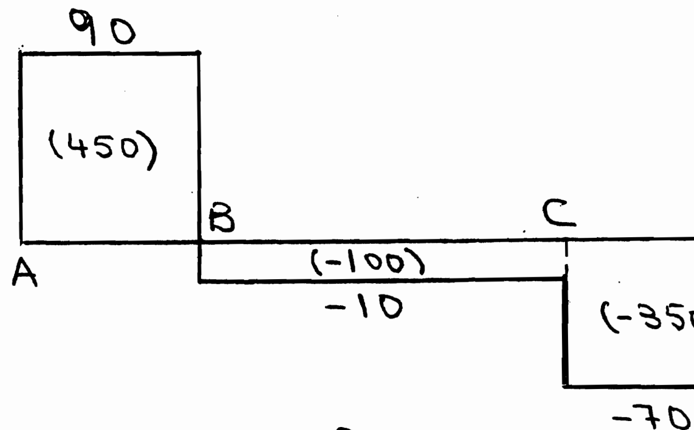
Shear Diagram (k)



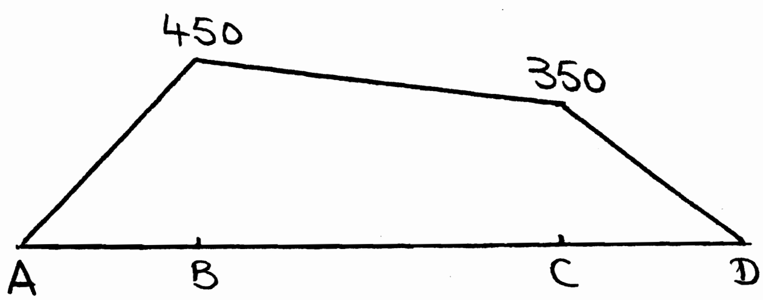
Bending Moment Diagram (k-ft)

5.29





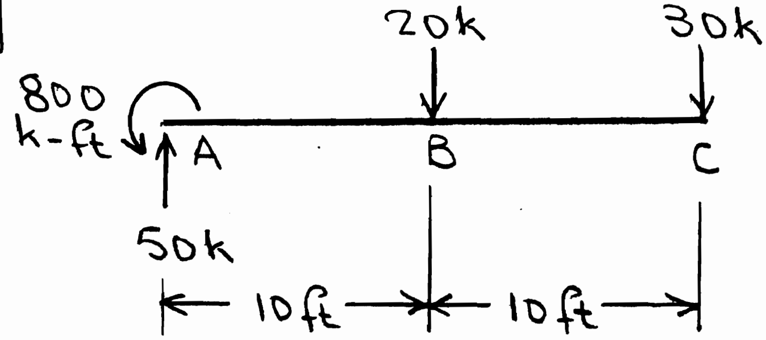
Shear Diagram (kN)

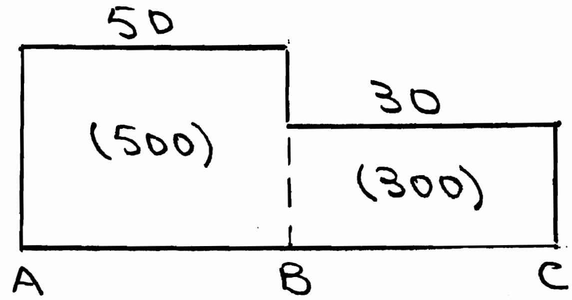


Bending Moment Diagram (kN.m)

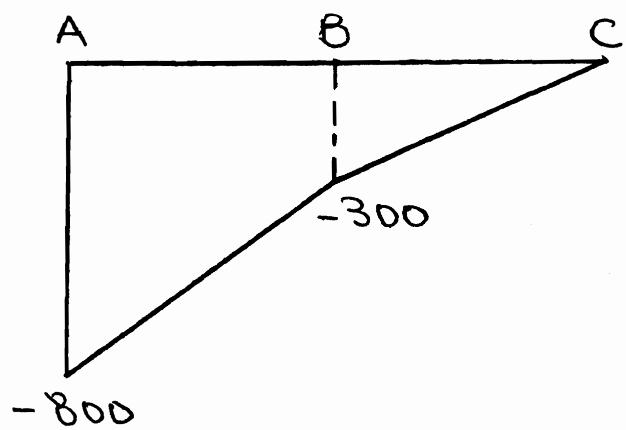
Qualitative Deflected Shape

5.30





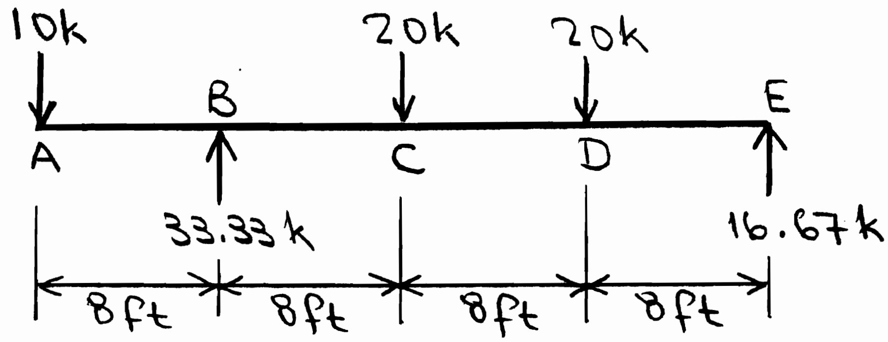
Shear Diagram (k)

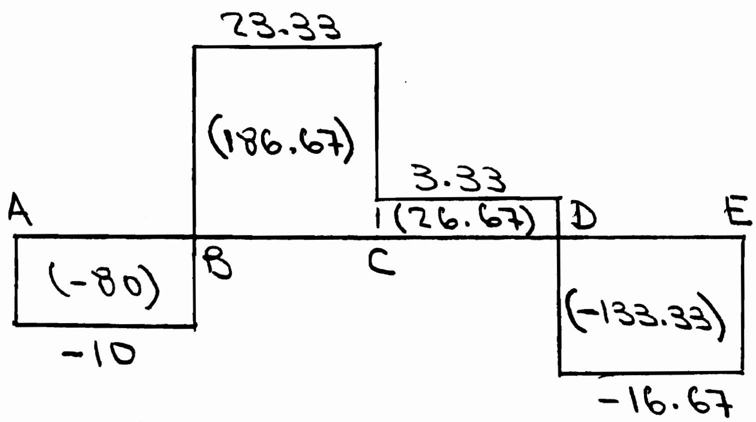


Bending Moment Diagram (k-ft)

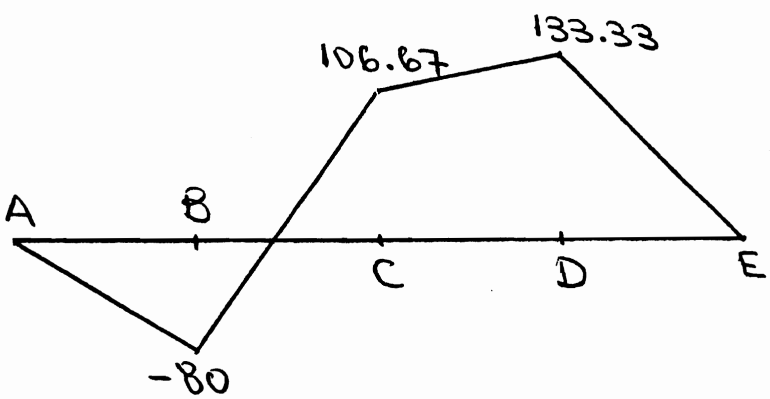
Qualitative Deflected Shape

5.31





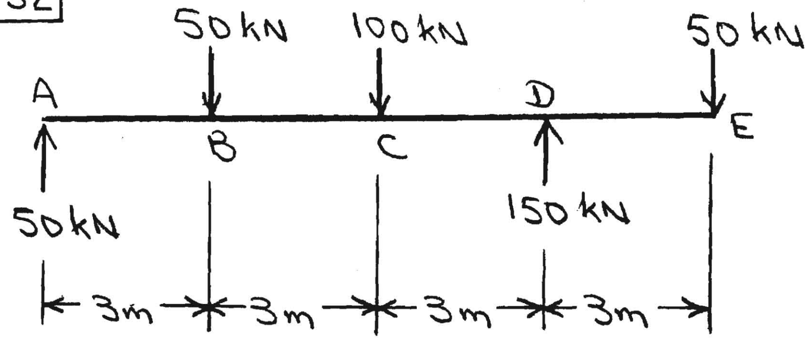
Shear Diagram (k)

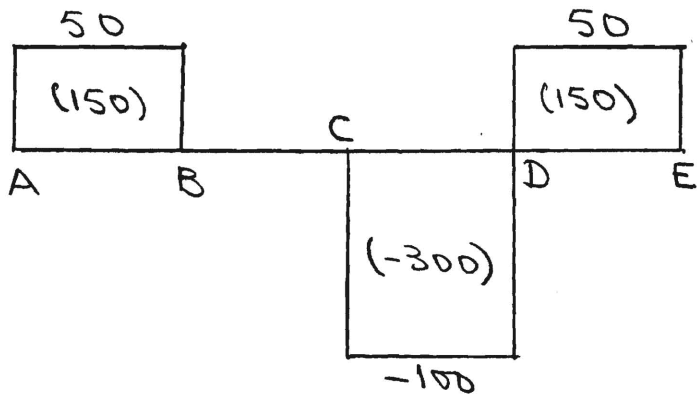


Bending Moment Diagram (k-ft)

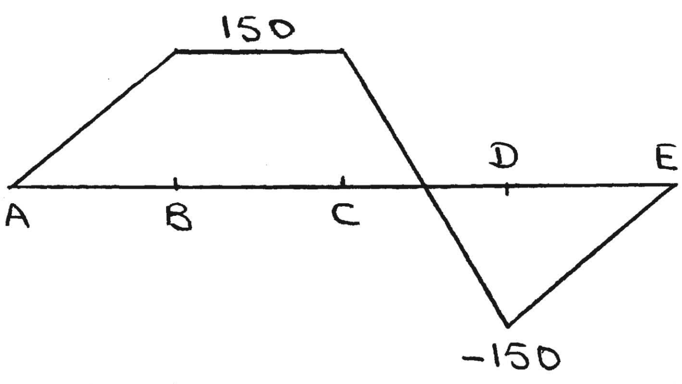
Qualitative Deflected Shape

5.32

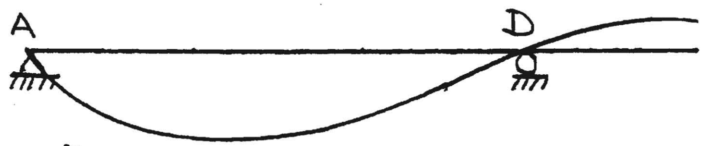




Shear Diagram (kN)

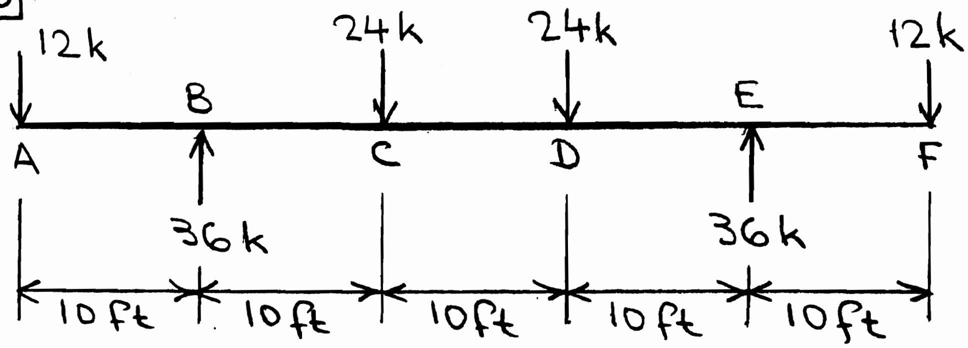


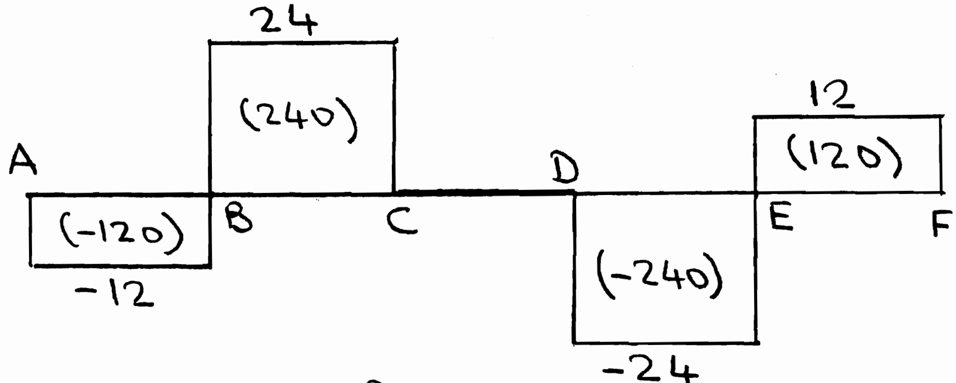
Bending Moment Diagram (kN.m)



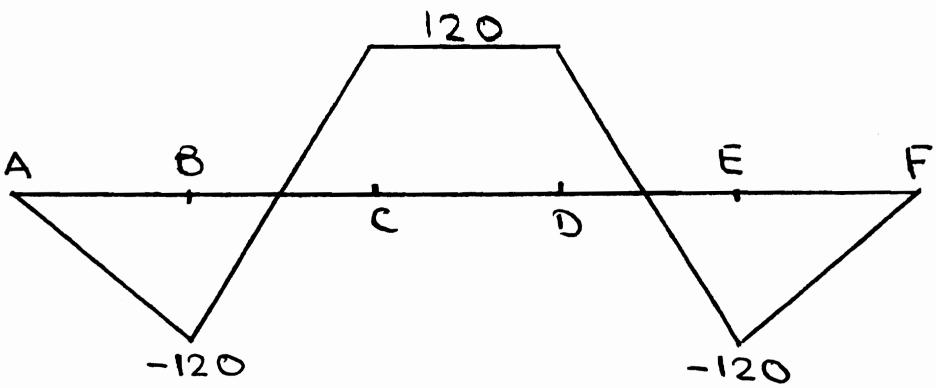
Qualitative Deflected Shape

5.33

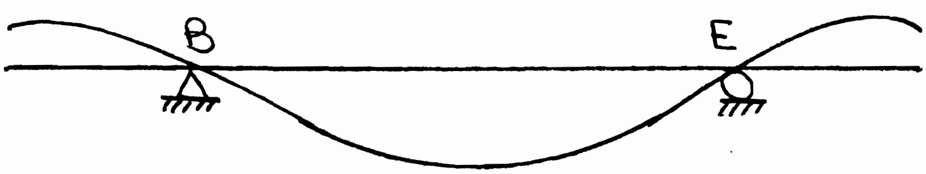




Shear Diagram (k)

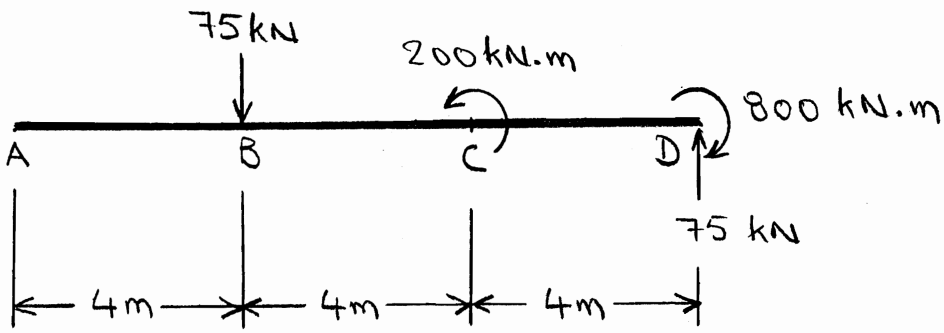


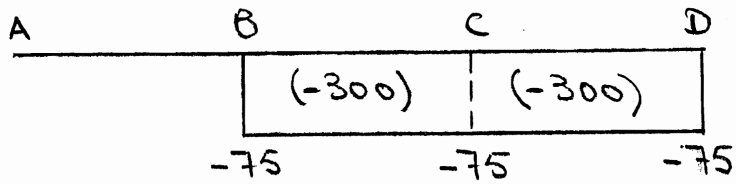
Bending Moment Diagram (k-ft)



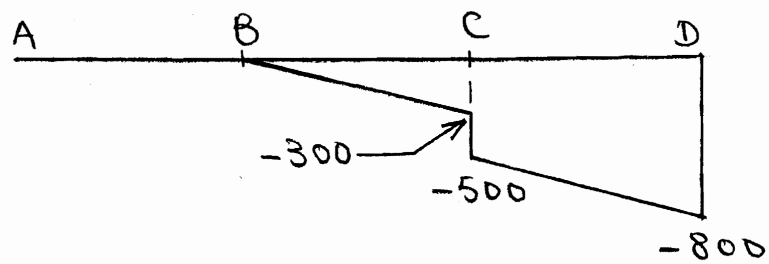
Qualitative Deflected Shape

5-34

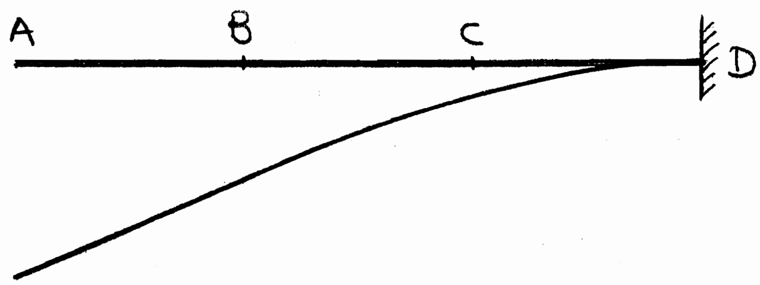




Shear Diagram (kN)

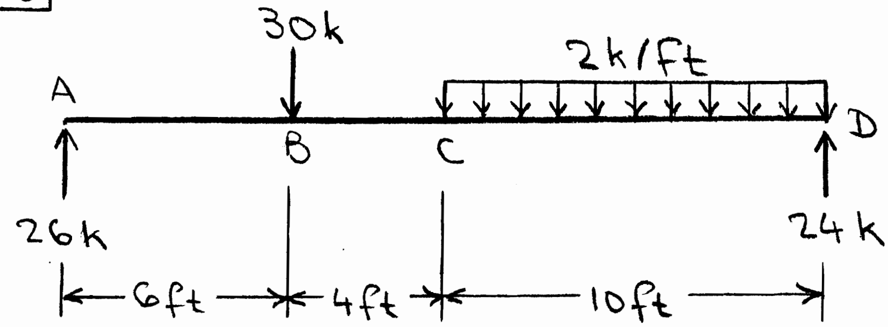


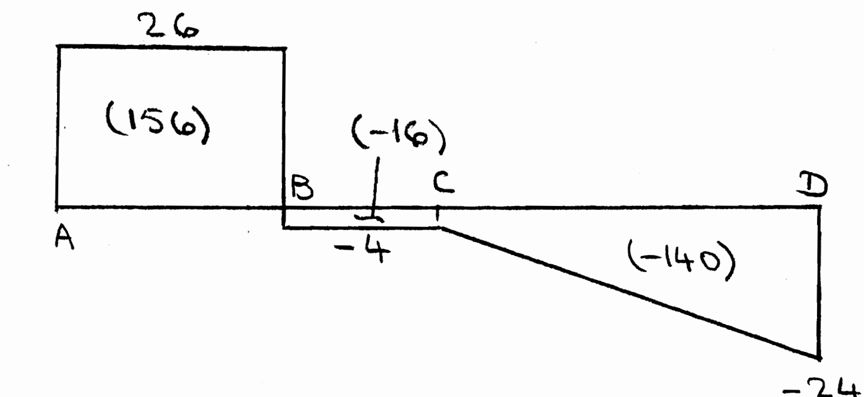
Bending Moment Diagram (kN.m)



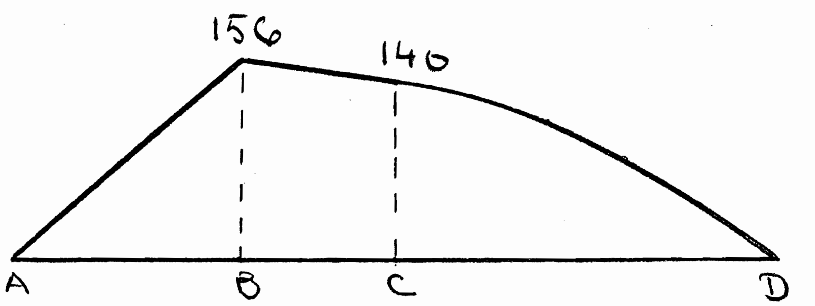
Qualitative Deflected Shape

5.35

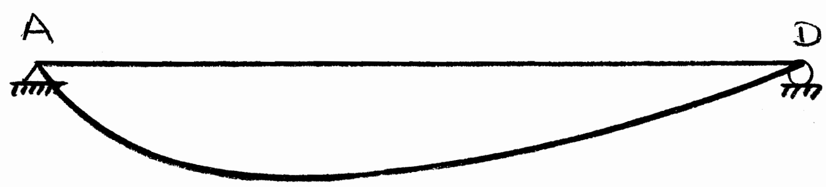




Shear Diagram (k)

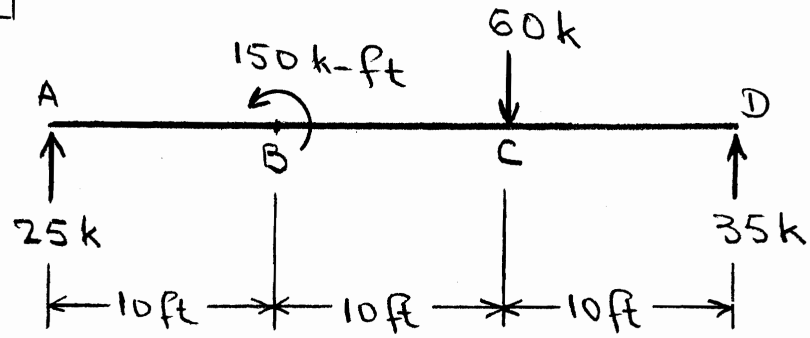


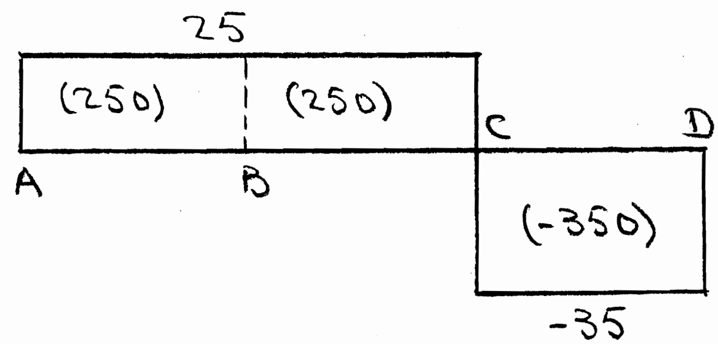
Bending Moment Diagram (k-ft)



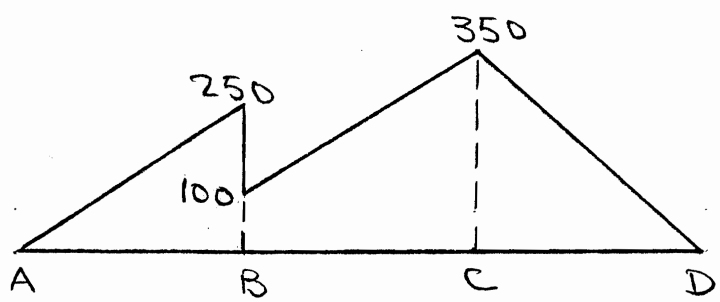
Qualitative Deflected Shape

5.36





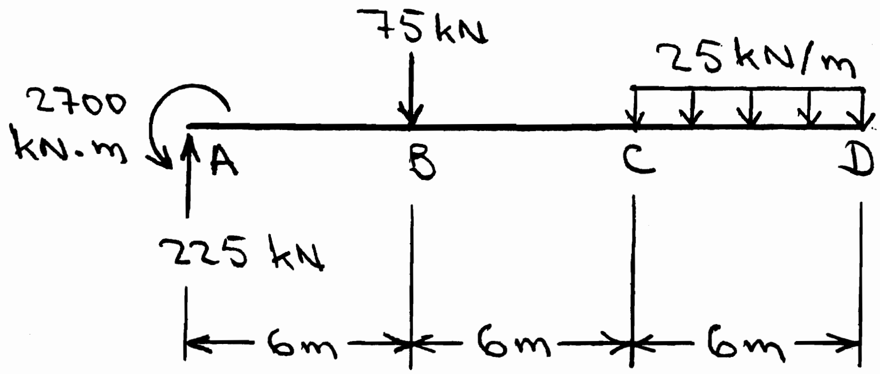
Shear Diagram (k)

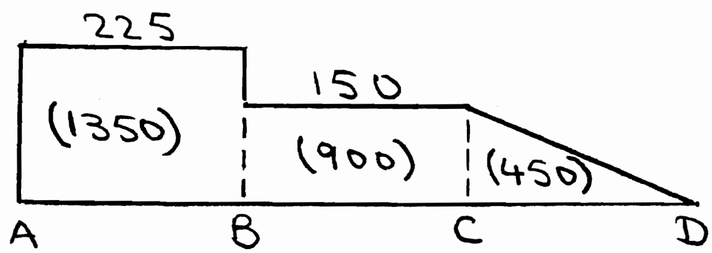


Bending Moment Diagram (k-ft)

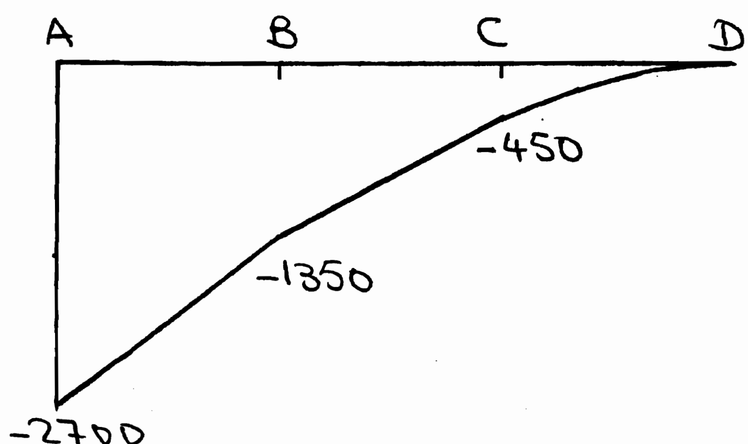
Qualitative Deflected Shape

5.37

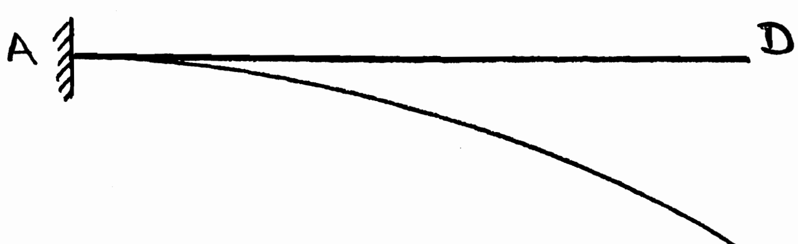




Shear Diagram (kN)

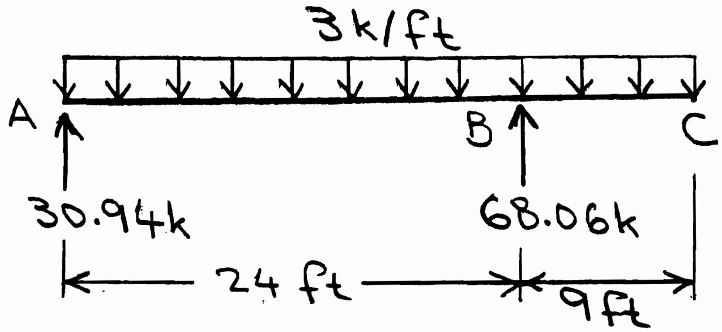


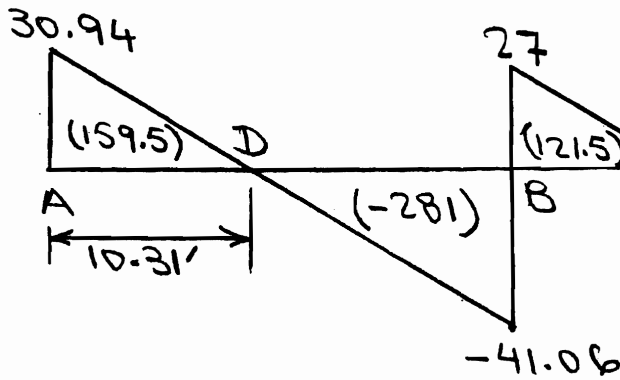
Bending Moment Diagram (kN.m)



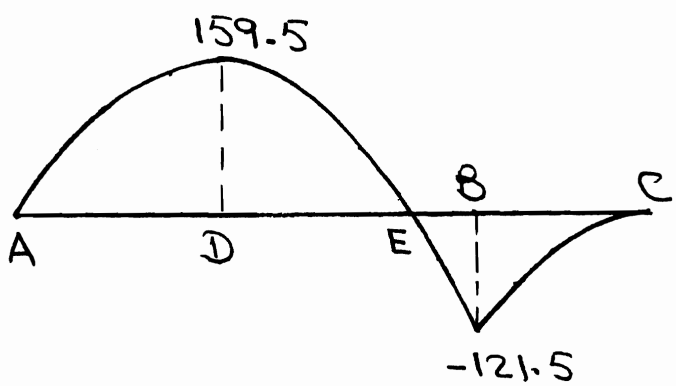
Qualitative Deflected Shape

5.38

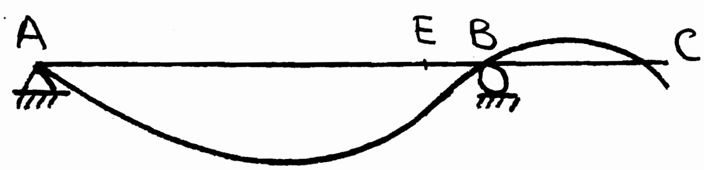




Shear Diagram (k)

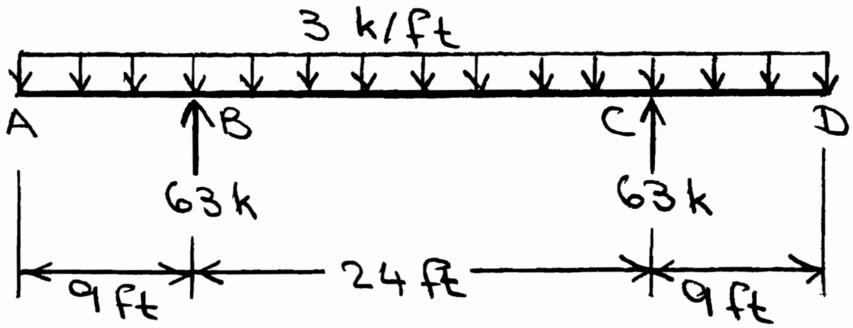


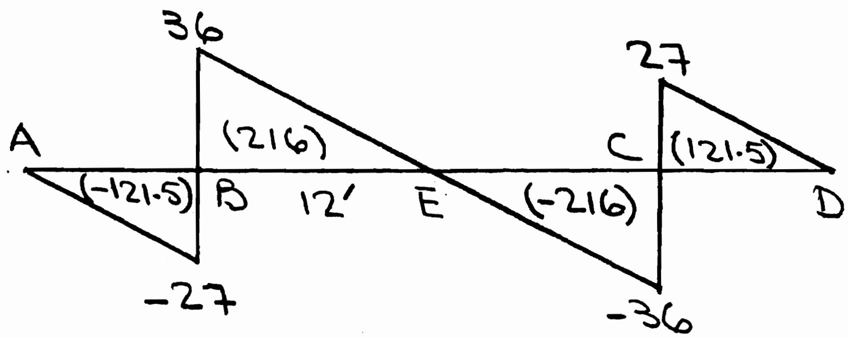
Bending Moment Diagram (k-ft)



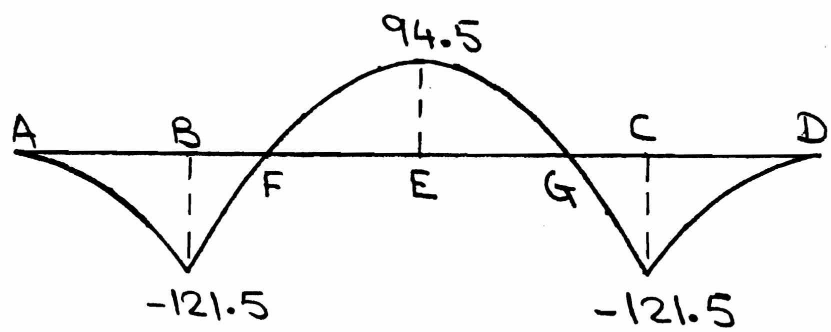
Qualitative Deflected Shape

5.39





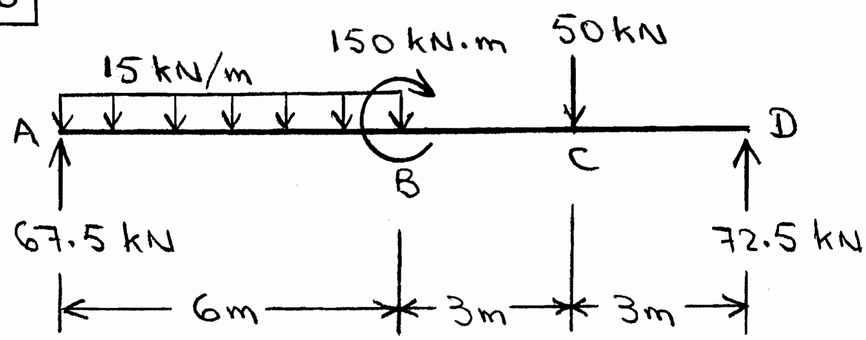
Shear Diagram (k)

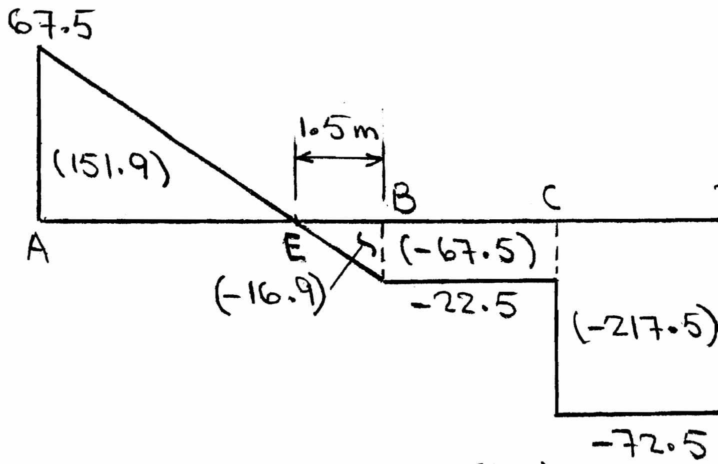


Bending Moment Diagram (k-ft)

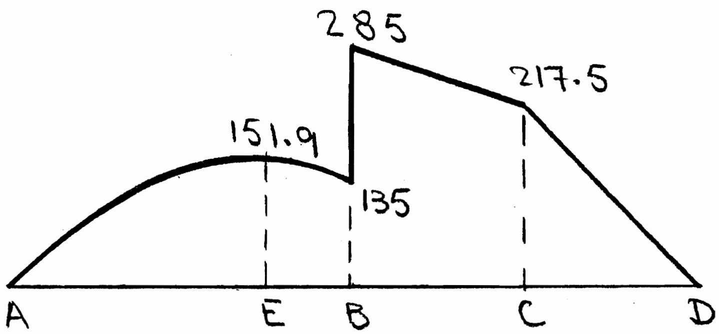
Qualitative Deflected Shape

5.40

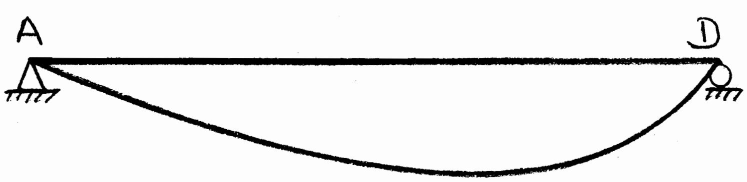




Shear Diagram (kN)

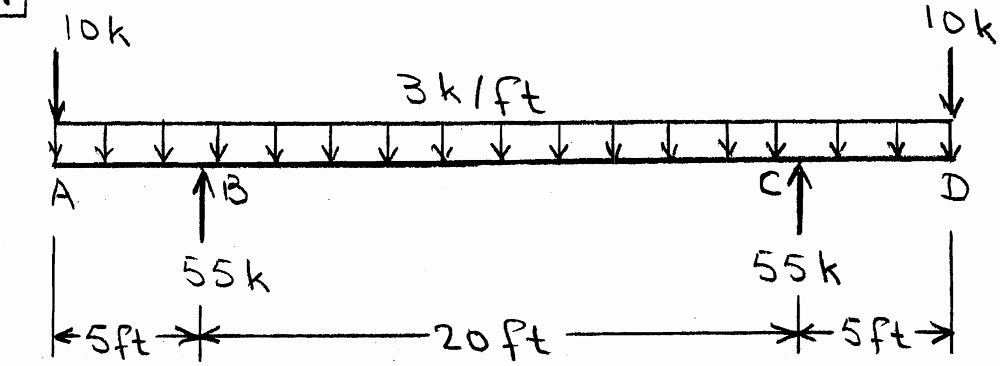


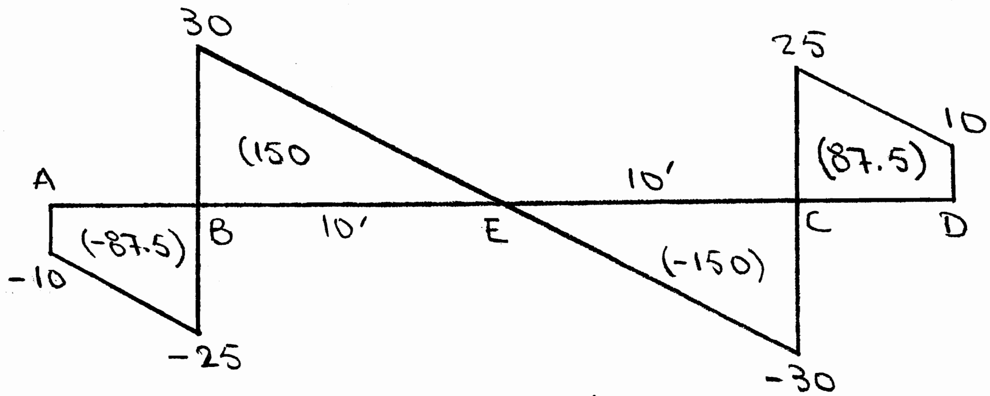
Bending Moment Diagram (kN.m)



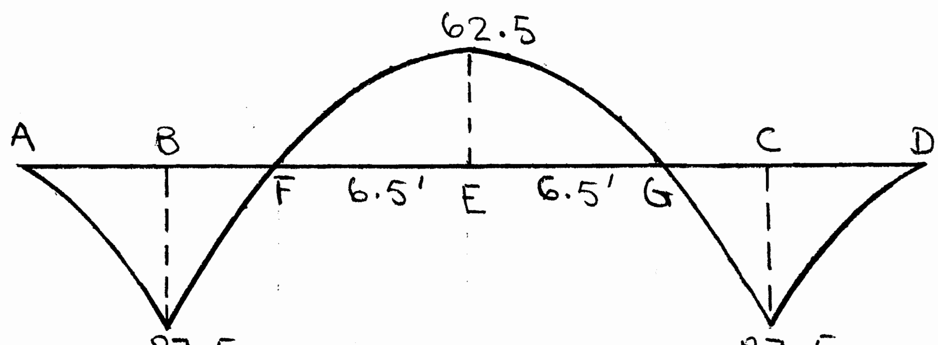
Qualitative Deflected Shape

5.41

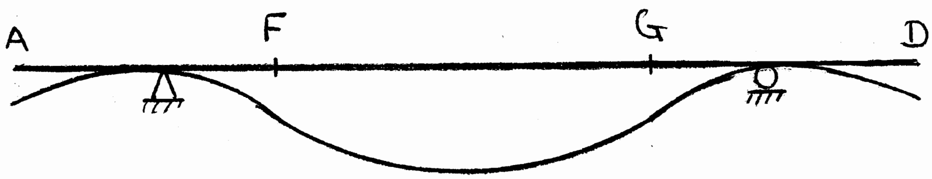




Shear Diagram (k)

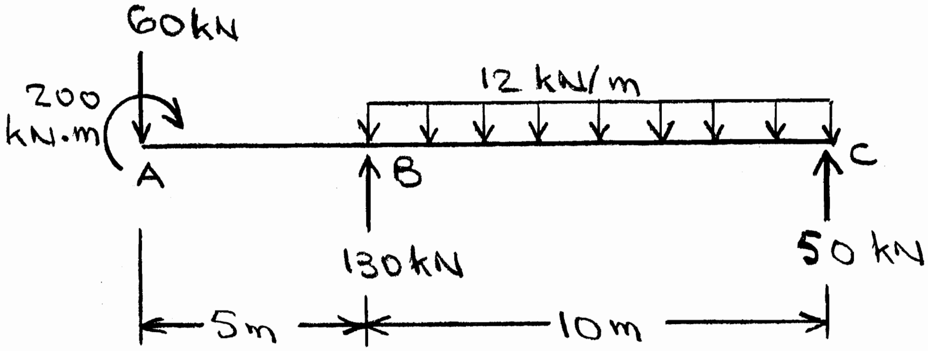


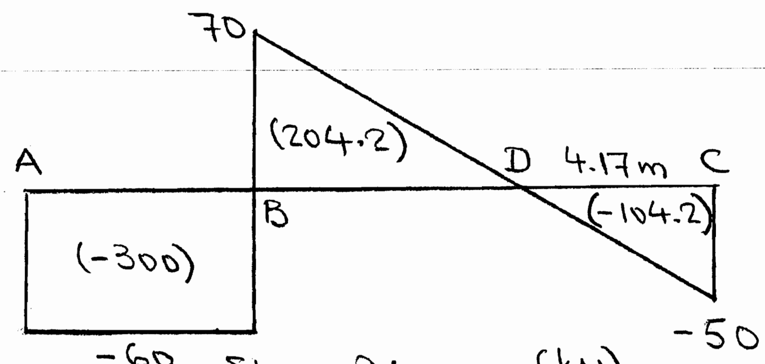
Bending Moment Diagram (k-ft)



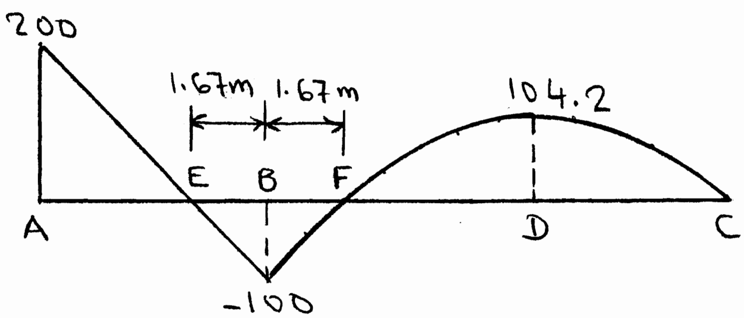
Qualitative Deflected Shape

5.42

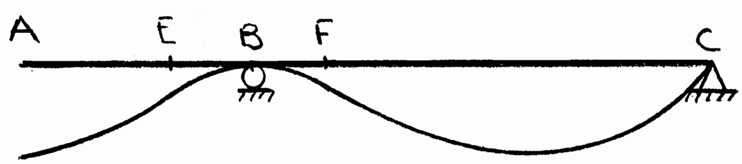




Shear Diagram (kN)

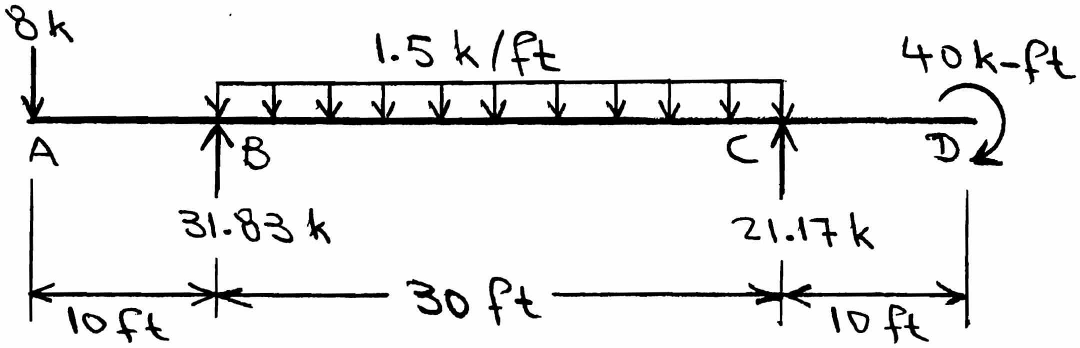


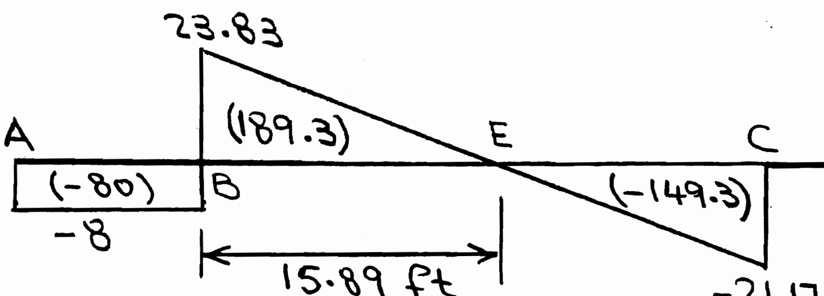
Bending Moment Diagram (kN.m)



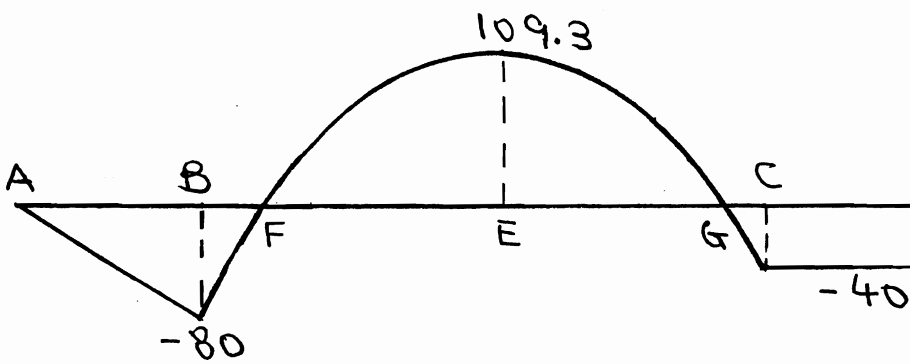
Qualitative Deflected Shape

5.43





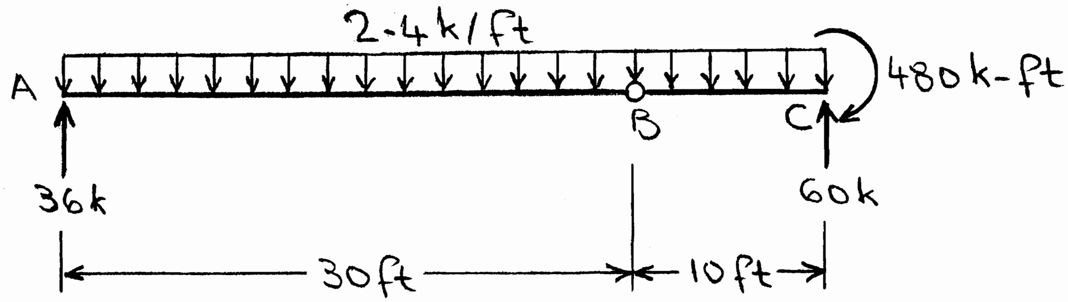
Shear Diagram (k)

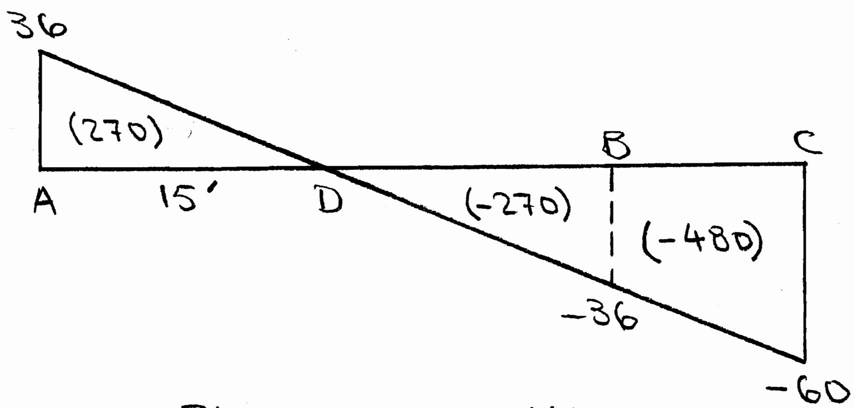


Bending Moment Diagram (k-ft)

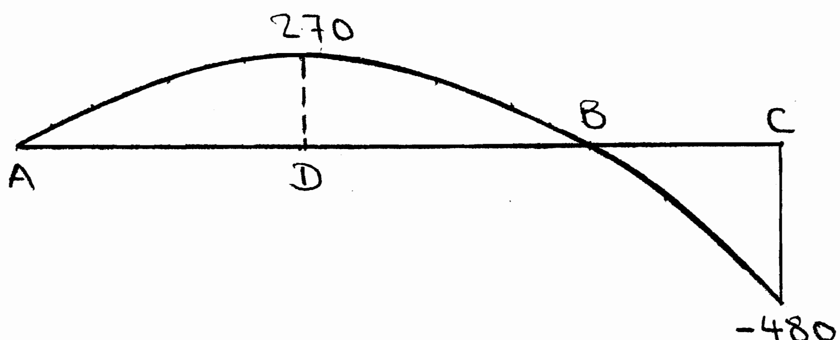
Qualitative Deflected Shape

5.44





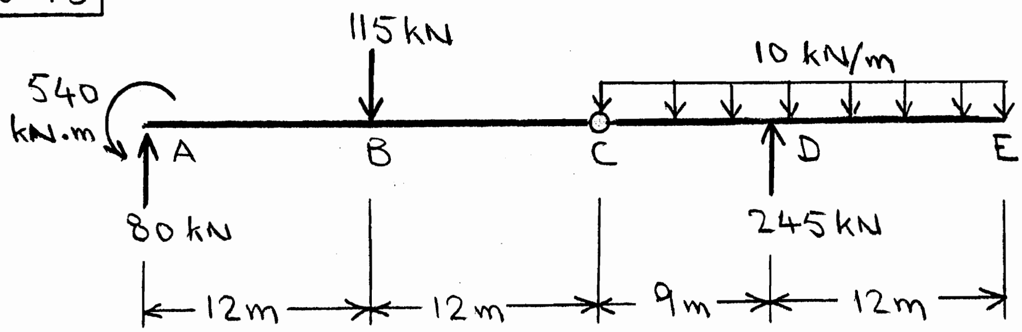
Shear Diagram (k)

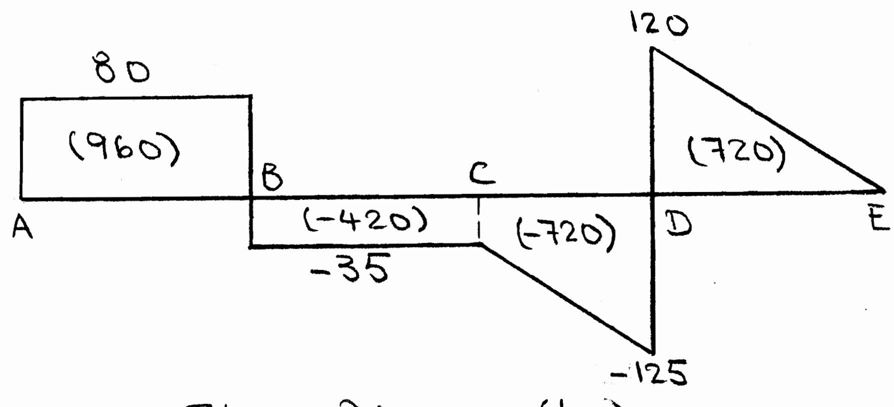


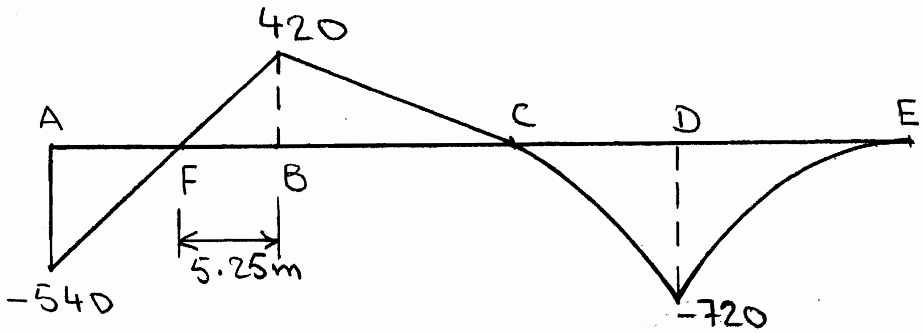
Bending Moment Diagram (k-ft)

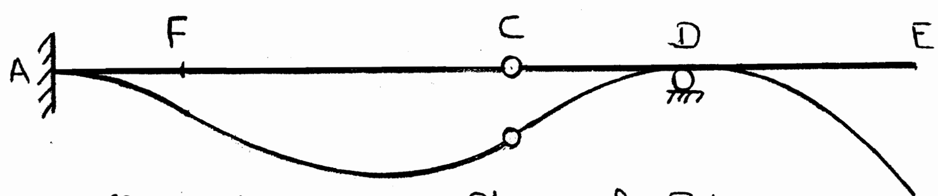
Qualitative Deflected Shape

5-45

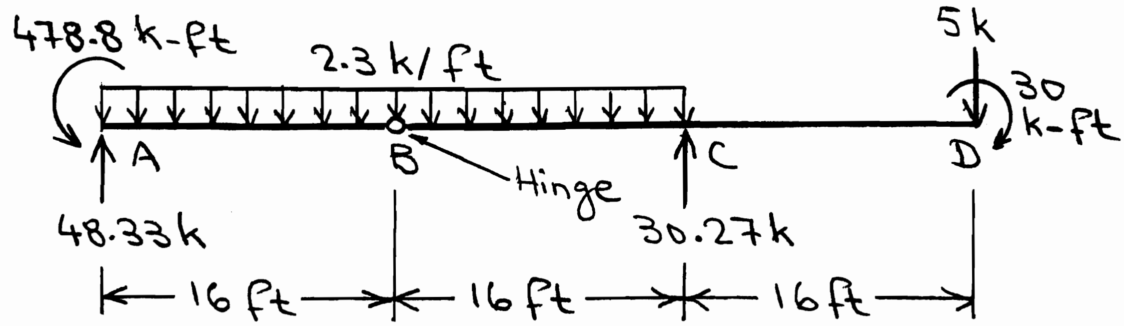

115 kN
10 kN/m
540 kN.m
A
B
C
D
E
80 kN
245 kN
12m
12m
9m
12m

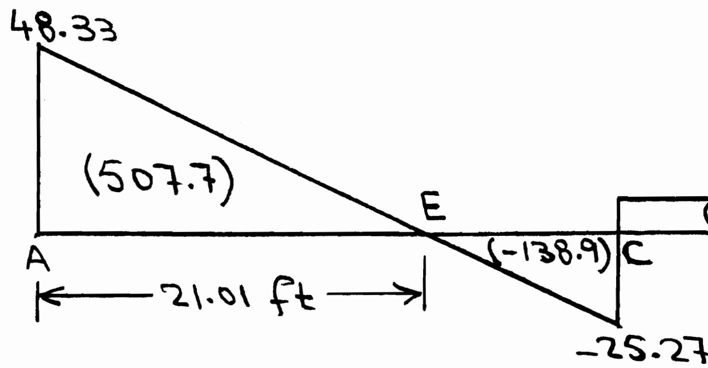

120
80
(960)
(720)
B
C
A
(-420)
(-720)
D
E
-35
-125
Shear Diagram (kN)


420
A
C
D
E
F
B
5.25m
-540
-720
Bending Moment Diagram (kN.m)

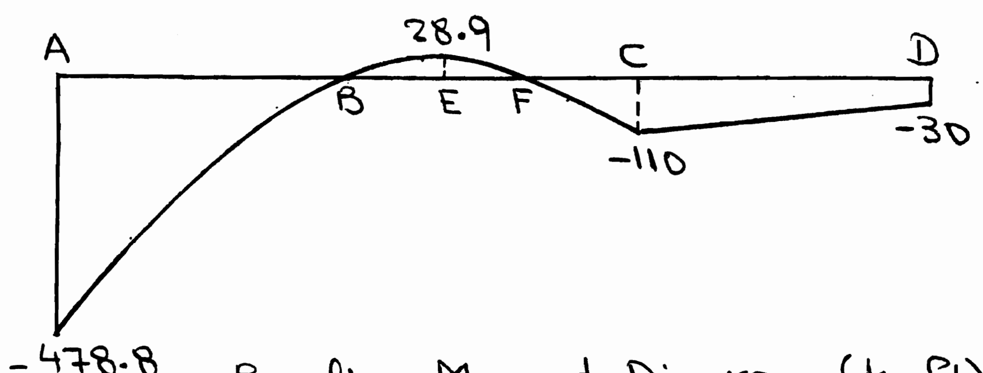

F
C
D
E
A
Qualitative Deflected Shape

5.46

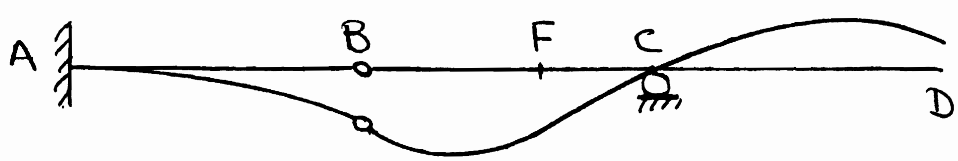




Shear Diagram (k)

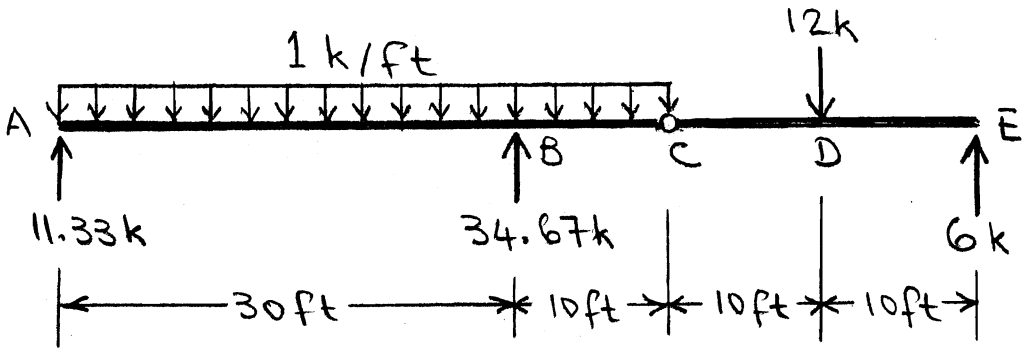


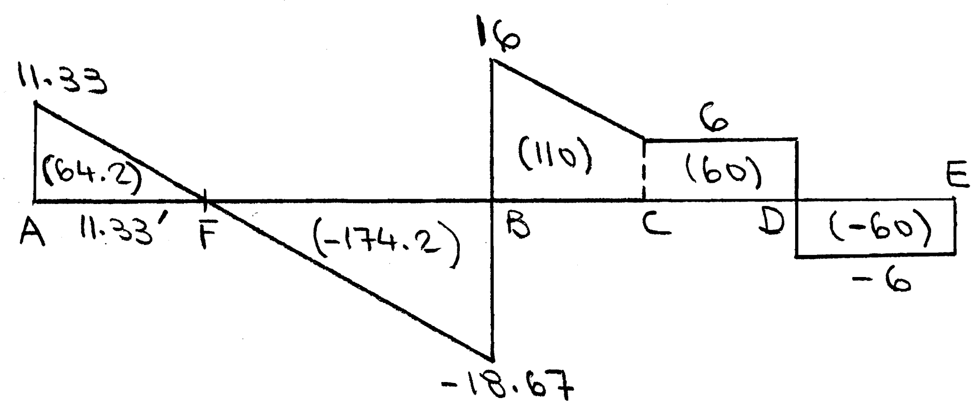
Bending Moment Diagram (k-ft)



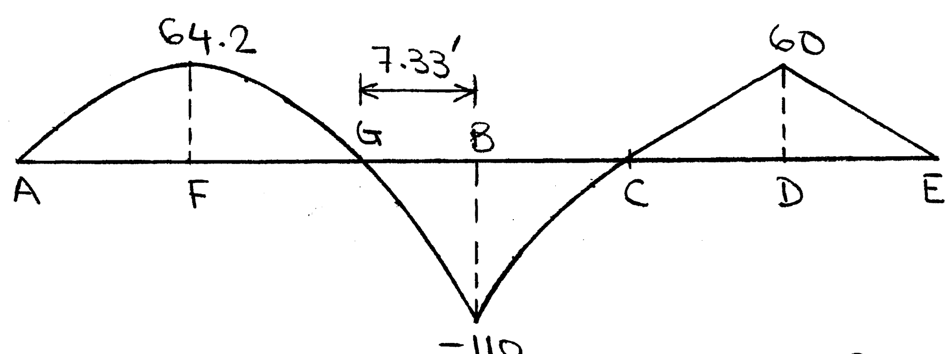
Qualitative Deflected Shape

5.47

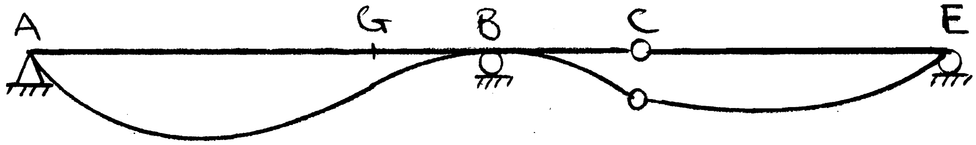




Shear Diagram (k)

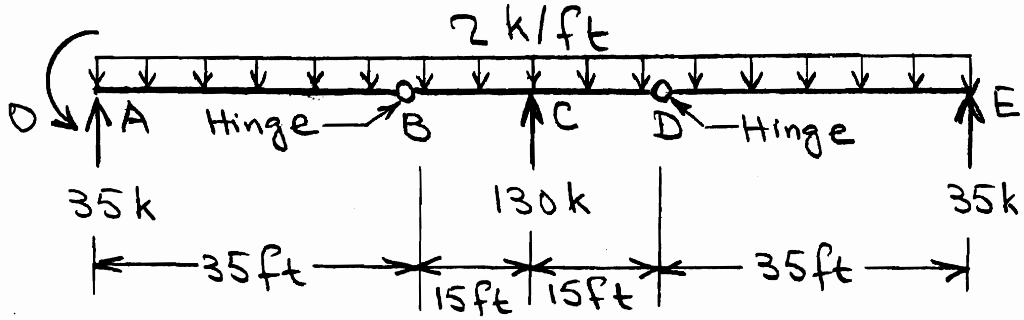


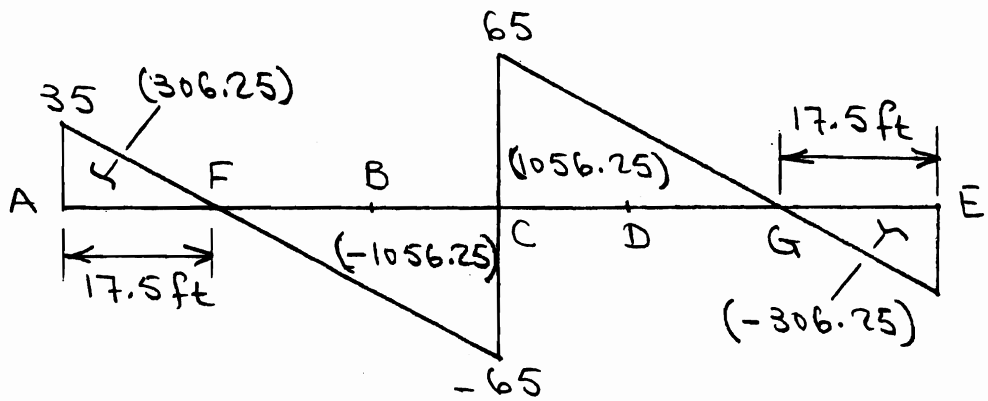
Bending Moment Diagram (k-ft)



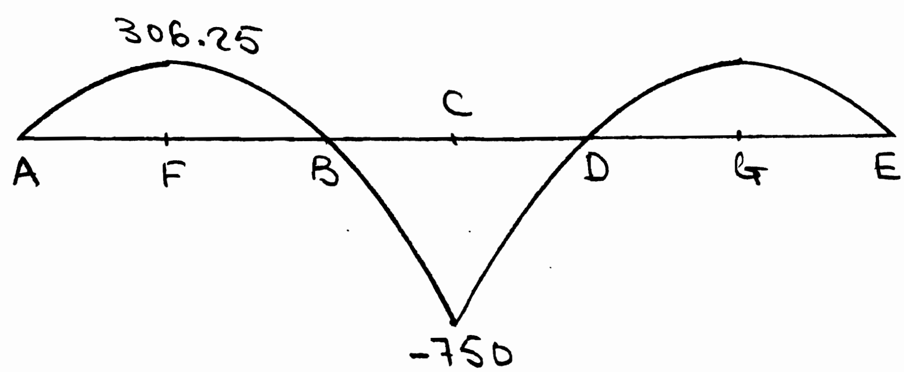
Qualitative Deflected Shape

5.48

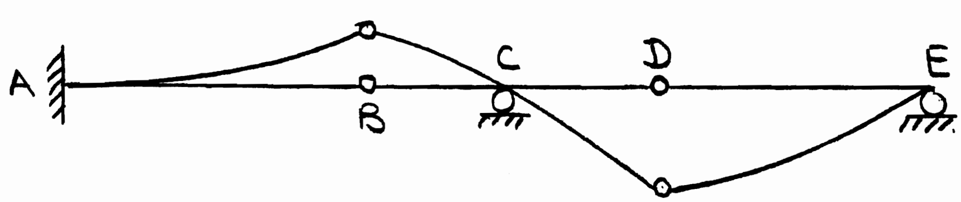




Shear Diagram (k)

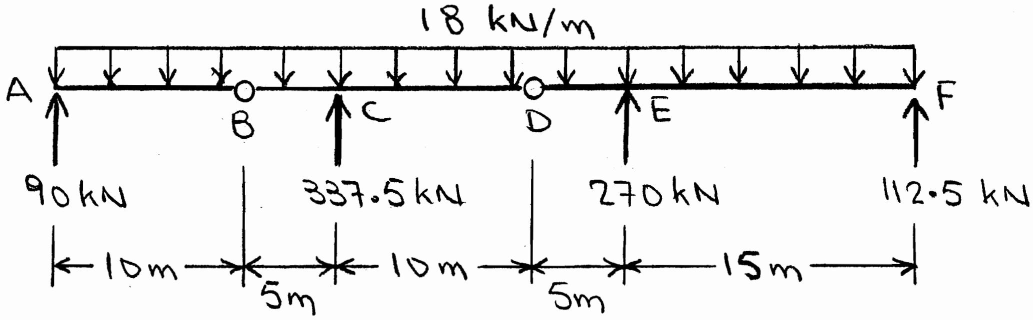


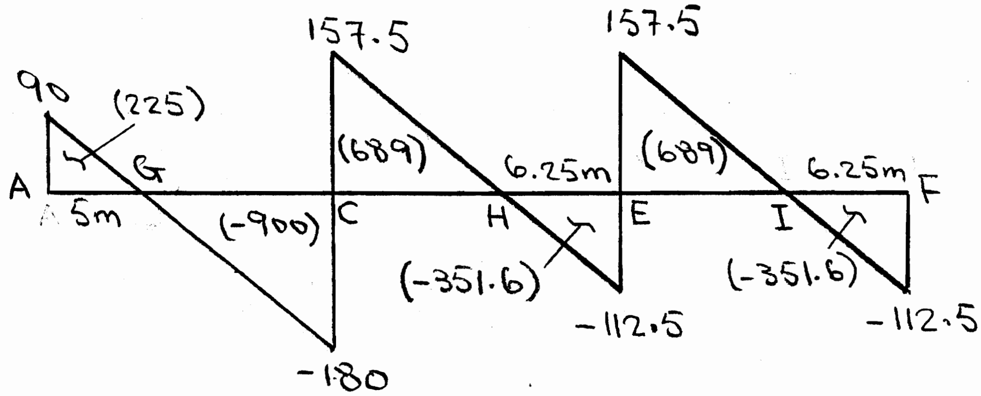
Bending Moment Diagram (k-ft)



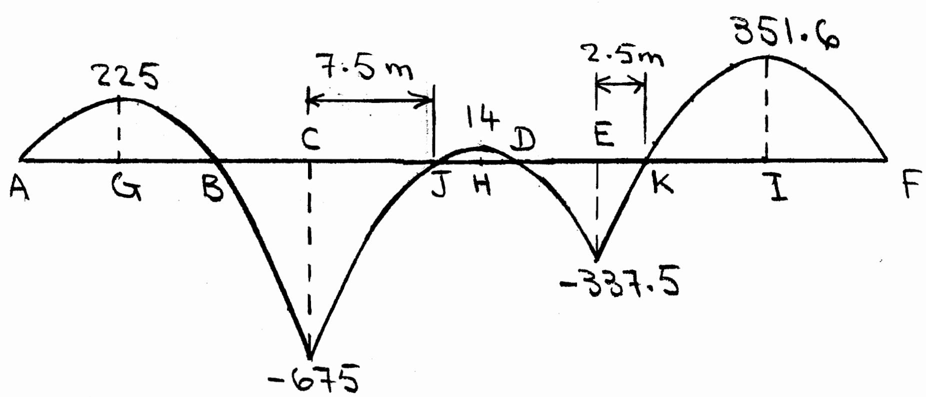
Qualitative Deflected Shape

5.49

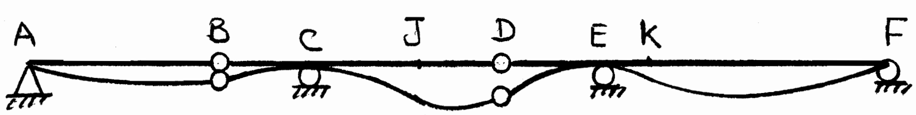




Shear Diagram (kN)

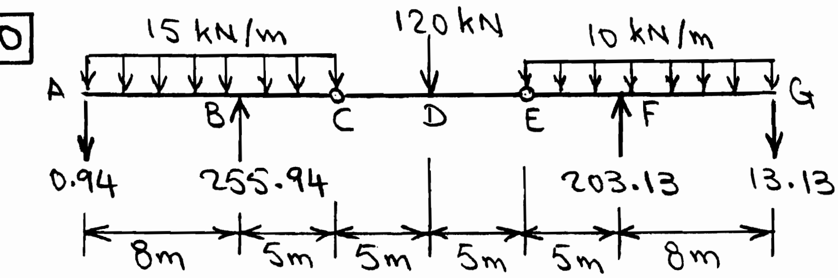


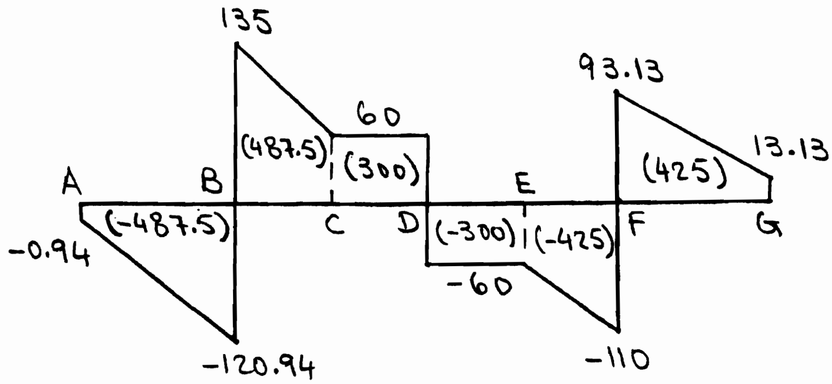
Bending Moment Diagram (kN.m)



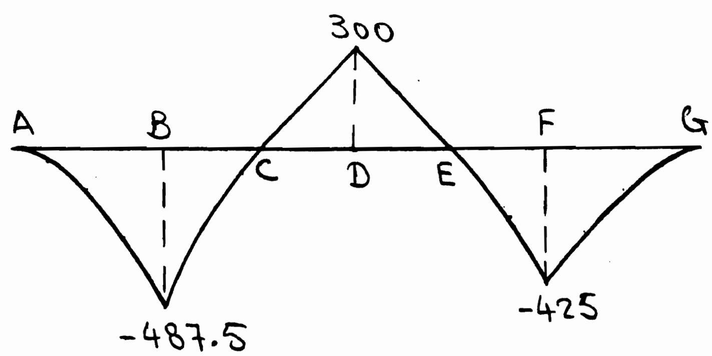
Qualitative Deflected Shape

5-50

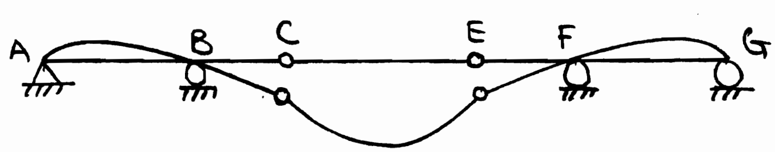




Shear Diagram (kN)

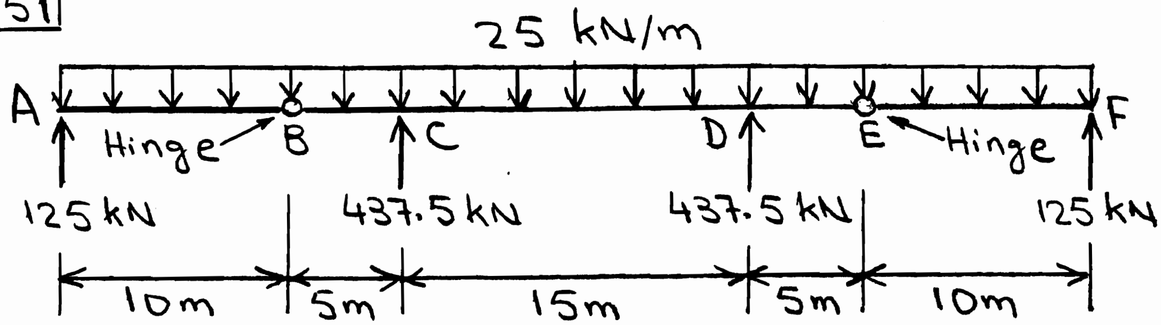


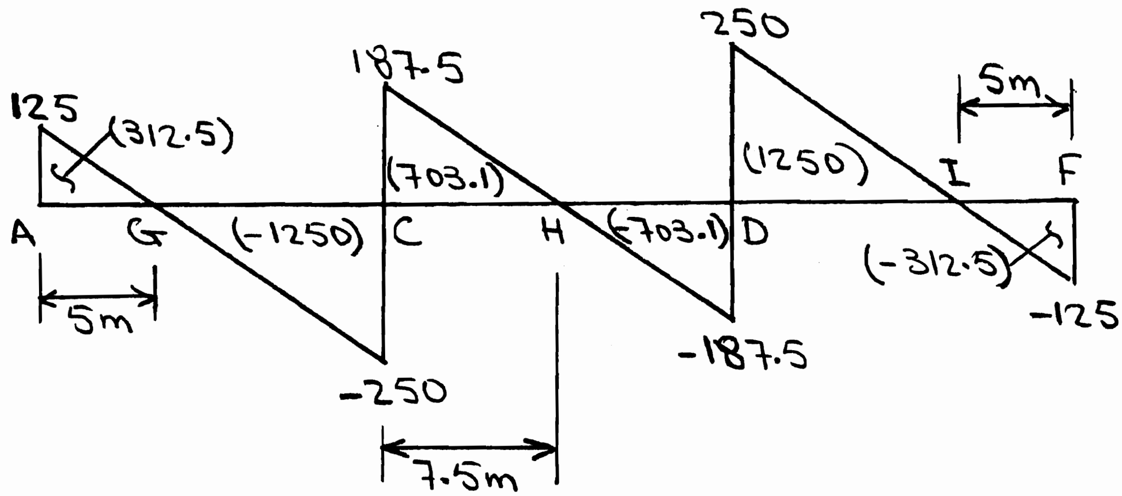
Bending Moment Diagram (kN-m)



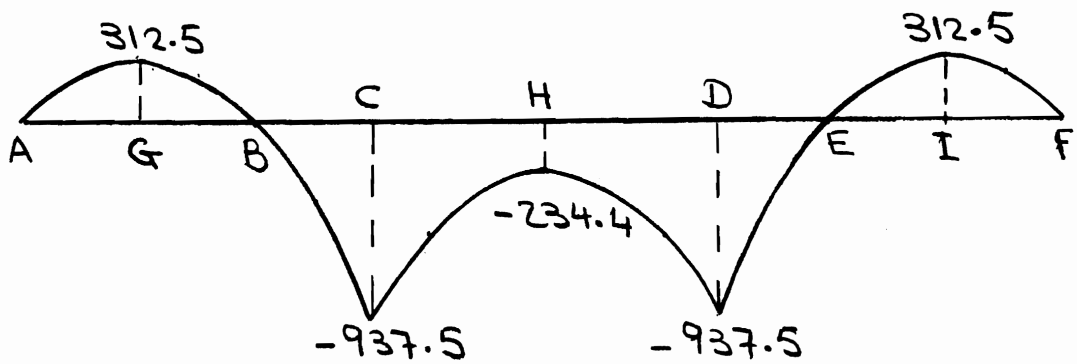
Qualitative Deflected Shape

5.51

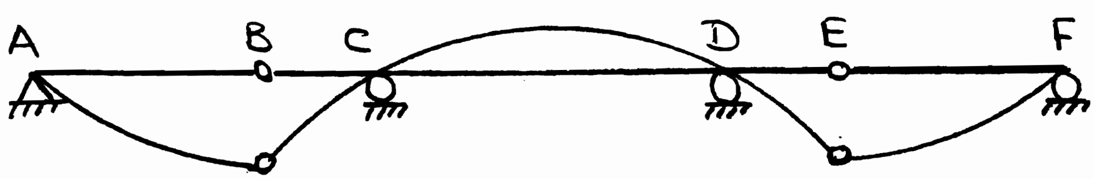




Shear Diagram (kN)

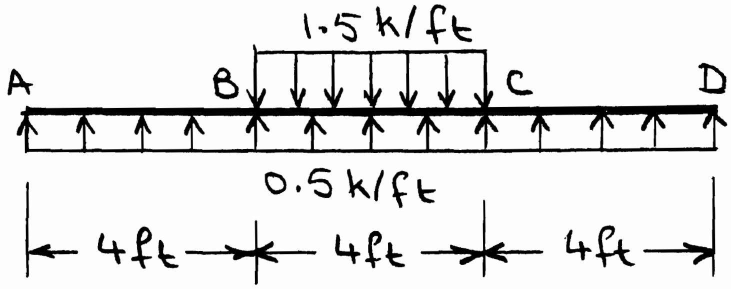


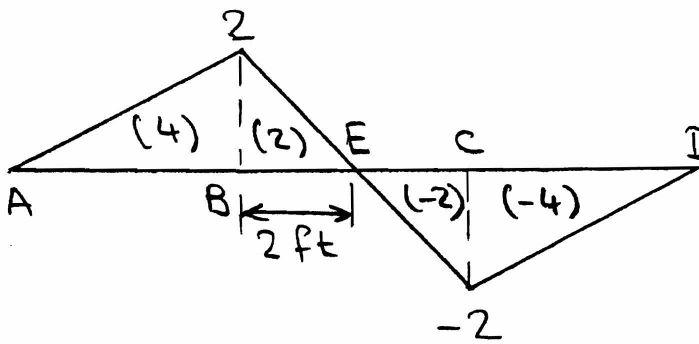
Bending Moment Diagram (kN.m)



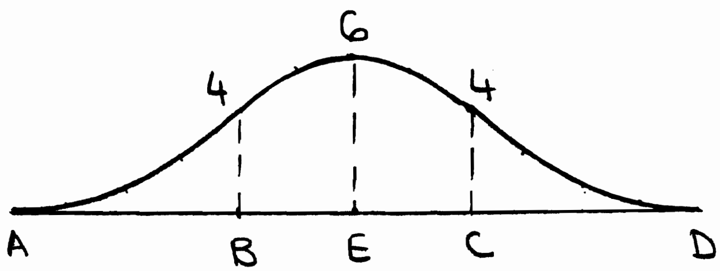
Qualitative Deflected Shape

5.52



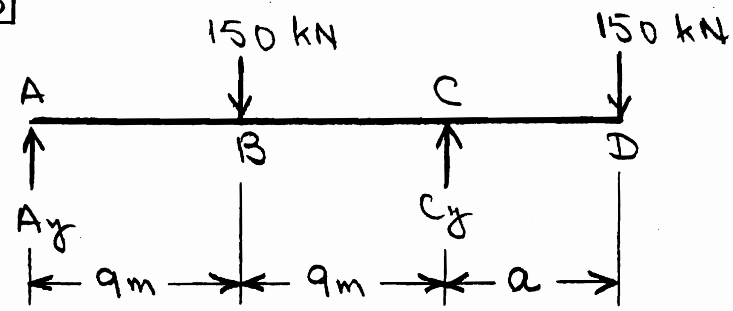


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.53



$+\circlearrowleft \Sigma M_C = 0 \qquad -A_y(18) + 150(9) - 150a = 0$

$$A_y = 75 - \frac{25}{3}a$$

Maximum positive bending moment occurs at B:

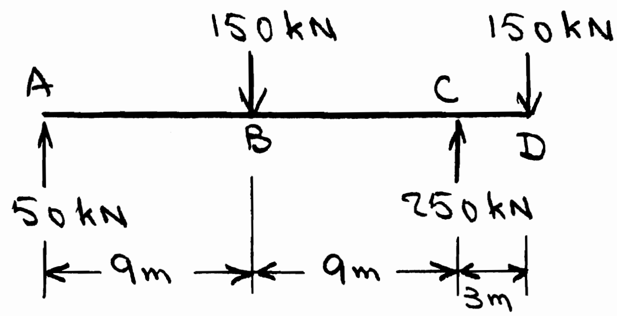
$$M_B = A_y(9) = 675 - 75a \qquad (1)$$

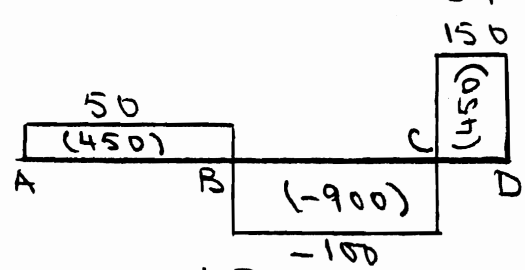
Maximum negative bending moment occurs at C:

$$M_C = 150a \qquad (2)$$

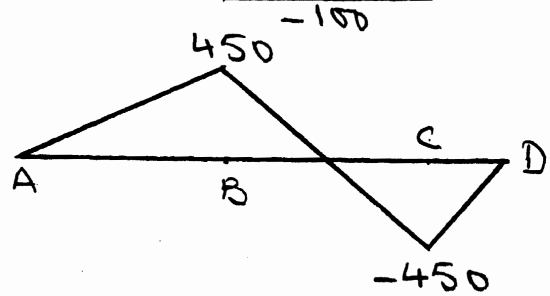
Equating Eqs. (1) and (2):

$$675 - 75a = 150a \qquad \underline{a = 3\text{ m}}$$



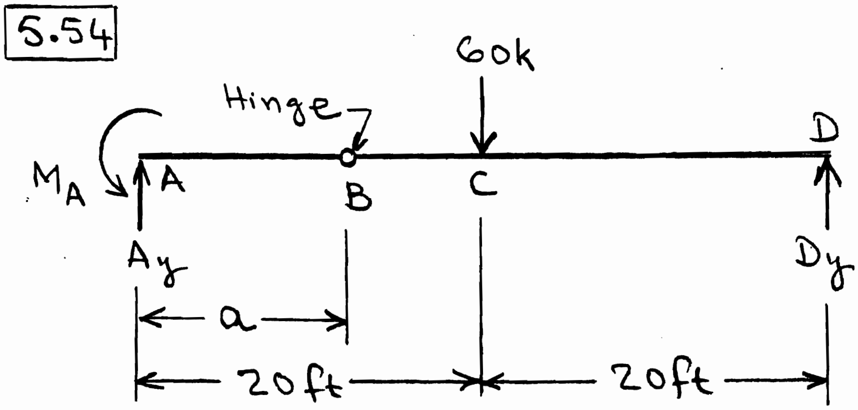


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.54



$$+\circlearrowleft \Sigma M_B^{BD} = 0 \qquad -60(20-a) + D_y(40-a) = 0$$

$$D_y = \frac{60(20-a)}{40-a}$$

Maximum positive bending moment occurs at C:

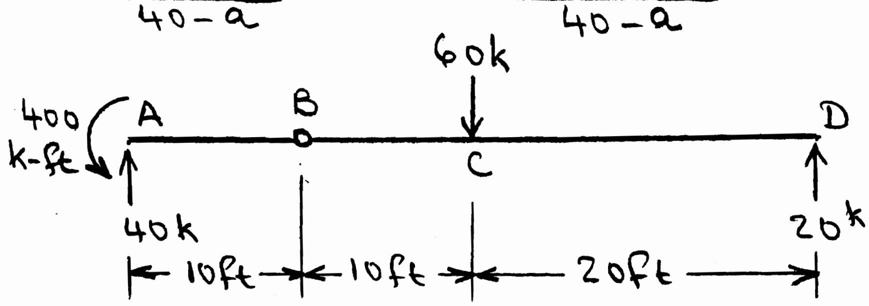
$$M_C = D_y(20) = \frac{1200(20-a)}{40-a} \qquad (1)$$

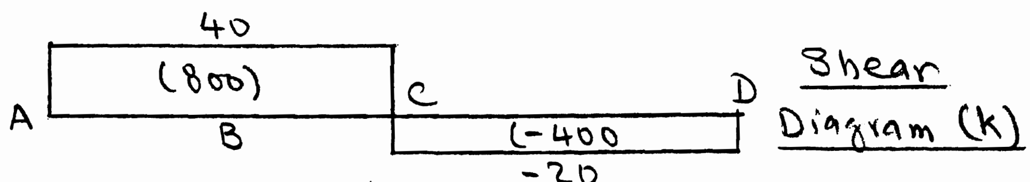
Maximum negative bending moment occurs at A:

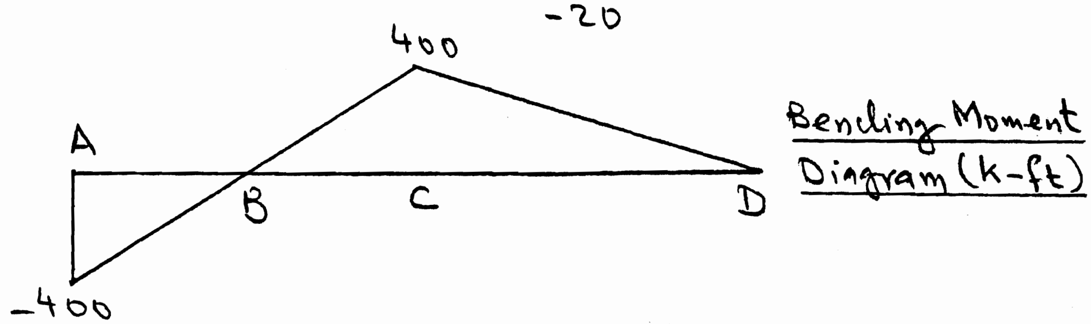
$$M_A = 60(20) - D_y(40) = 1200 - \frac{2400(20-a)}{40-a} \qquad (2)$$

Equating Eqs. (1) and (2):

$$\frac{1200(20-a)}{40-a} = 1200 - \frac{2400(20-a)}{40-a} \qquad \underline{a = 10\text{ ft}}$$







5.55 (a) $m = 4$, $j = 5$, $r = 3$, $e_c = 0$

$3m + r = 3j + e_c$; Statically determinate

(b) $m = 6$, $j = 7$, $r = 3$, $e_c = 1$

$3m + r < 3j + e_c$; Unstable

(c) $m = 7$, $j = 6$, $r = 3$, $e_c = 0$

$3m + r > 3j + e_c$; Statically indeterminate

$i = (21 + 3) - 18 = \underline{6}$

(d) $m = 7$, $j = 7$, $r = 6$, $e_c = 1$

$3m + r > 3j + e_c$; Statically indeterminate

$i = (21 + 6) - (21 + 1) = \underline{5}$

5.56 (a) $m = 6$, $j = 6$, $r = 4$, $e_c = 2$

$3m + r > 3j + e_c$; Statically indeterminate

$i = (18 + 4) - (18 + 2) = \underline{2}$

(b) $m = 6$, $j = 7$, $r = 6$, $e_c = 3$

$3m + r = 3j + e_c$; Statically determinate

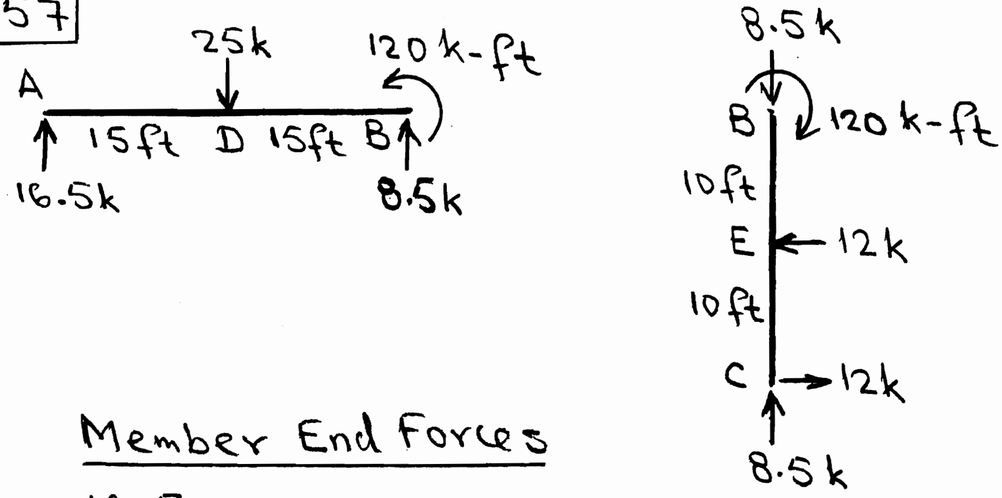
(c) $m = 5$, $j = 6$, $r = 4$, $e_c = 2$

$3m + r < 3j + e_c$; Unstable

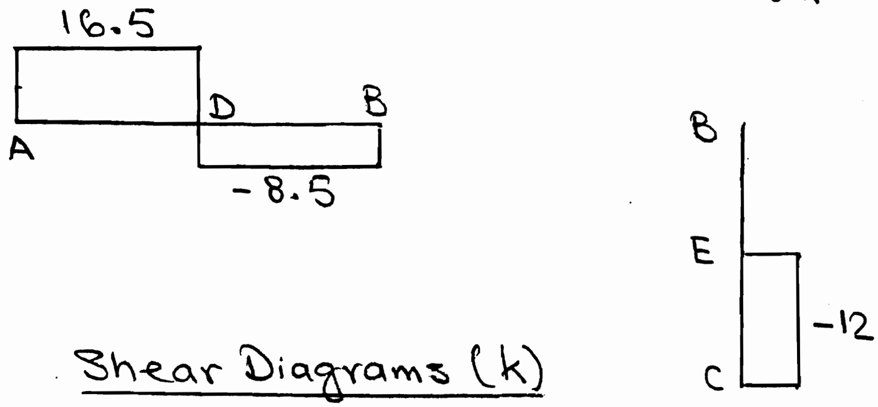
(d) Statically indeterminate

$i = 3(8) = \underline{24}$

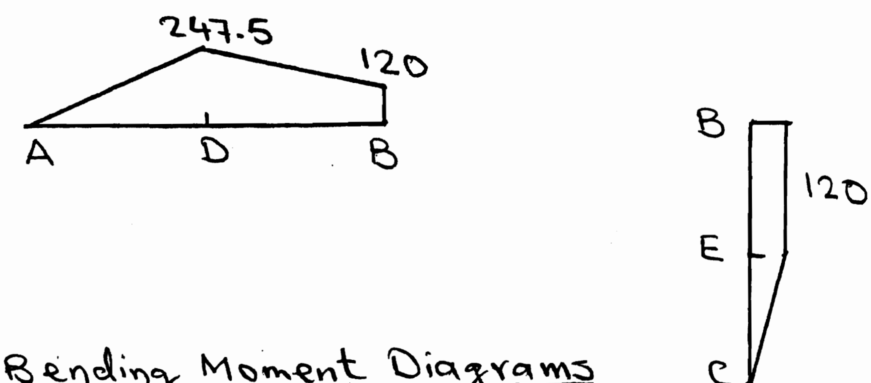
5.57



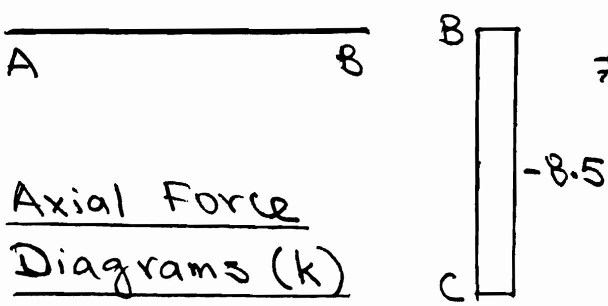
Member End Forces



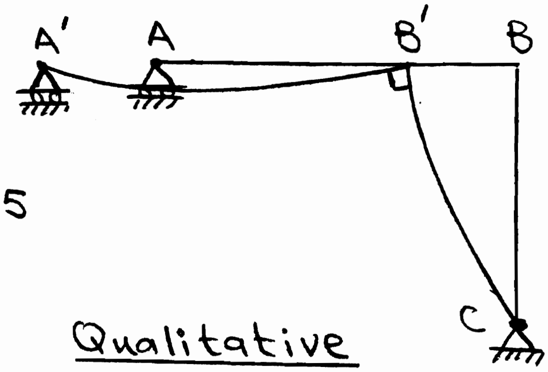
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

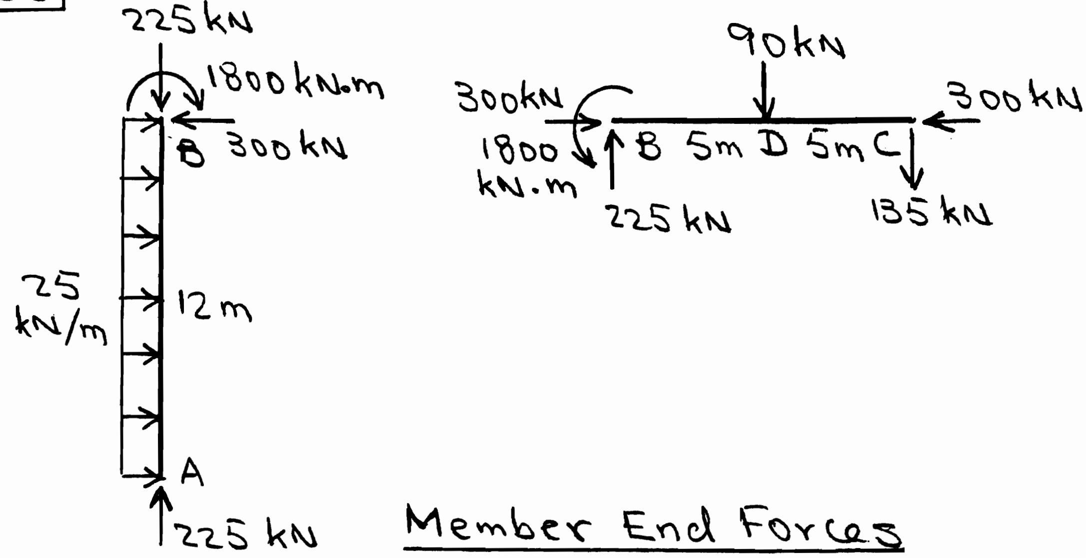


Axial Force Diagrams (k)

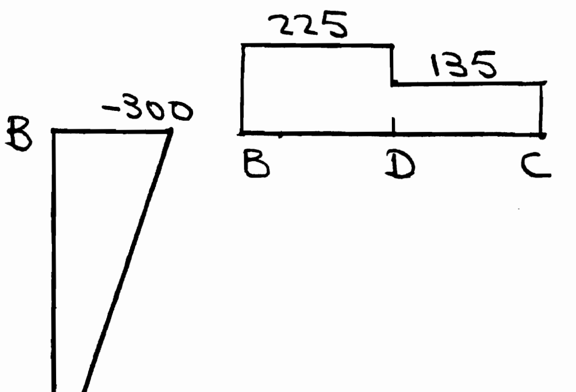


Qualitative Deflected Shape

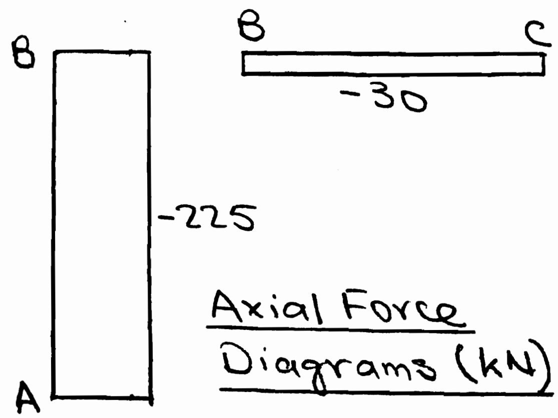
5.58



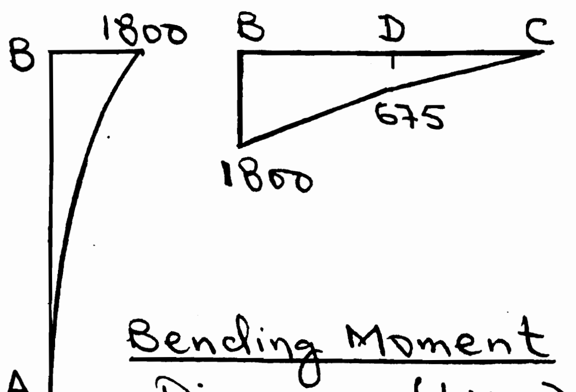
Member End Forces



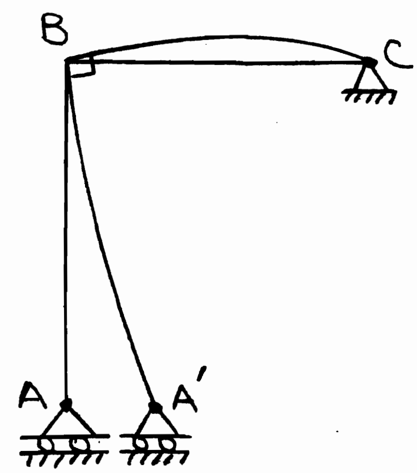
Shear Diagrams (kN)



Axial Force Diagrams (kN)

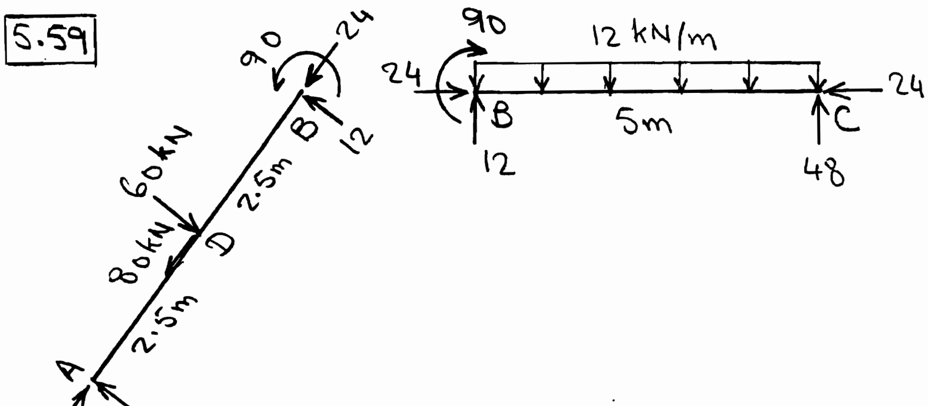


Bending Moment Diagrams (kN.m)

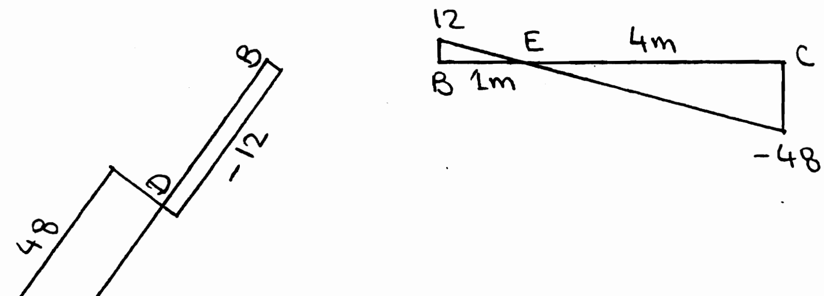


Qualitative Deflected Shape

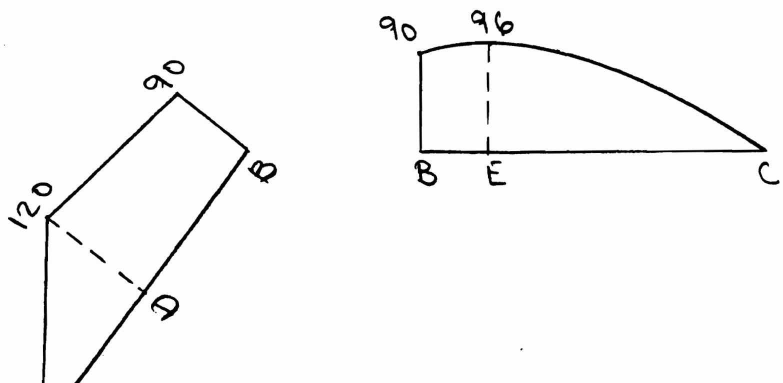
5.59



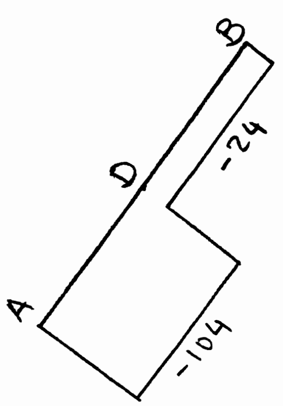
Member End Forces



Shear Diagrams (kN)

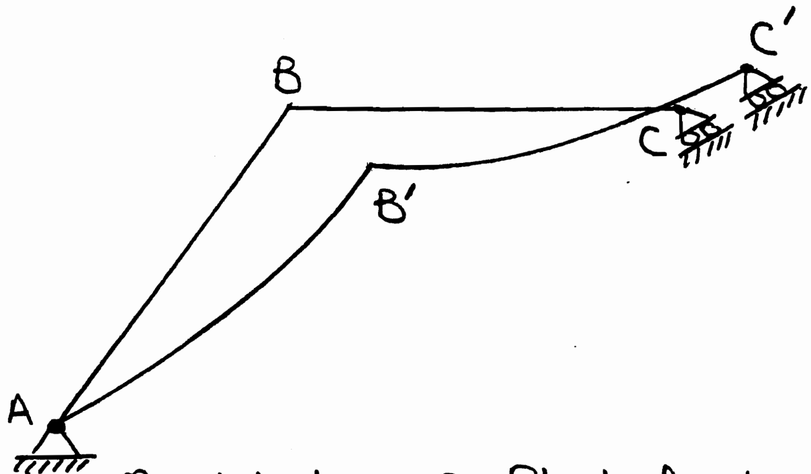


Bending Moment Diagrams (kN-m)



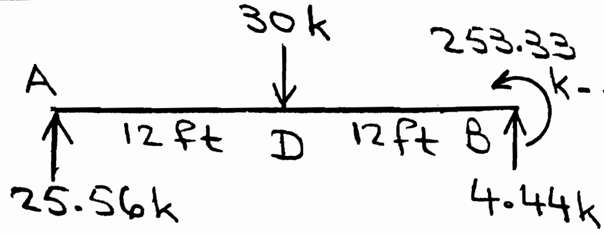
Axial Force Diagrams (kN)

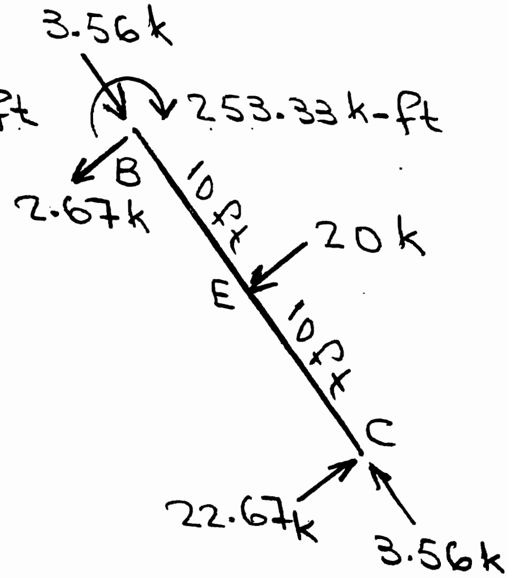
5.59 (contd.)



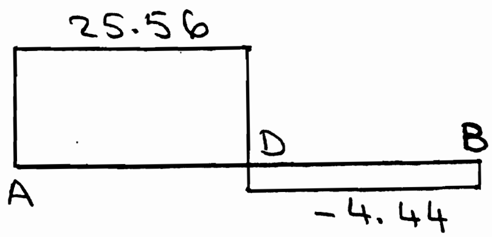
Qualitative Deflected Shape

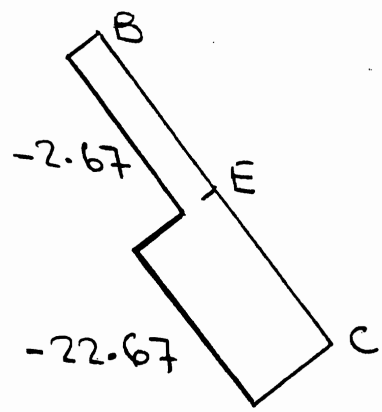
5.60



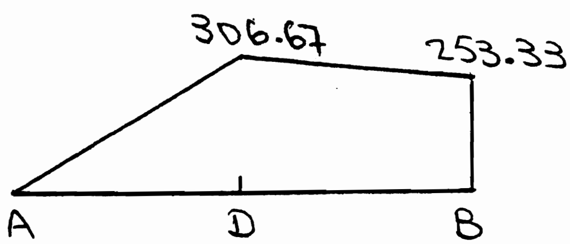


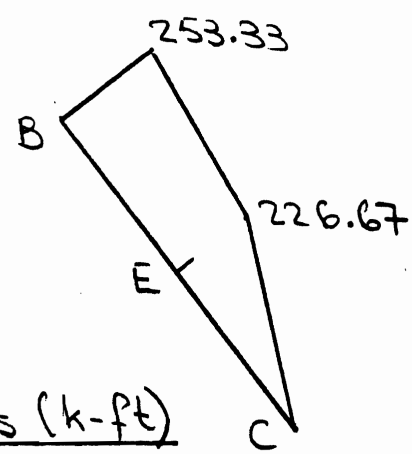
Member End Forces





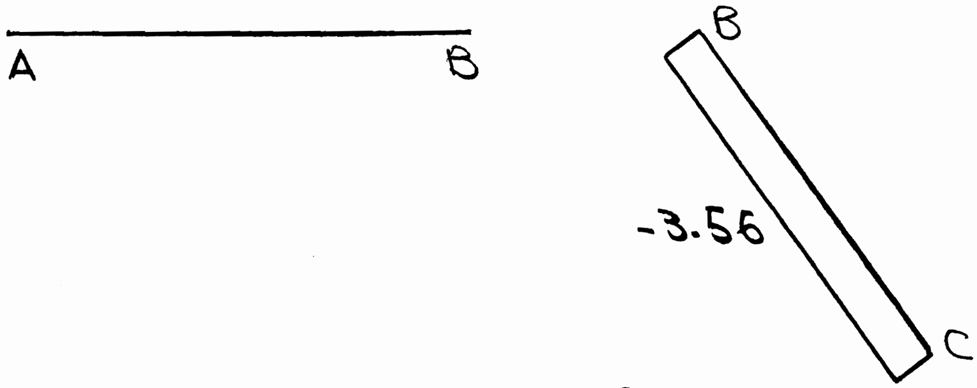
Shear Diagrams (k)



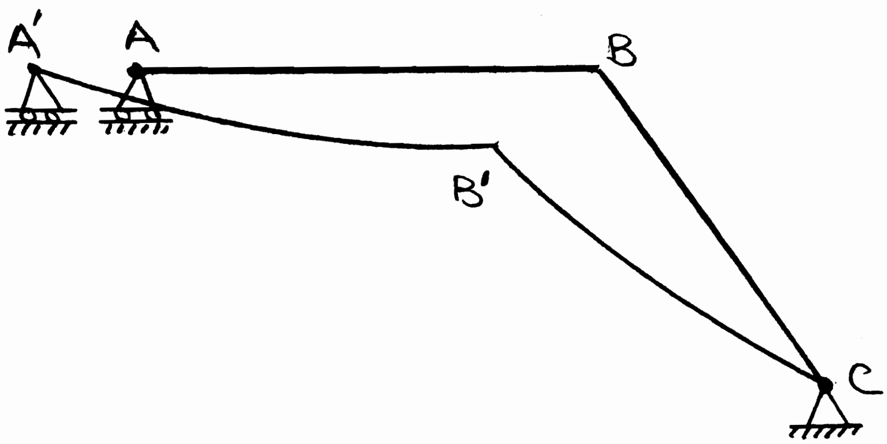


Bending Moment Diagrams (k-ft)

5.60 (contd.)

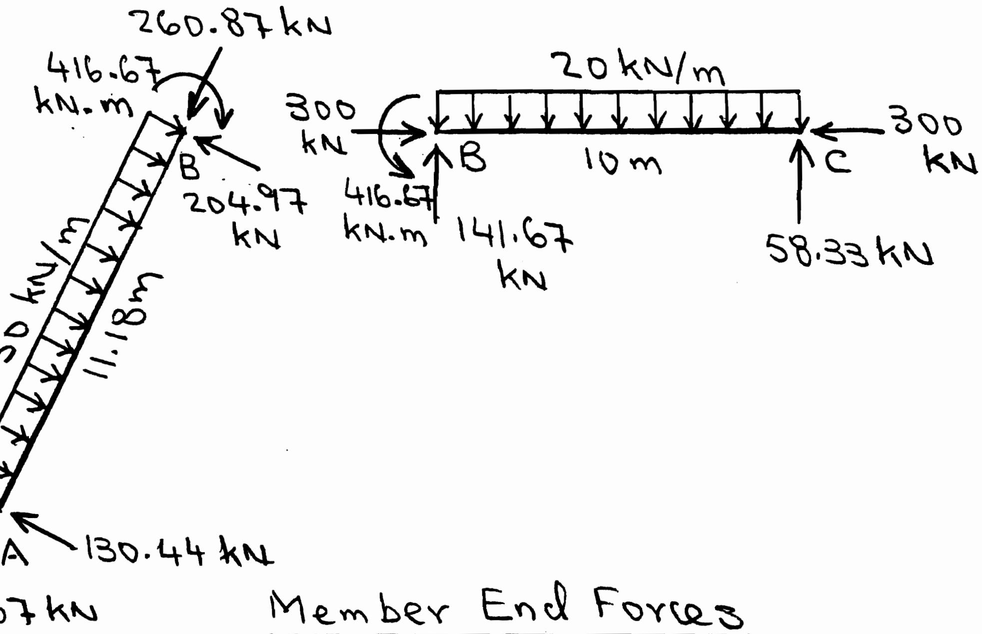


Axial Force Diagrams (k)

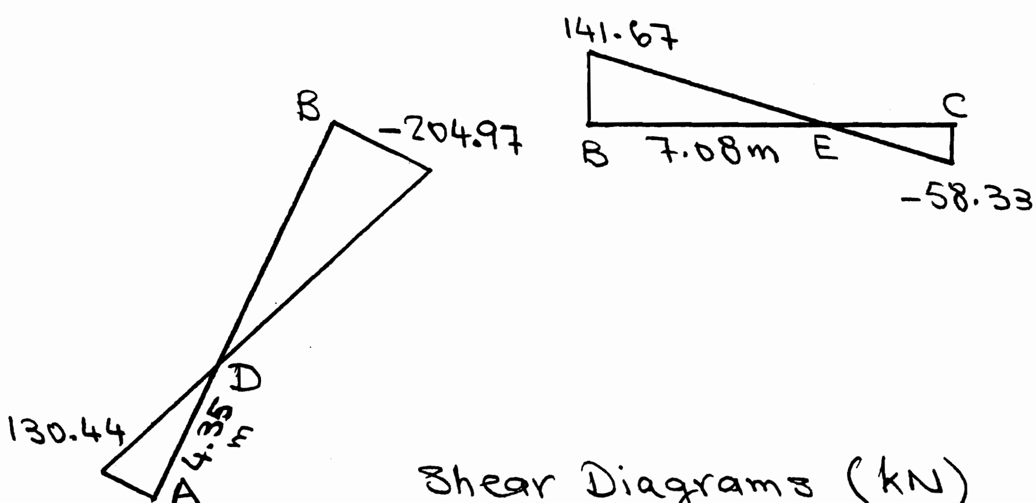


Qualitative Deflected Shape

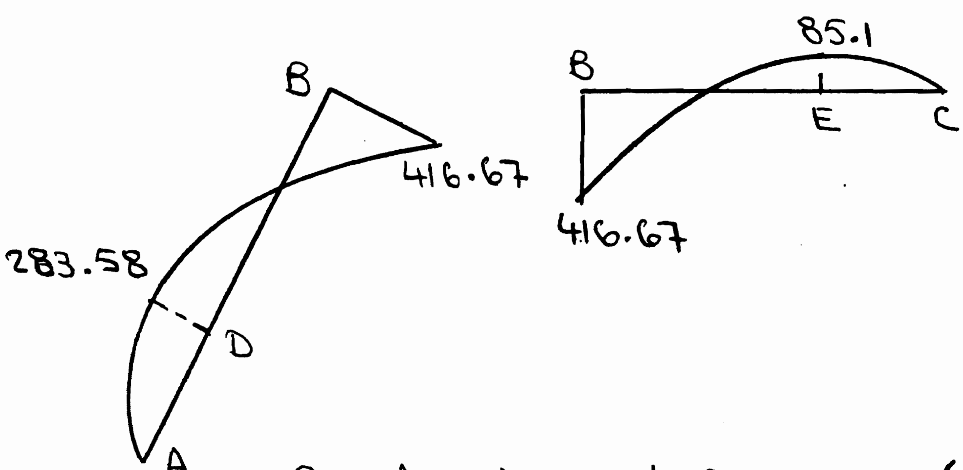
5.61



Member End Forces

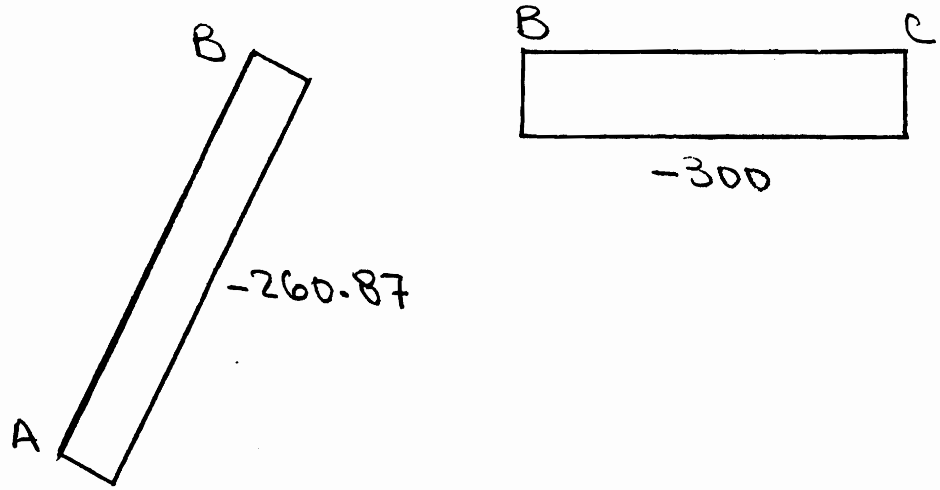


Shear Diagrams (kN)

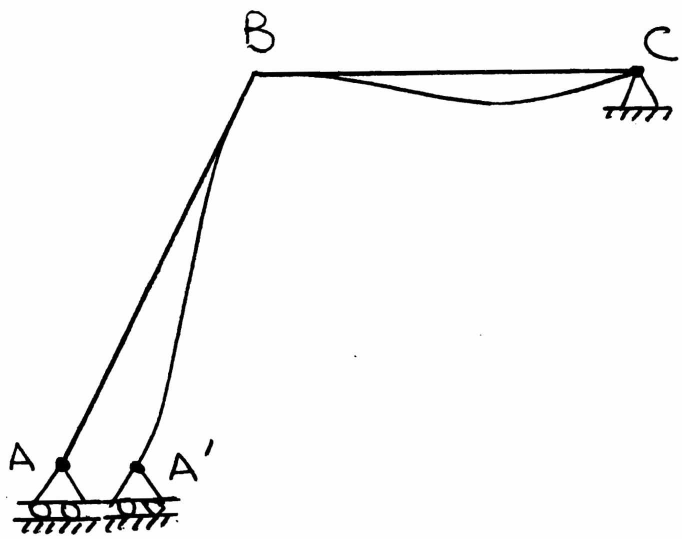


Bending Moment Diagrams (kN.m)

5.61 (contd.)

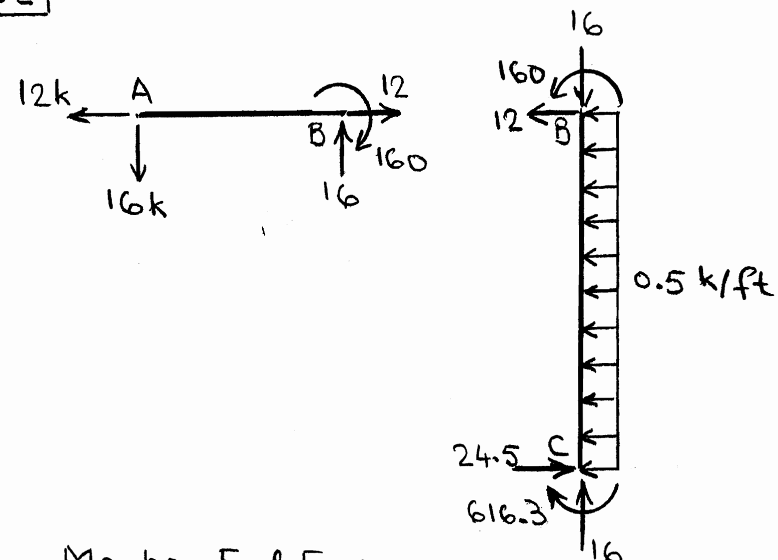


Axial Force Diagrams (kN)

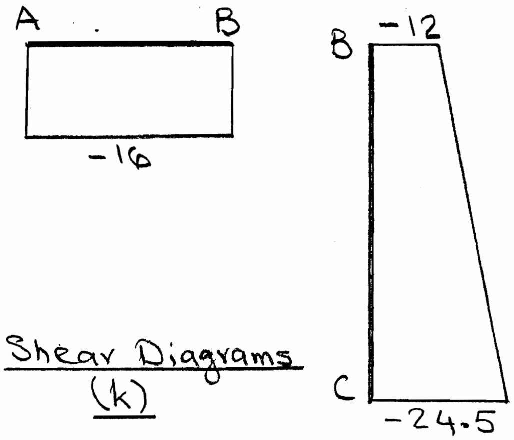


Qualitative Deflected Shape

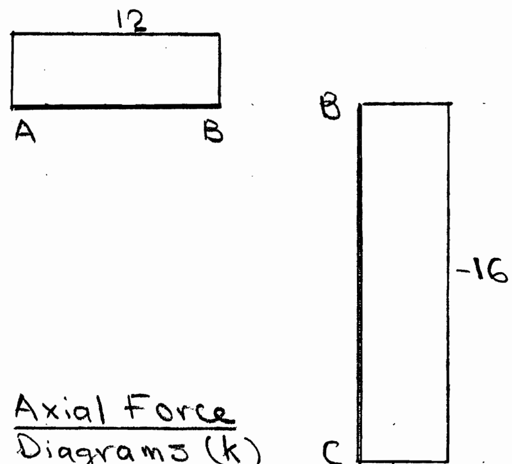
5.62



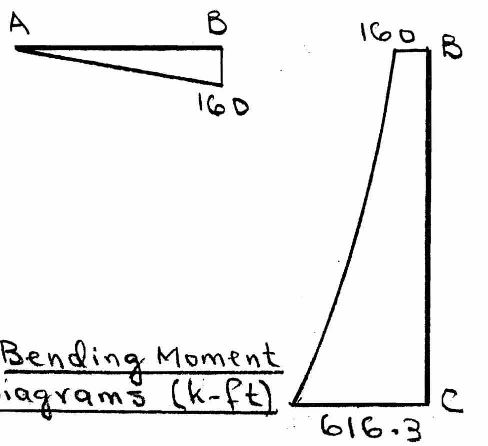
Member End Forces



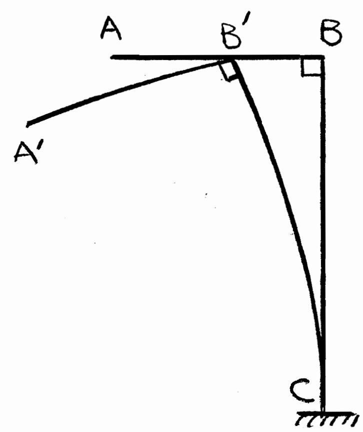
Shear Diagrams (k)



Axial Force Diagrams (k)

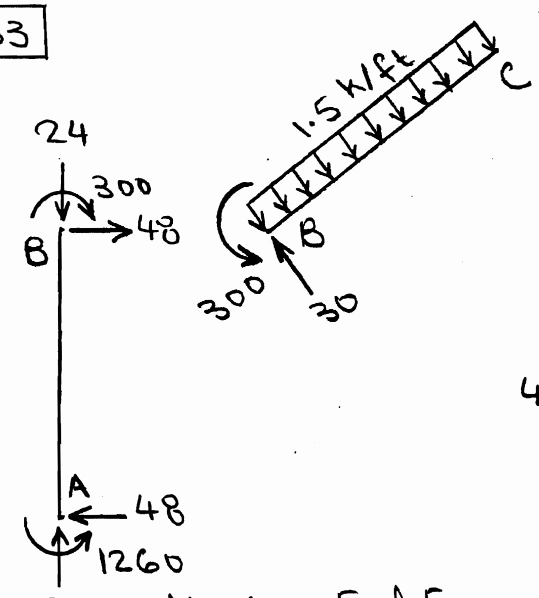


Bending Moment Diagrams (k-ft)

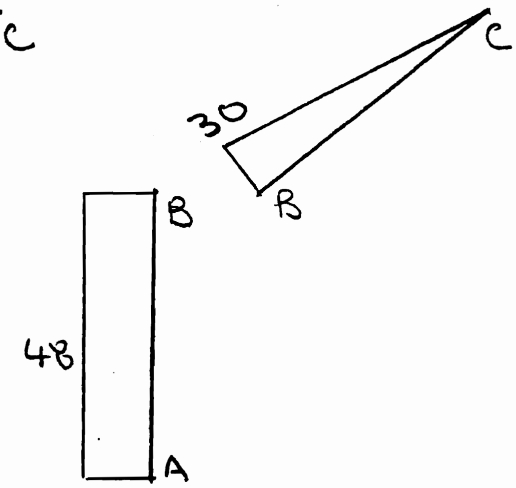


Qualitative Deflected Shape

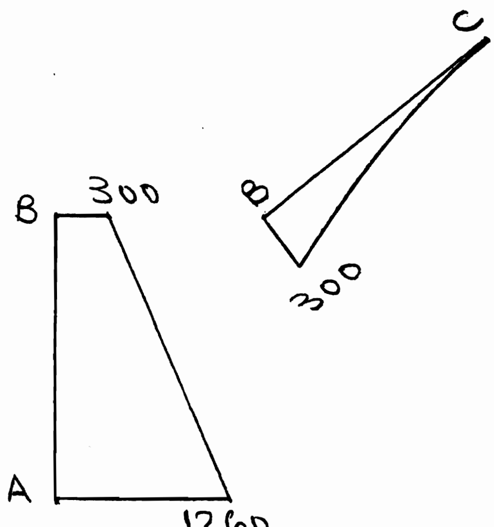
5.63



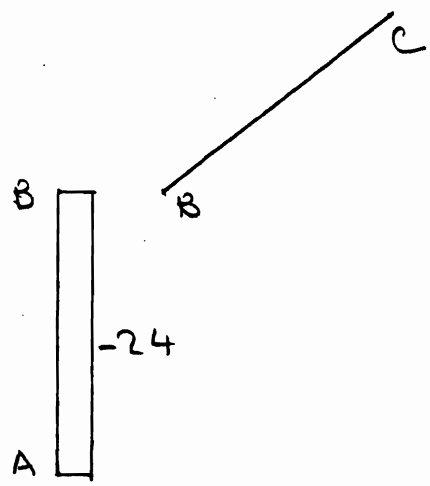
Member End Forces



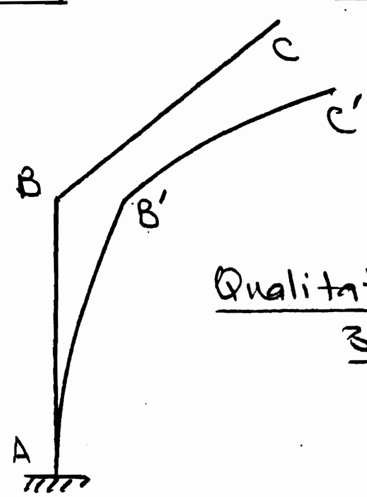
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

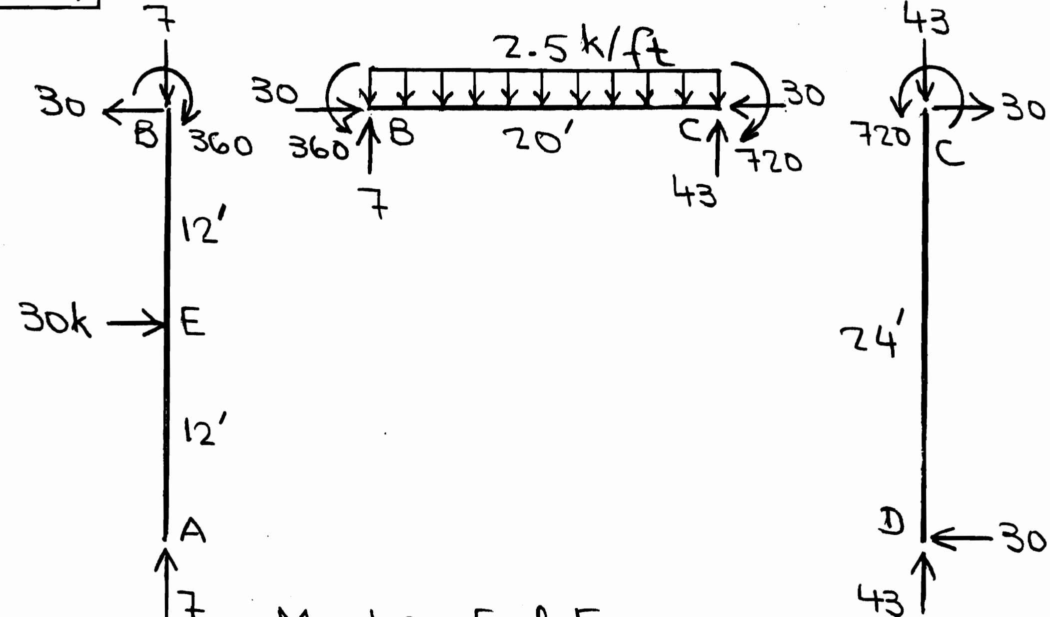


Axial Force Diagrams (kN)

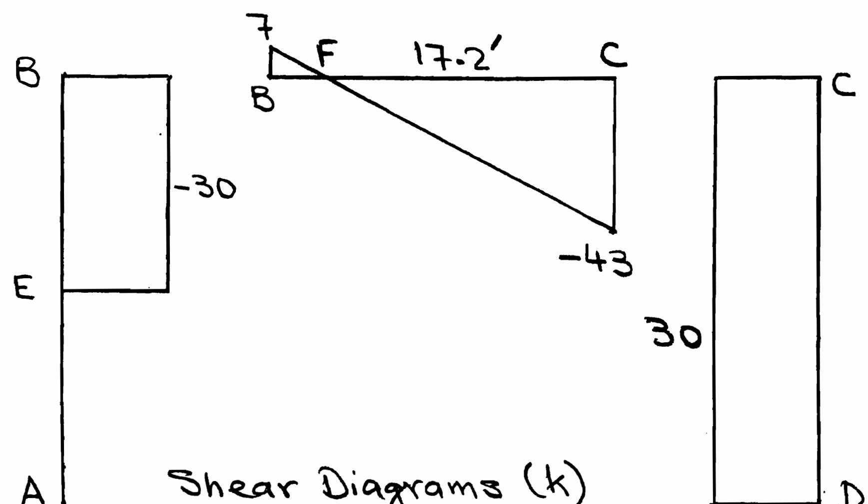


Qualitative Deflected Shape

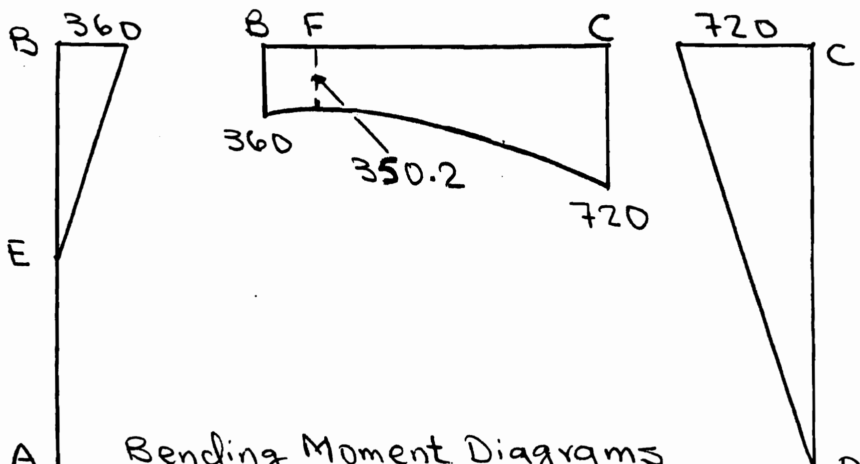
5.64



Member End Forces

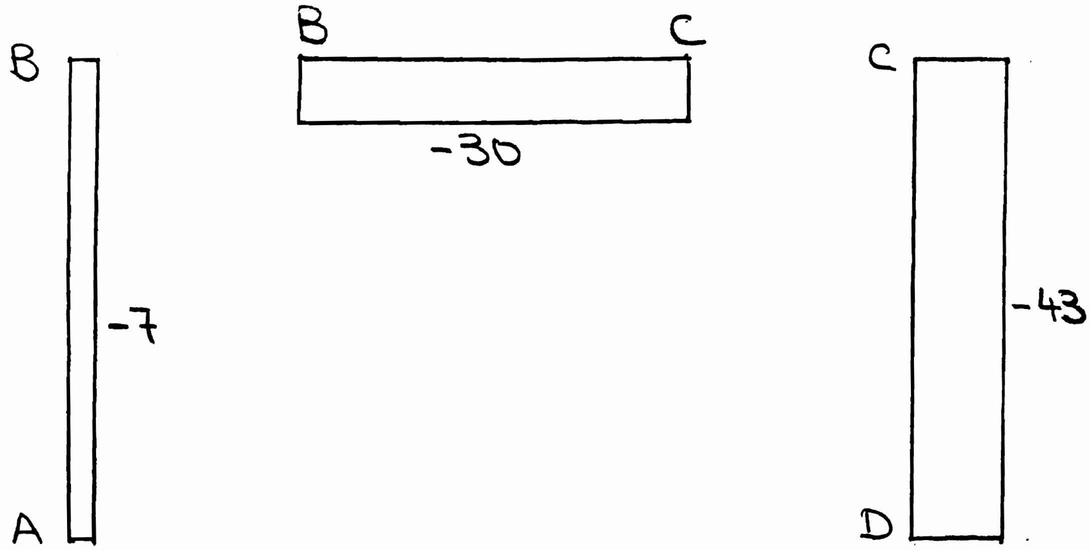


Shear Diagrams (k)

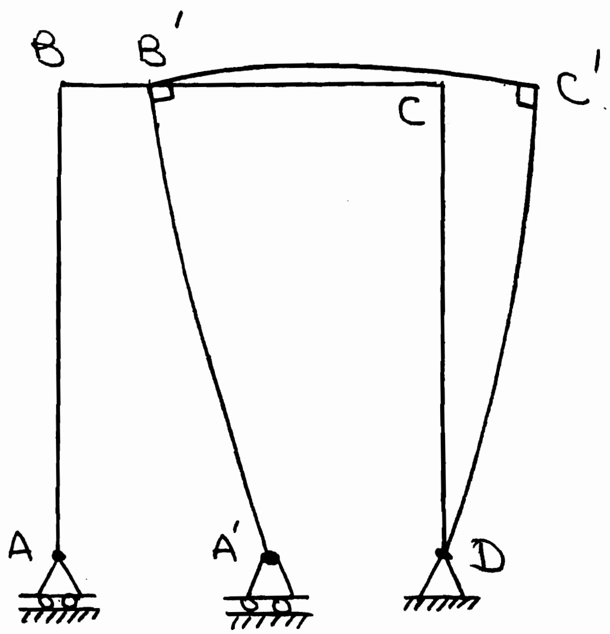


Bending Moment Diagrams (k-ft)

5-64 (contd.)

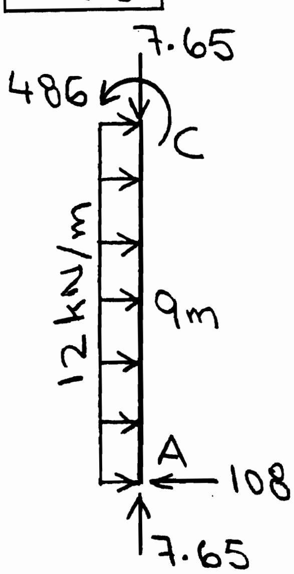


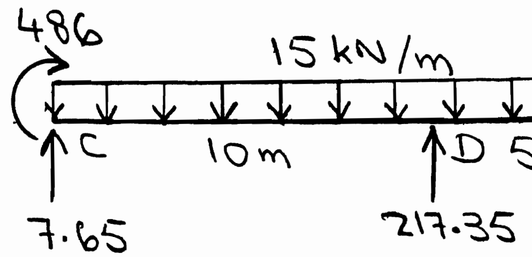
Axial Force Diagrams (k)

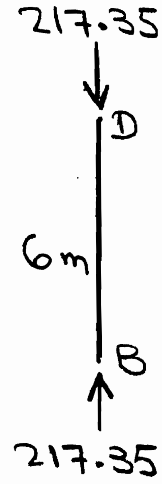


Qualitative Deflected Shape

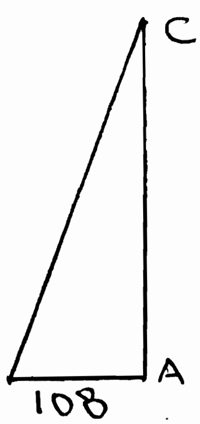
5.65

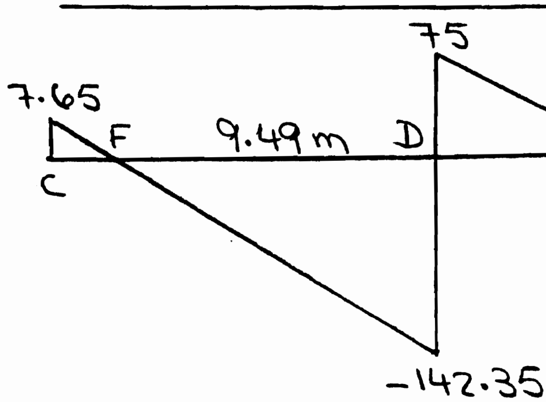




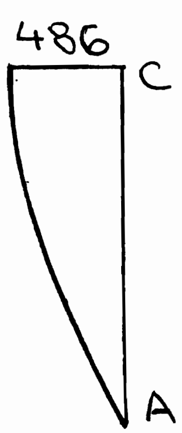


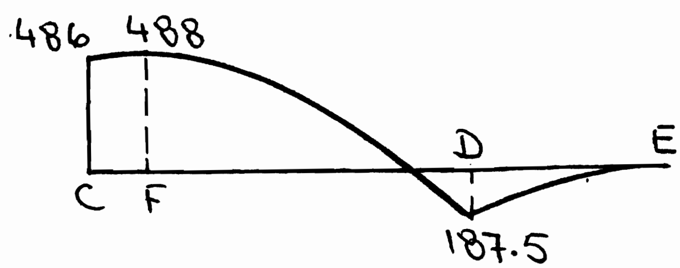
Member End Forces





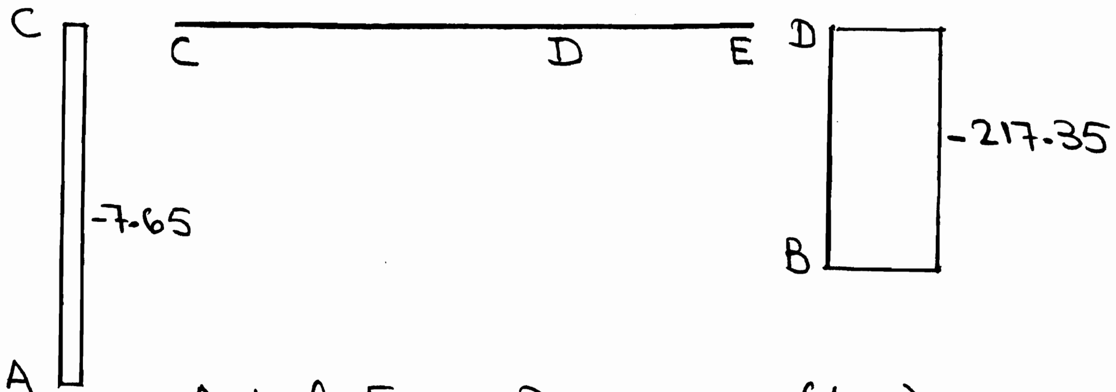
Shear Diagrams (kN)



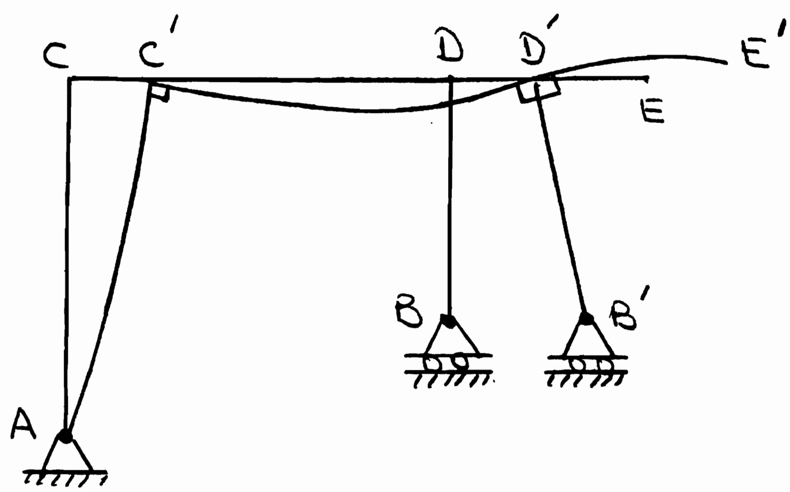


Bending Moment Diagrams (kN.m)

5.65 (contd.)

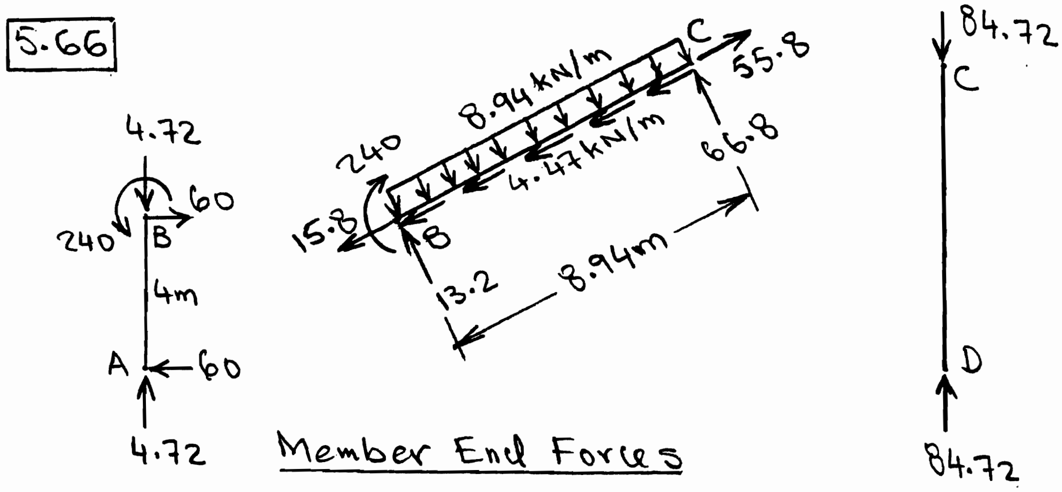


Axial Force Diagrams (kN)

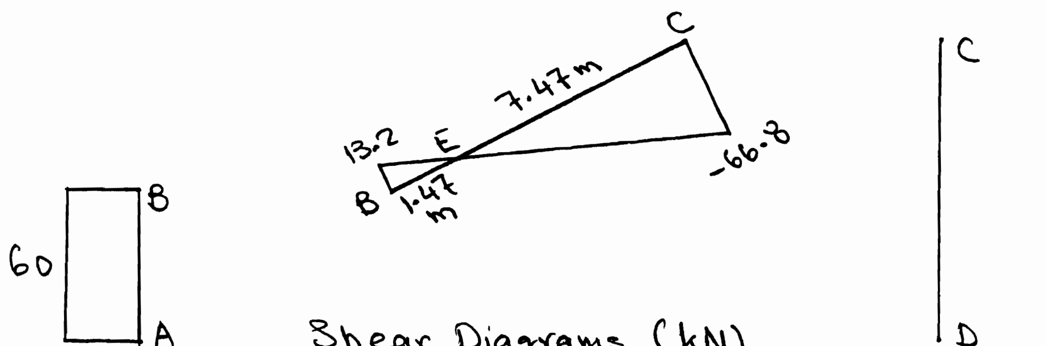


Qualitative Deflected Shape

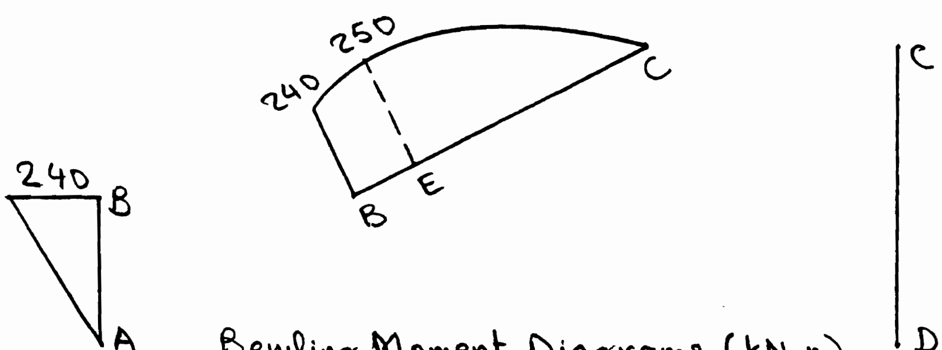
5.66



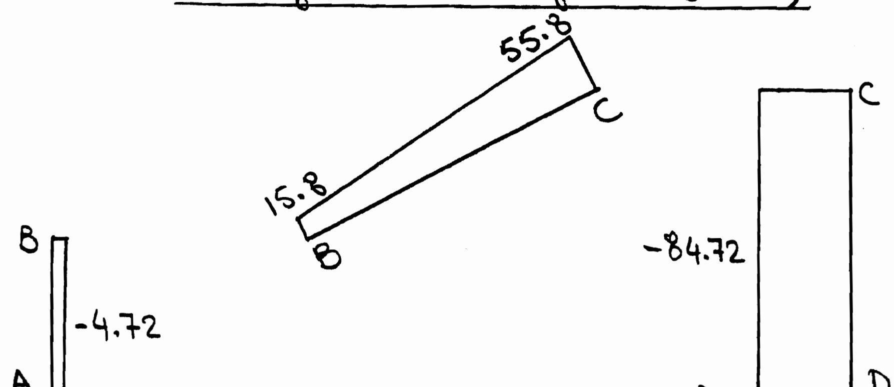
Member End Forces



Shear Diagrams (kN)

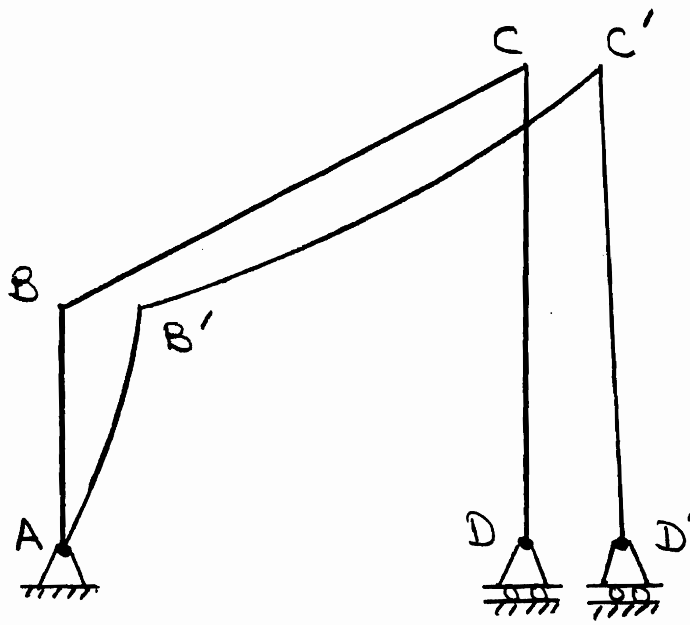


Bending Moment Diagrams (kN-m)



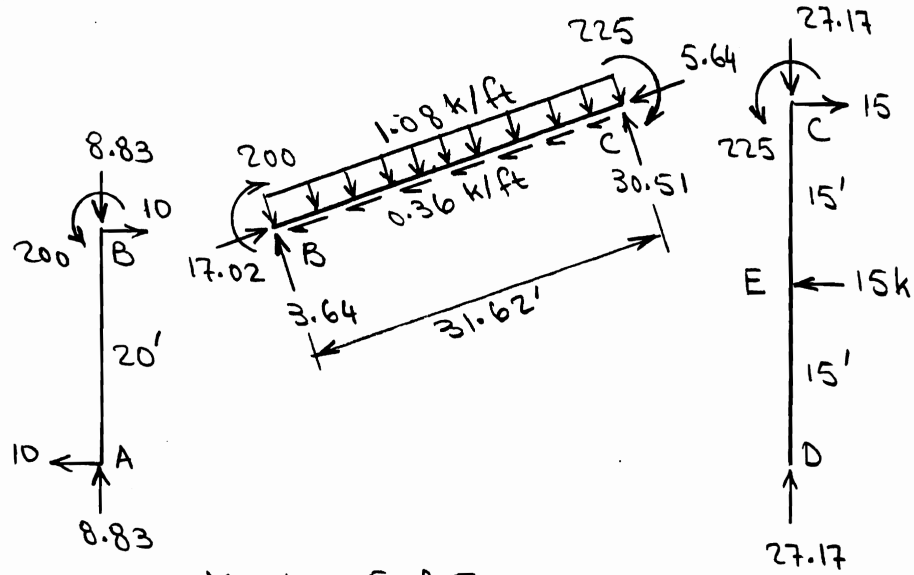
Axial Force Diagrams (kN)

5-66 (contd.)

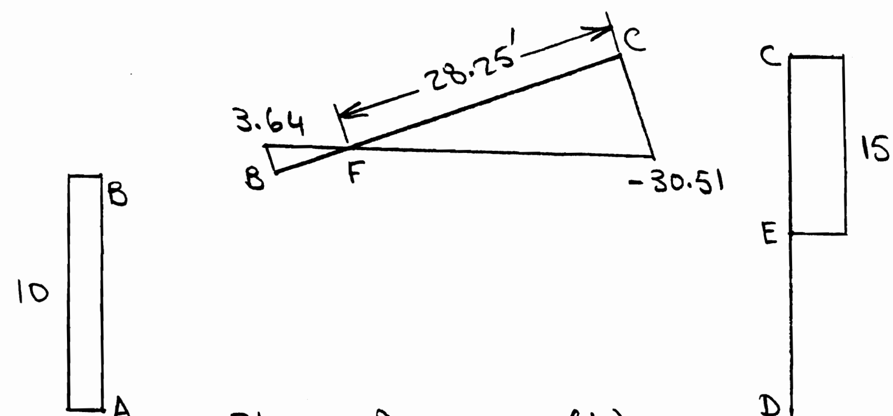


Qualitative Deflected Shape

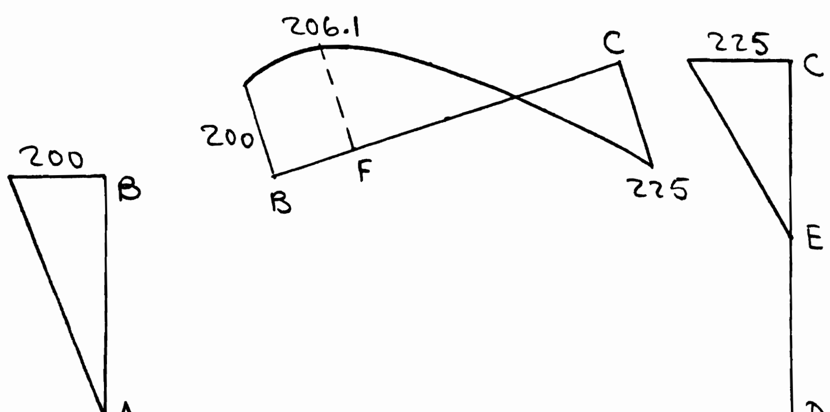
5.67



Member End Forces

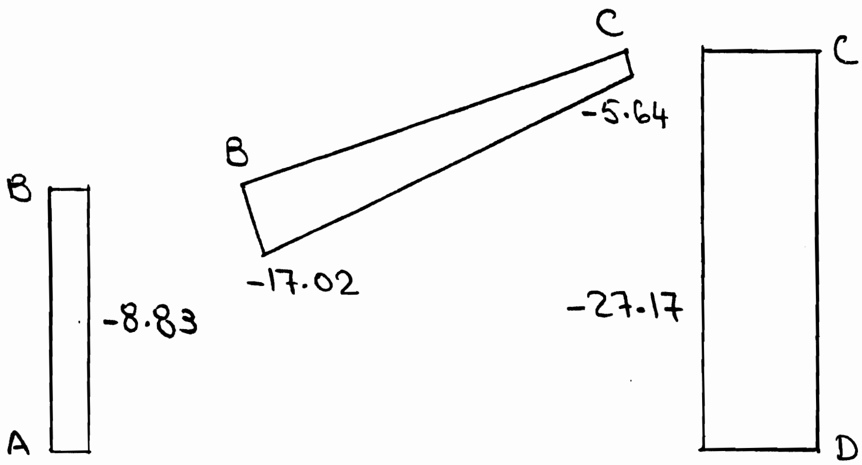


Shear Diagrams (k)

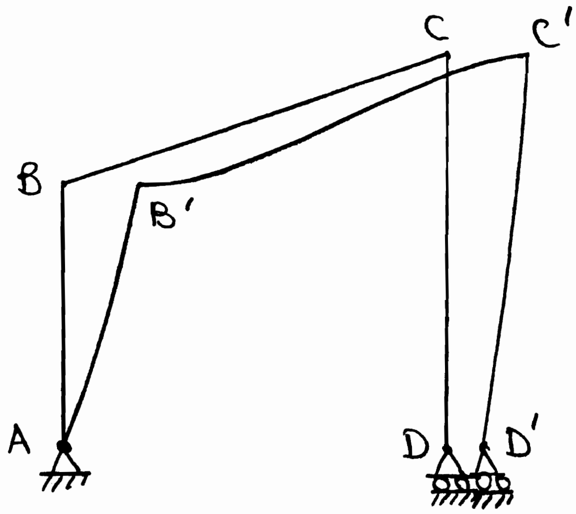


Bending Moment Diagrams (k-ft)

5.67 (Contd.)

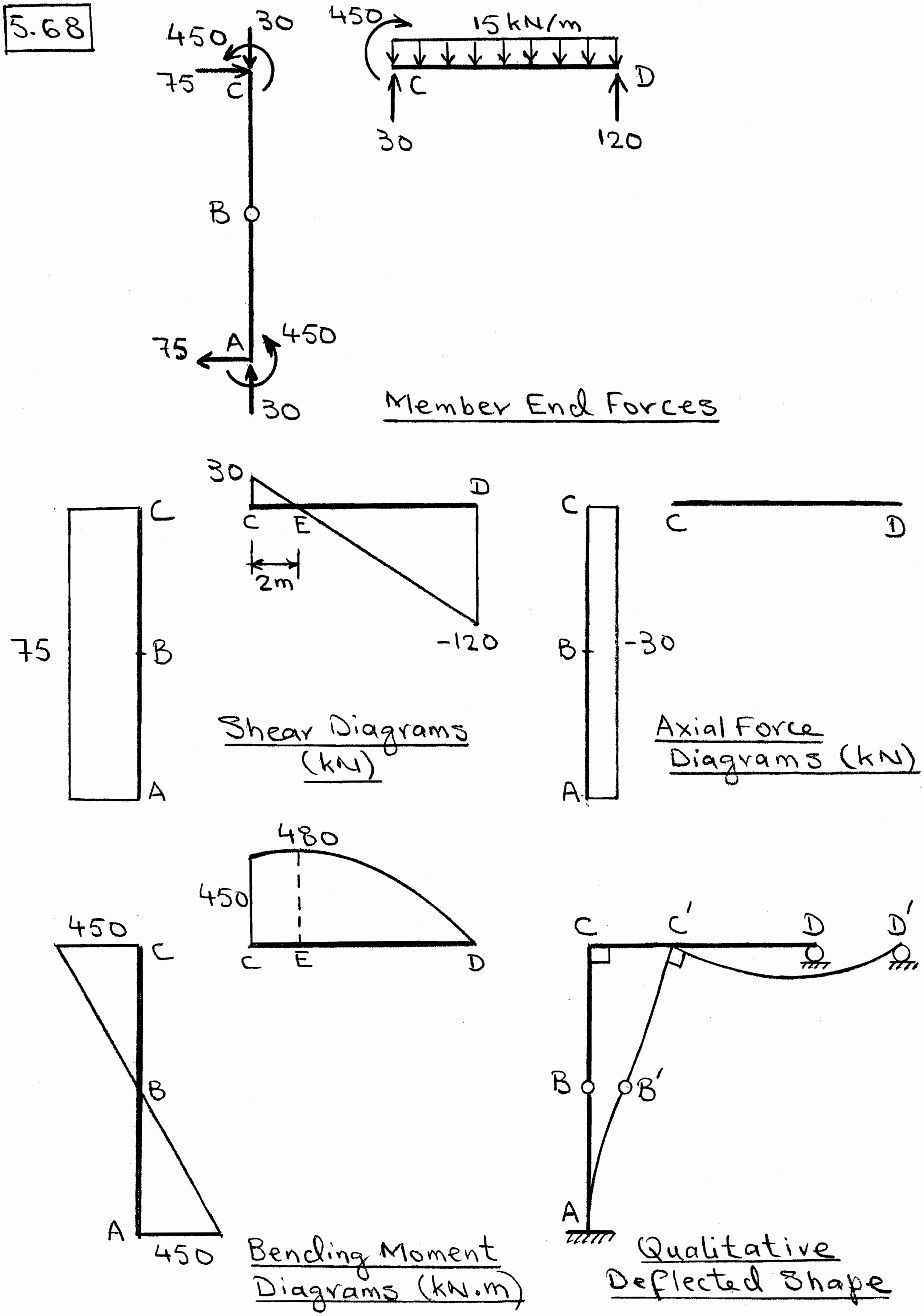


Axial Force Diagrams (k)



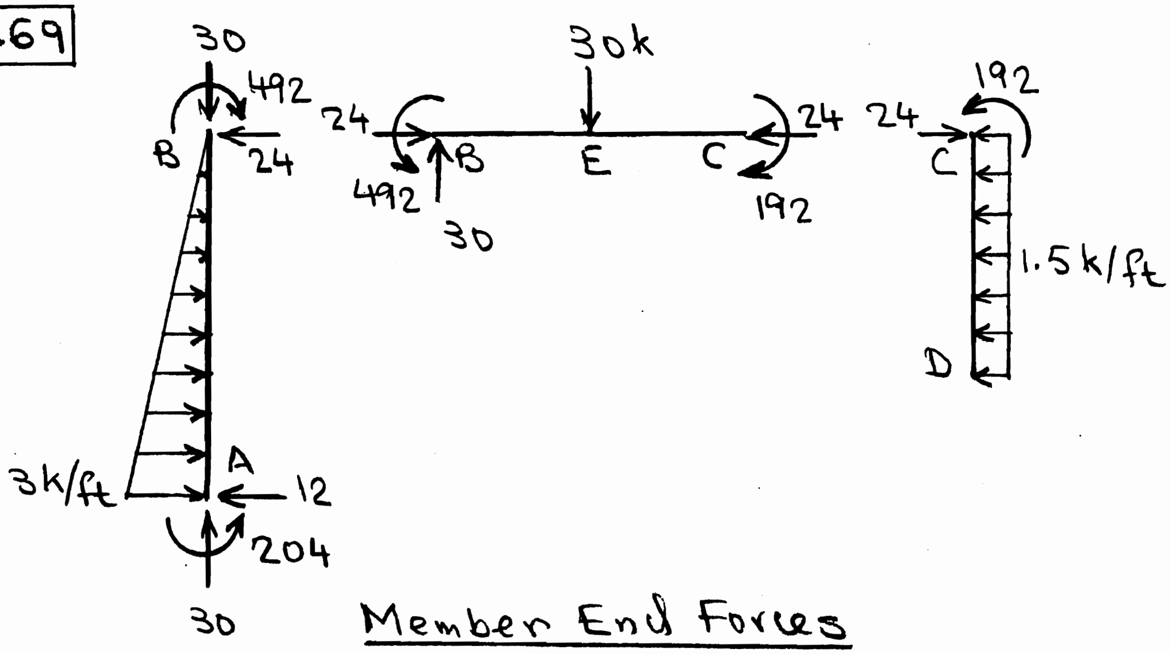
Qualitative Deflected Shape

5.68

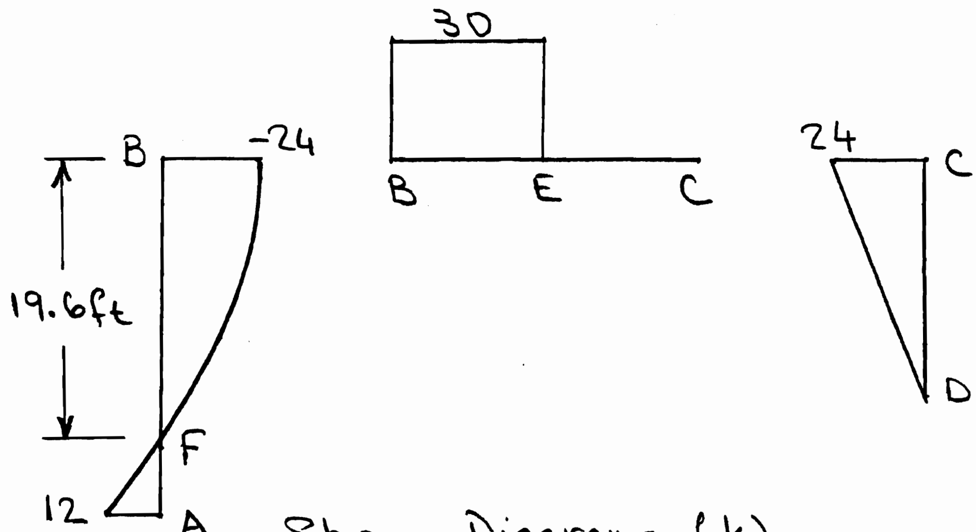


450
30
75
C
450
15 kN/m
C
D
30
120
B
75
A
450
30
Member End Forces
30
C
E
D
2m
-120
C
B
A
75
Shear Diagrams (kN)
C
B
A
-30
C
D
Axial Force Diagrams (kN)
480
450
C
E
D
450
C
B
A
450
Bending Moment Diagrams (kN.m)
C
C′
D
D′
B
B′
A
Qualitative Deflected Shape

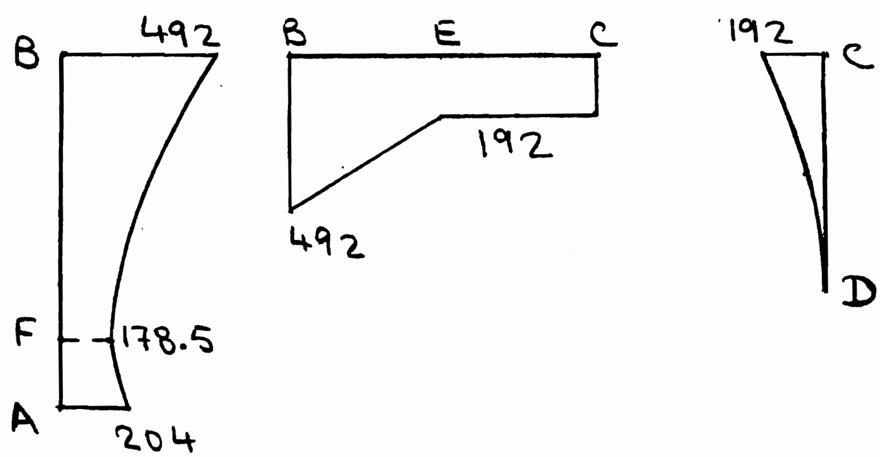
5.69



Member End Forces

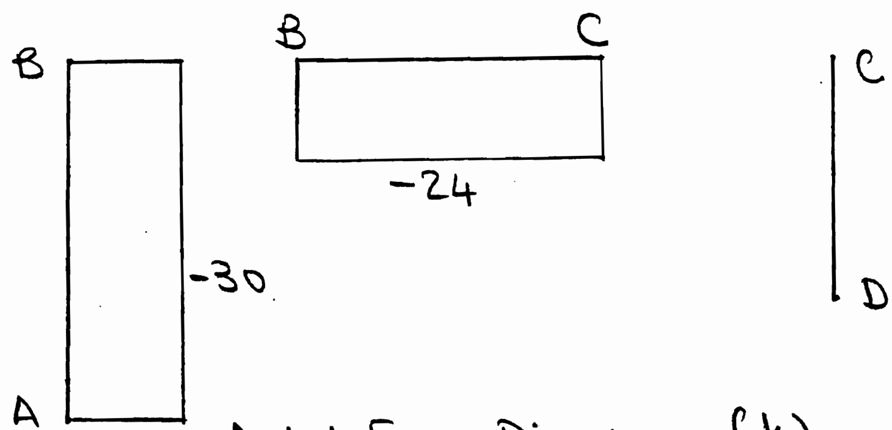


Shear Diagrams (k)

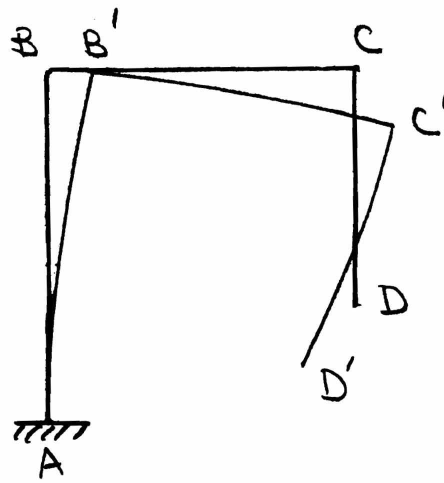


Bending Moment Diagrams (k-ft)

5.69 (contd.)

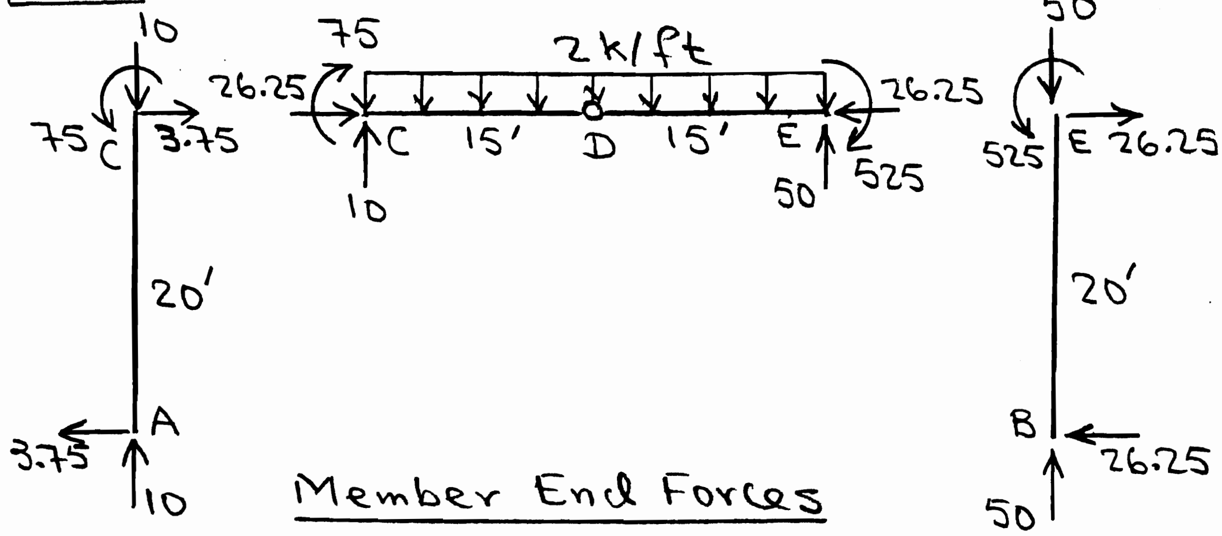


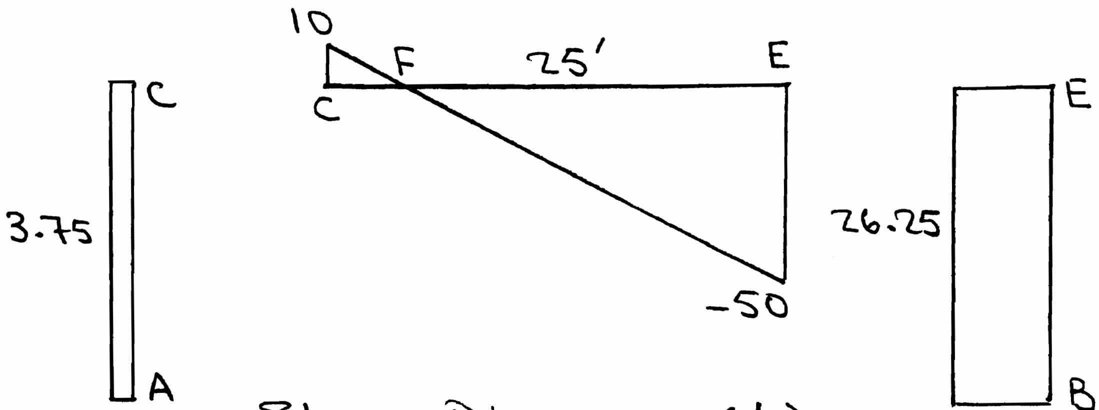
Axial Force Diagrams (k)



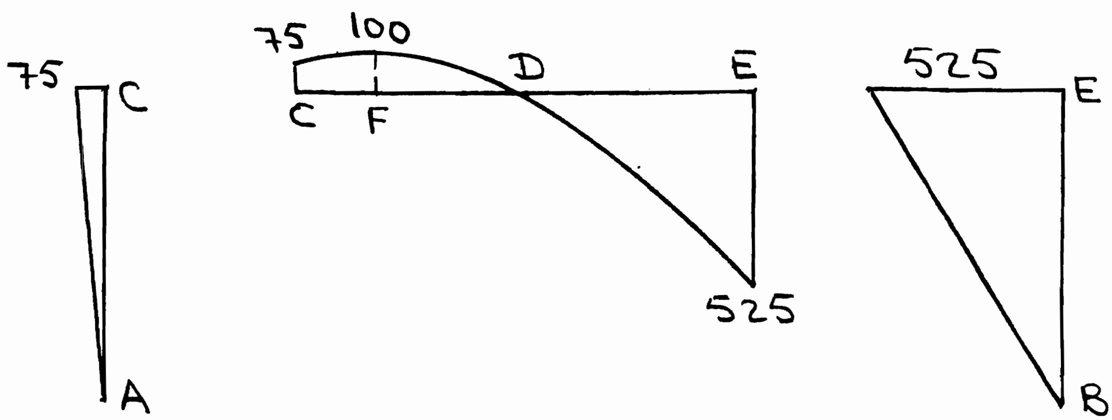
Qualitative Deflected Shape

5.70



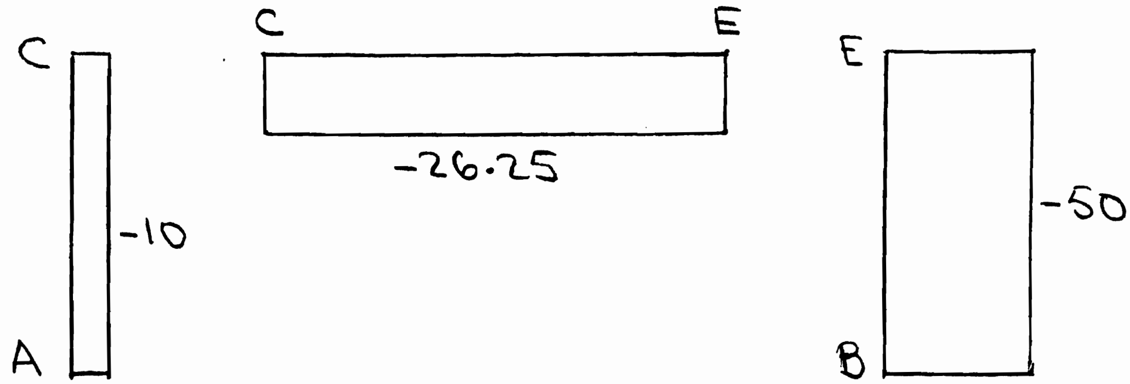


Shear Diagrams (k)

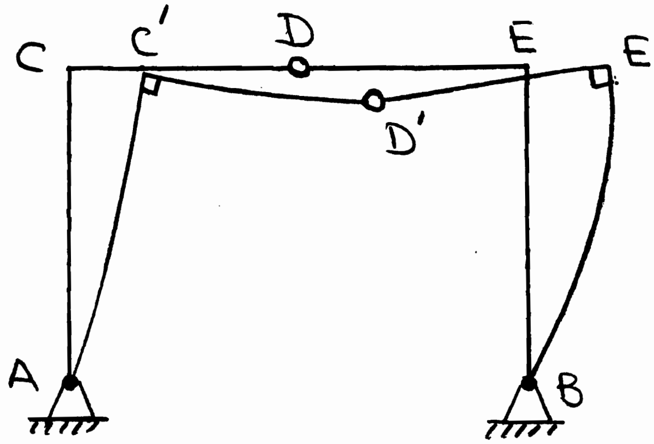


Bending Moment Diagrams (k-ft)

5.70 (contd.)

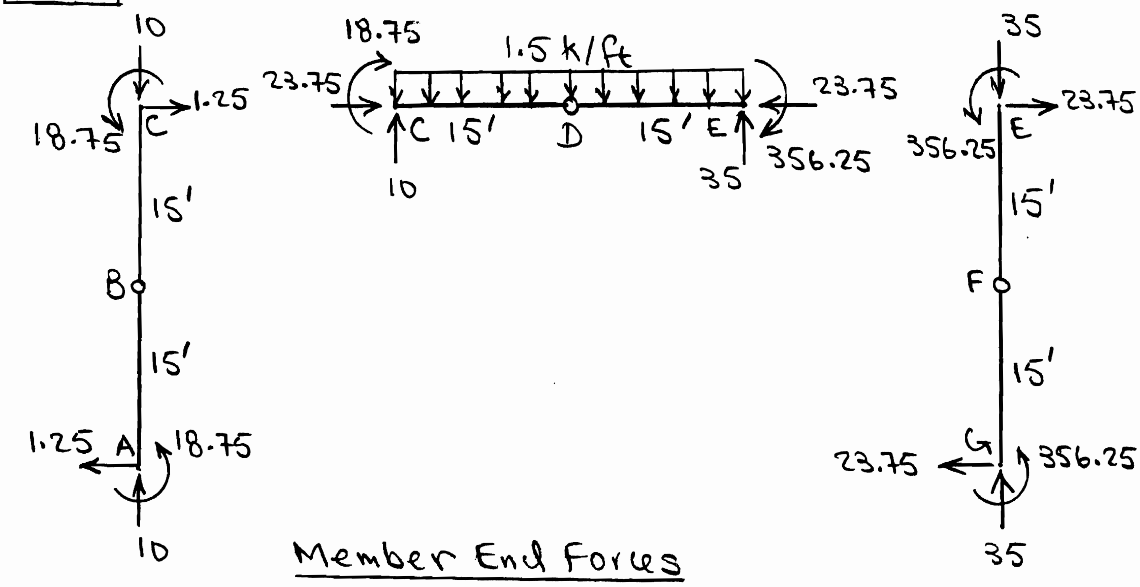


Axial Force Diagrams (k)

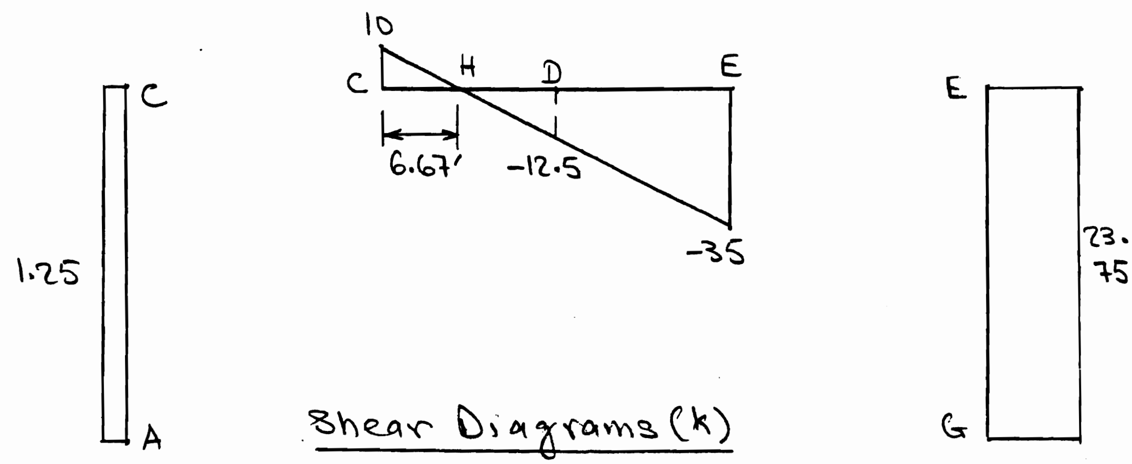


Qualitative Deflected Shape

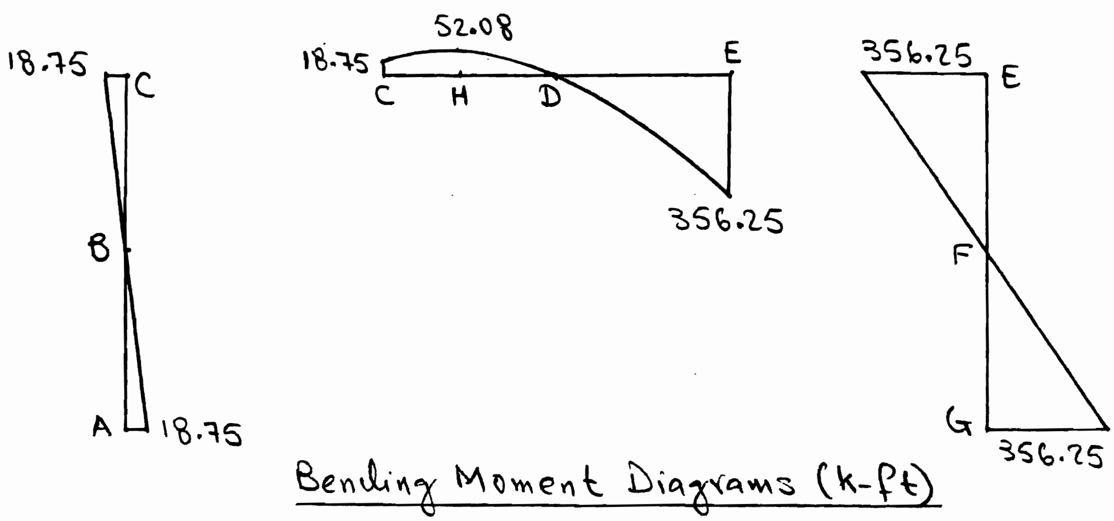
5.71



Member End Forces

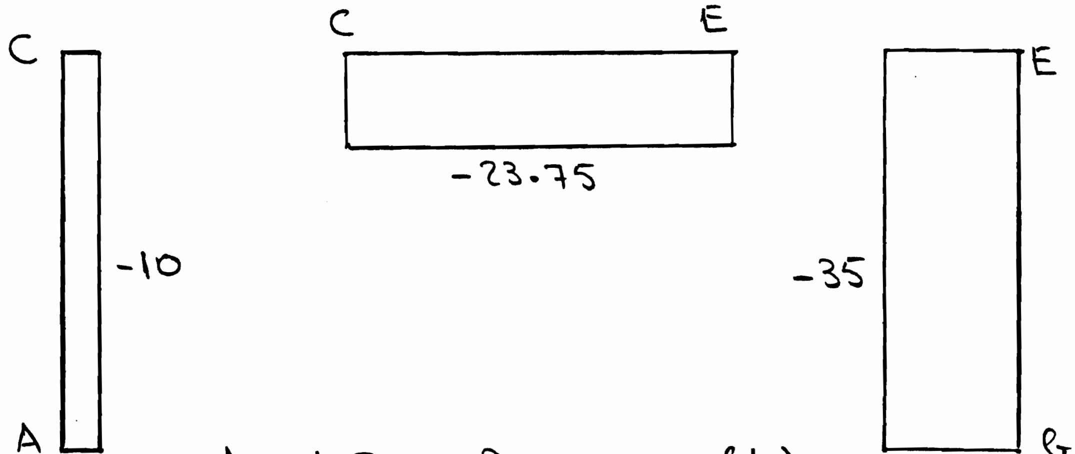


Shear Diagrams (k)

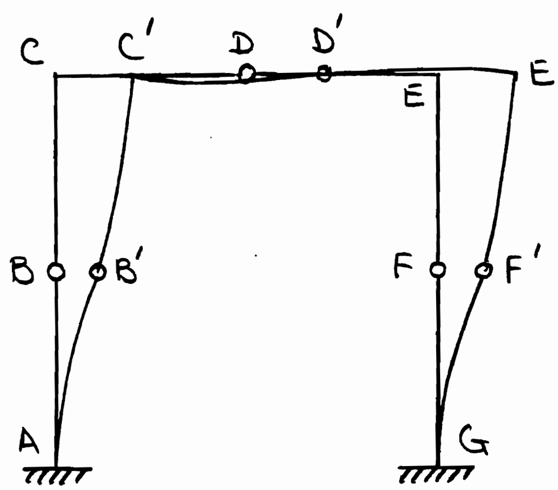


Bending Moment Diagrams (k-ft)

5.71 (Contd.)



Axial Force Diagrams (k)

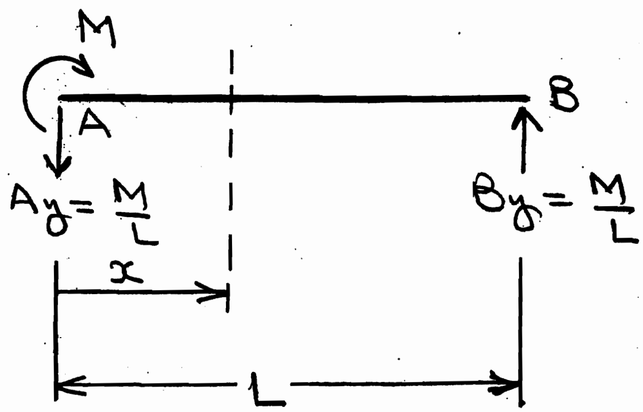


Qualitative Deflected Shape

Chapter Six
Deflections of Beams: Geometric Methods

CHAPTER 6

6.1



$$M_x = M - \frac{M}{L}(x) = \frac{M}{L}(L - x)$$

$$\frac{d^2y}{dx^2} = \frac{M}{EIL}(L-x)$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EIL}(L-x)\,dx = \frac{M}{EIL}\left(Lx - \frac{x^2}{2}\right) + C_1$$

$$y = \int \left[\frac{M}{EIL}\left(Lx - \frac{x^2}{2}\right) + C_1\right] dx$$

$$= \frac{M}{EIL}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + C_1 x + C_2$$

Boundary Conditions:

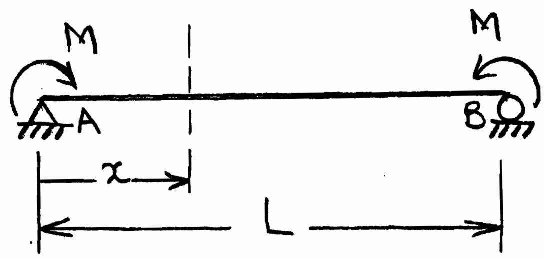
at $x = 0$, $y = 0$ $\therefore C_2 = 0$

at $x = L$, $y = 0$ $\therefore C_1 = -\frac{ML}{3EI}$

$$\theta = -\frac{M}{6EIL}(3x^2 - 6Lx + 2L^2)$$

$$y = -\frac{M}{6EIL}(x^3 - 3Lx^2 + 2L^2 x)$$

6.2



$$M_x = M$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI}\,dx = \frac{Mx}{EI} + C_1$$

$$y = \int \left(\frac{Mx}{EI} + C_1\right) dx = \frac{Mx^2}{2EI} + C_1 x + C_2$$

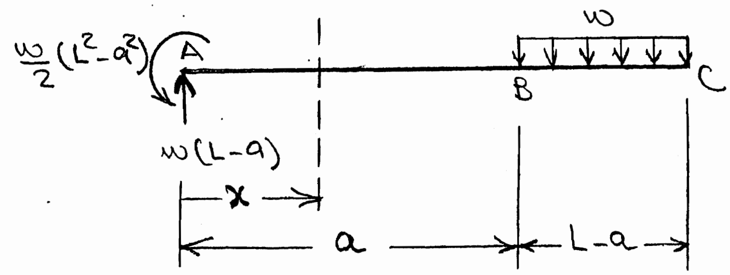
Boundary conditions:

at $x = 0$, $y = 0$ $\quad \therefore C_2 = 0$

at $x = L$, $y = 0$ $\quad \therefore C_1 = -\dfrac{ML}{2EI}$

$$\underline{\theta = \frac{M}{EI}\left(x - \frac{L}{2}\right)}\ ; \qquad \underline{y = \frac{Mx}{2EI}(x - L)}$$

6.3



Segment AB: $0 \leq x \leq a$

$$M = -\frac{w}{2}(L^2-a^2) + w(L-a)x$$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left[-\frac{w}{2}(L^2-a^2) + w(L-a)x\right]$$

$$EI\theta = -\frac{w}{2}(L^2-a^2)x + w(L-a)\frac{x^2}{2} + C_1$$

$$EIy = -\frac{w}{2}(L^2-a^2)\frac{x^2}{2} + w(L-a)\frac{x^3}{6} + C_1x + C_2$$

Using the boundary conditions, $\theta = 0$ and $y = 0$ at $x = 0$, we obtain $C_1 = 0$ and $C_2 = 0$

Thus,

$$\theta = \frac{wx}{2EI}\left[a^2 - L^2 + (L-a)x\right]$$

$$y = \frac{wx^2}{2EI}\left[\frac{a^2-L^2}{2} + \frac{(L-a)x}{3}\right]$$

Segment BC: $a \leq x \leq L$

$$M = -\frac{w}{2}(L^2-a^2) + w(L-a)x - \frac{w}{2}(x-a)^2$$

$$EI\theta = -\frac{w}{2}(L^2-a^2)x + w(L-a)\frac{x^2}{2} - \frac{w}{2}\left(\frac{x^3}{3} + a^2x - ax^2\right) + C_3$$

$$EIy = -\frac{w}{2}(L^2-a^2)\frac{x^2}{2} + w(L-a)\frac{x^3}{6} - \frac{w}{2}\left(\frac{x^4}{12} + \frac{ax^2}{2} - \frac{ax^3}{3}\right) + C_3x + C_4$$

By using the conditions that at $x = a$, $\theta_{B,Left} = \theta_{B,Right}$ and $y_{B,Left} = y_{B,Right}$, we obtain:

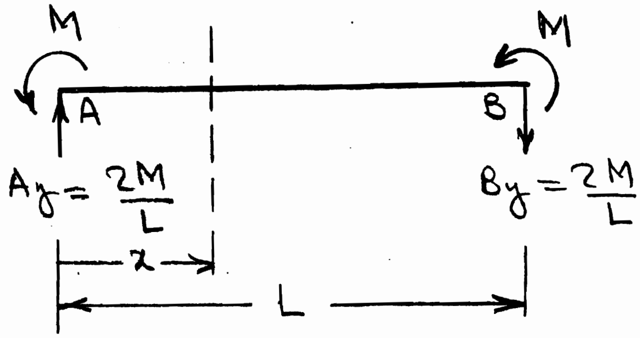
$$C_3 = \frac{wa^3}{6} \quad \text{and} \quad C_4 = -\frac{wa^4}{24}$$

Thus

$$\theta = \frac{w}{2EI}\left[xL(x-L) - \frac{x^3}{3} + \frac{a^3}{3}\right]$$

$$y = \frac{w}{2EI}\left[x^2L\left(\frac{x}{3} - \frac{L}{2}\right) - \frac{x^4}{12} - \frac{a^4}{12} + \frac{a^3x}{3}\right]$$

6.4



$$M_x = \frac{2M}{L}x - M$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}\left(\frac{2x}{L} - 1\right)$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI}\left(\frac{2x}{L} - 1\right)dx = \frac{Mx}{EI}\left(\frac{x}{L} - 1\right) + C_1$$

$$y = \int\left[\frac{Mx}{EI}\left(\frac{x}{L} - 1\right) + C_1\right]dx = \frac{Mx^2}{EI}\left(\frac{x}{3L} - \frac{1}{2}\right) + C_1x + C_2$$

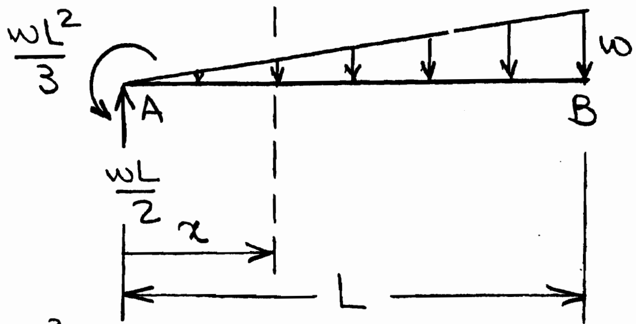
Boundary conditions:

at $x = 0$, $y = 0$ $\therefore C_2 = 0$

at $x = L$, $y = 0$ $\therefore C_1 = \frac{ML}{6EI}$

$$\theta = \frac{M}{6EIL}\left(6x^2 - 6Lx + L^2\right); \quad y = \frac{Mx}{6EIL}\left(2x^2 - 3Lx + L^2\right)$$

6.5



$$M = -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{1}{2}x\left(\frac{wx}{L}\right)\frac{x}{3}$$

$$EI\frac{d^2y}{dx^2} = -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{wx^3}{6L}$$

$$EI\theta = -\frac{wL^2}{3}x + \frac{wL}{2}\left(\frac{x^2}{2}\right) - \frac{w}{6L}\left(\frac{x^4}{4}\right) + C_1$$

$$EIy = -\frac{wL^2}{3}\left(\frac{x^2}{2}\right) + \frac{wL}{2}\left(\frac{x^3}{6}\right) + \frac{w}{6L}\left(\frac{x^5}{20}\right) + C_1x + C_2$$

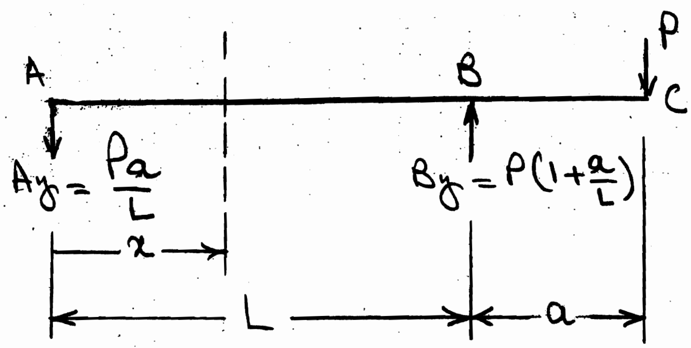
Using the boundary conditions that at $x = 0$, $\theta = 0$ and $y = 0$, we obtain $C_1 = 0$ and $C_2 = 0$.

Thus,

$$\theta = \frac{wx}{24EIL}\left(-8L^3 + 6L^2x - x^3\right)$$

$$y = \frac{wx^2}{120EIL}\left(-20L^3 + 10L^2x - x^3\right)$$

6.6



Segment AB: $0 \leqslant x \leqslant L$

$$M = -\frac{Pax}{L}$$

$$\frac{d^2y}{dx^2} = -\frac{Pax}{EIL}$$

$$\theta = -\frac{Pa}{EIL}\left(\frac{x^2}{2}\right) + C_1$$

$$y = -\frac{Pa}{2EIL}\left(\frac{x^3}{3}\right) + C_1 x + C_2$$

Applying the boundary conditions, $y = 0$ at $x = 0$ and $y = 0$ at $x = L$, we obtain

$$C_2 = 0 \qquad C_1 = \frac{PaL}{6EI}$$

Thus,

$$\theta = \frac{PaL}{6EI}\left(1 - 3\frac{x^2}{L^2}\right); \quad y = \frac{PaLx}{6EI}\left(1 - \frac{x^2}{L^2}\right)$$

Segment BC: $L \leqslant x \leqslant (L+a)$

$$M = -P(L + a - x)$$

$$\frac{d^2y}{dx^2} = -\frac{P}{EI}(L + a - x)$$

$$\theta = -\frac{P}{EI}\left(Lx + ax - \frac{x^2}{2}\right) + C_3$$

$$y = -\frac{P}{EI}\left(\frac{Lx^2}{2} + \frac{ax^2}{2} - \frac{x^3}{6}\right) + C_3 x + C_4$$

6.6 (Contd.)

By using the condition that at $x = L$, $\theta_{B,Left} = \theta_{B,Right}$, we obtain

$$C_3 = \frac{PL}{EI}\left(\frac{L}{2} + \frac{2a}{3}\right)$$

Next, by applying the condition that at $x = L$, $y = 0$, we obtain

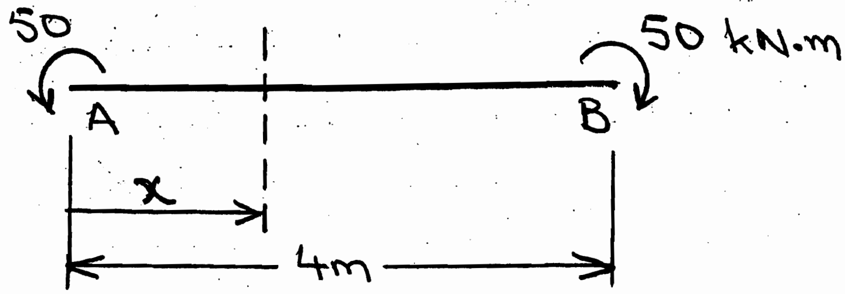
$$C_4 = -\frac{PL^2}{6EI}(L+a)$$

Thus,

$$\theta = \frac{PL}{EI}\left[\frac{x^2}{2L} - x\left(1+\frac{a}{L}\right) + \frac{L}{2} + \frac{2a}{3}\right]$$

$$y = \frac{PL}{EI}\left[\frac{x^3}{6L} - \frac{x^2}{2}\left(1+\frac{a}{L}\right) + x\left(\frac{L}{2} + \frac{2a}{3}\right) - \frac{L}{6}(L+a)\right]$$

6.7



$$M_x = -50$$

$$\frac{d^2y}{dx^2} = -\frac{50}{EI}$$

$$\theta = \frac{dy}{dx} = -\frac{50}{EI}x + C_1$$

$$y = -\frac{25}{EI}x^2 + C_1 x + C_2$$

Boundary Conditions:

at $x = 0$, $\theta = 0$ $\therefore C_1 = 0$

at $x = 0$, $y = 0$ $\therefore C_2 = 0$

Thus,

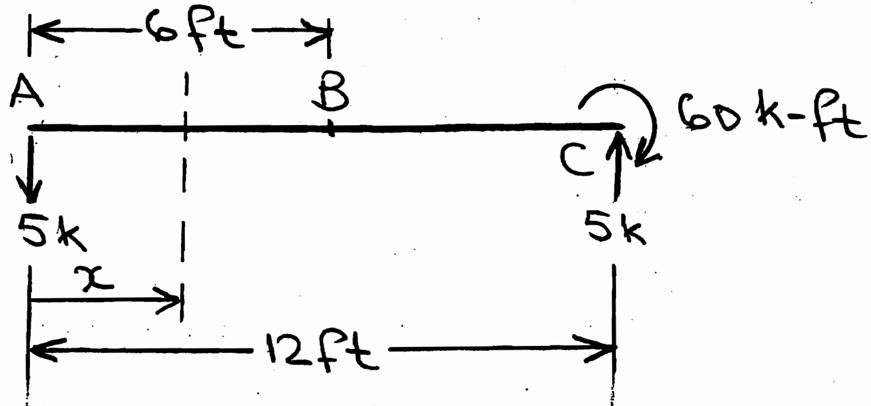
$$\theta = -\frac{50x}{EI} \; ; \quad y = -\frac{25x^2}{EI}$$

At $x = 4\text{m}$:

$$\theta = -\frac{50(4)}{70(164)} = -0.0174 \text{ rad} = \underline{0.0174 \text{ rad}}$$

$$y = -\frac{25(4)^2}{70(164)} = -0.0348 \text{ m} = \underline{34.8 \text{ mm} \downarrow}$$

6.8



$M_x = -5x$

$$\frac{d^2y}{dx^2} = -\frac{5x}{EI}$$

$$\theta = \frac{dy}{dx} = -\frac{5x^2}{2EI} + C_1$$

$$y = -\frac{5x^3}{6EI} + C_1 x + C_2$$

Boundary Conditions:

at $x = 0$, $y = 0$ $\therefore C_2 = 0$

at $x = 12'$, $y = 0$ $\therefore C_1 = \frac{120}{EI}$

Thus,

$$\theta = \frac{5}{2EI}(-x^2 + 48)$$

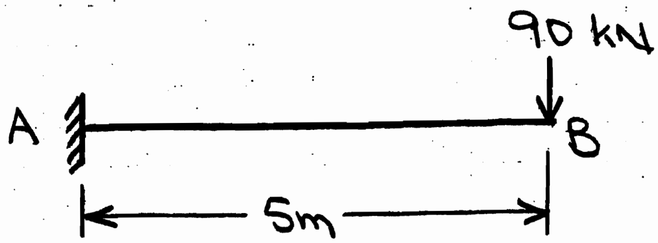
$$y = \frac{5x}{6EI}(-x^2 + 144)$$

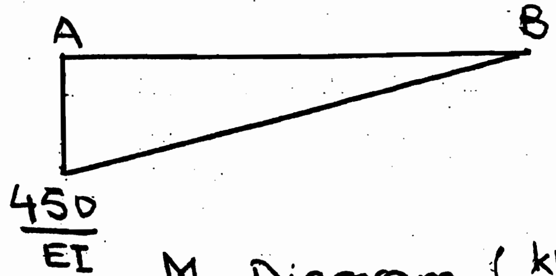
By substituting $x = 6$ ft, $E = 10000(144)$ ksf and $I = \frac{800}{(12)^4}$ ft.^{4}, we obtain

$\theta = 0.00054$ rad ↺

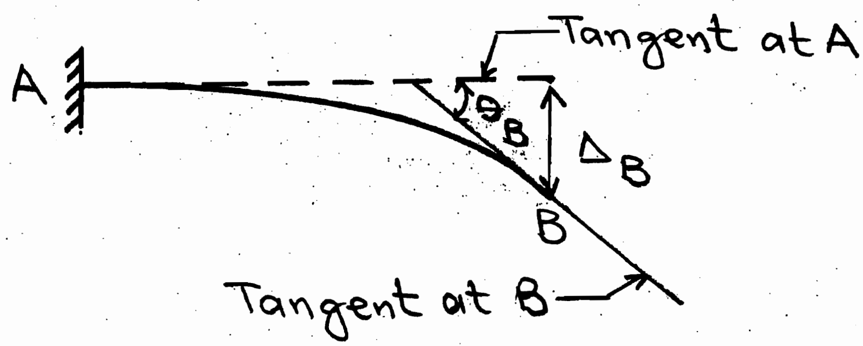
$y = 0.00972$ ft $=$ 0.117 in. ↑

6.9





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



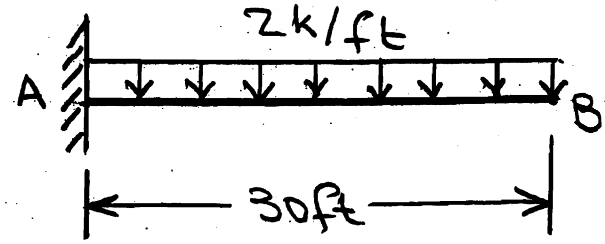
$$\theta_B = \theta_{BA} = \frac{1}{2}(5)\left(\frac{450}{EI}\right) = \frac{1125 \text{ kN.m}^2}{EI}$$

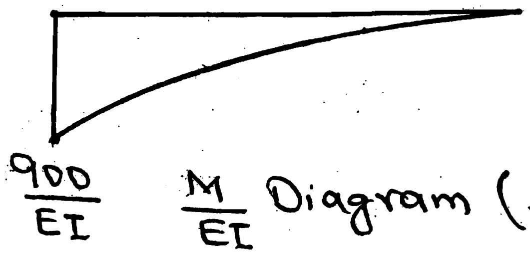
$$= \frac{1125}{200(800)} = \underline{0.00703 \text{ rad}}$$

$$\Delta_B = \Delta_{BA} = \frac{1125}{EI}\left(\frac{10}{3}\right) = \frac{3750 \text{ kN.m}^3}{EI}$$

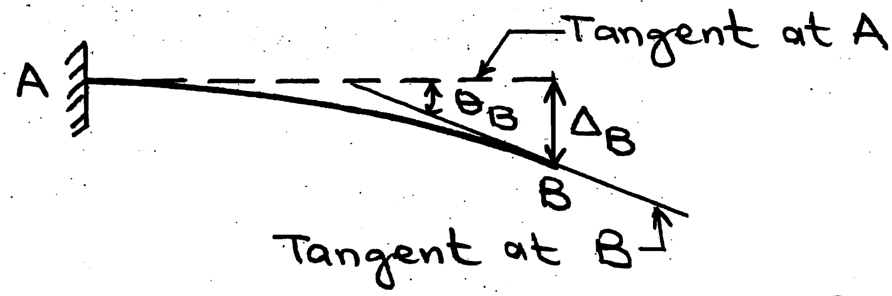
$$= \frac{3750}{200(800)} = 0.0234 \text{m} = \underline{23.4 \text{ mm} \downarrow}$$

6.10





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



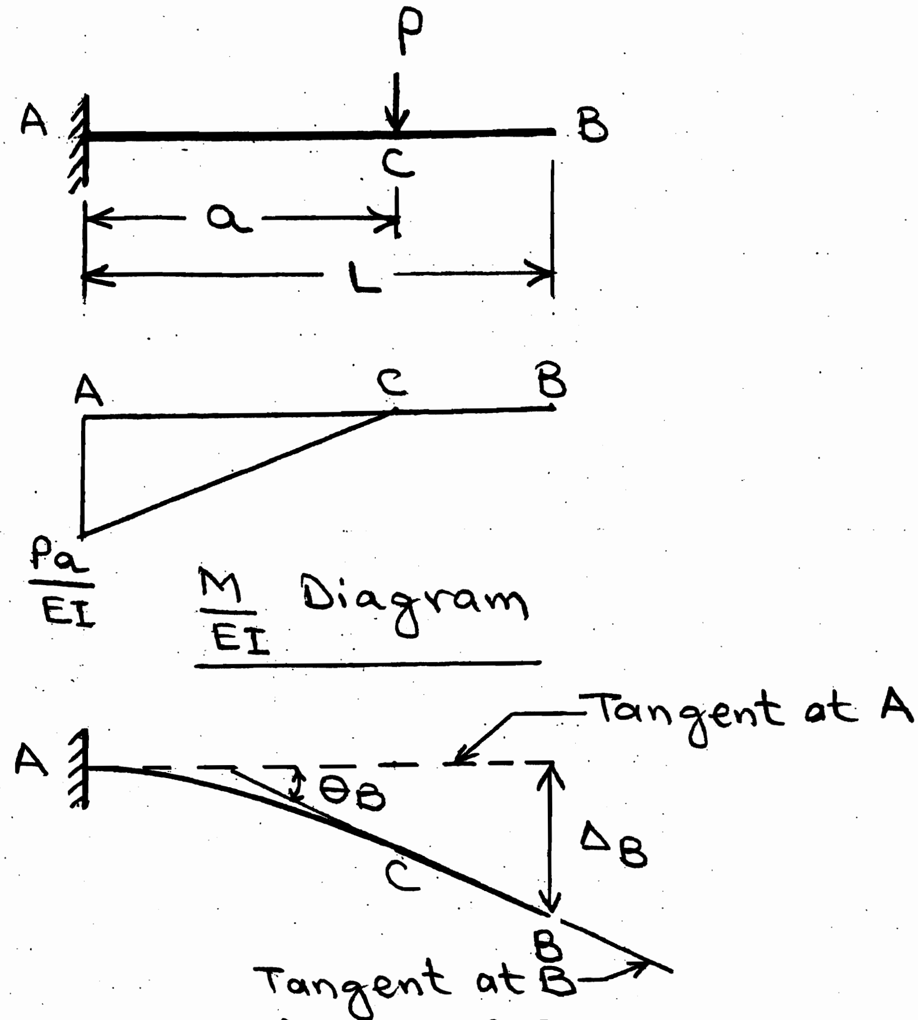
$$\theta_B = \theta_{BA} = \frac{1}{3}(30)\left(\frac{900}{EI}\right) = \frac{9000\ k\text{-}ft^2}{EI}$$

$$= \frac{9000(12)^2}{29000(3000)} = \underline{0.0149 \text{ rad}}$$

$$\Delta_B = \Delta_{BA} = \frac{9000}{EI}\left(\frac{90}{4}\right) = \frac{202500\ k\text{-}ft^3}{EI}$$

$$= \frac{202500(12)^3}{29000(3000)} = \underline{4.022 \text{ in.} \downarrow}$$

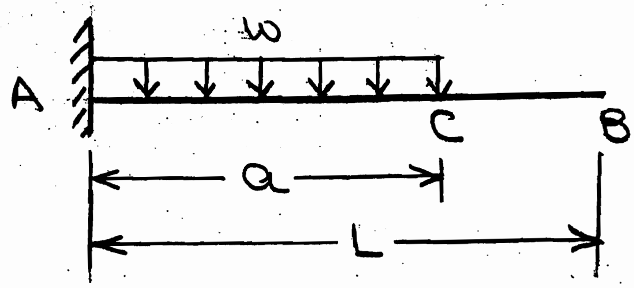
6.11

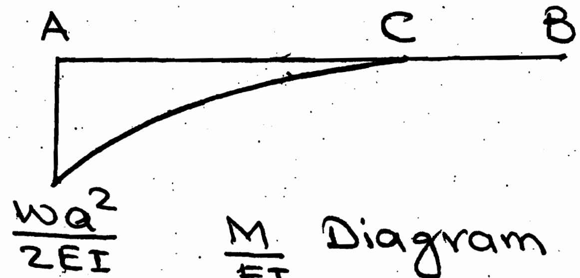


$$\theta_B = \theta_{BA} = \frac{1}{2}(a)\frac{Pa}{EI} = \frac{Pa^2}{2EI}$$

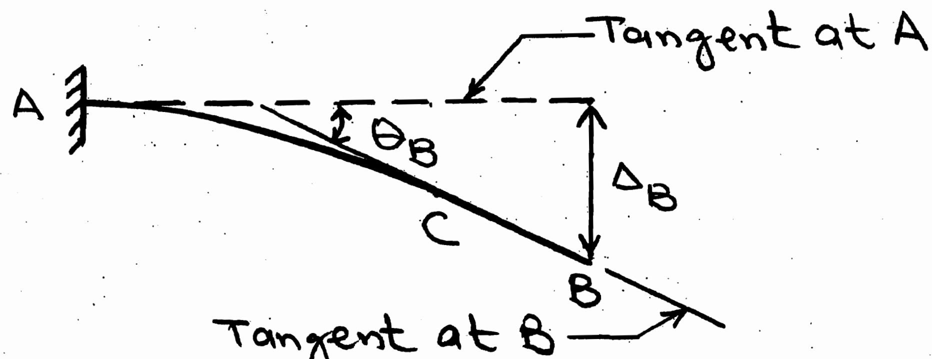
$$\Delta_B = \Delta_{BA} = \frac{Pa^2}{2EI}\left(L - \frac{a}{3}\right) = \frac{Pa^2}{6EI}(3L - a) \downarrow$$

6.12





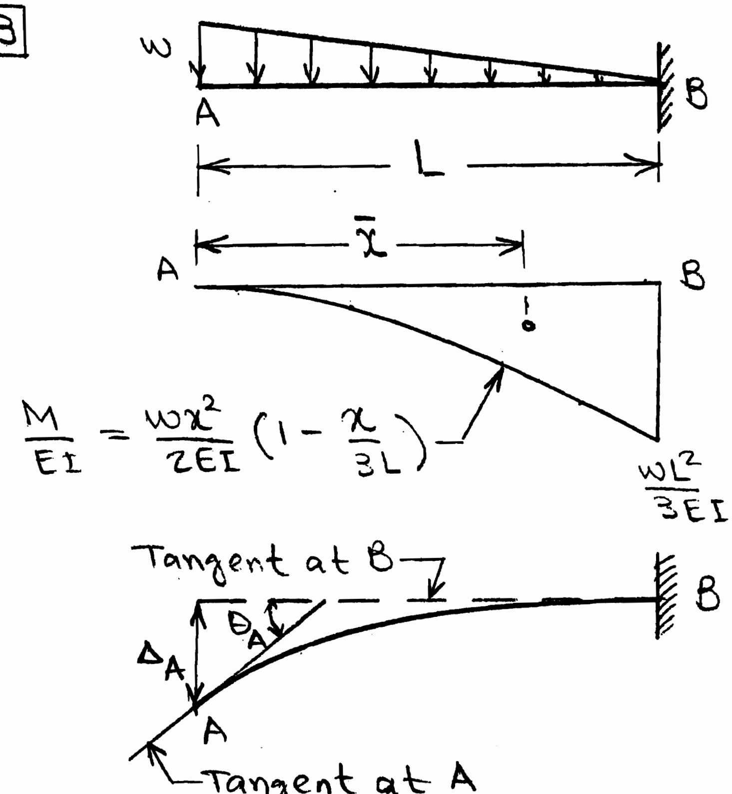
$\frac{M}{EI}$ Diagram



$$\theta_B = \theta_{BA} = \frac{1}{3}(a)\left(\frac{wa^2}{2EI}\right) = \frac{wa^3}{6EI}$$

$$\Delta_B = \Delta_{BA} = \frac{wa^3}{6EI}\left(L - \frac{a}{4}\right) = \frac{wa^3}{24EI}(4L - a) \downarrow$$

6.13

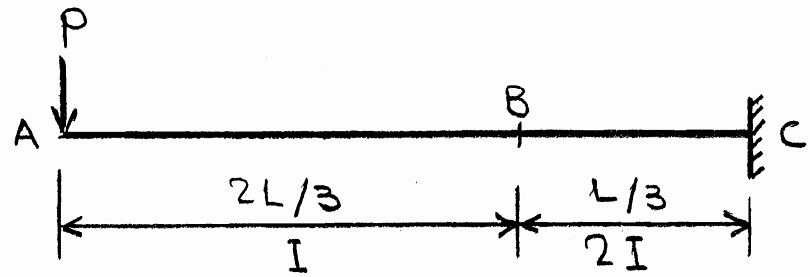


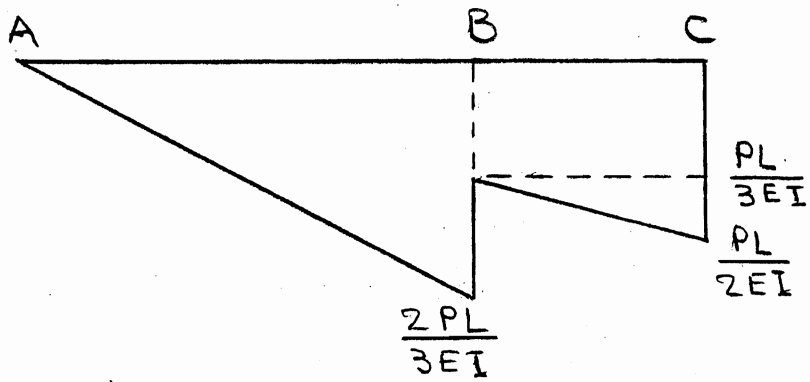
$$\theta_A = \theta_{AB} = \int_0^L \frac{M}{EI}\,dx = \int_0^L \frac{wx^2}{2EI}\left(1 - \frac{x}{3L}\right)dx = \underline{\frac{wL^3}{8EI}}$$

$$\Delta_A = \Delta_{AB} = \int_0^L \frac{M}{EI}(x)\,dx = \int_0^L \frac{wx^2}{2EI}\left(1 - \frac{x}{3L}\right)x\,dx$$

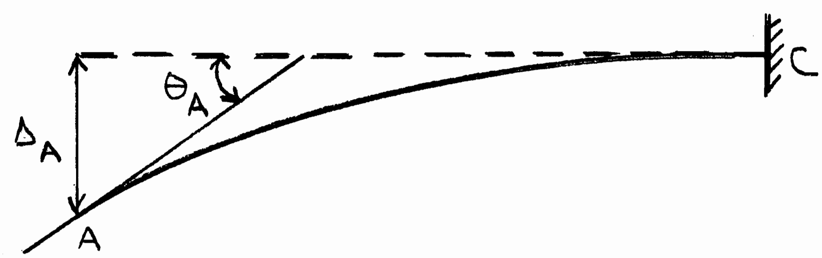
$$= \underline{\frac{11}{120}\frac{wL^4}{EI}} \downarrow$$

6.14





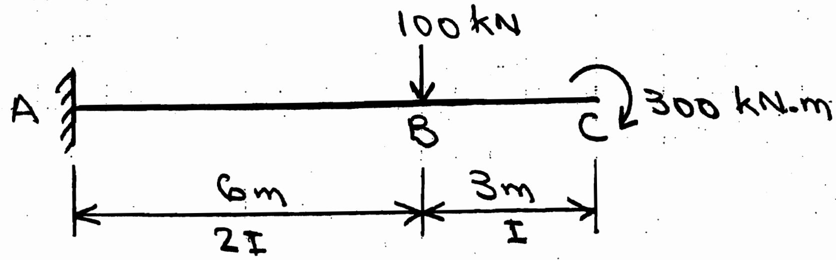
$\frac{M}{EI}$ Diagram



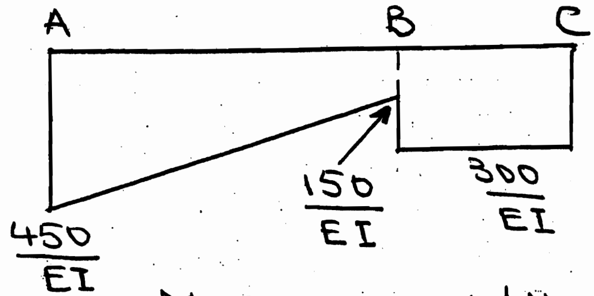
$$\theta_A = \theta_{AC} = \frac{1}{2}\left(\frac{2L}{3}\right)\left(\frac{2PL}{3EI}\right) + \frac{L}{3}\left(\frac{PL}{3EI}\right) + \frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{PL}{6EI}\right) = \frac{13PL^2}{36EI}$$

$$\Delta_A = \Delta_{AC} = \frac{2PL^2}{9EI}\left(\frac{4L}{9}\right) + \frac{PL^2}{9EI}\left(\frac{5L}{6}\right) + \frac{PL^2}{36EI}\left(\frac{8L}{9}\right) = \frac{35PL^3}{162EI} \downarrow$$

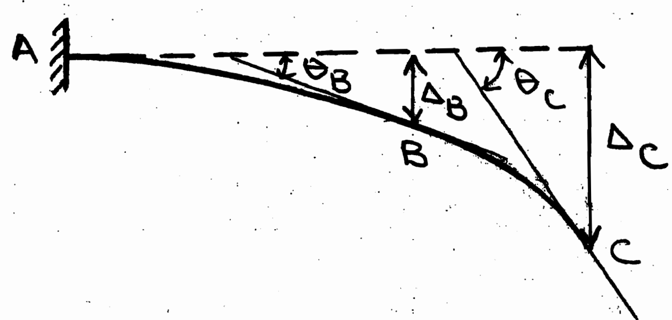
6.15



$I = 500(10^6)\ mm^4$



$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\theta_B = \theta_{BA} = \frac{1}{EI}\left[150(6) + \frac{1}{2}(300)6\right] = \frac{1800\ kN.m^2}{EI}$$

$$= \frac{1800}{70(500)} = \underline{0.0514\ rad}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[150(6)3 + \frac{1}{2}(300)(6)4\right] = \frac{6300\ kN.m^3}{EI}$$

$$= \frac{6300}{70(500)} = 0.18\ m = \underline{180\ mm\downarrow}$$

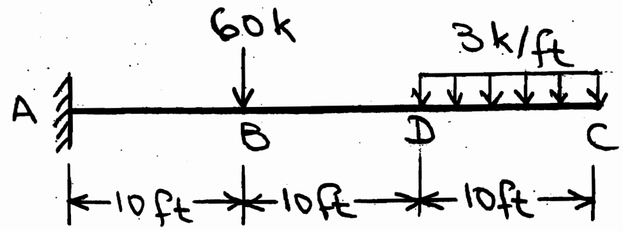
$$\theta_C = \theta_{CA} = \frac{1}{EI}\left[1800 + 300(3)\right] = \frac{2700\ kN.m^2}{EI}$$

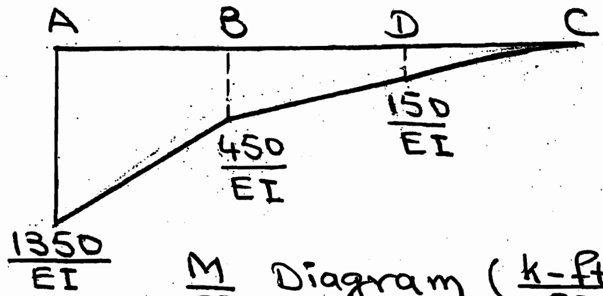
$$= \frac{2700}{70(500)} = \underline{0.0771\ rad}$$

$$\Delta_C = \Delta_{CA} = \frac{1}{EI}\left[150(6)6 + \frac{1}{2}(300)(6)7 + 300(3)(1.5)\right]$$

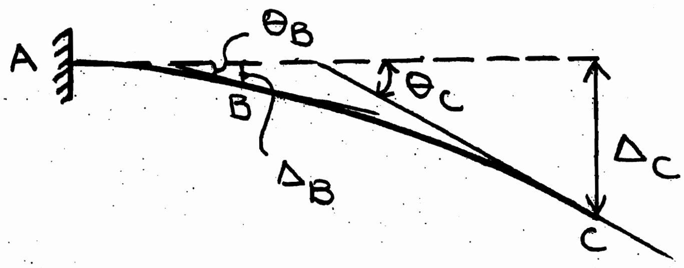
$$= \frac{13050\ kN.m^3}{EI} = \frac{13050}{70(500)} = 0.373\ m = \underline{373\ mm\downarrow}$$

6.16





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\theta_B = \theta_{BA} = \frac{1}{EI}\left[450(10) + \frac{1}{2}(900)10\right] = \frac{9000 \text{ k-ft}^2}{EI}$$

$$= \frac{9000(12)^2}{29000(4000)} = \underline{0.0112 \text{ rad}}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[450(10)5 + \frac{1}{2}(900)(10)\left(\frac{20}{3}\right)\right] = \frac{52500 \text{ k-ft}^3}{EI}$$

$$= \frac{52500(12)^3}{29000(4000)} = \underline{0.782 \text{ in.}\downarrow}$$

$$\theta_C = \theta_{CA} = \frac{1}{EI}\left[9000 + 150(10) + \frac{1}{2}(300)10 + \frac{1}{3}(150)10\right]$$

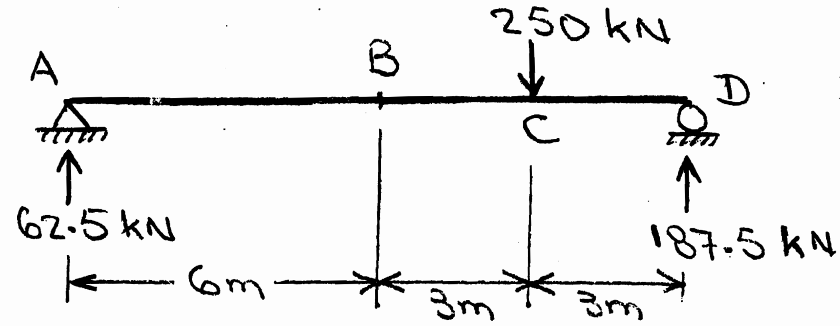
$$= \frac{12500 \text{ k-ft}^2}{EI} = \frac{12500(12)^2}{29000(4000)} = \underline{0.0155 \text{ rad}}$$

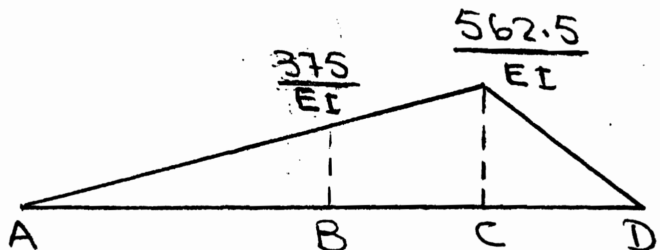
$$\Delta_C = \Delta_{CA} = \frac{1}{EI}\left[450(10)25 + \frac{1}{2}(900)10\left(\frac{20}{3} + 20\right)\right.$$

$$\left. + 150(10)15 + \frac{1}{2}(300)10\left(\frac{20}{3} + 10\right) + \frac{1}{3}(150)10\left(\frac{30}{4}\right)\right]$$

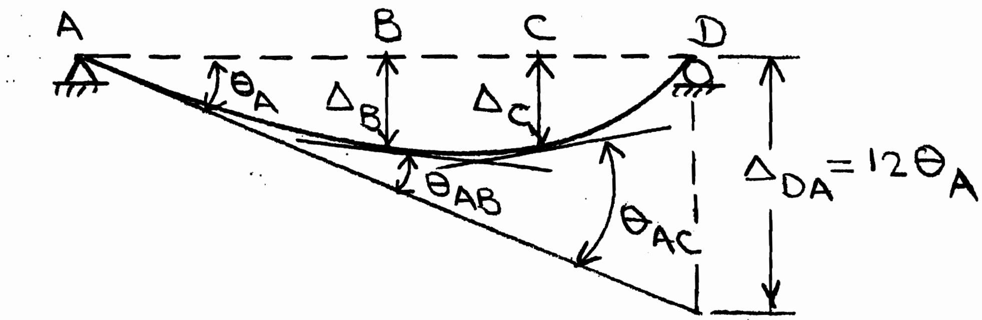
$$= \frac{283750 \text{ k-ft}^3}{EI} = \frac{283750(12)^3}{29000(4000)} = \underline{4.227 \text{ in.}\downarrow}$$

6.17





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(562.5)9(6) + \frac{1}{2}(562.5)3(2)\right] = \frac{16875}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{12} = \frac{1406.25}{EI}$$

$$\theta_B = \theta_A - \theta_{AB} = \frac{1}{EI}\left[1406.25 - \frac{1}{2}(375)6\right]$$
$$= \frac{281.25 \text{ kN.m}^2}{EI} = \frac{281.25}{200(462)} = \underline{0.00304 \text{ rad}}$$

$$\Delta_B = 6\theta_A - \Delta_{BA} = \frac{1}{EI}\left[6(1406.25) - \frac{1}{2}(375)6(2)\right]$$
$$= \frac{6187.5 \text{ kN.m}^3}{EI} = \frac{6187.5}{200(462)} = 0.067 \text{ m}$$

$$\underline{\Delta_B = 67 \text{ mm} \downarrow}$$

6.17 (contd.)

$$\theta_C = \theta_{AC} - \theta_A = \frac{1}{EI}\left[\frac{1}{2}(562.5)9 - 1406.25\right]$$

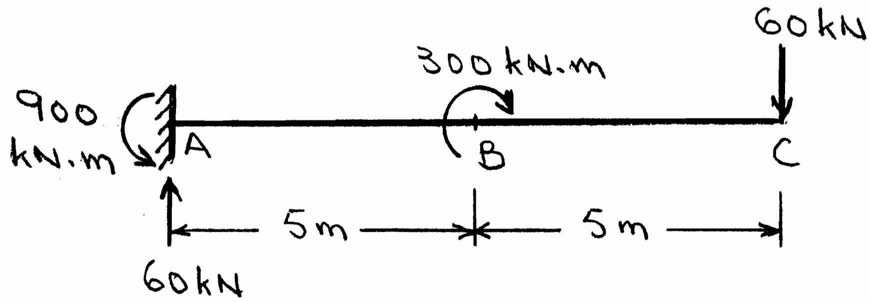
$$= \frac{1125 \text{ kN}\cdot\text{m}^2}{EI} = \frac{1125}{200(462)} = \underline{0.0122 \text{ rad}}$$

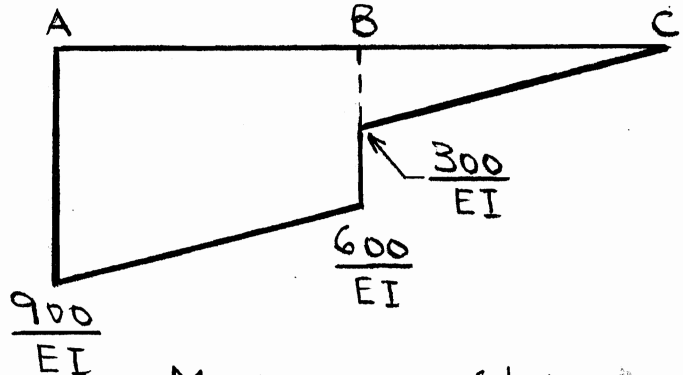
$$\Delta_C = 9\theta_A - \Delta_{CA} = \frac{1}{EI}\left[9(1406.25) - \frac{1}{2}(562.5)9(3)\right]$$

$$= \frac{5062.5 \text{ kN}\cdot\text{m}^3}{EI} = \frac{5062.5}{200(462)} = 0.0548 \text{ m}$$

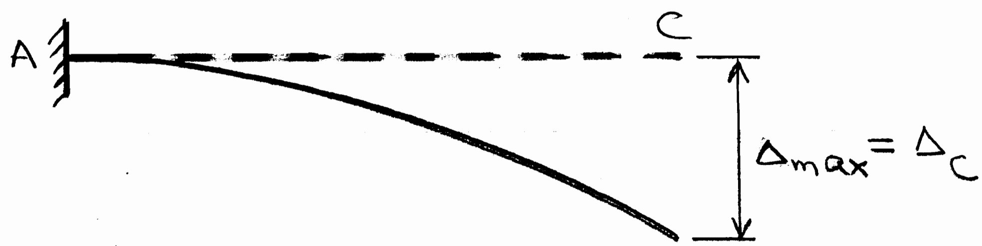
$$\underline{\Delta_C = 54.8 \text{ mm} \downarrow}$$

6.18





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



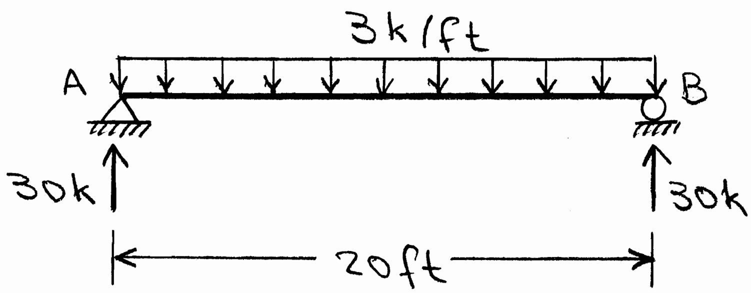
$$\Delta_{max} = \Delta_C = \Delta_{CA} = \frac{1}{EI}\left[600(5)(7.5) + \frac{1}{2}(300)(5)\left(\frac{25}{3}\right) + \frac{1}{2}(300)(5)\left(\frac{10}{3}\right)\right] = \frac{31250\ kN.m^3}{EI}$$

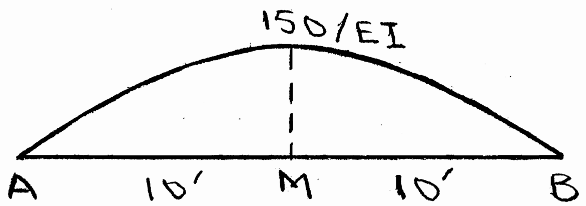
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

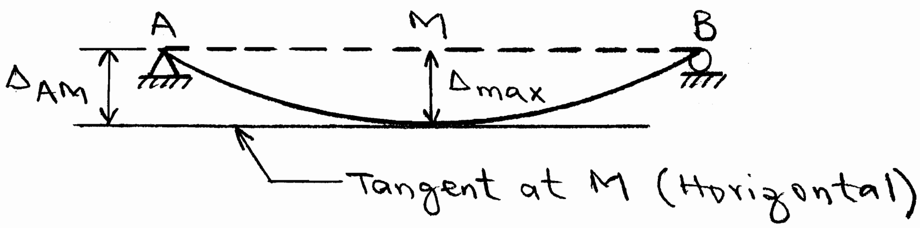
from which, $I = 5625(10^{-6})\ m^4 = \underline{5625(10^6)\ mm^4}$

6.19





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



The maximum deflection occurs at the beam midspan, M, where the tangent is horizontal.

$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{2}{3}(150)(10)\left(\frac{50}{8}\right)\right]$$

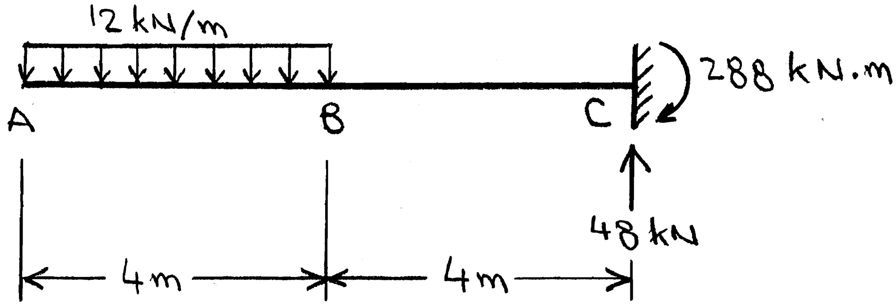
$$= \frac{6250\ k\text{-}ft^3}{EI}$$

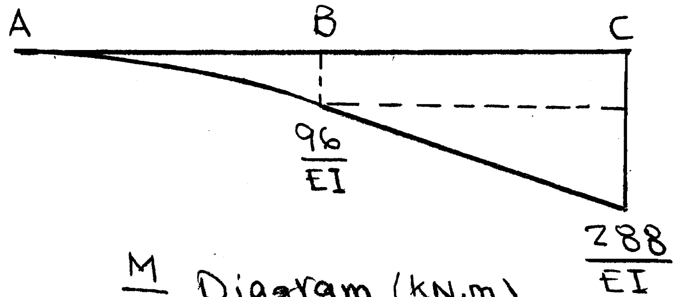
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{6250(12)^3}{29000(I)} = \frac{20(12)}{360}$$

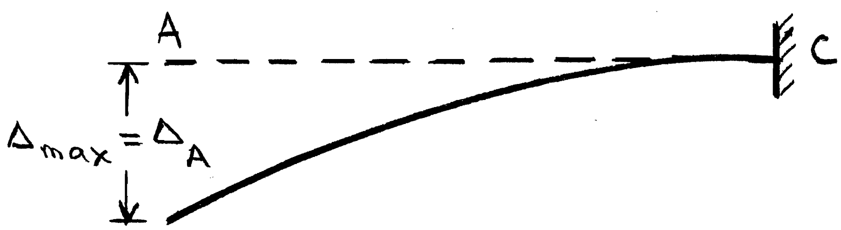
from which, $\underline{I = 559\ in^4}$

6.20





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



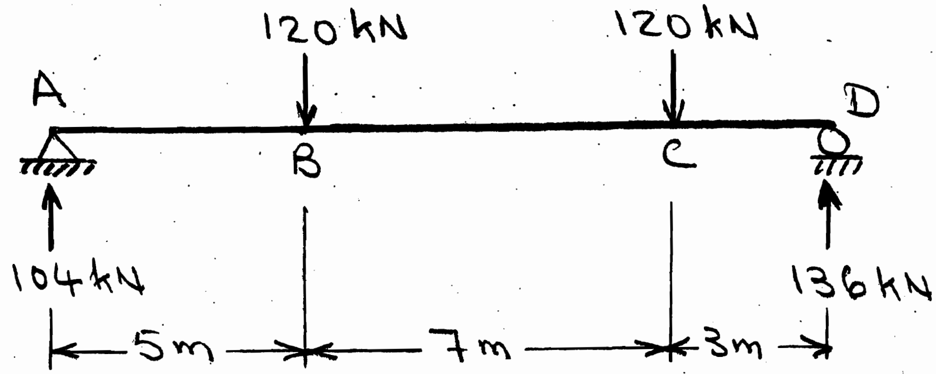
$$\Delta_{max} = \Delta_A = \Delta_{AC} = \frac{1}{EI}\left[\frac{1}{3}(96)(4)(3) + 96(4)(6) + \frac{1}{2}(192)4\left(\frac{8}{3}+4\right)\right] = \frac{5248 \text{ kN.m}^3}{EI}$$

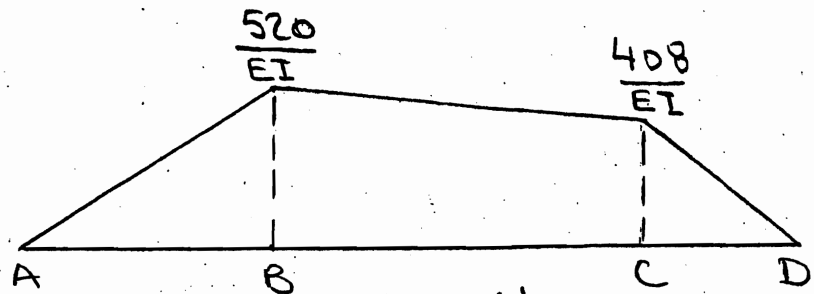
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{5248}{70(10^6)(I)} = \frac{8}{360}$$

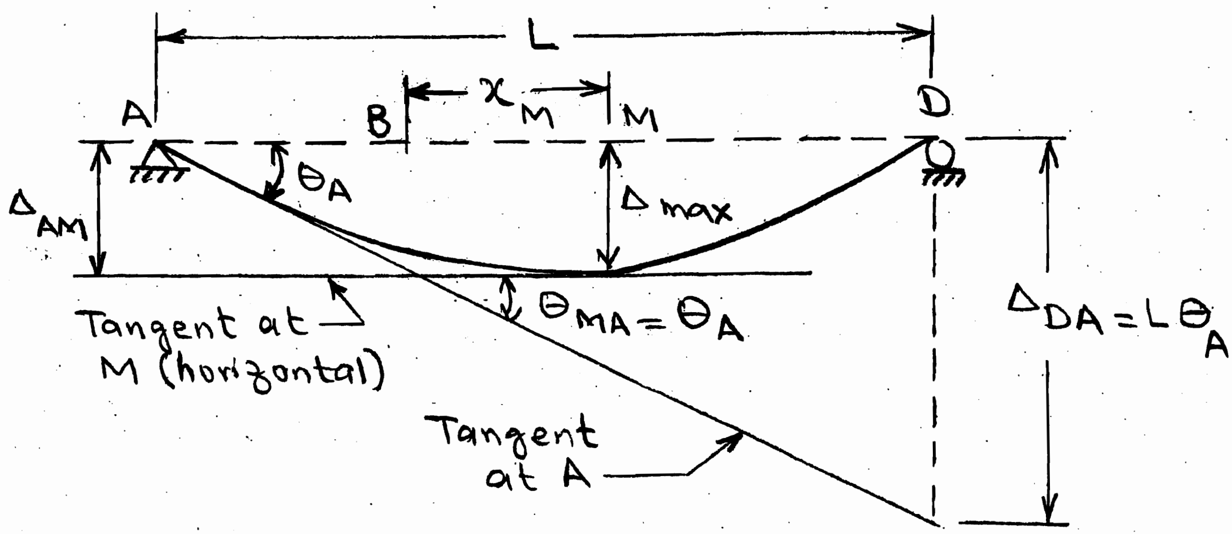
from which, $I = 3374(10^{-6})\text{m}^4 = \underline{3374(10^6)\text{mm}^4}$

6.21





$\frac{M}{EI}$ Diagram $\left(\frac{kN \cdot m}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(520)(5)\left(\frac{5}{3}+10\right) + (408)(7)(6.5) + \frac{1}{2}(112)(7)\left(\frac{14}{3}+3\right) + \frac{1}{2}(408)(3)(2)\right]$$

$$= \frac{37960 \text{ kN}\cdot\text{m}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{L} = \frac{37960}{15(EI)} = \frac{2530.67 \text{ kN}\cdot\text{m}^2}{EI}$$

6.21 (contd.)

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(520)(5) + 520x_M - \frac{1}{2}\left(\frac{112}{7}x_M\right)x_M\right] = \frac{2530.67}{EI}$$

$$8x_M^2 - 520x_M + 1230.67 = 0$$

$$x_M = 2.46 \text{ m}$$

$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(520)(5)\left(\frac{10}{3}\right)\right.$$

$$\left. + 520(2.46)\left(\frac{2.46}{2} + 5\right) - \frac{1}{2}\left(\frac{112}{7}\right)(2.46)(2.46)(6.64)\right]$$

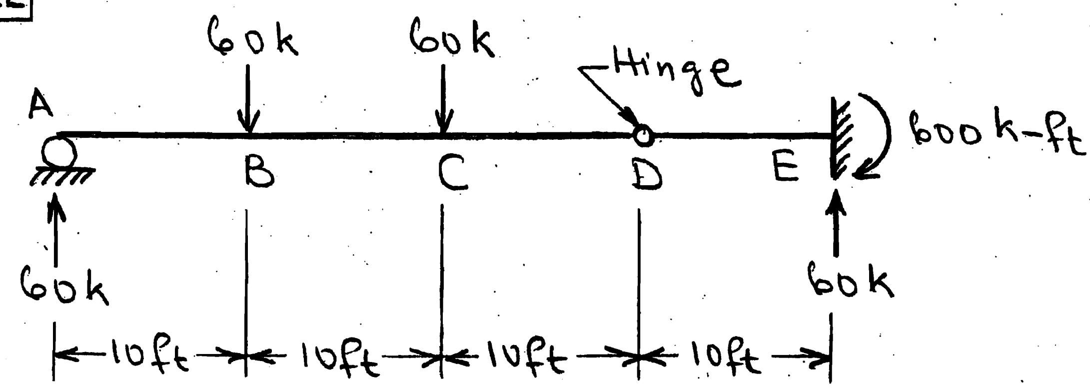
$$= \frac{11981.29 \text{ kN.m3}}{EI}$$

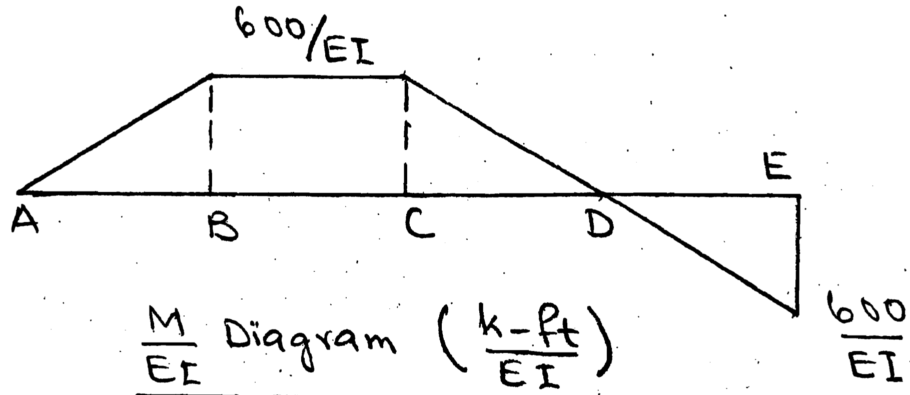
$$\Delta_{max.} = \frac{L}{360}$$

$$\frac{11981.29}{30(10^6)I} = \frac{15}{360}$$

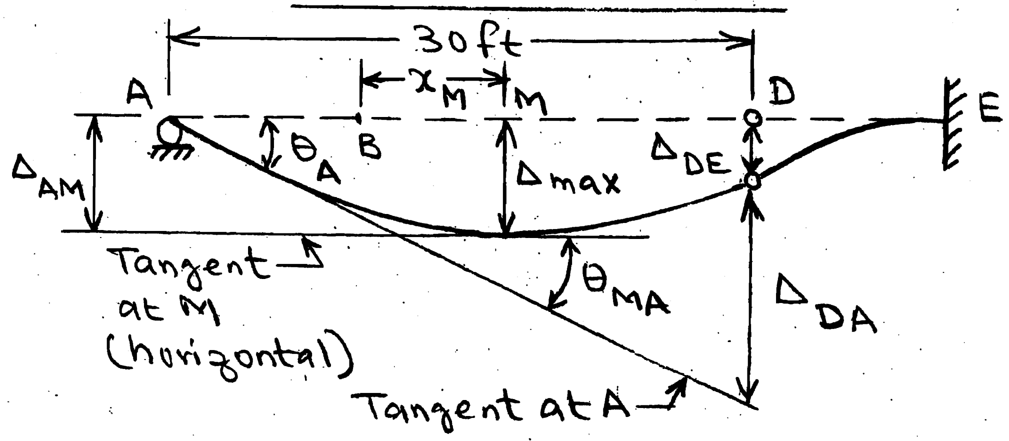
from which $I = 9585(10^{-6})\text{ m}^4 = \underline{9585(10^6)\text{ mm}^4}$

6.22





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(600)(10)\left(\frac{10}{3}+20\right) + 600(10)(15) + \frac{1}{2}(600)(10)\left(\frac{20}{3}\right)\right] = \frac{180000\ k\text{-}ft^3}{EI}$$

$$\Delta_{DE} = \frac{1}{EI}\left[\frac{1}{2}(600)(10)\left(\frac{20}{3}\right)\right] = \frac{20000\ k\text{-}ft^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}+\Delta_{DE}}{30} = \frac{180000+20000}{30} = \frac{6666.67}{EI}$$

6.22 (contd.)

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(600)(10) + 600x_M\right] = \frac{6666.67}{EI}$$

$$x_M = 6.11 \text{ ft}$$

$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(600)(10)\left(\frac{20}{3}\right)\right.$$

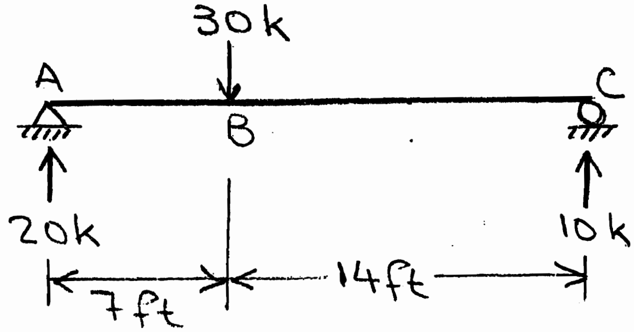
$$\left. + (600)(6.11)(13.06)\right] = \frac{67878 \text{ k-ft}^3}{EI}$$

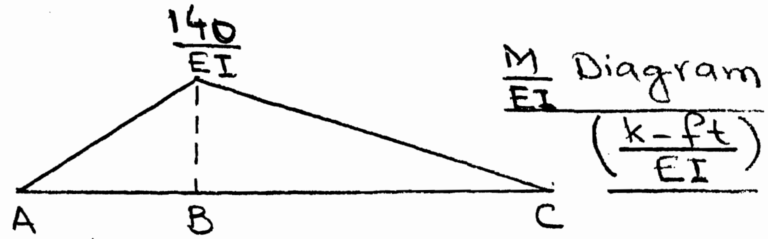
$$\Delta_{max} = \frac{L}{360}$$

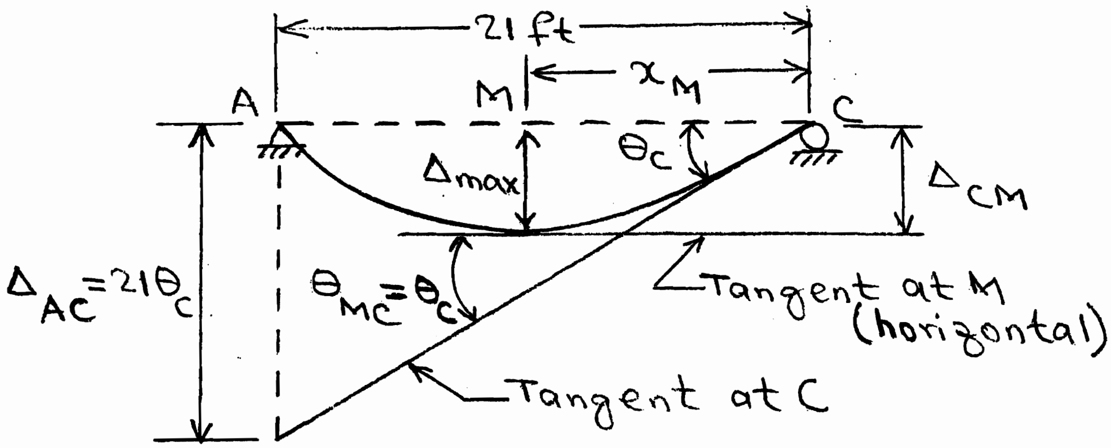
$$\frac{67878(12)^3}{29000(I)} = \frac{40(12)}{360}$$

from which $\underline{I = 3033 \text{ in}^4}$

6.23







$$\Delta_{AC} = \frac{1}{EI}\left[\frac{1}{2}(140)7\left(\frac{14}{3}\right) + \frac{1}{2}(140)14\left(7+\frac{14}{3}\right)\right] = \frac{13720}{EI}$$

$$\theta_C = \frac{\Delta_{AC}}{21} = \frac{653.33}{EI}$$

$$\theta_{MC} = \theta_C$$

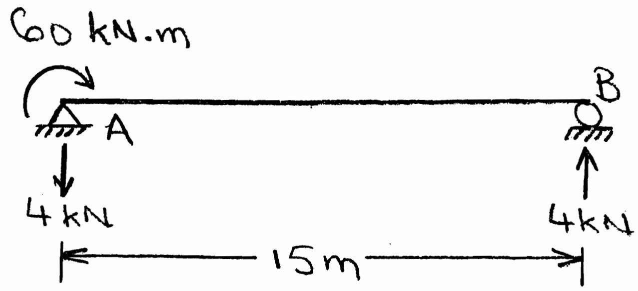
$$\frac{1}{EI}\left[\frac{1}{2}\left(\frac{140}{14}x_M\right)x_M\right] = \frac{653.33}{EI}$$

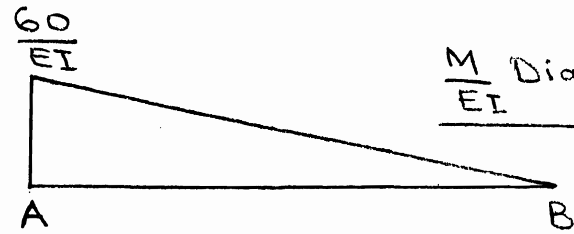
from which, $x_M = 11.43$ ft

$$\Delta_{max} = \Delta_{CM} = \frac{1}{EI}\left[\frac{1}{2}(114.3)(11.43)\frac{2}{3}(11.43)\right]$$

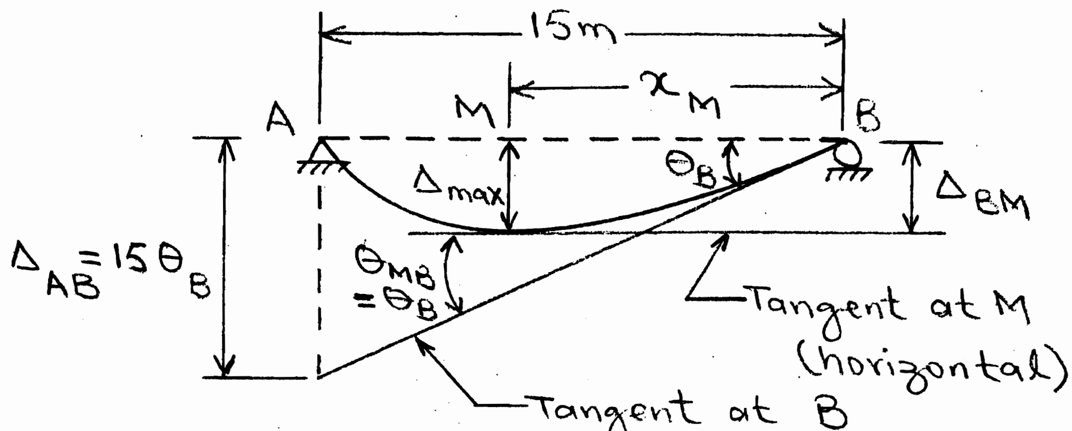
$$= \frac{4978.81 \text{ k-ft}^3}{EI} = \frac{4978.81(12)^3}{10000(500)} = \underline{1.72 \text{ in.}\downarrow}$$

6.24





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{AB} = \frac{1}{EI}\left[\frac{1}{2}(60)15(5)\right] = \frac{2250}{EI}$$

$$\theta_B = \frac{\Delta_{AB}}{15} = \frac{150}{EI}$$

$$\theta_{MB} = \theta_B$$

$$\frac{1}{EI}\left[\frac{1}{2}\left(\frac{60}{15}x_M\right)x_M\right] = \frac{150}{EI}$$

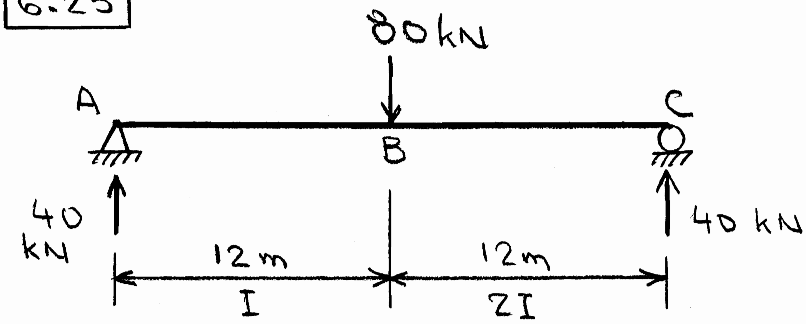
from which $x_M = 8.66$ m

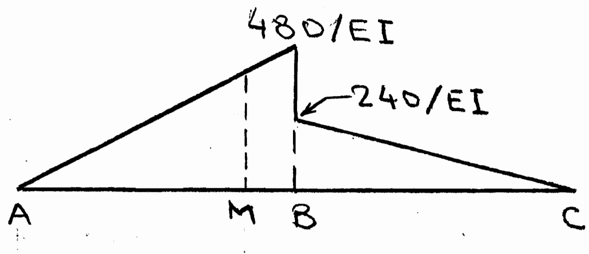
$$\Delta_{max} = \Delta_{BM} = \frac{1}{EI}\left[\frac{1}{2}(34.64)(8.66)\frac{2}{3}(8.66)\right]$$

$$= \frac{866 \text{ kN.m}^3}{EI} = \frac{866}{70(712)} = 0.0174 \text{ m}$$

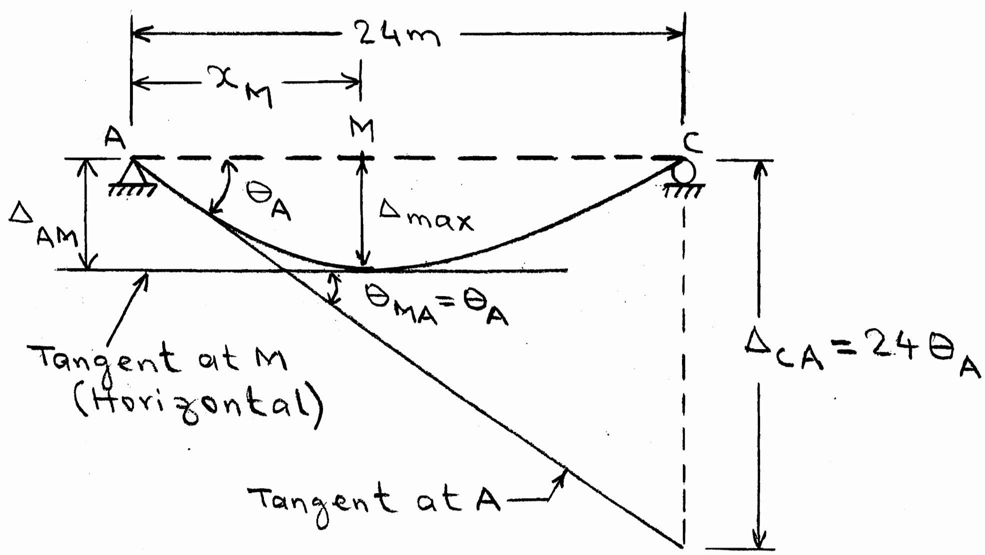
$$\Delta_{max} = 17.4 \text{ mm} \downarrow$$

6.25





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{CA}=\frac{1}{EI}\left[\frac{1}{2}(480)(12)(16)+\frac{1}{2}(240)(12)(8)\right]=\frac{57600 \text{ kN.m}^3}{EI}$$

$$\theta_A=\frac{\Delta_{CA}}{24}=\frac{57600}{24(EI)}=\frac{2400 \text{ kN.m}^2}{EI}$$

If the maximum deflection occurs at M, then

$$\theta_{MA}=\theta_A$$

6.25 (contd.)

$$\frac{1}{EI}\left[\frac{1}{2}(40x_M)x_M\right] = \frac{2400}{EI}$$

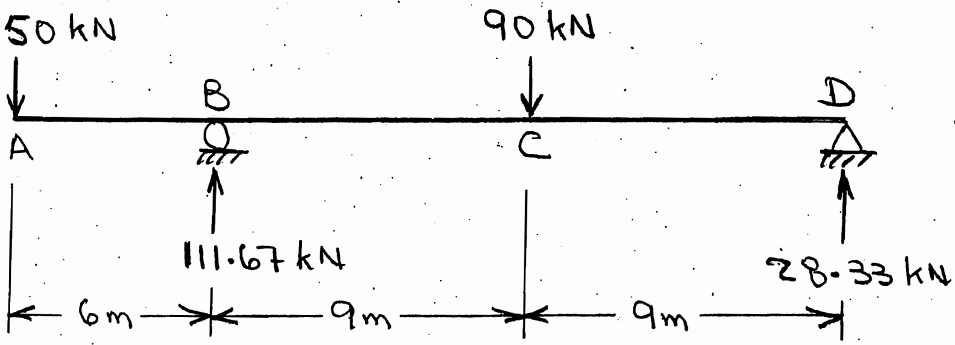
from which, $x_M = 10.95 \text{ m}$

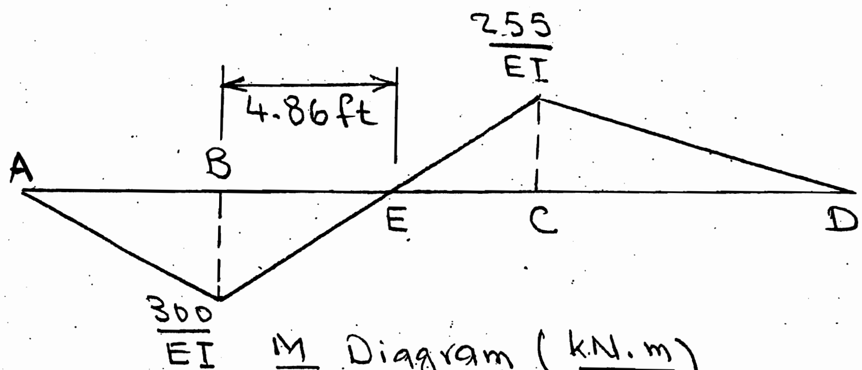
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(438)(10.95)\frac{2}{3}(10.95)\right]$$

$$= \frac{17506 \text{ kN.m}^3}{EI} = \frac{17506}{200(600)} = 0.146 \text{ m}$$

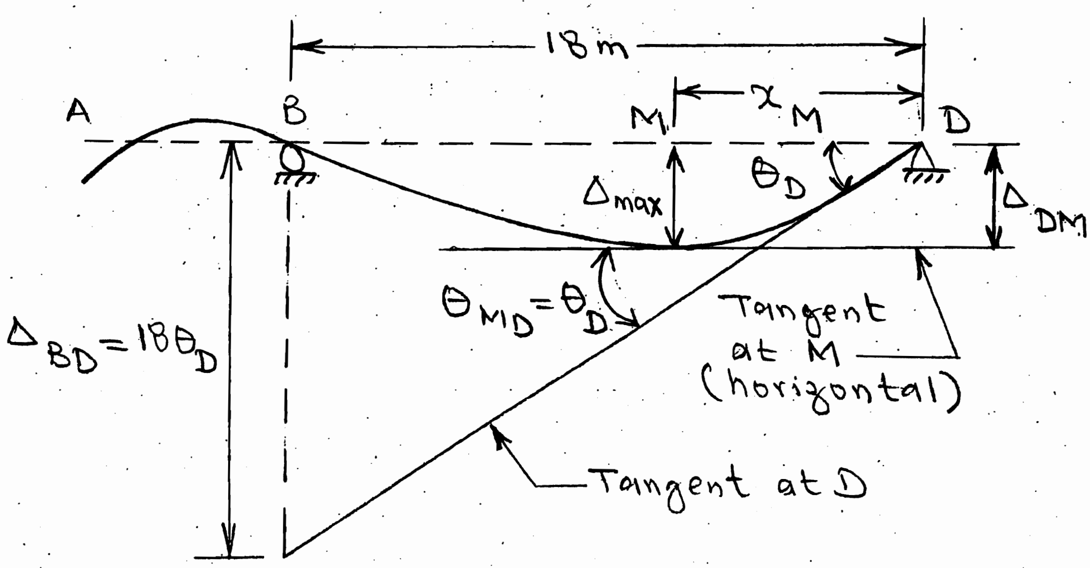
$$\Delta_{max} = 146 \text{ mm} \downarrow$$

6.26





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{BD} = \frac{1}{EI}\left[-\frac{1}{2}(300)(4.86)\left(\frac{4.86}{3}\right) + \frac{1}{2}(255)(4.14)(7.62) + \frac{1}{2}(255)(9)(12)\right] = \frac{16611.24 \text{ kN.m}^3}{EI}$$

$$\theta_D = \frac{\Delta_{BD}}{18} = \frac{16611.24}{18(EI)} = \frac{922.85 \text{ kN.m}^2}{EI}$$

6.26 (contd.) $\theta_{MD} = \theta_D$

$$\frac{1}{EI}\left[\frac{1}{2}\left(\frac{255}{9}x_M\right)x_M\right] = \frac{922.85}{EI}$$

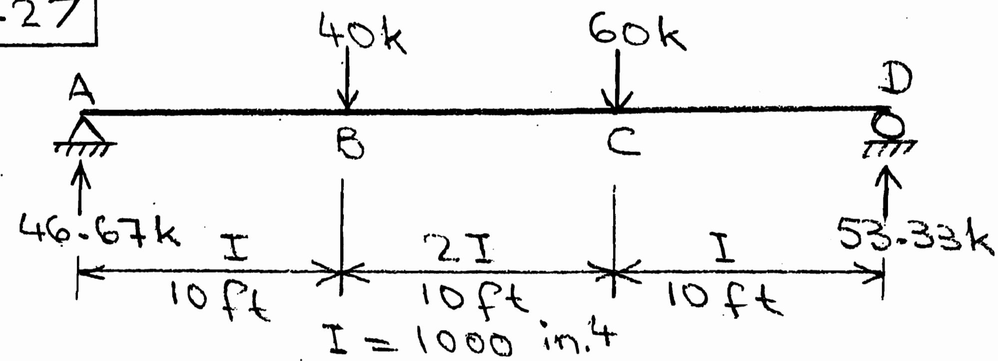
from which, $x_M = 8.07$ m

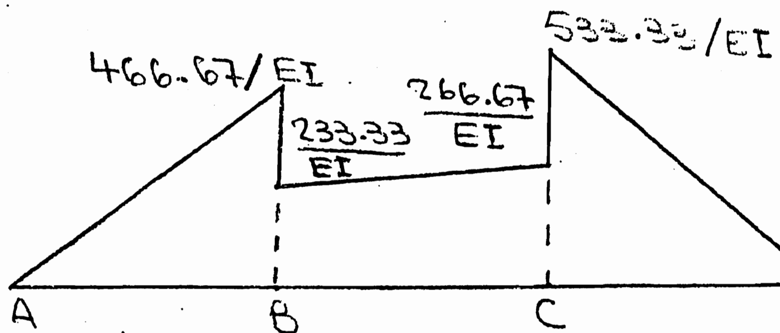
$$\Delta_{max} = \Delta_{DM} = \frac{1}{EI}\left[\frac{1}{2}(228.65)(8.07)\frac{2}{3}(8.07)\right]$$

$$= \frac{4963.6 \text{ kN.m}^3}{EI} = \frac{4963.6}{70(95)} = 0.746 \text{m}$$

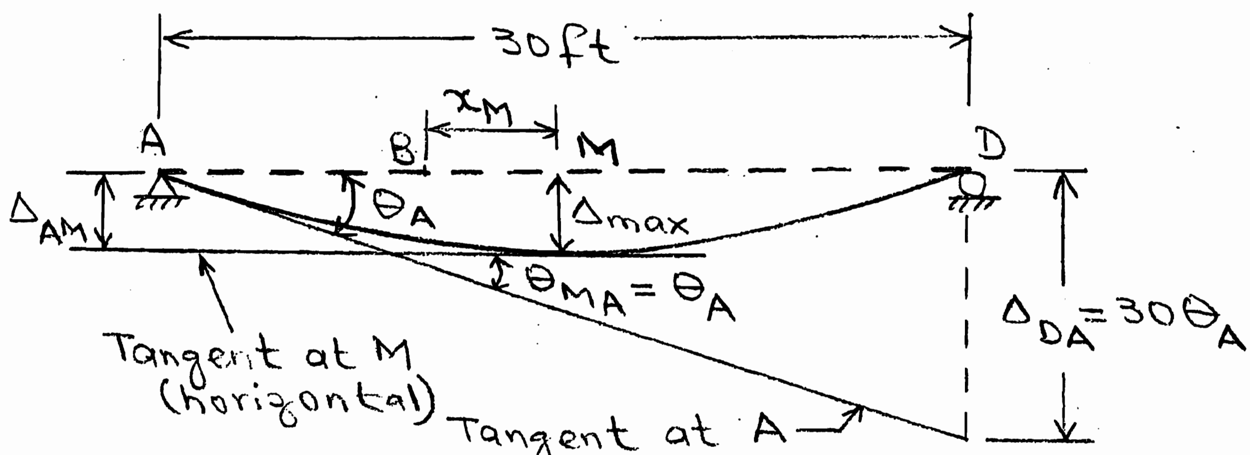
$$\underline{\Delta_{max} = 746 \text{ mm}\downarrow}$$

6.27





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(466.67)10\left(\frac{10}{3}+20\right) + \frac{1}{2}(33.33)10\left(\frac{10}{3}+10\right) + 233.33(10)15 + \frac{1}{2}(533.33)10\left(\frac{20}{3}\right)\right]$$

$$= \frac{109444.4 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{30} = \frac{3648.15 \text{ k-ft}^2}{EI}$$

6.27 (contd.)

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(466.67)10 + 233.33x_M + \frac{1}{2}\left(\frac{33.33}{10}x_M\right)x_M\right] = \frac{3648.15}{EI}$$

$$1.67x_M^2 + 233.33x_M - 1314.8 = 0$$

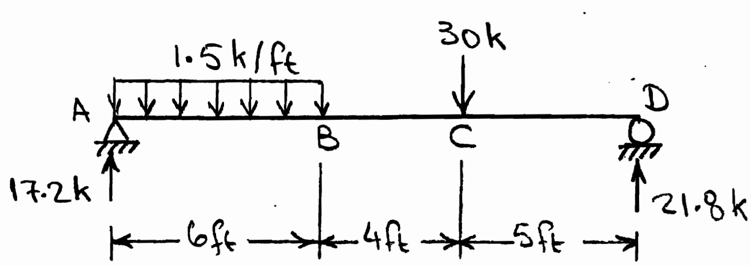
$$x_M = 5.42 \text{ ft}$$

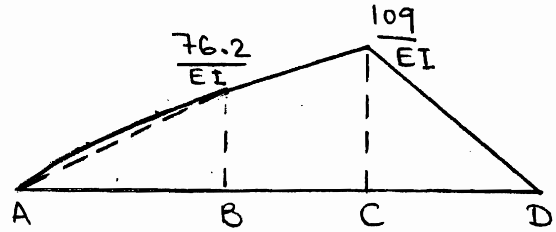
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(466.67)10\left(\frac{20}{3}\right) + 233.33(5.42)\left(\frac{5.42}{2} + 10\right) + \frac{1}{2}(18.06)(5.42)(3.61 + 10)\right]$$

$$= \frac{32296 \text{ k-ft}^3}{EI}$$

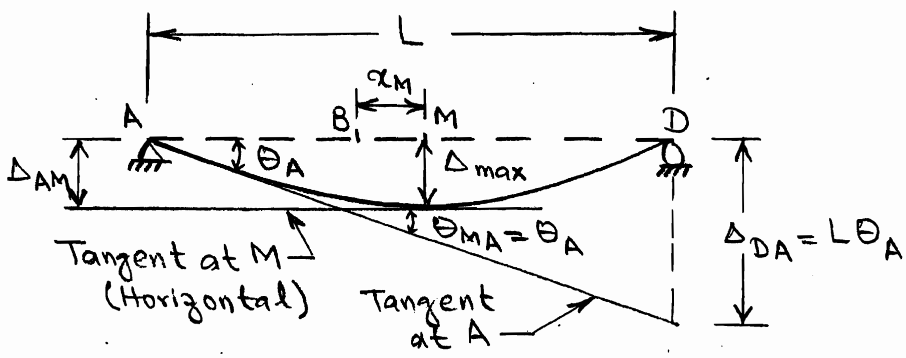
$$= \frac{32296(12)^3}{29000(1000)} = \underline{1.92 \text{ in.}\downarrow}$$

6.28





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(76.2)(6)(11) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6)(12) + (76.2)(4)(7) + \frac{1}{2}(32.8)(4)\left(\frac{4}{3}+5\right) + \frac{1}{2}(109)(5)\left(\frac{10}{3}\right)\right]$$

$$= \frac{6296 \text{ k-ft}^3}{EI}$$

$$\Theta_A = \frac{6296}{15\,EI} = \frac{419.73 \text{ k-ft}^2}{EI}$$

By setting $\Theta_{MA} = \Theta_A$, we write

$$\frac{1}{EI}\left[\frac{1}{2}(76.2)(6) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6) + \frac{1}{2}(152.4 + 8.2x_M)x_M\right] = \frac{419.73}{EI}$$

$$4.1x_M^2 + 76.2x_M - 164.13 = 0$$

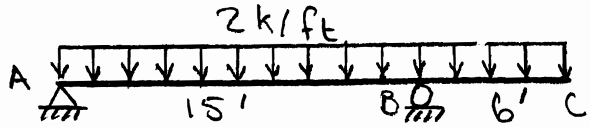
$$x_M = 1.95 \text{ ft}$$

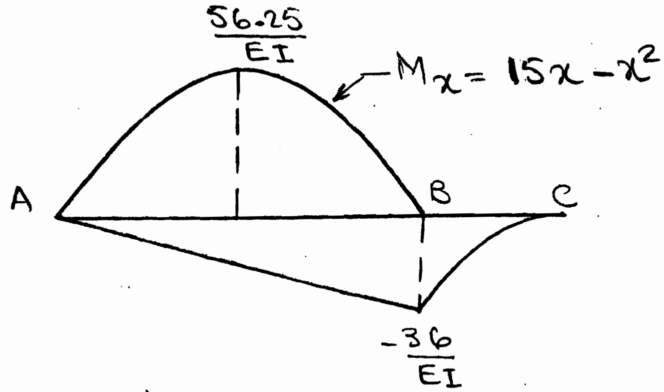
6.28 (contd.)

$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(76.2)(6)(4) + \frac{2}{3}\frac{(1.5)(6)^3}{8}(3) + 76.2(1.95)\left(\frac{1.95}{2}+6\right) + \frac{1}{2}(16)(1.95)(1.3+6)\right]$$

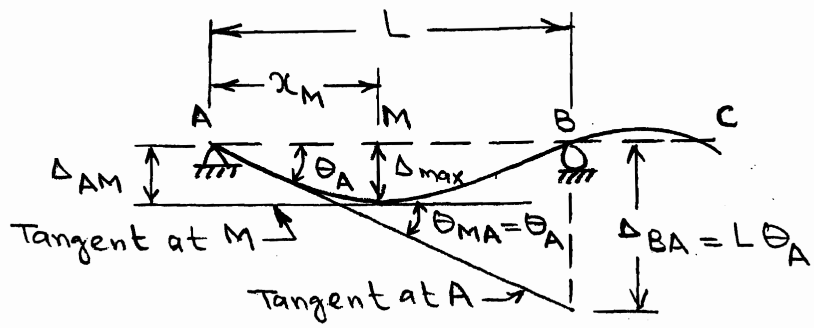
$$= \frac{2145.7 \text{ k-ft}^3}{EI} = \frac{2145.7(12)^3}{(1500)(20000)} = \underline{0.124 \text{ in.}\downarrow}$$

6.29





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{BA} = \frac{1}{EI}\left[\frac{2}{3}(56.25)15(7.5) - \frac{1}{2}(36)(15)5\right]$$

$$= \frac{2868.75 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{2868.75}{15\,EI} = \frac{191.25 \text{ k-ft}^2}{EI}$$

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(15x_M - x_M^2)x_M + \frac{2}{3}\left(\frac{2x_M^2}{8}\right)x_M - \frac{1}{2}(2.4x_M)x_M\right] = \frac{191.25}{EI}$$

$$-\frac{x_M^3}{3} + 6.3x_M^2 = 191.25$$

from which $x_M = 6.92$ ft

6.29 (contd.)

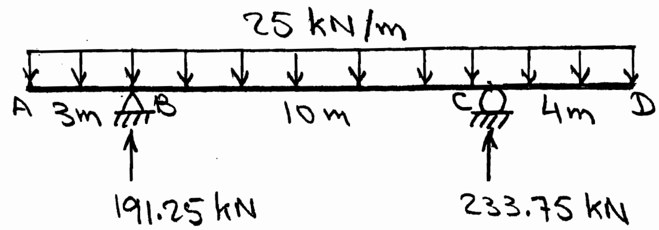
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(55.91)(6.92)(4.61)\right.$$

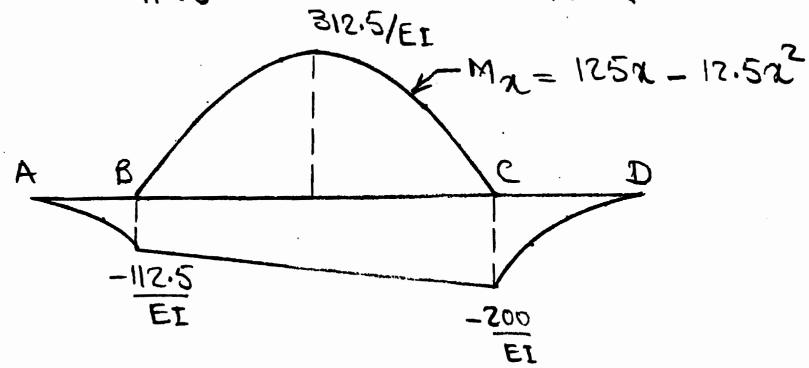
$$\left. + \frac{2}{3}(11.97)\frac{(6.92)^2}{2} - \frac{1}{2}(16.61)(6.92)\frac{2}{3}(6.92)\right]$$

$$= \frac{817.73 \text{ k-ft}^3}{EI} = \frac{817.73(12)^3}{29000(3500)}$$

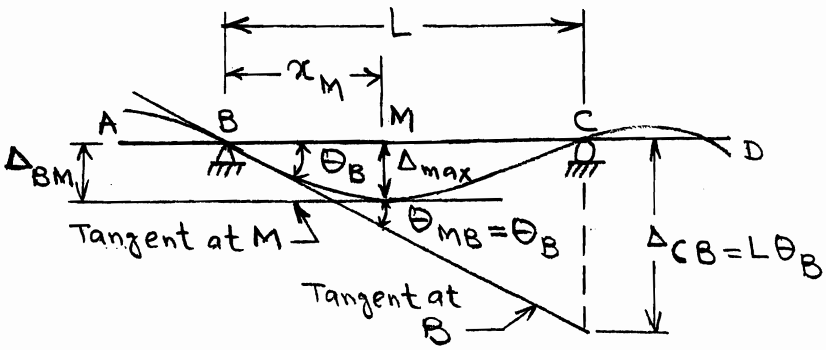
$$= \underline{0.0139 \text{ in.}\downarrow}$$

6.30





$\frac{M}{EI}$ Diagram $\left(\frac{\text{kN-m}}{EI}\right)$



$$\Delta_{CB} = \frac{1}{EI}\left[\frac{2}{3}(312.5)(10)(5) - (112.5)(10)(5) - \frac{1}{2}(87.5)(10)\left(\frac{10}{3}\right)\right]$$

$$= \frac{3333.33 \text{ kN-m}^3}{EI}$$

$$\theta_B = \frac{3333.33}{10EI} = \frac{333.33 \text{ kN-m}^2}{EI}$$

$$\theta_{MB} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(125x_M - 12.5x_M^2)x_M + \frac{2}{3}\left(\frac{25x_M^2}{8}\right)x_M - 112.5x_M - \frac{1}{2}(8.75x_M)x_M\right] = \frac{333.33}{EI}$$

$$-4.167x_M^3 + 58.125x_M^2 - 112.5x_M = 333.33$$

From which $x_M = 4.77$ m

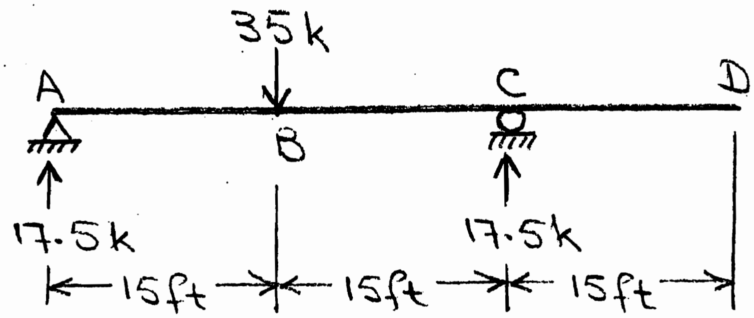
6.30 (Contd.)

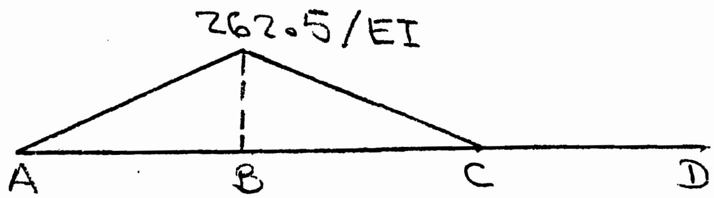
$$\Delta_{max} = \Delta_{BM} = \frac{1}{EI}\left[\frac{1}{2}(311.84)(4.77)(3.18) + \frac{2}{3}(71.1) \times \frac{(4.77)^2}{2} - (112.5)\frac{(4.77)^2}{2} - \frac{1}{2}(41.74)(4.77)(3.18)\right]$$

$$= \frac{1307.91 \text{ kN-m}^3}{EI} = \frac{1307.91}{(200)(500)} = 0.0131 \text{ m}$$

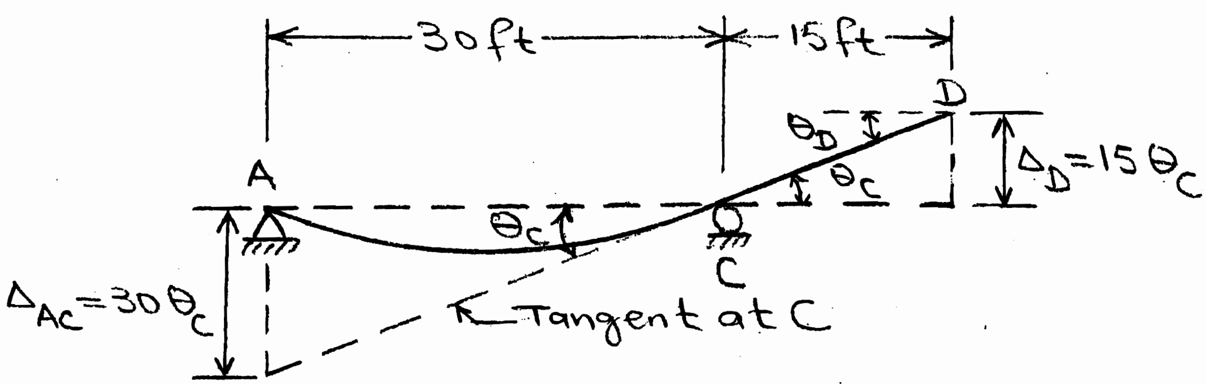
$$\Delta_{max} = 13.1 \text{ mm} \downarrow$$

6.31





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{AC} = \frac{1}{EI}\left[\frac{1}{2}(262.5)30(15)\right] = \frac{59062.5}{EI}$$

$$\theta_C = \frac{\Delta_{AC}}{30} = \frac{1968.75}{EI}$$

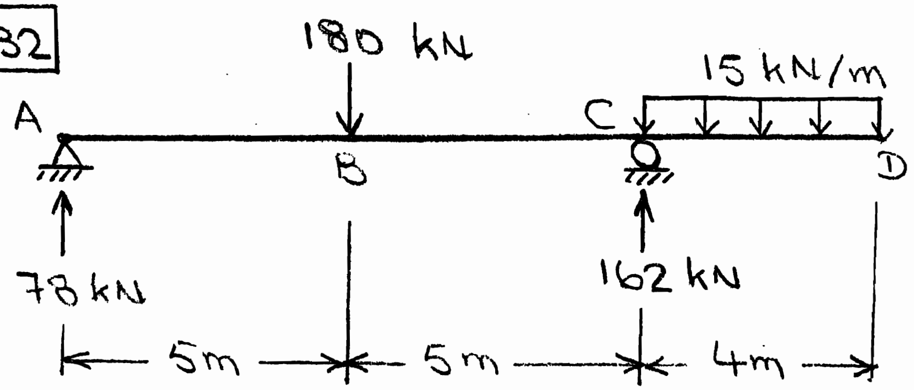
$$\theta_D = \theta_C = \frac{1968.75 \text{ k-ft}^2}{EI} = \frac{1968.75(12)^2}{10000(2500)}$$

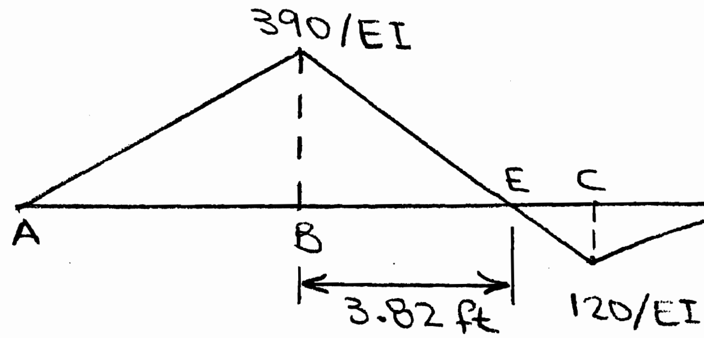
$$\underline{\theta_D = 0.01134 \text{ rad}}$$

$$\Delta_D = 15\theta_C = \frac{29531.25 \text{ k-ft}^3}{EI} = \frac{29531.25(12)^3}{10000(2500)}$$

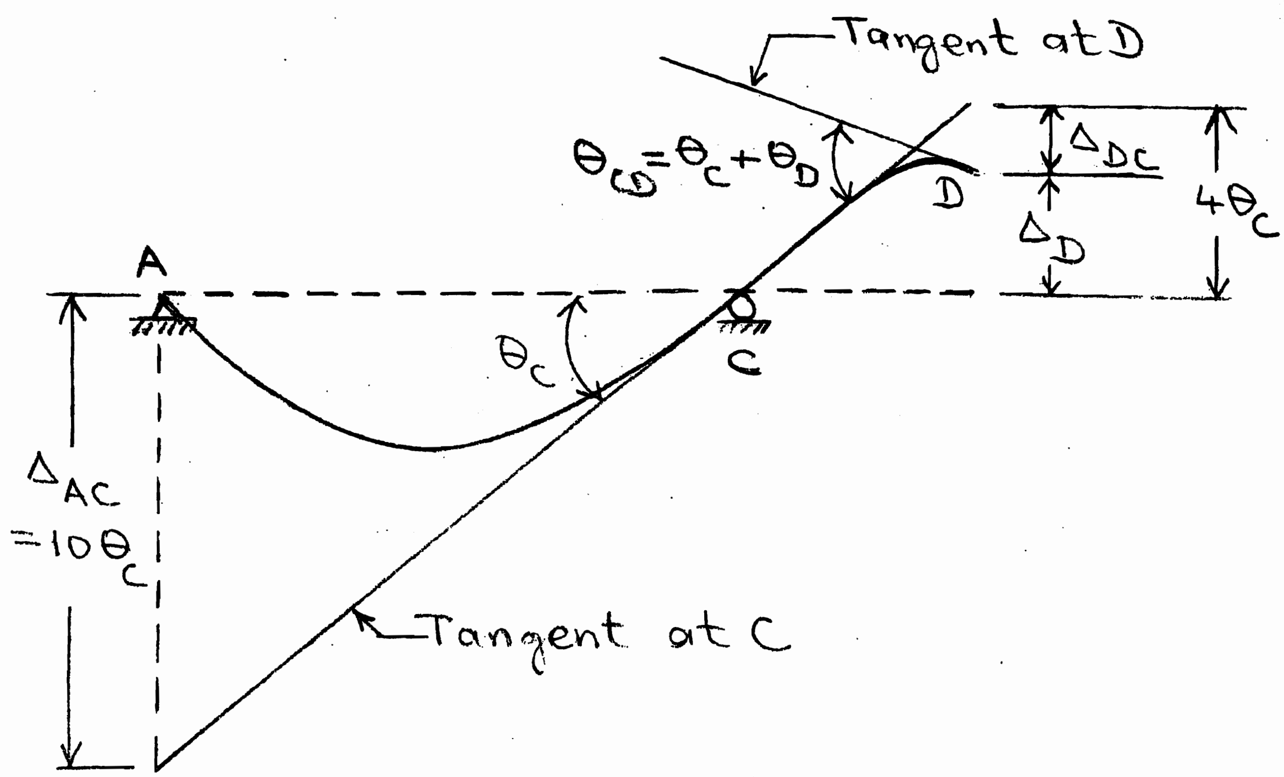
$$\underline{\Delta_D = 2.04 \text{ in.} \uparrow}$$

6.32





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



6-32 (contd.)

$$\Delta_{AC} = \frac{1}{EI}\left[\frac{1}{2}(390)5\left(\frac{10}{3}\right) + \frac{1}{2}(390)3.82\left(\frac{3.82}{3}+5\right) - \frac{1}{2}(120)(1.18)(9.607)\right] = \frac{7243}{EI}$$

$$\theta_C = \frac{\Delta_{AC}}{10} = \frac{724.3}{EI}$$

$$\theta_{CD} = \frac{1}{EI}\left[\frac{1}{3}(120)4\right] = \frac{160}{EI}$$

$$\theta_D = \theta_{CD} - \theta_C = \frac{1}{EI}(160 - 724.3) = -\frac{564.3 \text{ kN.m}^2}{EI}$$

$$= -\frac{564.3}{70(2340)} = -0.00345 \text{ rad}$$

$$\underline{\theta_D = 0.00345 \text{ rad}}$$

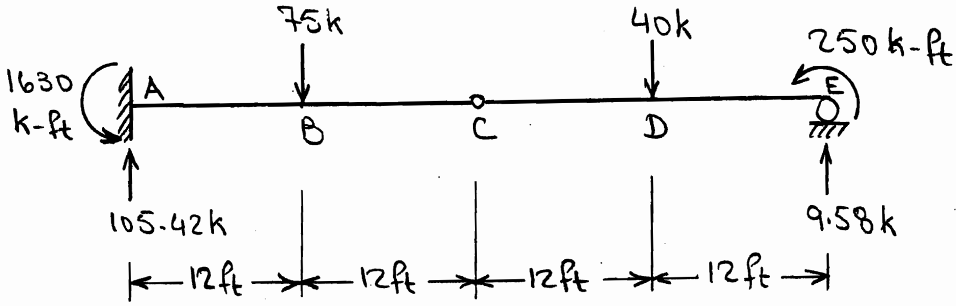
$$\Delta_{DC} = \frac{1}{EI}\left[160(3)\right] = \frac{480}{EI}$$

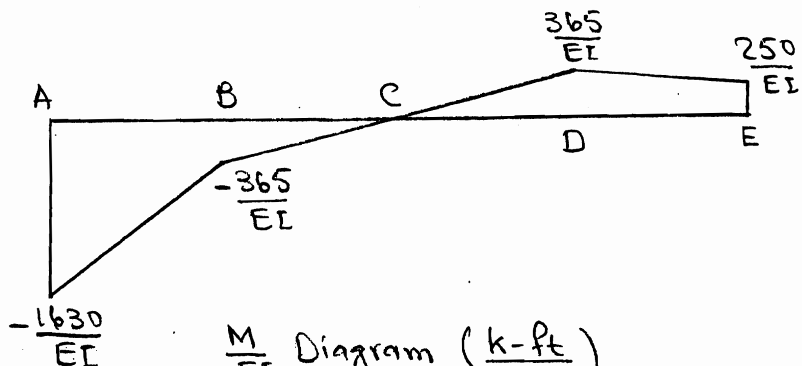
$$\Delta_D = 4\theta_C - \Delta_{DC} = \frac{1}{EI}\left[4(724.3) - 480\right]$$

$$= \frac{2417.2 \text{ kN.m}^3}{EI} = \frac{2417.2}{70(2340)} = 0.01476 \text{ m}$$

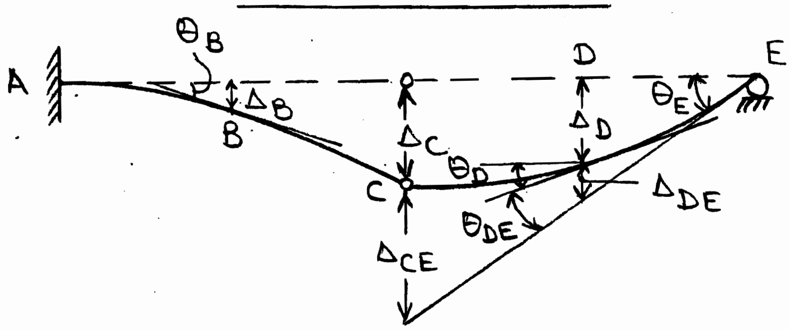
$$\underline{\Delta_D = 14.76 \text{ mm} \uparrow}$$

6.33





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\theta_B = \theta_{BA} = \frac{1}{EI}\left[\left(\frac{1630+365}{2}\right)12\right] = \frac{11970 \text{ k-ft}^2}{EI} = \underline{0.0099 \text{ rad.}}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[(365)12(6) + \frac{1}{2}(1265)(12)(8)\right] = \frac{87000}{EI} = \underline{0.86 \text{ in.}\downarrow}$$

$$\Delta_C = \Delta_{CA} = \frac{1}{EI}\left[365(12)(18) + \frac{1}{2}(1265)(12)(20) + \frac{1}{2}(365) \times (12)8\right] = \frac{248160 \text{ k-ft}^3}{EI}$$

$$\Delta_{CE} = \frac{1}{EI}\left[\frac{1}{2}(365)(12)(8) + (250)(12)(18) + \frac{1}{2}(115)(12)(16)\right] = \frac{82560 \text{ k-ft}^3}{EI}$$

$$\theta_E = \frac{\Delta_C + \Delta_{CE}}{24} = \frac{248160 + 82560}{24EI} = \frac{13780 \text{ k-ft}^2}{EI}$$

6.33 (contd.)

$$\theta_{DE} = \frac{1}{EI}\left[\left(\frac{365+250}{2}\right)12\right] = \frac{3690 \text{ k-ft}^2}{EI}$$

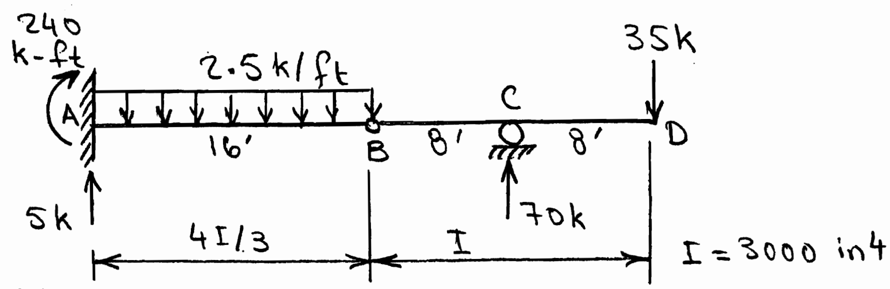
$$\theta_D = \theta_E - \theta_{DE} = \frac{13780 - 3690}{EI} = \frac{10090 \text{ k-ft}^2}{EI} = \underline{0.0084 \text{ rad.} \circlearrowright}$$

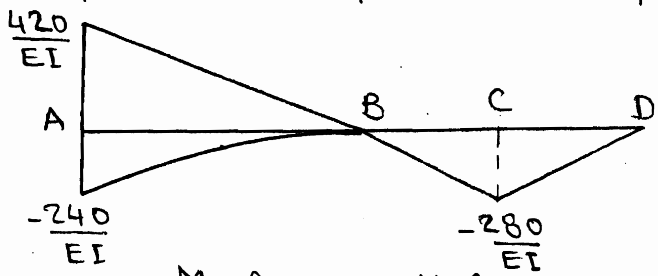
$$\Delta_{DE} = \frac{1}{EI}\left[(250)(12)(6) + \frac{1}{2}(115)(12)(4)\right] = \frac{20760 \text{ k-ft}^3}{EI}$$

$$\Delta_D = 12\theta_E - \Delta_{DE} = \frac{1}{EI}\left[12(13780) - 20760\right] = \frac{144600 \text{ k-ft}^3}{EI}$$

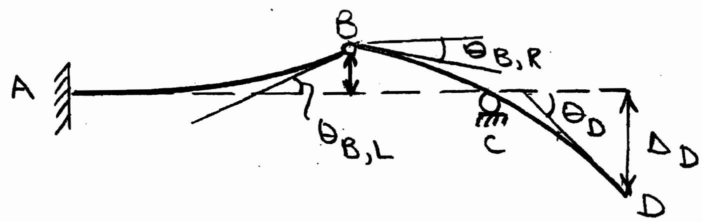
$$\underline{\Delta_D = 1.44 \text{ in.} \downarrow}$$

6.34





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\theta_{B,Left} = \theta_{AB} = \frac{1}{EI}\left[\frac{1}{2}(420)16 - \frac{1}{3}(240)16\right]$$

$$= \frac{2080\ k\text{-}ft^2}{EI} = \frac{2080\,(12)^2}{30000\,(3000)} = \underline{0.0033\ rad.}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[\frac{1}{2}(420)16\,(10.67) - \frac{1}{3}(240)16\,(12)\right]$$

$$= \frac{20480\ k\text{-}ft^3}{EI} = \frac{20480\,(12)^3}{30000\,(3000)} = \underline{0.39\ in \uparrow}$$

$$\Delta_{BC} = \frac{1}{EI}\left[\frac{1}{2}(280)8\left(\frac{16}{3}\right)\right] = \frac{5973.33}{EI}$$

$$\Delta_{BC} + \Delta_B = 8\,\theta_C$$

$$\theta_C = \frac{1}{8EI}(5973.33 + 20480) = \frac{3306.67}{EI}$$

$$\theta_{BC} = \frac{1}{EI}\left[\frac{1}{2}(280)8\right] = \frac{1120}{EI}$$

$$\theta_{B,Right} = \theta_C - \theta_{BC} = \frac{1}{EI}(3306.67 - 1120)$$

$$= \frac{2186.67\ k\text{-}ft^2}{EI} = \underline{0.0035\ rad.}$$

6.34 (contd.)

$$\theta_{CD} = \frac{1}{EI}\left[\frac{1}{2}(280)8\right] = \frac{1120}{EI}$$

$$\theta_D = \theta_C + \theta_{CD} = \frac{1}{EI}(3306.67 + 1120)$$

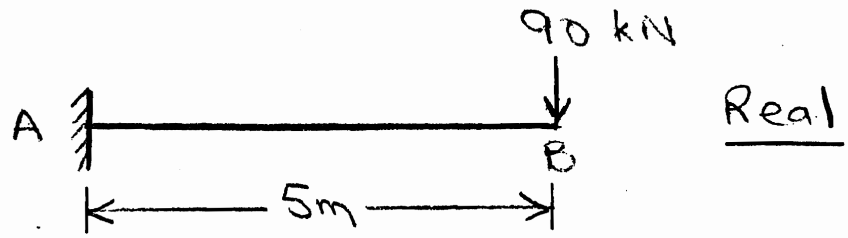
$$= \frac{4426.67\ \text{k-ft}^2}{EI} = \underline{0.0071\ \text{rad.}}$$

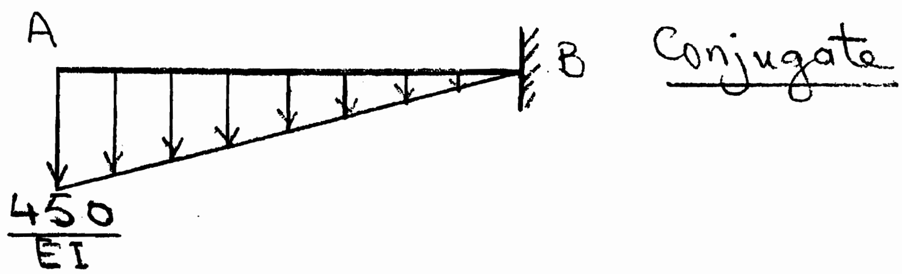
$$\Delta_{DC} = \frac{1}{EI}\left[1120\left(\frac{16}{3}\right)\right] = \frac{5973.33}{EI}$$

$$\Delta_D = 8\theta_C + \Delta_{DC} = \frac{1}{EI}\left[8(3306.67) + 5973.33\right]$$

$$= \frac{32426.67\ \text{k-ft}^3}{EI} = \underline{0.62\ \text{in} \downarrow}$$

6.35





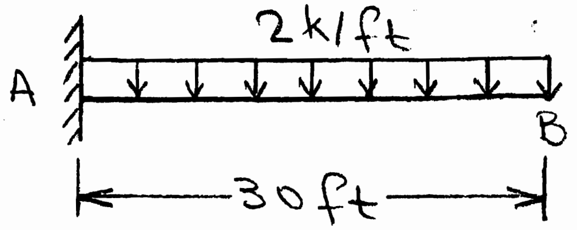
$$\theta_B = -\frac{1}{2}(5)\left(\frac{450}{EI}\right) = -\frac{1125\ \text{kN.m}^2}{EI} = -\frac{1125}{200(800)}$$

$$= -0.00703\ \text{rad} = \underline{0.00703\ \text{rad}}$$

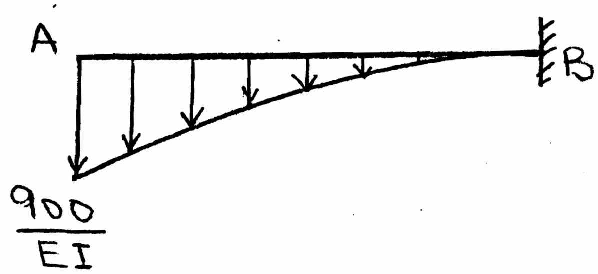
$$\Delta_B = -\frac{1}{2}(5)\left(\frac{450}{EI}\right)\left(\frac{10}{3}\right) = -\frac{3750\ \text{kN.m}^3}{EI}$$

$$= -\frac{3750}{200(800)} = -0.0234\text{m} = \underline{23.4\ \text{mm}\ \downarrow}$$

6.36



Real



Conjugate

$$\theta_B = -\frac{1}{3}\left(\frac{900}{EI}\right)30 = -\frac{9000 \text{ k-ft}^2}{EI}$$

$$= -\frac{9000(12)^2}{29000(3000)} = -0.0149 \text{ rad}$$

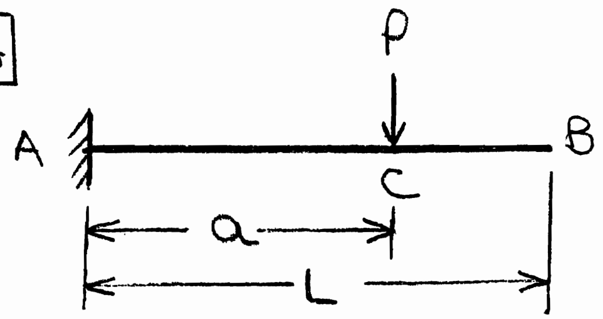
$$\underline{\theta_B = 0.0149 \text{ rad} \curvearrowright}$$

$$\Delta_B = -\frac{9000}{EI}\left(\frac{3}{4}\right)30 = -\frac{202500 \text{ k-ft}^3}{EI}$$

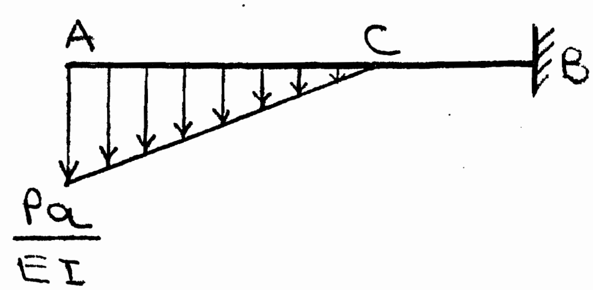
$$= -\frac{202500(12)^3}{29000(3000)} = -4.022 \text{ in.}$$

$$\underline{\Delta_B = 4.022 \text{ in.} \downarrow}$$

6.37



Real



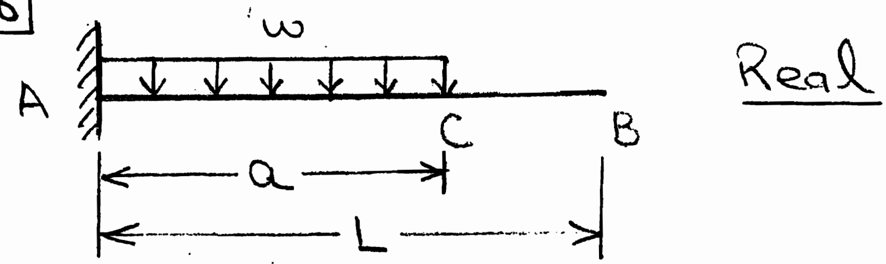
Conjugate

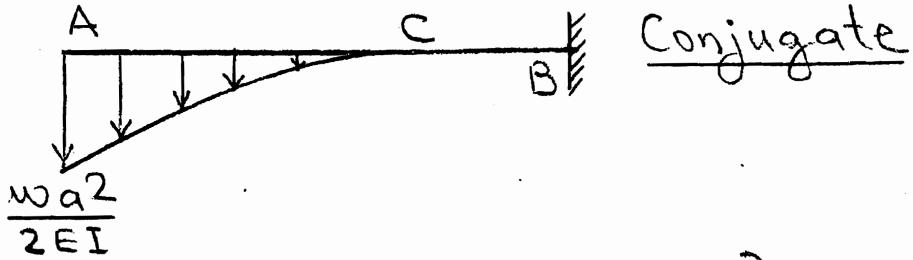
$$\theta_B = -\frac{1}{2}\left(\frac{Pa}{EI}\right)a = -\frac{Pa^2}{2EI} = \frac{Pa^2}{EI} \curvearrowright$$

$$\Delta_B = -\frac{Pa^2}{2EI}\left(L - \frac{a}{3}\right) = -\frac{Pa^2}{6EI}(3L-a)$$

$$= \frac{Pa^2}{6EI}(3L-a) \downarrow$$

6.38



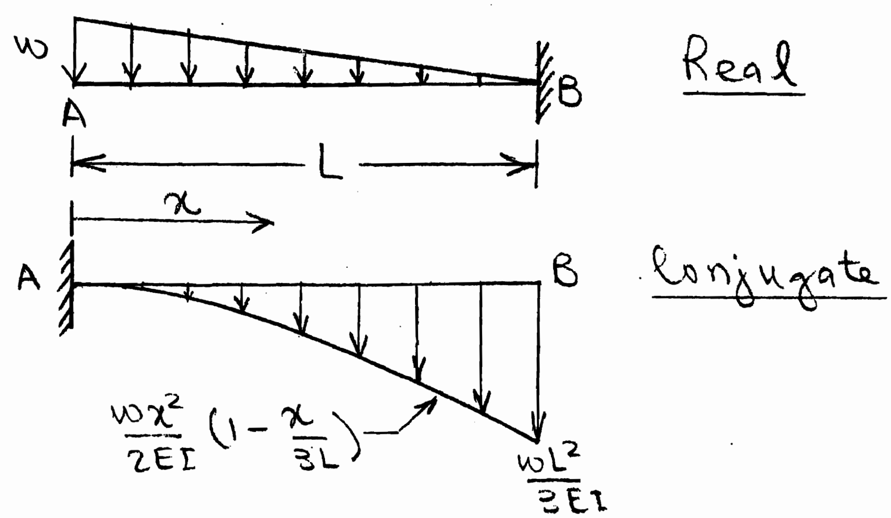


$$\theta_B = -\frac{1}{3}\left(\frac{wa^2}{2EI}\right)a = -\frac{wa^3}{6EI} = \frac{wa^3}{6EI} \curvearrowright$$

$$\Delta_B = -\frac{wa^3}{6EI}\left(L - \frac{a}{4}\right) = -\frac{wa^3}{24EI}(4L - a)$$

$$= \frac{wa^3}{24EI}(4L - a)\downarrow$$

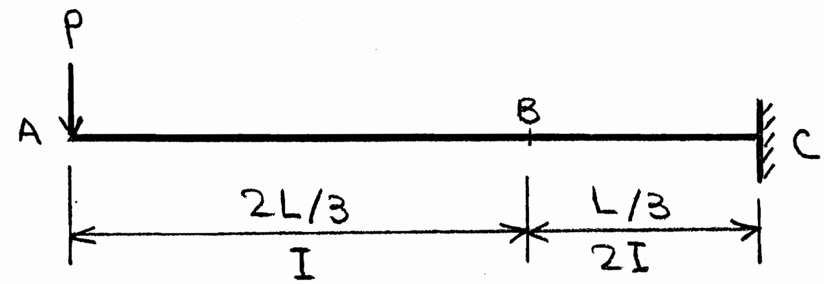
6.39



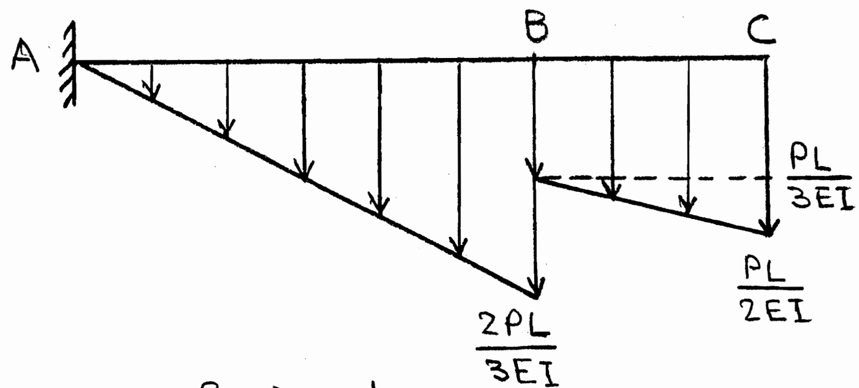
$$\theta_A = \int_0^L \frac{wx^2}{2EI}\left(1-\frac{x}{3L}\right)dx = \frac{wL^3}{8EI}$$

$$\Delta_A = \int_0^L \frac{wx^2}{2EI}\left(1-\frac{x}{3L}\right)x\,dx = -\frac{11}{120}\frac{wL^4}{EI} = \frac{11}{120}\frac{wL^4}{EI}\downarrow$$

6.40



Real



Conjugate

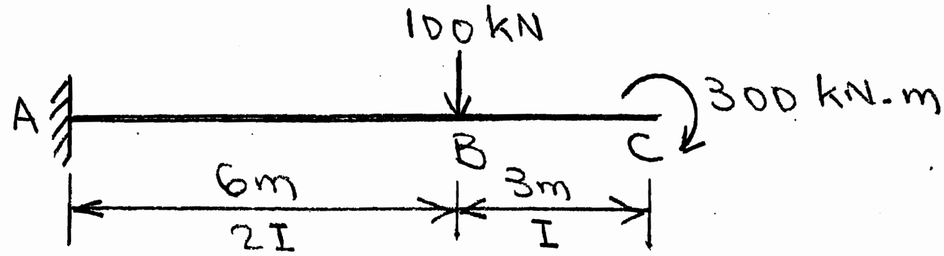
$$\theta_A = \frac{1}{2}\left(\frac{2L}{3}\right)\left(\frac{2PL}{3EI}\right) + \frac{L}{3}\left(\frac{PL}{3EI}\right) + \frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{PL}{6EI}\right) = \frac{13PL^2}{36EI}$$

$$\Delta_A = -\frac{2PL^2}{9EI}\left(\frac{4L}{9}\right) - \frac{PL^2}{9EI}\left(\frac{5L}{6}\right) - \frac{PL^2}{36EI}\left(\frac{8L}{9}\right) = -\frac{35PL^3}{162EI}$$

$$= \frac{35PL^3}{162EI} \downarrow$$

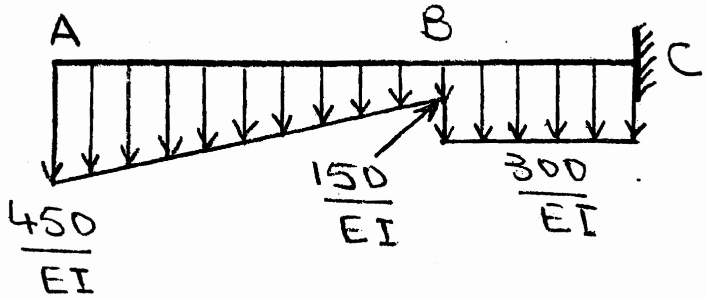
6.41

Real



$I = 500(10^6)\ mm^4$

Conjugate



$$\theta_B = \frac{1}{EI}\left[-150(6) - \frac{1}{2}(300)6\right] = -\frac{1800\ kN.m^2}{EI}$$

$$= -\frac{1800}{70(500)} = -0.0514\ rad = \underline{0.0514\ rad}$$

$$\Delta_B = \frac{1}{EI}\left[-150(6)3 - \frac{1}{2}(300)6(4)\right] = -\frac{6300\ kN.m^3}{EI}$$

$$= -\frac{6300}{70(500)} = -0.18\ m = \underline{180\ mm\downarrow}$$

$$\theta_C = \frac{1}{EI}\left[-1800 - 300(3)\right] = -\frac{2700\ kN.m^2}{EI}$$

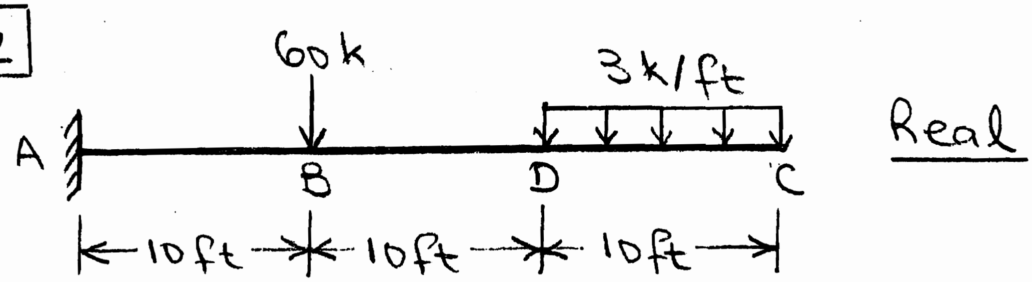
$$= -\frac{2700}{70(500)} = -0.0771\ rad = \underline{0.0771\ rad}$$

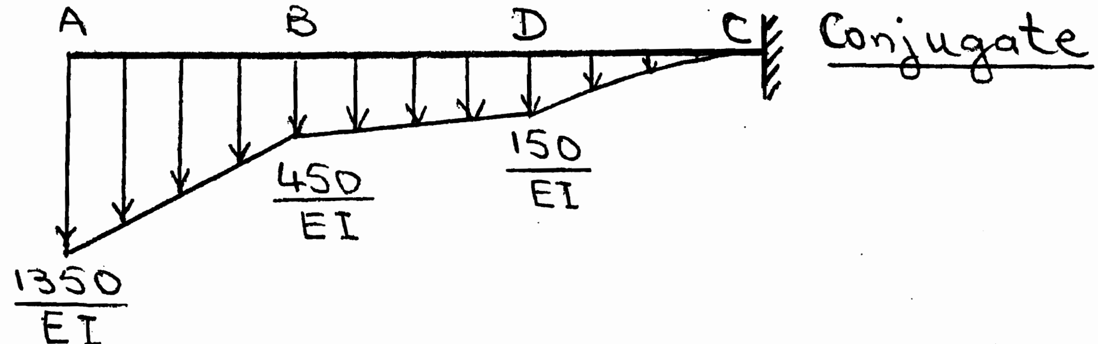
$$\Delta_C = \frac{1}{EI}\left[-150(6)(6) - \frac{1}{2}(300)6(7) - 300(3)(1.5)\right]$$

$$= -\frac{13050\ kN.m^3}{EI} = -\frac{13050}{70(500)} = -0.373\ m$$

$$= \underline{373\ mm\downarrow}$$

6.42





$$\theta_B = \frac{1}{EI}\left[-450(10) - \frac{1}{2}(900)10\right] = -\frac{9000 \text{ k-ft}^2}{EI}$$

$$= -\frac{9000(12)^2}{29000(4000)} = -0.0112 \text{ rad} = \underline{0.0112 \text{ rad}}$$

$$\Delta_B = \frac{1}{EI}\left[-450(10)5 - \frac{1}{2}(900)10\left(\frac{20}{3}\right)\right] = -\frac{52500 \text{ k-ft}^3}{EI}$$

$$= -\frac{52500(12)^3}{29000(4000)} = -0.782 \text{ in.} = \underline{0.782 \text{ in.}\downarrow}$$

$$\theta_C = \frac{1}{EI}\left[-9000 - 150(10) - \frac{1}{2}(300)10 - \frac{1}{3}(150)10\right]$$

$$= -\frac{12500 \text{ k-ft}^2}{EI} = -\frac{12500(12)^2}{29000(4000)} = -0.0155 \text{ rad}$$

$$= \underline{0.0155 \text{ rad}}$$

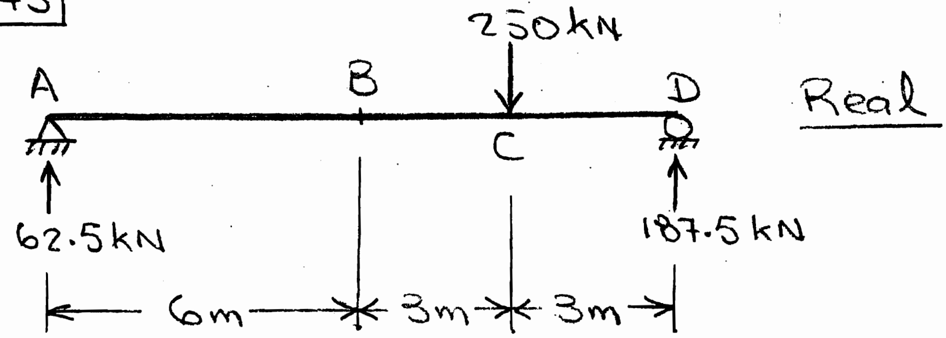
$$\Delta_C = \frac{1}{EI}\left[-450(10)25 - \frac{1}{2}(900)10\left(\frac{20}{3}+20\right)\right.$$

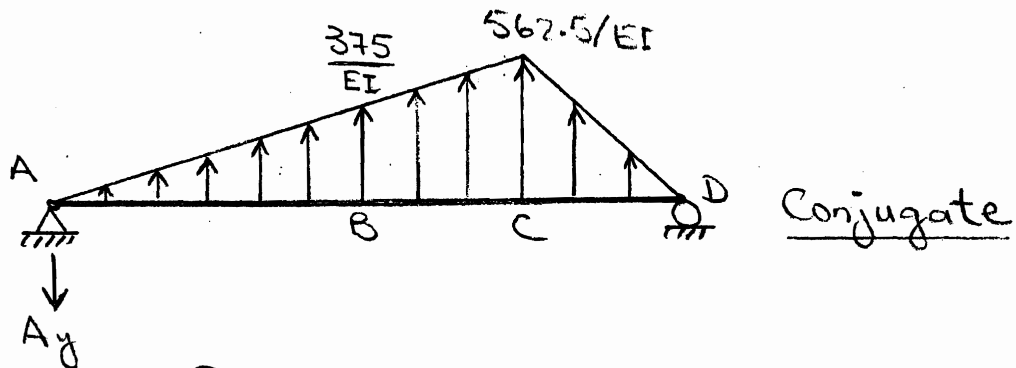
$$\left. -150(10)15 - \frac{1}{2}(300)10\left(\frac{20}{3}+10\right) - \frac{1}{3}(150)10\left(\frac{30}{4}\right)\right]$$

$$= -\frac{283750 \text{ k-ft}^3}{EI} = -\frac{283750(12)^3}{29000(4000)}$$

$$= -4.227 \text{ in.} = \underline{4.227 \text{ in.}\downarrow}$$

6.43





Reaction for conjugate beam:

$+\circlearrowright \Sigma M_D = 0$

$$A_y(12) - \frac{1}{2}\left(\frac{562.5}{EI}\right)9(6) - \frac{1}{2}\left(\frac{562.5}{EI}\right)3(2) = 0$$

$$A_y = \frac{1406.25 \text{ kN.m}^2}{EI}$$

$$\theta_B = \frac{1}{EI}\left[-1406.25 + \frac{1}{2}(375)6\right] = -\frac{281.25 \text{ kN.m}^2}{EI}$$

$$= -\frac{281.25}{200(462)} = -0.00304 \text{ rad} = \underline{0.00304 \text{ rad}}$$

$$\Delta_B = \frac{1}{EI}\left[-1406.25(6) + \frac{1}{2}(375)6(2)\right] = -\frac{6187.5 \text{ kN.m}^3}{EI}$$

$$= -\frac{6187.5}{200(462)} = -0.067 \text{ m} = \underline{67 \text{ mm}\downarrow}$$

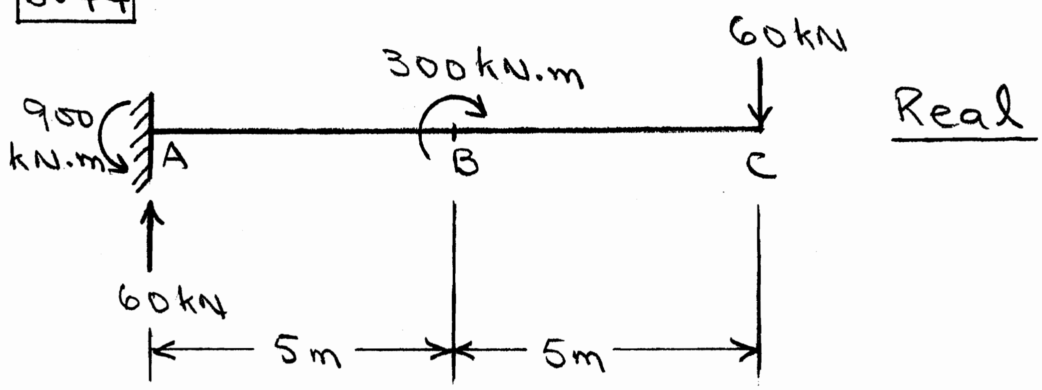
$$\theta_C = \frac{1}{EI}\left[-1406.25 + \frac{1}{2}(562.5)9\right] = \frac{1125 \text{ kN.m}^2}{EI}$$

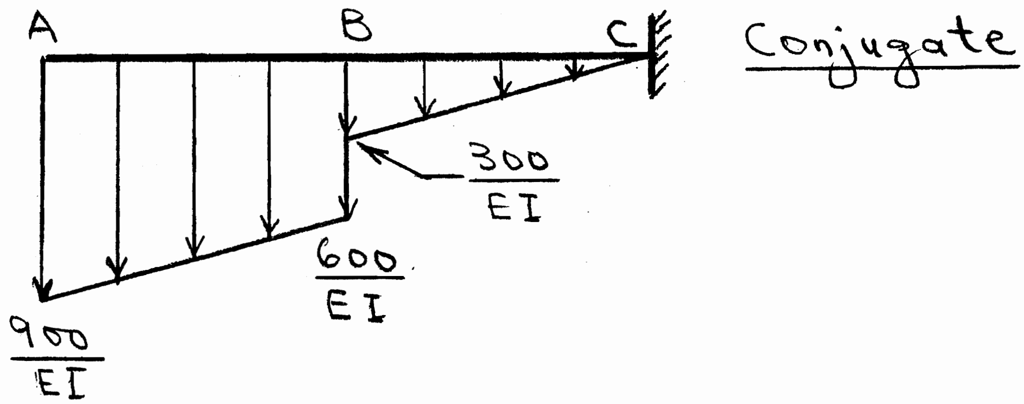
$$= \frac{1125}{200(462)} = \underline{0.0122 \text{ rad}}$$

$$\Delta_C = \frac{1}{EI}\left[-1406.25(9) + \frac{1}{2}(562.5)9(3)\right] = -\frac{5062.5 \text{ kN.m}^3}{EI}$$

$$= -\frac{5062.5}{200(462)} = -0.0548 \text{ m} = \underline{54.8 \text{ mm}\downarrow}$$

6.44





$$\Delta_{max} = \Delta_C = -\frac{1}{EI}\left[600(5)(7.5) + \frac{1}{2}(300)(5)\left(\frac{25}{3}\right) + \frac{1}{2}(300)(5)\left(\frac{10}{3}\right)\right] = \frac{-31250 \text{ kN.m}^3}{EI}$$

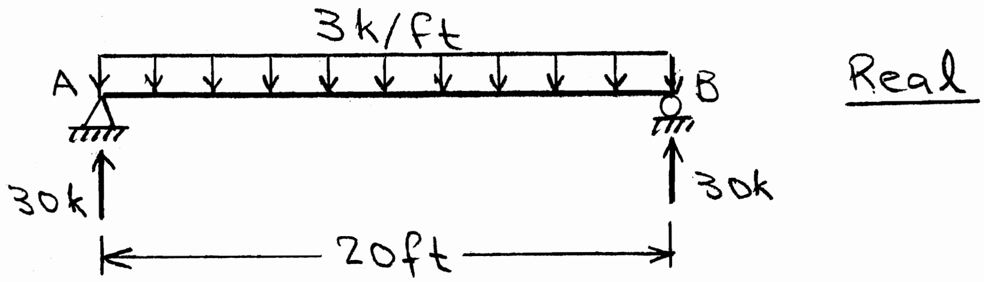
$$= \frac{31250 \text{ kN.m}^3}{EI} \downarrow$$

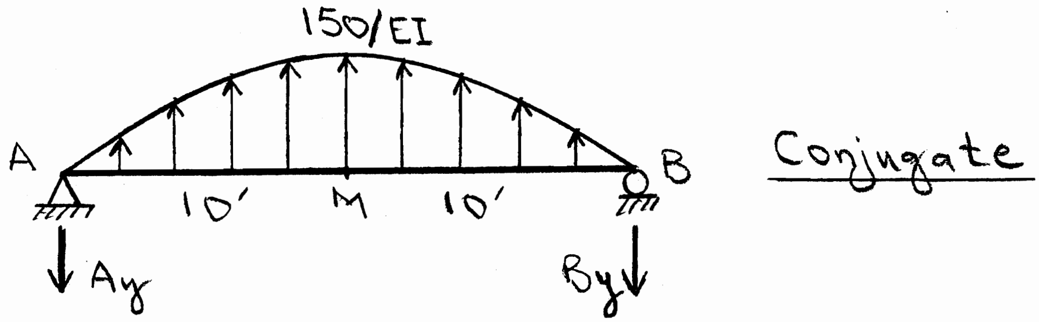
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

from which, $I = 5625(10^{-6})\text{m}^4 = \underline{5625(10^6)\text{ mm}^4}$

6.45





Reactions for conjugate beam:

$$A_y = B_y = \frac{1}{2}\left(\frac{2}{3}\right)\left(\frac{150}{EI}\right)(20) = \frac{1000 \text{ k-ft}^2}{EI}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-1000(10) + \frac{2}{3}(150)(10)\left(\frac{30}{8}\right)\right]$$

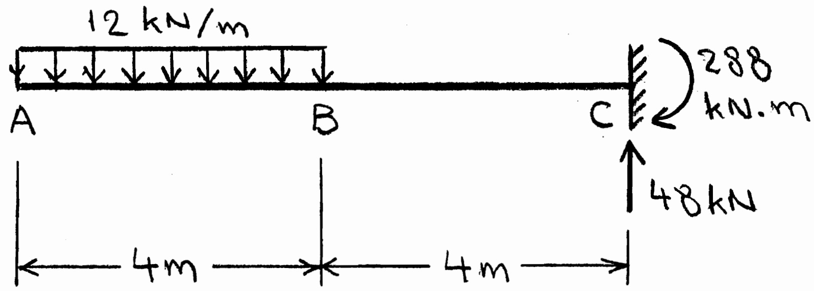
$$= -\frac{6250 \text{ k-ft}^3}{EI} = \frac{6250 \text{ k-ft}^3}{EI} \downarrow$$

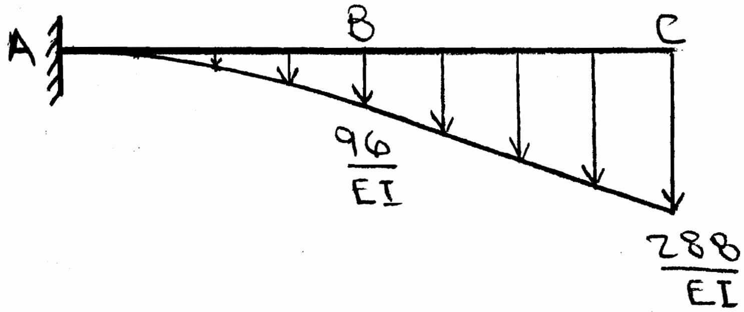
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{6250(12)^3}{29000(I)} = \frac{20(12)}{360}$$

from which, $\underline{I = 559 \text{ in}^4}$

6.46





$$\Delta_{max} = \Delta_A = \frac{1}{EI}\left[-\frac{1}{3}(96)(4)(3) - 96(4)(6) - \frac{1}{2}(192)(4)\left(\frac{8}{3}+4\right)\right] = -\frac{5248 \text{ kN.m}^3}{EI}$$

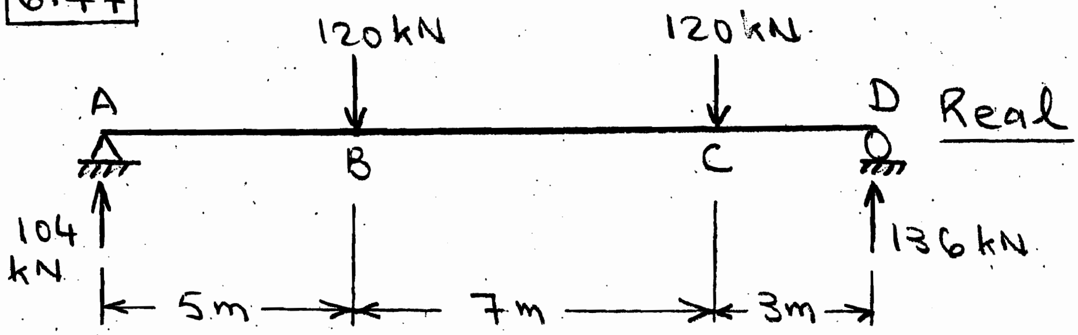
$$= \frac{5248 \text{ kN.m}^3}{EI} \downarrow$$

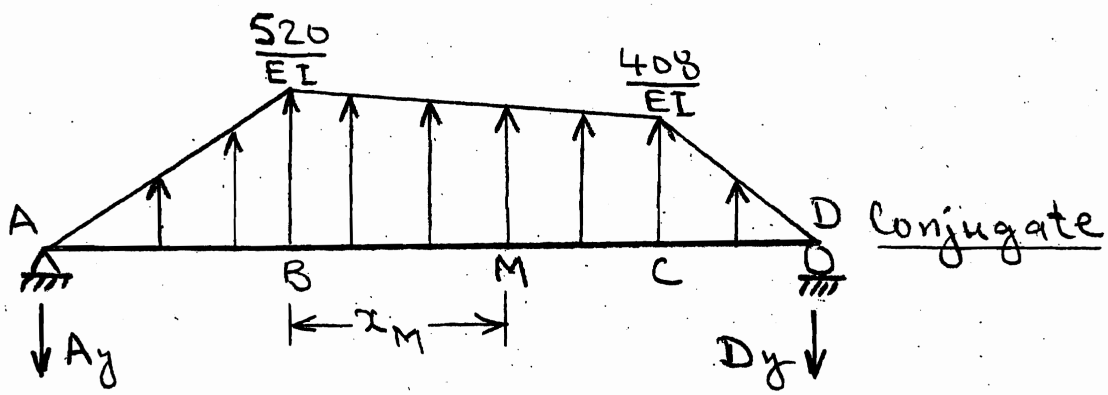
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{5248}{70(10^6)I} = \frac{8}{360}$$

from which, $I = 3374(10^{-6})\text{ m}^4 = \underline{3374(10^6)\text{ mm}^4}$

6.47





$+\circlearrowleft \Sigma M_D = 0$

$$A_y(15) - \frac{1}{2}\left(\frac{520}{EI}\right)5\left(\frac{5}{3}+10\right) - \left(\frac{408}{EI}\right)(7)\left(\frac{7}{2}+3\right)$$
$$-\frac{1}{2}\left(\frac{112}{EI}\right)7\left(\frac{14}{3}+3\right) - \frac{1}{2}\left(\frac{408}{EI}\right)3\,(2) = 0$$

$$A_y = \frac{2530.67 \text{ kN.m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[-2530.67 + \frac{1}{2}(520)(5) + (520)x_M - \frac{1}{2}\left(\frac{112}{7}x_M\right)x_M\right] = 0$$

$$8x_M^2 - 520x_M + 1230.67 = 0$$

$$x_M = 2.46 \text{ m}$$

6.47 (Contd.)

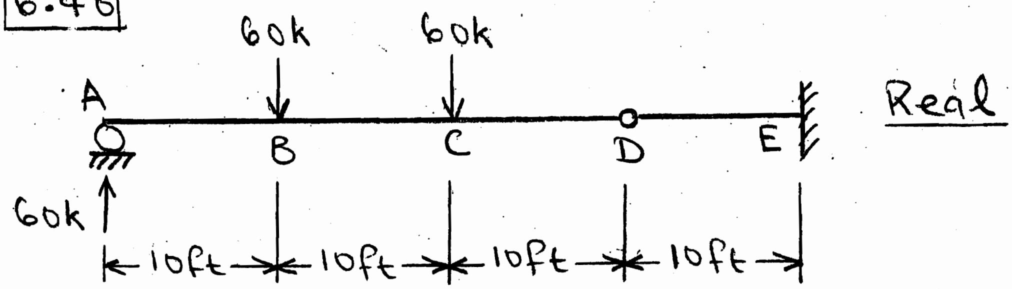
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\Big[-2530.67(7.46) + \frac{1}{2}(520)(5)\left(\frac{5}{3} + 2.46\right) + (520)(2.46)\left(\frac{2.46}{2}\right) - \frac{1}{2}(39.36)(2.46)\left(\frac{2.46}{3}\right)\Big] = \frac{-11980.41}{EI} = \frac{11980.41 \text{ kN.m}^3}{EI} \downarrow$$

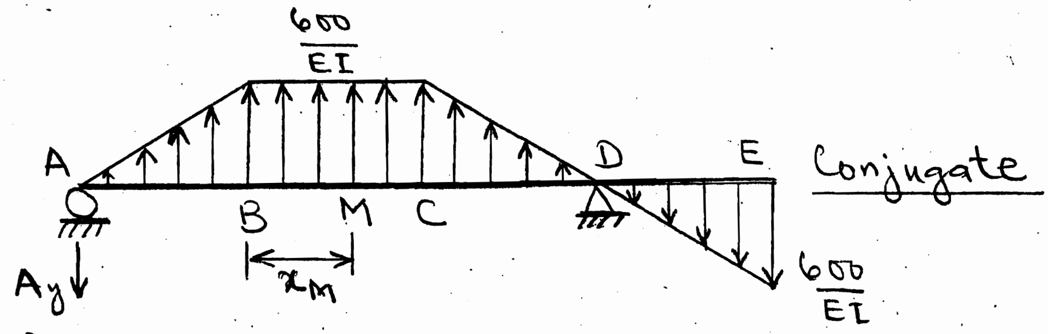
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{11980.41}{30(10^6)I} = \frac{15}{360}$$

from which, $I = 9584(10^{-6})\,m^4 = \underline{9584(10^6)\,mm^4}$

6.48





$+\circlearrowright \sum M_D = 0$

$$A_y(30) - \frac{1}{2}\left(\frac{600}{EI}\right)10\left(\frac{10}{3}+20\right) - \left(\frac{600}{EI}\right)(10)(15)$$

$$-\frac{1}{2}\left(\frac{600}{EI}\right)(10)\left(\frac{20}{3}\right) - \frac{1}{2}\left(\frac{600}{EI}\right)10\left(\frac{20}{3}\right) = 0$$

$$A_y = \frac{6666.67 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[-6666.67 + \frac{1}{2}(600)10 + (600)x_M\right] = 0$$

$$x_M = 6.11 \text{ ft}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-6666.67(16.11) + \frac{1}{2}(600)(10)\left(\frac{10}{3} + 6.11\right) + (600)(6.11)\left(\frac{6.11}{2}\right)\right]$$

$$= -\frac{67870}{EI} = \frac{67870 \text{ k-ft}^3}{EI} \downarrow$$

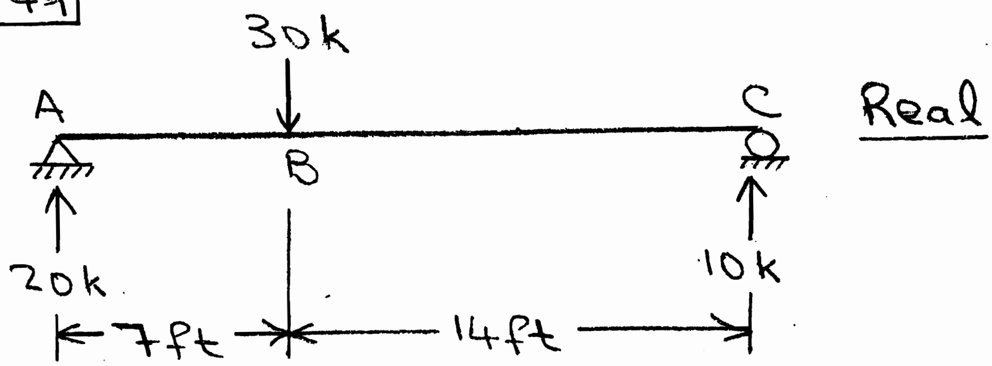
$$\Delta_{max} = \frac{L}{360}$$

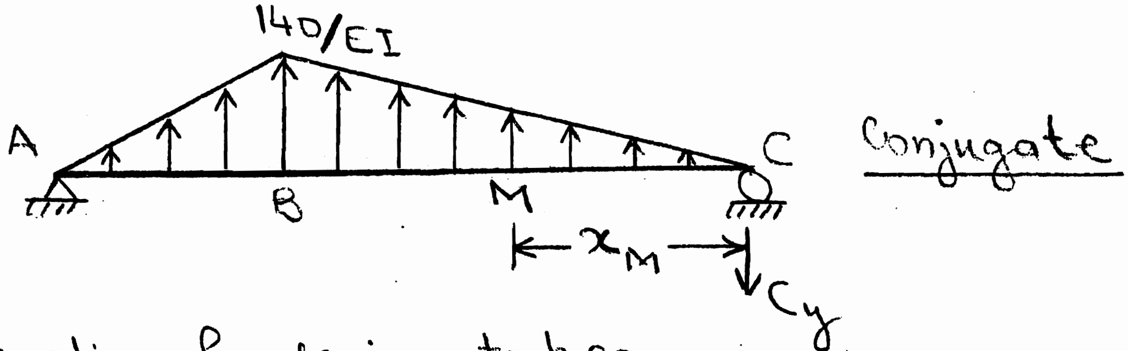
$$\frac{67870(12)^3}{29000(I)} = \frac{40(12)}{360}$$

from which

$$\underline{I = 3033 \text{ in}^4}$$

6.49





Reaction for conjugate beam:

$$+\circlearrowleft \Sigma M_A = 0$$

$$-C_y(21) + \frac{1}{2}\left(\frac{140}{EI}\right)14\left(\frac{14}{3}+7\right) + \frac{1}{2}\left(\frac{140}{EI}\right)7\left(\frac{14}{3}\right) = 0$$

$$C_y = \frac{653.33 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point C. Then, the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[653.33 - \frac{1}{2}(10x_M)x_M\right] = 0$$

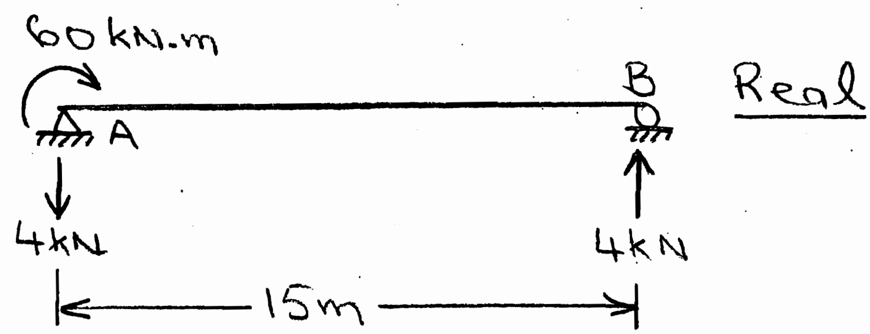
$$x_M = 11.43 \text{ ft}$$

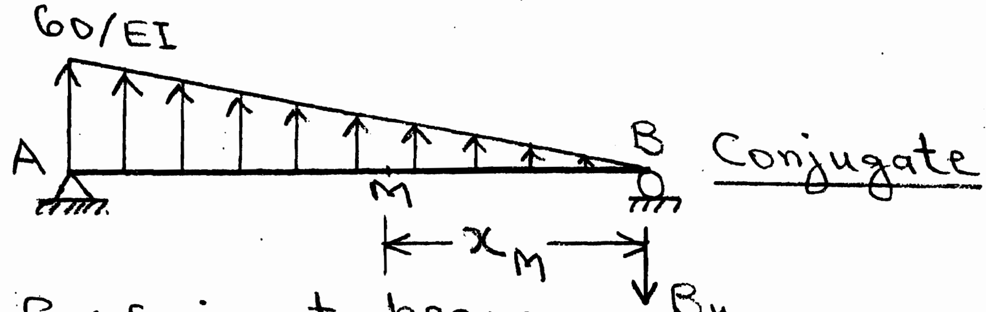
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-653.33(11.43) + \frac{1}{2}(114.3)11.43\left(\frac{11.43}{3}\right)\right]$$

$$= -\frac{4978.81 \text{ k-ft}^3}{EI} = -\frac{4978.81(12)^3}{10000(500)}$$

$$= -1.72 \text{ in.} = \underline{1.72 \text{ in.}\downarrow}$$

6.50





Reaction for conjugate beam:

$$+\circlearrowright \Sigma M_A = 0$$

$$-B_y(15) + \frac{1}{2}\left(\frac{60}{EI}\right)15(5) = 0$$

$$B_y = \frac{150 \text{ kN.m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[150 - \frac{1}{2}(4x_M)x_M\right] = 0$$

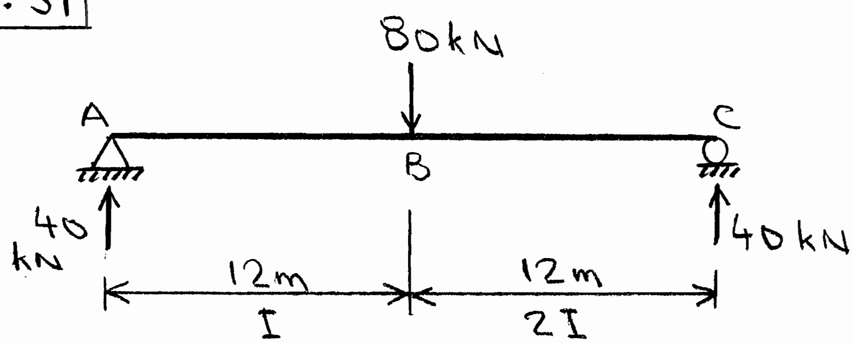
$$x_M = 8.66 \text{ m}$$

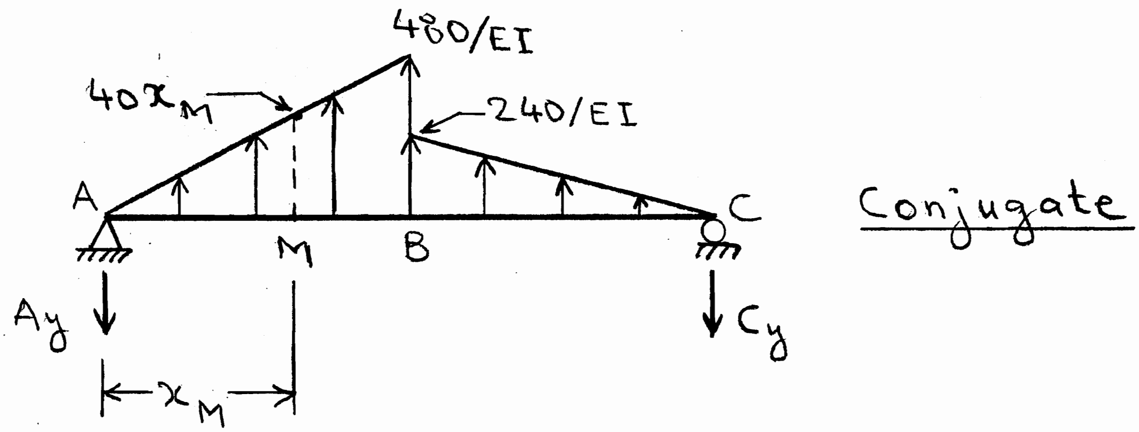
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-150(8.66) + \frac{1}{2}(34.64)(8.66)\left(\frac{8.66}{3}\right)\right]$$

$$= -\frac{866 \text{ kN.m}^3}{EI} = -\frac{866}{70(712)} = -0.0174 \text{m}$$

$$= 17.4 \text{ mm} \downarrow$$

6.51





$+\circlearrowleft \Sigma M_C = 0$

$$A_y(24) - \frac{1}{2}\left(\frac{480}{EI}\right)(12)(16) - \frac{1}{2}\left(\frac{240}{EI}\right)(12)(8) = 0$$

$$A_y = \frac{2400 \text{ kN.m}^2}{EI}$$

$S_M = 0$

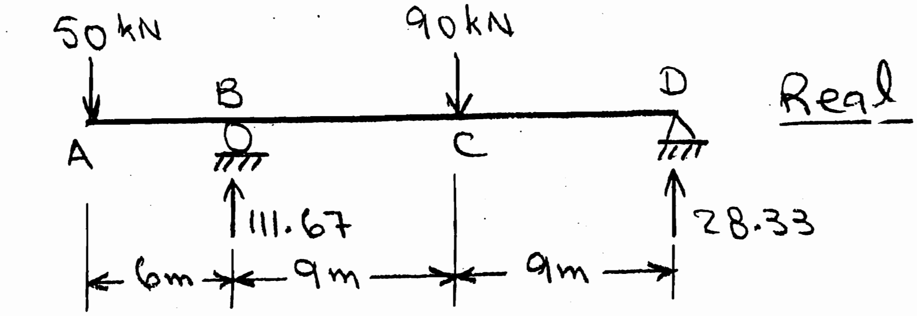
$$\frac{1}{EI}\left[-2400 + \frac{1}{2}(40x_M)x_M\right] = 0 \qquad \underline{x_M = 10.95 \text{ m}}$$

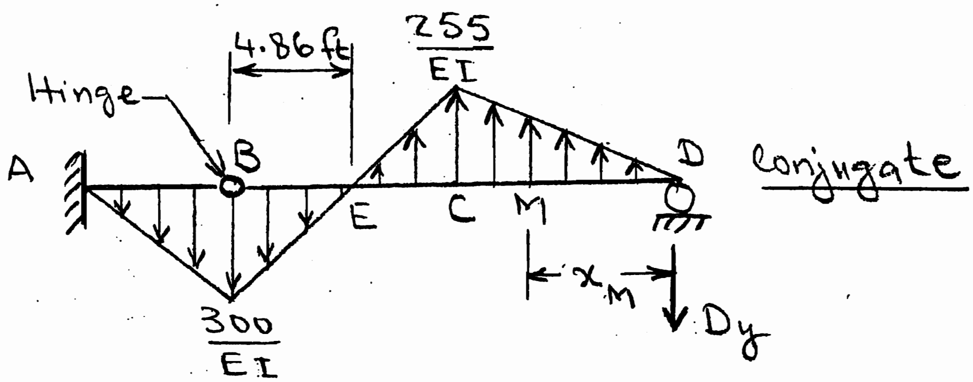
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-2400(10.95) + \frac{1}{2}(438)(10.95)\left(\frac{10.95}{3}\right)\right]$$

$$= -\frac{17527 \text{ kN.m}^3}{EI} = -\frac{17527}{200(600)} = -0.146 \text{ m}$$

$$\underline{\Delta_{max} = 146 \text{ mm} \downarrow}$$

6.52





$$+\circlearrowleft \sum M_B^{BD} = 0$$

$$-D_y(18) + \frac{1}{2}\left(\frac{255}{EI}\right)(9)(12) + \frac{1}{2}\left(\frac{255}{EI}\right)(4.14)(7.62)$$

$$-\frac{1}{2}\left(\frac{300}{EI}\right)(4.86)\left(\frac{4.86}{3}\right) = 0$$

$$D_y = \frac{922.85 \text{ kN}\cdot\text{m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point D. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[922.85 - \frac{1}{2}\left(\frac{255}{9}x_M\right)x_M\right] = 0$$

$$x_M = 8.07 \text{ m}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-922.85(8.07) + \frac{1}{2}\left(\frac{255}{9}\right)(8.07)\cdot\right.$$

$$\left.(8.07)\left(\frac{8.07}{3}\right)\right] = -\frac{4965 \text{ kN}\cdot\text{m}^3}{EI}$$

$$= -\frac{4965}{70(95)} = -0.746 \text{ m} = \underline{746 \text{ mm} \downarrow}$$

6.52 (contd.)

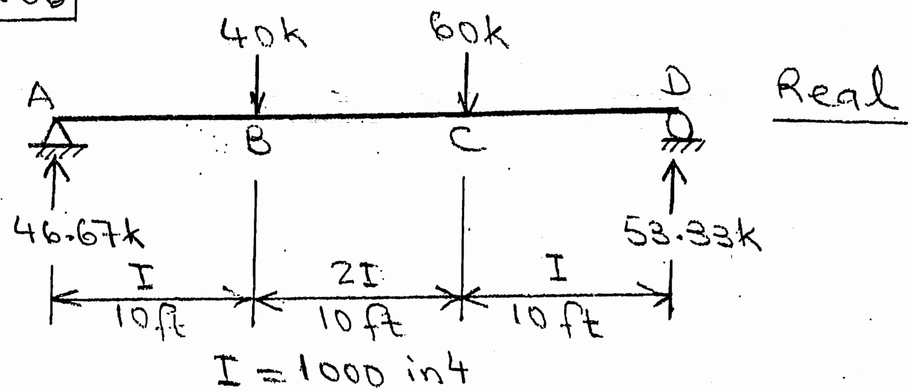
$$1.67\,x_M^2 + 233.33\,x_M - 1314.8 = 0$$

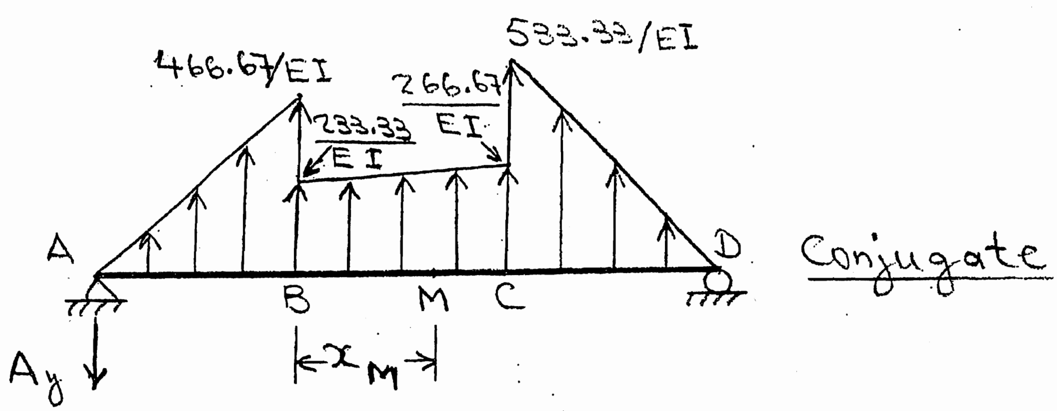
$$x_M = 5.42 \text{ ft}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\Big[-3648.15(15.42) + \frac{1}{2}(466.67)10\left(\frac{10}{3}+5.42\right) + 233.33(5.42)\left(\frac{5.42}{2}\right) + \frac{1}{2}(18.06)5.42\left(\frac{5.42}{3}\right)\Big] = -\frac{32314 \text{ k-ft}^3}{EI}$$

$$= -\frac{32314(12)^3}{29000(1000)} = -1.92 \text{ in.} = \underline{1.92 \text{ in.}\downarrow}$$

6.53





6.53 (contd.)

Reaction for conjugate beam:

$+\circlearrowleft \Sigma M_D = 0$

$$A_y(30) - \frac{1}{EI}\left[\frac{1}{2}(466.67)10\left(\frac{10}{3}+20\right) + 233.33(10)15 + \frac{1}{2}(33.33)10\left(\frac{10}{3}+10\right) + \frac{1}{2}(533.33)10\left(\frac{20}{3}\right)\right] = 0$$

$$A_y = \frac{3648.15 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[-3648.15 + \frac{1}{2}(466.67)10 + 233.33\, x_M + \frac{1}{2}(3.33\, x_M)\, x_M\right] = 0$$

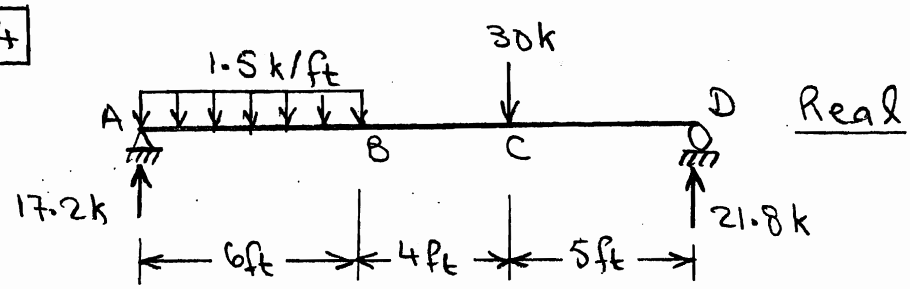
$$1.67\, x_M^2 + 233.33\, x_M - 1314.8 = 0$$

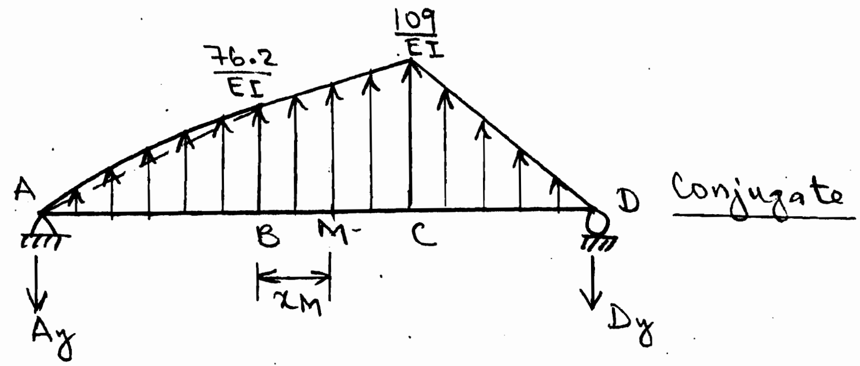
$$x_M = 5.42 \text{ ft}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-3648.15(15.42) + \frac{1}{2}(466.67)10\left(\frac{10}{3}+5.42\right) + 233.33(5.42)\left(\frac{5.42}{2}\right) + \frac{1}{2}(18.06)5.42\left(\frac{5.42}{3}\right)\right] = -\frac{32314 \text{ k-ft}^3}{EI}$$

$$= -\frac{32314(12)^3}{29000(1000)} = -1.92 \text{ in.} = 1.92 \text{ in.}\downarrow$$

6.54





$+\circlearrowleft \Sigma M_D = 0$

$$A_y(15) - \frac{1}{EI}\left[\frac{1}{2}(76.2)6(11) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6)(12) + 76.2(4)7 + \frac{1}{2}(32.8)(4)\left(\frac{4}{3}+5\right) + \frac{1}{2}(109)5\left(\frac{10}{3}\right)\right] = 0$$

$$A_y = \frac{419.73 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M. Thus,

$$S_M = \frac{1}{EI}\left[-419.73 + \frac{1}{2}(76.2)6 + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6) + 76.2x_M + \frac{1}{2}(8.2x_M)x_M\right] = 0$$

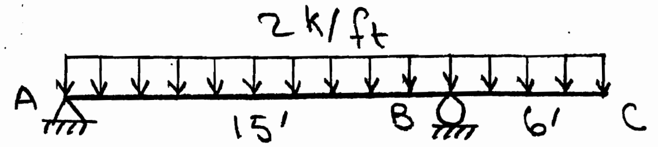
$$4.1x_M^2 + 76.2x_M - 164.13 = 0$$

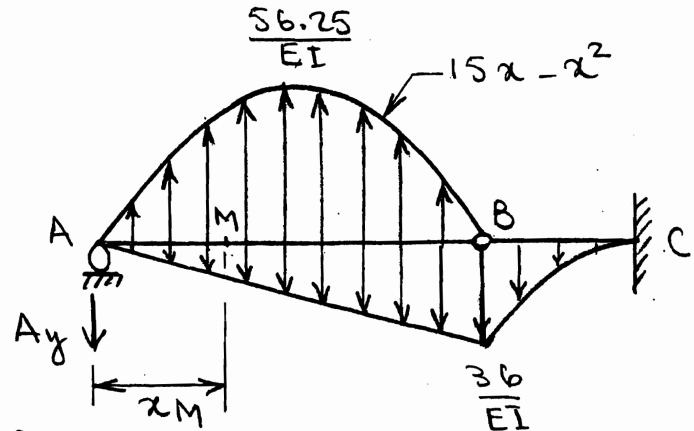
$$x_M = 1.95 \text{ ft}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-419.73(7.95) + \frac{1}{2}(76.2)6(3.95) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6)(4.95) + 76.2\frac{(1.95)^2}{2} + \frac{1}{2}(15.99)(1.95)\left(\frac{1.95}{3}\right)\right] = -\frac{2145.2 \text{ k-ft}^3}{EI}$$

$$= -\frac{2145.2(12)^3}{1500(20000)} = -0.124 \text{ in} = \underline{0.124 \text{ in.}\downarrow}$$

6.55





$+\circlearrowleft \sum M_B^{AB} = 0$

$$A_y(15) - \frac{1}{EI}\left[\frac{2}{3}(56.25)15(7.5) - \frac{1}{2}(36)15(5)\right] = 0$$

$$A_y = \frac{191.25 \text{ k-ft}^2}{EI}$$

$$S_M = 0$$

$$\frac{1}{EI}\left[-191.25 + \frac{1}{2}(15x_M - x_M^2)x_M + \frac{2}{3}\left(\frac{2x_M^2}{8}\right)x_M - \frac{1}{2}(2.4x_M)x_M\right] = 0$$

$$-\frac{x_M^3}{3} + 6.3x_M^2 - 191.25 = 0$$

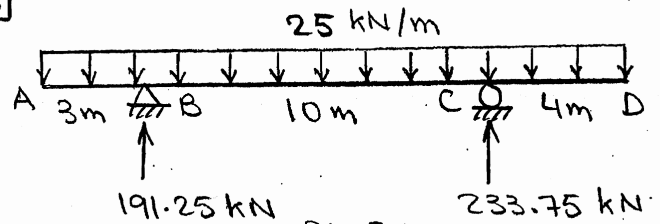
from which $x_M = 6.92$ ft

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-191.25(6.92) + \frac{1}{2}(55.91)\frac{(6.92)^2}{3} + \frac{2}{3}(11.97)\frac{(6.92)^2}{2} - \frac{1}{2}(16.61)6.92\left(\frac{6.92}{3}\right)\right]$$

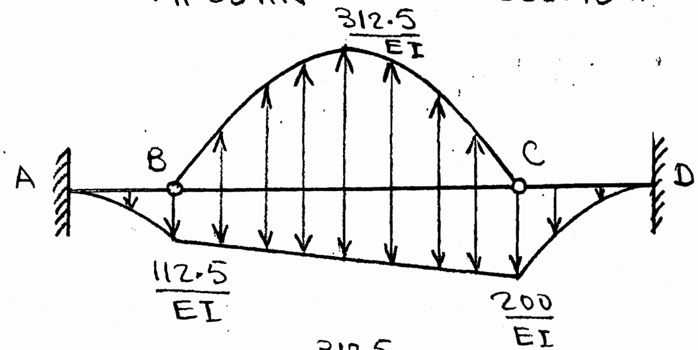
$$= -\frac{818.73 \text{ k-ft}^3}{EI} = -0.0139 \text{ in}$$

$$= \underline{0.0139 \text{ in.} \downarrow}$$

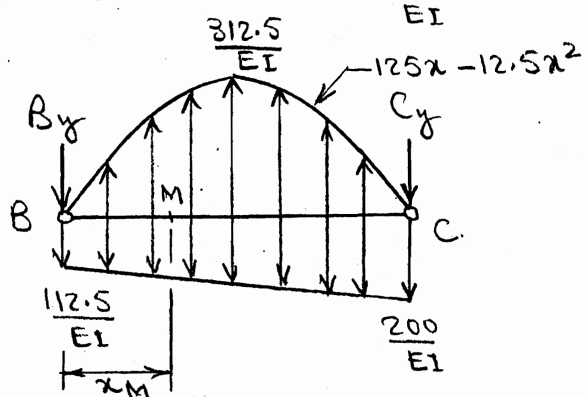
6.56



Real



Conjugate



$$+\circlearrowleft \Sigma M_C^{BC} = 0$$

$$B_y(10) - \frac{1}{EI}\left[\frac{2}{3}(312.5)10(5) - 112.5(10)5 - \frac{1}{2}(87.5)10\left(\frac{10}{3}\right)\right] = 0$$

$$B_y = \frac{333.33 \text{ kN-m}^2}{EI}$$

$$\delta_M = 0$$

$$\frac{1}{EI}\left[-333.33 + \frac{1}{2}(125x_M - 12.5x_M^2)x_M + \frac{2}{3}\left(\frac{25x_M^2}{8}\right)x_M - 112.5x_M - \frac{1}{2}(8.75x_M)x_M\right] = 0$$

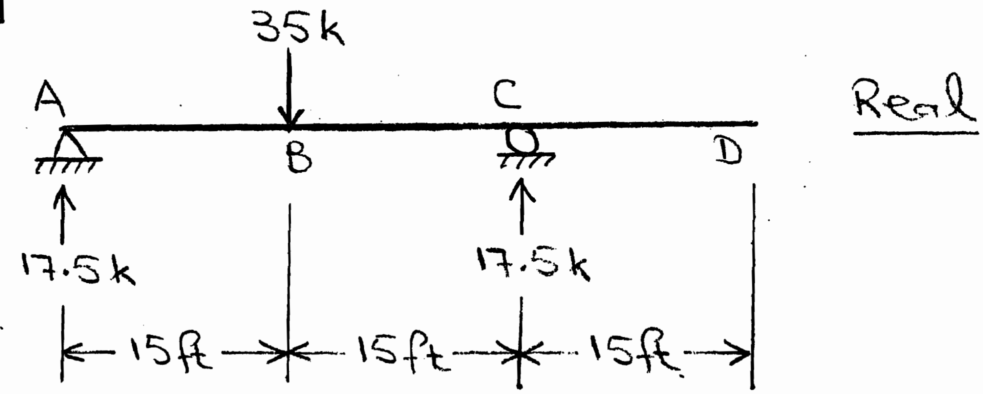
$$-4.167x_M^3 + 58.125x_M^2 - 112.5x_M - 333.33 = 0$$

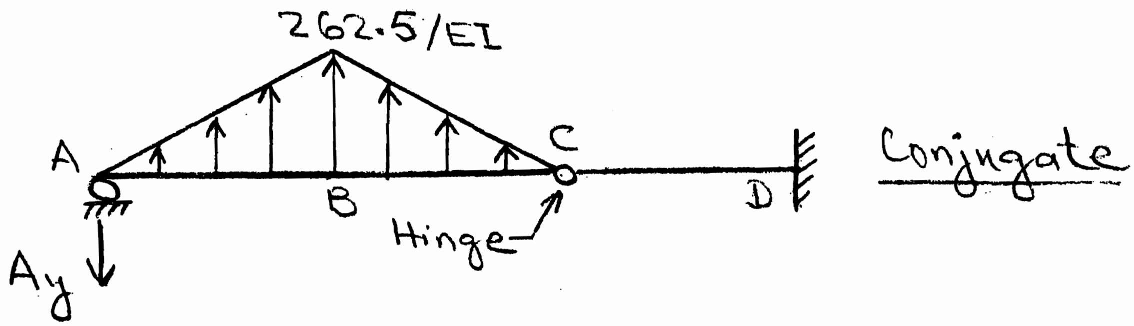
from which $x_M = 4.77$ m

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-333.33(4.77) + \frac{1}{2}(311.84)\frac{(4.77)^2}{3} + \frac{2}{3}(71.1)\frac{(4.77)^2}{2} - 112.5\frac{(4.77)^2}{2} - \frac{1}{2}(41.74)\frac{(4.77)^2}{3}\right]$$

$$= -\frac{1306.3 \text{ kN-m}^3}{EI} = -0.0131\text{m} = \underline{13.1 \text{ mm}\downarrow}$$

6.57





$$+\circlearrowleft \sum M_C^{AC} = 0$$

$$A_y(30) - \frac{1}{2}\left(\frac{262.5}{EI}\right)30\,(15) = 0$$

$$A_y = \frac{1968.75 \text{ k-ft}^2}{EI}$$

$$\theta_D = \frac{1}{EI}\left[-1968.75 + \frac{1}{2}(262.5)30\right] = \frac{1968.75 \text{ k-ft}^2}{EI}$$

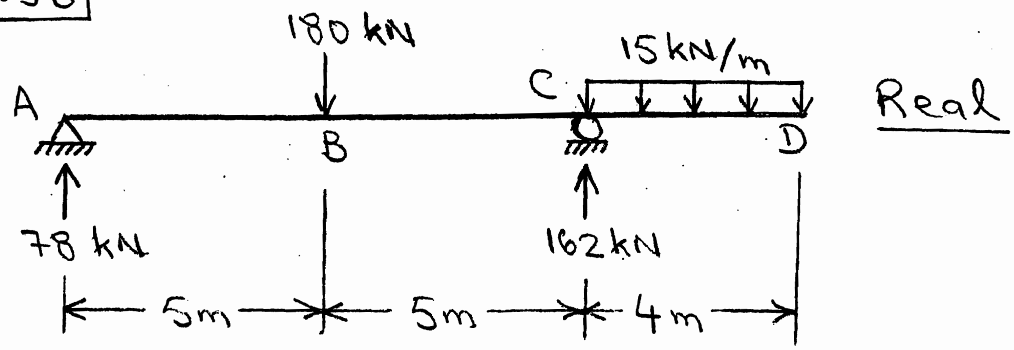
$$= \frac{1968.75\,(12)^2}{10000\,(2500)} = \underline{0.01134 \text{ rad}}$$

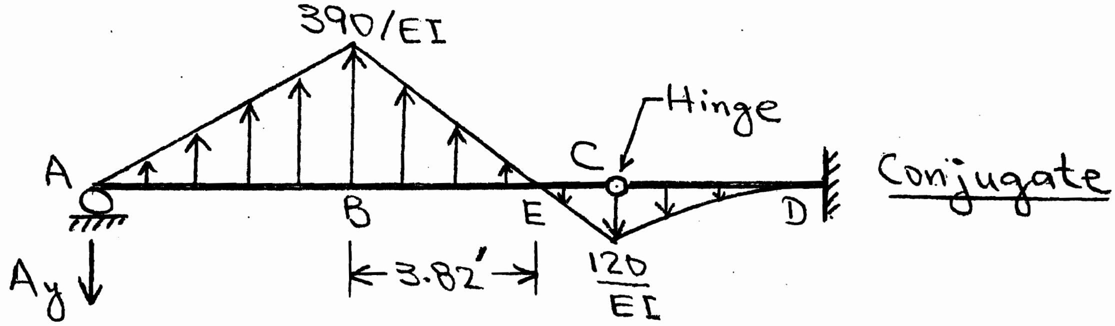
$$\Delta_D = \frac{1}{EI}\left[-1968.75(45) + \frac{1}{2}(262.5)30(30)\right]$$

$$= \frac{29531.25 \text{ k-ft}^3}{EI} = \frac{29531.25\,(12)^3}{10000\,(2500)}$$

$$= \underline{2.04 \text{ in.} \uparrow}$$

6.58





$+\circlearrowleft \sum M_C^{AC} = 0$

$$A_y(10) - \frac{1}{EI}\left[\frac{1}{2}(390)5\left(\frac{5}{3}+5\right) + \frac{1}{2}(390)(3.82)(3.73) - \frac{1}{2}(120)1.18\left(\frac{1.18}{3}\right)\right] = 0$$

$$A_y = \frac{925.1 \text{ kN}\cdot\text{m}^2}{EI}$$

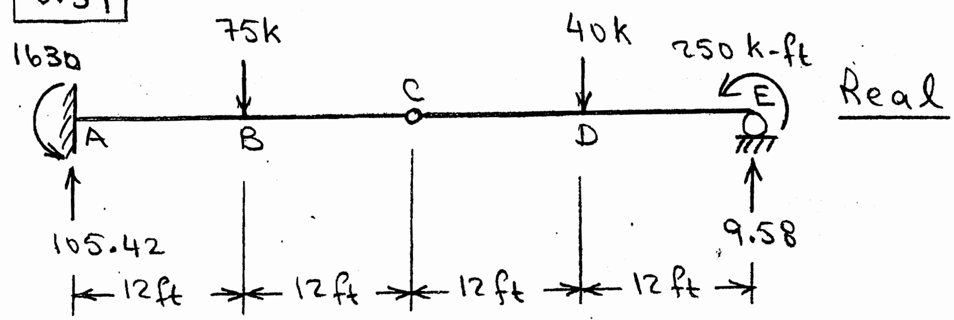
$$\theta_D = \frac{1}{EI}\left[-925.1 + \frac{1}{2}(390)8.82 - \frac{1}{2}(120)1.18 - \frac{1}{3}(120)4\right]$$

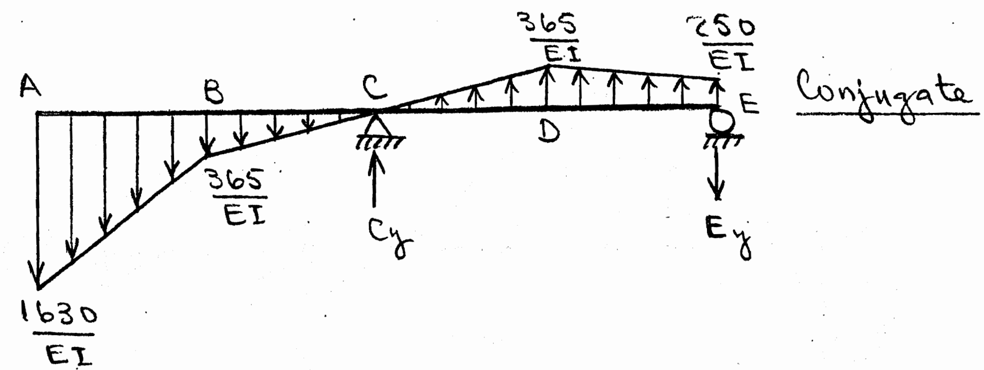
$$= \frac{564 \text{ kN}\cdot\text{m}^2}{EI} = \frac{564}{70(2340)} = \underline{0.00344 \text{ rad}}$$

$$\Delta_D = \frac{1}{EI}\left[-925.1(14) + \frac{1}{2}(390)5(10.67) + \frac{1}{2}(390)(3.82)(7.727) - \frac{1}{2}(120)1.18(4.393) - \frac{1}{3}(120)4(3)\right] = \frac{2417 \text{ kN}\cdot\text{m}^3}{EI}$$

$$= \frac{2417}{70(2340)} = 0.01476 \text{ m} = \underline{14.76 \text{ mm} \uparrow}$$

6.59





$+\circlearrowright \Sigma M_E = 0$

$$\frac{1}{EI}\left[\frac{1}{2}(1265)12(44) + 365(12)42 + \frac{1}{2}(365)12(32) - \frac{1}{2}(365)12(16) - \frac{1}{2}(115)12(8) - 250(12)6\right] - C_y(24) = 0$$

$$C_y = \frac{22060\ \text{k-ft}^2}{EI}\ \uparrow$$

$+\uparrow \Sigma F_y = 0$

$$\frac{1}{EI}\left[-\frac{1}{2}(1630+365)12 + \frac{1}{2}(365+250)12 + 22060\right] - E_y = 0$$

$$E_y = \frac{13780\ \text{k-ft}^2}{EI}\ \downarrow$$

$$\theta_B = -\frac{1}{EI}\left[\frac{1}{2}(1630+365)12\right] = -\frac{11970\ \text{k-ft}^2}{EI} = \underline{0.0099\ \text{rad}\ \circlearrowright}$$

$$\Delta_B = -\frac{1}{EI}\left[365(12)6 + \frac{1}{2}(1265)12(8)\right] = -\frac{87000\ \text{k-ft}^3}{EI}$$

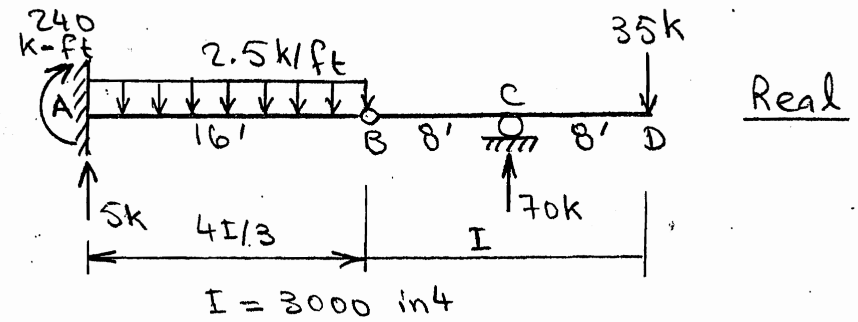
$$\underline{\Delta_B = 0.86\ \text{in}\ \downarrow}$$

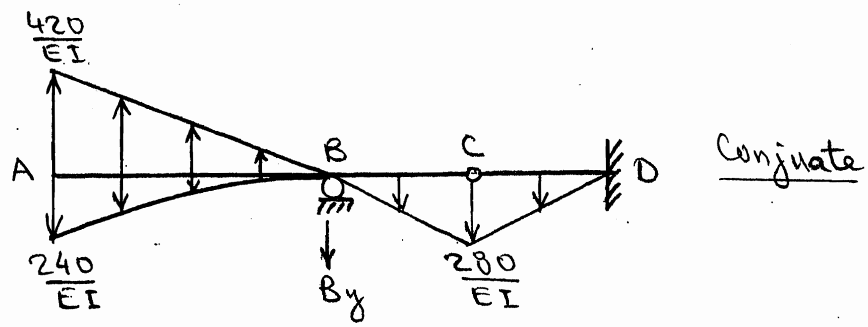
$$\theta_D = \frac{1}{EI}\left[13780 - \frac{1}{2}(365+250)12\right] = \frac{10090}{EI} = \underline{0.0084\ \text{rad}\ \circlearrowleft}$$

$$\Delta_D = \frac{1}{EI}\left[-13780(12) + 250(12)6 + \frac{1}{2}(115)12(4)\right]$$

$$= -\frac{144600\ \text{k-ft}^3}{EI} = 1.44\ \text{in}\ \downarrow$$

6.60





$+\circlearrowleft \sum M_C^{AC} = 0$

$$\frac{1}{EI}\left[\frac{1}{3}(240)16(20) - \frac{1}{2}(420)16(18.67) + \frac{1}{2}(280)8\left(\frac{8}{3}\right)\right] + B_y(8) = 0$$

$$B_y = \frac{4266.67 \text{ k-ft}^2}{EI} \downarrow$$

$$\theta_{B,Left} = \frac{1}{EI}\left[\frac{1}{2}(420)16 - \frac{1}{3}(240)16\right] = \frac{2080 \text{ k-ft}^2}{EI}$$

$$= \frac{2080(12)^2}{30000(3000)} = \underline{0.0033 \text{ rad.}}$$

$$\theta_{B,Right} = \frac{1}{EI}(2080 - 4266.67) = -\frac{2186.67 \text{ k-ft}^2}{EI}$$

$$= \underline{-0.0035 \text{ rad.}}$$

$$\Delta_B = \frac{1}{EI}\left[\frac{1}{2}(420)16(10.67) - \frac{1}{3}(240)16(12)\right]$$

$$= \frac{20480 \text{ k-ft}^3}{EI} = \frac{20480(12)^3}{30000(3000)} = \underline{0.39 \text{ in} \uparrow}$$

$$\theta_D = \frac{1}{EI}\left[\frac{1}{2}(420)16 - \frac{1}{3}(240)16 - 4266.67 - \frac{1}{2}(280)16\right]$$

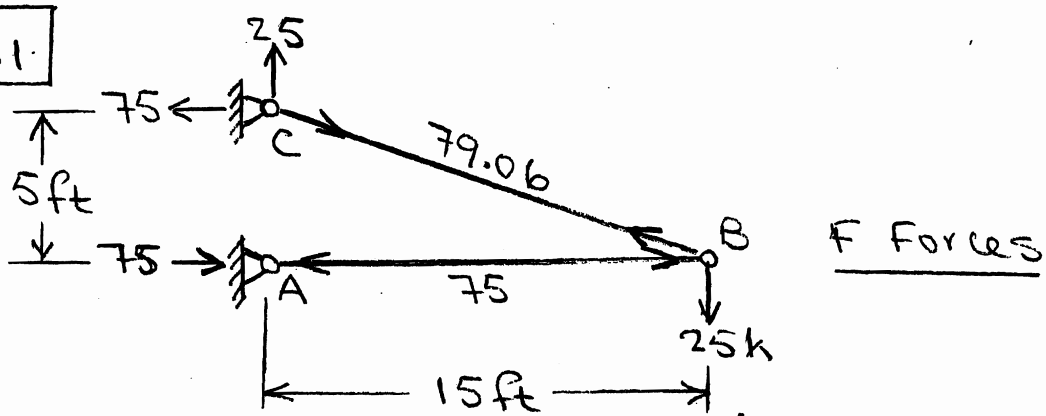
$$= -\frac{4426.67 \text{ k-ft}^2}{EI} = -0.0071 \text{ rad.} = \underline{0.0071 \text{ rad.}}$$

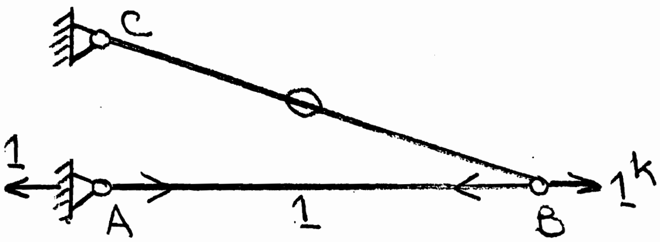
$$\Delta_D = \frac{1}{EI}\left[\frac{1}{2}(420)16(26.67) - \frac{1}{3}(240)16(28) - 4266.67(16) - \frac{1}{2}(280)16(8)\right] = -\frac{32426.67}{EI} = \underline{0.62 \text{ in} \downarrow}$$

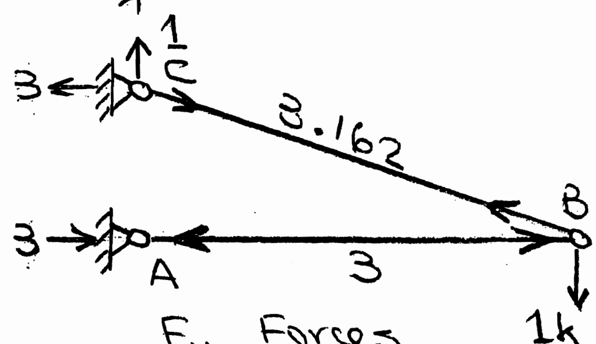
Chapter Seven
Deflections of Trusses, Beams, and Frames: Work-Energy Methods

CHAPTER 7

7.1







Member	L (in)	F (k)	F_{v1} (k)	$F_{v1}(FL)$ (k^2-in)	F_{v2} (k)	$F_{v2}(FL)$ (k^2-in)
AB	180	-75	1	-13500	-3	40500
BC	189.74	79.06	0	0	3.162	47432.67
			Σ	-13500		87932.67

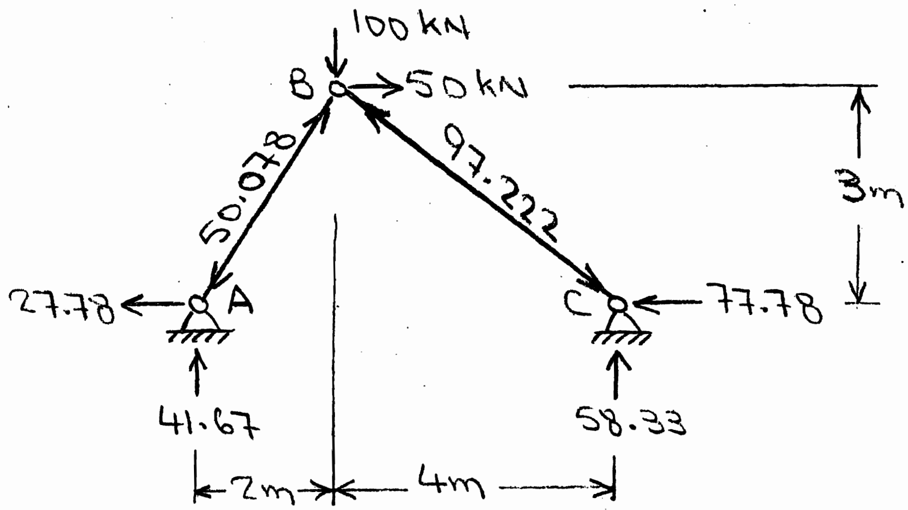
$$(1k)\,\Delta_{BH} = -\frac{13500}{10000(6)} = -0.225 \text{ k-in.}$$

$$\Delta_{BH} = -0.225 \text{ in.} = \underline{0.225 \text{ in.} \leftarrow}$$

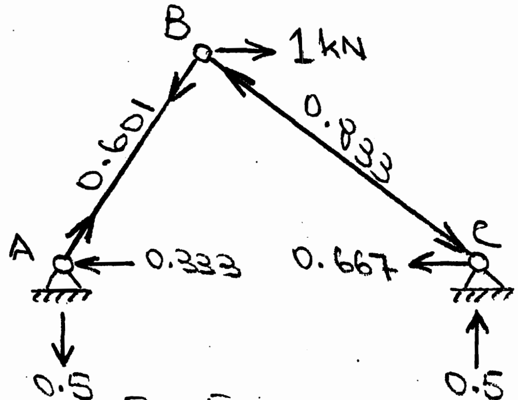
$$(1k)\,\Delta_{BV} = \frac{87932.67}{10000(6)} = 1.466 \text{ k-in.}$$

$$\underline{\Delta_{BV} = 1.466 \text{ in.} \downarrow}$$

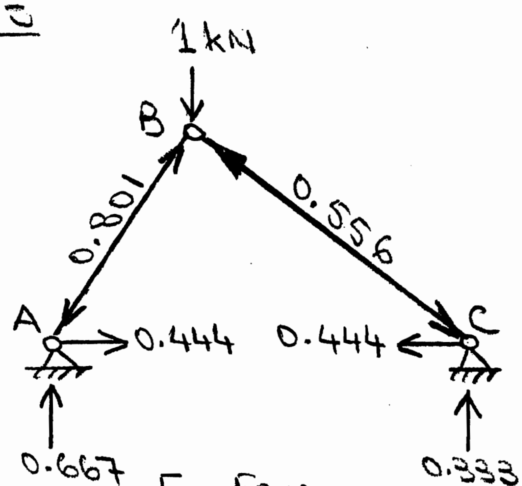
7.2



F Forces



F_{V1} Forces



F_{V2} Forces

Member	L (m)	F (kN)	F_{V1} (kN)	$F_{V1}(FL)$ (kN^2-m)	F_{V2} (kN)	$F_{V2}(FL)$ (kN^2-m)
AB	3.606	-50.078	0.601	-108.53	-0.801	144.65
BC	5	-97.222	-0.833	405.09	-0.556	270.06
			Σ	296.56		414.71

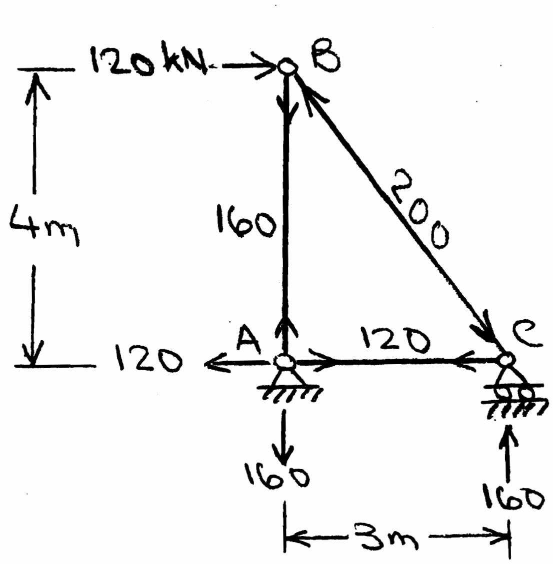
$$(1\,kN)\,\Delta_{BH} = \frac{296.56}{70(10^6)(0.001)} = 0.00424\ kN\text{-}m$$

$$\Delta_{BH} = 0.00424\ m = \underline{4.24\ mm \rightarrow}$$

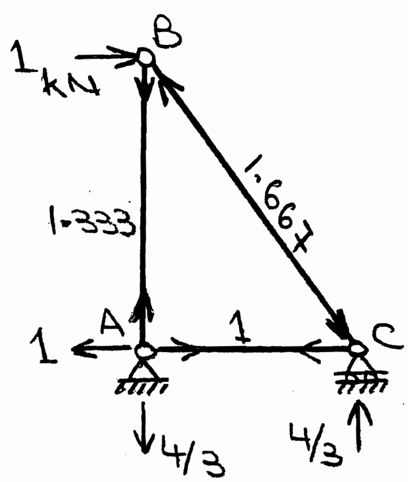
$$(1\,kN)\,\Delta_{BV} = \frac{414.71}{70(10^6)(0.001)} = 0.00592\ kN\text{-}m$$

$$\Delta_{BV} = 0.00592\ m = \underline{5.92\ mm \downarrow}$$

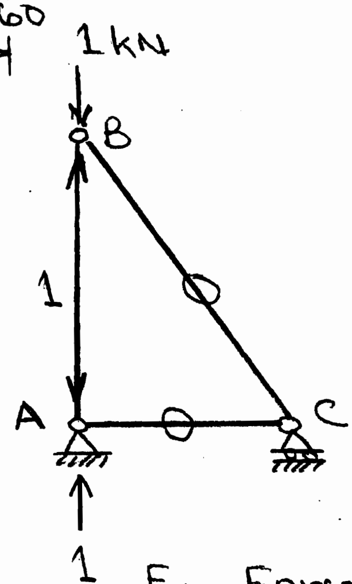
7.3



F Forces



F_{v1} Forces



F_{v2} Forces

Member	L (m)	F (kN)	F_{v1} (kN)	$F_{v1}(FL)$ (kN^2-m)	F_{v2} (kN)	$F_{v2}(FL)$ (kN^2-m)
AC	3	120	1	360	0	0
AB	4	160	1.333	853.33	−1	−640
BC	5	−200	−1.667	1666.67	0	0
			Σ	2880		−640

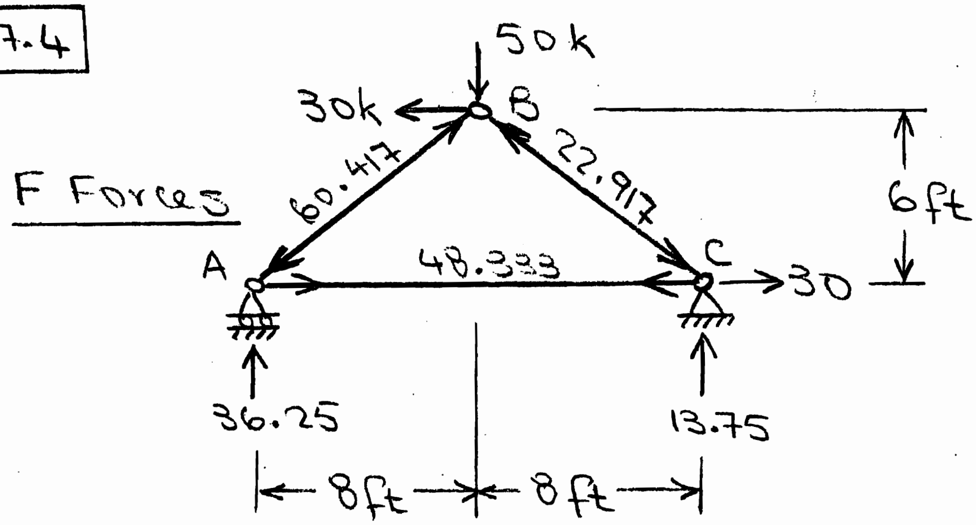
$$(1\text{kN})\,\Delta_{BH} = \frac{2880}{200(10^6)(0.0015)} = 0.0096 \text{ kN.m}$$

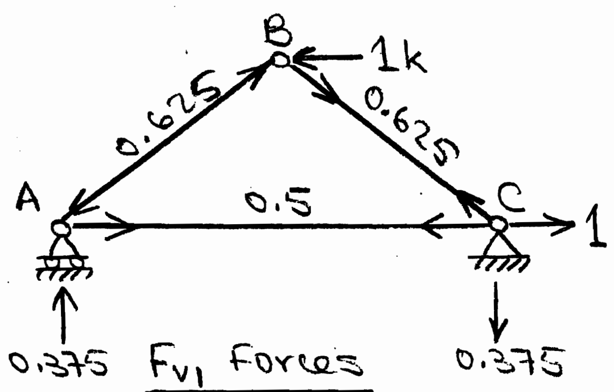
$$\Delta_{BH} = 0.0096 \text{ m} = \underline{9.6 \text{ mm} \rightarrow}$$

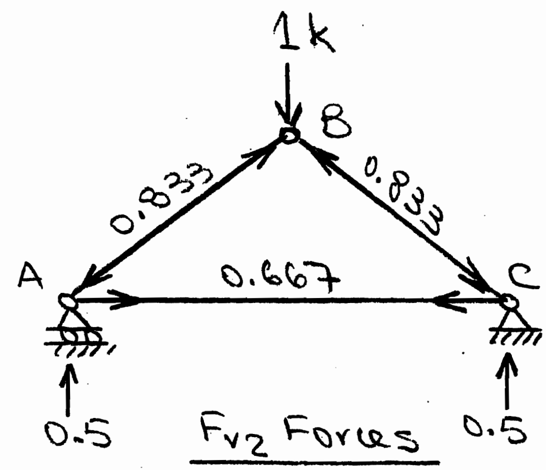
$$(1\text{kN})\,\Delta_{BV} = \frac{-640}{200(10^6)(0.0015)} = -0.00213 \text{ kN.m}$$

$$\Delta_{BV} = -0.00213 \text{ m} = \underline{2.13 \text{ mm} \uparrow}$$

7.4







Member	L (in.)	F (k)	F_{v1} (k)	F_{v1} (FL) (k^2-in.)	F_{v2} (k)	F_{v2} (FL) (k^2-in.)
AC	192	48.333	0.5	4640	0.667	6186.67
AB	120	-60.417	-0.625	4531.25	-0.833	6041.67
BC	120	-22.917	0.625	-1718.75	-0.833	2291.67
			Σ	7452.5		14520

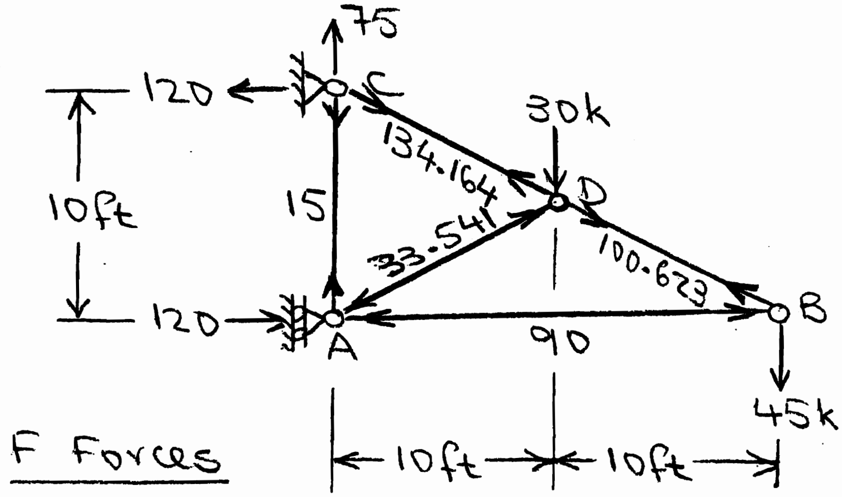
$$(1k)\Delta_{BH} = \frac{7452.5}{29000(3)} = 0.0857 \text{ k-in.}$$

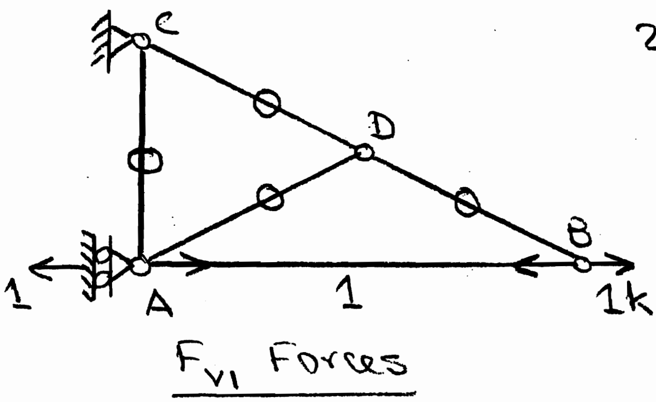
$$\Delta_{BH} = 0.0857 \text{ in.} \leftarrow$$

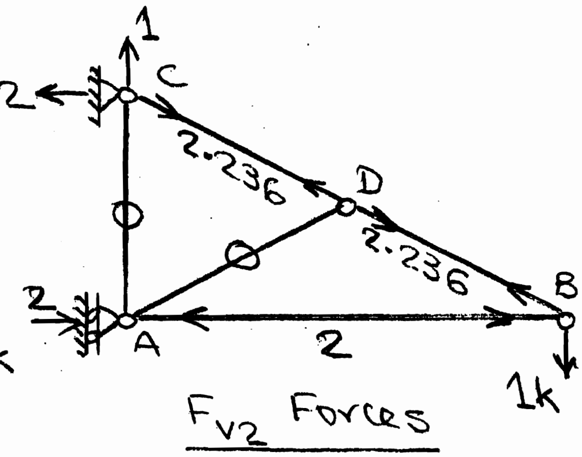
$$(1k)\Delta_{BV} = \frac{14520}{29000(3)} = 0.167 \text{ k-in.}$$

$$\Delta_{BV} = 0.167 \text{ in.} \downarrow$$

7.5







Member	L (in.)	A (in.²)	F (k)	F_{V1} (k)	$F_{V1}(FL/A)$ (k²/in.)	F_{V2} (k)	$F_{V2}(FL/A)$ (k²/in.)
AB	240	6	−90	1	−3600	−2	7200
AC	120	4	15	0	0	0	0
AD	134.16	4	−33.541	0	0	0	0
CD	134.16	6	134.164	0	0	2.236	6707.99
BD	134.16	6	100.623	0	0	2.236	5030.99
				Σ	−3600		18938.98

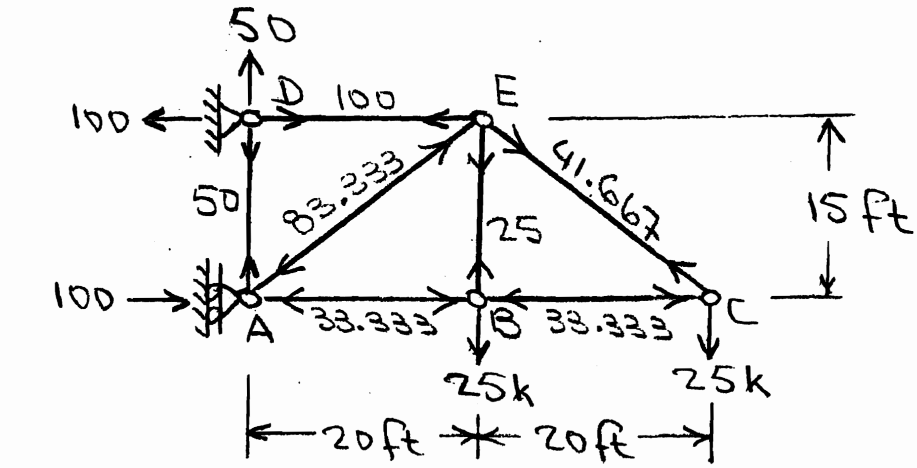
$$(1k)\Delta_{BH} = \frac{-3600}{10000} = -0.36 \text{ k-in}$$

$$\Delta_{BH} = 0.36 \text{ in.} \leftarrow$$

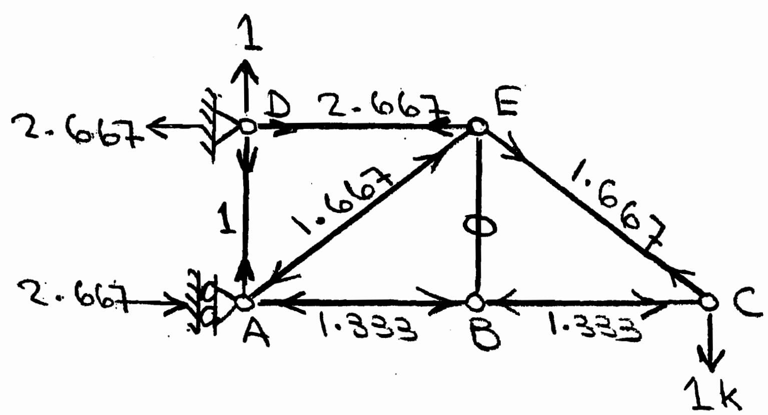
$$(1k)\Delta_{BV} = \frac{18938.99}{10000} = 1.894 \text{ k-in.}$$

$$\Delta_{BV} = 1.894 \text{ in.} \downarrow$$

7.6



F Forces

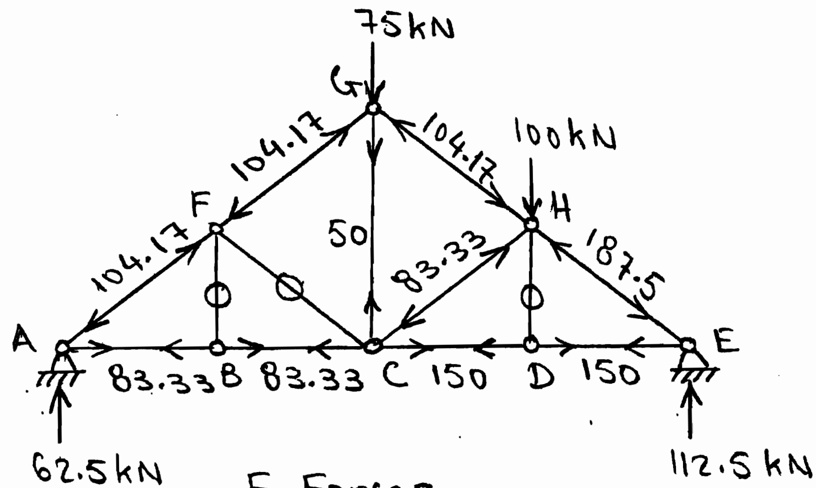


F_v Forces

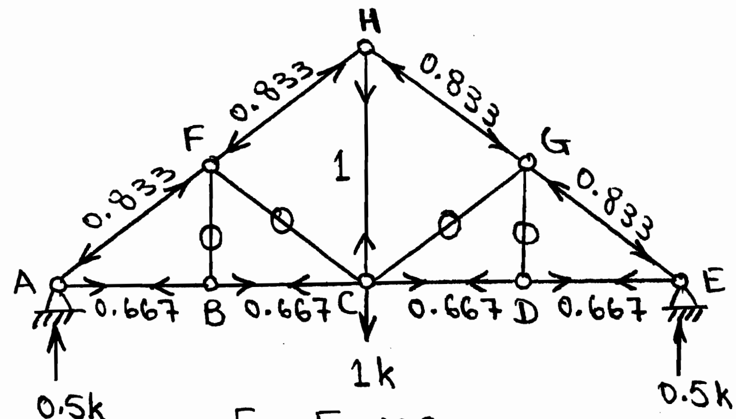
Member	L (in.)	A (in².)	F (k)	F_v (k)	F_v (FL/A) (k²/in.)
AB	240	4	-33.333	-1.333	2666.67
BC	240	4	-33.333	-1.333	2666.67
DE	240	4	100	2.667	16000
AD	180	3	50	1	3000
BE	180	3	25	0	0
AE	300	3	-83.333	-1.667	13888.89
CE	300	4	41.667	1.667	5208.33
				Σ	43430.56

$$\Delta_C = \frac{43430.56}{29000} = 1.498 \text{ in.} \downarrow$$

7.7



F Forces



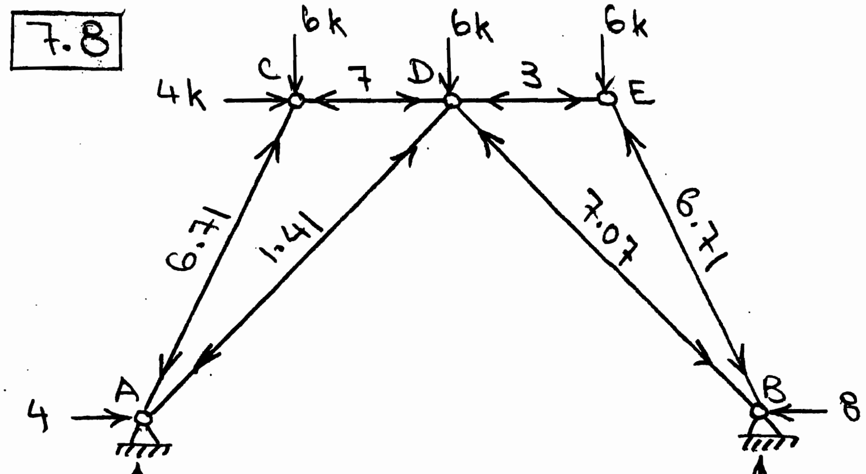
F_v Forces

Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ (kN^2-m)
AB	6	83.33	0.667	333.5
BC	6	83.33	0.667	333.5
CD	6	150	0.667	600.3
DE	6	150	0.667	600.3
AF	7.5	-104.17	-0.833	650.8
FG	7.5	-104.17	-0.833	650.8
GH	7.5	-104.17	-0.833	650.8
EH	7.5	-187.5	-0.833	1171.4
CG	9	50	1	450
			Σ	5441.4

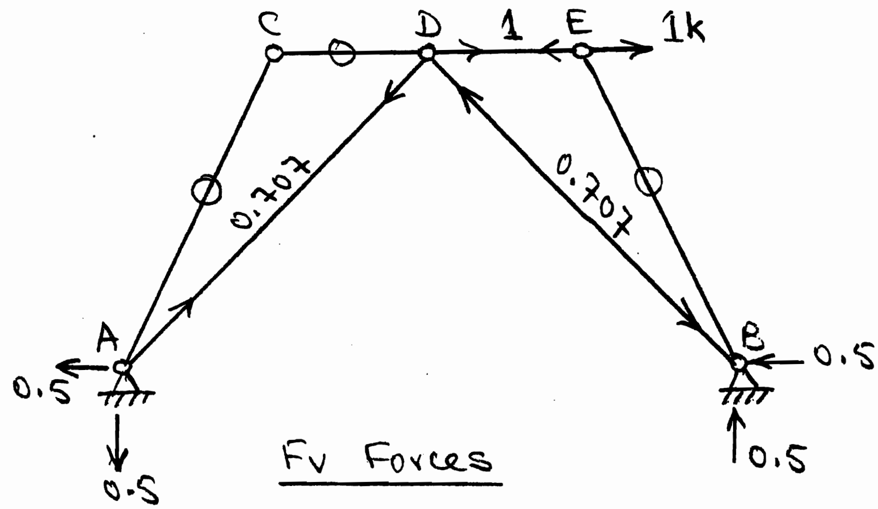
$$(1\text{kN})\,\Delta_C = \frac{5441.4}{200(10^6)(0.003)} = 0.0091 \text{ kN-m}$$

$$\Delta_C = 0.0091\text{ m} = \underline{9.1\text{ mm}\downarrow}$$

7.8



F Forces



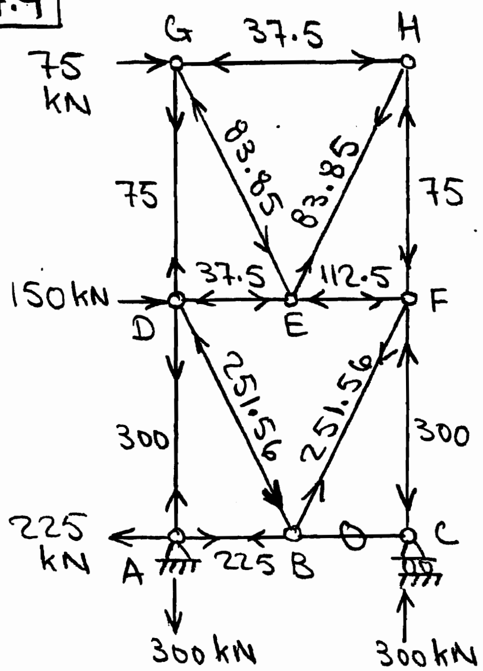
Fv Forces

Member	L (ft)	F (k)	Fv (k)	Fv (FL) (k^2-ft)
AD	14.14	-1.41	0.707	-14.1
BD	14.14	-7.07	-0.707	70.68
DE	5	-3	1	-15
			Σ	41.58

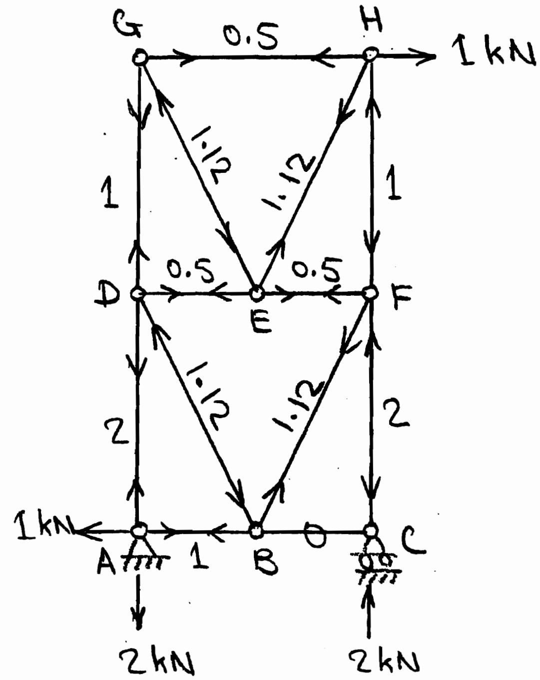
$$\Delta_{EH} = \frac{41.58}{(29000)6} = 0.000\,239 \text{ ft.}$$

$$= \underline{0.0029 \text{ in.} \rightarrow}$$

7.9



F Forces



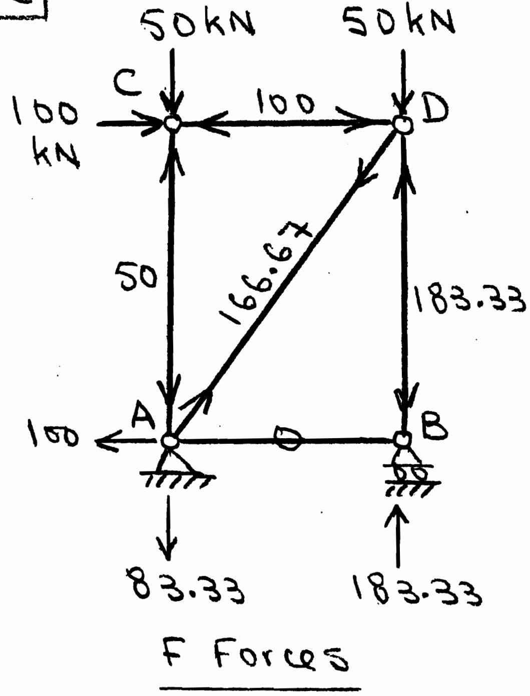
F_v Forces

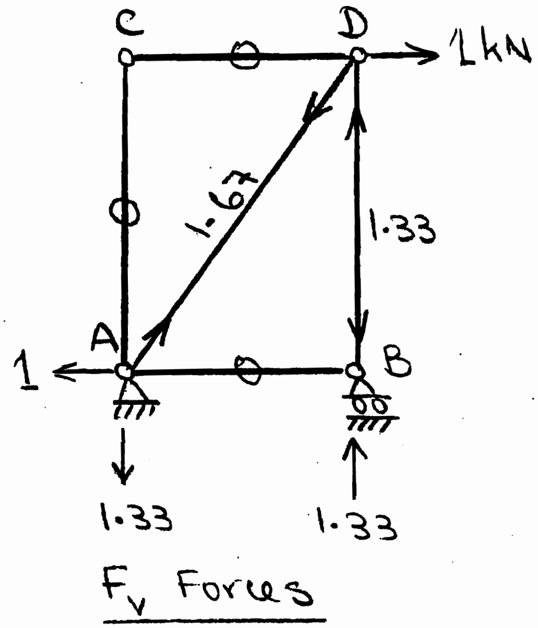
Member	L (m)	A (m^2)	F (kN)	F_v (kN)	$F_v(FL/A)$ (kN^2/m)
AD	4	0.0025	300	2	960 000
CF	4	0.0025	−300	−2	960 000
DG	4	0.0025	75	1	120 000
FH	4	0.0025	−75	−1	120 000
GH	4	0.0015	−37.5	0.5	−50 000
AB	2	0.0015	225	1	300 000
DE	2	0.0015	−37.5	0.5	−25 000
EF	2	0.0015	−112.5	0.5	−75 000
BD	4.47	0.0015	−251.56	−1.12	839607
BF	4.47	0.0015	251.56	1.12	839607
EG	4.47	0.0015	−83.85	−1.12	279858
EH	4.47	0.0015	83.85	1.12	279858
				Σ	4548930

$$(1\,kN)\,\Delta_H = \frac{4548930}{200(10^6)} = 0.023 \text{ kN-m}$$

$$\Delta_H = 0.023 \text{ m} = \underline{23.\text{ mm} \rightarrow}$$

7.10





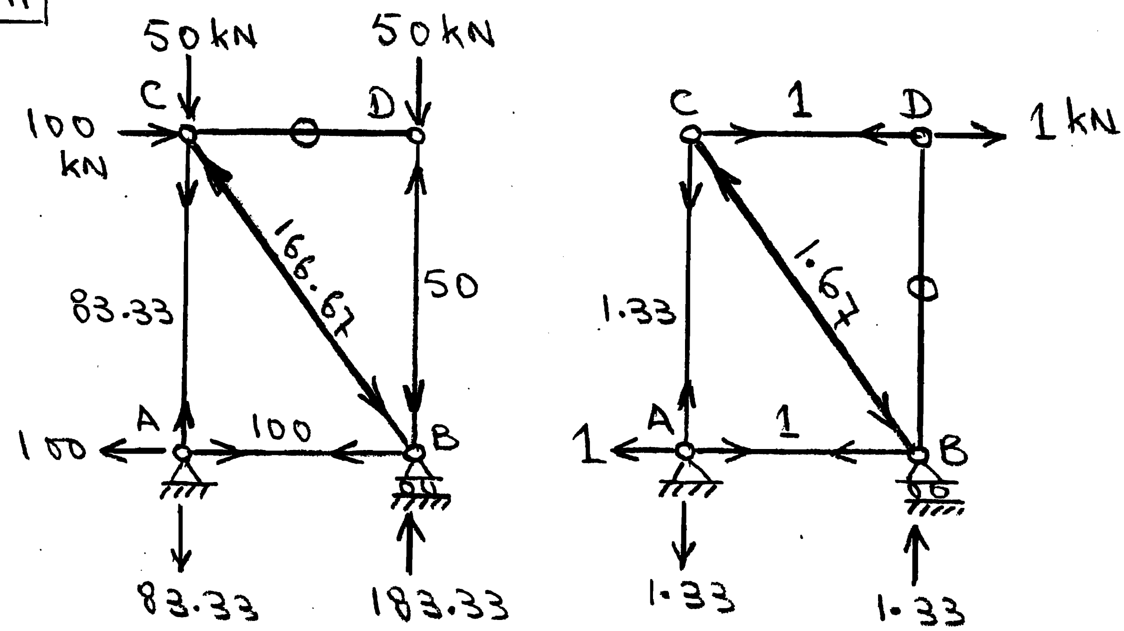
Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ ($kN^2 \cdot m$)
AB	3	0	0	0
CD	3	-100	0	0
AC	4	-50	0	0
BD	4	-183.33	-1.33	975.32
AD	5	166.67	1.67	1391.69
			Σ	2367

$$(1\,kN)\Delta_D = \frac{2367\,kN^2 \cdot m}{EA}$$

$$\Delta_D = \frac{2367\,kN \cdot m}{EA} = \frac{2367}{70(10^6)A} = 0.01\,m$$

from which, $A = 0.003381\,m^2 = \underline{3381\,mm^2}$

7.11



F Forces

F_v Forces

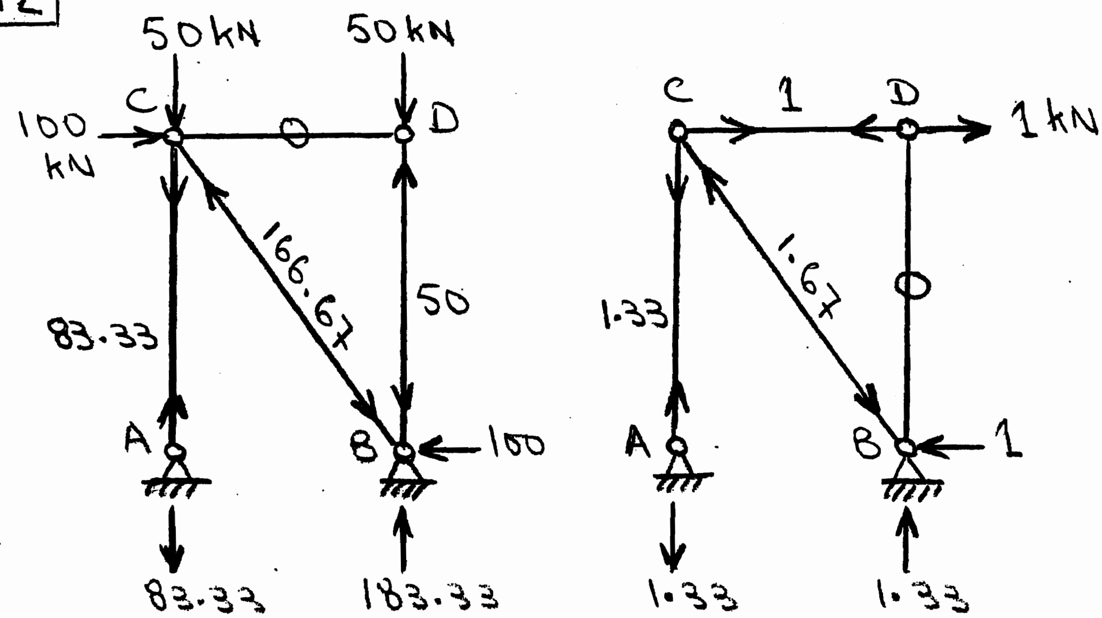
Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ ($kN^2 \cdot m$)
AB	3	100	1	300
CD	3	0	1	0
AC	4	83.33	1.33	443.32
BD	4	-50	0	0
BC	5	-166.67	-1.67	1391.69
			Σ	2135

$$(1\text{ kN})\Delta_D = \frac{2135\text{ kN}^2\cdot\text{m}}{EA}$$

$$\Delta_D = \frac{2135\text{ kN}\cdot\text{m}}{EA} = \frac{2135}{70(10^6)A} = 0.01\text{ m}$$

from which, $A = 0.00305\text{ m}^2 = \underline{3050\text{ mm}^2}$

7.12



F Forces $\qquad$ F_v Forces

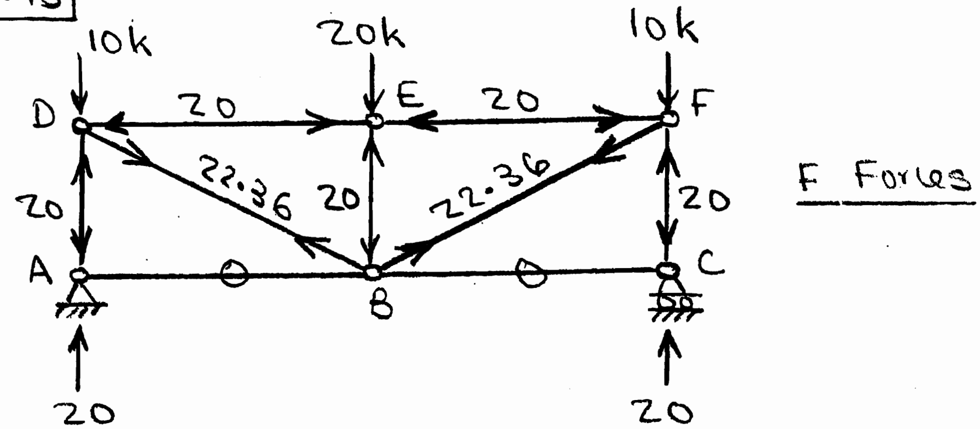
Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ ($kN^2 \cdot m$)
CD	3	0	1	0
AC	4	83.33	1.33	443.32
BD	4	−50	0	0
BC	5	−166.67	−1.67	1391.69
			Σ	1835

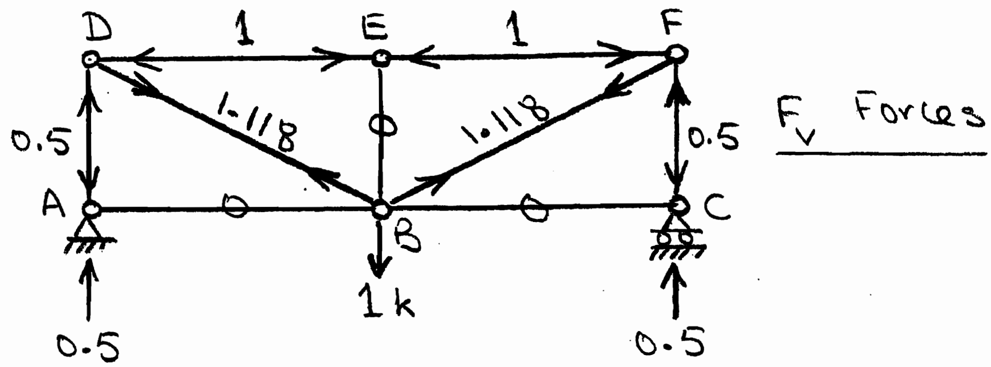
$$(1\text{ kN})\Delta_D = \frac{1835 \text{ kN}^2\cdot\text{m}}{EA}$$

$$\Delta_D = \frac{1835 \text{ kN}\cdot\text{m}}{EA} = \frac{1835}{70(10^6)A} = 0.01 \text{ m}$$

from which, $A = 0.002621 \text{ m}^2 = \underline{2621 \text{ mm}^2}$

7.13





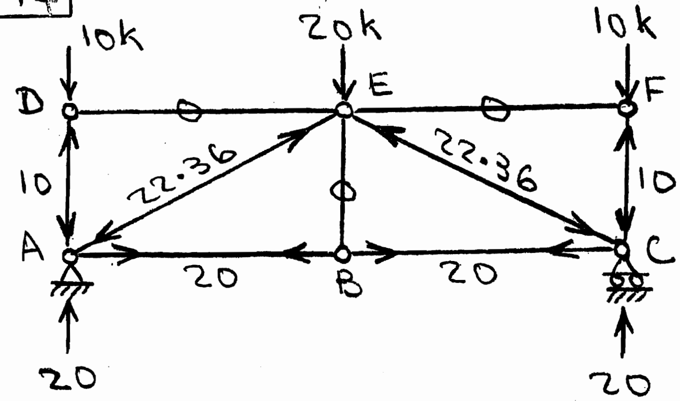
Member	L (in)	F (k)	F_v (k)	F_v(FL) (k^2-in)
AB	72	0	0	0
BC	72	0	0	0
DE	72	-20	-1	1440
EF	72	-20	-1	1440
AD	36	-20	-0.5	360
BE	36	-20	0	0
CF	36	-20	-0.5	360
BD	80.5	22.36	1.118	2012.4
BF	80.5	22.36	1.118	2012.4
			Σ	7624.8

$$(1k)\Delta_B = \frac{7624.8\ k^2\text{-in}}{EA}$$

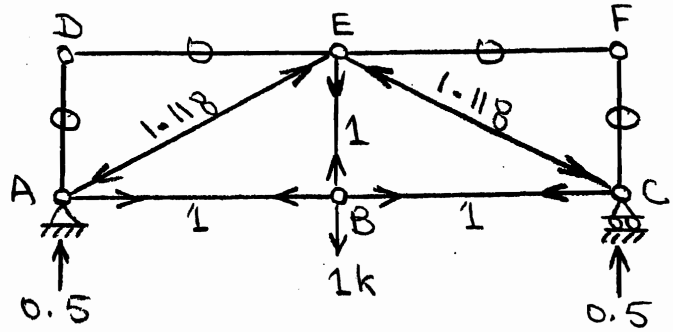
$$\Delta_B = \frac{7624.8\ k\text{-in}}{EA} = \frac{7624.8}{1600\,(A)} = 0.4 \text{ in}$$

from which, $\underline{A = 11.91 \text{ in}^2}$

7-14



F Forces



F_v Forces

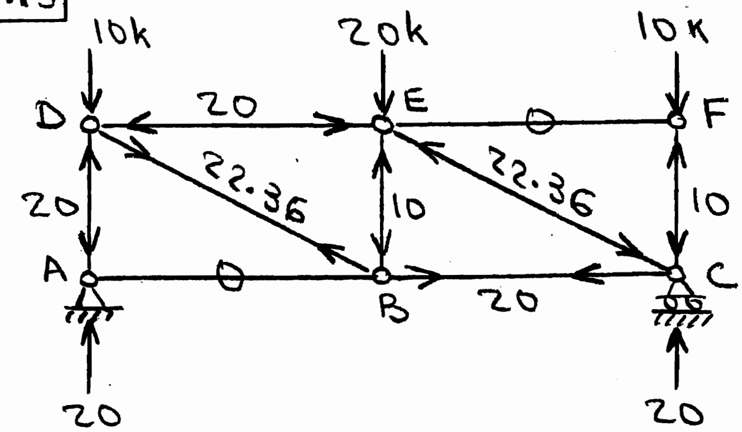
Member	L (in.)	F (k)	F_v (k)	$F_v(FL)$ (k^2-in)
AB	72	20	1	1440
BC	72	20	1	1440
DE	72	0	0	0
EF	72	0	0	0
AD	36	-10	0	0
BE	36	0	1	0
CF	36	-10	0	0
AE	80.5	-22.36	-1.118	2012.4
CE	80.5	-22.36	-1.118	2012.4
			Σ	6904.8

$$(1k)\Delta_B = \frac{6904.8 \text{ k}^2\text{-in}}{EA}$$

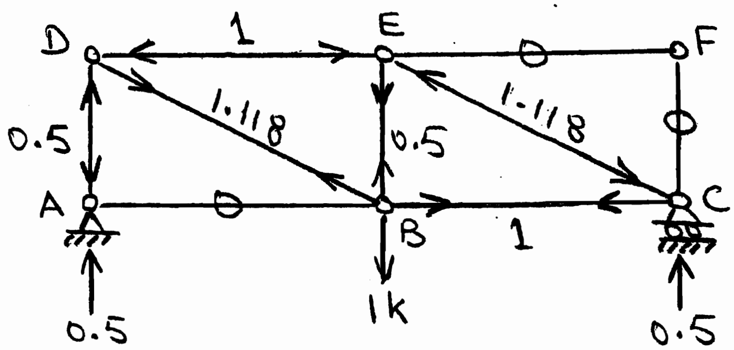
$$\Delta_B = \frac{6904.8 \text{ k-in}}{EA} = \frac{6904.8}{1600(A)} = 0.4 \text{ in}$$

from which, $A = \underline{10.79 \text{ in}^2}$

7.15



F Forces



F_v Forces

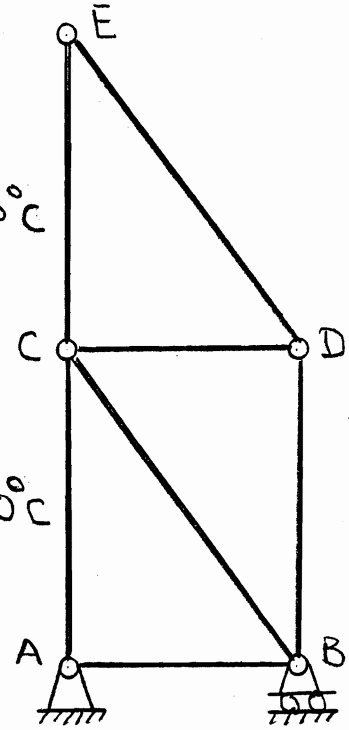
Member	L (in.)	F (k)	F_v (k)	$F_v(FL)$ (k^2-in.)
AB	72	0	0	0
BC	72	20	1	1440
DE	72	-20	-1	1440
EF	72	0	0	0
AD	36	-20	-0.5	360
BE	36	-10	0.5	-180
CF	36	-10	0	0
BD	80.5	22.36	1.118	2012.4
CE	80.5	-22.36	-1.118	2012.4
			Σ	7084.8

$$(1k)\Delta_B = \frac{7084.8 \text{ k}^2\text{-in.}}{EA}$$

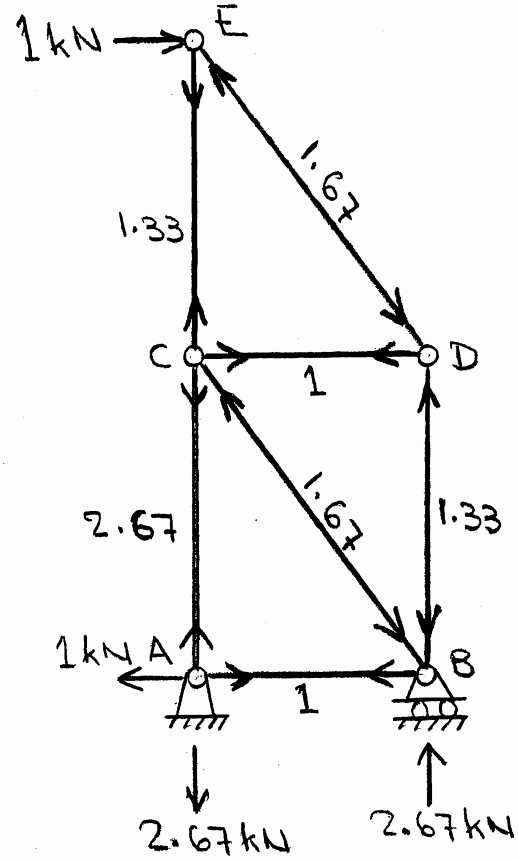
$$\Delta_B = \frac{7084.8 \text{ k-in.}}{EA} = \frac{7084.8}{1600(A)} = 0.4 \text{ in}$$

from which, $A = 11.07 \text{ in}^2$

7.16



Real System – ΔT



Virtual System – F_v Forces

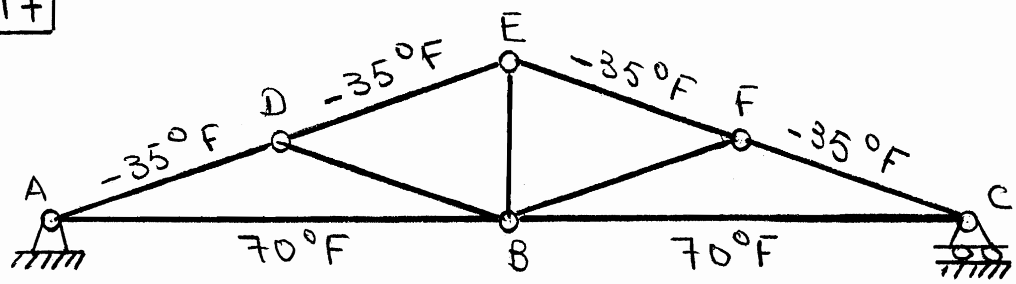
Member	F_v (kN)
AC	2.67
CE	1.33
Σ	4

$$(1\,kN)\,\Delta_E = \alpha(\Delta T)L\sum F_v = 1.2(10^{-5})(50)(4)(4)$$

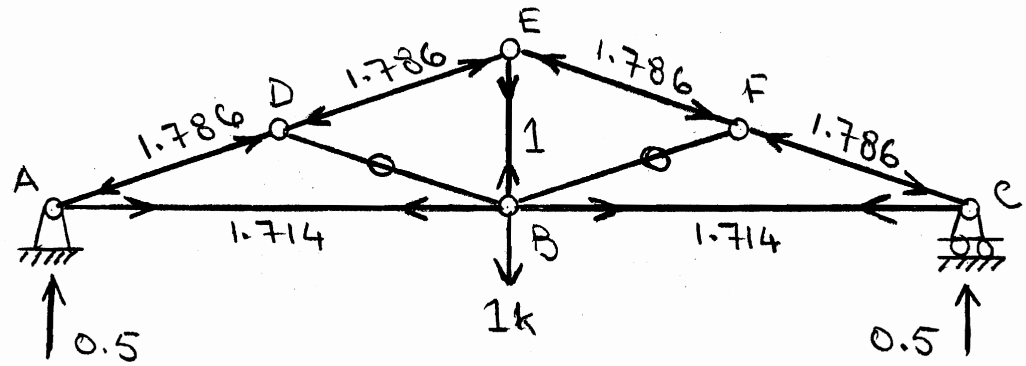
$$= 0.0096 \text{ kN.m}$$

$$\Delta_E = 0.0096\text{ m} = \underline{9.6\text{ mm}} \rightarrow$$

7.17



Real System – ΔT



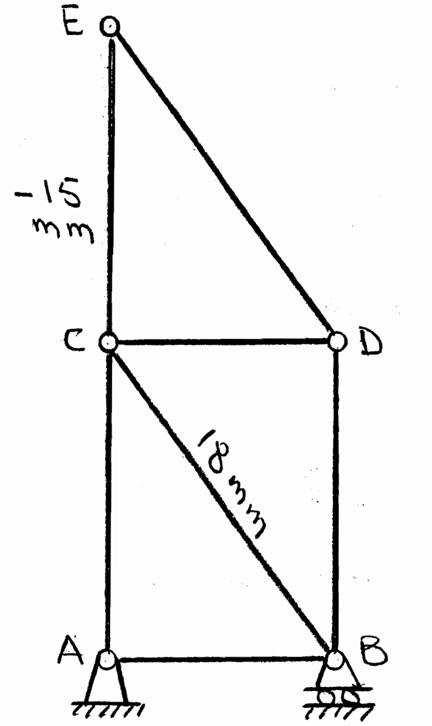
Virtual System – F_v Forces

Member	L (in.)	ΔT (°F)	F_v (k)	$F_v(\Delta T)L$ (k-in-°F)
AB	288	70	1.714	34560
BC	288	70	1.714	34560
AD	150	-35	-1.786	9375
DE	150	-35	-1.786	9375
EF	150	-35	-1.786	9375
CF	150	-35	-1.786	9375
			Σ	106620

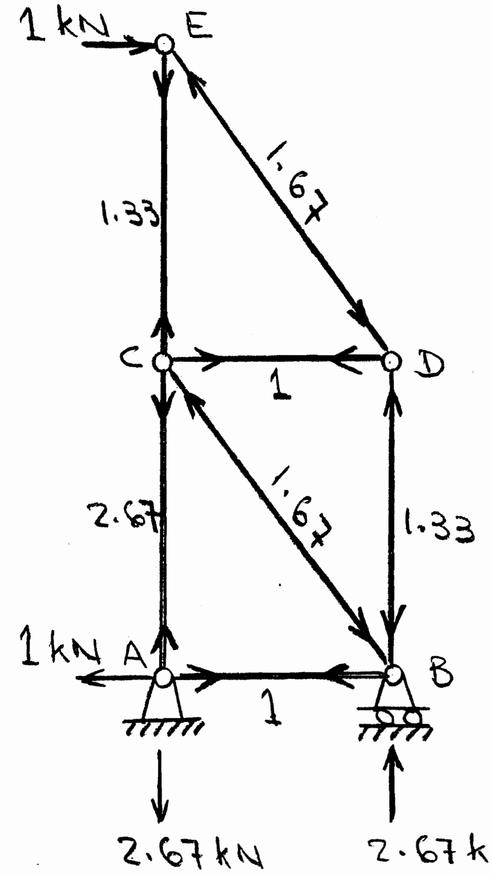
$$(1k)\Delta_B = \alpha \sum F_v(\Delta T)L = 6.5(10^{-6})(106620)$$

$$\Delta_B = 0.693 \text{ in.}\downarrow$$

7.18



Real System – δ



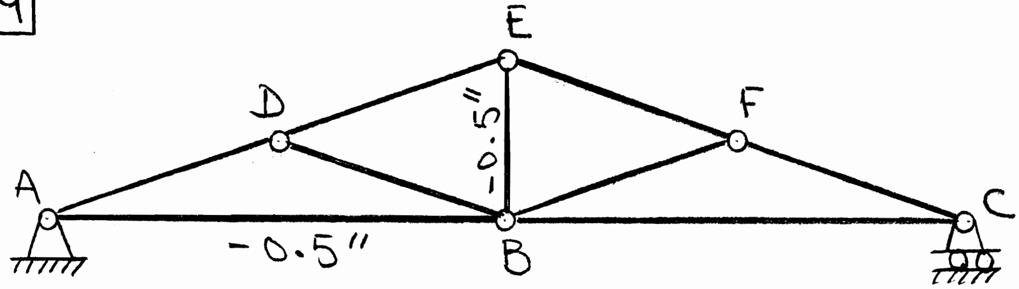
Virtual System – F_v Forces

Member	δ (mm)	F_v (kN)	$F_v(\delta)$ (kN·mm)
BC	18	-1.67	-30
CE	-15	1.33	-20
		Σ	-50

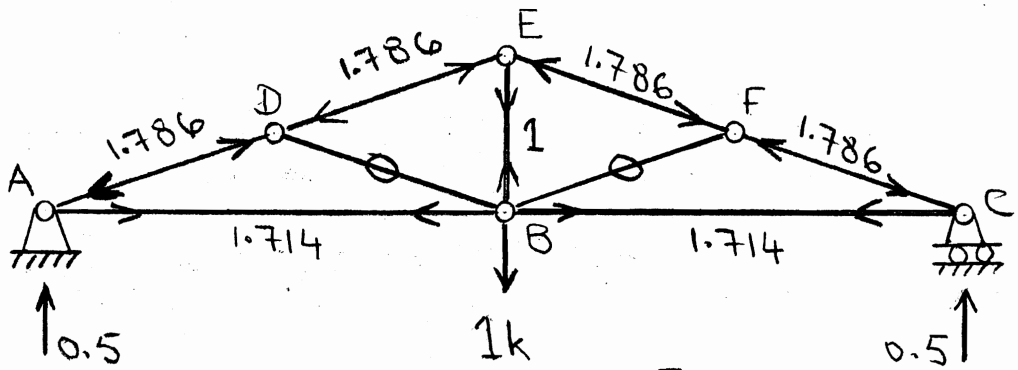
$$(1\text{ kN})\Delta_E = \Sigma F_v(\delta) = -50 \text{ kN}\cdot\text{mm}$$

$$\Delta_E = -50\text{ mm} = \underline{50 \text{ mm} \leftarrow}$$

7.19



Real System – δ



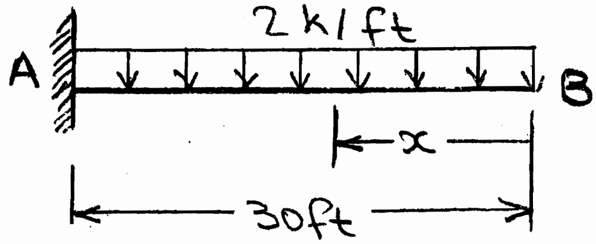
Virtual System – F_v Forces

Member	F_v (k)
AB	1.714
BE	1
Σ	2.714

$$(1k)\,\Delta_B = \delta \Sigma F_v = (-0.5)2.714 = -1.357 \text{ k-in.}$$

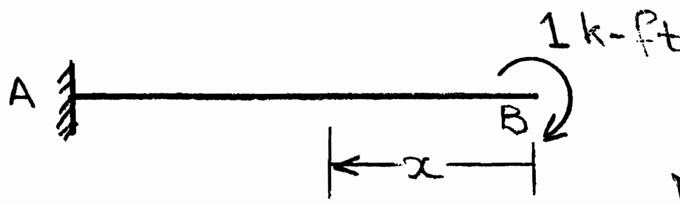
$$\Delta_B = -1.357 \text{ in.} = \underline{1.357 \text{ in.}\uparrow}$$

7.20



$$M = -\frac{2x^2}{2} = -x^2$$

Real System – M

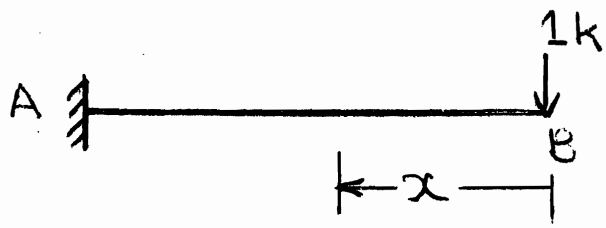


$$M_{v1} = -1$$

Virtual System – M_{v1}

$$\theta_B = \frac{1}{EI}\int_0^{30} -1(-x^2)\,dx = \frac{9000 \text{ k-ft}^2}{EI}$$

$$= \frac{9000(12)^2}{29000(3000)} = \underline{0.0149 \text{ rad}}$$



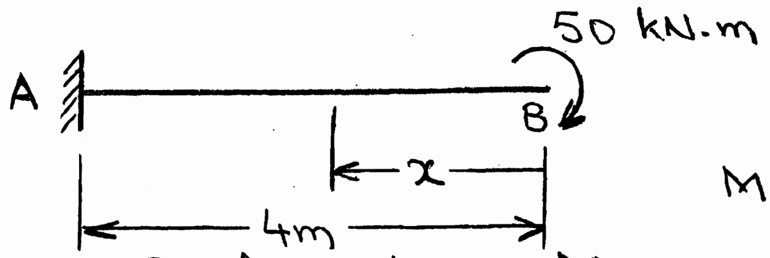
$$M_{v2} = -1(x) = -x$$

Virtal System M_{v2}

$$\Delta_B = \frac{1}{EI}\int_0^{30} (-x)(-x^2)\,dx = \frac{202500 \text{ k-ft}^3}{EI}$$

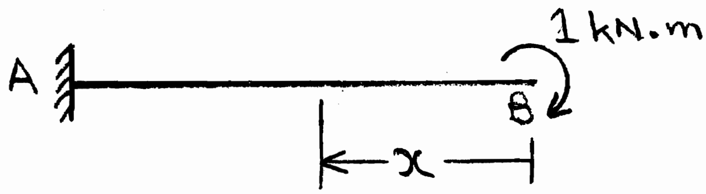
$$= \frac{202500(12)^3}{29000(3000)} = \underline{4.022 \text{ in.}\downarrow}$$

7.21



$M = -50$

Real System – M

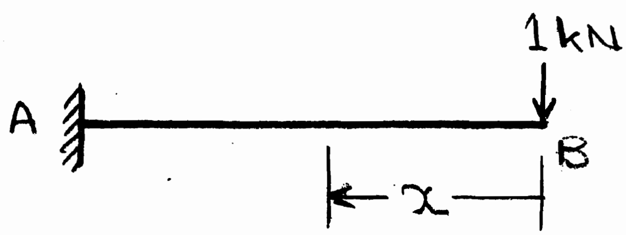


$M_{V1} = -1$

Virtual System – M_{V1}

$$\theta_B = \frac{1}{EI}\int_0^4 (-1)(-50)\,dx = \frac{200 \text{ kN.m}^2}{EI}$$

$$= \frac{200}{70(164)} = \underline{0.0174 \text{ rad}}$$



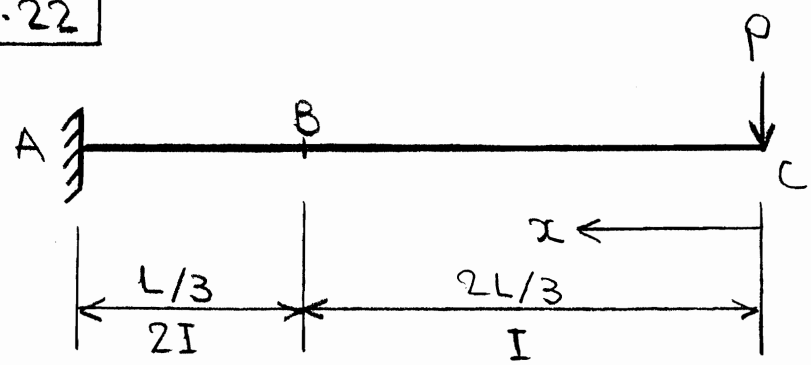
$M_{V2} = -1(x) = -x$

Virtual System – M_{V2}

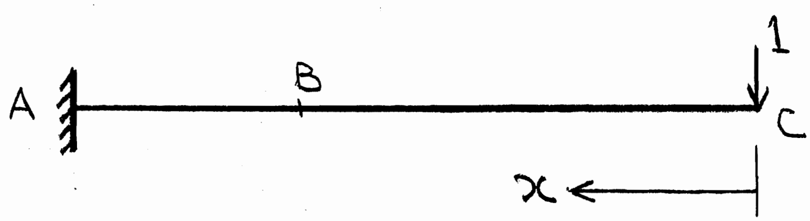
$$\Delta_B = \frac{1}{EI}\int_0^4 (-x)(-50)\,dx = \frac{400 \text{ kN.m}^3}{EI}$$

$$= \frac{400}{70(164)} = 0.0348 \text{ m} = \underline{34.8 \text{ mm}\downarrow}$$

7.22



Real (M)



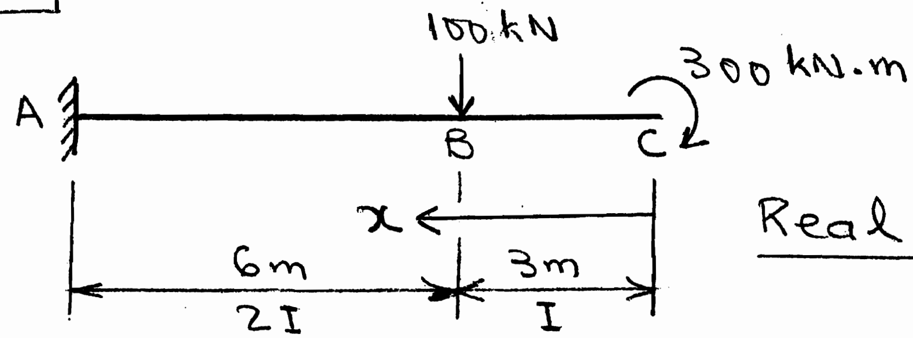
Virtual (M_v)

Segment	x Coordinate		M	M_v
	Origin	Limits		
CB	C	$0 - \frac{2L}{3}$	$-Px$	$-1x$
BA	C	$\frac{2L}{3} - L$	$-Px$	$-1x$

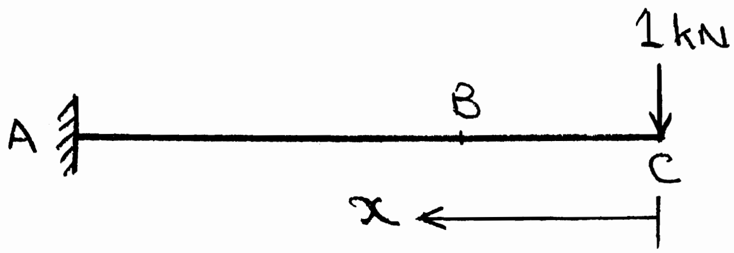
$$\Delta_C = \frac{1}{EI}\left[\int_0^{\frac{2L}{3}} (-x)(-Px)\,dx + \frac{1}{2}\int_{\frac{2L}{3}}^{L} (-x)(-Px)\,dx\right]$$

$$= \frac{35PL^3}{162EI}\downarrow$$

7.23



Real (M)



Virtual (M_v)

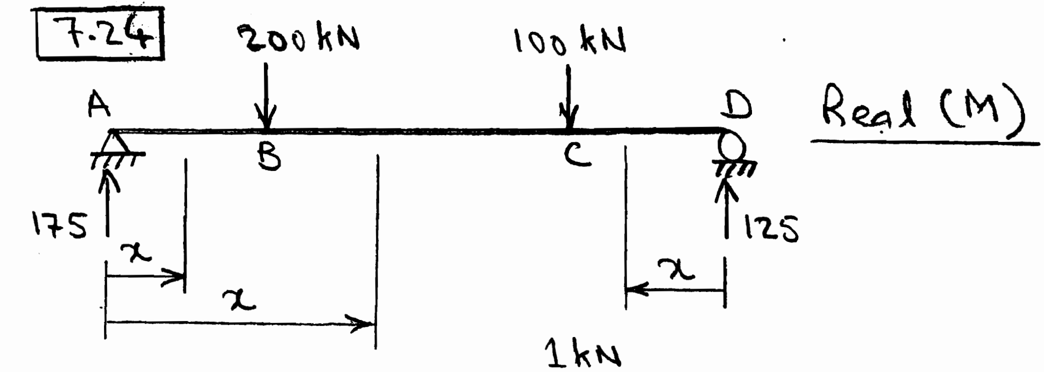
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
CB	C	0 – 3	−300	−x
BA	C	3 – 9	−300 −100(x−3)	−x

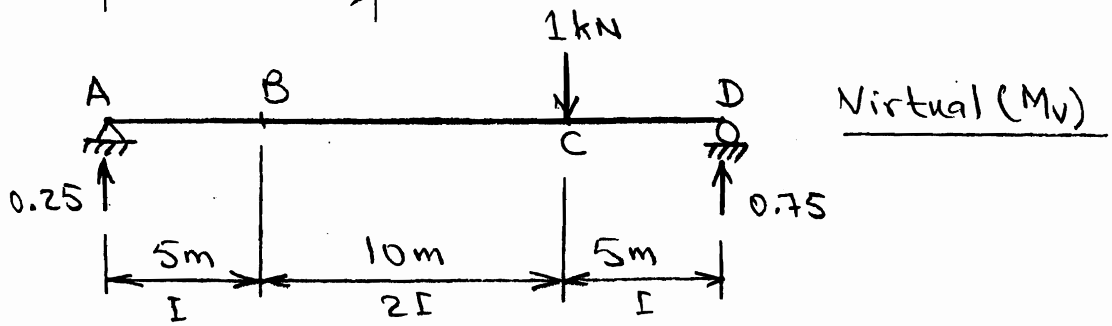
$$\Delta_C = \frac{1}{EI}\left[\int_0^3 (-x)(-300)\,dx + \frac{1}{2}\int_3^9 (-x)(-100x)\,dx\right]$$

$$= \frac{13050 \text{ kN.m}^3}{EI} = \frac{13050}{70(500)} = 0.373 \text{ m}$$

$$\Delta_C = 373 \text{ mm} \downarrow$$

7.24





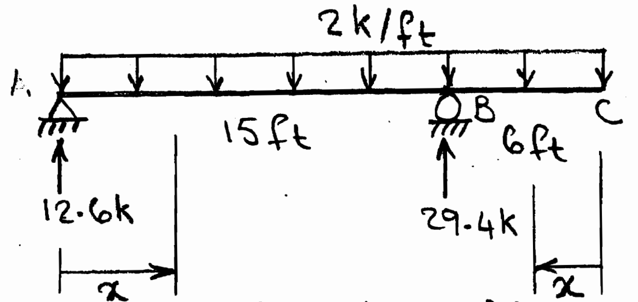
Segment	x Coordinate		M (kN-m)	M_v (kN-m)
	Origin	Limits (m)		
AB	A	0 – 5	$175x$	$0.25x$
BC	A	5 – 15	$175x - 200(x-5)$	$0.25x$
DC	D	0 – 5	$125x$	$0.75x$

$$\Delta_C = \frac{1}{EI}\Big[\int_0^5 (0.25x)(175x)\,dx + \frac{1}{2}\int_5^{15} 0.25x(-25x+1000)\,dx + \int_0^5 0.75x(125x)\,dx\Big]$$

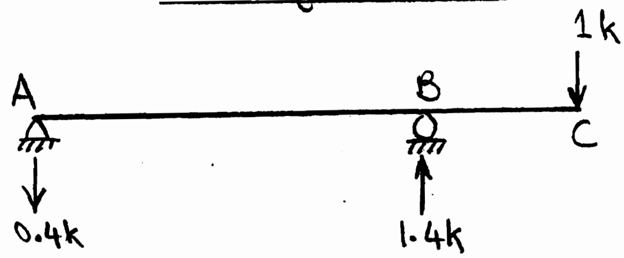
$$= \frac{14843.75 \text{ kN-m}^3}{EI} = \frac{14843.75}{250(600)}$$

$$= 0.099\text{m} = \underline{99\text{ mm}\downarrow}$$

7.25



Real System – M



Virtual System – M_v

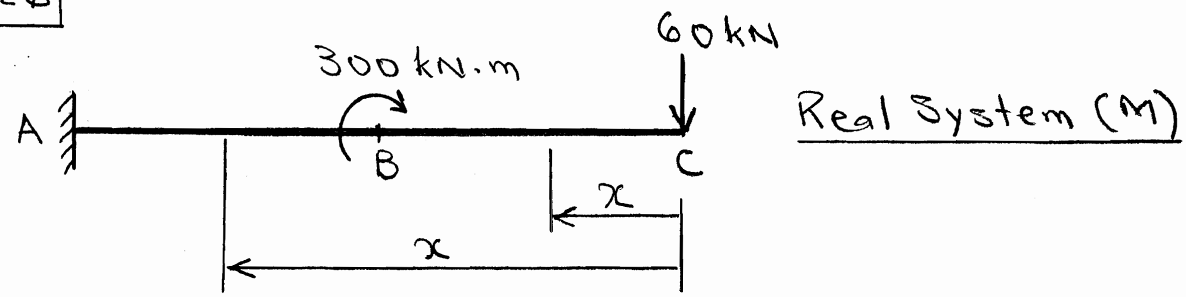
Segment	x Coordinate: Origin	x Coordinate: Limits (ft)	M (k-ft)	M_v (k-ft)
AB	A	0 – 15	$12.6x - x^2$	$-0.4x$
CB	C	0 – 6	$-x^2$	$-x$

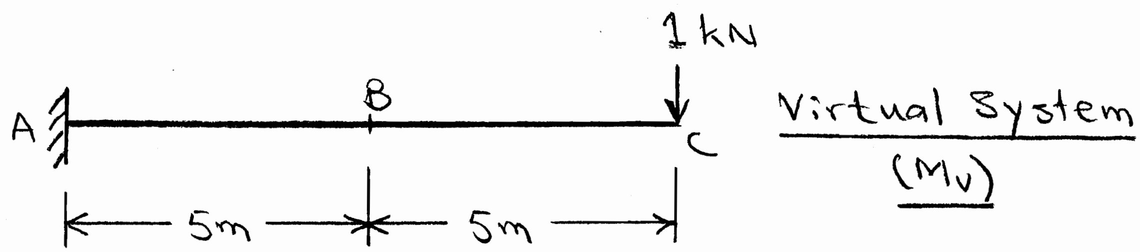
$$(1k)\Delta_C = \frac{1}{EI}\left[\int_0^{15} -0.4x(12.6x - x^2)\,dx + \int_0^{6} -x(-x^2)\,dx\right]$$

$$= -\frac{283.5 \text{ k}^2\text{-ft}^3}{EI}$$

$$\Delta_C = -\frac{283.5(12)^3}{(29000)(3500)} = -0.0048 \text{ in.} = \underline{0.0048 \text{ in.}\uparrow}$$

7.26





Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
CB	C	0 – 5	$-60x$	$-1x$
BA	C	5 – 10	$-60x-300$	$-1x$

$$\Delta_{max} = \Delta_C = \frac{1}{EI}\left[\int_0^5 (-1x)(-60x)\,dx + \int_5^{10}(-1x)(-60x-300)\,dx\right]$$

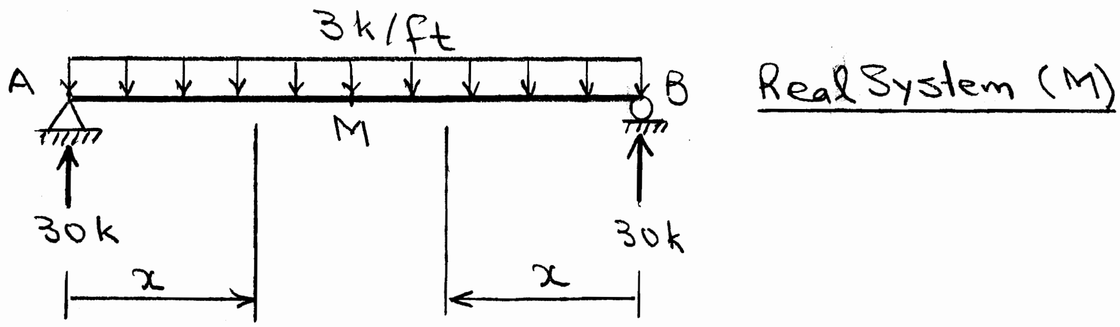
$$= \frac{31250 \text{ kN.m}^3}{EI} \downarrow$$

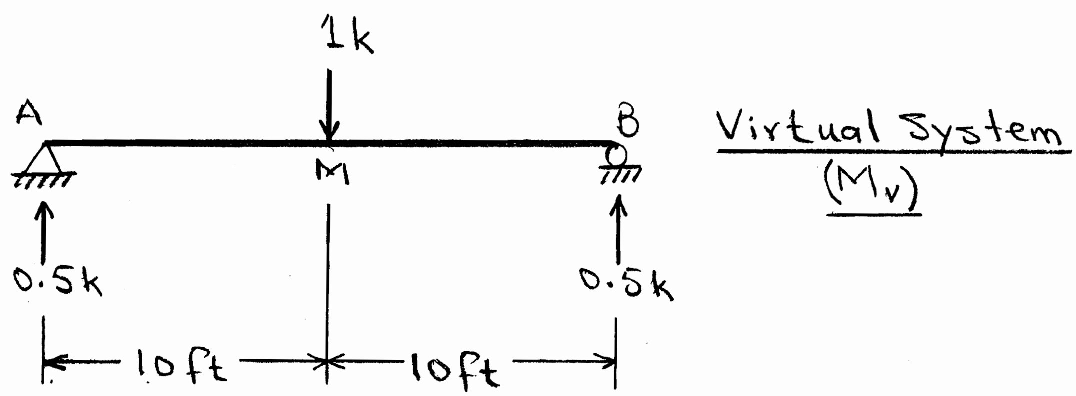
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

from which, $I = 5625(10^{-6})\,m^4 = \underline{5625(10^6)\,mm^4}$

7.27





Segment	x Coordinate: Origin	x Coordinate: Limits (ft)	M (k-ft)	M_v (k-ft)
AM	A	0 – 10	$30x-\frac{3x^2}{2}$	$0.5x$
BM	B	0 – 10	$30x-3\frac{x^2}{2}$	$0.5x$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[2\int_0^{10}(0.5x)\left(30x-\frac{3x^2}{2}\right)dx\right]$$

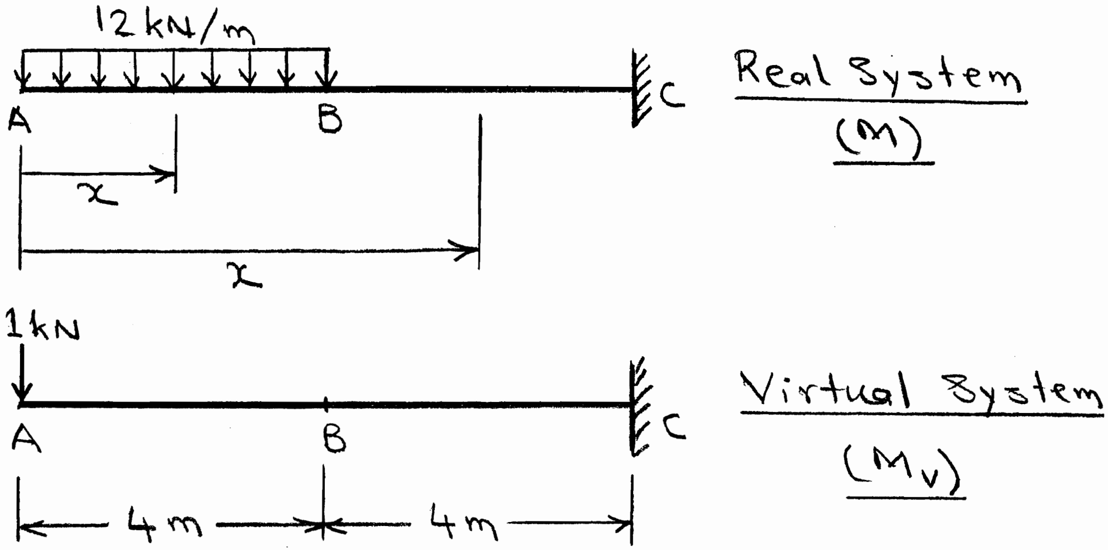
$$= \frac{6250 \text{ k-ft}^3}{EI}$$

$$\Delta_{max} = \frac{L}{360}$$

$$\frac{6250\,(12)^3}{29000\,(I)} = \frac{20(12)}{360}$$

from which, $\underline{I = 559 \text{ in}^4}$

7.28



Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0 – 4	$-12x^2/2$	$-1x$
BC	A	4 – 8	$-12(4)(x-2)$	$-1x$

$$\Delta_{max} = \Delta_A = \frac{1}{EI}\left[\int_0^4 (-1x)(-6x^2)\,dx + \int_4^8 (-1x)(-48)(x-2)\,dx\right]$$

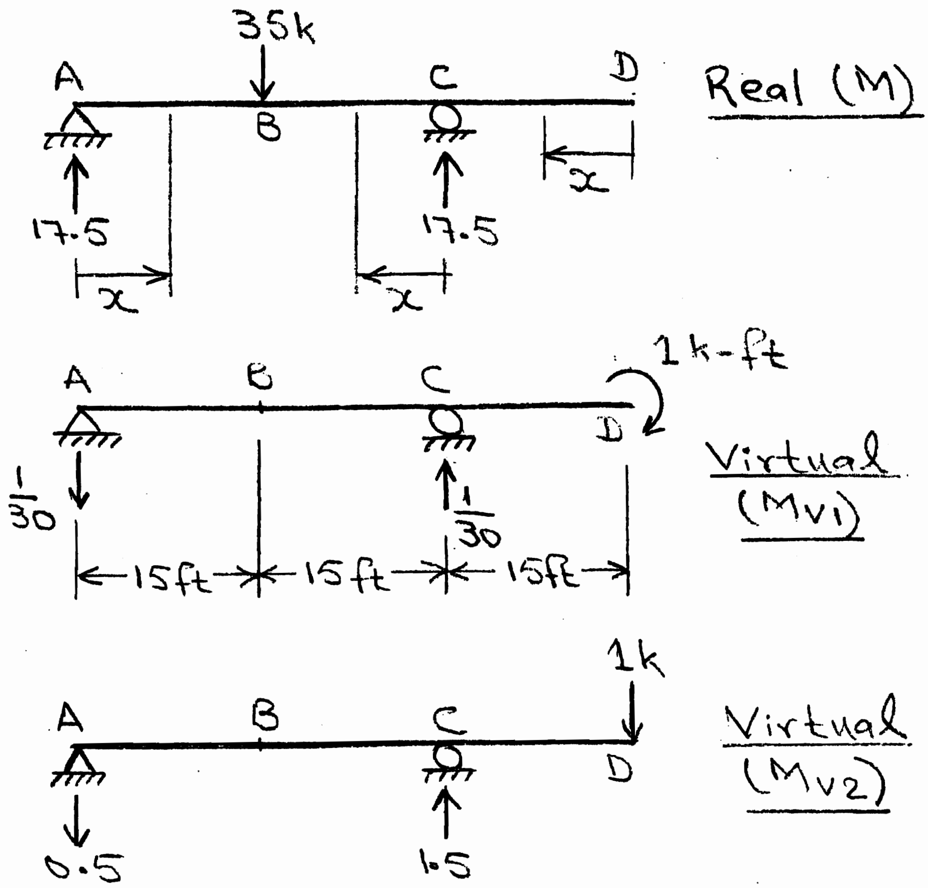
$$= \frac{5248 \text{ kN.m}^3}{EI}$$

$$\Delta_{max} = \frac{L}{360}$$

$$\frac{5248}{70(10^6)I} = \frac{8}{360}$$

from which, $I = 3374(10^{-6})\text{ m}^4 = \underline{3374(10^6)\text{ mm}^4}$

7.29



Segment	x Coordinate		M (k-ft)	Mv1 (k-ft)	Mv2 (k-ft)
	Origin	Limits (ft)			
AB	A	0–15	17.5x	−x/30	−0.5x
CB	C	0–15	17.5x	−1+(x/30)	−1(x+15)+1.5x
DC	D	0–15	0	−1	−1x

$$\theta_D = \frac{1}{EI}\left[\int_0^{15} -\frac{x}{30}(17.5x)\,dx + \int_0^{15}\left(-1+\frac{x}{30}\right)(17.5x)\,dx\right]$$

$$= -\frac{1968.75\text{ k-ft}^2}{EI} = -\frac{1968.75(12)^2}{10000(2500)} = -0.01134\text{ rad}$$

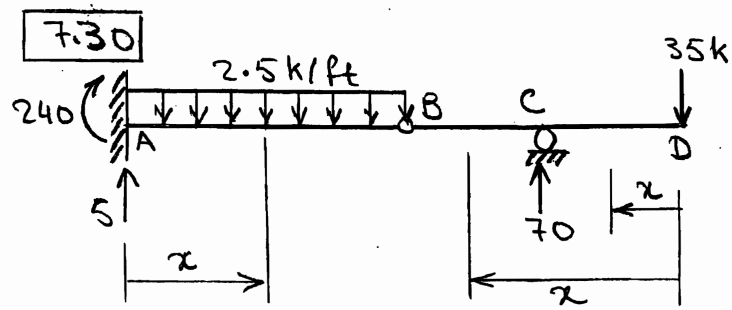
$$\underline{\theta_D = 0.01134\text{ rad}}$$

$$\Delta_D = \frac{1}{EI}\left[\int_0^{15}(-0.5x)(17.5x)\,dx + \int_0^{15}(0.5x-15)(17.5x)\,dx\right]$$

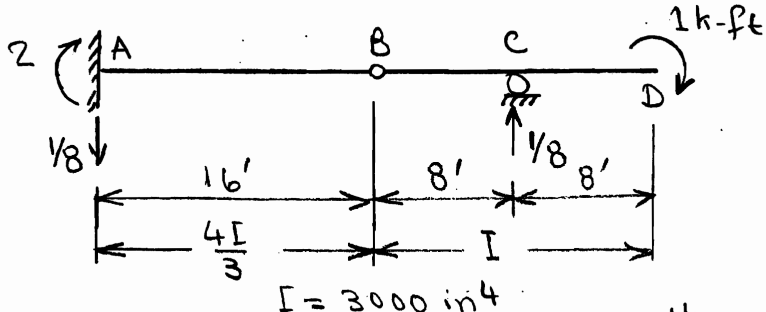
$$= -\frac{29531.25\text{ k-ft}^3}{EI} = -\frac{29531.25(12)^3}{10000(2500)}$$

$$= -2.04\text{ in.} = \underline{2.04\text{ in.}\uparrow}$$

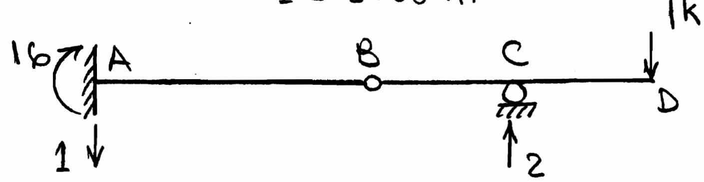
7.30



Real (M)



Virtual (M_{v1})



Virtual (M_{v2})

Segment	x Coordinate Origin	x Coordinate Limits (ft)	M (k-ft)	M_{v1} (k-ft)	M_{v2} (k-ft)
DC	D	0 – 8	$-35x$	-1	$-1x$
CB	D	8 – 16	$-35x+70(x-8)$	$-1+\frac{1}{8}(x-8)$	$x-16$
AB	A	0 – 16	$240+5x-1.25x^2$	$2-x/8$	$16-x$

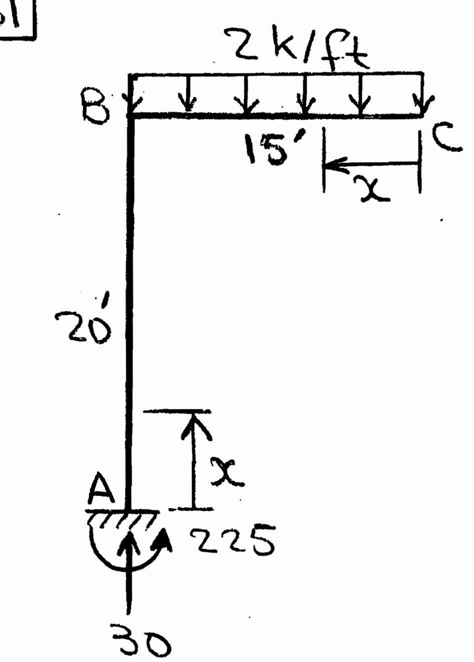
$$\theta_D = \frac{1}{EI}\left[\int_0^8 -1(-35x)\,dx + \int_8^{16}\left(\frac{x}{8}-2\right)(35x-560)\,dx + \frac{3}{4}\int_0^{16}\left(2-\frac{x}{8}\right)(240+5x-1.25x^2)\,dx\right]$$

$$= \frac{4426.67\text{ k-ft}^2}{EI} = \frac{4426.67(12)^2}{30\,000(3000)} = 0.0071\text{ rad.}$$

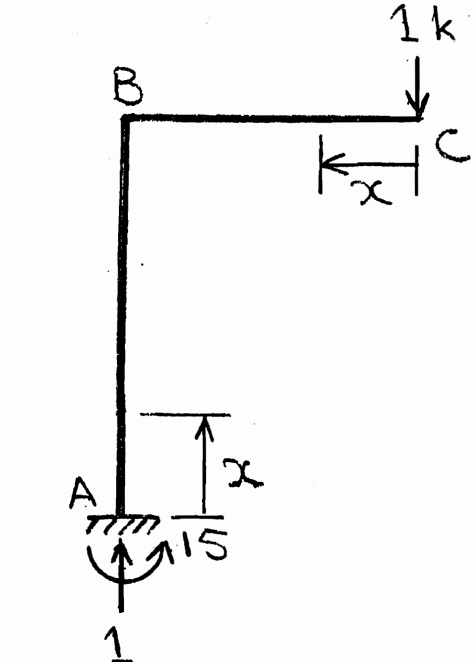
$$\Delta_D = \frac{1}{EI}\left[\int_0^8 -x(-35x)\,dx + \int_8^{16}(x-16)(35x-560)\,dx + \frac{3}{4}\int_0^{16}(16-x)(240+5x-1.25x^2)\,dx\right]$$

$$= \frac{32426.67\text{ k-ft}^3}{EI} = \frac{32426.67(12)^3}{30000(3000)} = 0.62\text{ in.}\downarrow$$

7.31



Real System – M



Virtual System – M_v

Segment	x Coordinate: Origin	x Coordinate: Limits (ft.)	M (k-ft)	M_v (k-ft)
AB	A	0–20	−225	−15
CB	C	0–15	$-x^2$	$-x$

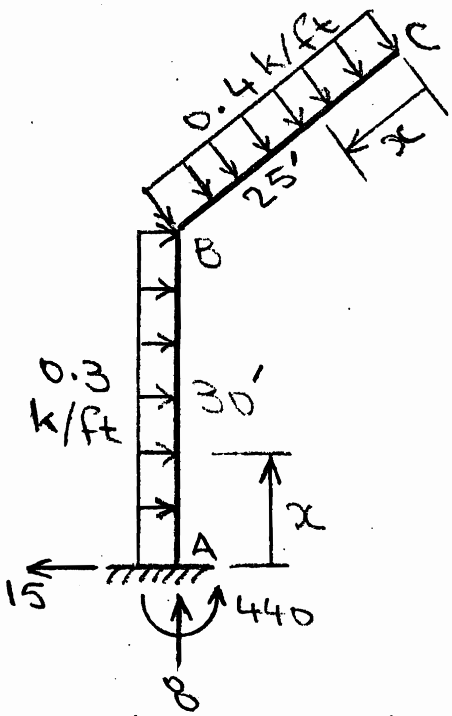
$$(1k)\,\Delta_C = \frac{1}{EI}\left[\int_0^{20}(-15)(-225)\,dx + \int_0^{15}(-x)(-x^2)\,dx\right]$$

$$= \frac{80156.25\ k^2\text{-}ft^3}{EI}$$

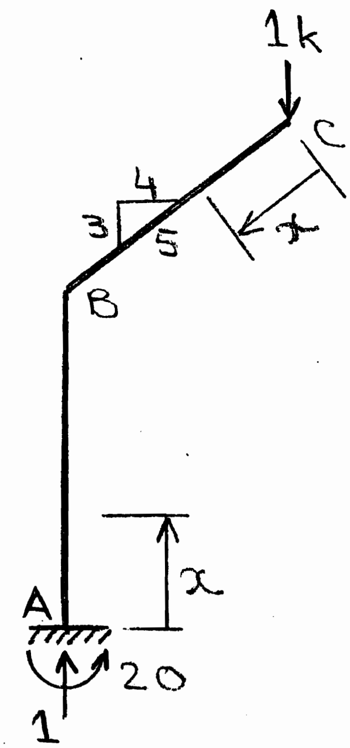
$$\Delta_C = \frac{80156.25\ k\text{-}ft^3}{EI} = \frac{80156.25\,(12)^3}{29000\,(2000)}$$

$$= 2.388\ \text{in.}\downarrow$$

7.32



Real System – M



Virtual System – M_v

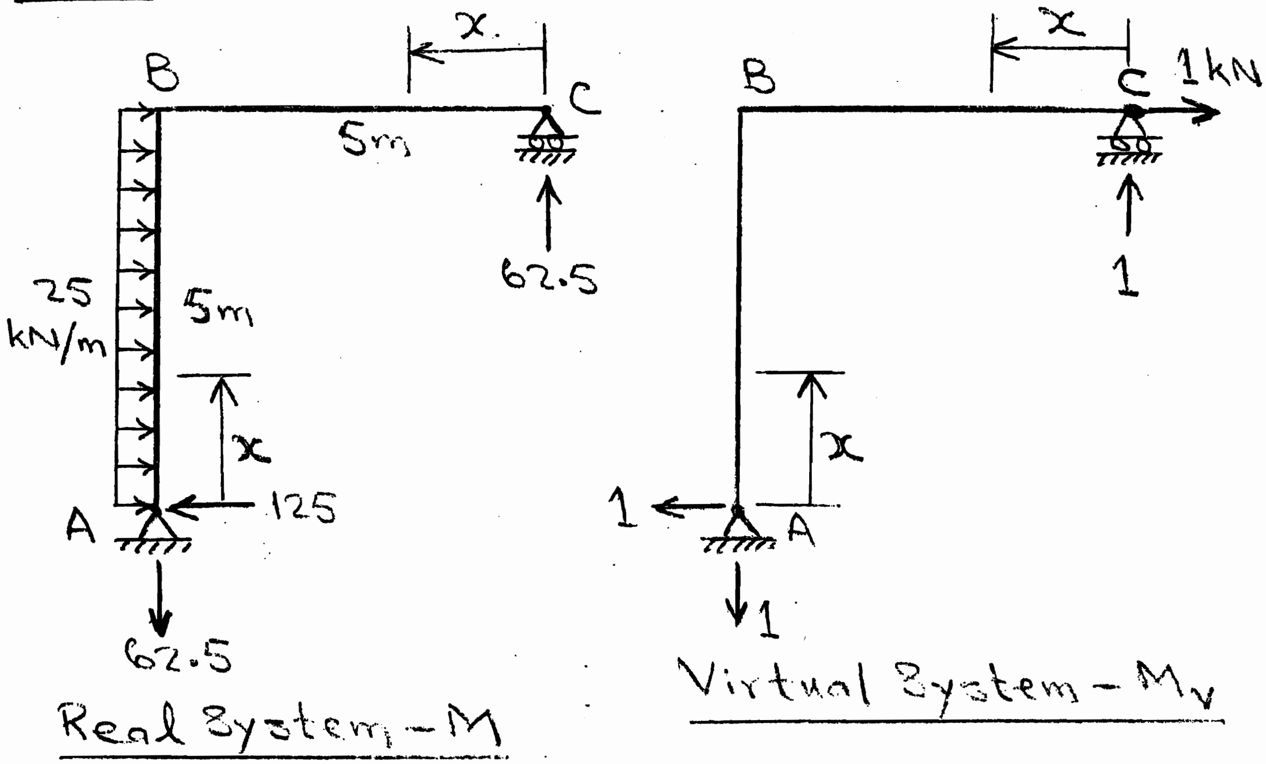
Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–30	$-440+15x-0.15x^2$	-20
CB	C	0–25	$-0.2x^2$	$-\frac{4}{5}x$

$$(1\text{k})\,\Delta_C = \frac{1}{EI}\left[\int_0^{30}(-20)(-440+15x-0.15x^2)\,dx + \int_0^{25}\left(-\frac{4}{5}x\right)(-0.2x^2)\,dx\right]$$

$$= \frac{171625\ \text{k}^2\text{-ft}^3}{EI}$$

$$\Delta_C = \frac{171625\ \text{k-ft}^3}{EI} = \frac{171625(12)^3}{10000(8160)} = 3.63\ \text{in.}\downarrow$$

7.33



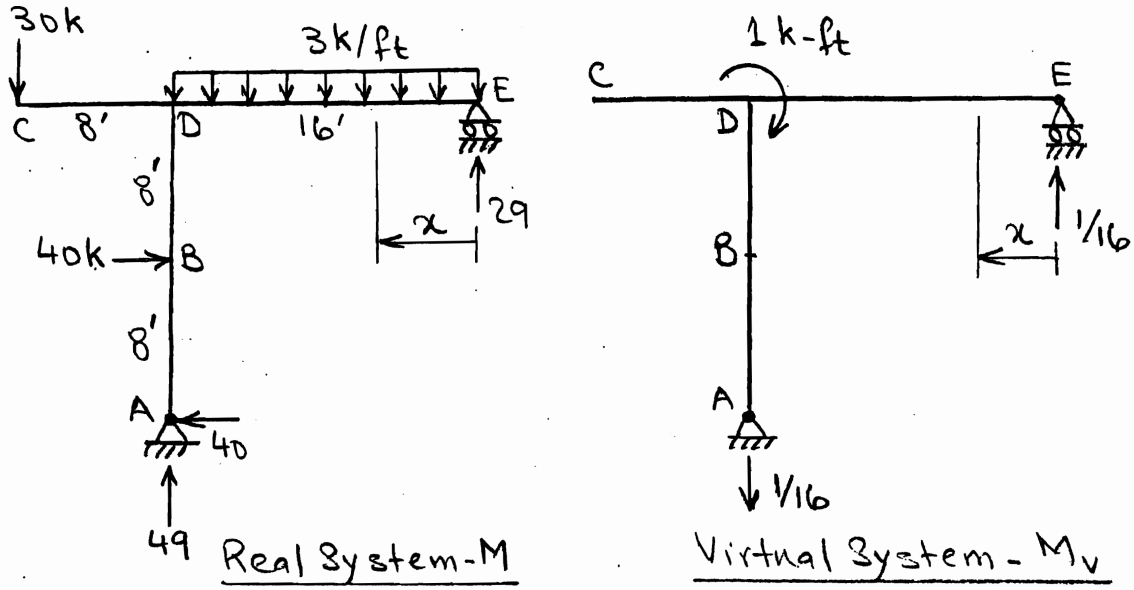
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0 – 5	$125x - 12.5x^2$	$1x$
CB	C	0 – 5	$62.5x$	$1x$

$$\Delta_C = \frac{1}{EI}\left[\int_0^5 x(125x - 12.5x^2)\,dx + \int_0^5 x(62.5x)\,dx\right]$$

$$= \frac{5859.375 \text{ kN.m}^3}{EI} = \frac{5859.375}{70(1030)}$$

$$= 0.0813 \text{ m} = 81.3 \text{ mm} \rightarrow$$

7.34



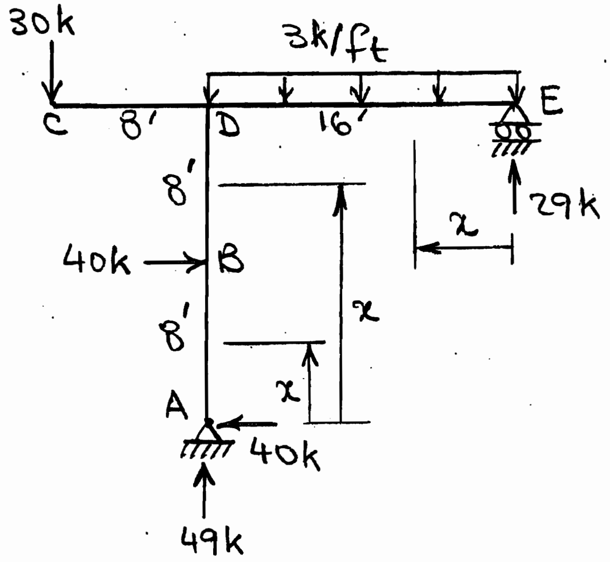
$M = 29x - 1.5x^2$

$M_v = x/16$

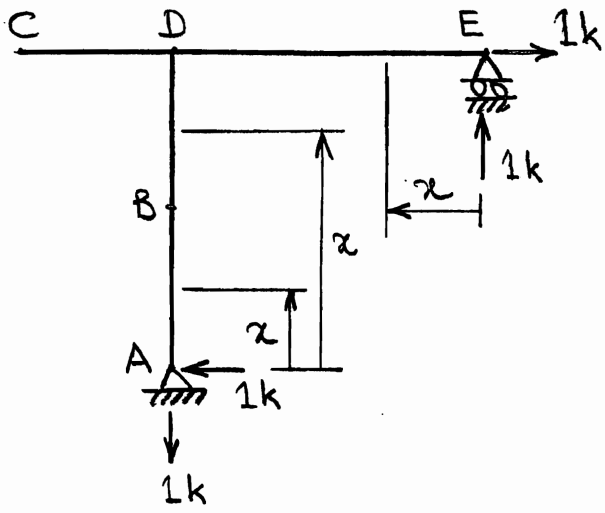
$$\theta_D = \frac{1}{2EI}\int_0^{16}\left(\frac{x}{16}\right)(29x - 1.5x^2)\,dx = \frac{469.33 \text{ k-ft}^2}{EI}$$

$$= \frac{469.33(12)^2}{2000(10000)} = \underline{0.0034 \text{ rad}}$$

7.35



Real System – M



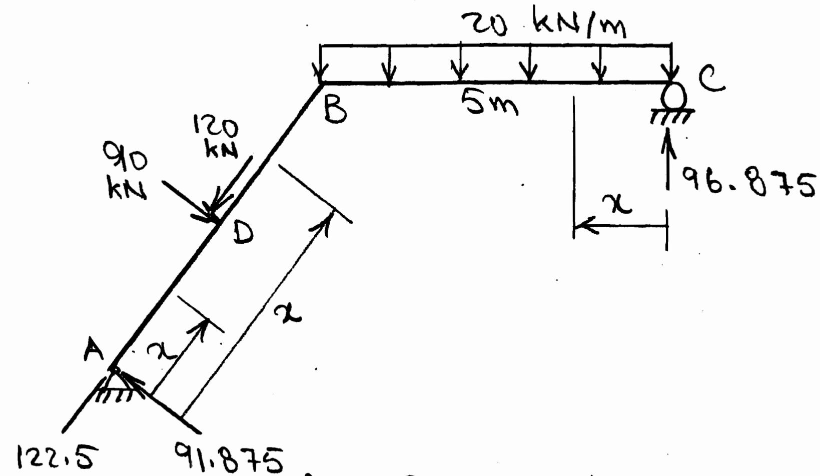
Virtual System – M_v

Segment	x coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–8	$40x$	$1x$
BD	A	8–16	320	$1x$
ED	E	0–16	$29x - \frac{3}{2}x^2$	$1x$

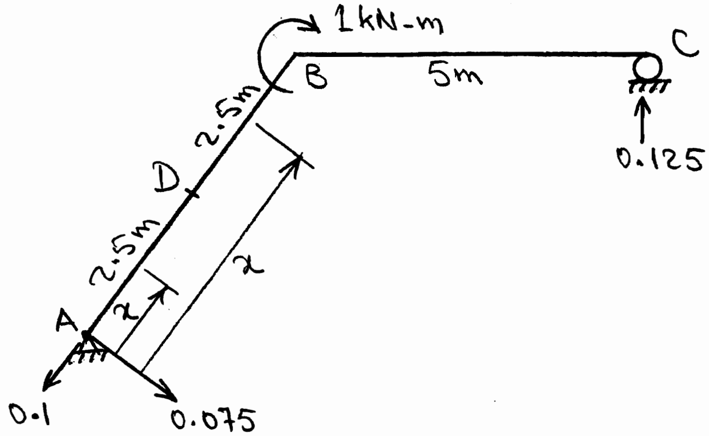
$$(1k)\,\Delta_E = \frac{1}{2EI}\left[\int_0^8 x(40x)\,dx + \int_8^{16} x(320)\,dx + \int_0^{16} x\left(29x - \frac{3}{2}x^2\right)dx\right] = \frac{26282.67\ k^2\text{-}ft^3}{EI}$$

$$\Delta_E = \frac{26282.67\,(12)^3}{(2000)(10000)} = 2.27\ \text{in.} \rightarrow$$

7.36



Real System – M



Virtual System – M_v

Segment	x Coordinate Origin	Limits (m)	M (kN.m)	M_v (kN-m)
AD	A	0–2.5	$91.875x$	$-0.075x$
DB	A	2.5–5	$1.875x+225$	$-0.075x$
CB	C	0–5	$96.875x-10x^2$	$0.125x$

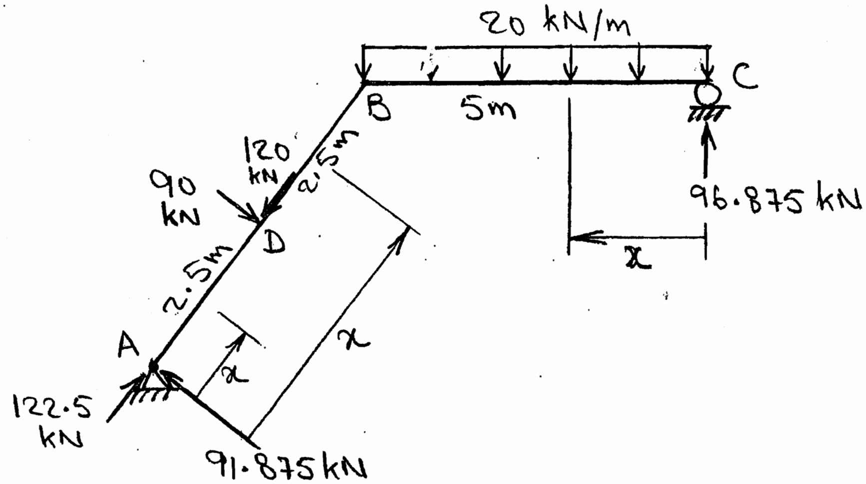
$$\theta_B = \frac{1}{EI}\left[\int_0^{2.5} -0.075x(91.875x)\,dx \right.$$

$$+ \int_{2.5}^{5} -0.075x\,(1.875x+225)\,dx$$

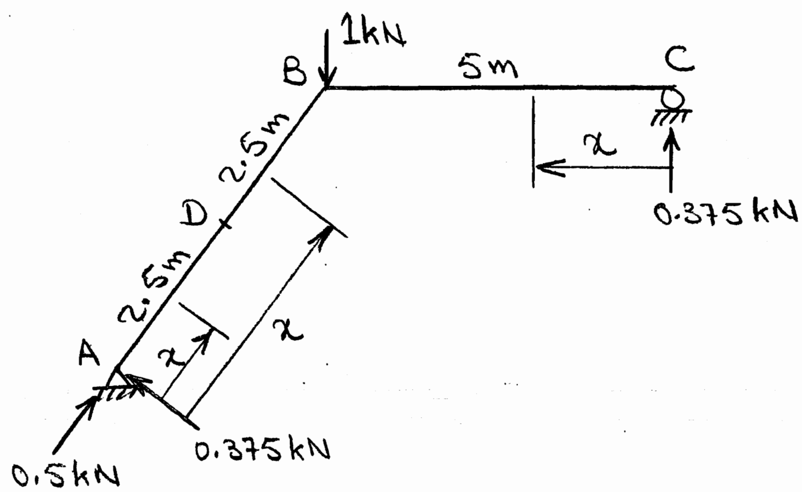
$$\left. + \int_0^5 0.125x\,(96.875x-10x^2)\,dx \right]$$

$$= \frac{110.01\ \text{kN-m}^2}{EI} = \frac{110.01}{200(500)} = \underline{0.0011\ \text{rad.}} \ \text{↻}$$

7.37



Real System – M



Virtual System – M_v

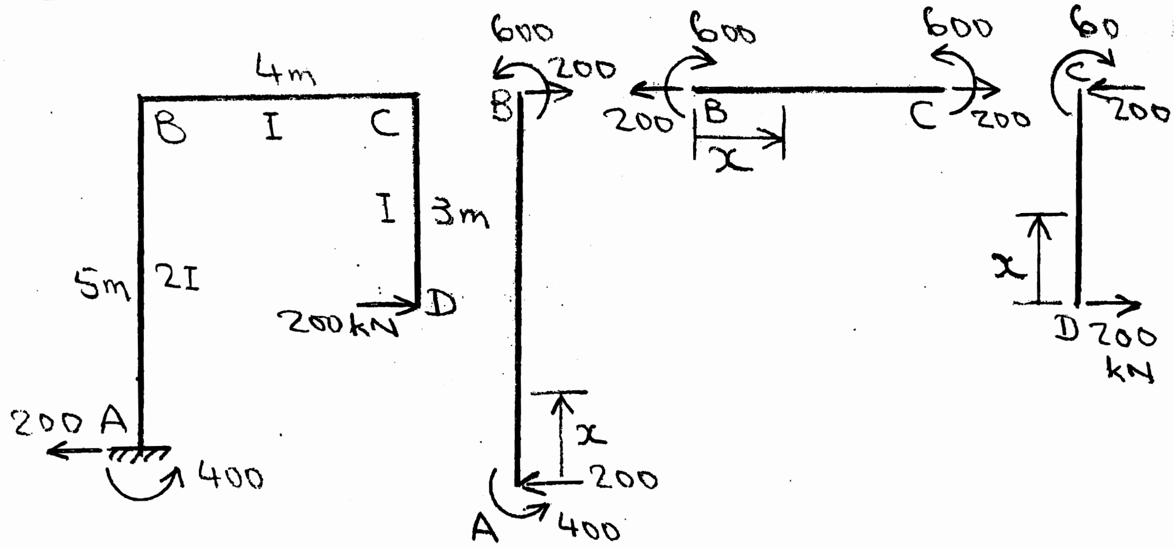
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AD	A	0 – 2.5	$91.875x$	$0.375x$
DB	A	2.5 – 5	$1.875x + 225$	$0.375x$
CB	C	0 – 5	$96.875x - 10x^2$	$0.375x$

$$(1\,\text{kN})\,\Delta_B = \frac{1}{EI}\left[\int_0^{2.5} 0.375x(91.875x)\,dx + \int_{2.5}^{5} 0.375x(1.875x + 225)\,dx + \int_0^{5} 0.375x(96.875x - 10x^2)\,dx\right]$$

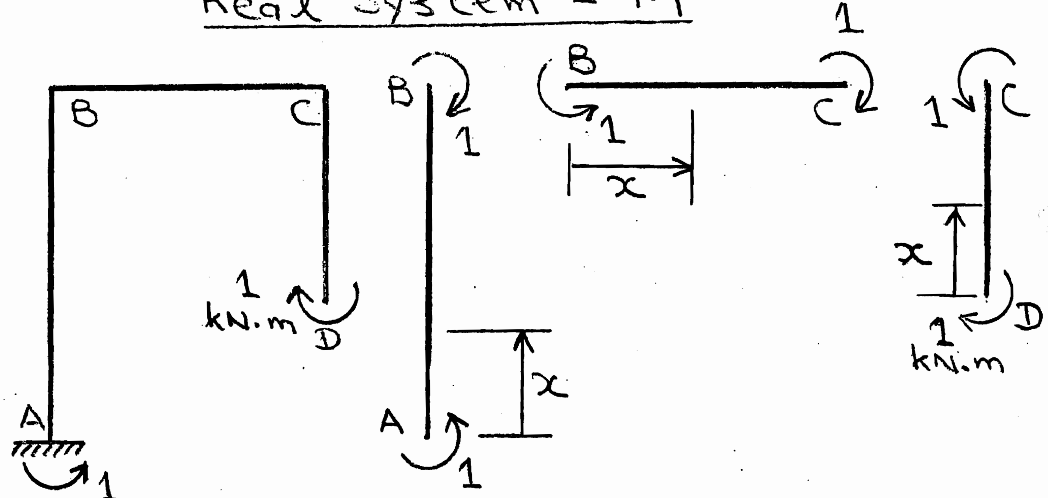
$$= \frac{1923.83\ \text{kN}^2\cdot\text{m}^3}{EI}$$

$$\Delta_B = \frac{1923.83}{200(500)} = 0.0192\,\text{m} = \underline{19.2\ \text{mm}\downarrow}$$

7.38



Real System – M



Virtual System – M_v

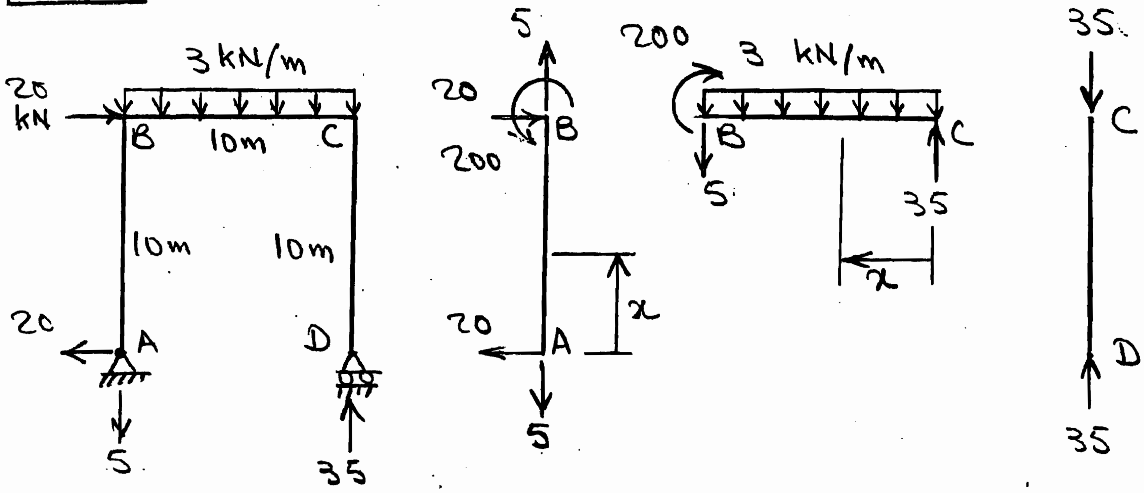
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0–5	$-400+200x$	−1
BC	B	0–4	600	−1
DC	D	0–3	$-200x$	1

$$\theta_D = \frac{1}{EI}\left[\frac{1}{2}\int_0^5 -1(-400+200x)\,dx + \int_0^4 -1(600)\,dx + \int_0^3 1(-200x)\,dx\right]$$

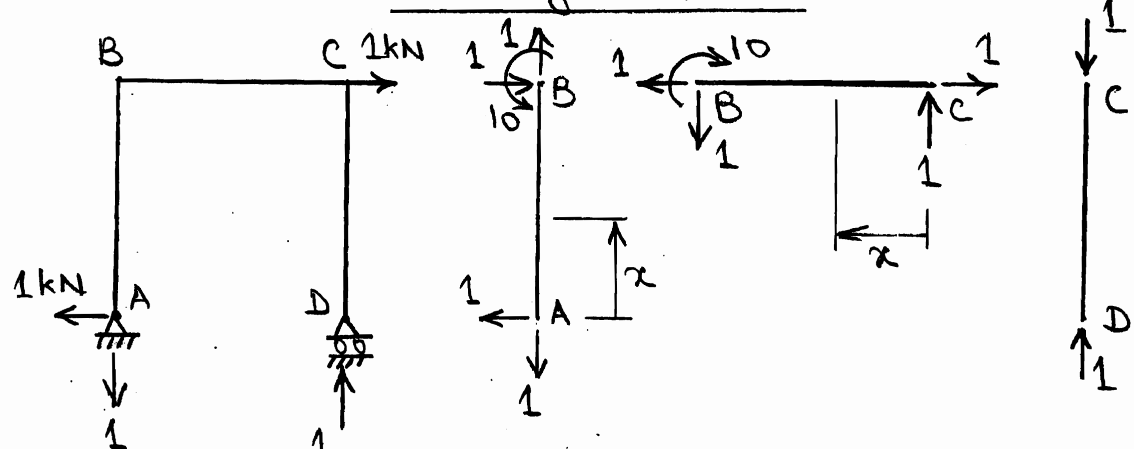
$$= -\frac{3550\ \text{kN}\cdot\text{m}^2}{EI} = -\frac{3550}{70(1290)} = -0.0393\ \text{rad}$$

$$= \underline{0.0393\ \text{rad}}$$

7.39



Real System – M



Virtual System – M_v

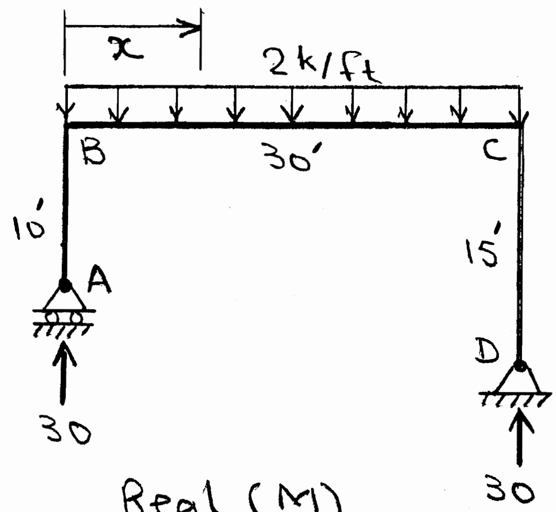
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0–10	$20x$	$1x$
CB	C	0–10	$35x-1.5x^2$	$1x$

$$(1\text{kN})\,\Delta_C = \frac{1}{EI}\left[\int_0^{10} x(20x)\,dx + \int_0^{10} x(35x-1.5x^2)\,dx\right]$$

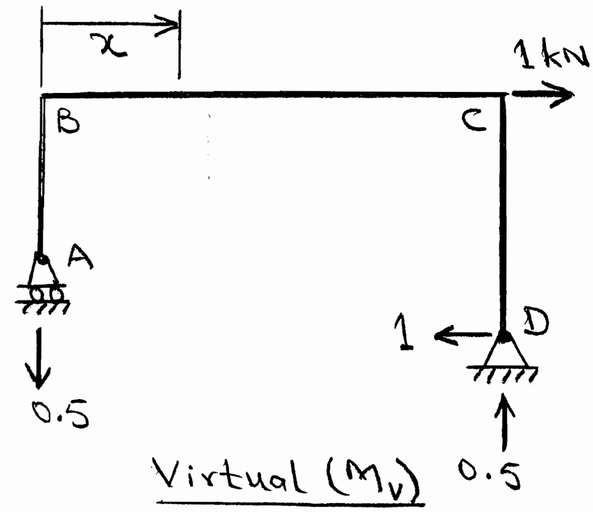
$$= \frac{14583.33\ \text{kN}^2.\text{m}^3}{EI}$$

$$\Delta_C = \frac{14583.33}{200(400)} = \underline{0.182\ \text{m} \rightarrow}$$

7.40



Real (M)



Virtual (M_v)

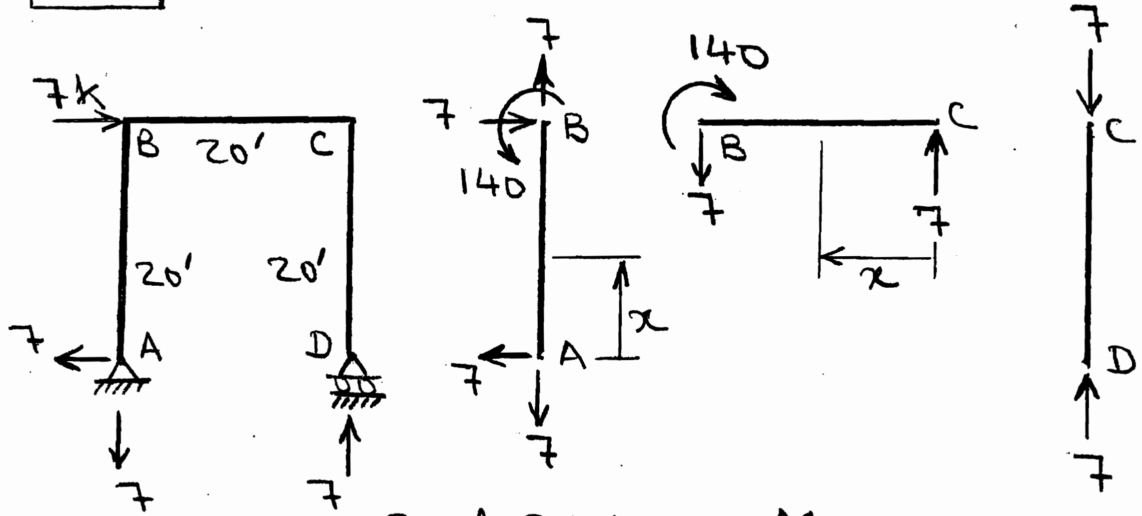
$$M = 30x - \frac{2x^2}{2}$$

$$M_v = -0.5x$$

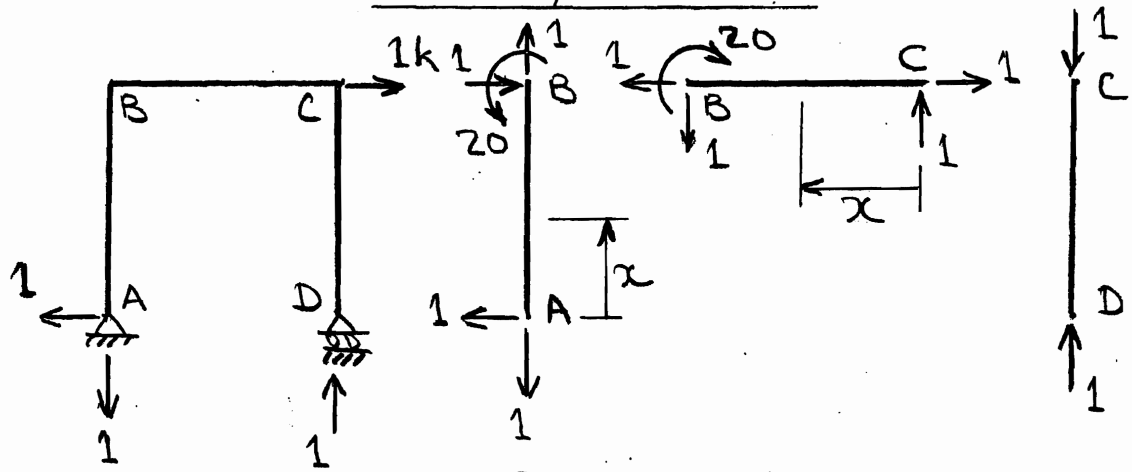
$$\Delta_C = \frac{1}{EI}\int_0^{30} (-0.5x)(30x - x^2)\,dx = -\frac{33750\ \text{k-ft}^3}{EI}$$

$$= -\frac{33750\,(12)^3}{29000\,(1500)} = -1.34\ \text{in.} = 1.34\ \text{in.} \leftarrow$$

7.41



Real System – M



Virtual System – M_v

Segment	x coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–20	$7x$	$1x$
CB	C	0–20	$7x$	$1x$

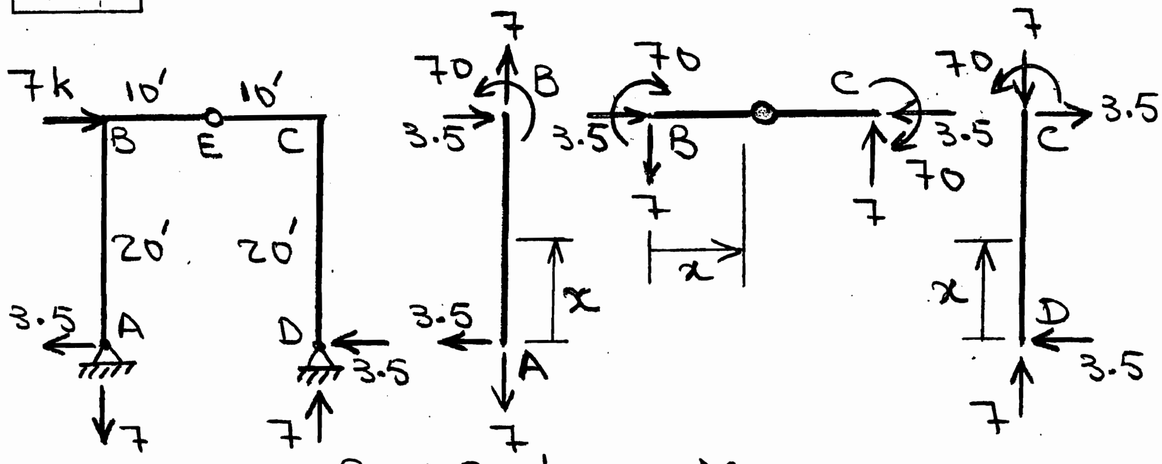
$$(1k)\Delta_C = \frac{1}{EI}\left[\int_0^{20}(1x)(7x)\,dx + \int_0^{20}(1x)(7x)\,dx\right]$$

$$= \frac{37333\ k^2\text{-}ft^3}{EI}$$

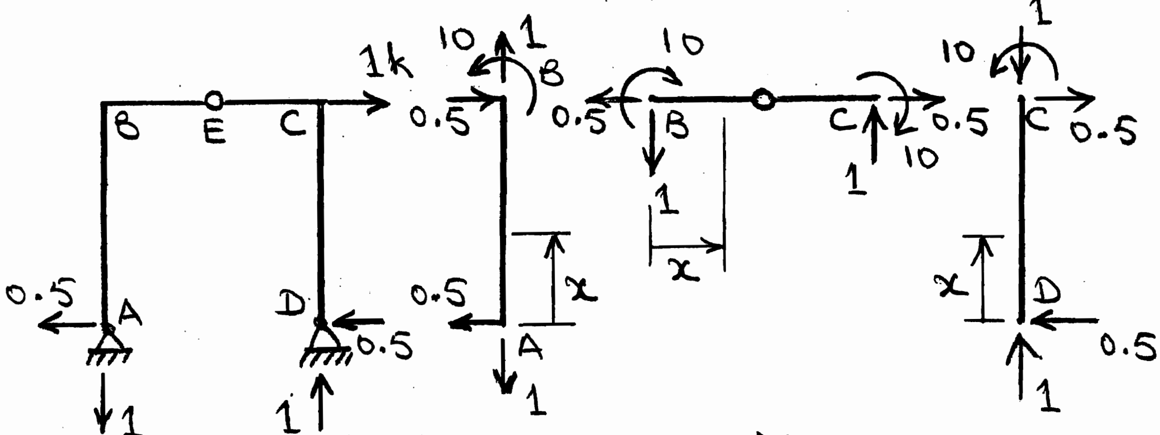
$$\Delta_C = \frac{37333\ k\text{-}ft^3}{EI} = \frac{37333\,(12)^3}{29000\,(I)} = 1\ in.$$

from which, $\underline{I = 2225\ in^4}$

7.42



Real System – M



Virtual System – M_v

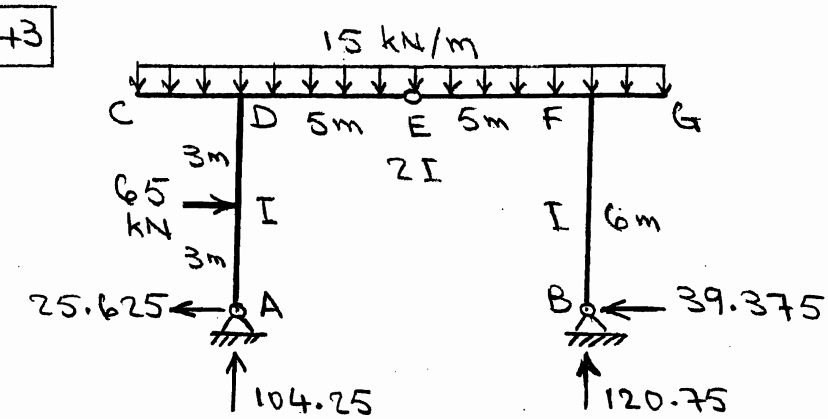
Segment	x coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–20	$3.5x$	$0.5x$
BC	B	0–20	$70-7x$	$10-1x$
DC	D	0–20	$3.5x$	$0.5x$

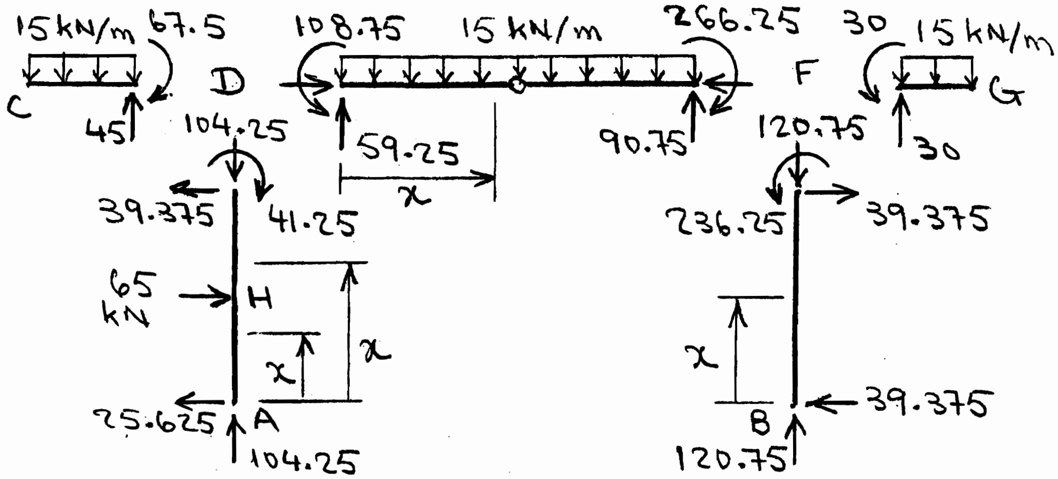
$$(1k)\Delta_C = \frac{1}{EI}\left[2\int_0^{20}(0.5x)(3.5x)\,dx + \int_0^{20}(10-x)(70-7x)\,dx\right] = \frac{14000\ k^2\text{-}ft^3}{EI}$$

$$\Delta_C = \frac{14000\ k\text{-}ft^3}{EI} = \frac{14000(12)^3}{29000(I)} = 1\ \text{in.}$$

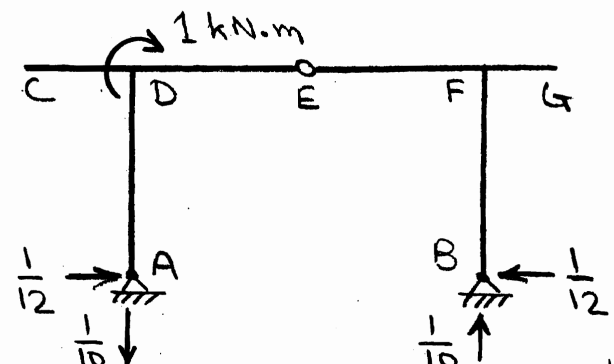
from which, $I = 834\ in^4$

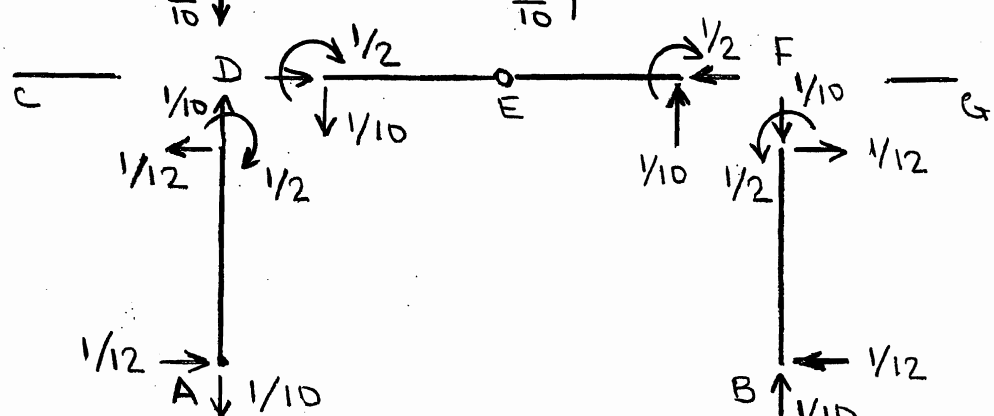
7.43





Real System – M





Virtual System – Mv

7.43 (contd.)

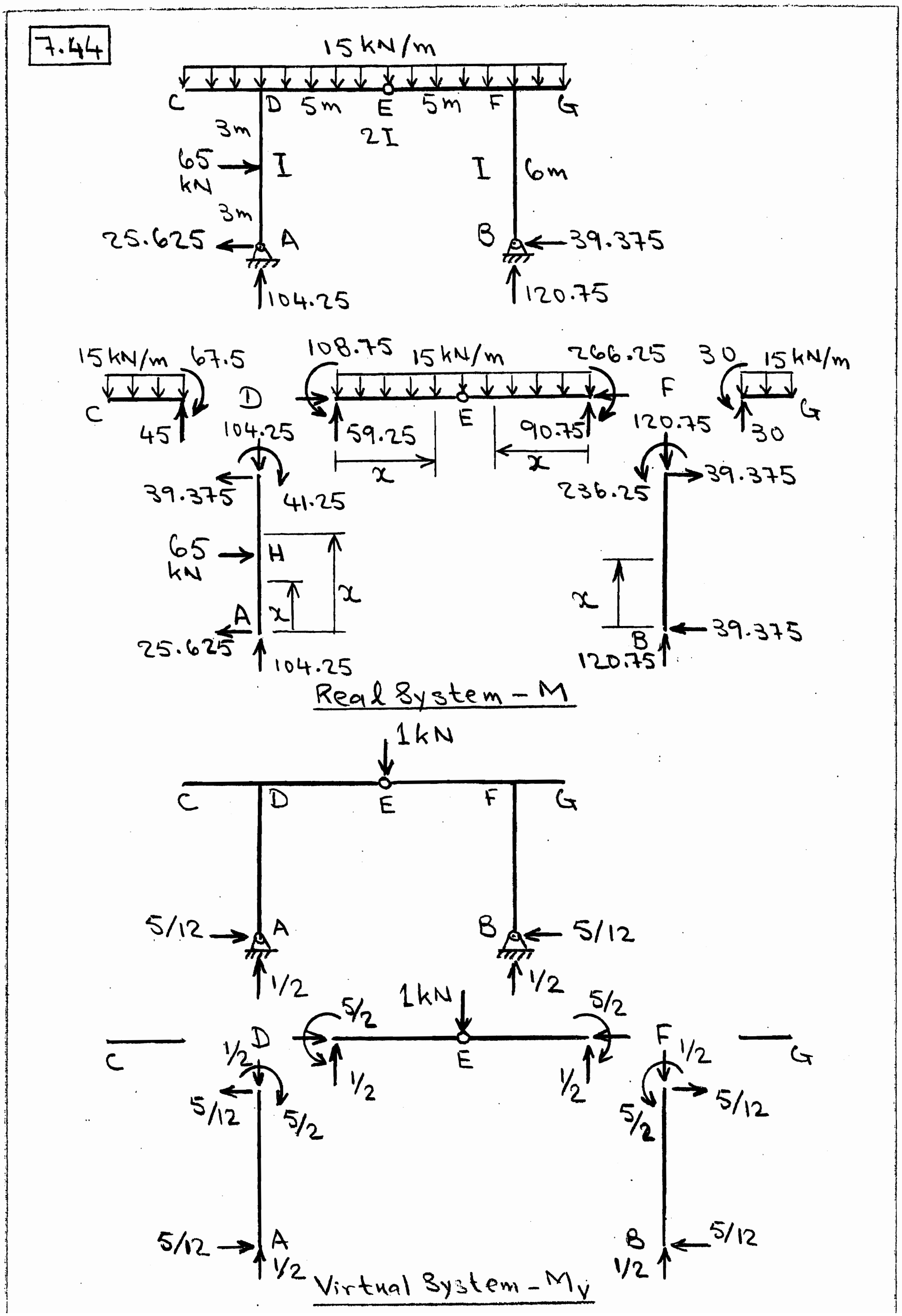
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AH	A	0–3	$25.625x$	$-x/12$
HD	A	3–6	$25.625x - 65(x-3)$	$-x/12$
BF	B	0–6	$39.375x$	$x/12$
DF	D	0–10	$59.25x - 108.75 - 7.5x^2$	$\frac{1}{2} - \frac{x}{10}$

$$(1\text{kN.m})\,\theta_D = \frac{1}{EI}\Big[\int_0^3 \left(-\frac{x}{12}\right)(25.625x)\,dx + \int_3^6 \left(-\frac{x}{12}\right)(25.625x - 65x + 195)\,dx + \int_0^6 \frac{x}{12}(39.375x)\,dx$$

$$+ \frac{1}{2}\int_0^{10} \left(\frac{1}{2} - \frac{x}{10}\right)(59.25x - 108.75 - 7.5x^2)\,dx\Big]$$

$$= \frac{270\ \text{kN}^2\text{.m}^3}{EI}$$

$$\theta_D = \frac{270}{200(350)} = \underline{0.00386\ \text{rad.}\ ↻}$$


7.44
15 kN/m
C
D
5m
E
5m
F
G
3m
2I
65 kN
I
I
6m
3m
25.625
A
B
39.375
104.25
120.75
15 kN/m
67.5
108.75
15 kN/m
266.25
30
15 kN/m
C
D
E
F
G
45
104.25
59.25
90.75
120.75
30
x
x
39.375
41.25
236.25
39.375
65 kN
H
x
x
x
A
B
39.375
25.625
104.25
120.75
Real System – M
1kN
C
D
E
F
G
5/12
A
B
5/12
1/2
1/2
1kN
5/2
5/2
D
E
F
C
G
1/2
1/2
1/2
1/2
5/12
5/12
5/2
5/2
5/12
A
B
5/12
1/2
1/2
Virtual System – Mv

7.44 (contd.)

Segment	x coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AH	A	0 – 3	$25.625x$	$-5x/12$
HD	A	3 – 6	$25.625x - 65(x-3)$	$-5x/12$
BF	B	0 – 6	$39.375x$	$5x/12$
DE	D	0 – 5	$59.25x - 108.75 - 7.5x^2$	$\frac{x}{2} - \frac{5}{2}$
FE	F	0 – 5	$90.75x - 266.25 - 7.5x^2$	$\frac{x}{2} - \frac{5}{2}$

$$(1\,kN)\,\Delta_E = \frac{1}{EI}\Bigg[\int_0^3 \left(-\frac{5x}{12}\right)(25.625x)\,dx$$

$$+ \int_3^6 \left(-\frac{5x}{12}\right)(25.625x - 65x + 195)\,dx$$

$$+ \int_0^6 \frac{5x}{12}(39.375x)\,dx + \frac{1}{2}\int_0^5 \left(\frac{x}{2} - \frac{5}{2}\right)(59.25x - 108.75$$

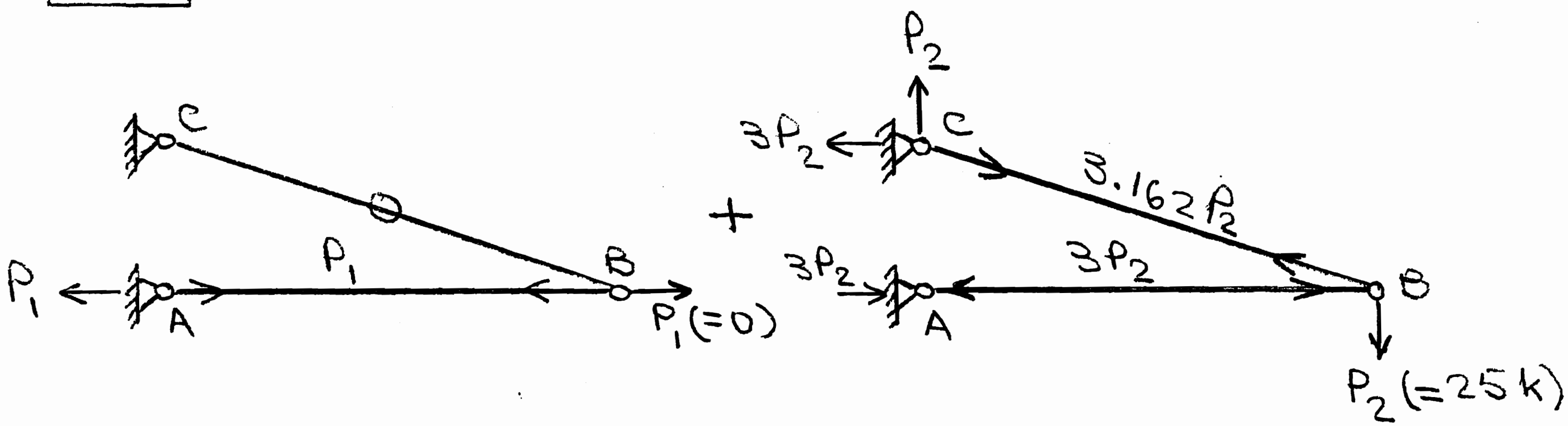
$$-7.5x^2)\,dx + \frac{1}{2}\int_0^5 \left(\frac{x}{2} - \frac{5}{2}\right)(90.75x - 266.25 - 7.5x^2)\,dx\Bigg]$$

$$= \frac{1607.8125\ kN^2\text{-}m^3}{EI}$$

$$\Delta_E = \frac{1607.8125}{200(350)} = 0.023\ m \downarrow$$

$$\Delta_E = 23\ mm \downarrow$$

7.45

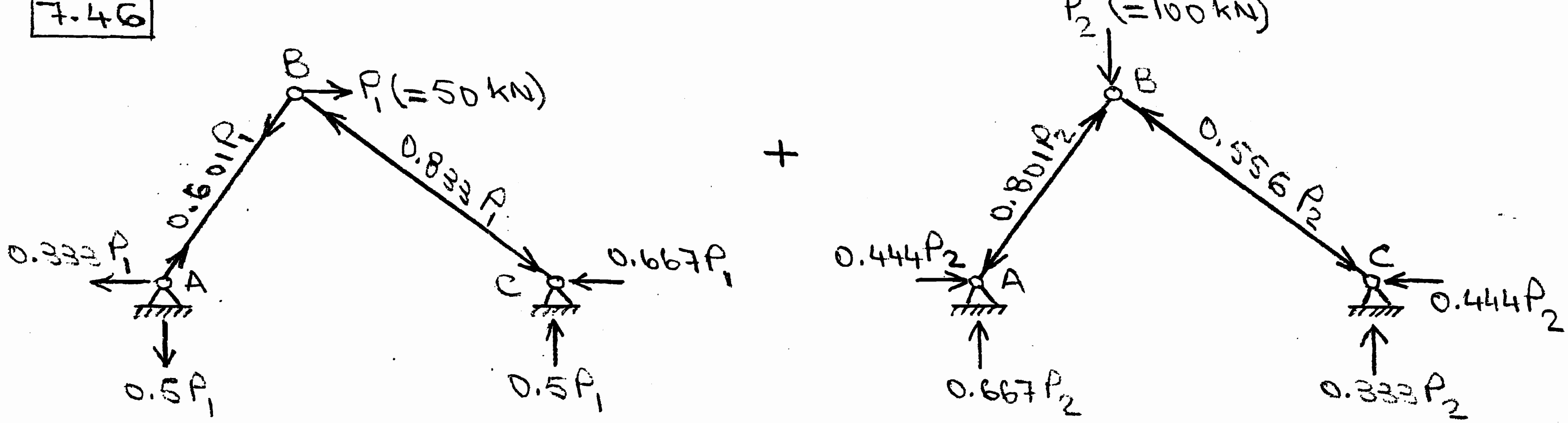


Member	L (in.)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 0,\ P_2 = 25\text{k}$: $\frac{\partial F}{\partial P_1} FL$	$P_1 = 0,\ P_2 = 25\text{k}$: $\frac{\partial F}{\partial P_2} FL$
AB	180	$P_1 - 3P_2$	1	−3	−13500	40500
BC	189.74	$3.162\,P_2$	0	3.162	0	47432.67
				Σ	−13500	87932.67

$$\Delta_{BH} = -\frac{13500}{10000(6)} = -0.225 \text{ in.} = \underline{0.225 \text{ in.} \rightarrow}$$

$$\Delta_{BV} = \frac{87932.67}{10000(6)} = \underline{1.466 \text{ in.} \downarrow}$$

7.46

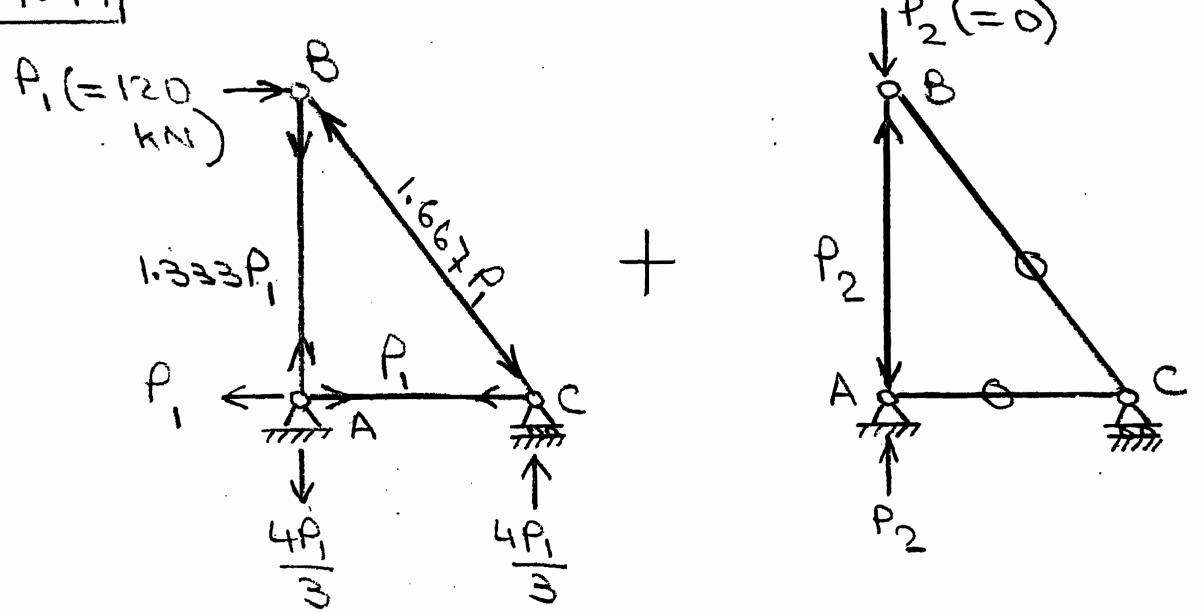


Member	L (m)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 50$ kN, $P_2 = 100$ kN: $\frac{\partial F}{\partial P_1} FL$	$\frac{\partial F}{\partial P_2} FL$
AB	3.606	$0.601P_1 - 0.801P_2$	0.601	-0.801	-108.53	144.65
BC	5	$-0.833P_1 - 0.556P_2$	-0.833	-0.556	405.09	270.06
				Σ	296.56	414.71

$$\Delta_{BH} = \frac{296.56}{70(10^6)(0.001)} = 0.00424 \text{ m} = \underline{4.24 \text{ mm} \rightarrow}$$

$$\Delta_{BV} = \frac{414.71}{70(10^6)(0.001)} = 0.00592 \text{ m} = \underline{5.92 \text{ mm} \downarrow}$$

7.47

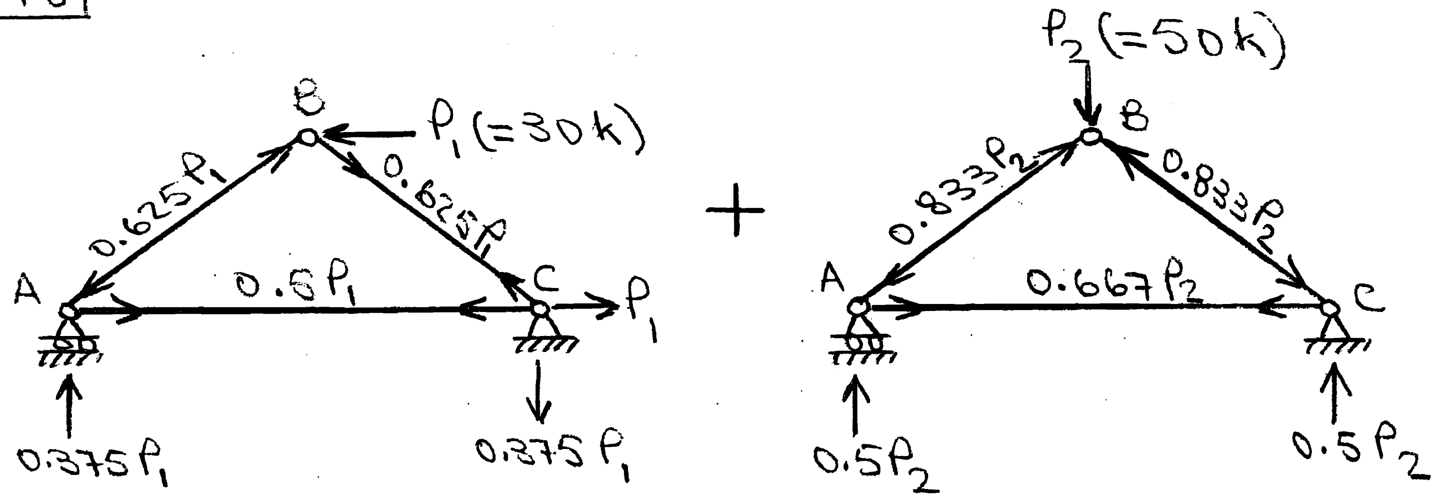


Member	L (m)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 120$ kN, $P_2 = 0$: $\frac{\partial F}{\partial P_1} FL$	$P_1 = 120$ kN, $P_2 = 0$: $\frac{\partial F}{\partial P_2} FL$
AC	3	P_1	1	0	360	0
AB	4	$1.333P_1 - P_2$	1.333	−1	853.33	−640
BC	5	$-1.667P_1$	−1.667	0	1666.67	0
				Σ	2880	−640

$$\Delta_{BH} = \frac{2880}{200(10^6)(0.0015)} = 0.0096 \text{ m} = \underline{9.6 \text{ mm} \rightarrow}$$

$$\Delta_{BV} = -\frac{640}{200(10^6)(0.0015)} = -0.00213 \text{ m} = \underline{2.13 \text{ mm} \uparrow}$$

7.48

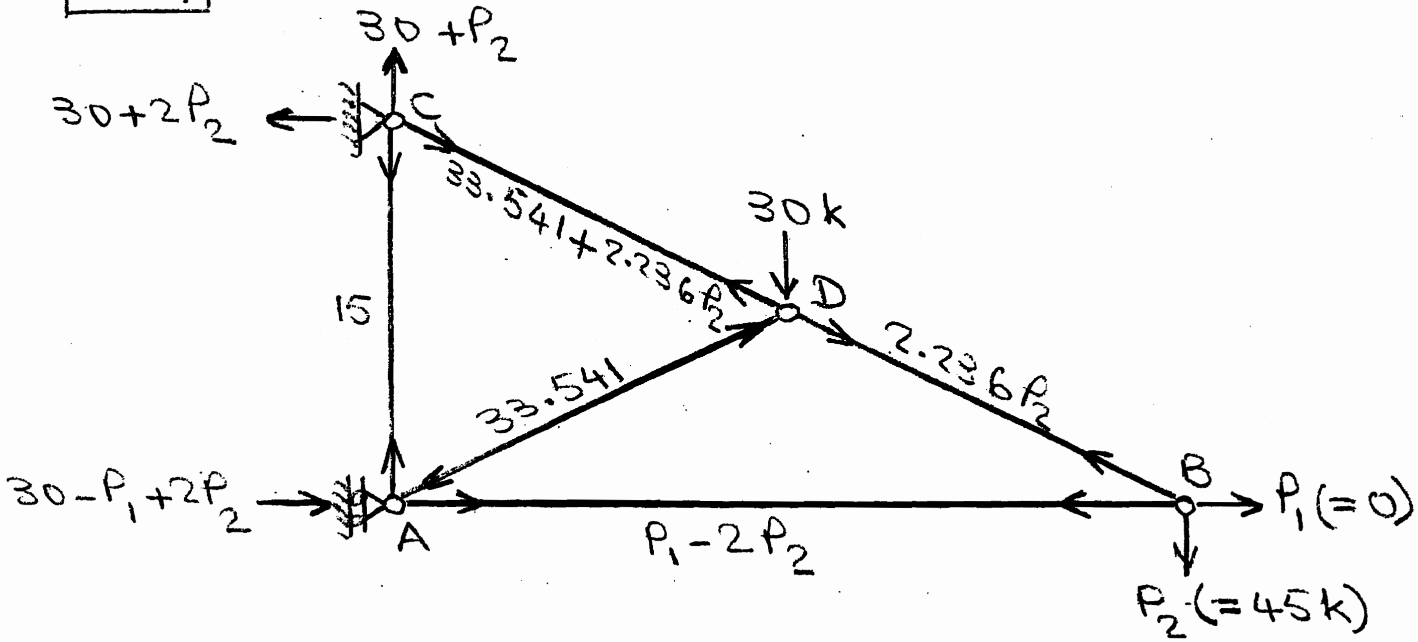


Member	L (in.)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 30$ k, $P_2 = 50$ k	
AC	192	$0.5P_1 + 0.667P_2$	0.5	0.667	4640	6186.67
AB	120	$-0.625P_1 - 0.833P_2$	-0.625	-0.833	4531.25	6041.67
BC	120	$0.625P_1 - 0.833P_2$	0.625	-0.833	-1718.75	2291.67
				Σ	7452.5	14520

$$\Delta_{BH} = \frac{7452.5}{29000(3)} = \underline{0.0857 \text{ in.} \leftarrow}$$

$$\Delta_{BV} = \frac{14520}{29000(3)} = \underline{0.167 \text{ in.} \downarrow}$$

7.49

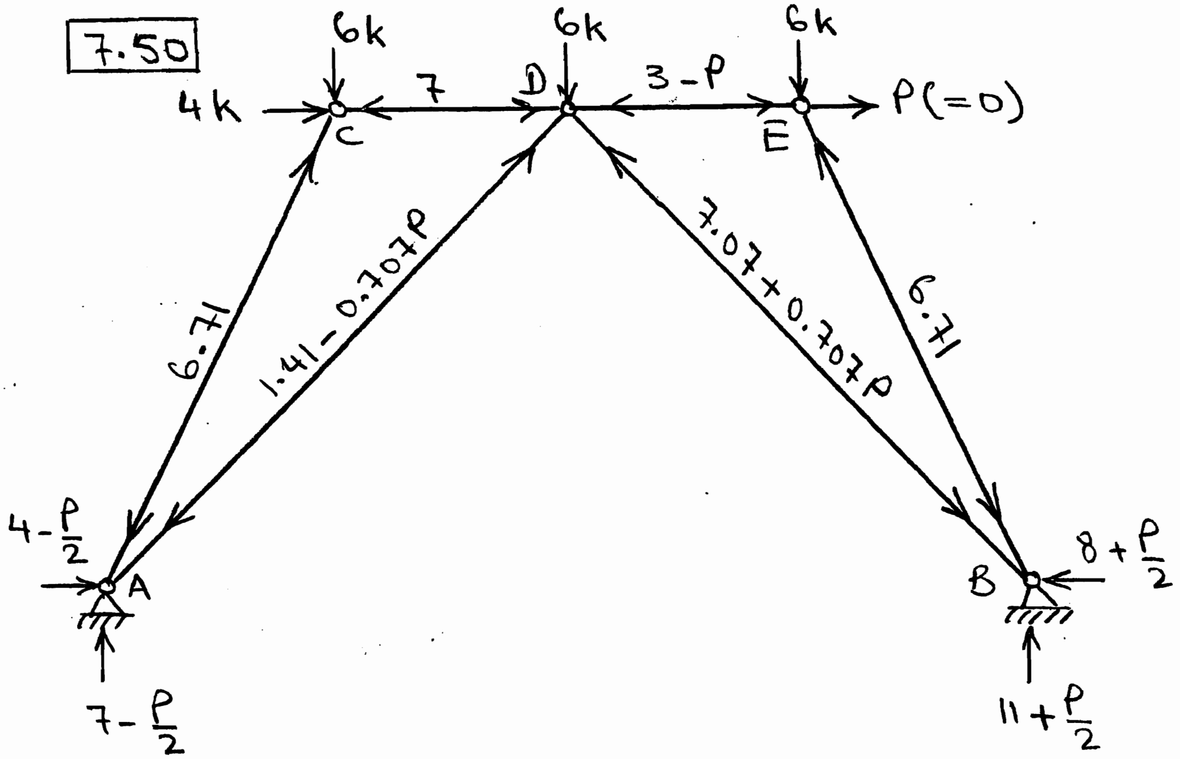


Member	L (in.)	A (in.²)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 0,\ P_2 = 45k$: $\frac{\partial F}{\partial P_1}\left(\frac{FL}{A}\right)$	$P_1 = 0,\ P_2 = 45k$: $\frac{\partial F}{\partial P_2}\left(\frac{FL}{A}\right)$
AB	240	6	$P_1 - 2P_2$	1	−2	−3600	7200
AC	120	4	15	0	0	0	0
AD	134.16	4	−33.541	0	0	0	0
CD	134.16	6	$33.541 + 2.236P_2$	0	2.236	0	6707.99
BD	134.16	6	$2.236P_2$	0	2.236	0	5030.99
					Σ	−3600	18938.98

$$\Delta_{BH} = \frac{-3600}{10000} = -0.36 \text{ in.} = \underline{0.36 \text{ in} \leftarrow}$$

$$\Delta_{BV} = \frac{18938.98}{10000} = \underline{1.894 \text{ in.} \downarrow}$$

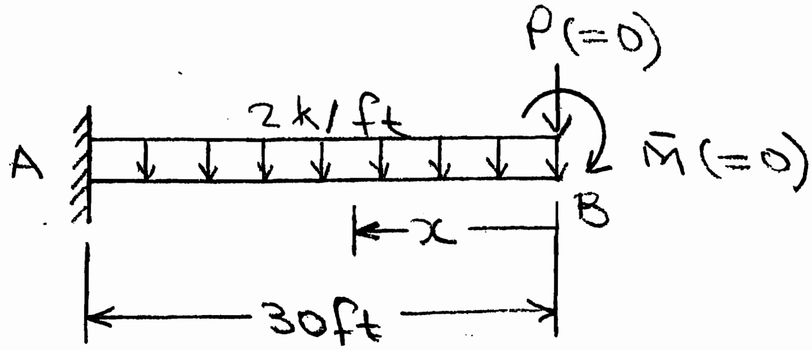
7.50



Member	L (in.)	F	$\frac{\partial F}{\partial P}$	P = 0: $\frac{\partial F}{\partial P}(FL)$
AC	134.16	−6.71	0	0
BE	134.16	−6.71	0	0
AD	169.71	−1.41 + 0.707P	0.707	−169.18
BD	169.71	−7.07 − 0.707P	−0.707	848.29
CD	60	−7	0	0
DE	60	−3 + P	1	−180
			Σ	499.11

$$\Delta_{EH} = \frac{499.11}{29000(6)} = \underline{0.0029 \text{ in.} \rightarrow}$$

7.51



$$M = -x^2 - \bar{M} - Px$$

$$\frac{\partial M}{\partial \bar{M}} = -1$$

$$\theta_B = \frac{1}{EI}\left[\int_0^{30} (-1)(-x^2)\,dx\right] = \frac{9000 \text{ k-ft}^2}{EI}$$

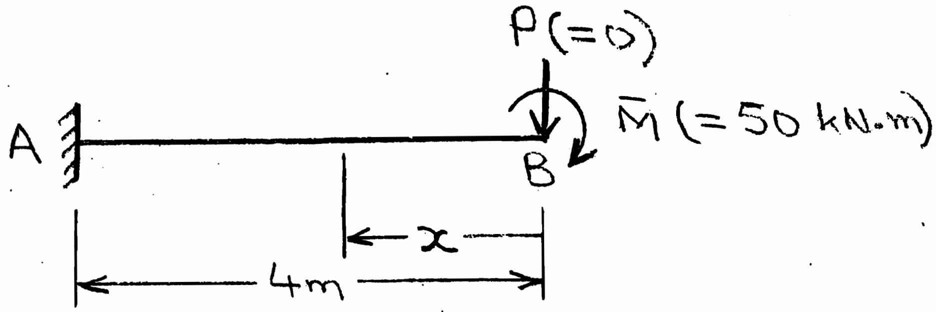
$$= \frac{9000(12)^2}{29000(3000)} = \underline{0.0149 \text{ rad}}$$

$$\frac{\partial M}{\partial P} = -x$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{30} (-x)(-x^2)\,dx\right] = \frac{202500 \text{ k-ft}^3}{EI}$$

$$= \frac{202500(12)^3}{29000(3000)} = \underline{4.022 \text{ in.}\downarrow}$$

7.52



$$M = -\bar{M} - Px$$

$$\frac{\partial M}{\partial \bar{M}} = -1$$

$$\theta_B = \frac{1}{EI}\left[\int_0^4 (-1)(-50)\,dx\right] = \frac{200 \text{ kN.m}^2}{EI}$$

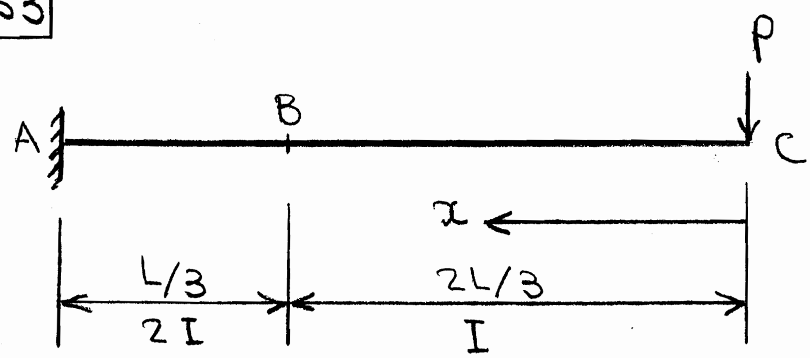
$$= \frac{200}{70(164)} = \underline{0.0174 \text{ rad}}$$

$$\frac{\partial M}{\partial P} = -x$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^4 (-x)(-50)\,dx\right] = \frac{400 \text{ kN.m}^3}{EI}$$

$$= \frac{400}{70(164)} = 0.0348 \text{ m} = \underline{34.8 \text{ mm}} \downarrow$$

7.53

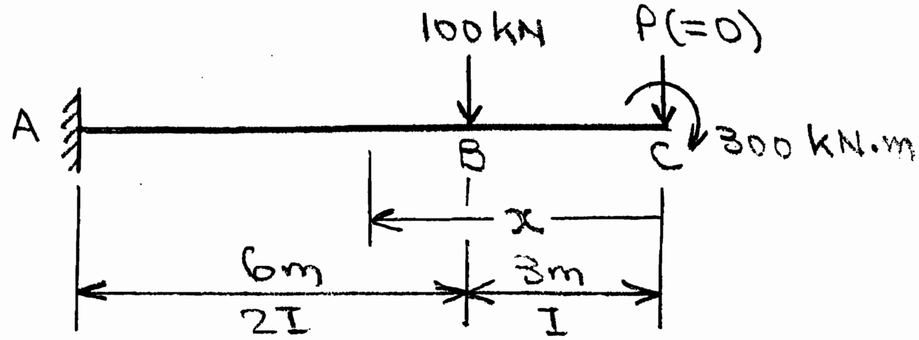


Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
CB	C	$0 - \frac{2L}{3}$	$-Px$	$-x$
BA	C	$\frac{2L}{3} - L$	$-Px$	$-x$

$$\Delta_C = \frac{1}{EI}\left[\int_0^{\frac{2L}{3}} (-x)(-Px)\,dx + \frac{1}{2}\int_{\frac{2L}{3}}^{L} (-x)(-Px)\,dx\right]$$

$$= \frac{35PL^3}{162EI}\downarrow$$

7.54



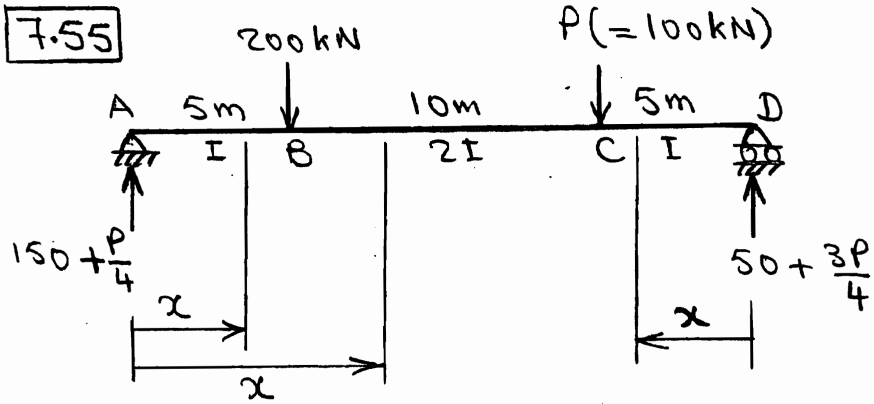
Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits (m)		
CB	C	0–3	$-300 - Px$	$-x$
BA	C	3–9	$-300 - Px - 100(x-3)$	$-x$

$$\Delta_C = \frac{1}{EI}\left[\int_0^3 (-x)(-300)\,dx + \frac{1}{2}\int_3^9 (-x)(-100x)\,dx\right]$$

$$= \frac{13050 \text{ kN}\cdot\text{m}^3}{EI} = \frac{13050}{70(500)}$$

$$= 0.373 \text{ m} = \underline{373 \text{ mm} \downarrow}$$

7.55



Segment	x coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
AB	A	0 – 5	$(150 + P/4)x$	$0.25x$
BC	A	5 – 15	$(150 + P/4)x - 200(x-5)$	$0.25x$
DC	D	0 – 5	$(50 + 3/4P)x$	$0.75x$

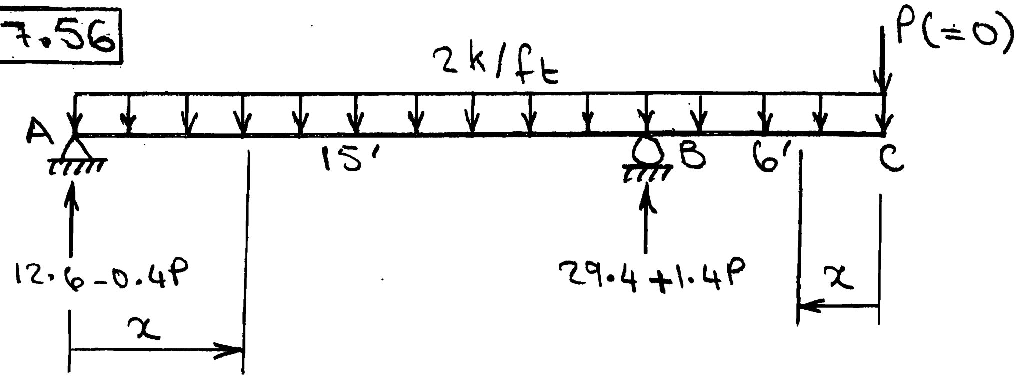
Substituting $P = 100$ kN and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^5 (0.25x)(175x)\,dx + \frac{1}{2}\int_5^{15} 0.25x(-25x+1000)\,dx + \int_0^5 0.75x(125x)\,dx\right]$$

$$= \frac{14843.75 \text{ kN-m}^3}{EI} = \frac{14843.75}{250(600)} = 0.099 \text{ m}$$

$$\Delta_C = 99 \text{ mm} \downarrow$$

7.56



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
AB	A	0-15	$(12.6-0.4P)x - x^2$	$-0.4x$
CD	C	0-6	$-Px - x^2$	$-x$

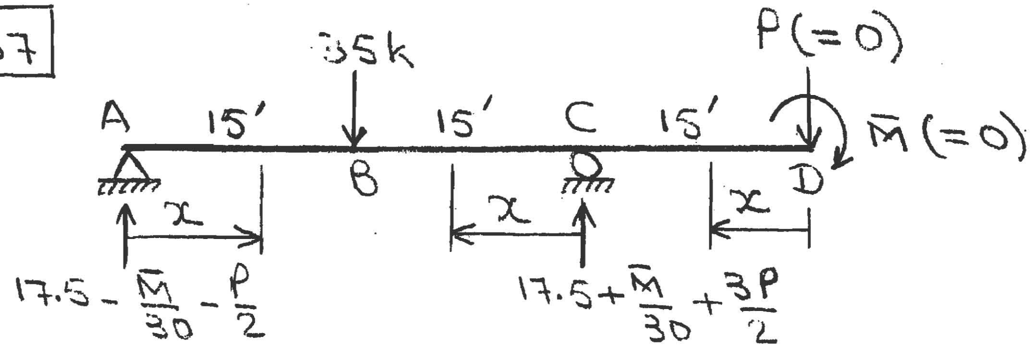
Substituting $P=0$ and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^{15}(-0.4x)(12.6x - x^2)\,dx + \int_0^{6} -x(-x^2)\,dx\right]$$

$$= -\frac{283.5 \text{ k-ft}^3}{EI} = -\frac{283.5(12)^3}{29000(3500)}$$

$$= -0.0048 \text{ in.} = \underline{0.0048 \text{ in.}\uparrow}$$

7.57



Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$	$\frac{\partial M}{\partial P}$
	Origin	Limits (ft)			
AB	A	0–15	$(17.5 - \frac{\bar{M}}{30} - \frac{P}{2})x$	$-x/30$	$-x/2$
CB	C	0–15	$(17.5 + \frac{\bar{M}}{30} + \frac{3P}{2})x - \bar{M} - P(x+15)$	$\frac{x}{30} - 1$	$\frac{x}{2} - 15$
DC	D	0–15	$-\bar{M} - Px$	-1	$-x$

Substituting $\bar{M} = P = 0$, and integrating, we obtain

$$\theta_D = \frac{1}{EI}\left[\int_0^{15} -\frac{x}{30}(17.5x)\,dx + \int_0^{15}\left(\frac{x}{30} - 1\right)(17.5x)\,dx\right]$$

$$= -\frac{1968.75\text{ k-ft}^2}{EI} = -\frac{1968.75(12)^2}{10000(2500)}$$

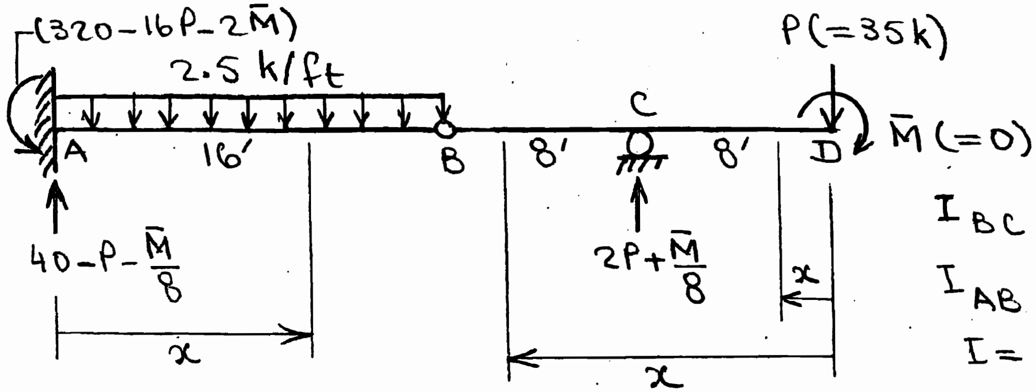
$$= -0.01134\text{ rad} = \underline{0.01134\text{ rad}}$$

$$\Delta_D = \frac{1}{EI}\left[\int_0^{15} -\frac{x}{2}(17.5x)\,dx + \int_0^{15}\left(\frac{x}{2} - 15\right)(17.5x)\,dx\right]$$

$$= -\frac{29531.25\text{ k-ft}^3}{EI} = -\frac{29531.25(12)^3}{10000(2500)}$$

$$= -2.04\text{ in.} = \underline{2.04\text{ in.}\uparrow}$$

7.58



$I_{BC} = I_{CD} = I$

$I_{AB} = \frac{4}{3} I$

$I = 3000 \text{ in.}^4$

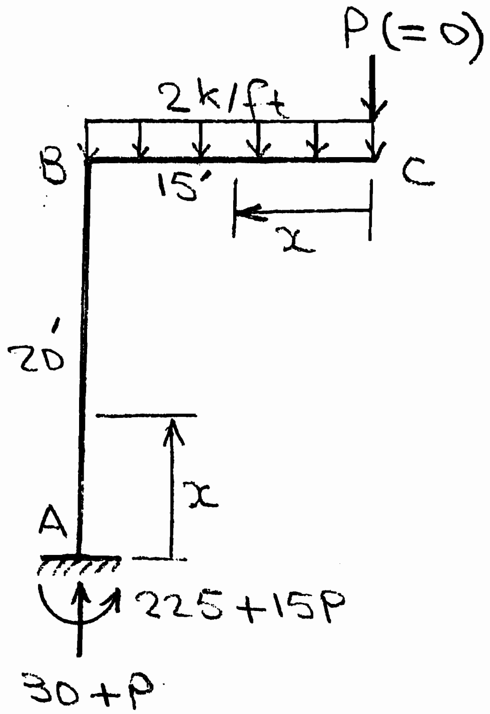
Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$	$\frac{\partial M}{\partial P}$
	Origin	Limits			
DC	D	0–8	$-Px - \bar{M}$	-1	$-x$
CB	D	8–16	$-Px - \bar{M} + (2P + \frac{\bar{M}}{8})(x-8)$	$\frac{x}{8} - 2$	$x - 16$
AB	A	0–16	$-(320 - 16P - 2\bar{M}) + (40 - P - \frac{\bar{M}}{8})x - 1.25x^2$	$2 - \frac{x}{8}$	$16 - x$

Substituting $\bar{M} = 0$ and $P = 35$ k, and integrating, we obtain

$$\theta_D = \frac{1}{EI}\left[\int_0^8 -1(-35x)\,dx + \int_8^{16}\left(\frac{x}{8} - 2\right)(35x - 560)\,dx + \frac{3}{4}\int_0^{16}\left(2 - \frac{x}{8}\right)(240 + 5x - 1.25x^2)\,dx\right] = \frac{4426.67 \text{ k-ft}^2}{EI} = \underline{0.0071 \text{ rad.}}$$

$$\Delta_D = \frac{1}{EI}\left[\int_0^8 -x(-35x)\,dx + \int_8^{16}(x - 16)(35x - 560)\,dx + \frac{3}{4}\int_0^{16}(16 - x)(240 + 5x - 1.25x^2)\,dx\right] = \frac{32426.67 \text{ k-ft}^3}{EI} = \underline{0.62 \text{ in.} \downarrow}$$

7.59



Segment	x Coordinate Origin	x Coordinate Limits (ft)	M	$\frac{\partial M}{\partial P}$
AB	A	0–20	$-(225+15P)$	-15
CB	C	0–15	$-x^2 - Px$	$-x$

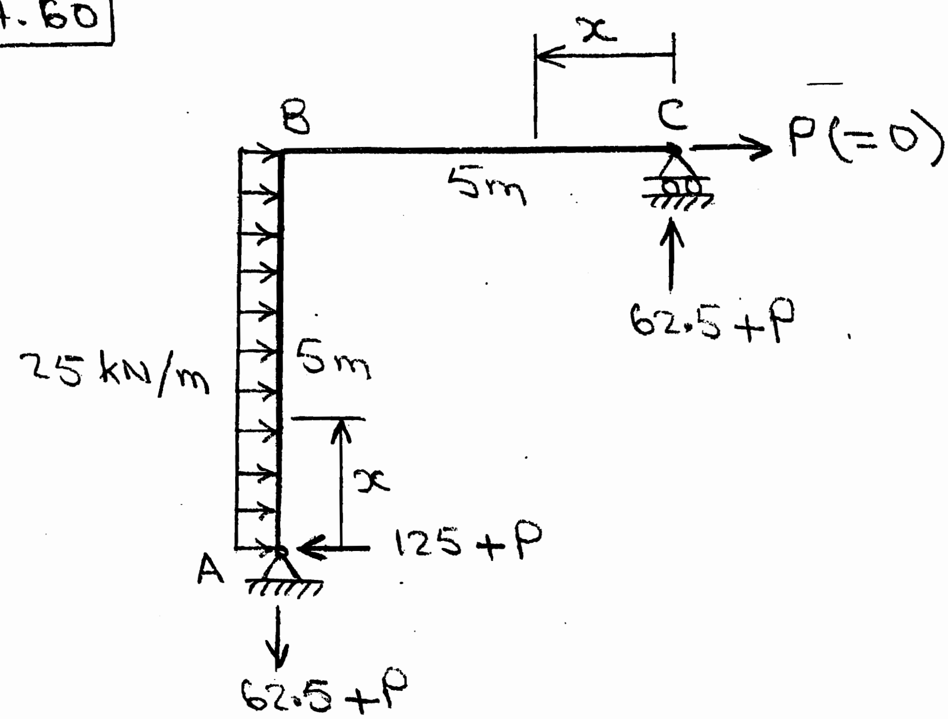
Substituting P = 0 and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^{20}(-15)(-225)\,dx + \int_0^{15}(-x)(-x^2)\,dx\right]$$

$$= \frac{80156.25 \text{ k-ft}^3}{EI} = \frac{80156.25(12)^3}{29000(2000)}$$

$$= \underline{2.388 \text{ in.} \downarrow}$$

7.60



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits (m)		
AB	A	0-5	$(125+P)x-12.5x^2$	x
CB	C	0-5	$(62.5+P)x$	x

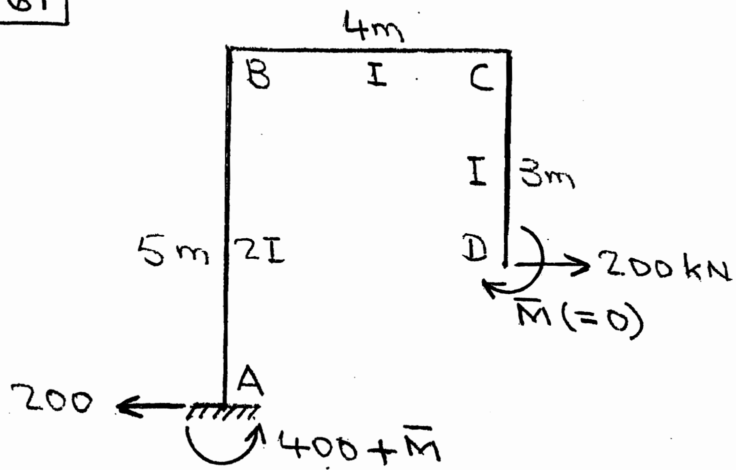
Substituting $P=0$ and integrating, we obtain

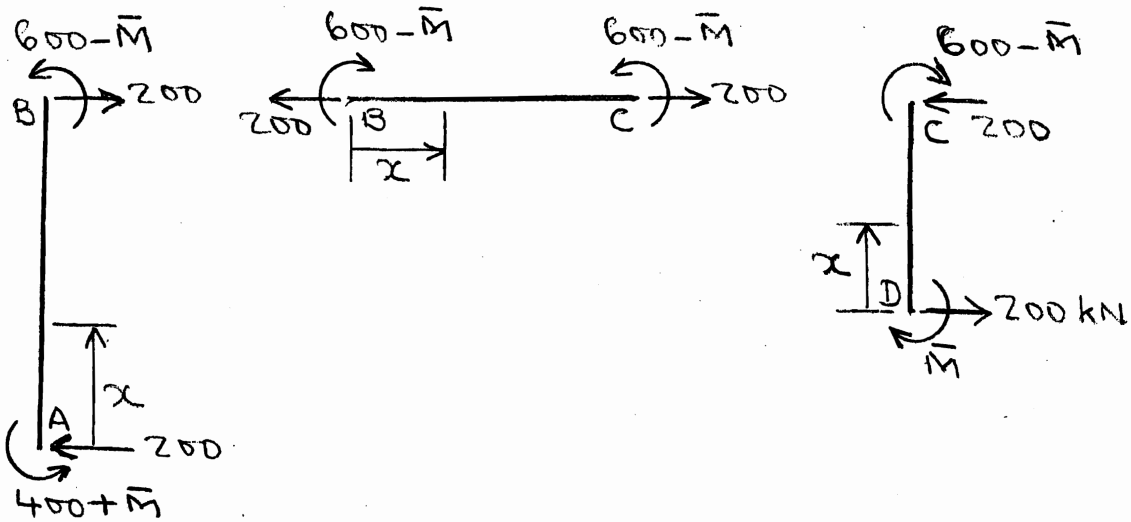
$$\Delta_C = \frac{1}{EI}\left[\int_0^5 x(125x-12.5x^2)\,dx + \int_0^5 x(62.5x)\,dx\right]$$

$$= \frac{5859.375 \text{ kN}\cdot\text{m}^3}{EI} = \frac{5859.375}{70(1030)}$$

$$= 0.0813 \text{ m} = \underline{81.3 \text{ mm} \rightarrow}$$

7.61





Segment	x Coordinate Origin	x Coordinate Limits (m)	M	$\frac{\partial M}{\partial \bar{M}}$
AB	A	0 − 5	$-(400+\bar{M})+200x$	−1
BC	B	0 − 4	$600-\bar{M}$	−1
DC	D	0 − 3	$-200x+\bar{M}$	1

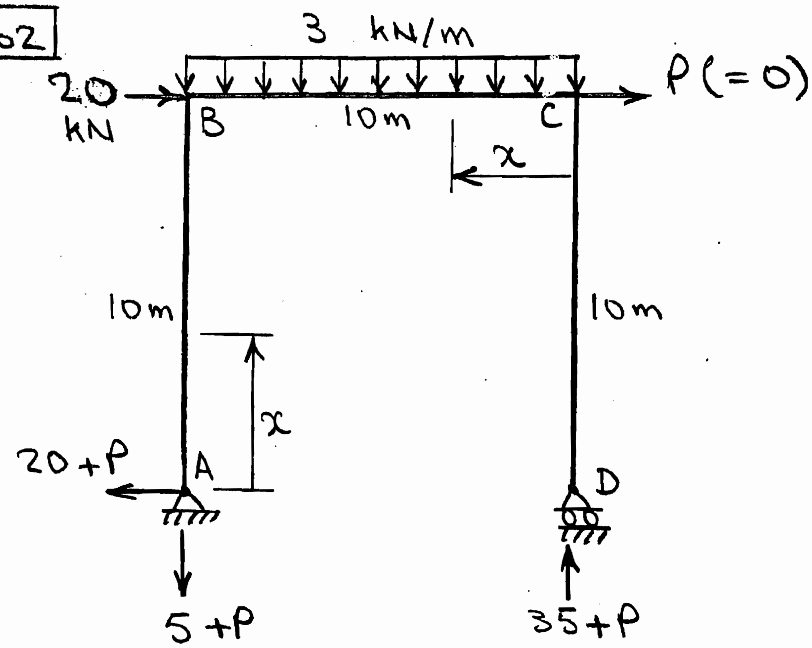
Substituting $\bar{M}=0$ and integrating, we obtain

$$\theta_D = \frac{1}{EI}\left[\frac{1}{2}\int_0^5 -1(-400+200x)\,dx + \int_0^4 -1(600)\,dx + \int_0^3 1(-200x)\,dx\right]$$

$$= -\frac{3550 \text{ kN.m}^2}{EI} = -\frac{3550}{70(1290)} = -0.0393 \text{ rad}$$

$$= \underline{0.0393 \text{ rad}}$$

7.62



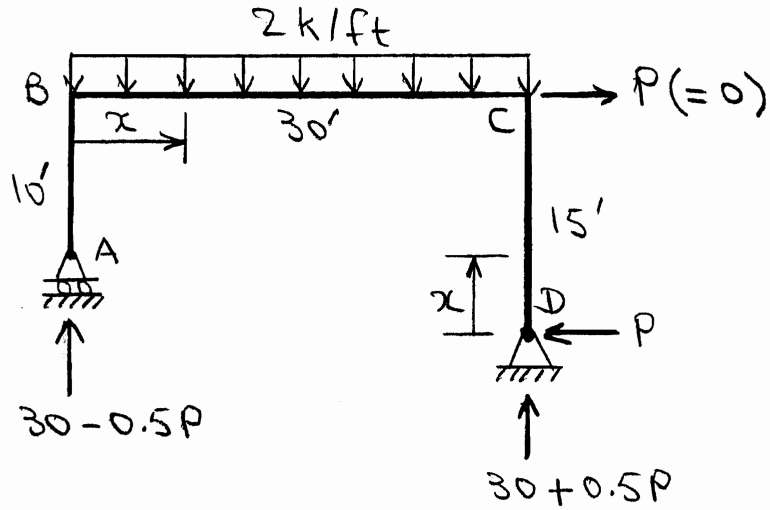
Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
AB	A	0–10	$(20+P)x$	x
CB	C	0–10	$(35+P)x-1.5x^2$	x

Substituting $P=0$ and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^{10} x(20x)\,dx + \int_0^{10} x(35x-1.5x^2)\,dx\right]$$

$$= \frac{14583.33 \text{ kN-m}^3}{EI} = \underline{0.182 \text{ m} \rightarrow}$$

7.63



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
BC	B	0 − 30	$(30-0.5P)x - x^2$	$-0.5x$
DC	D	0 − 15	Px	x

Substituting $P = 0$ and integrating, we obtain

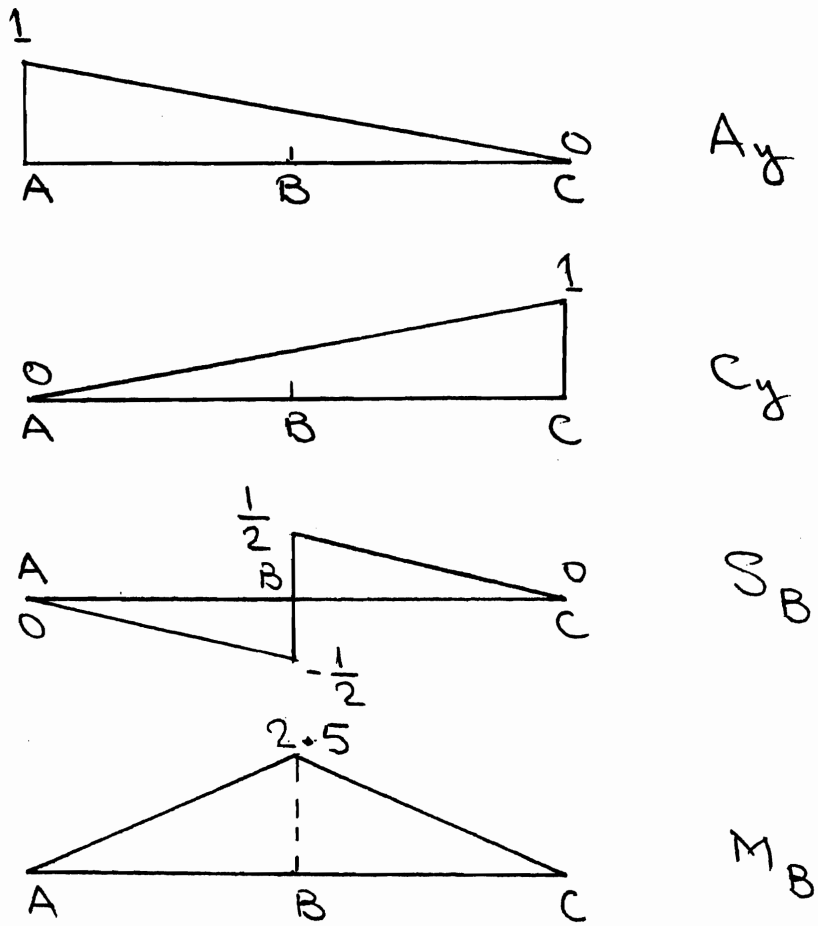
$$\Delta_C = \frac{1}{EI}\int_0^{30}(-0.5x)(30x - x^2)\,dx = -\frac{33750 \text{ k-ft}^3}{EI}$$

$$= -\frac{33750\,(12)^3}{29000\,(1500)} = -1.34 \text{ in.} = \underline{1.34 \text{ in.} \leftarrow}$$

Chapter Eight
Influence Lines

CHAPTER 8

8-1



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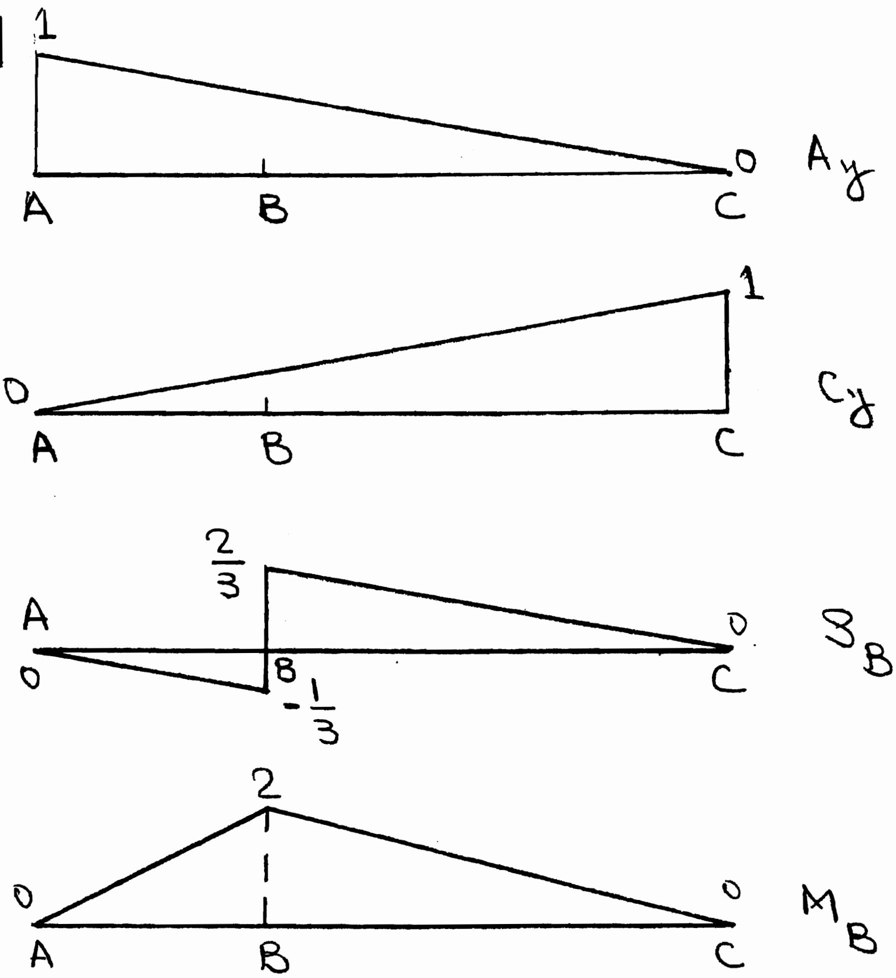

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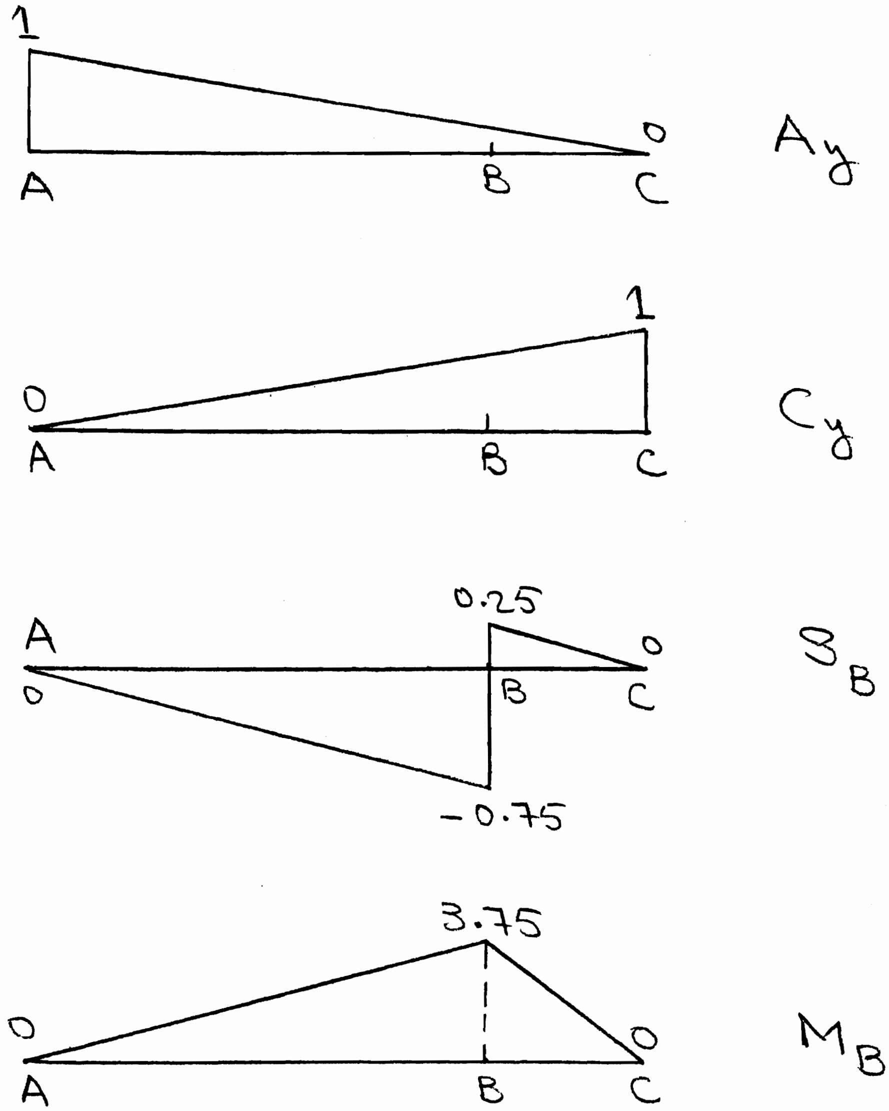

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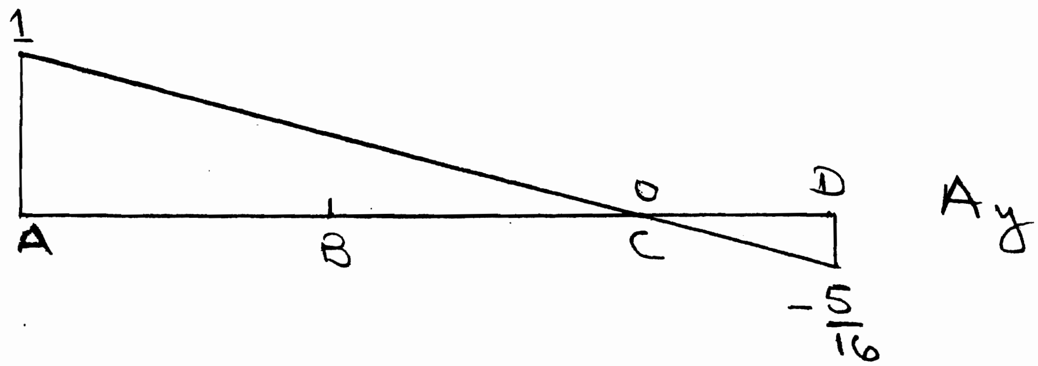

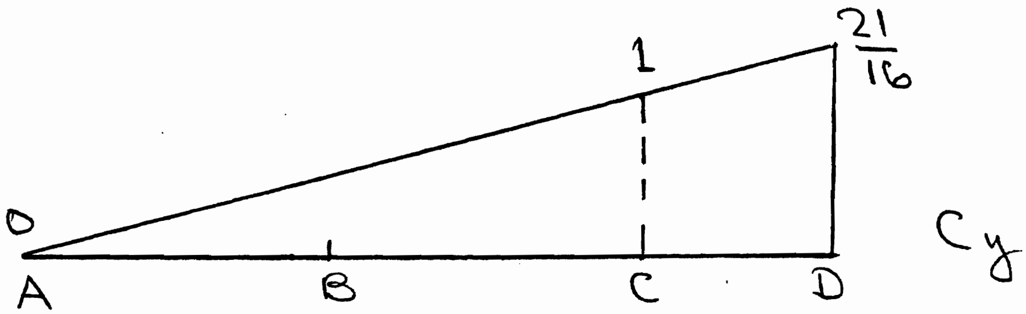

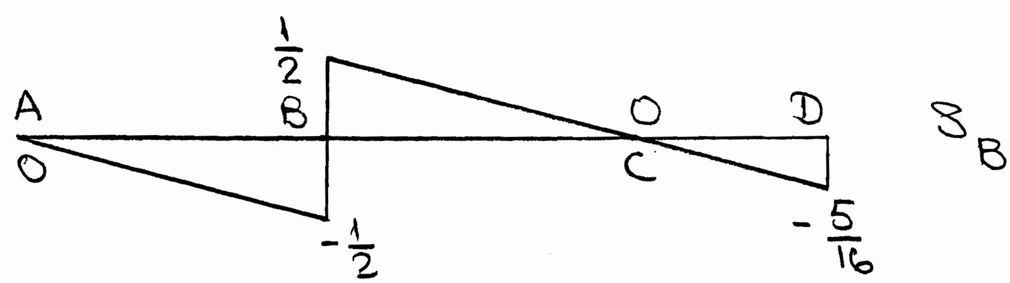

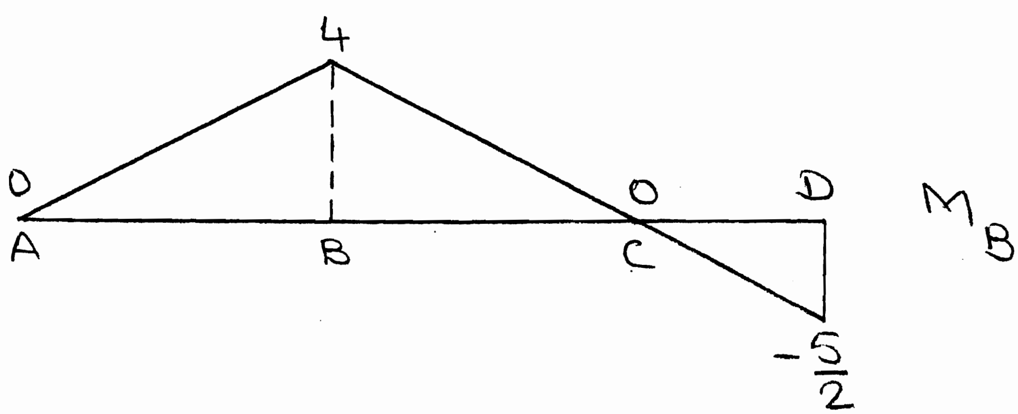

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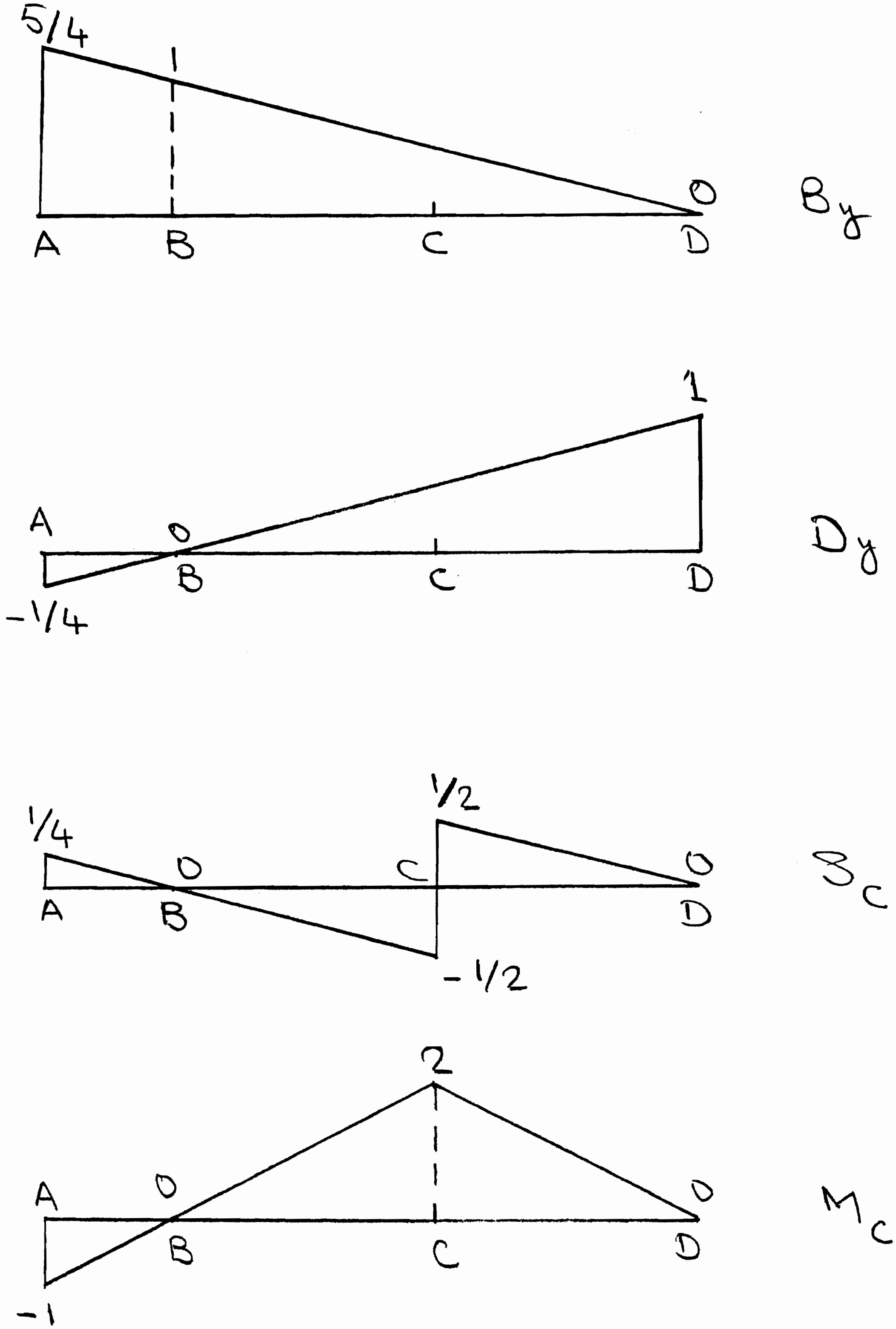

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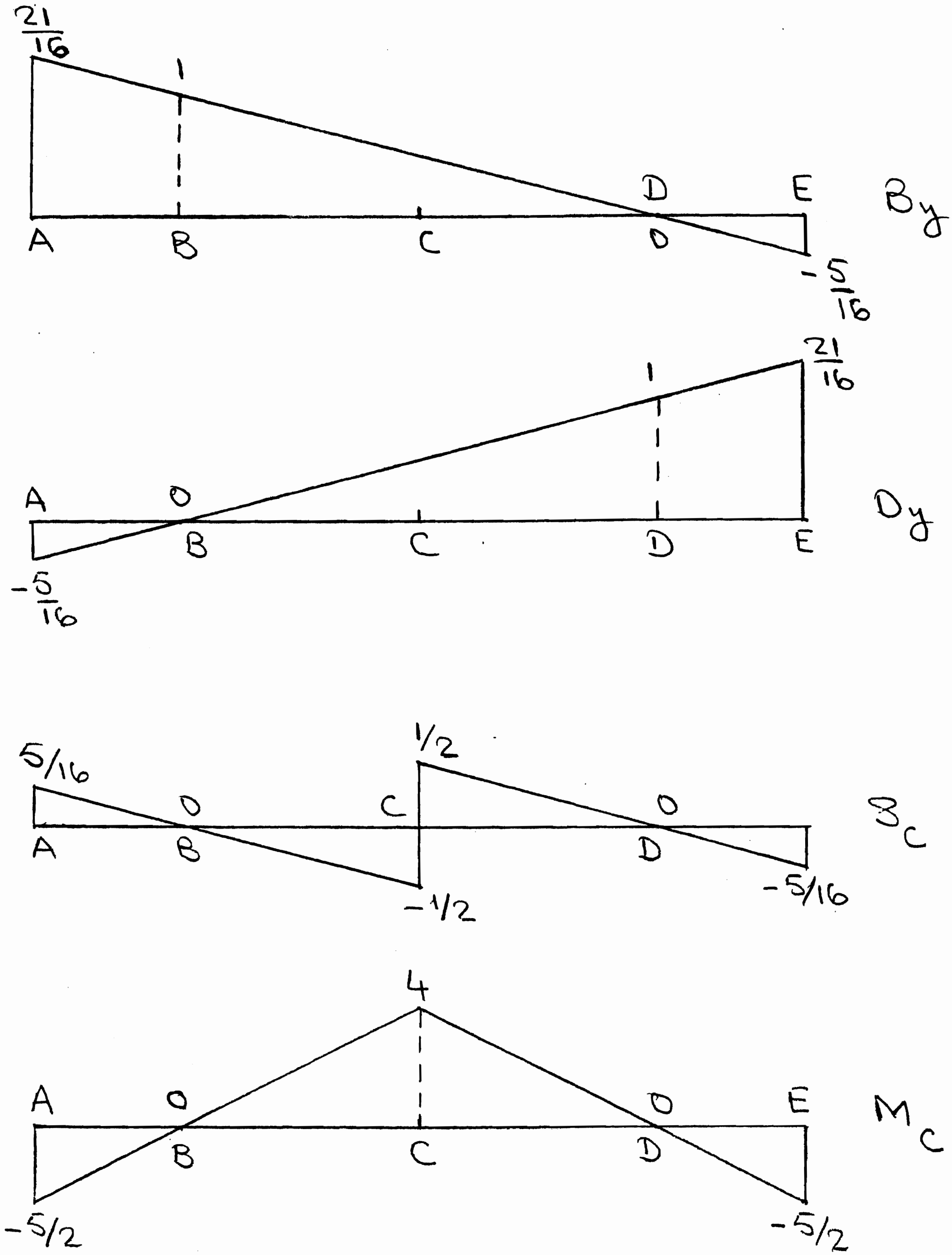

21/16
1
D
E
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A
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C
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-5/16
21/16
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A
0
Dy
B
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E
-5/16
1/2
5/16
0
C
0
Sc
A
B
D
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0
0
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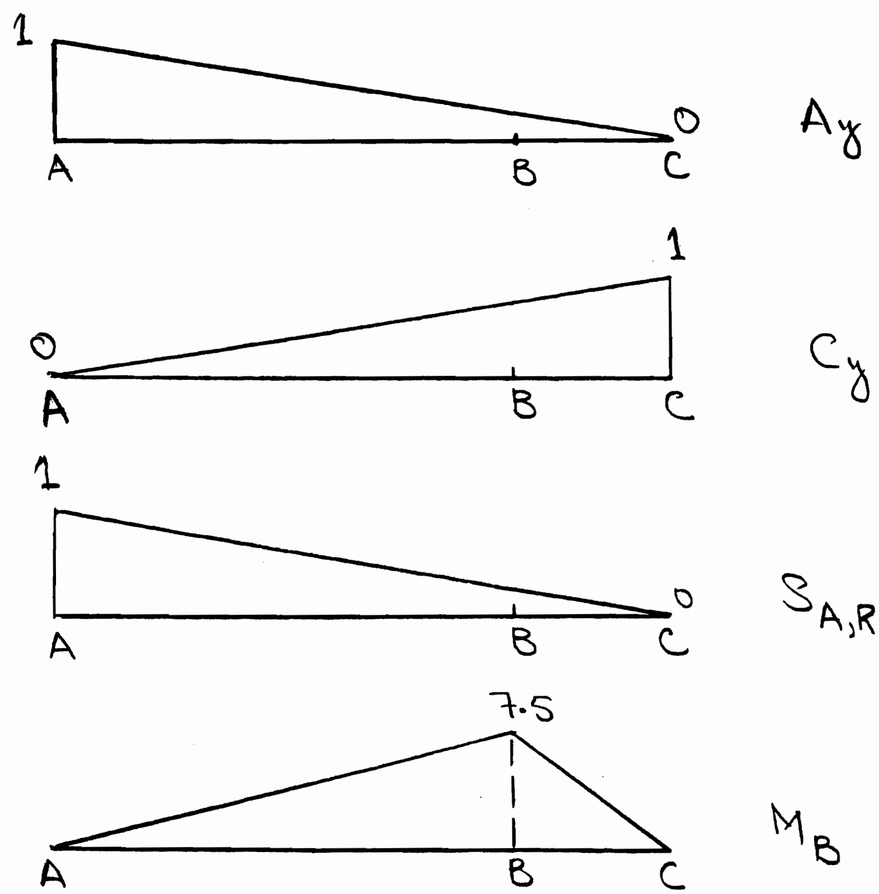
Mc
B
C
D
-5/2
-5/2

8.7



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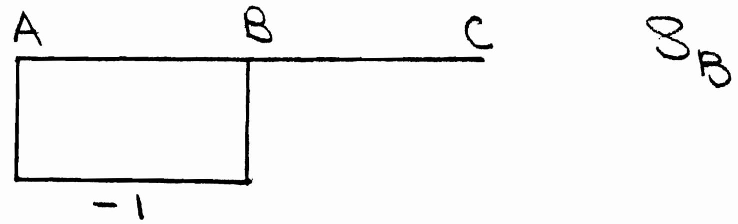

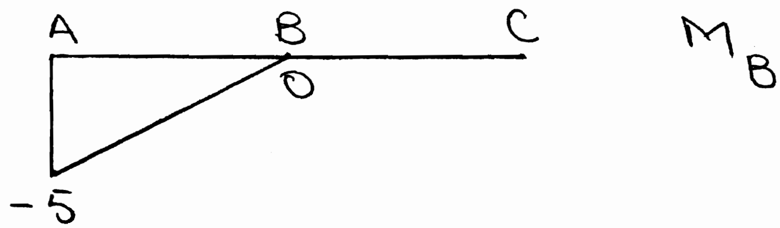

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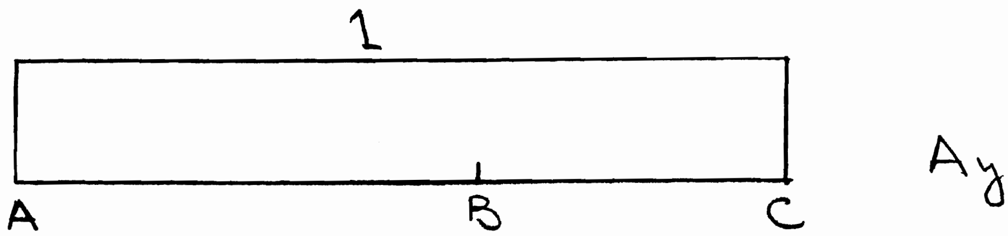

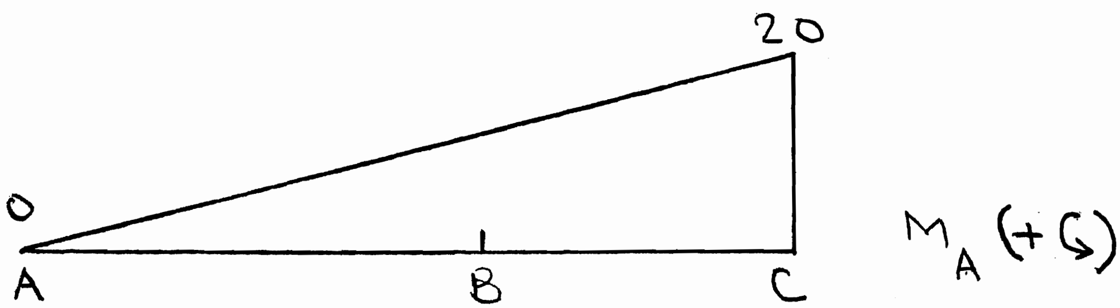

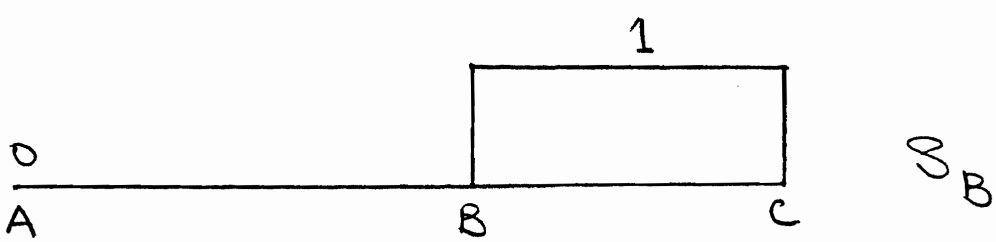

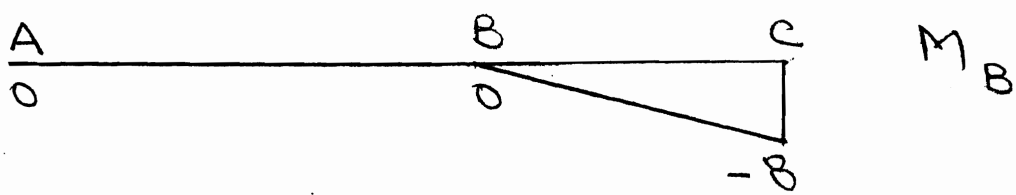

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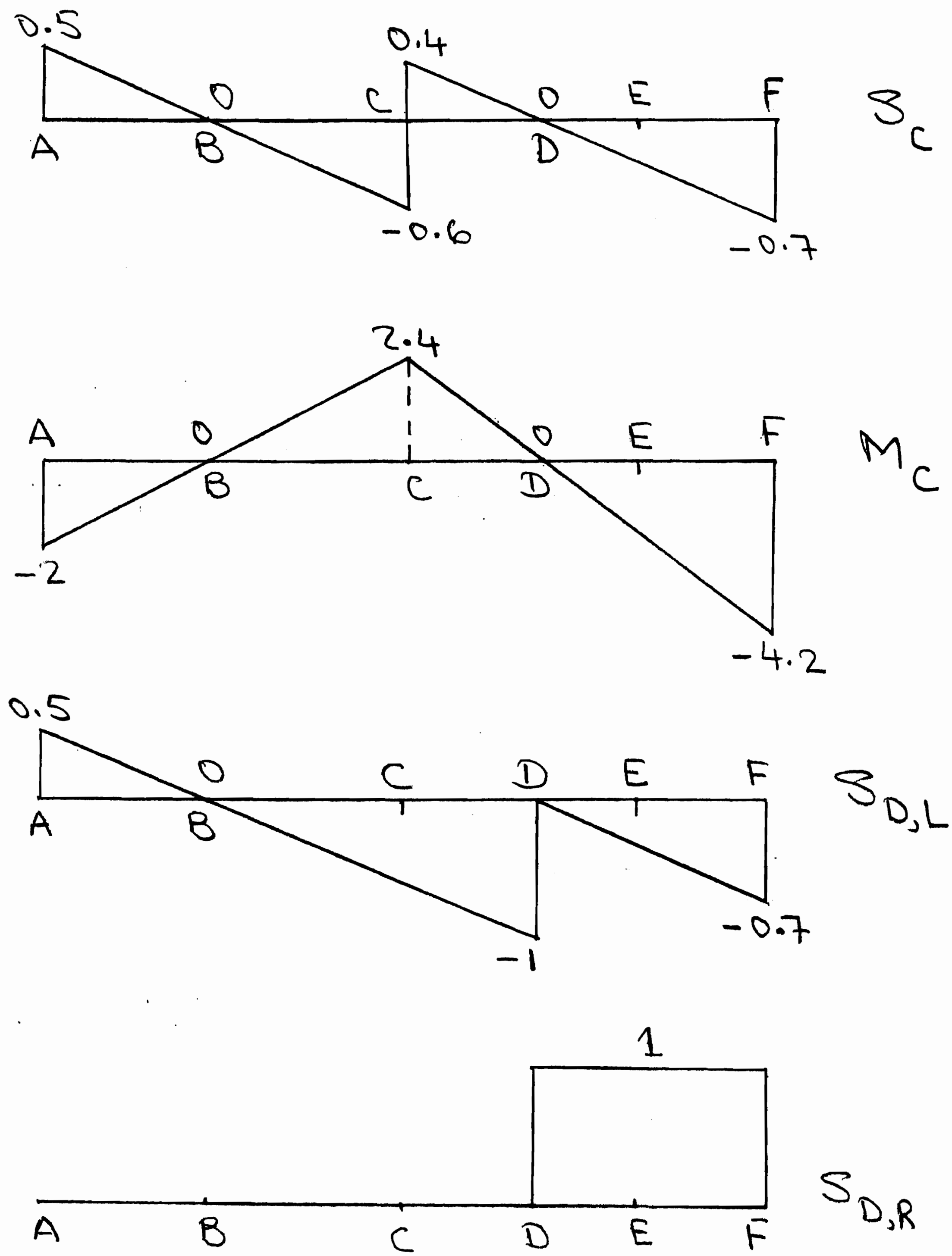
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0.4
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C
0
E
F
A
B
D
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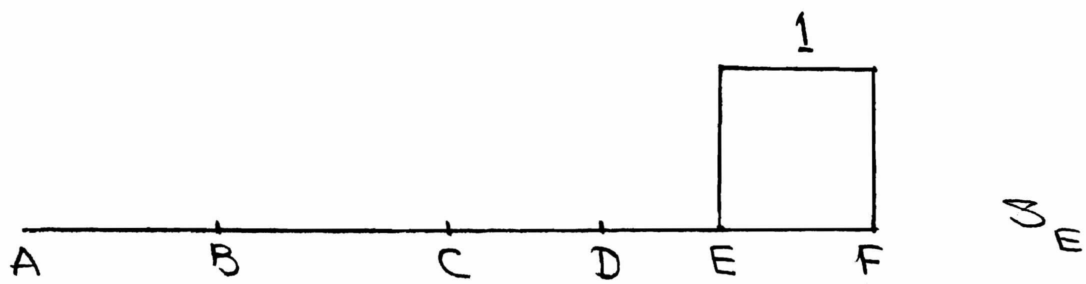
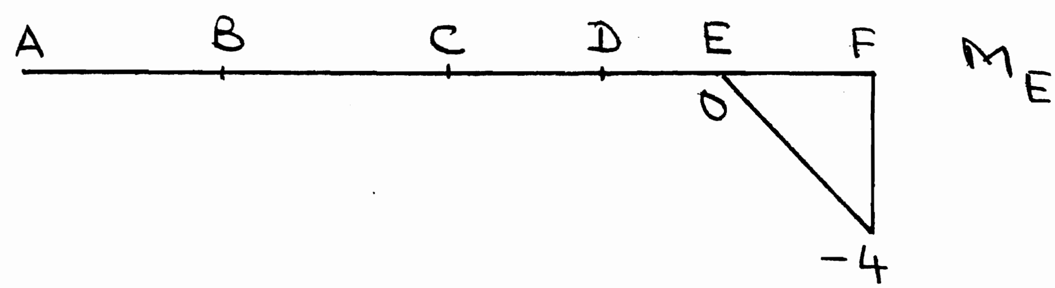
-0.7
S_C
2.4
A
0
0
E
F
B
C
D
-2
-4.2
M_C
0.5
0
C
D
E
F
A
B
-1
-0.7
$S_{D,L}$
1
A
B
C
D
E
F
$S_{D,R}$

8.11





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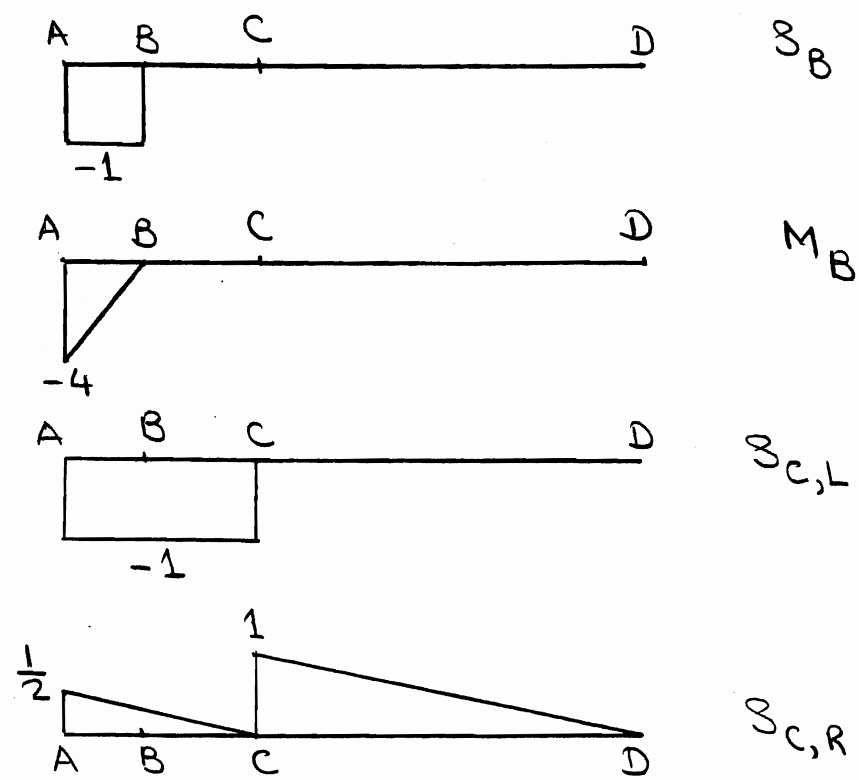

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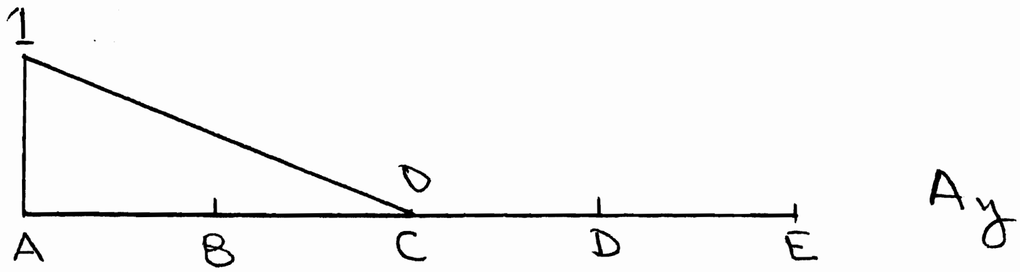

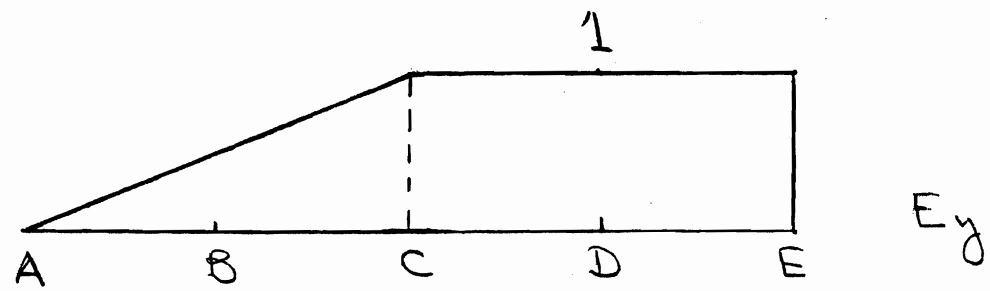

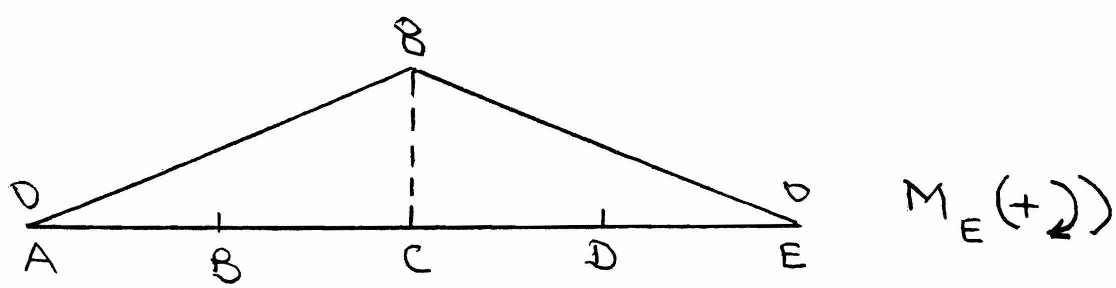

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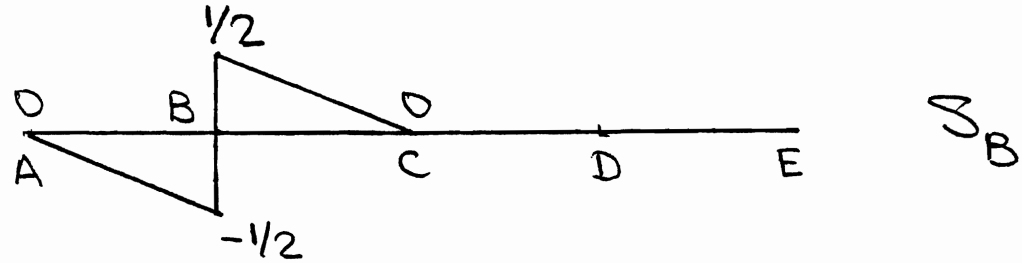

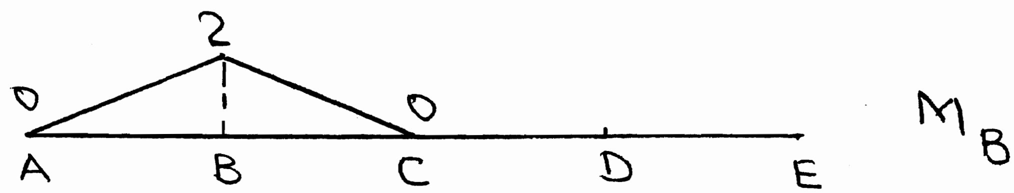

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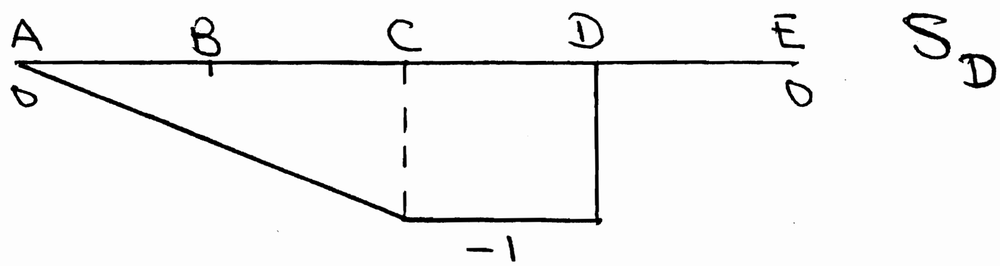

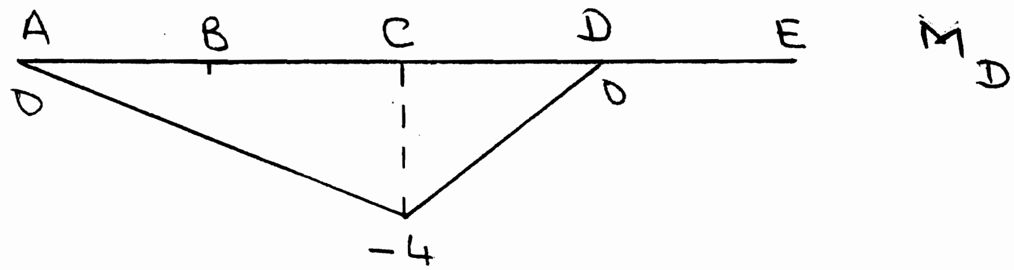

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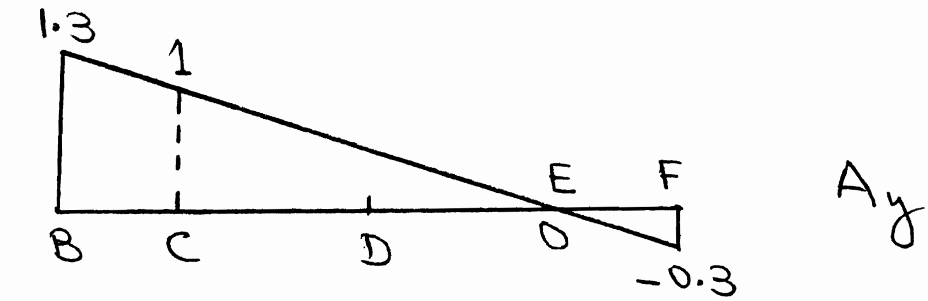
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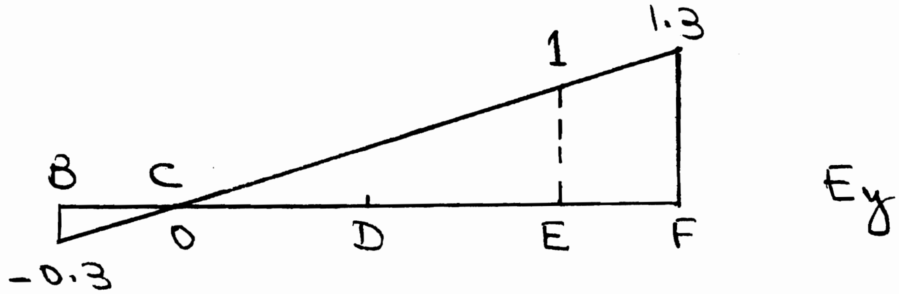
F
B
C
D
0
-0.3
Ay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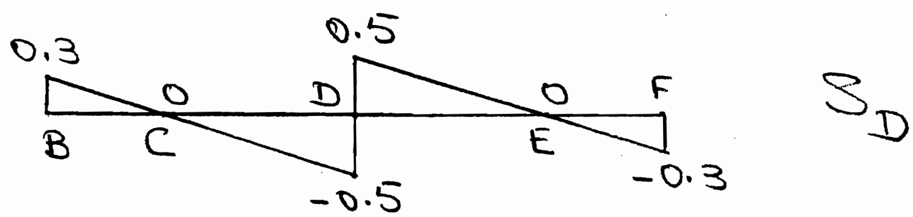
1
B
C
-0.3
0
D
E
F
Ey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0
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0
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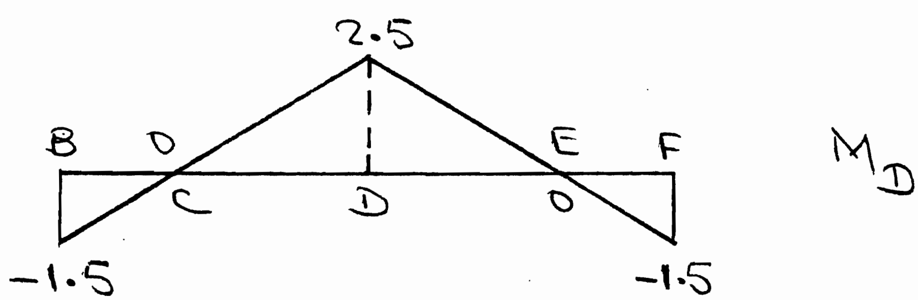
B
C
E
-0.5
-0.3
SD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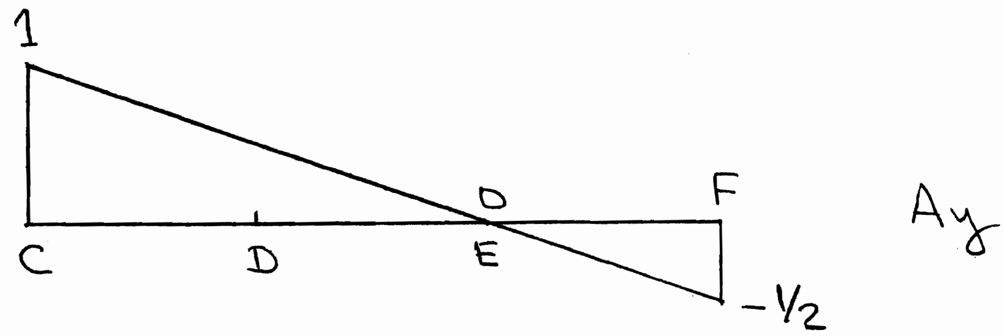
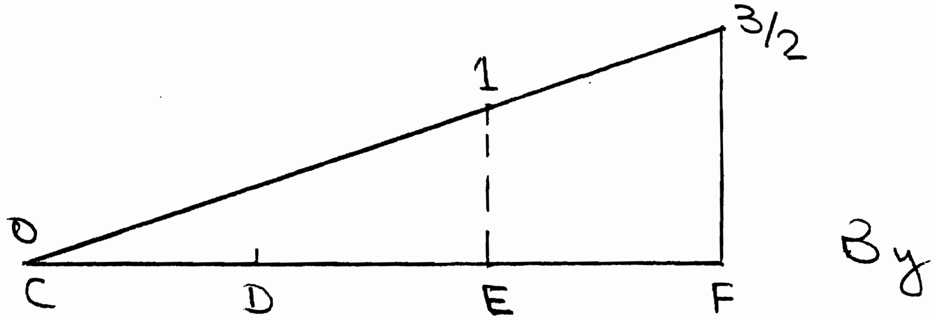
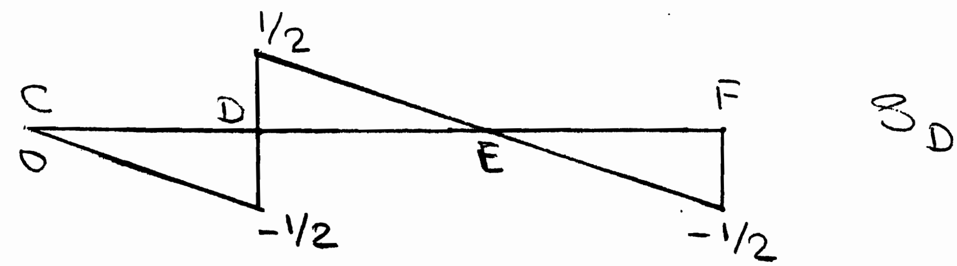
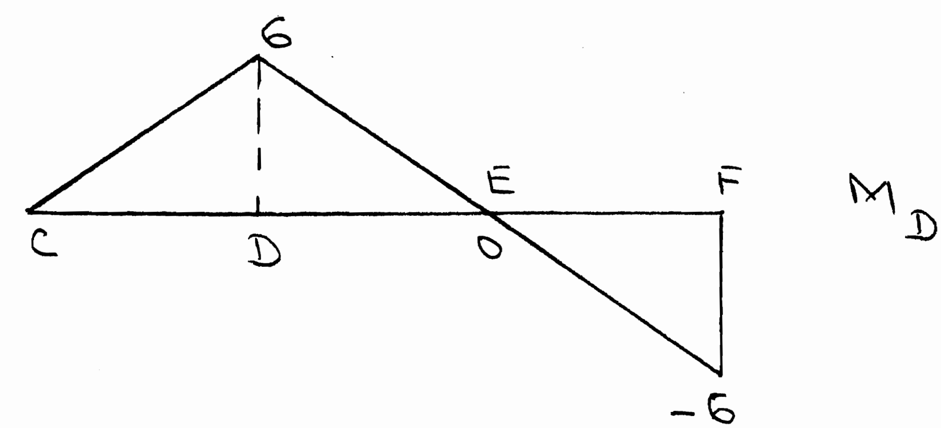
0
E
F
C
D
0
-1.5
-1.5
MD

8.17









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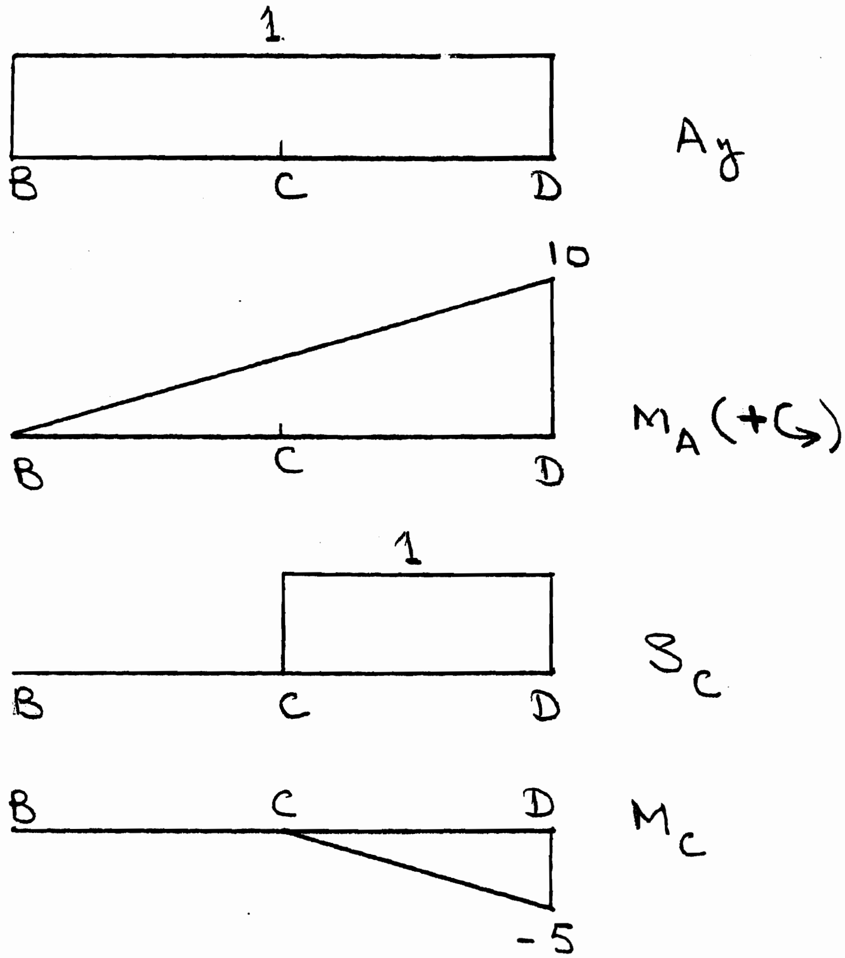

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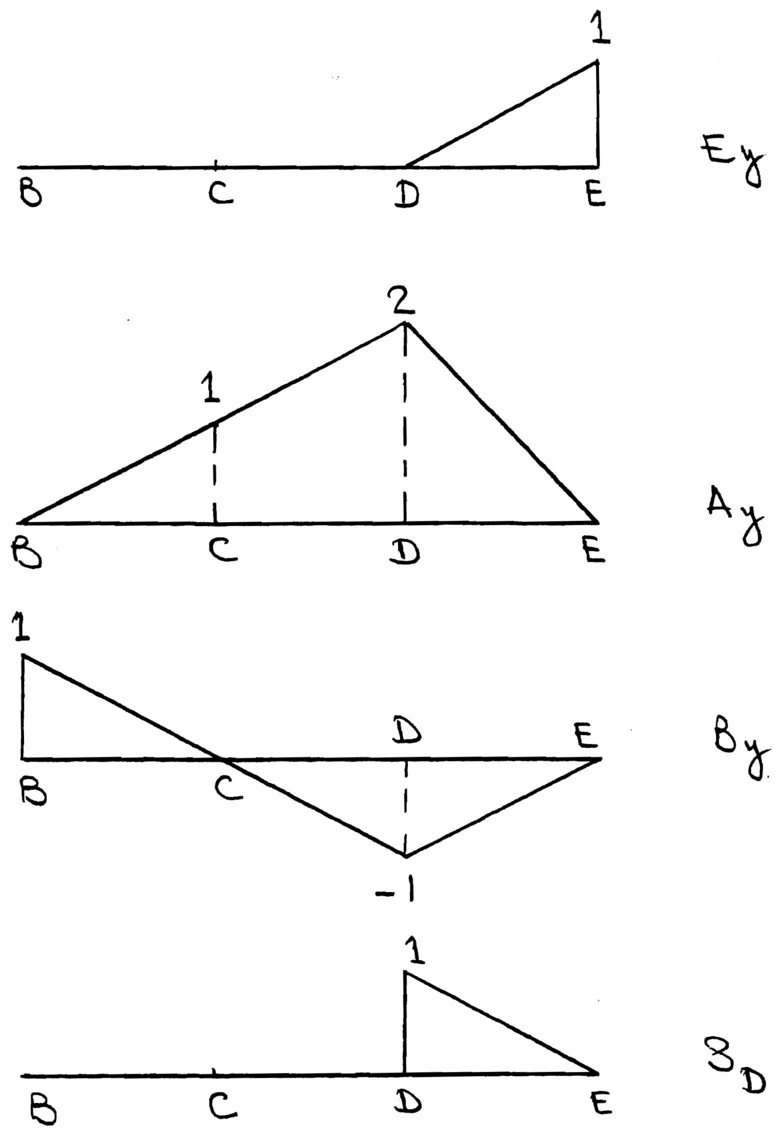

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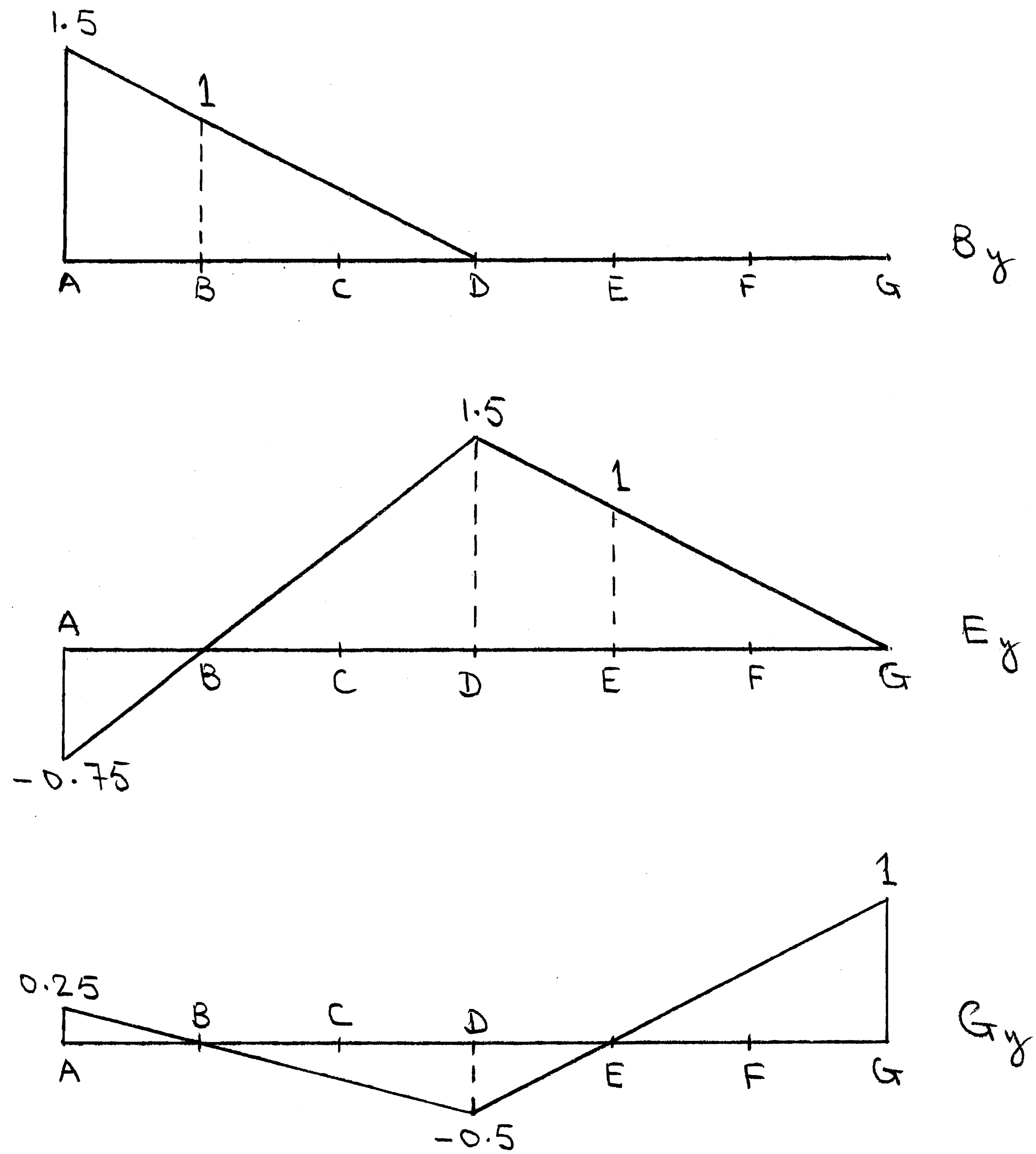

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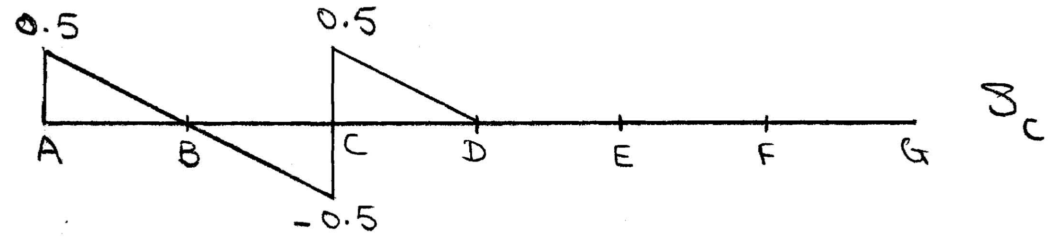

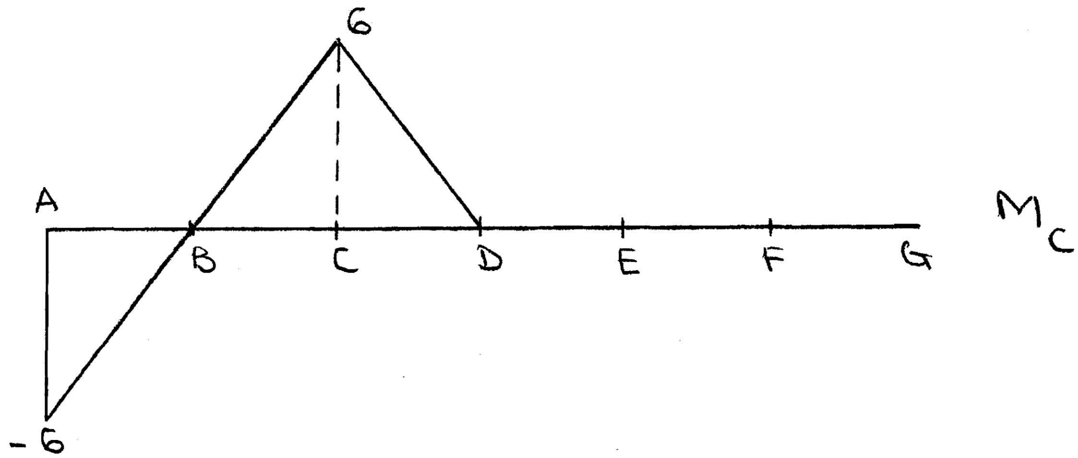

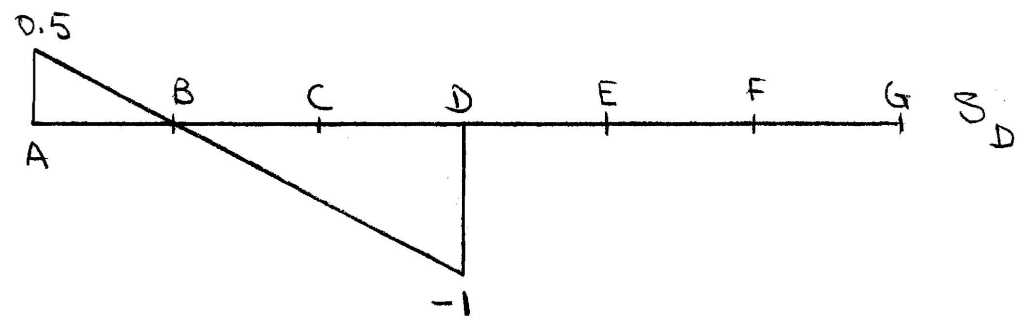

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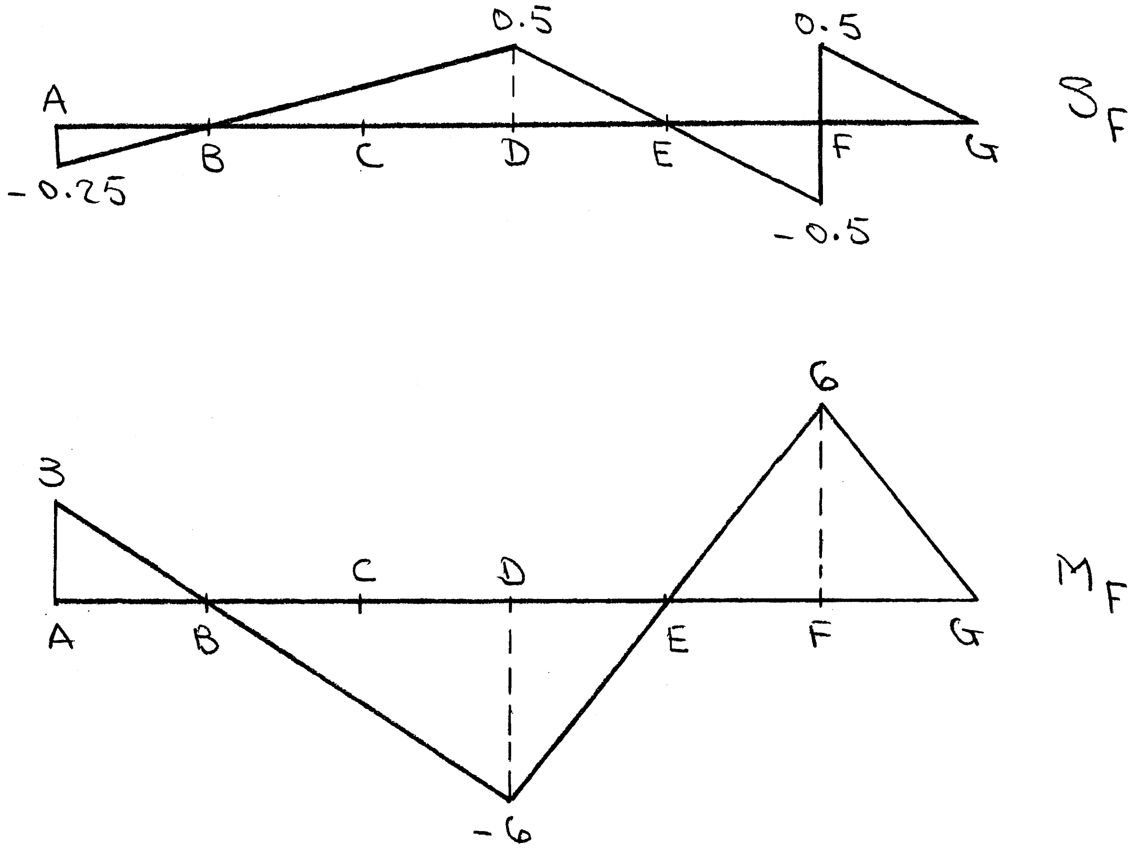

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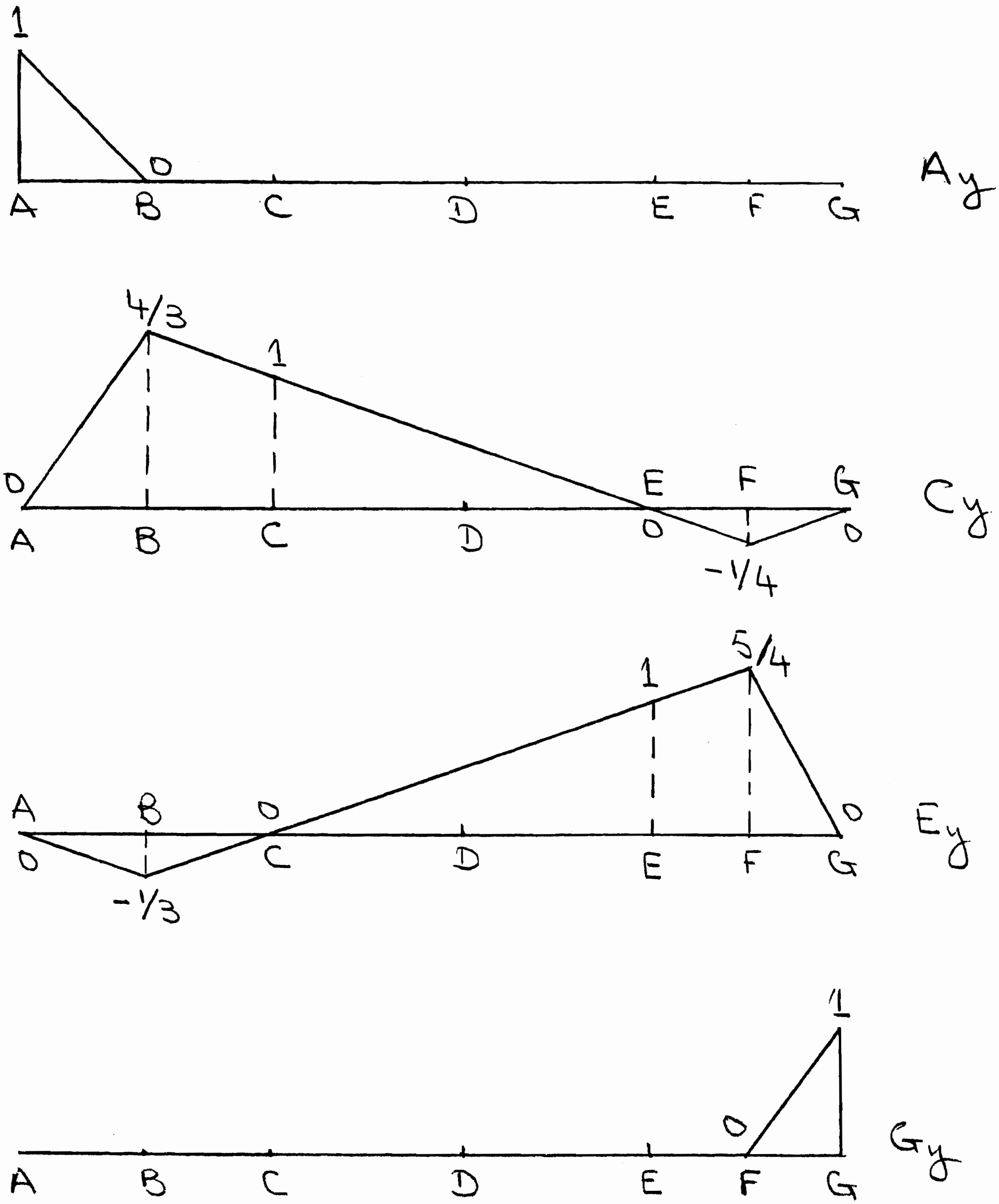

1
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-1/4
Cy
5/4
1
A
B
0
0
0
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Ey
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B
C
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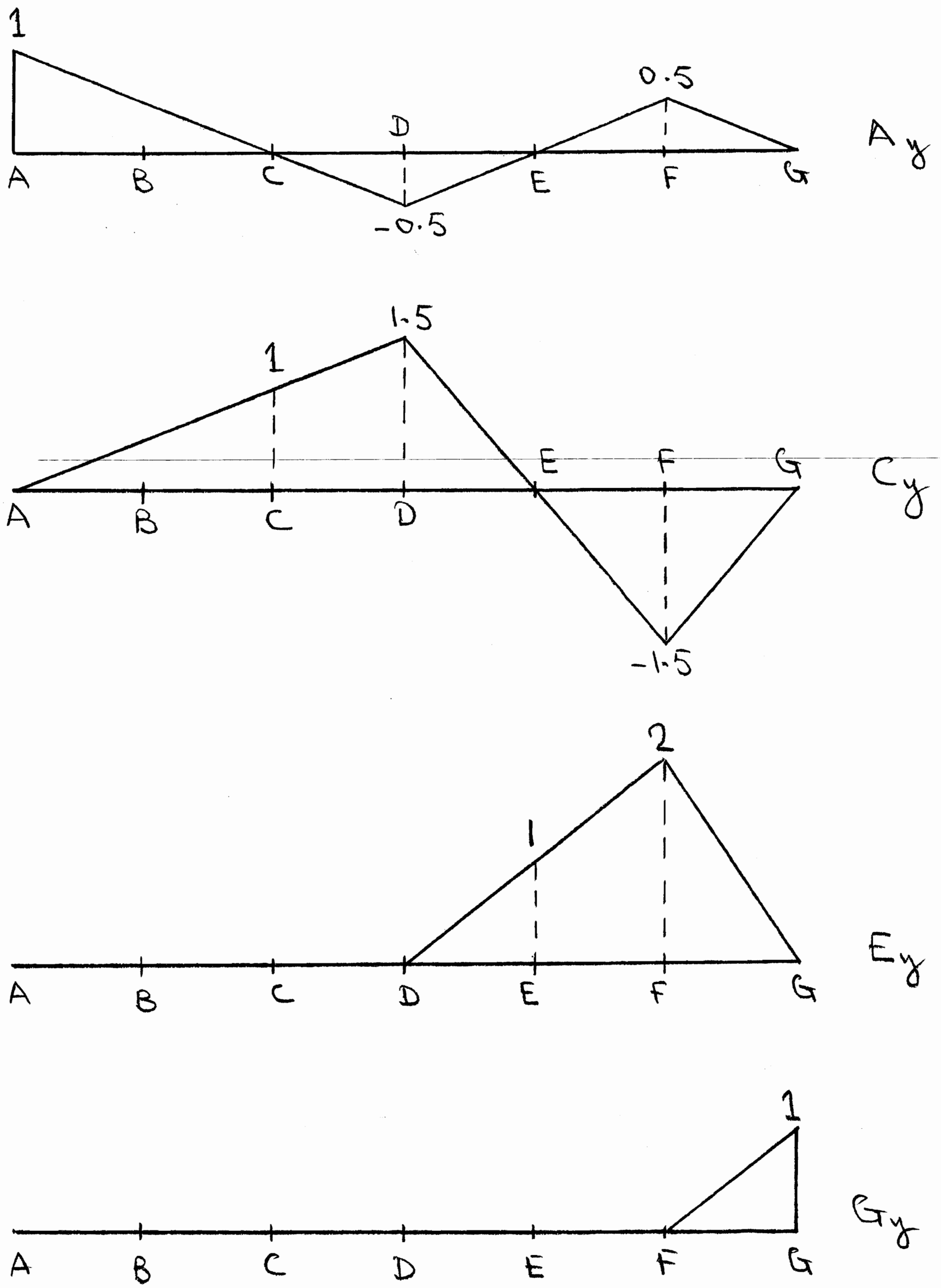
G
Gy

8.24



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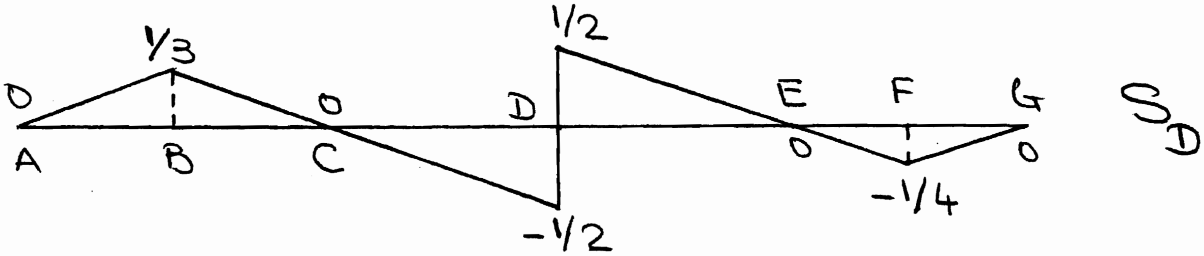

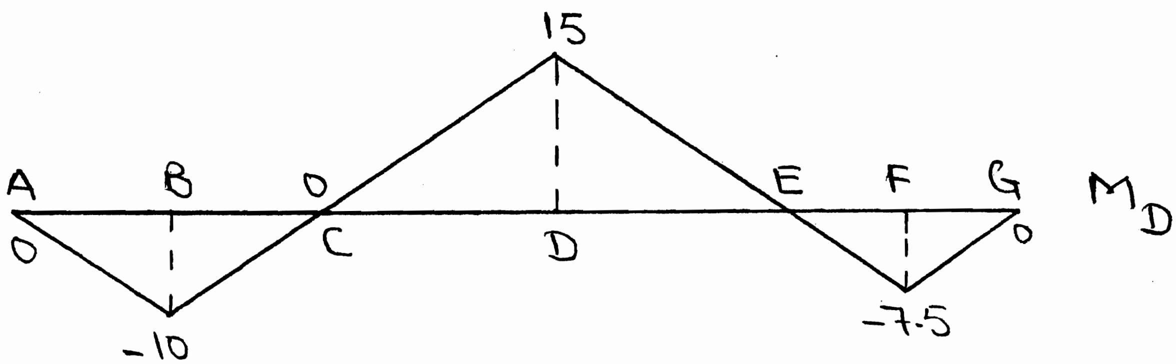

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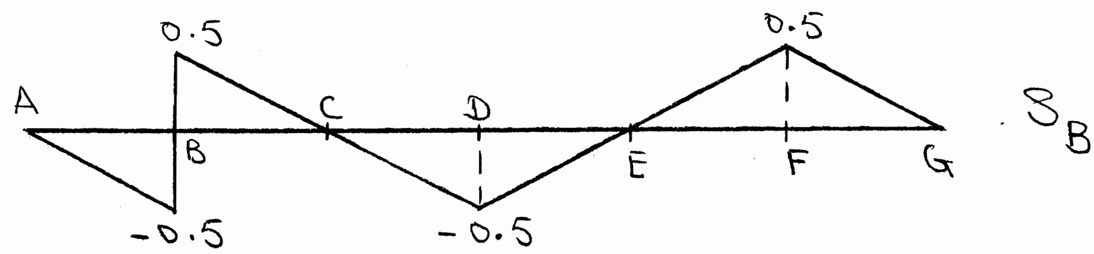

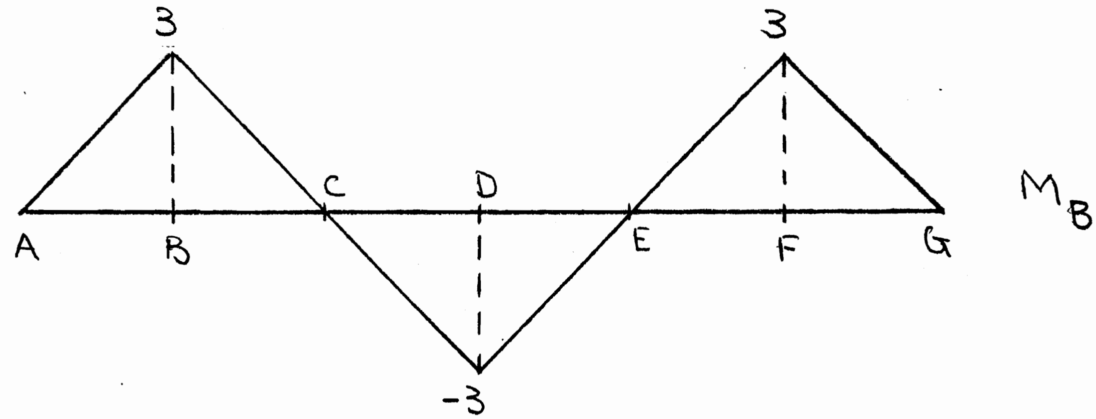

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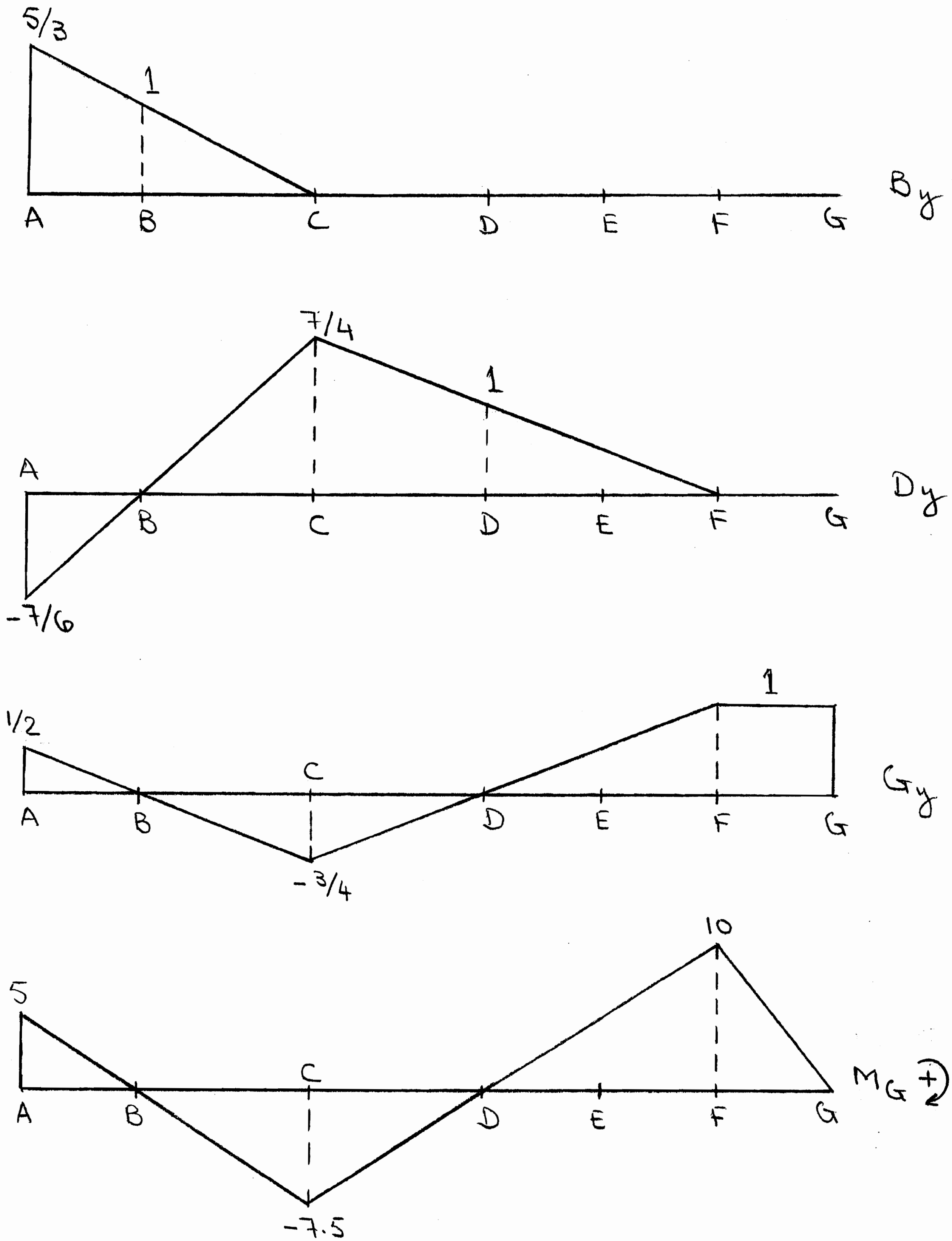
5/3
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A
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C
D
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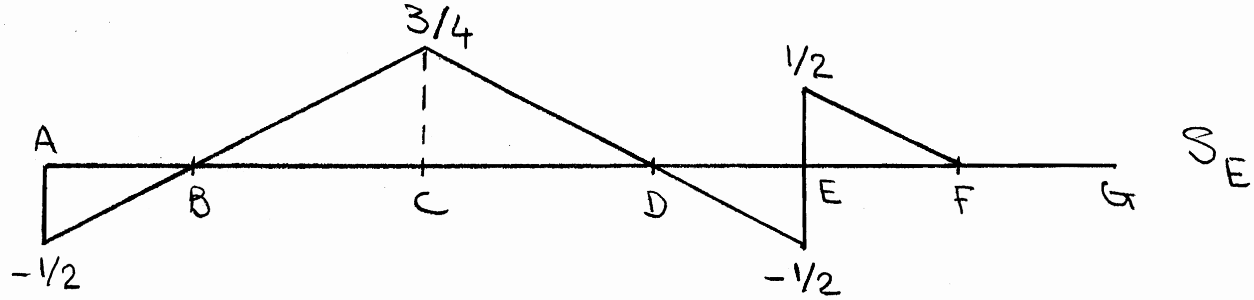
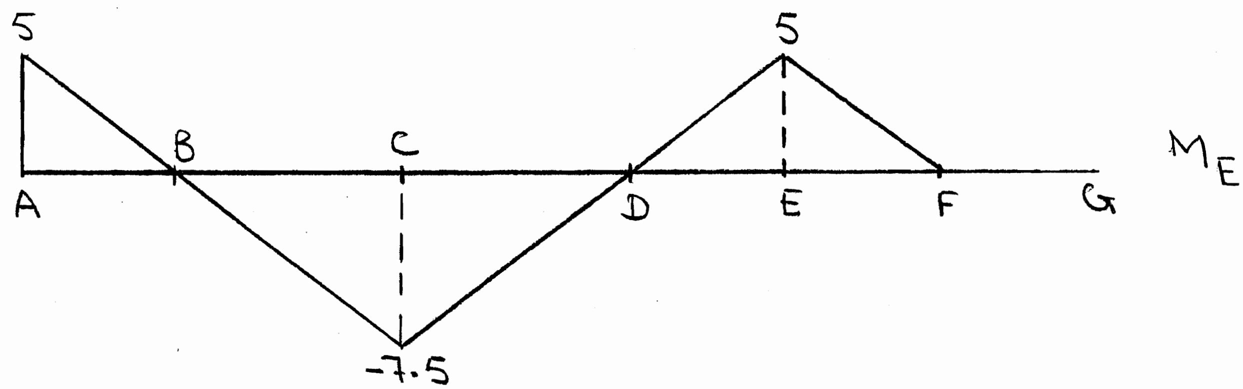
F
G
By
7/4
1
A
B
C
D
E
F
G
Dy
-7/6
1/2
1
C
A
B
D
E
F
G
Gy
-3/4
10
5
C
A
B
D
E
F
G
MG
+
-7.5

8.28





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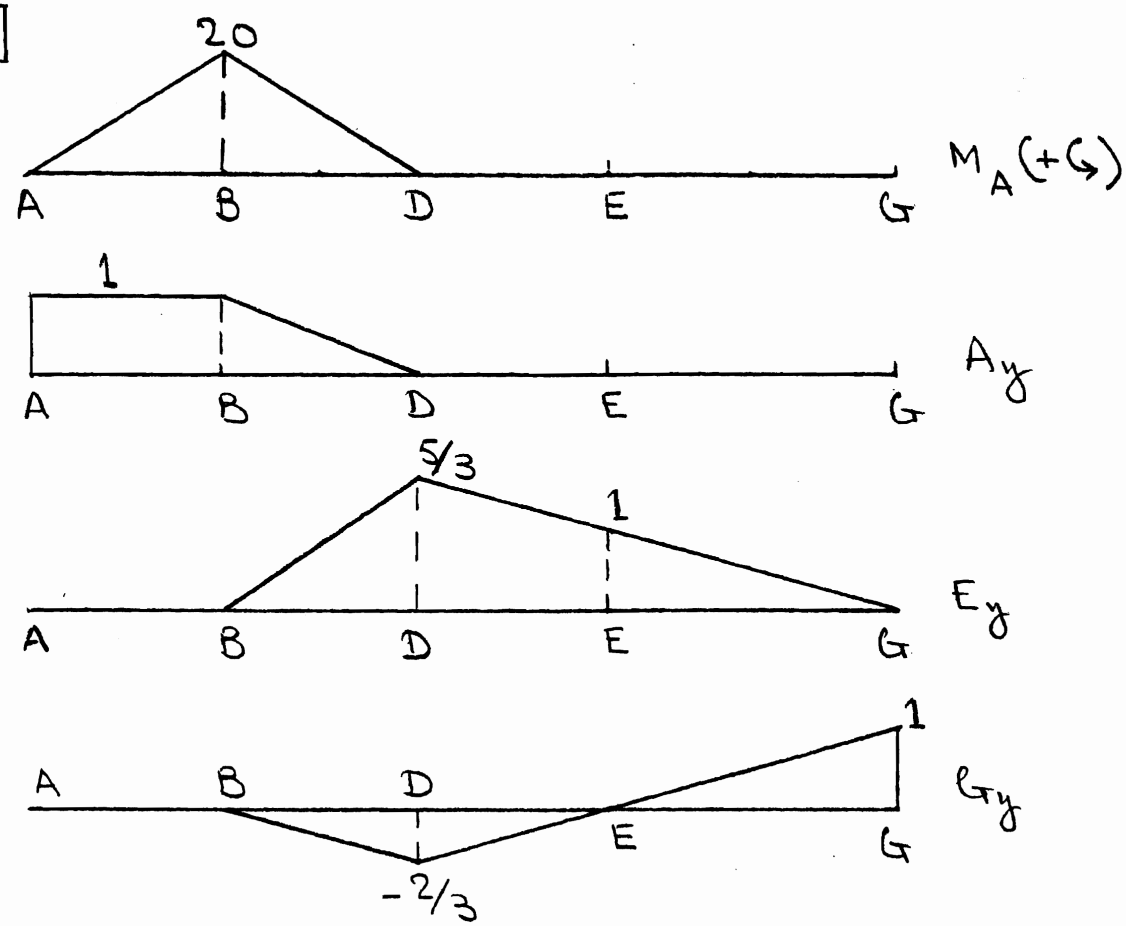

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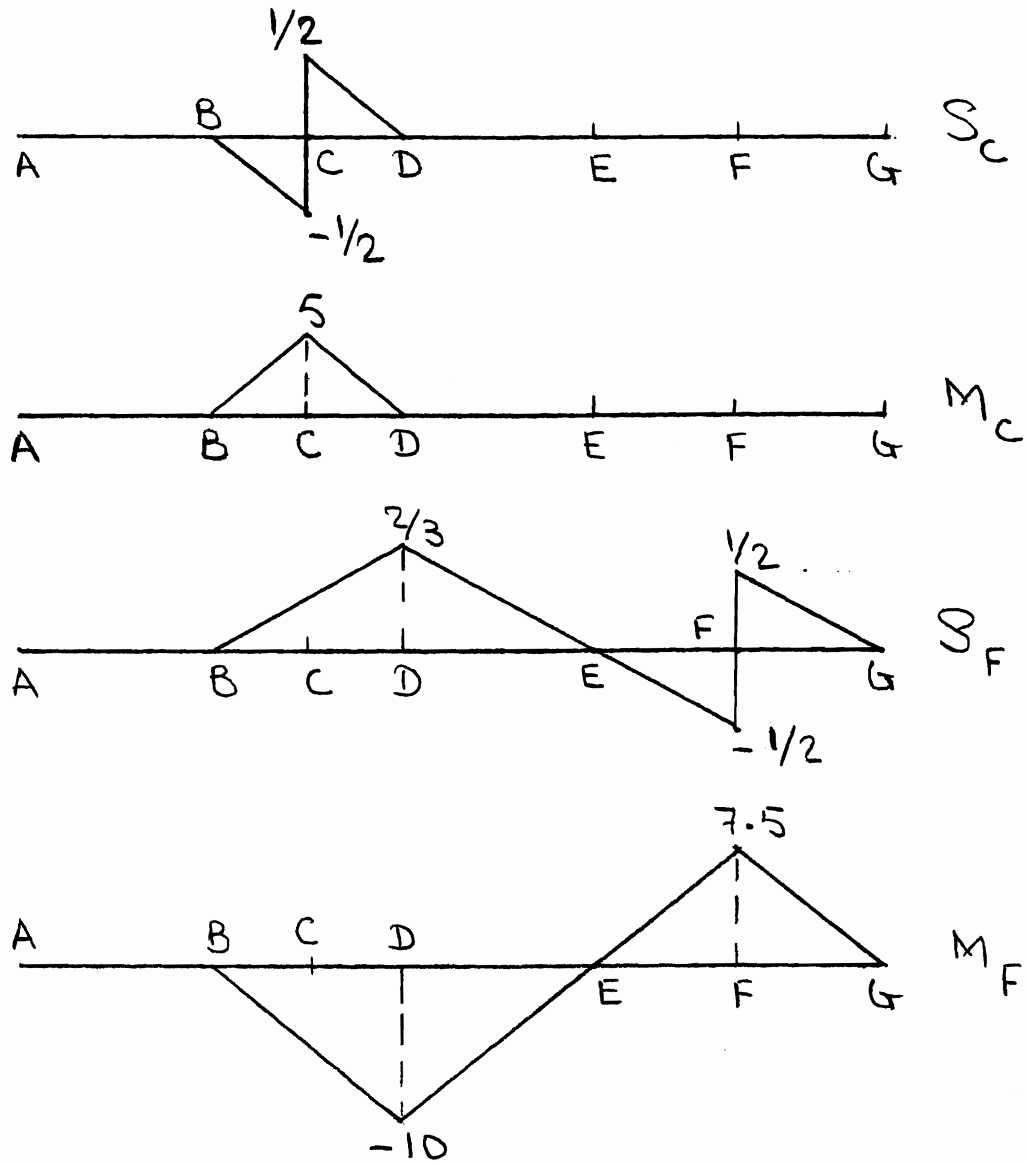

1/2
B
A
C
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G
S_C
-1/2
5
A
B
C
D
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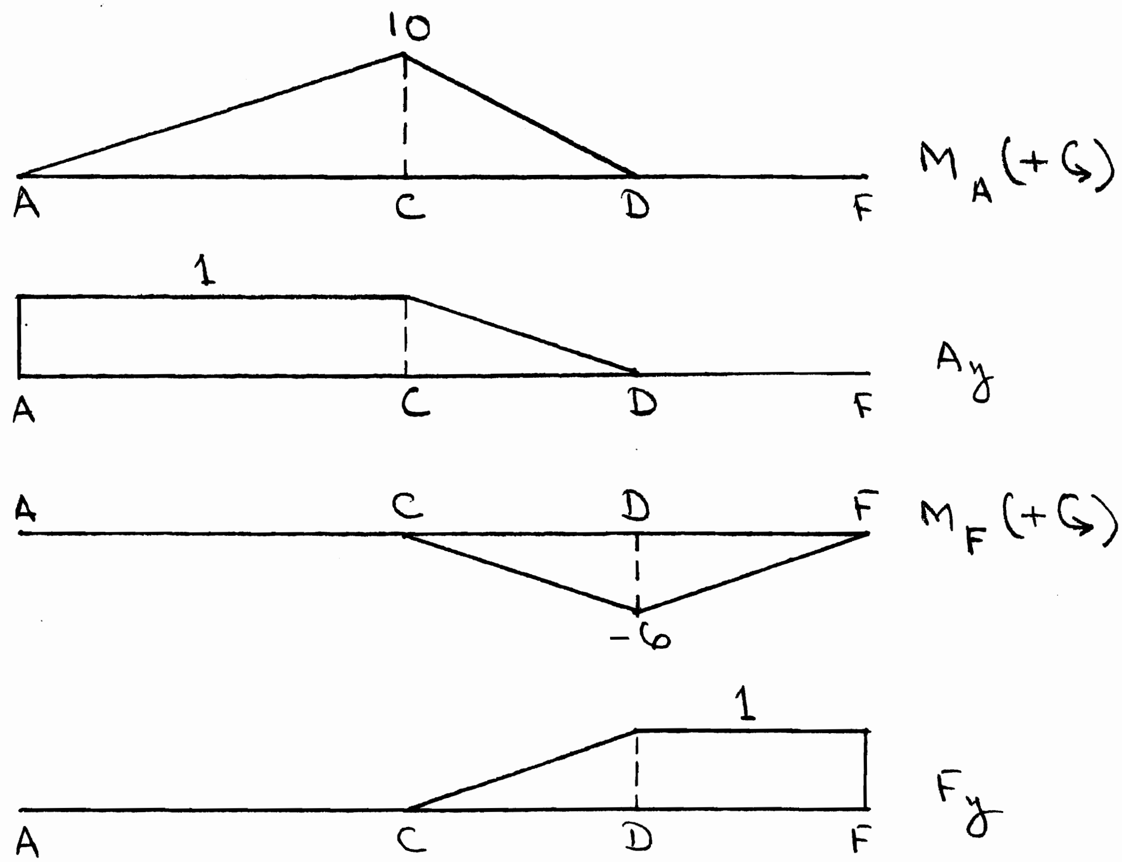
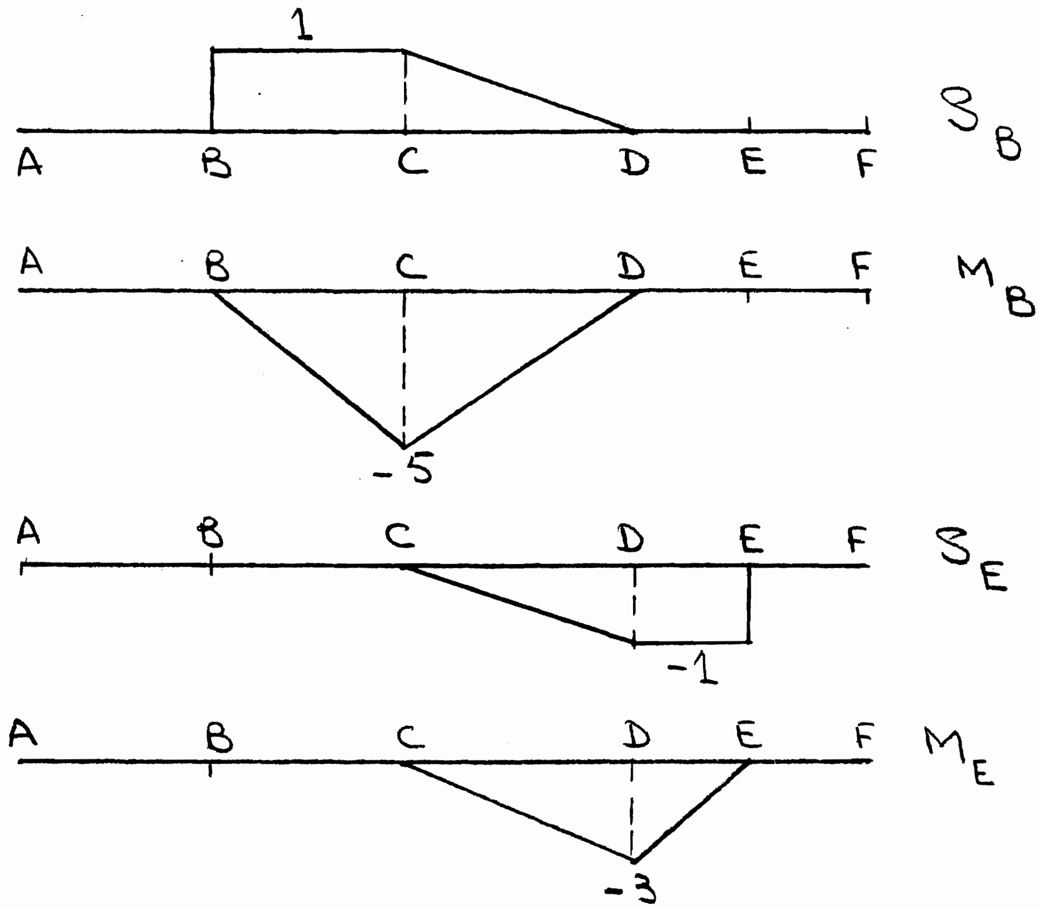
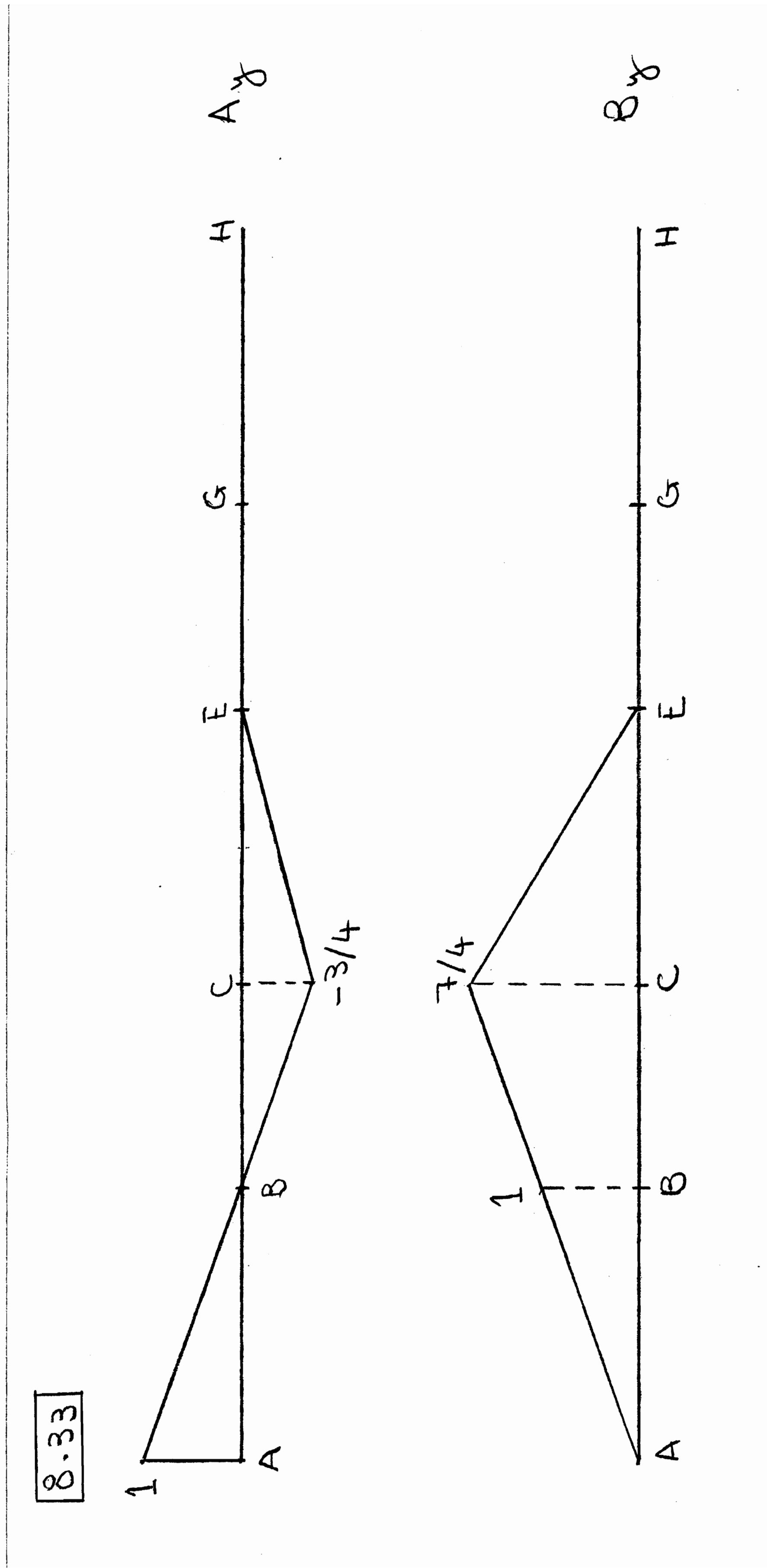
F
G
M_C
2/3
1/2
F
A
B
C
D
E
G
S_F
-1/2
7.5
A
B
C
D
E
F
G
M_F
-10

8.31

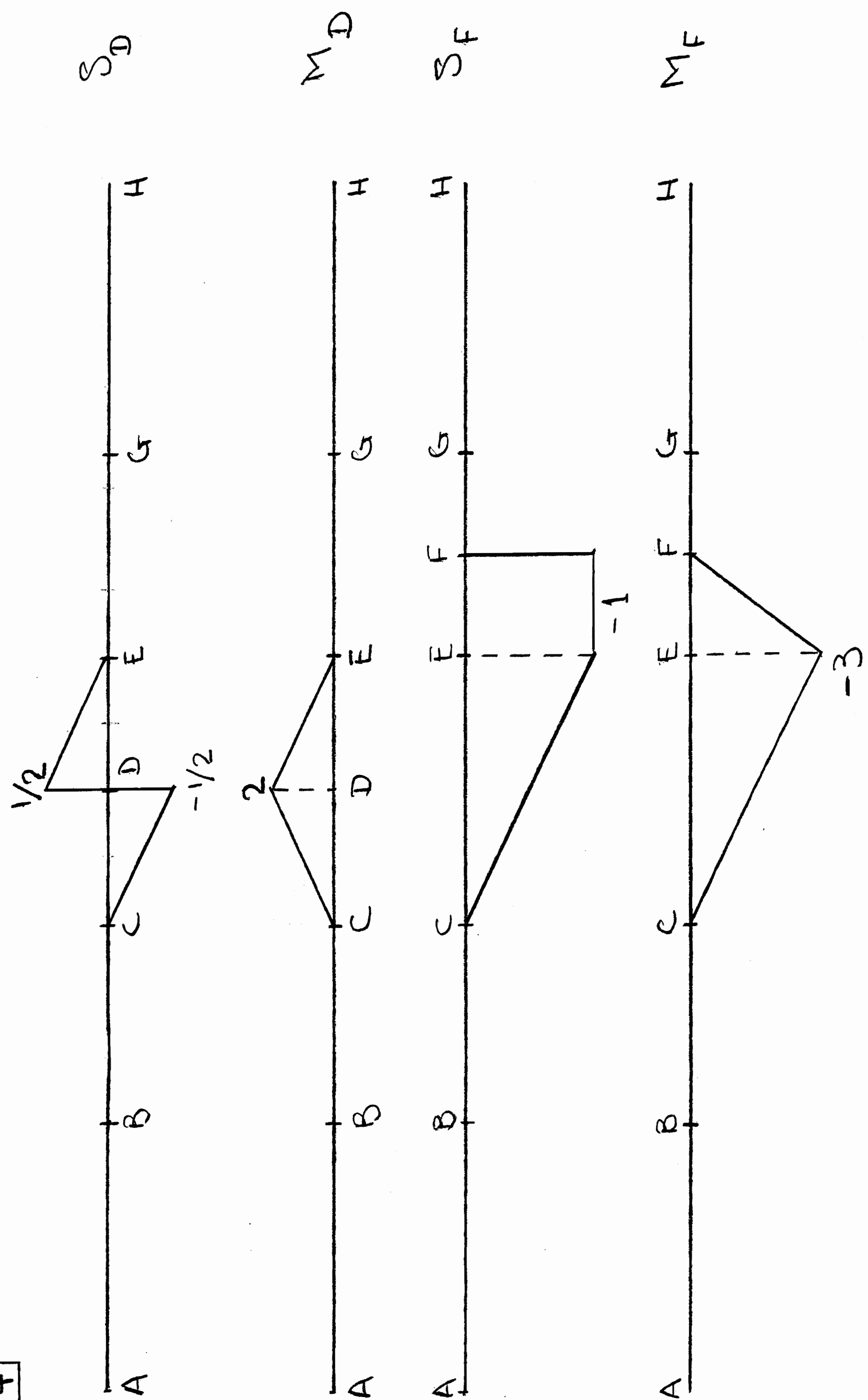


8.32




8.33
Ay
By
A
B
C
E
G
H
1
-3/4
7/4
1

8.34



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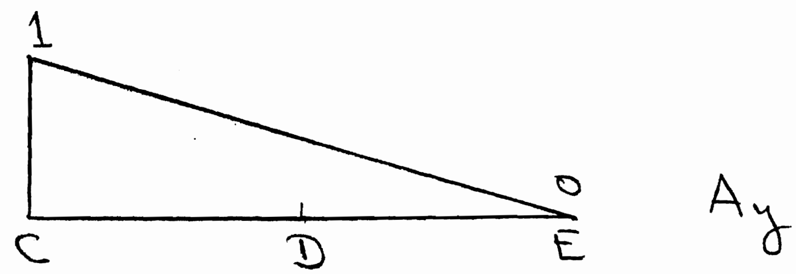

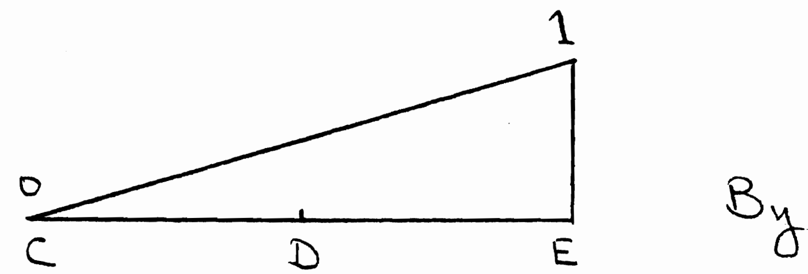

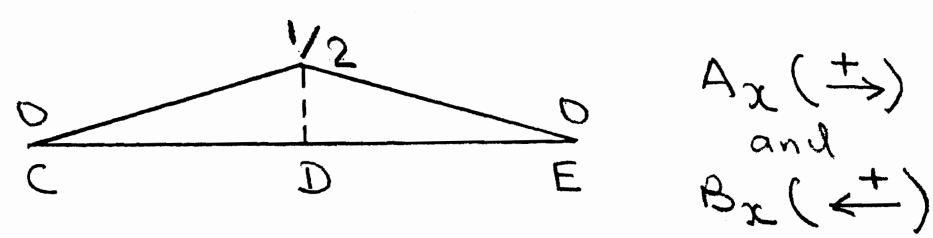

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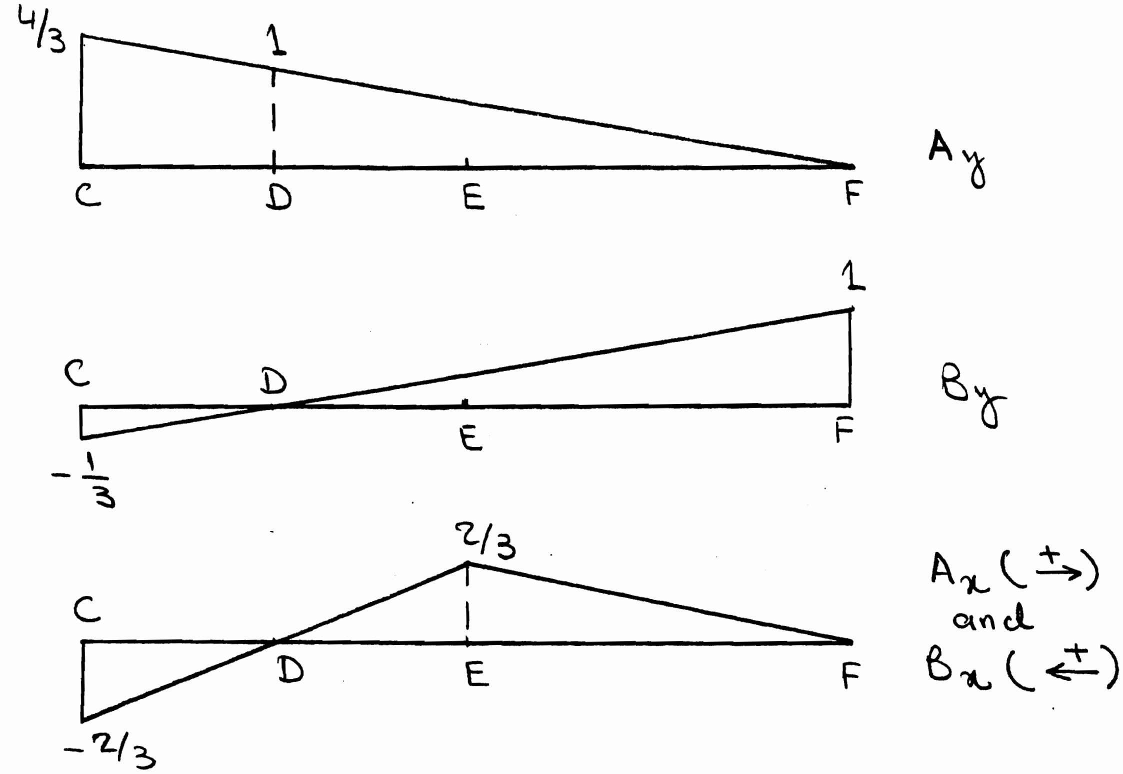
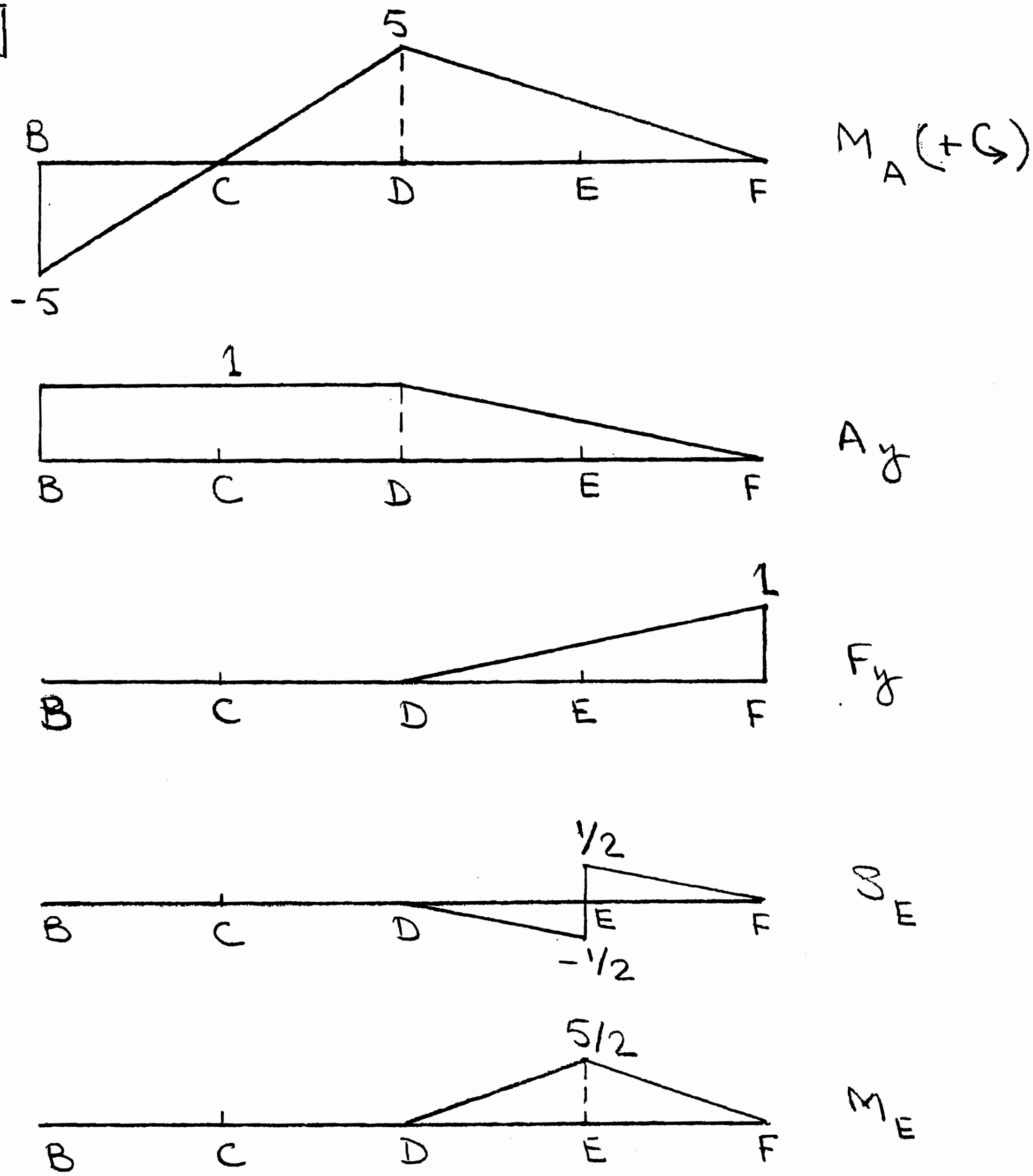


8.37



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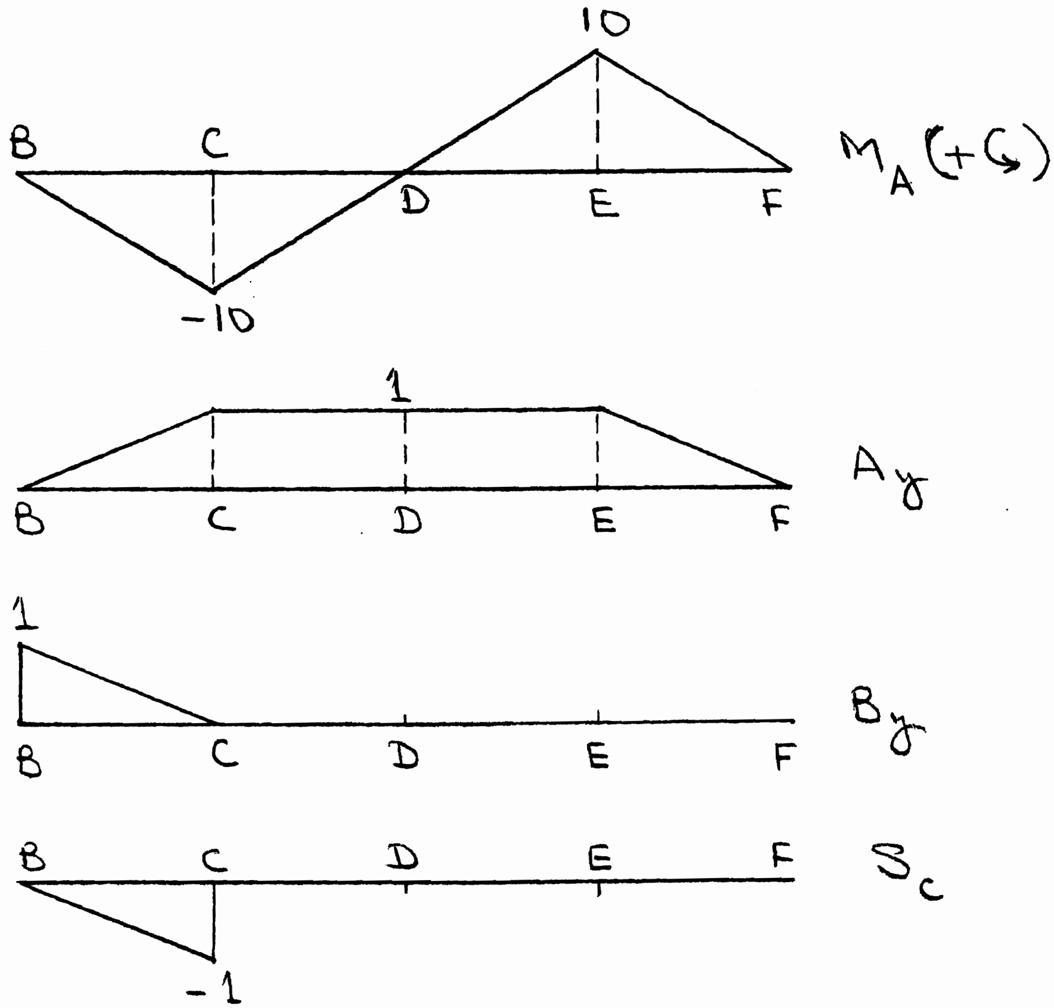
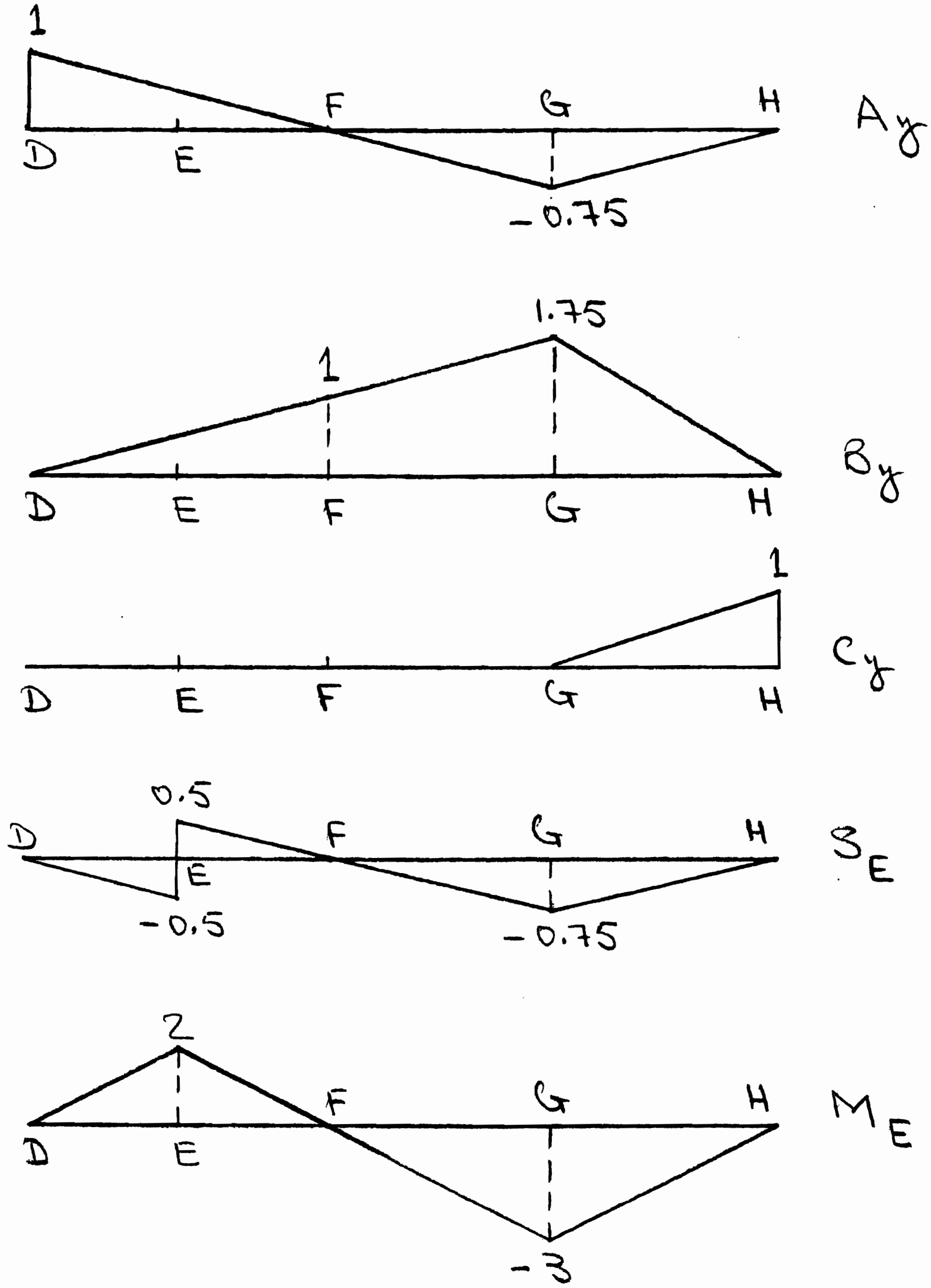
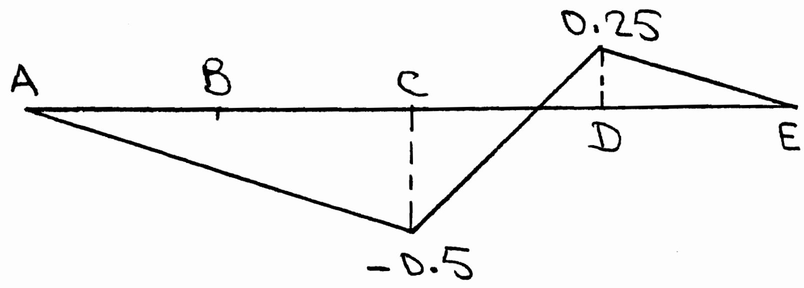


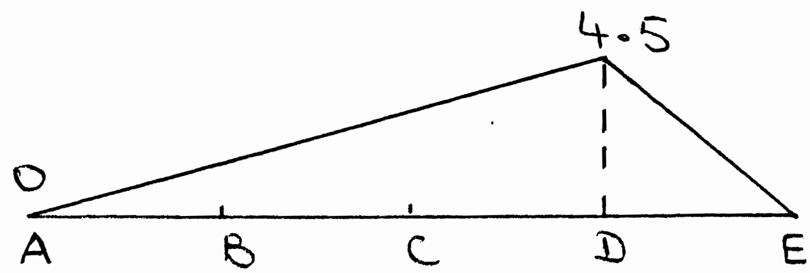
8.39


1
F
G
H
Ay
D
E
-0.75
1.75
1
By
D
E
F
G
H
1
Cy
D
E
F
G
H
0.5
D
F
G
H
SE
E
-0.5
-0.75
2
F
G
H
ME
D
E
-3

8.40

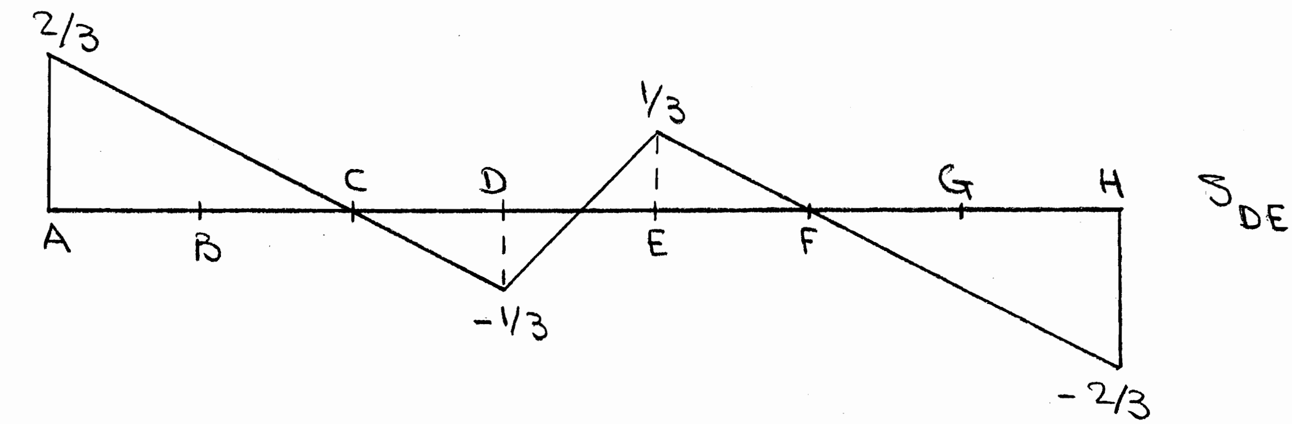


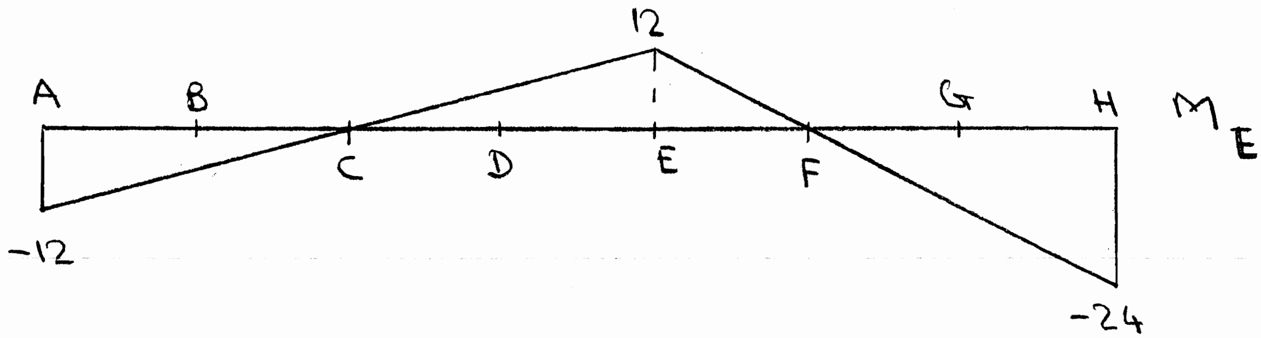
δ_{CD}



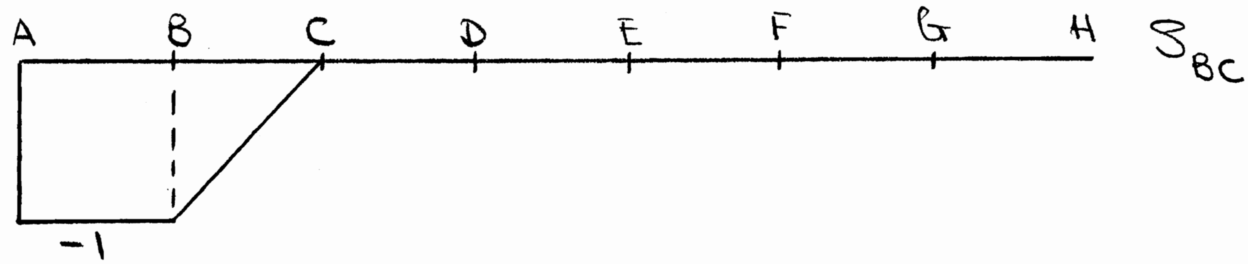
M_D

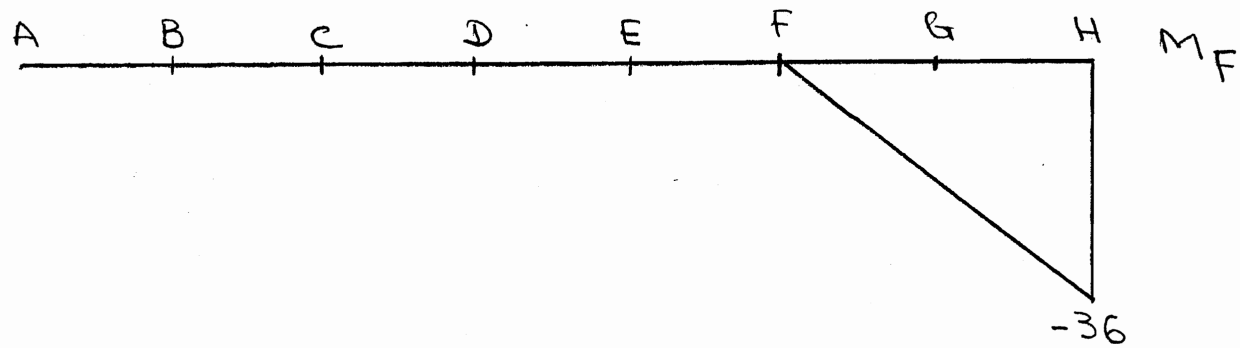
8.41





8.42





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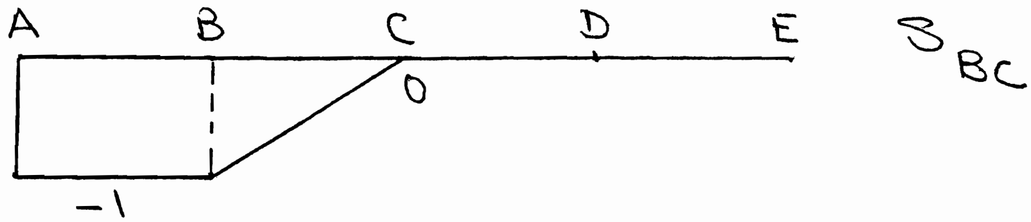

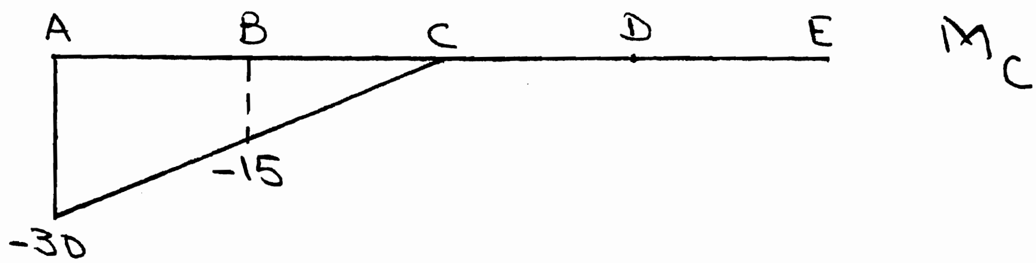

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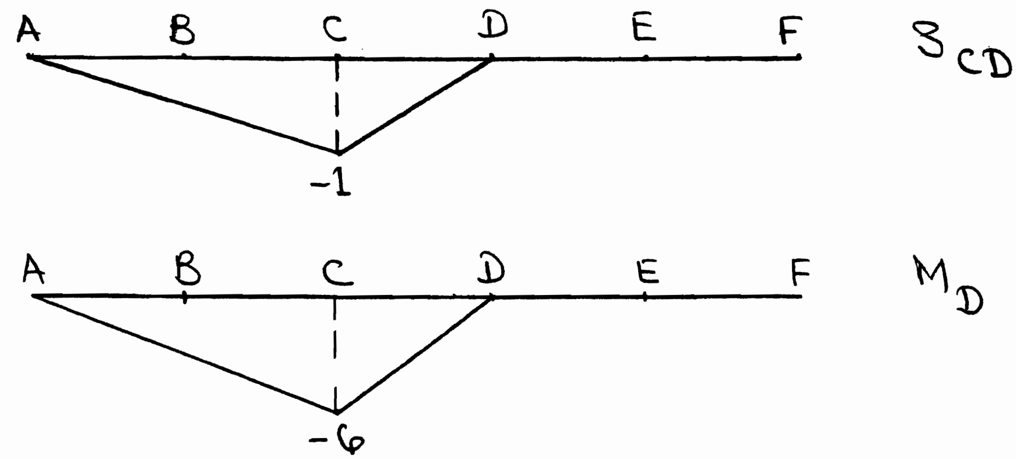
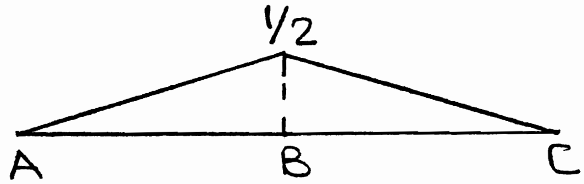
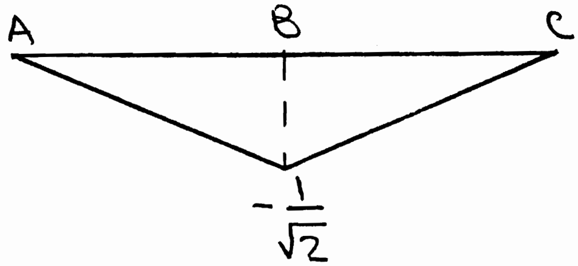


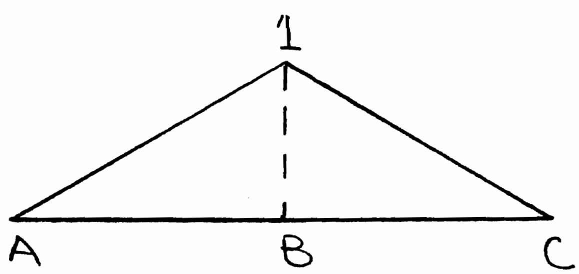
8.45



F_{AB}

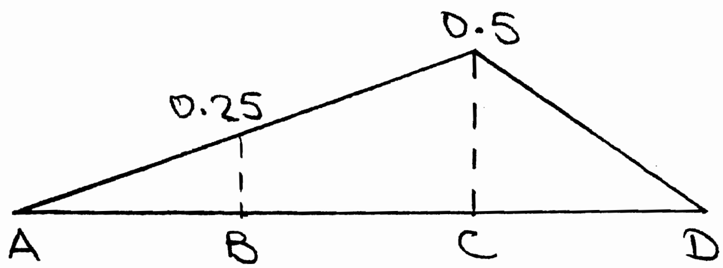


F_{AD}

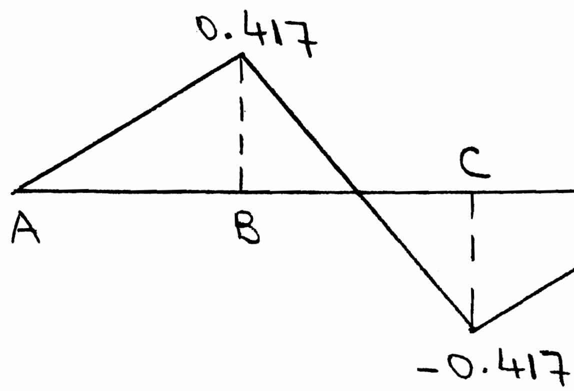


F_{BD}

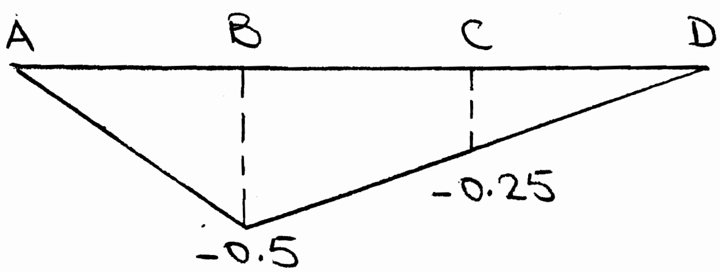
8.46



F_{BC}

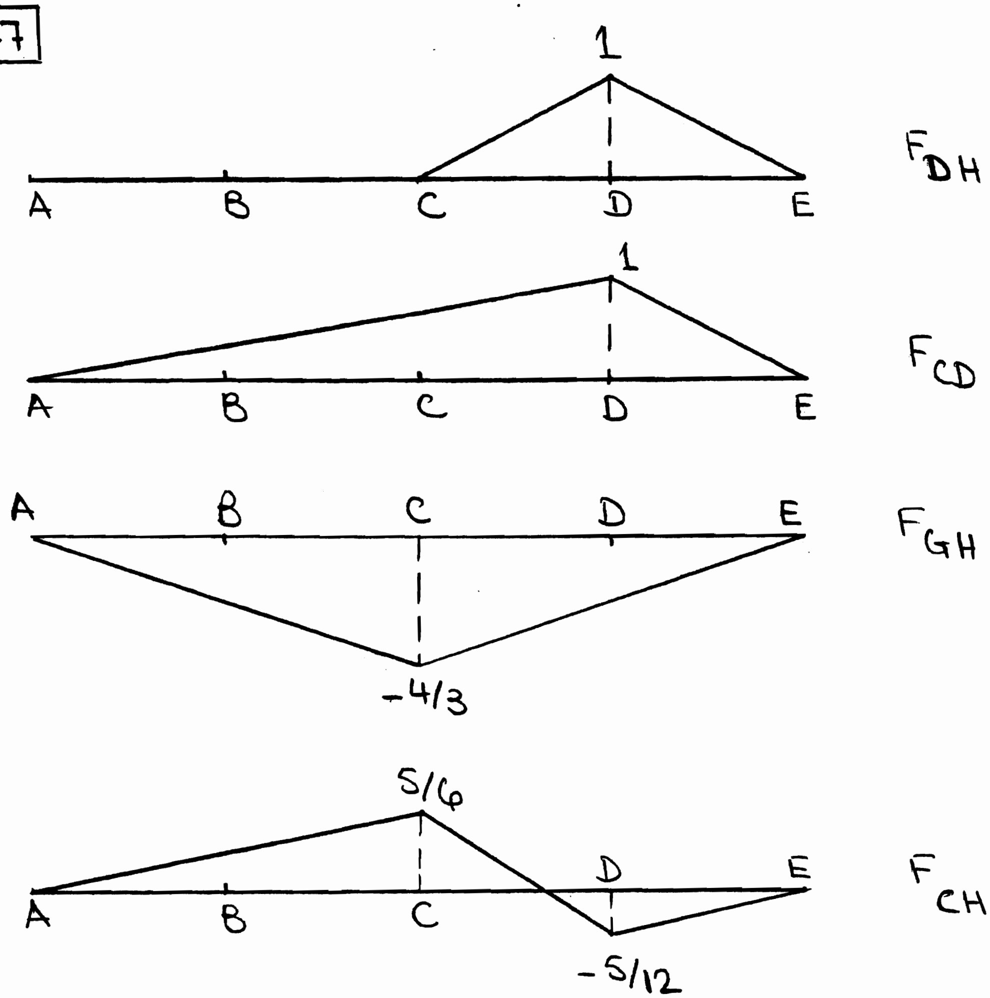


F_{BF}

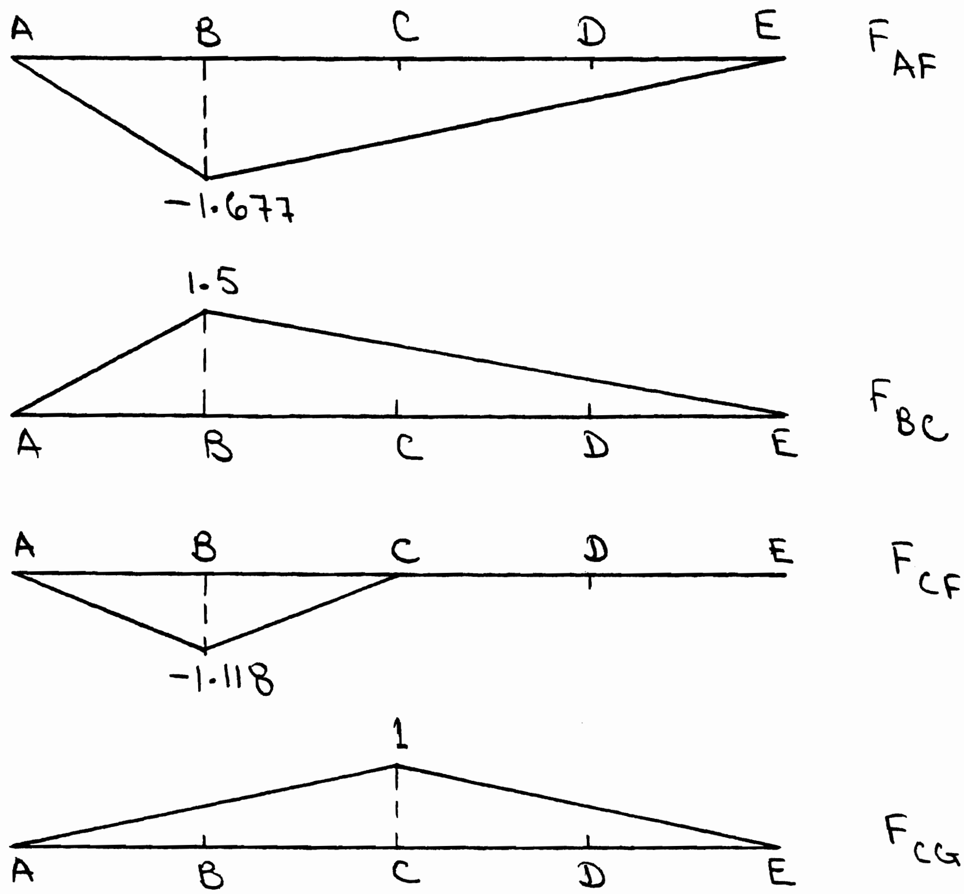


F_{EF}

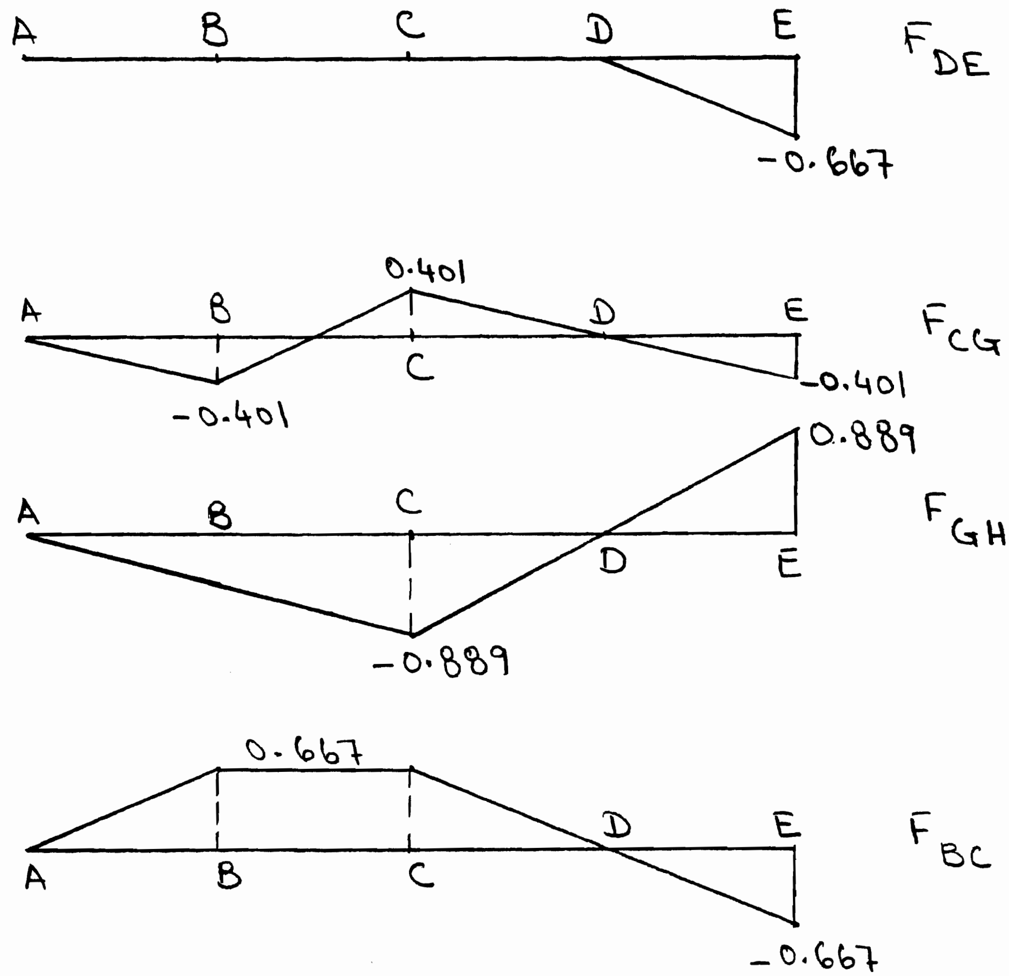
8.47



8.48

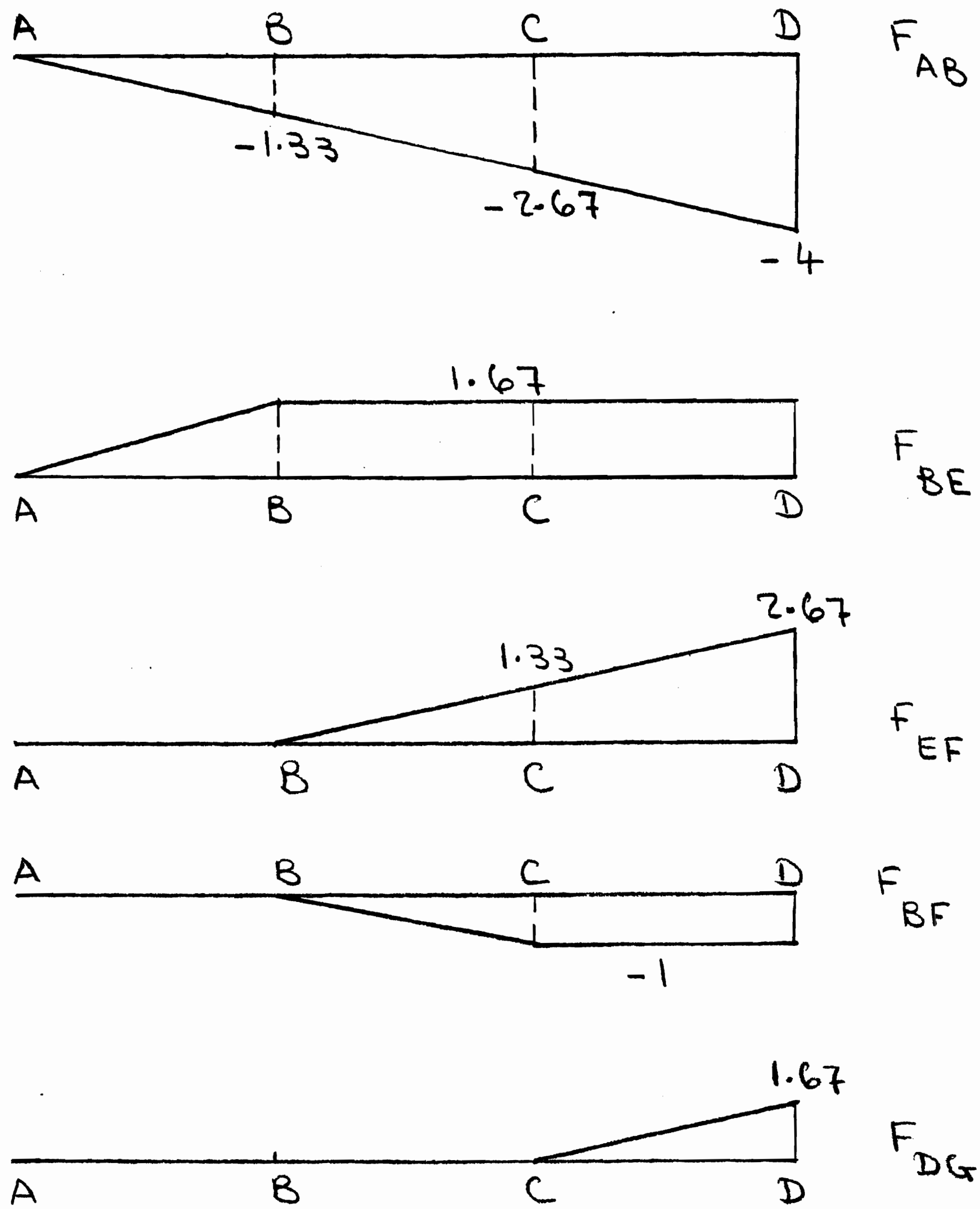


8.49

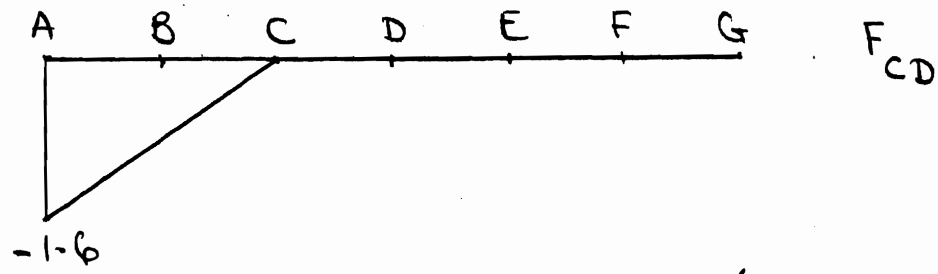


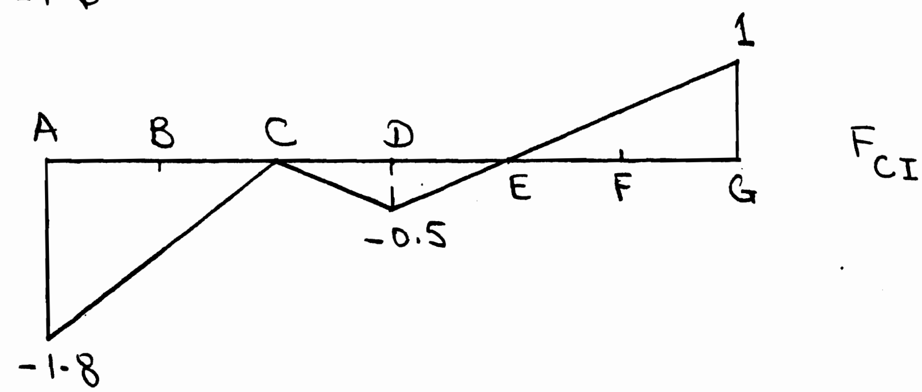
A
B
C
D
E
F DE
-0.667
0.401
A
B
D
E
F CG
C
-0.401
-0.401
0.889
A
B
C
F GH
D
E
-0.889
0.667
D
E
F BC
A
B
C
-0.667

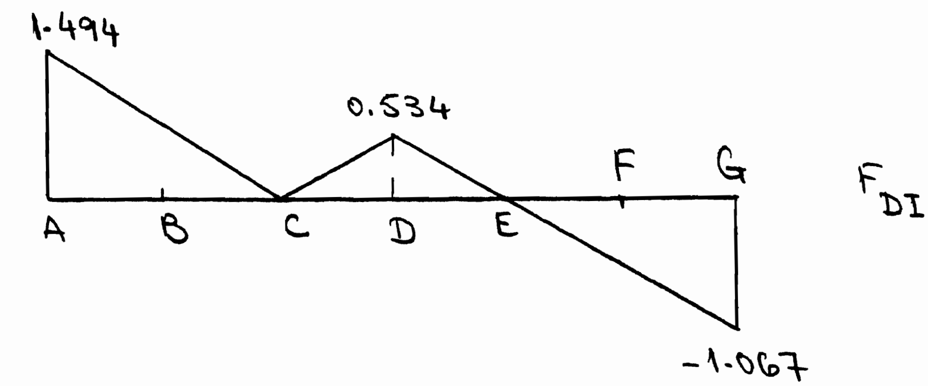
8.50

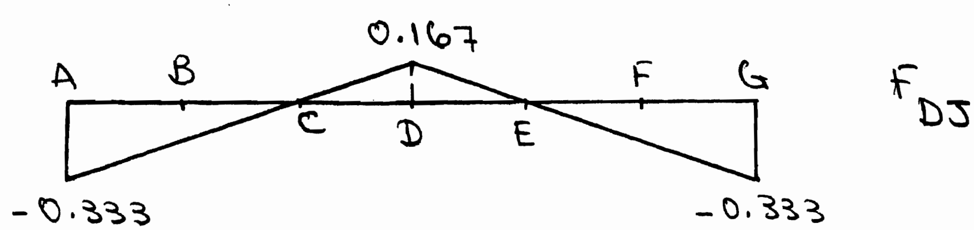


8.51

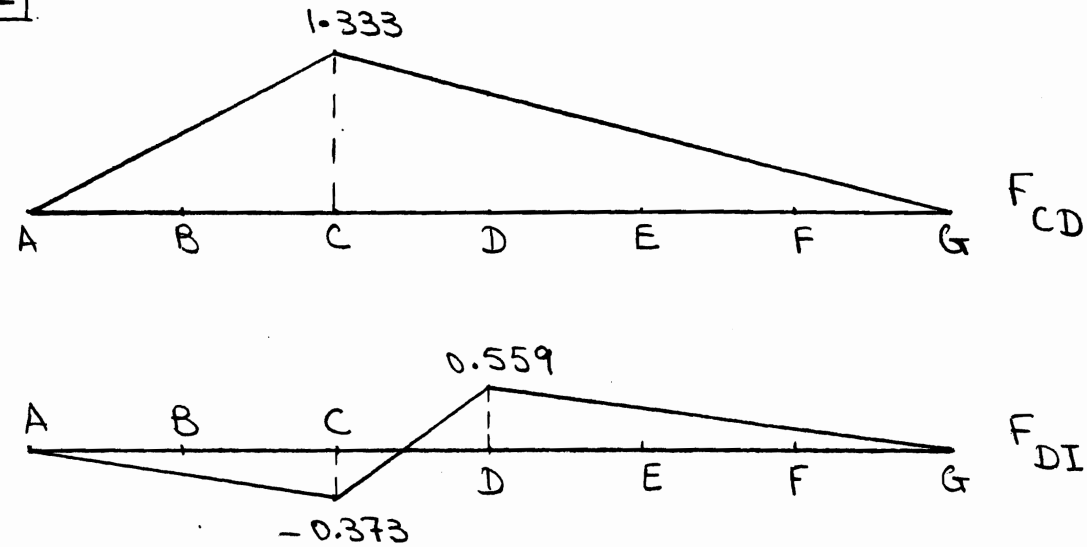

A B C D E F G
F_{CD}
-1.6


1
A B C D
F_{CI}
E F G
-0.5
-1.8


1.494
0.534
F G
F_{DI}
A B C D E
-1.067


0.167
A B F G
F_{DJ}
C D E
-0.333
-0.333

8.52



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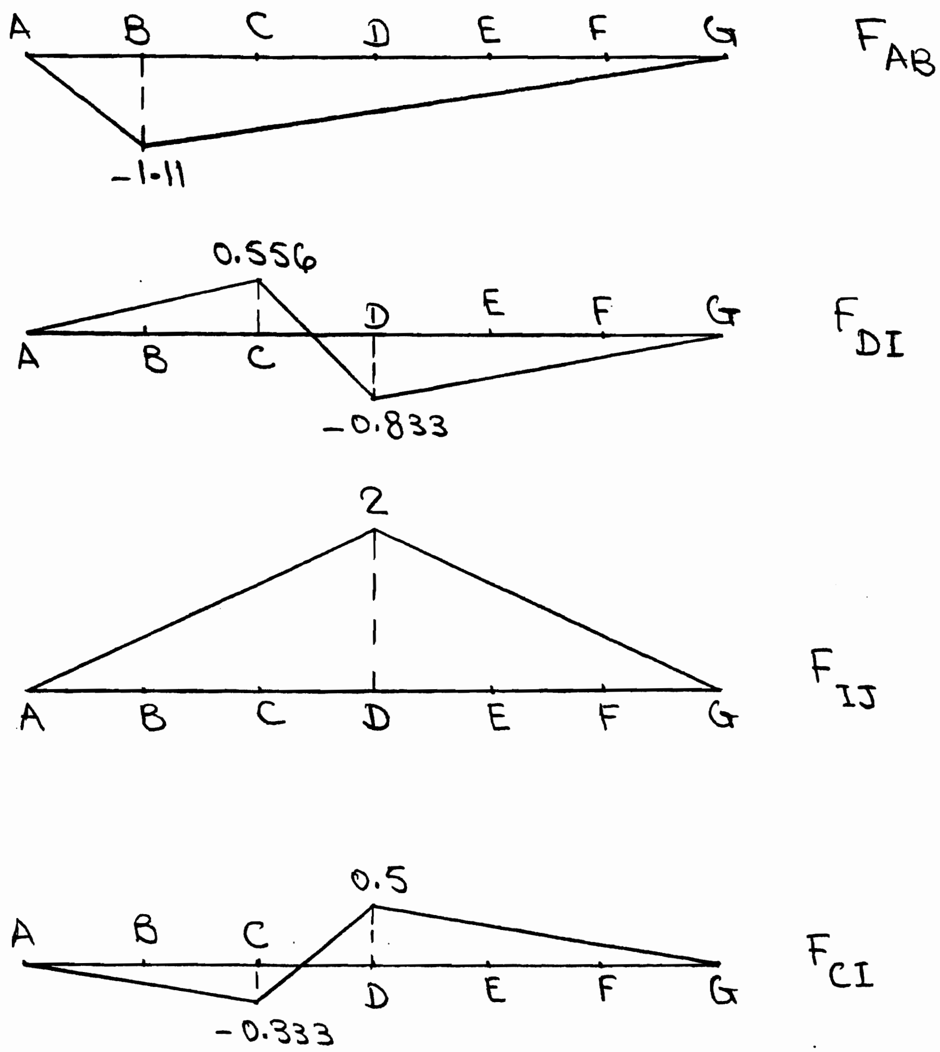
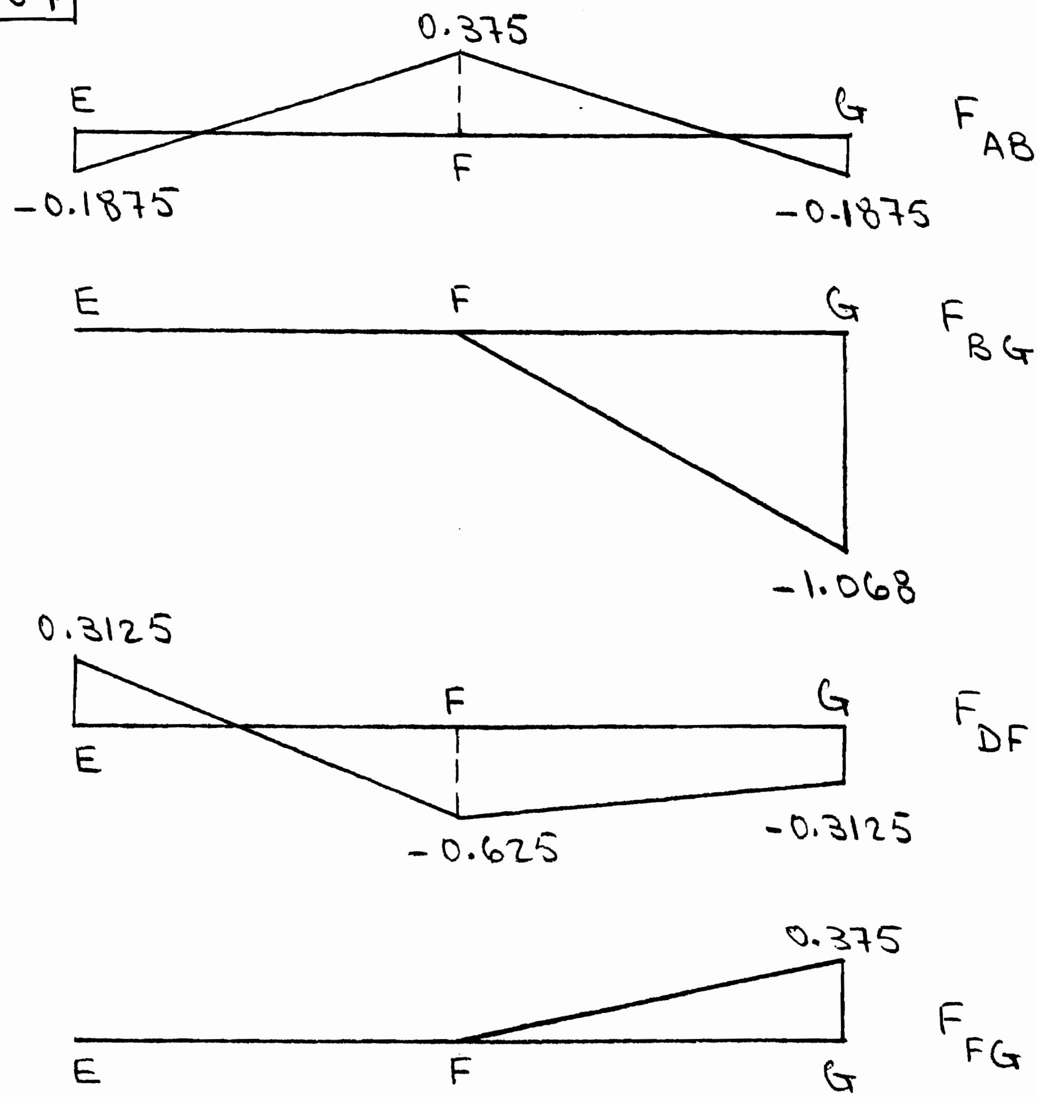
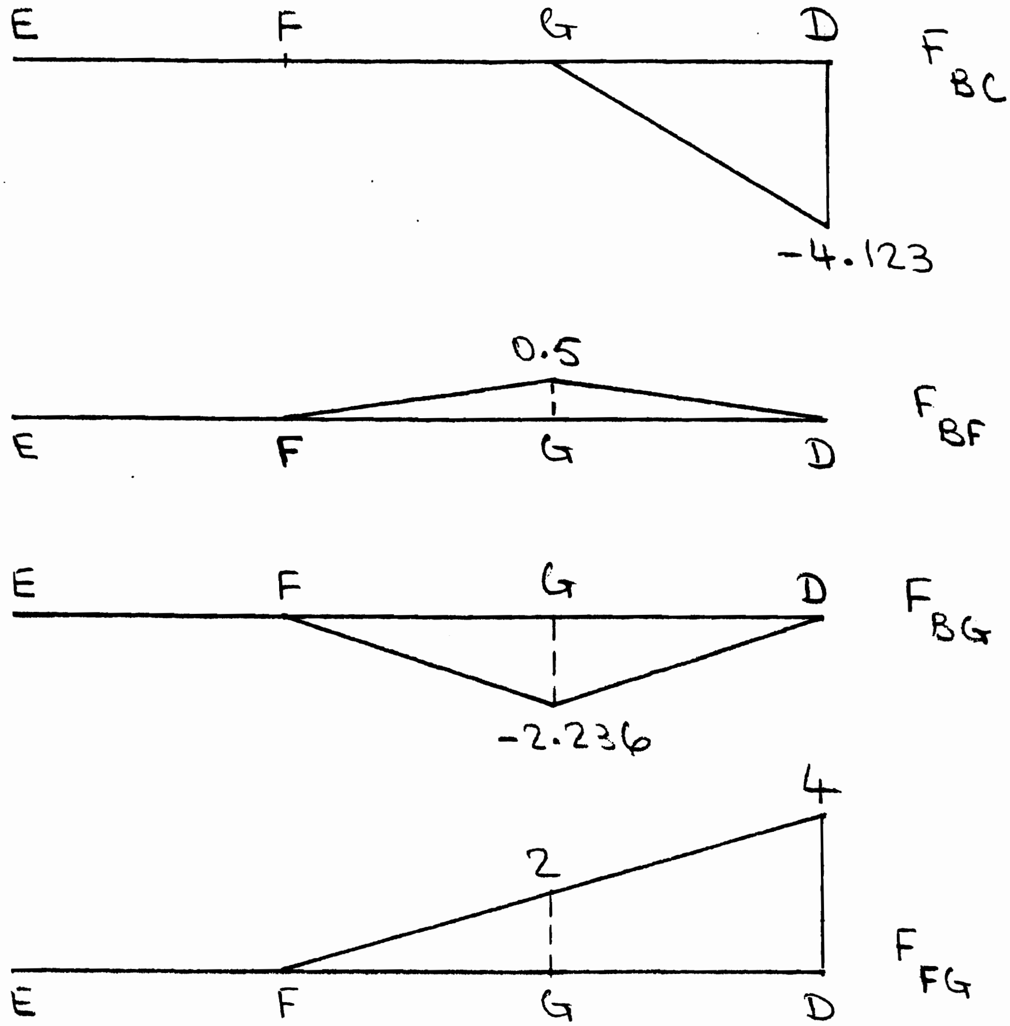


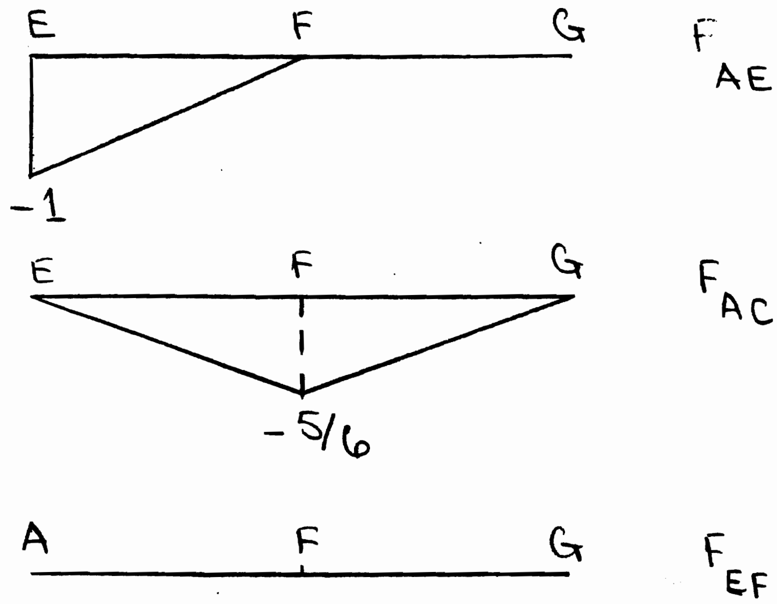
8.54


0.375
E
G
F
F_{AB}
-0.1875
-0.1875
E
F
G
F_{BG}
-1.068
0.3125
F
G
F_{DF}
E
-0.625
-0.3125
0.375
F_{FG}
E
F
G

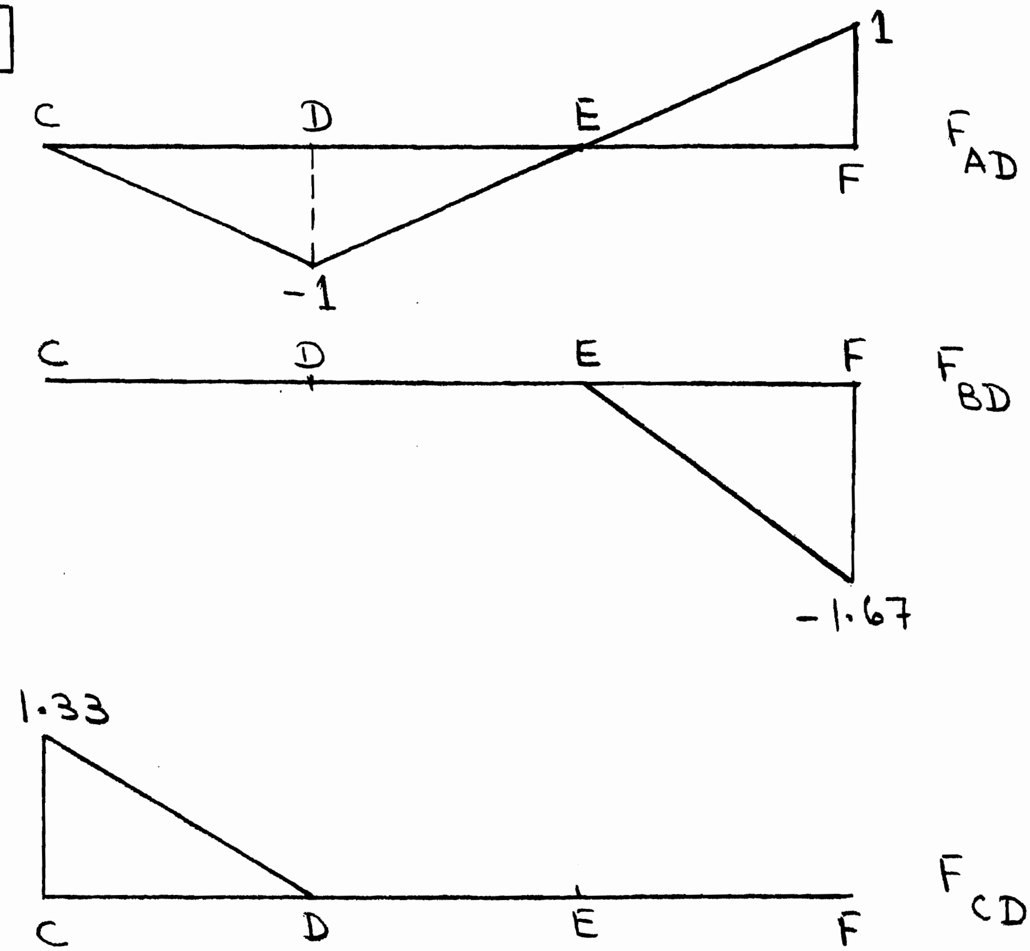
8.55



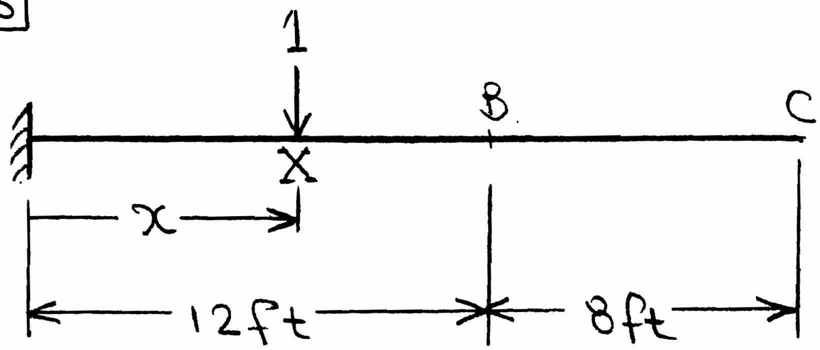
8.56



8.57

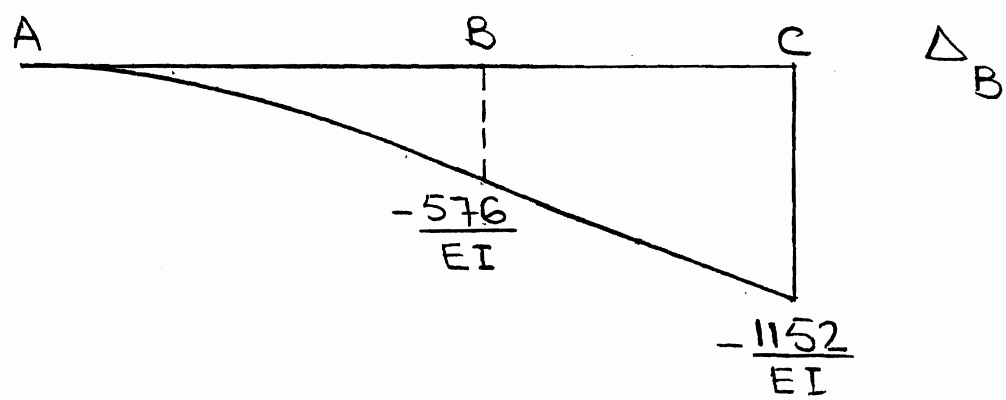


8.58

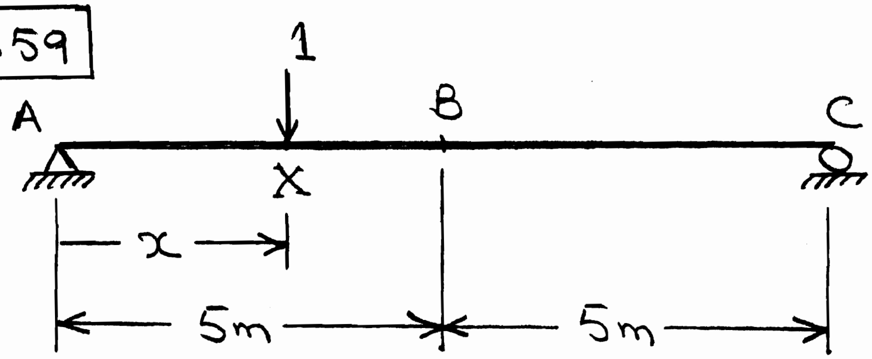


$0 \leq x \leq 12'$: $\quad f_{BX} = \frac{1}{6EI}(x^3 - 36x^2)$

$12' \leq x \leq 20'$: $\quad f_{BX} = \frac{72}{EI}(4 - x)$

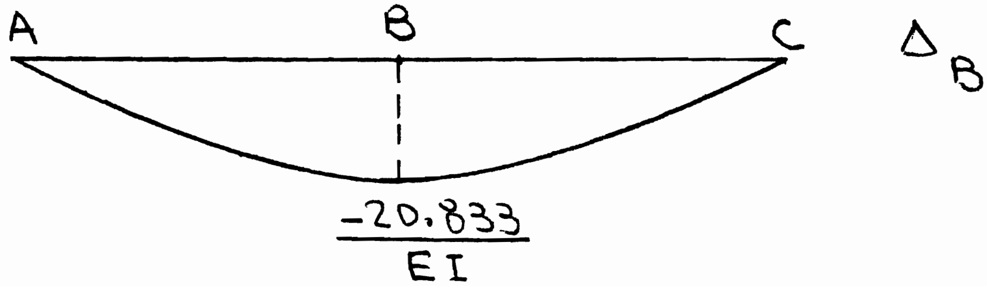


8.59

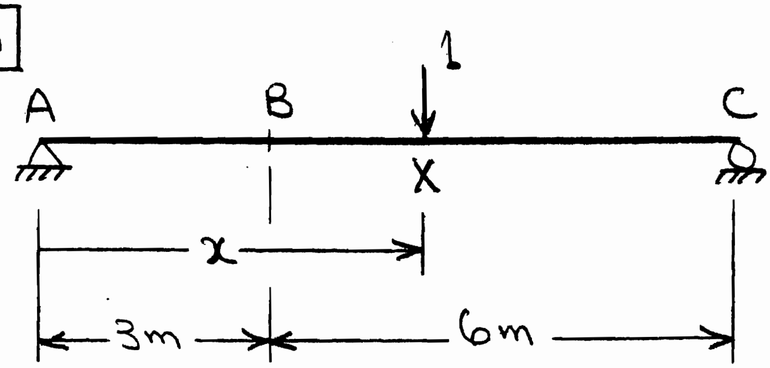


$$0 \leq x \leq 5\text{m}: \quad f_{BX} = \frac{1}{12EI}\left(x^3 - 75x\right)$$

$$5\text{m} \leq x \leq 10\text{m}: \quad f_{BX} = \frac{1}{12EI}\left[(10-x)^3 - 75(10-x)\right]$$

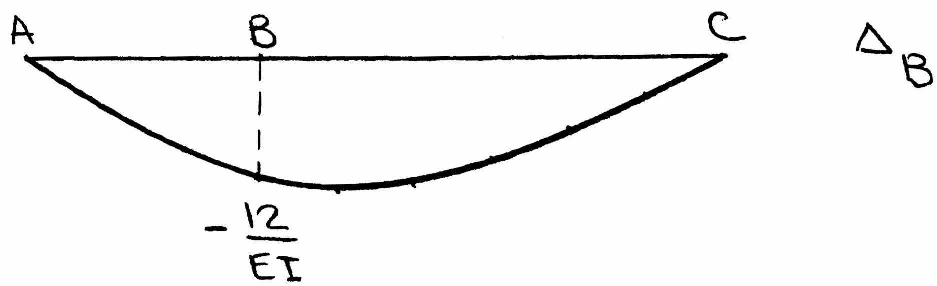


8.60

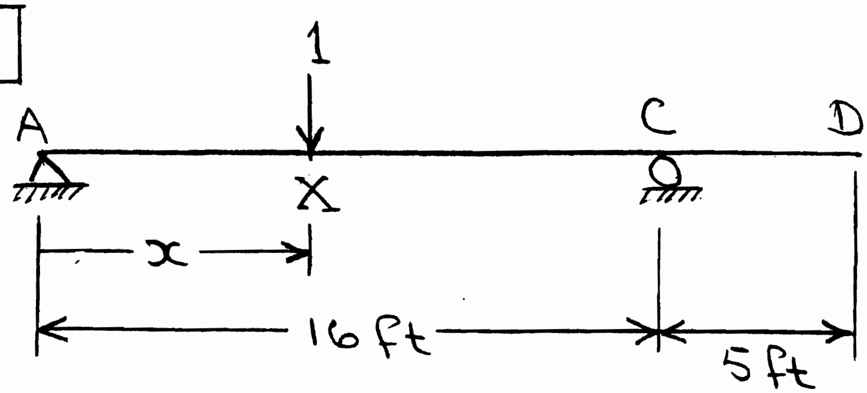


$0 \leq x \leq 3\,\text{m} \quad f_{BX} = \frac{1}{9EI}(x^3 - 45x)$

$3\,\text{m} \leq x \leq 9\,\text{m} \quad f_{BX} = \frac{1}{18EI}(-x^3 + 27x^2 - 171x + 81)$

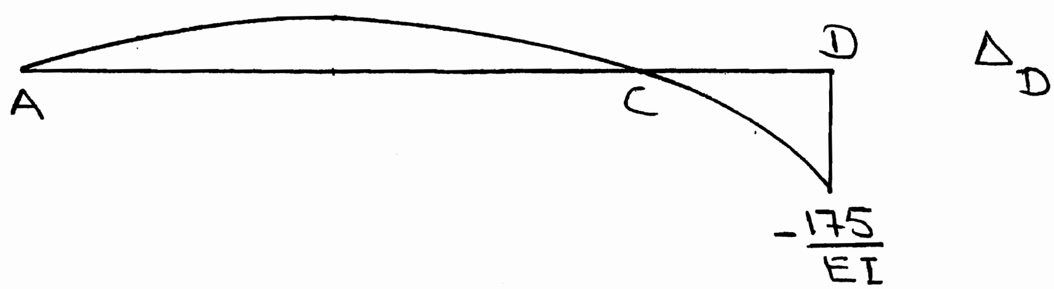


8.61



$0 \leq x \leq 16'$: $f_{DX} = \frac{5x}{96EI}(256 - x^2)$

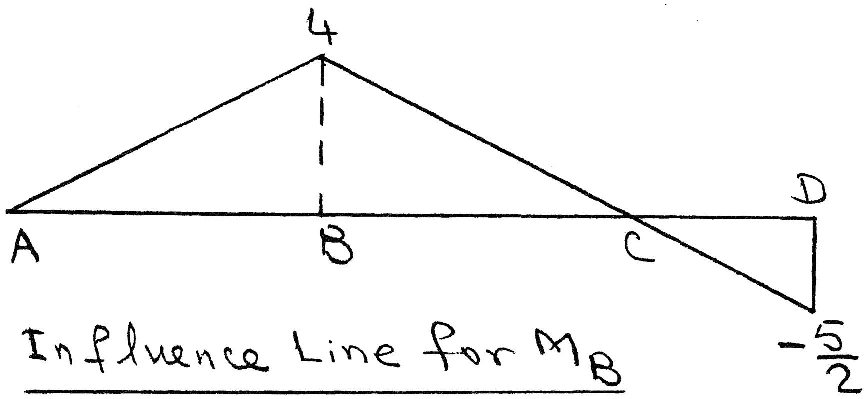
$16' \leq x \leq 21'$: $f_{DX} = \frac{1}{6EI}(-5376 + 1088x - 63x^2 + x^3)$



Chapter Nine
Application of Influence Lines

CHAPTER 9

9.1

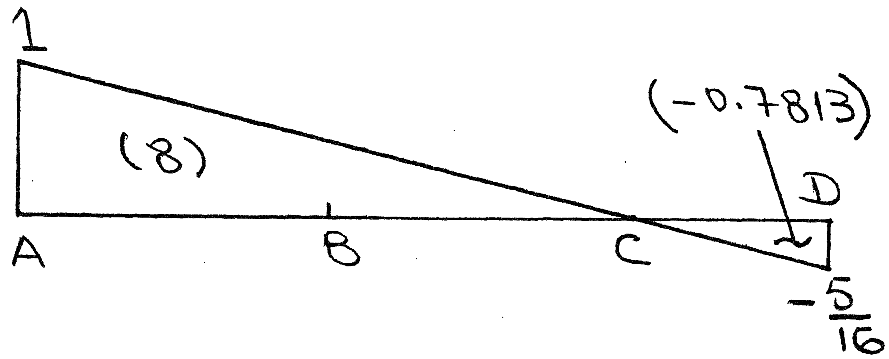


Influence Line for M_B

Maximum Negative $M_B = 15\left(-\frac{5}{2}\right)$

$= -37.5$ k-ft

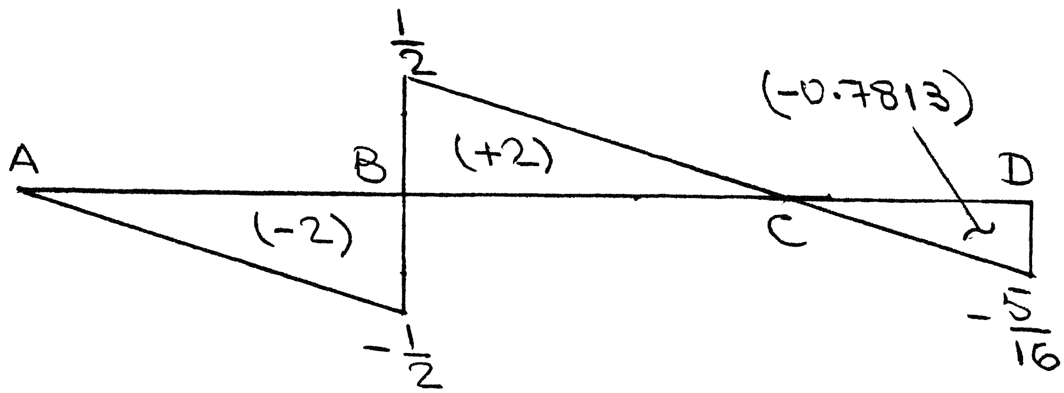
9.2



Influence Line for A_y

Maximum Upward $A_y = 3(8) = 24\text{ k}\uparrow$

9.3

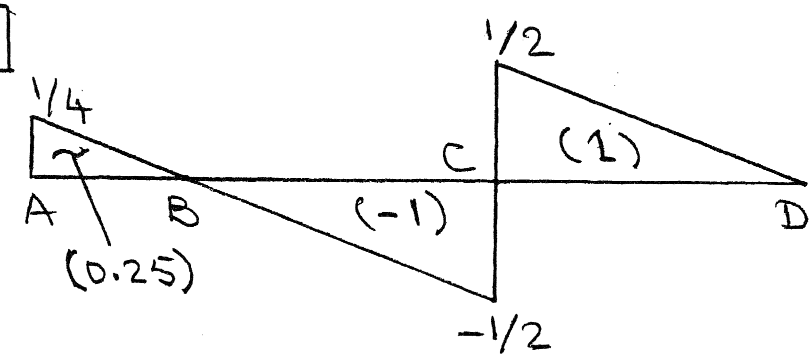


Influence Line for S_B

Maximum Negative $S_B = 3(-2 - 0.7813)$

$= \underline{-8.344 \text{ k}}$

9.4



Influence Line for S_C

Maximum Positive $S_C = 100\left(\frac{1}{2}\right) + 50(0.25 + 1)$

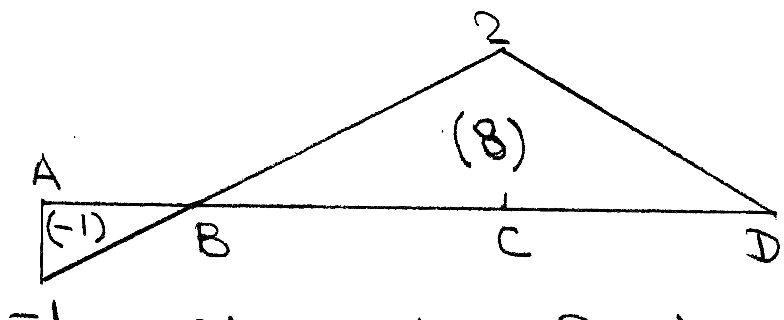
$+ 20(0.25 - 1 + 1)$

$= \underline{117.5 \text{ kN}}$

Maximum Negative $S_C = 100\left(-\frac{1}{2}\right) + 50(-1)$

$+ 20(0.25 - 1 + 1)$

$= \underline{-95 \text{ kN}}$



Influence Line for M_C

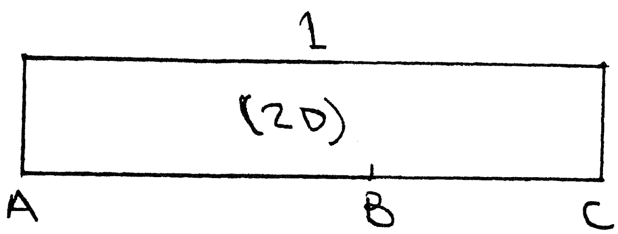
Maximum Positive $M_C = 100(2) + 50(8) + 20(7)$

$= \underline{740 \text{ kN.m}}$

Maximum Negative $M_C = 100(-1) + 50(-1) + 20(7)$

$= \underline{-10 \text{ kN.m}}$

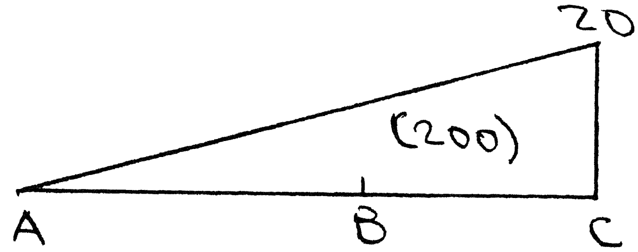
9.5



Influence Line for A_y

Maximum Upward $A_y = 25(1) + 2(20) + 0.5(20)$

$= 75\text{ k}\uparrow$



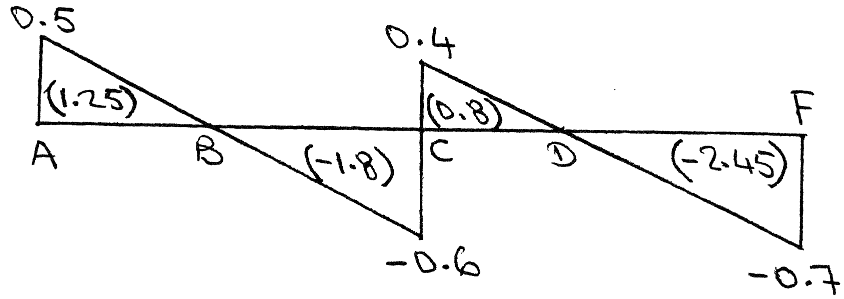
Influence Line for M_A (+↺)

Maximum Counterclockwise M_A

$= 25(20) + 2(200) + 0.5(200)$

$= 1000\text{ k-ft}$ ↺

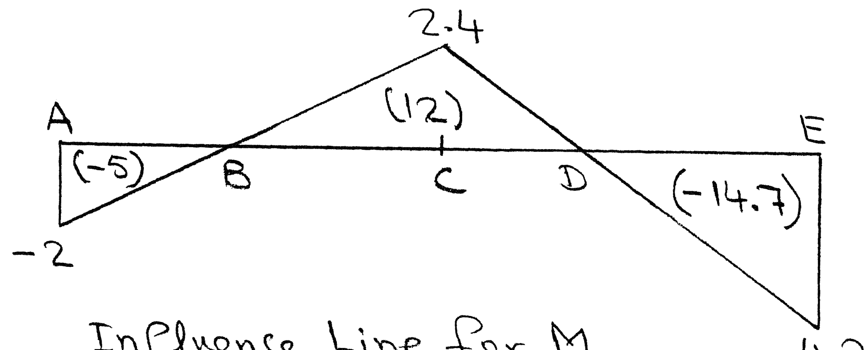
9.6



Influence Line for S_C

Maximum Positive $S_C = 150(0.5) + 50(1.25 + 0.8)$
$+25(1.25 - 1.8 + 0.8 - 2.45)$
$= \underline{122.5 \text{ kN}}$

Maximum Negative $S_C = 150(-0.7) + 50(-1.8 - 2.45)$
$+25(1.25 - 1.8 + 0.8 - 2.45)$
$= \underline{-372.5 \text{ kN}}$

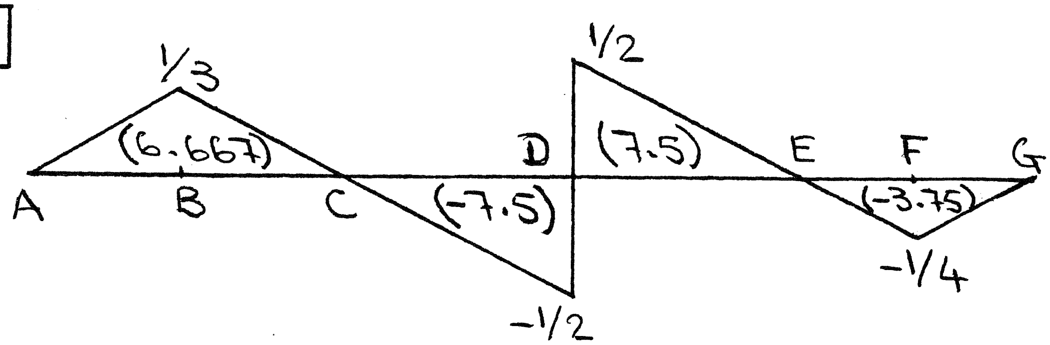


Influence Line for M_C

Maximum Positive $M_C = 150(2.4) + 50(12)$
$+25(-5 + 12 - 14.7)$
$= \underline{767.5 \text{ kN.m}}$

Maximum Negative $M_C = 150(-4.2) + 50(-5 - 14.7)$
$+25(-5 + 12 - 14.7)$
$= \underline{-1807.5 \text{ kN.m}}$

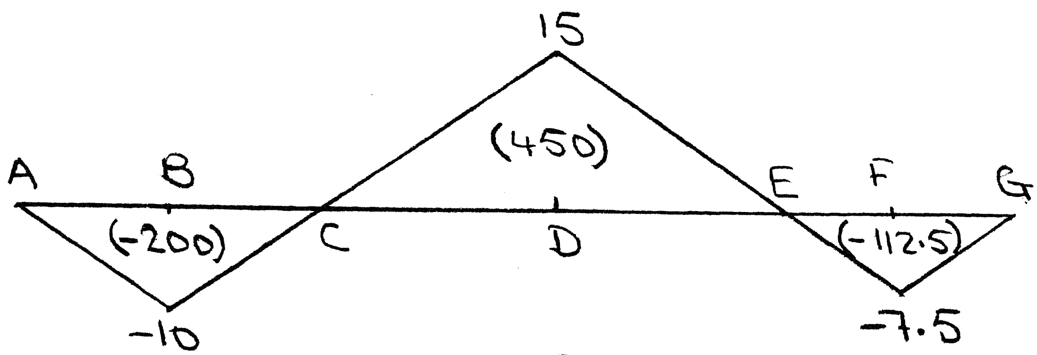
9.7



Influence Line for S_D

Maximum Positive $S_D = 30(\frac{1}{2}) + 3(6.667 + 7.5)$
$+ 1(6.667 - 7.5 + 7.5 - 3.75)$
$= \underline{60.417\ k}$

Maximum Negative $S_D = 30(-\frac{1}{2}) + 3(-7.5 - 3.75)$
$+ 1(6.667 - 7.5 + 7.5 - 3.75)$
$= \underline{-45.833\ k}$

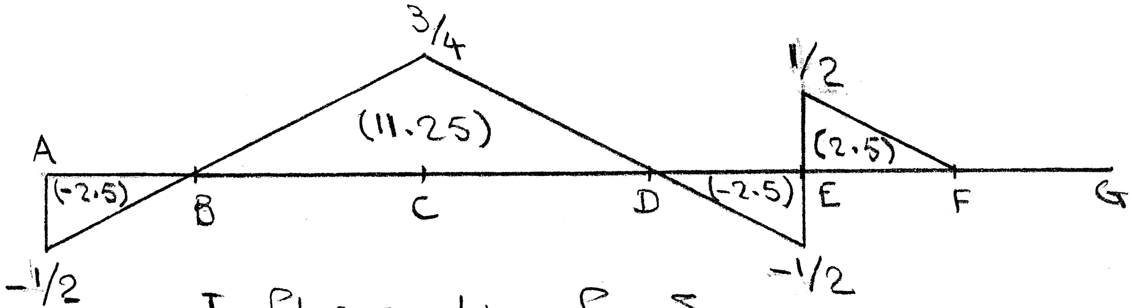


Influence Line for M_D

Maximum Positive $M_D = 30(15) + 3(450)$
$+ 1(-200 + 450 - 112.5)$
$= \underline{1937.5\ k\text{-}ft}$

Maximum Negative $M_D = 30(-10) + 3(-200 - 112.5)$
$+ 1(-200 + 450 - 112.5)$
$= \underline{-1100\ k\text{-}ft}$

9.8



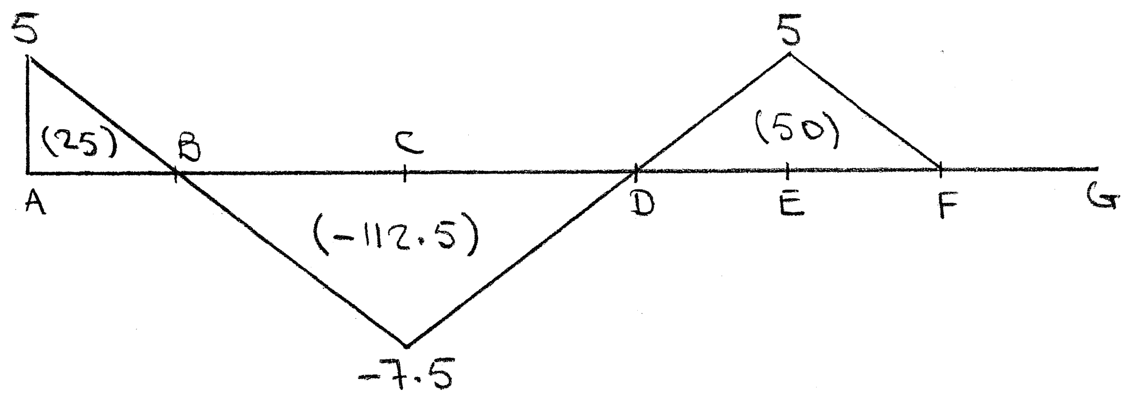
<u>Influence Line for S_E</u>

Maximum Positive $S_E = 40\left(\frac{3}{4}\right) + 2(11.25 + 2.5) + 1(-2.5 + 11.25)$

$= \underline{66.25\ k}$

Maximum Negative $S_E = 40\left(-\frac{1}{2}\right) + 2(-2.5 - 2.5) + 1(-2.5 + 11.25)$

$= \underline{-21.25\ k}$



<u>Influence Line for M_E</u>

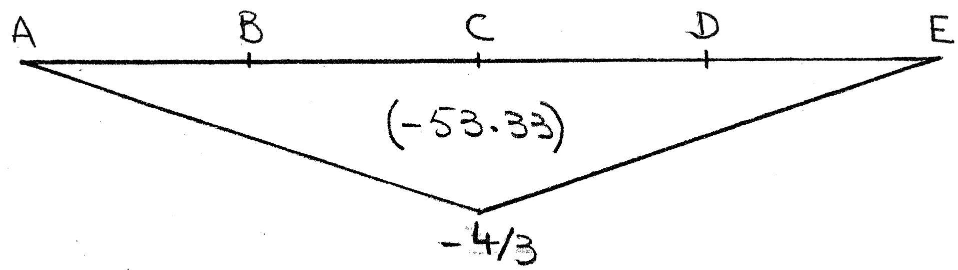
Maximum Positive $M_E = 40(5) + 2(25 + 50) + 1(25 - 112.5 + 50)$

$= \underline{312.5\ k\text{-}ft}$

Maximum Negative $M_E = 40(-7.5) + 2(-112.5) + 1(25 - 112.5 + 50)$

$= \underline{-562.5\ k\text{-}ft}$

9.9

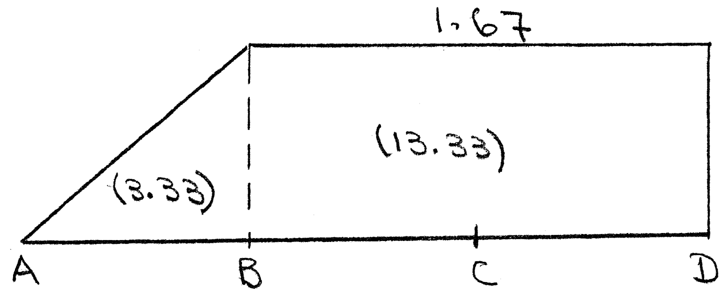


Influence Line for F_{GH}

Maximum Compressive $F_{GH} = 30\left(-\frac{4}{3}\right) + 3(-53.33)$

$= -200\text{ k} = \underline{200\text{ k (C)}}$

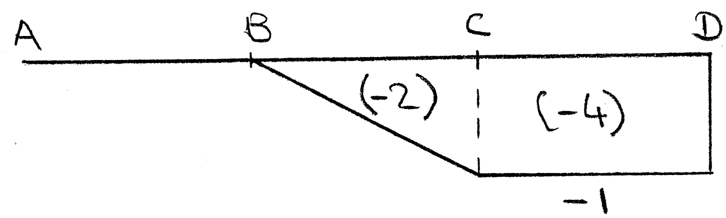
9.10



Influence Line for F_{BE}

Maximum Tensile $F_{BE} = 120(1.67) + 60(3.33 + 13.33)$

$= \underline{1200 \text{ kN (T)}}$



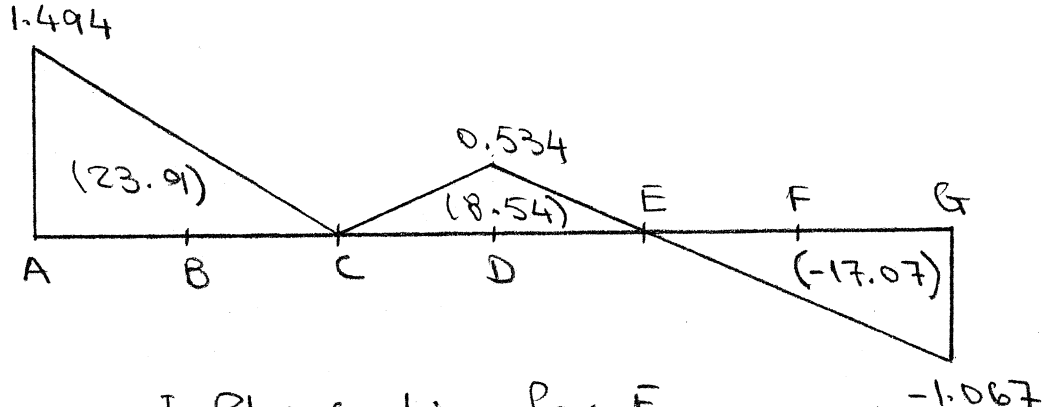
Influence Line for F_{BF}

Maximum Compressive $F_{BF} = 120(-1) + 60(-2-4)$

$= -480 \text{ kN}$

$= \underline{480 \text{ kN (C)}}$

9.11

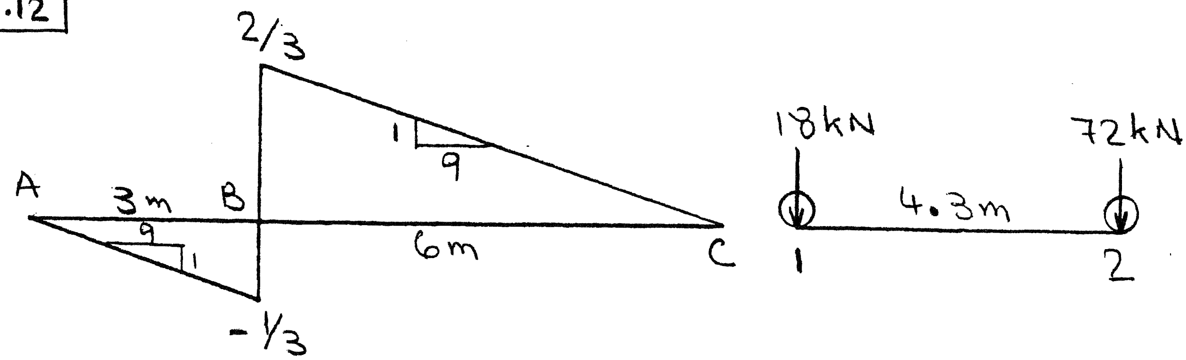


Influence Line for F_{DI}

Maximum Tensile $F_{DI} = 40(1.494) + 4(23.9 + 8.54)$
$+ 2(23.9 + 8.54 - 17.07) = \underline{220.3\text{ k (T)}}$

Maximum Compressive $F_{DI} = 40(-1.067) + 4(-17.07)$
$+2(23.9 + 8.54 - 17.07) = -80.2\text{ k}$
$= \underline{80.2\text{ k (C)}}$

9.12



Influence Line for S_B

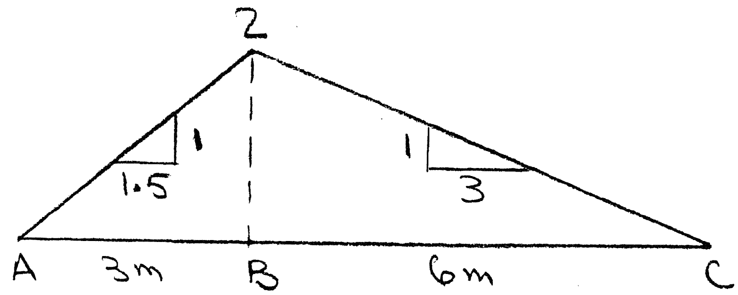
Loading position 1:

$$S_B = 18\left(\frac{2}{3}\right) + 72\left(\frac{1.7}{9}\right) = 25.6 \text{ kN}$$

Loading position 2:

$$S_B = 72\left(\frac{2}{3}\right) = 48 \text{ kN}$$

Max. Positive S_B = **48 kN**



Influence Line for M_B

Loading position 1:

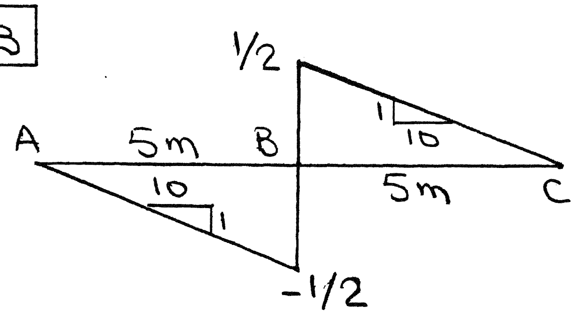
$$M_B = 18(2) + 72\left(\frac{1.7}{3}\right) = 76.8 \text{ kN.m}$$

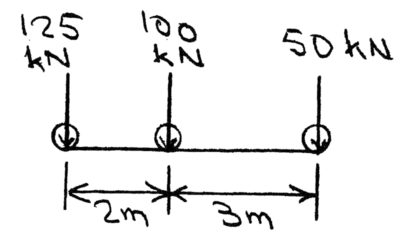
Loading position 2:

$$M_B = 72(2) = 144 \text{ kN.m}$$

Max. Positive M_B = **144 kN.m**

9.13





Influence Line for S_B

Loading position 1:

$$S_B = 125\left(\frac{1}{2}\right) + 100\left(\frac{3}{10}\right) = 92.5 \text{ kN}$$

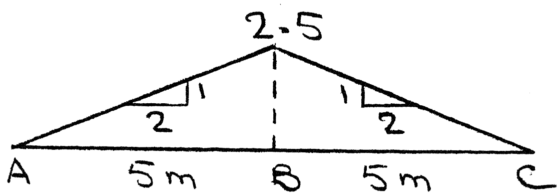
Loading position 2:

$$S_B = 125\left(-\frac{3}{10}\right) + 100\left(\frac{1}{2}\right) + 50\left(\frac{2}{10}\right) = 22.5 \text{ kN}$$

Loading position 3:

$$S_B = 100\left(-\frac{2}{10}\right) + 50\left(\frac{1}{2}\right) = 5 \text{ kN}$$

Max. Positive $S_B = \underline{92.5 \text{ kN}}$



Influence Line for M_B

Loading position 1:

$$M_B = 125(2.5) + 100(1.5) = 462.5 \text{ kN.m}$$

Loading position 2:

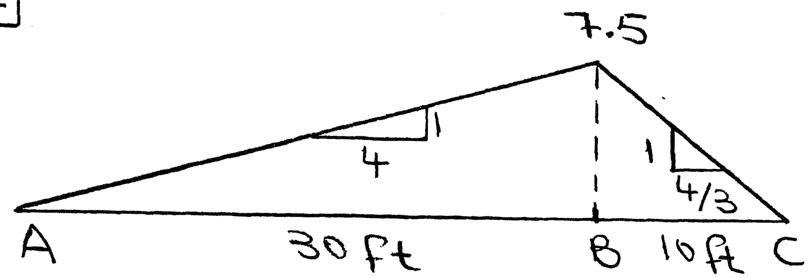
$$M_B = 125(1.5) + 100(2.5) + 50(1) = 487.5 \text{ kN.m}$$

Loading position 3:

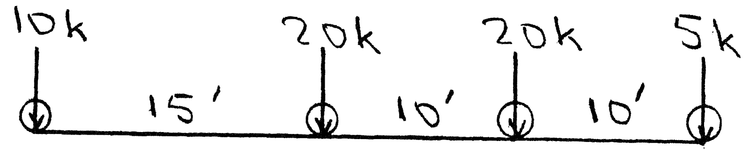
$$M_B = 100(1) + 50(2.5) = 225 \text{ kN.m}$$

Max. Positive $\underline{M_B = 487.5 \text{ kN.m}}$

9.14



Influence Line for M_B



Loading position 1:

$$M_B = 10(7.5) = 75 \text{ k-ft}$$

Loading position 2:

$$M_B = 10\left(\frac{15}{4}\right) + 20(7.5) = 187.5 \text{ k-ft}$$

Loading position 3:

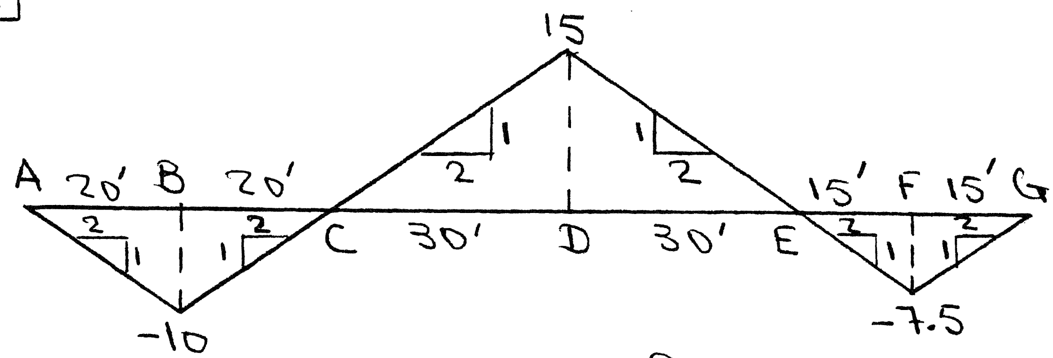
$$M_B = 10\left(\frac{5}{4}\right) + 20(5) + 20(7.5) = 262.5 \text{ k-ft}$$

Loading position 4:

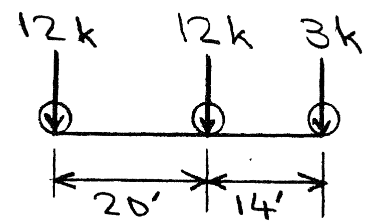
$$M_B = 20\left(\frac{10}{4}\right) + 20(5) + 5(7.5) = 187.5 \text{ k-ft}$$

Max. Positive $M_B = 262.5$ k-ft

9.15



Influence Line for M_D



Loading position 1:

$$M_D = 12(15) + 12(5) + 3\left(-\frac{4}{2}\right) = 234 \text{ k-ft}$$

Loading position 2:

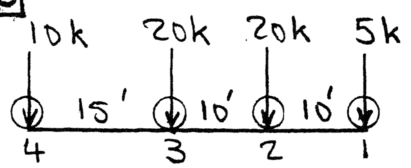
$$M_D = 12(5) + 12(15) + 3\left(\frac{16}{2}\right) = 264 \text{ k-ft}$$

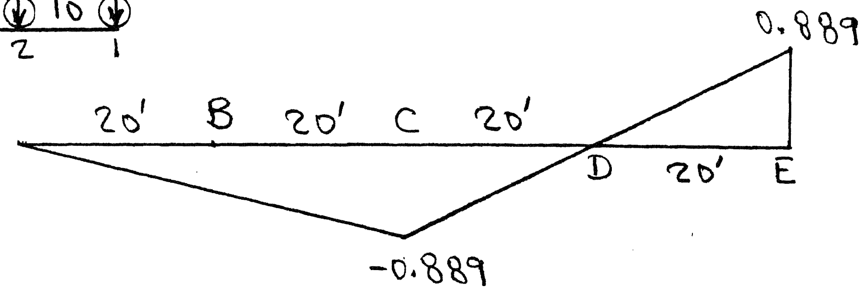
Loading position 3:

$$M_D = 12\left(-\frac{4}{2}\right) + 12\left(\frac{16}{2}\right) + 3(15) = 117 \text{ k-ft}$$

Max. Positive $M_D = 264$ k-ft

9.16





Influence Line for F_{GH}

Loading position 1:

$$F_{GH} = [5(40)+20(30+20)+10(5)]\left(\frac{-0.889}{40}\right) = -27.78\text{ k}$$

Loading position 2:

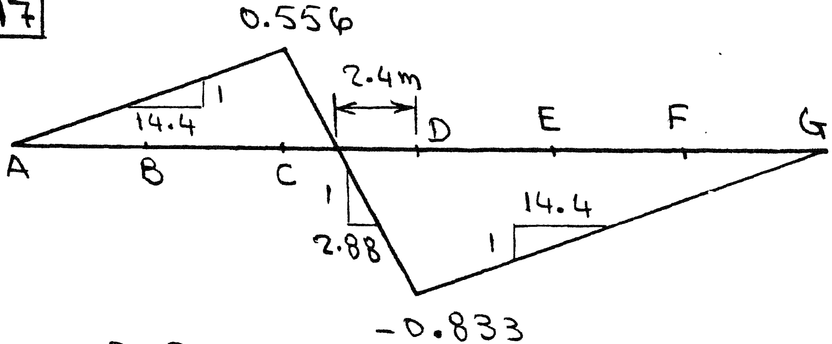
$$F_{GH} = 5(10)\left(\frac{-0.889}{20}\right) + [20(40+30)+10(15)]\left(\frac{-0.889}{40}\right)$$
$$= -36.67\text{ k}$$

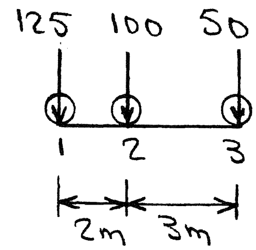
Loading position 3:

$$F_{GH} = [5(0)+20(10)]\left(\frac{-0.889}{20}\right) + [20(40)+10(25)]\left(\frac{-0.889}{40}\right)$$
$$= -32.23\text{ k}$$

Maximum compressive $F_{GH} = -36.67\text{k} = 36.67\text{k (C)}$

9.17





Influence Line for F_{DI}

Loading position 1:

$$F_{DI} = 125(0.556) - 100\left(\frac{0.4}{2.88}\right) - 50\left(\frac{11}{14.4}\right) = 17.42 \text{ kN}$$

Loading position 2:

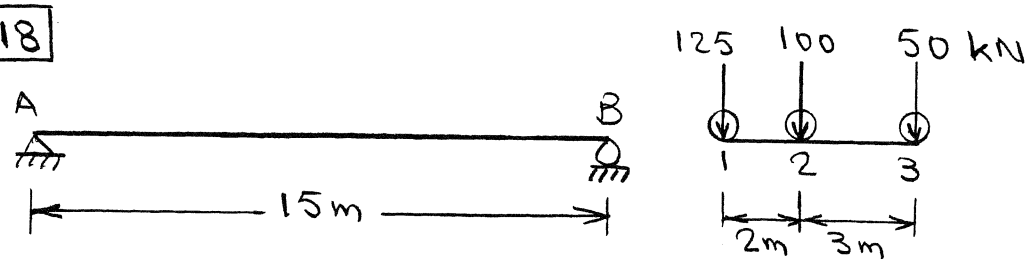
$$F_{DI} = 125\left(\frac{6}{14.4}\right) + 100(0.556) - 50\left(\frac{1.4}{2.88}\right) = 83.38 \text{ kN}$$

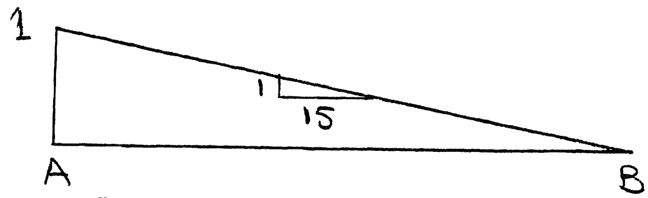
Loading position 3:

$$F_{DI} = 125\left(\frac{3}{14.4}\right) + 100\left(\frac{5}{14.4}\right) + 50(0.556) = 88.56 \text{ kN}$$

Maximum Tensile F_{DI} = 88.56 kN (T)

9.18





Influence Line $S_{A,R}$

Loading position 1:

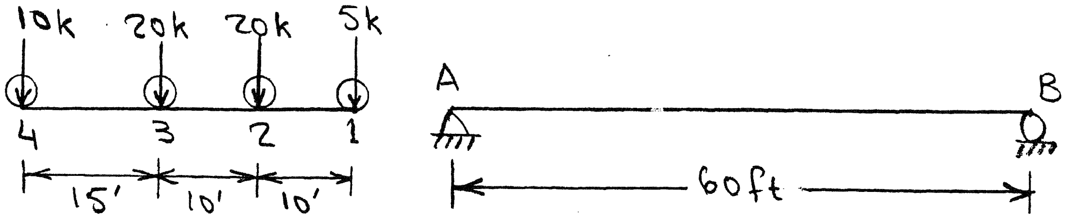
$$S_{A,R} = [125(15) + 100(13) + 50(10)]\frac{1}{15} = 245 \text{ kN}$$

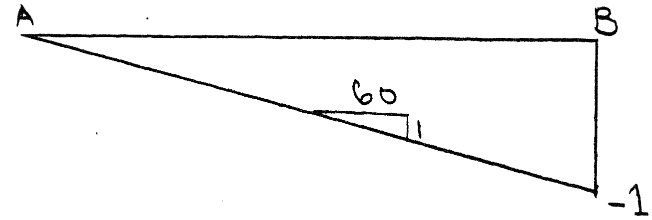
Loading position 2:

$$S_{A,R} = [100(15) + 50(12)]\frac{1}{15} = 140 \text{ kN}$$

Absolute maximum shear = 245 kN

9.19





Influence Line for $S_{B,L}$

Loading position 1:

$$S_{B,L} = [5(60) + 20(50+40) + 10(25)]\left(-\frac{1}{60}\right) = -39.17\text{ k}$$

Loading position 2:

$$S_{B,L} = [20(60+50) + 10(35)]\left(\frac{-1}{60}\right) = -42.5\text{ k}$$

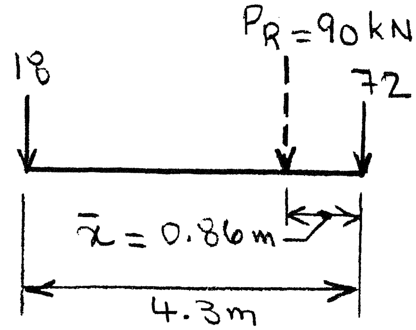
Loading position 3:

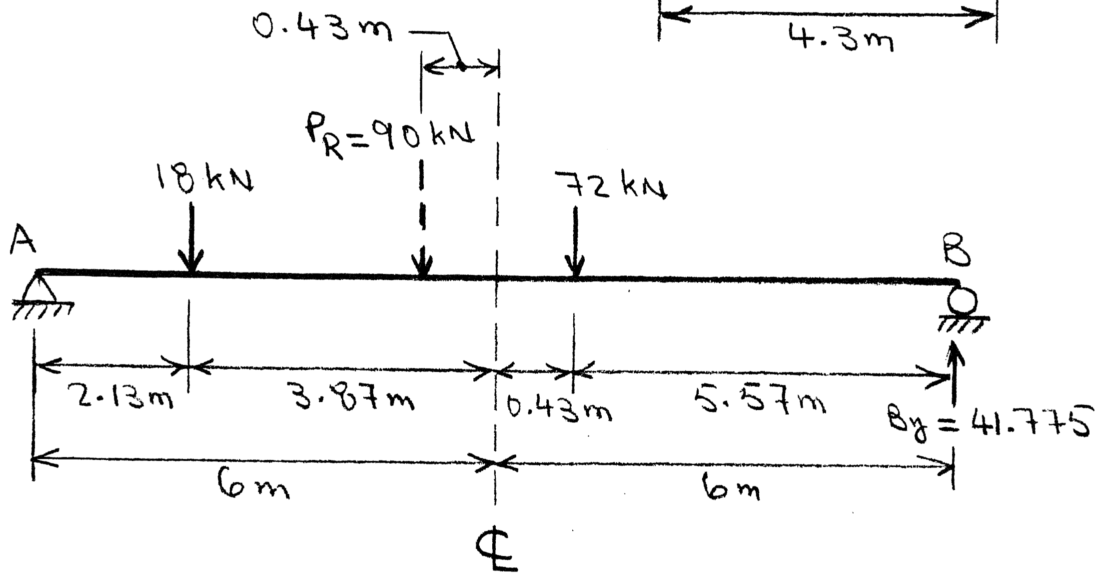
$$S_{B,L} = [20(60) + 10(45)]\left(\frac{-1}{60}\right) = -27.5\text{ k}$$

Absolute maximum shear = 42.5 k

9.20

$$\bar{x} = \frac{18(4.3)}{90} = 0.86\ m$$



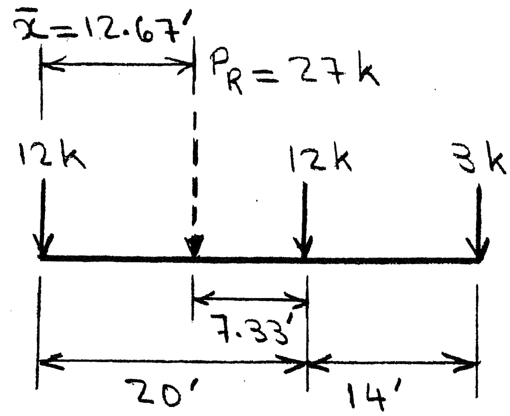


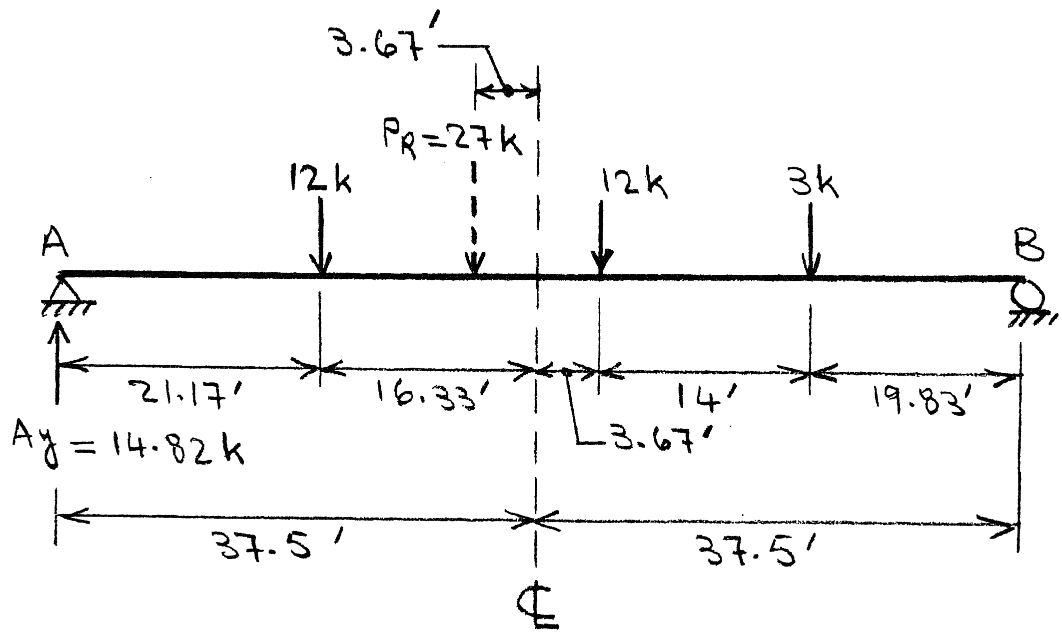
$$B_y = \frac{90(6-0.43)}{12} = 41.775\ \text{kN}$$

$$M_{max} = 41.775\,(5.57) = \underline{232.7\ \text{kN.m}}$$

9.21

$$\bar{x} = \frac{12(20)+3(34)}{27} = 12.67 \text{ ft}$$

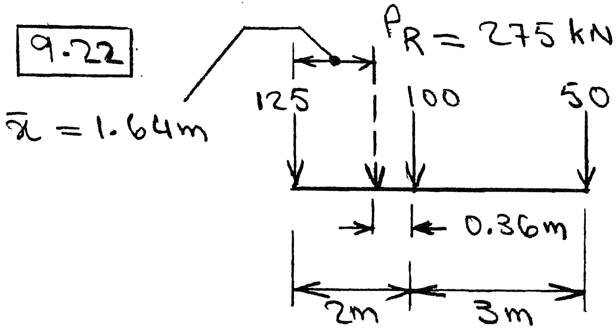




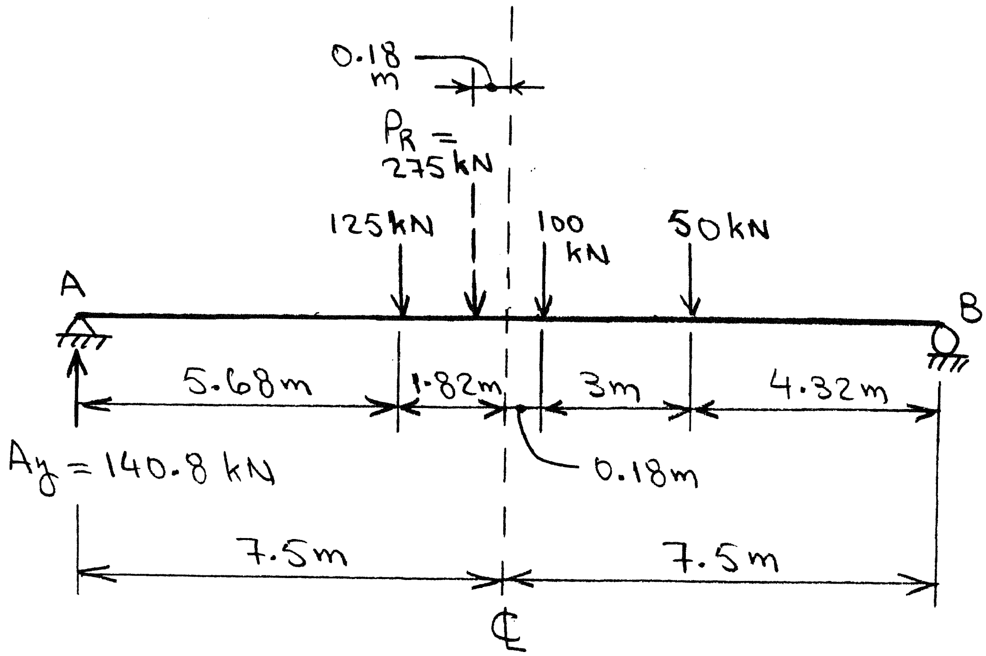
$$A_y = \frac{27(37.5+3.67)}{75} = 14.82 \text{ k}$$

$$M_{max} = 14.82(37.5+3.67) - 12(20) = \underline{370.1 \text{ k-ft}}$$

9.22



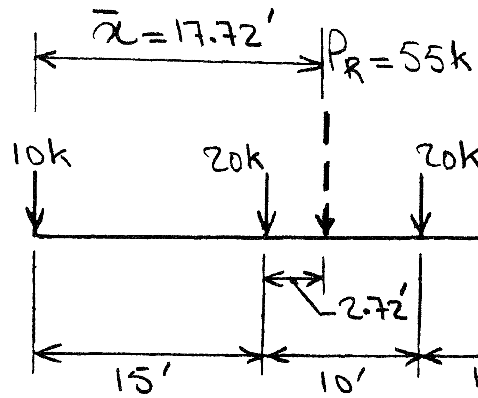
$$\bar{x} = \frac{100(2) + 50(5)}{275} = 1.64 \text{ m}$$



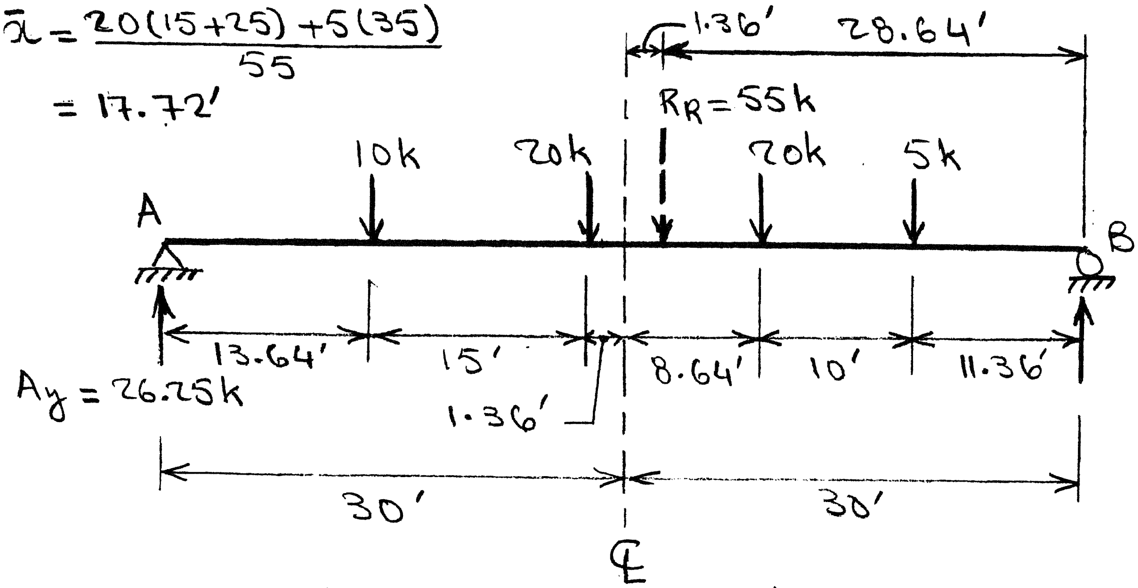
$$A_y = 275\left(\frac{0.18 + 7.5}{15}\right) = 140.8 \text{ kN}$$

$$M_{max} = 140.8(7.5 + 0.18) - 125(2) = \underline{831.3 \text{ kN.m}}$$

9.23



$$\bar{x} = \frac{20(15+25)+5(35)}{55}$$
$$= 17.72'$$

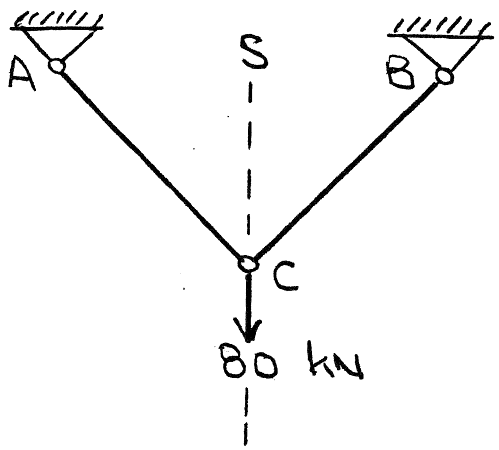


$$A_y = 55(28.64/60) = 26.25\text{k}$$
$$M_{max} = 26.25(28.64) - 10(15) = \underline{601.8 \text{ k-ft}}$$

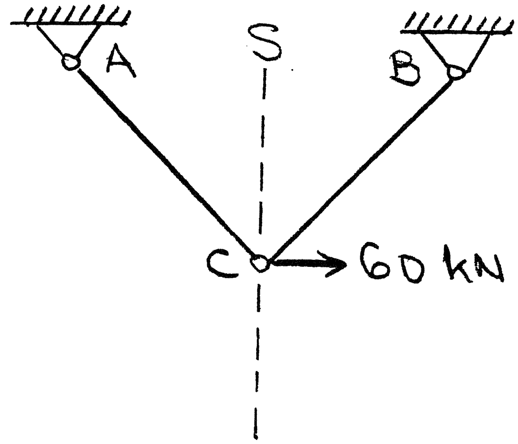
Chapter Ten
Analysis of Symmetric Structures

CHAPTER 10

10.1

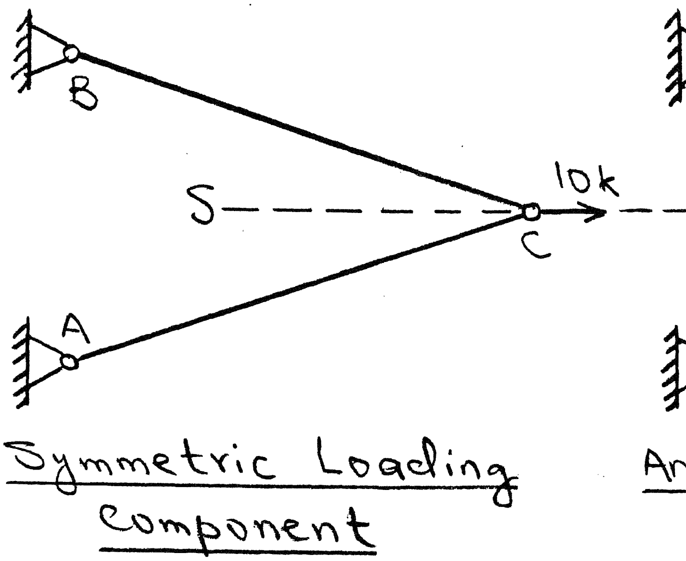


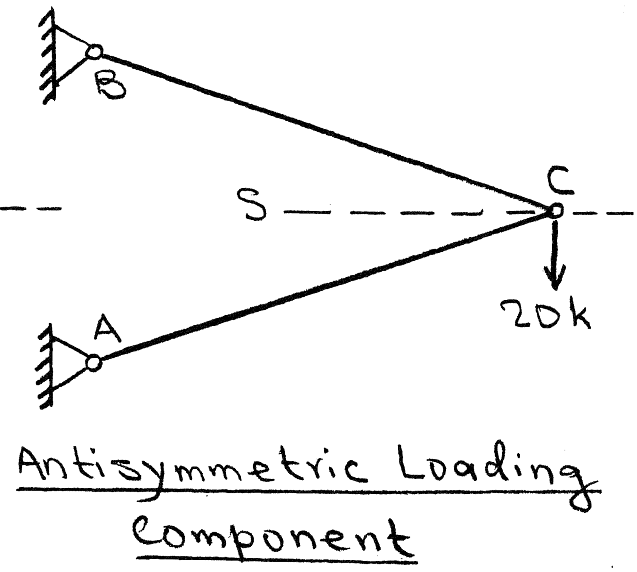
Symmetric Loading Component



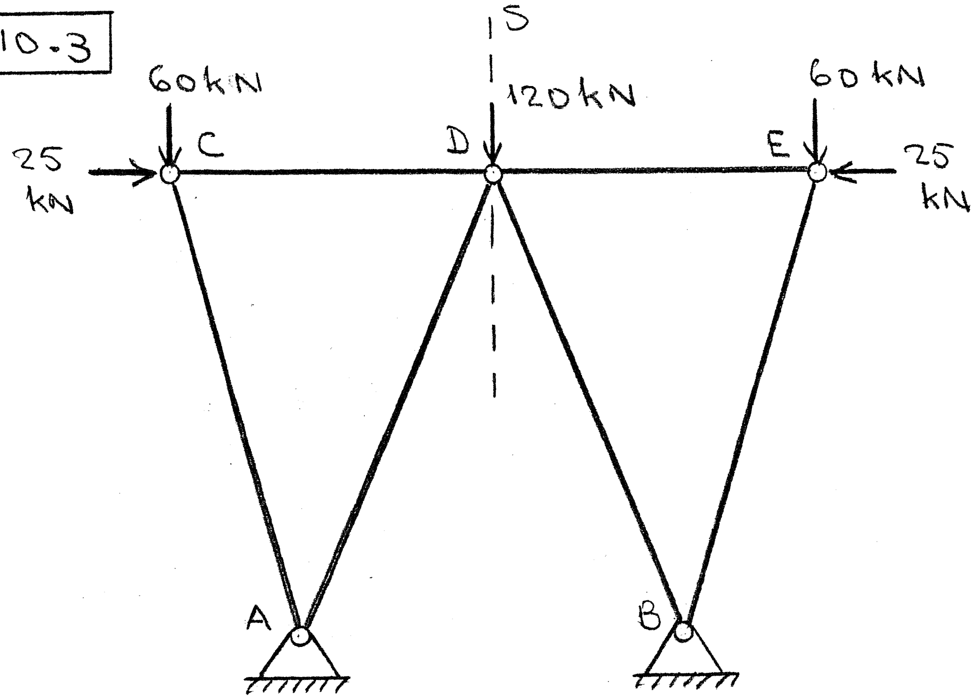
Antisymmetric Loading Component

10.2

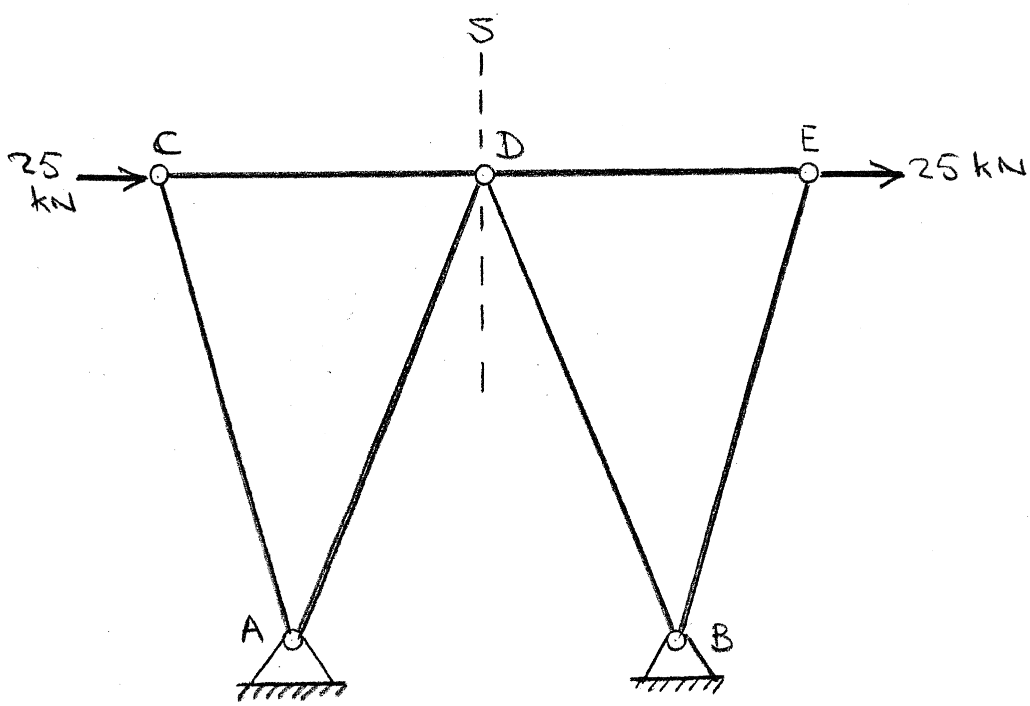

B
S
10k
C
A
Symmetric Loading Component


B
C
S
20k
A
Antisymmetric Loading Component

10-3

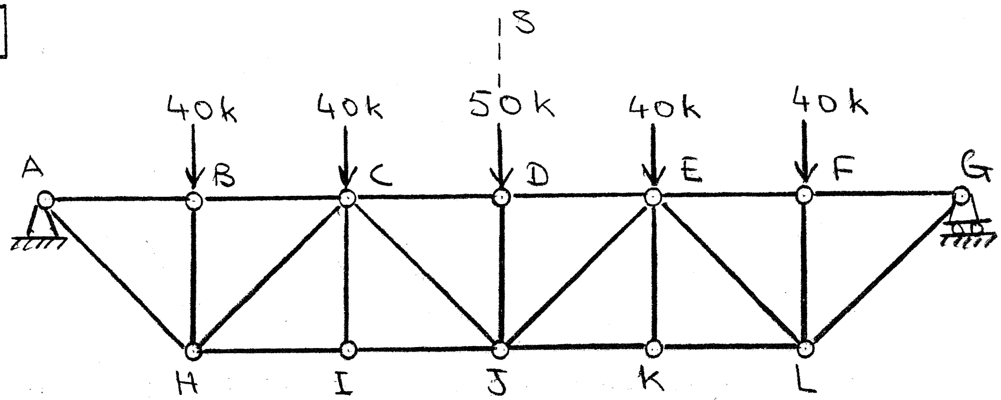


Symmetric Loading Component

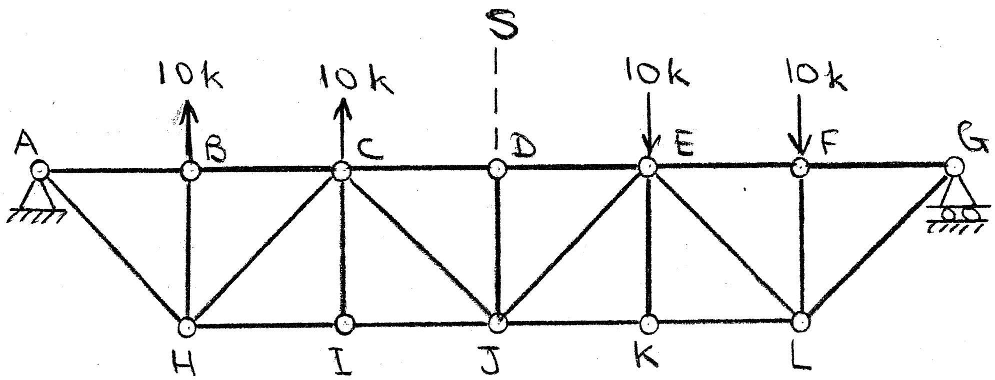


Antisymmetric Loading Component

10.4

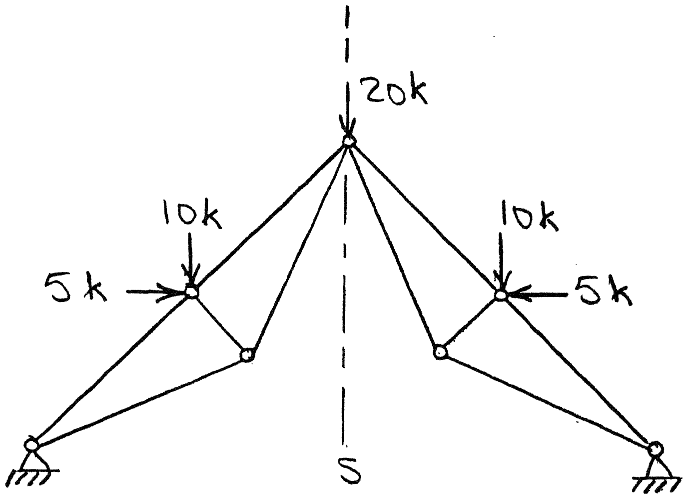


Symmetric Loading Component

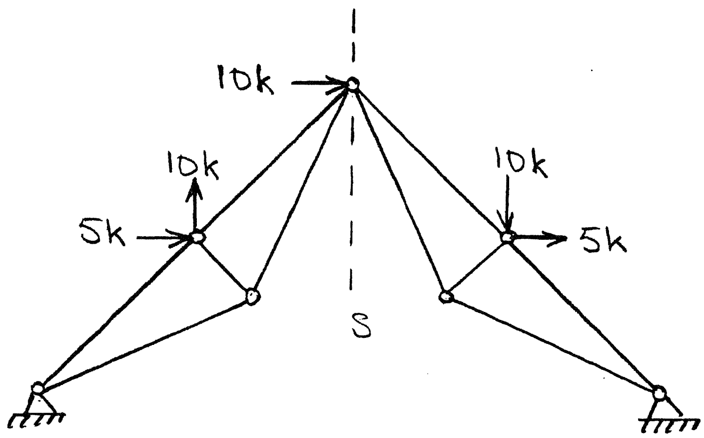


Antisymmetric Loading Component

10.5

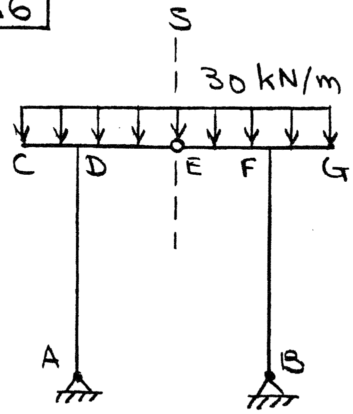


Symmetric Loading Component

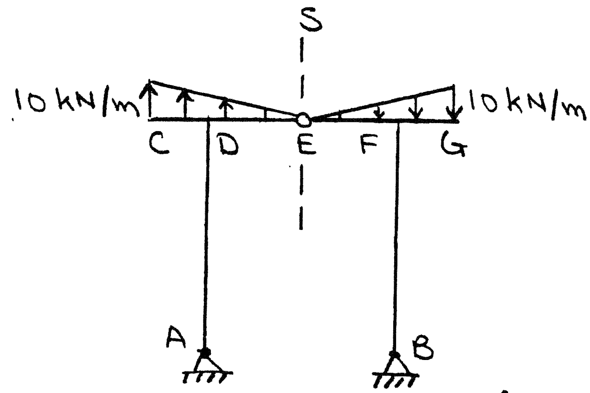


Antisymmetric Loading Component

10.6

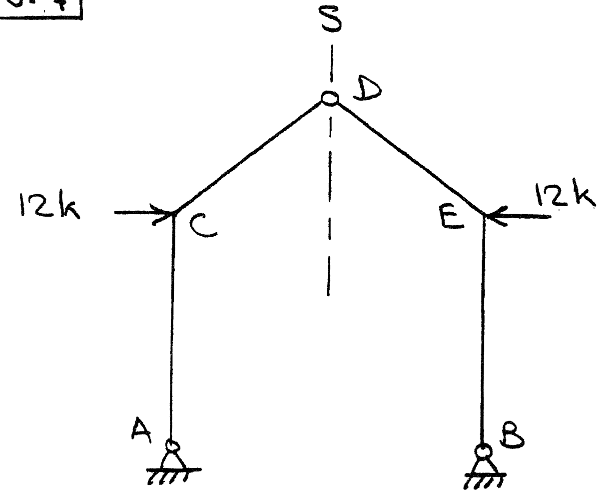


Symmetric Loading Component

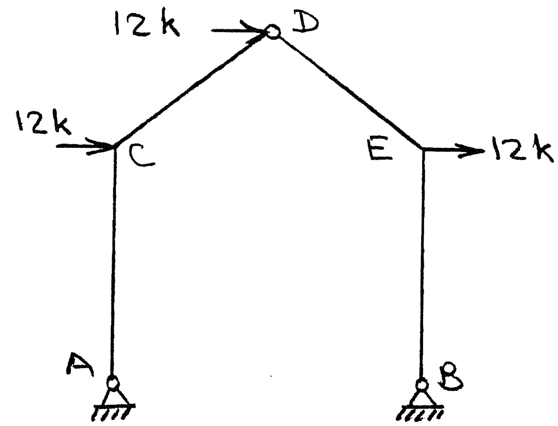


Antisymmetric Loading Component

10.7

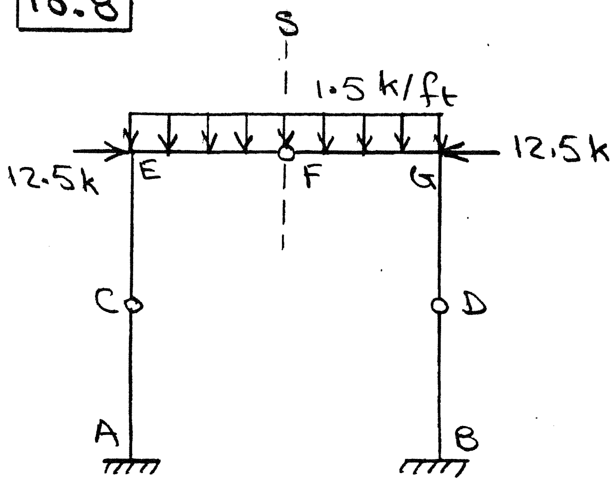


Symmetric Loading Component

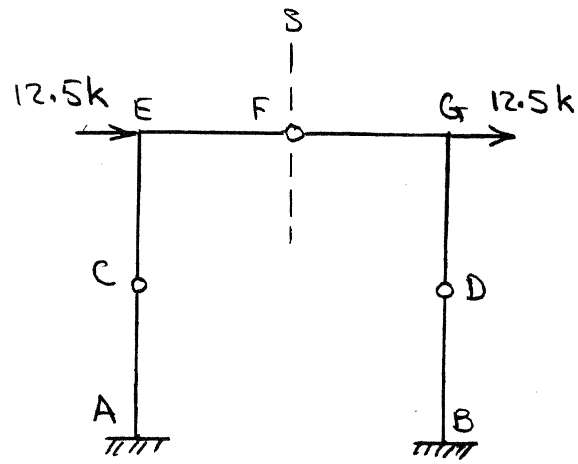


Antisymmetric Loading Component

10.8

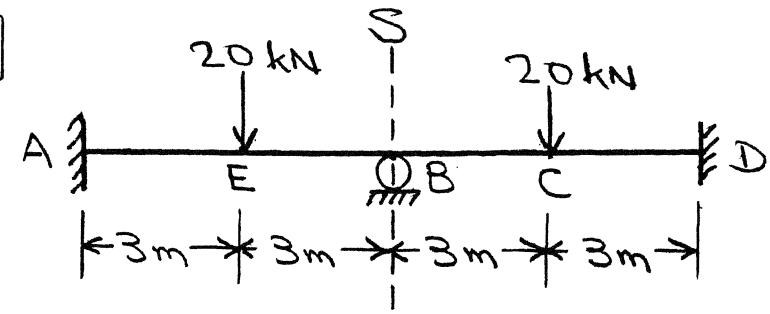


Symmetric Loading Component

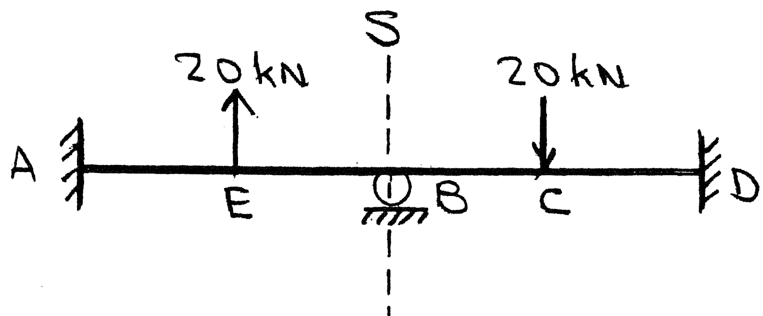


Antisymmetric Loading Component

10.9

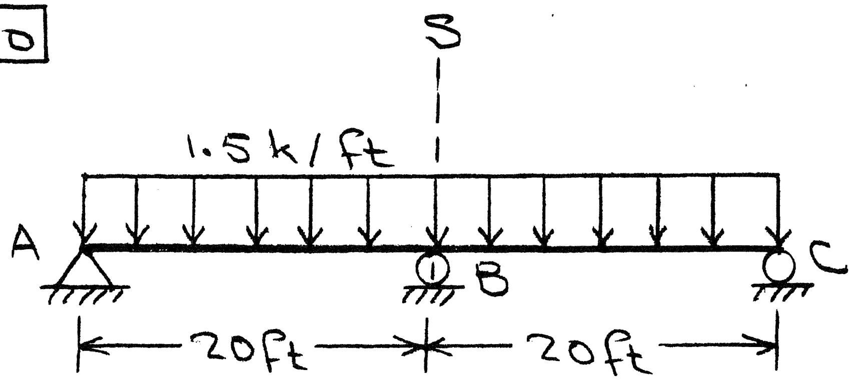


Symmetric Loading Component

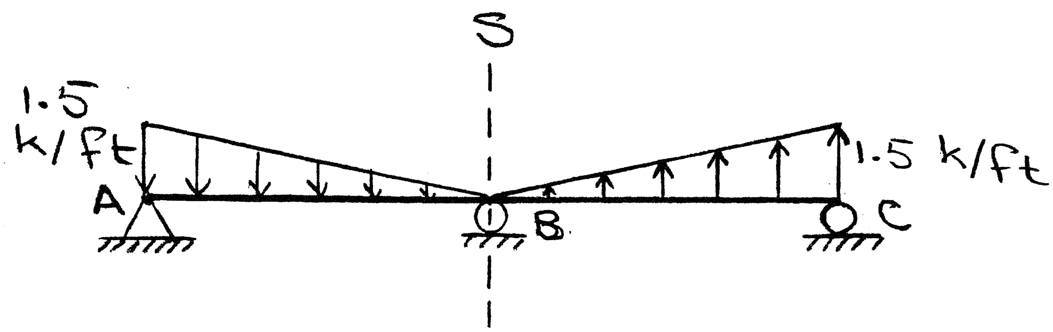


Antisymmetric Loading Component

10.10

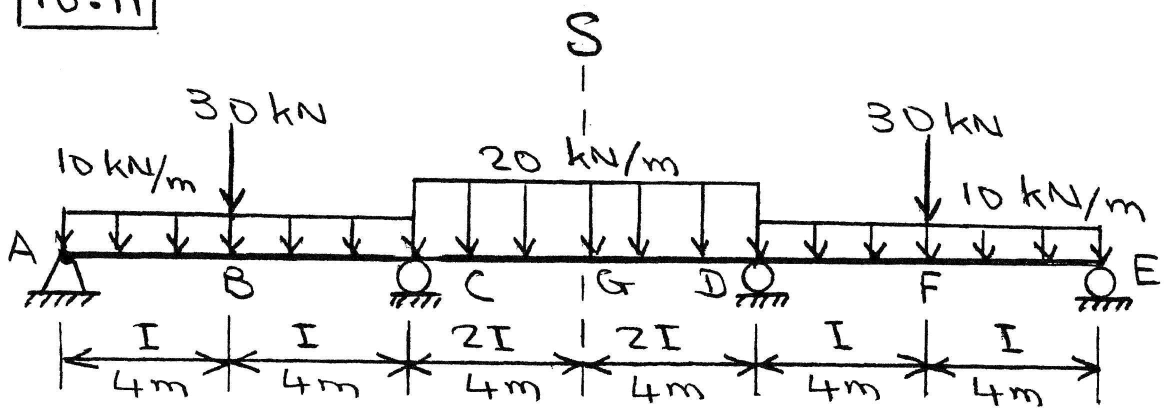


Symmetric Loading Component

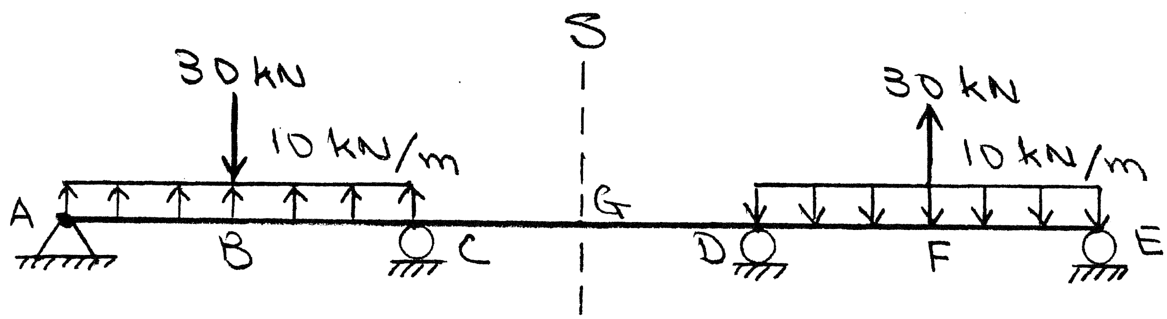


Antisymmetric Loading Component

10.11

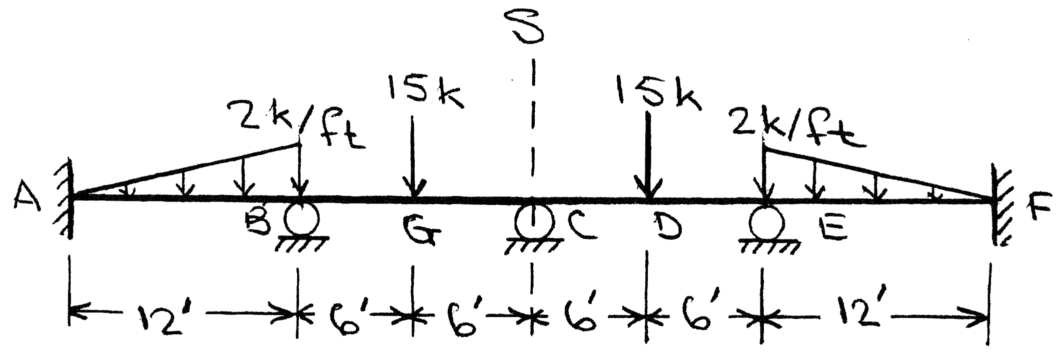


Symmetric Loading Component

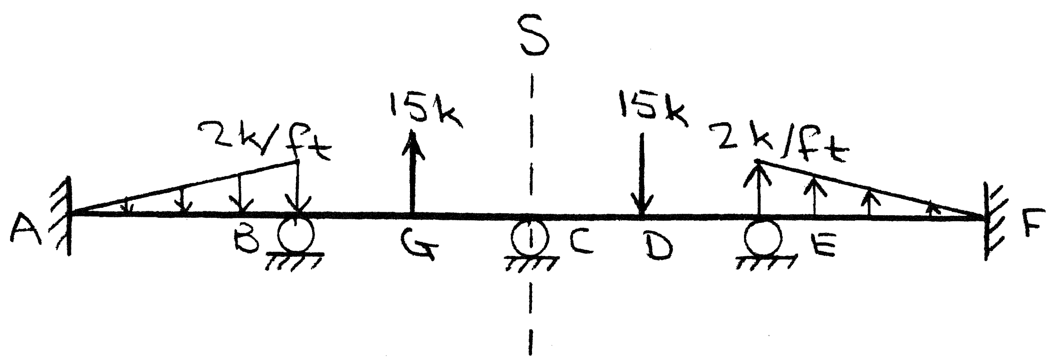


Antisymmetric Loading Component

10.12

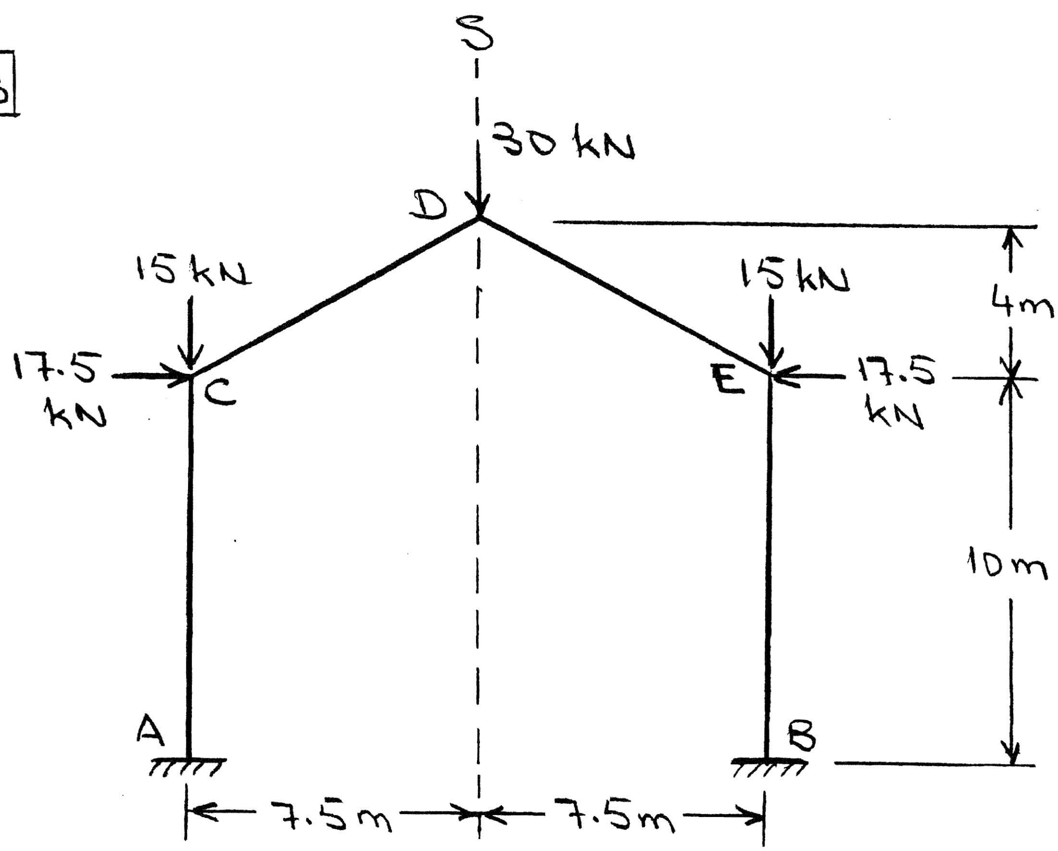


Symmetric Loading Component

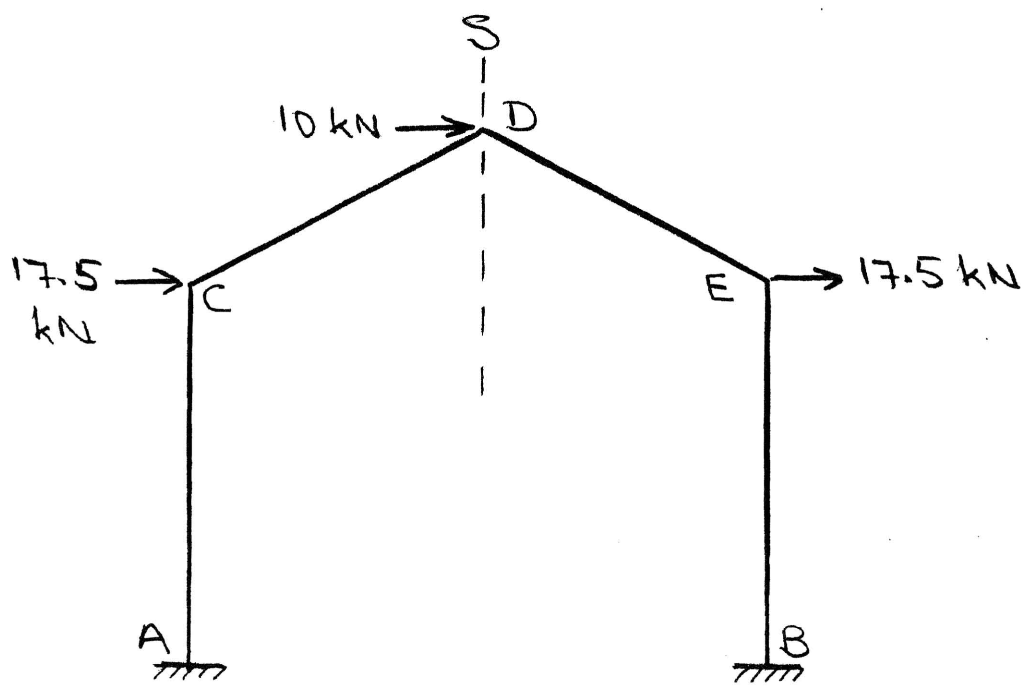


Antisymmetric Loading Component

10.13

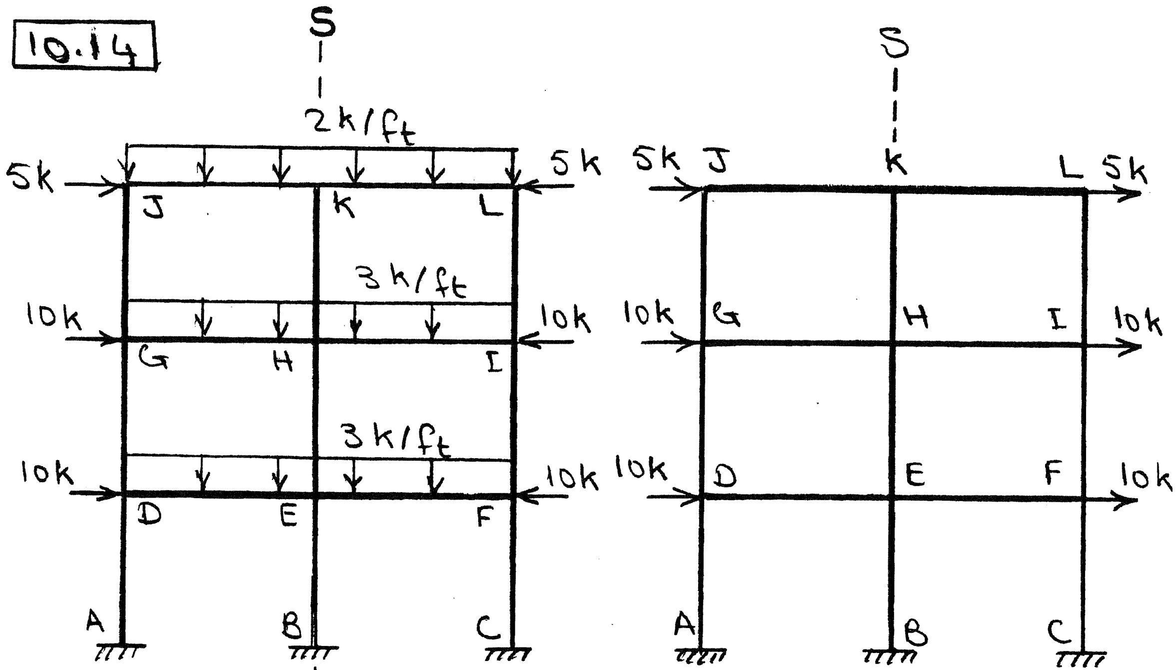


Symmetric Loading Component



Antisymmetric Loading Component

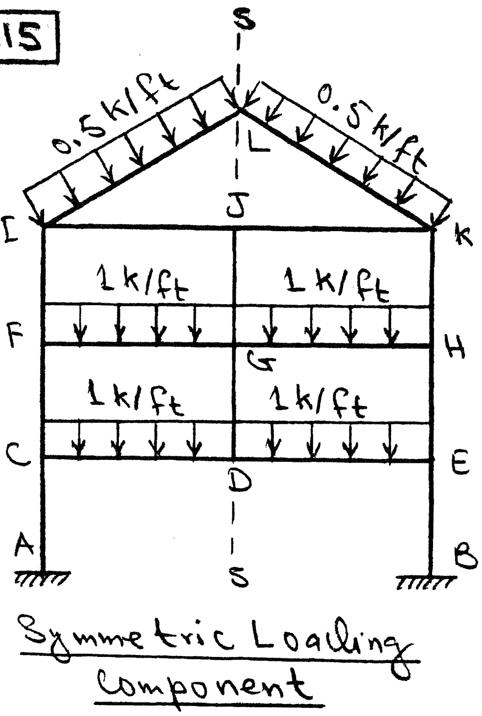
10.14



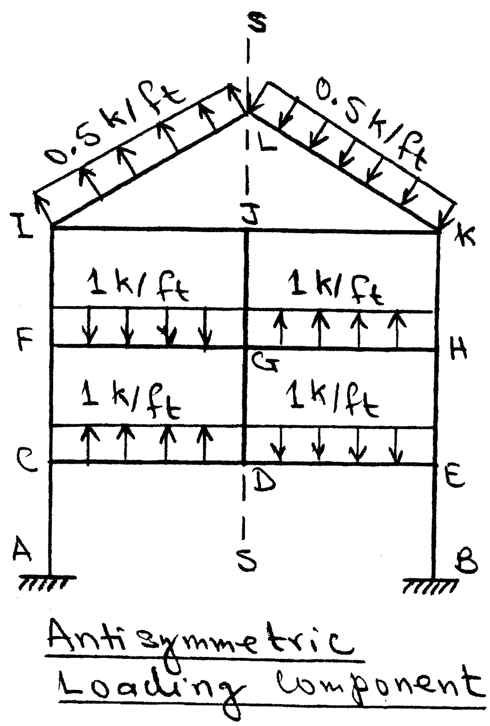
Symmetric Loading Component

Antisymmetric Loading Component

10.15

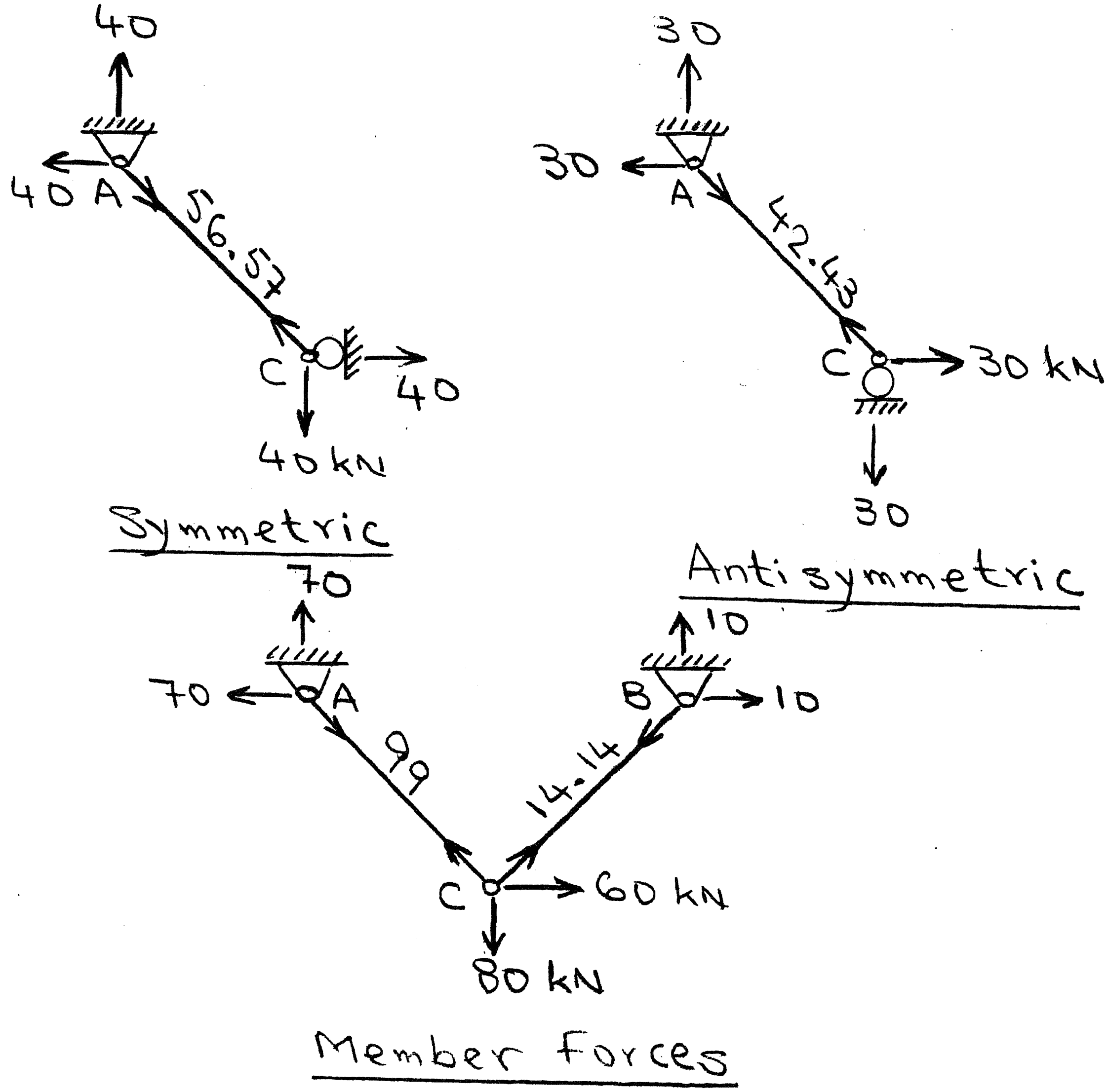


S
0.5 k/ft
0.5 k/ft
L
J
I
K
1 k/ft
1 k/ft
F
H
G
1 k/ft
1 k/ft
C
E
D
A
B
S
Symmetric Loading Component

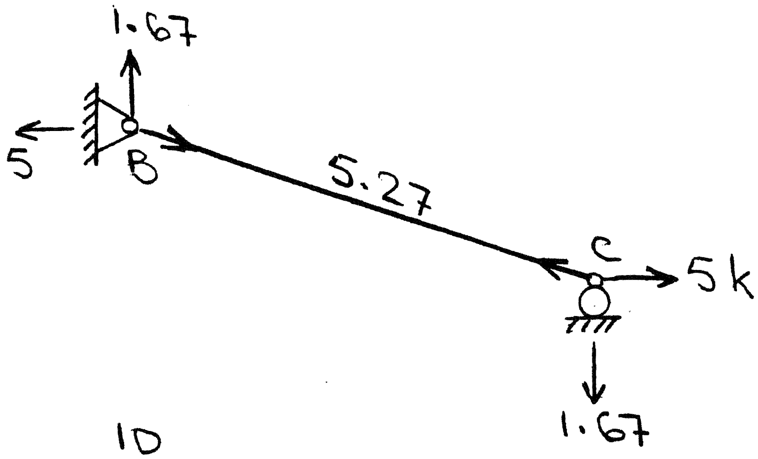


S
0.5 k/ft
0.5 k/ft
L
J
I
K
1 k/ft
1 k/ft
F
H
G
1 k/ft
1 k/ft
C
E
D
A
B
S
Antisymmetric Loading Component

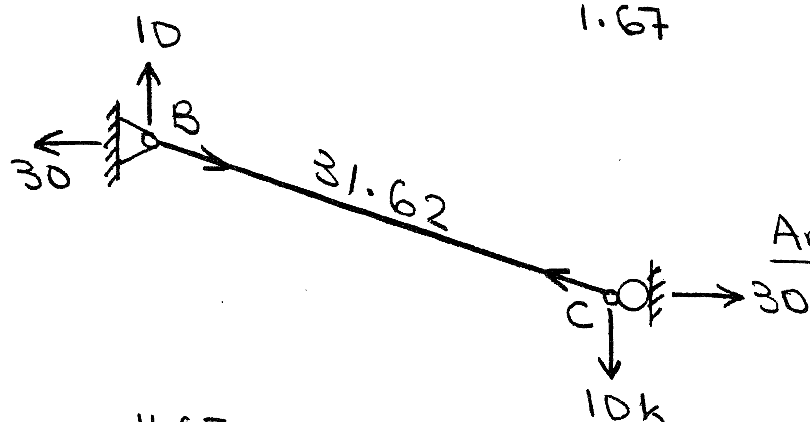
10.16


40
40
A
56.57
C
40
40 kN
Symmetric
30
30
A
42.43
C
30 kN
30
Antisymmetric
70
70
A
99
10
B
10
14.14
C
60 kN
80 kN
Member Forces

10.17



Symmetric



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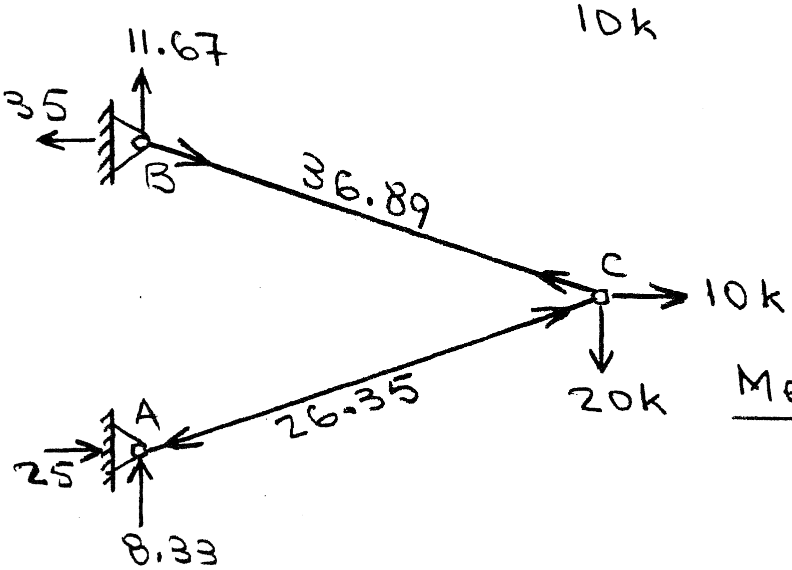
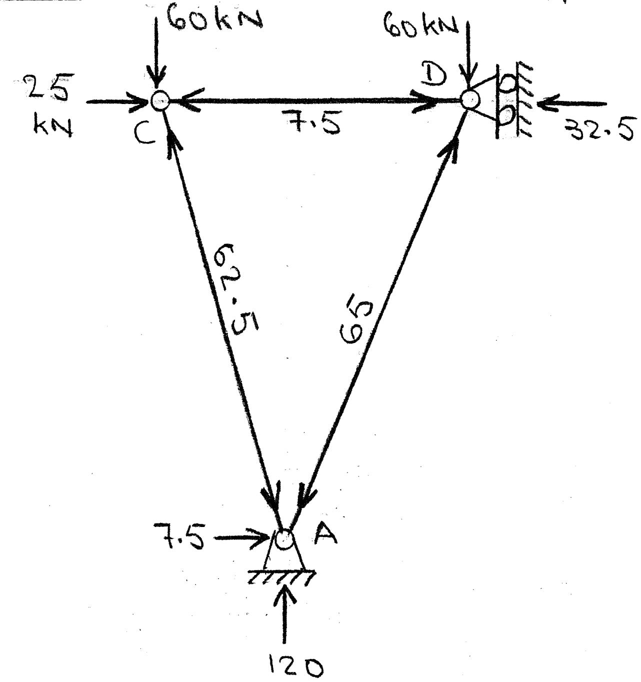
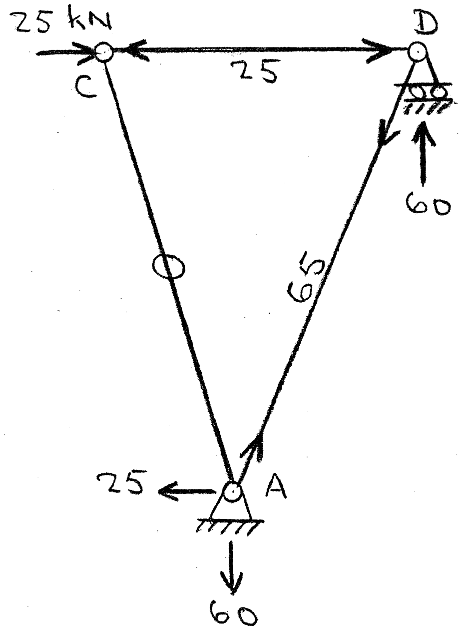


Member Forces

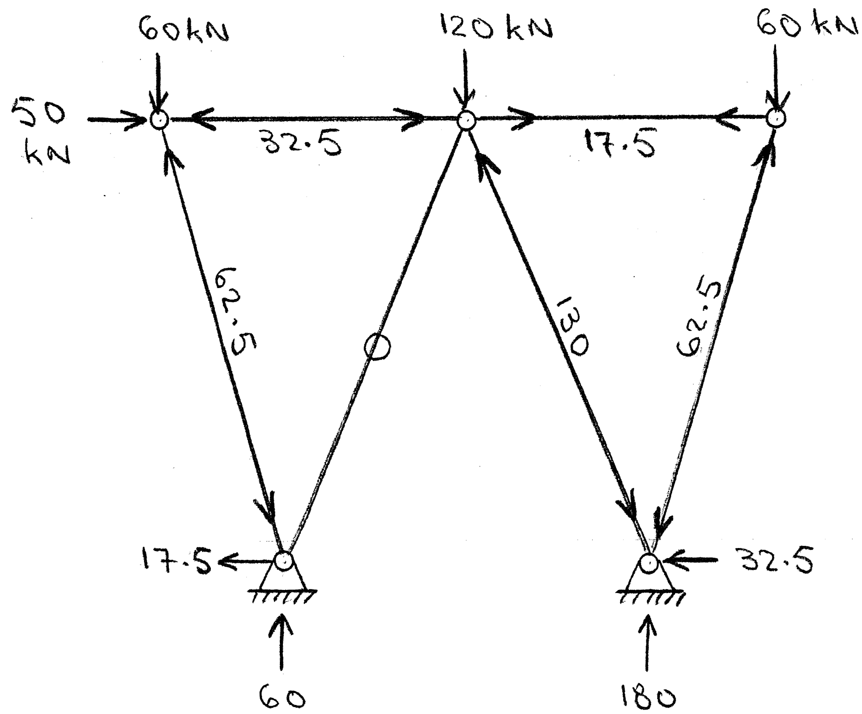
10.18 See solution of problem 10.3



Symmetric

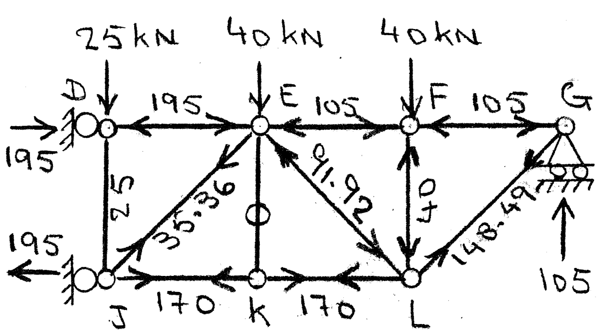


Antisymmetric

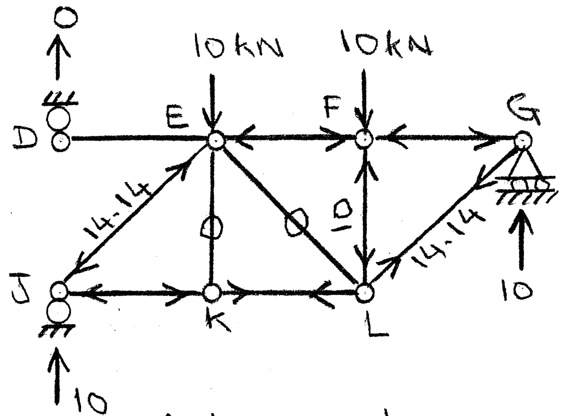


Member Forces

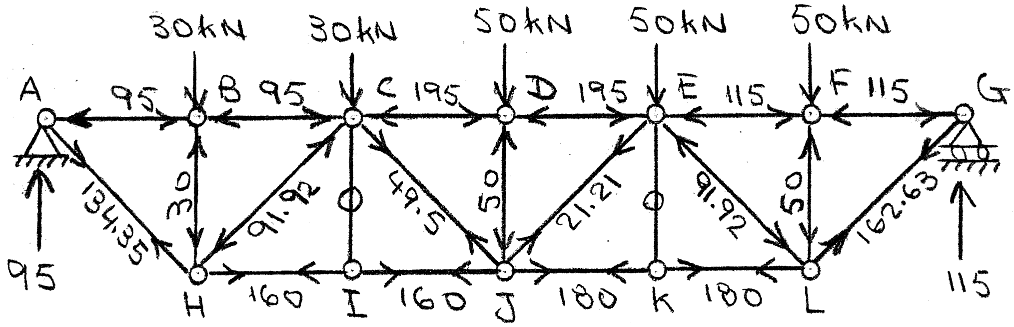
10.19 See solution of Problem 10.4.



Symmetric

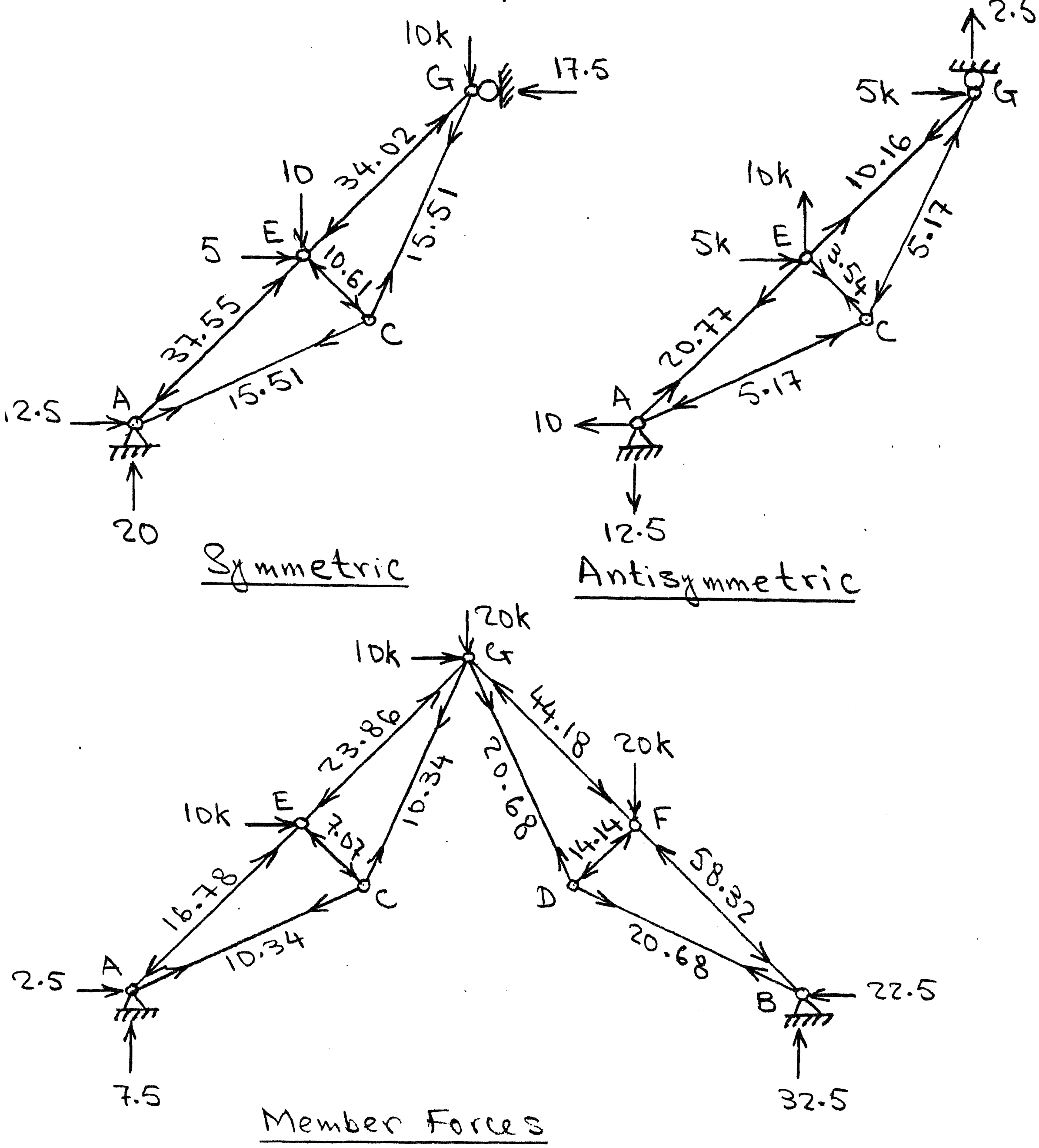


Antisymmetric

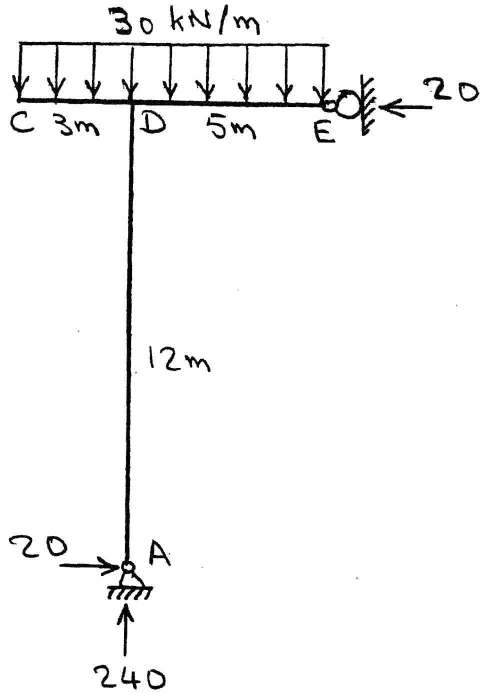


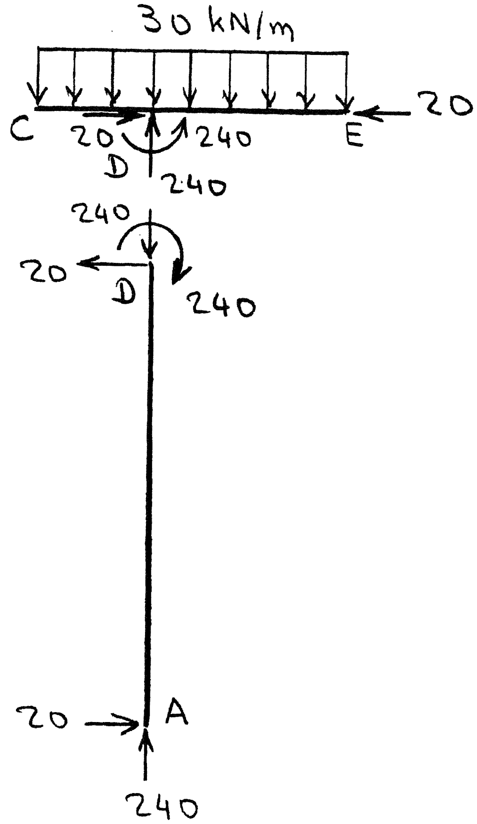
Member Forces

10.20 See solution of Problem 10.5.

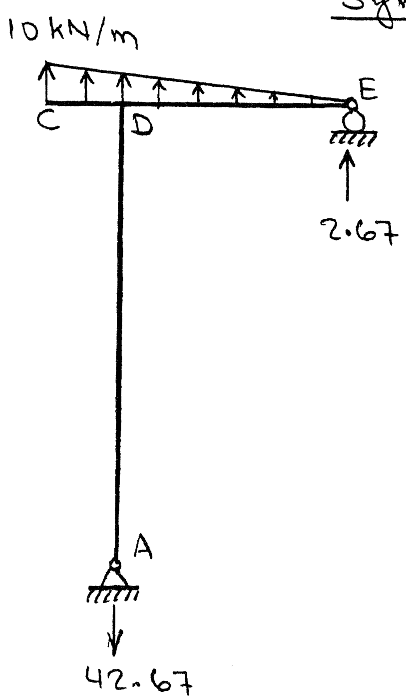


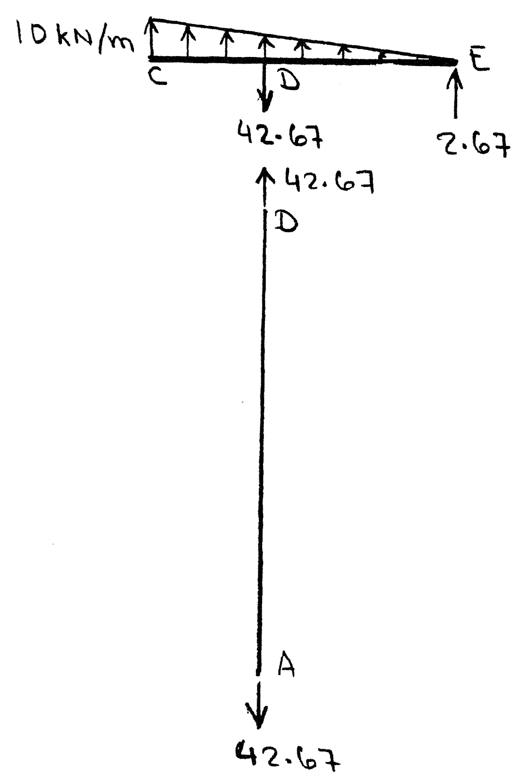
10.21 See solution of Problem 10.6.





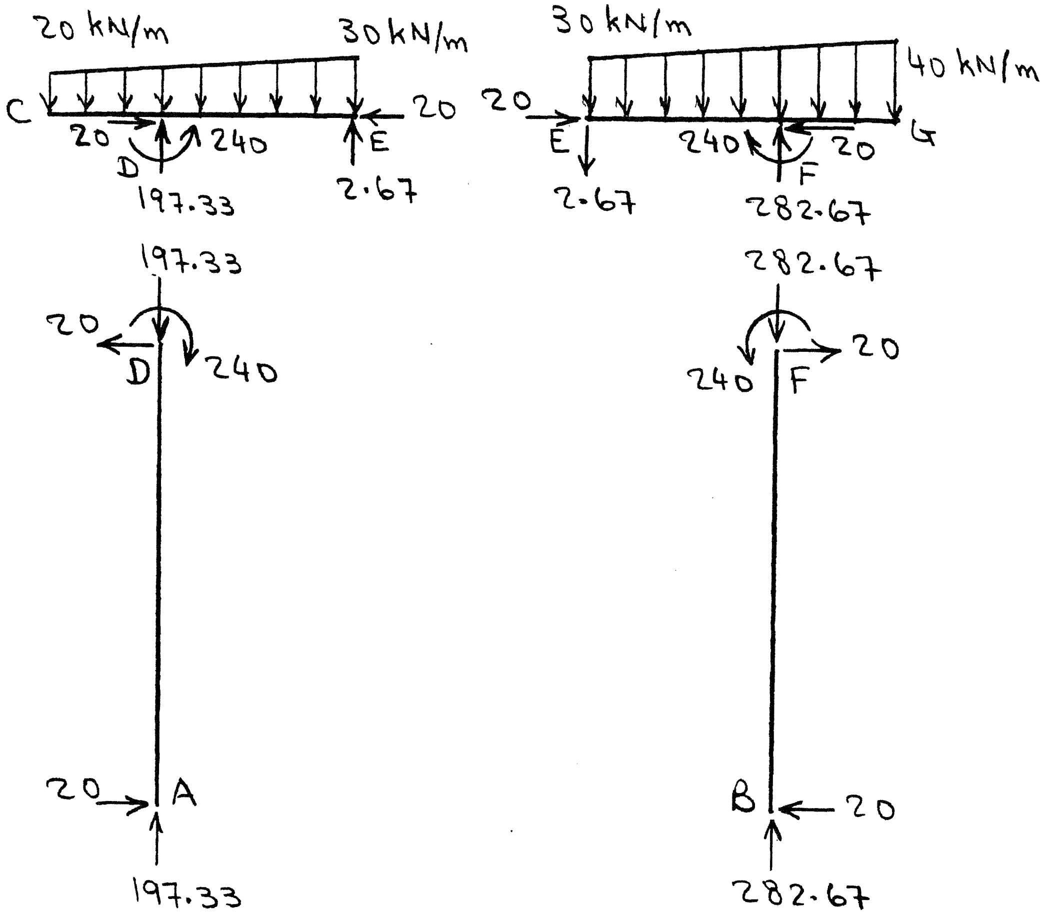
Symmetric





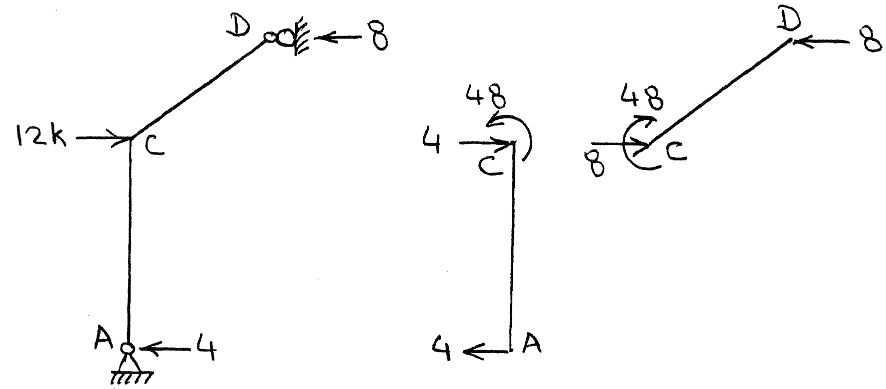
Antisymmetric

10.21 (Contd.)

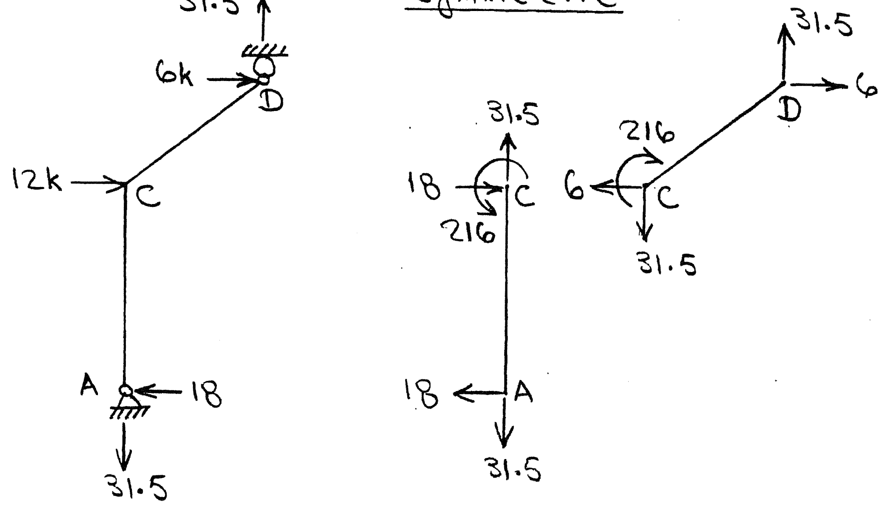


Member End Forces

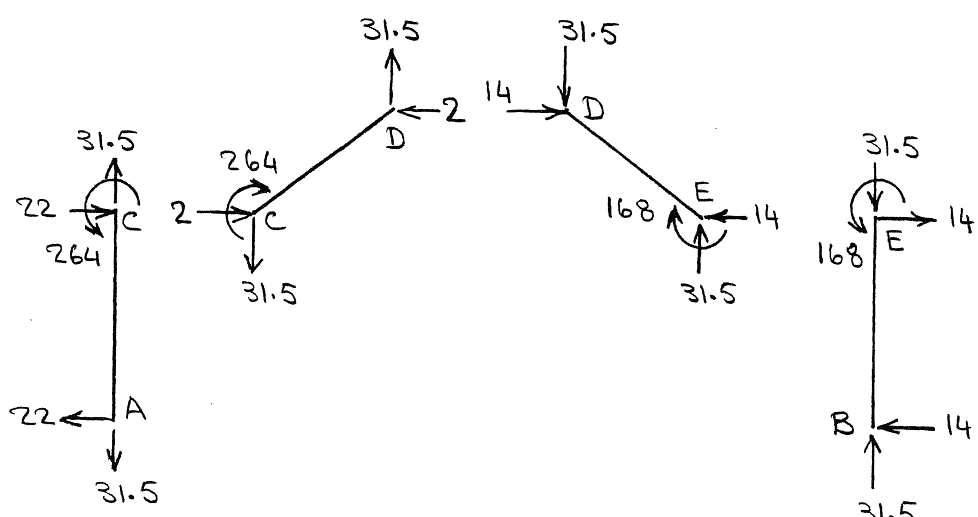
10.22 See solution of Problem 10.7.



Symmetric

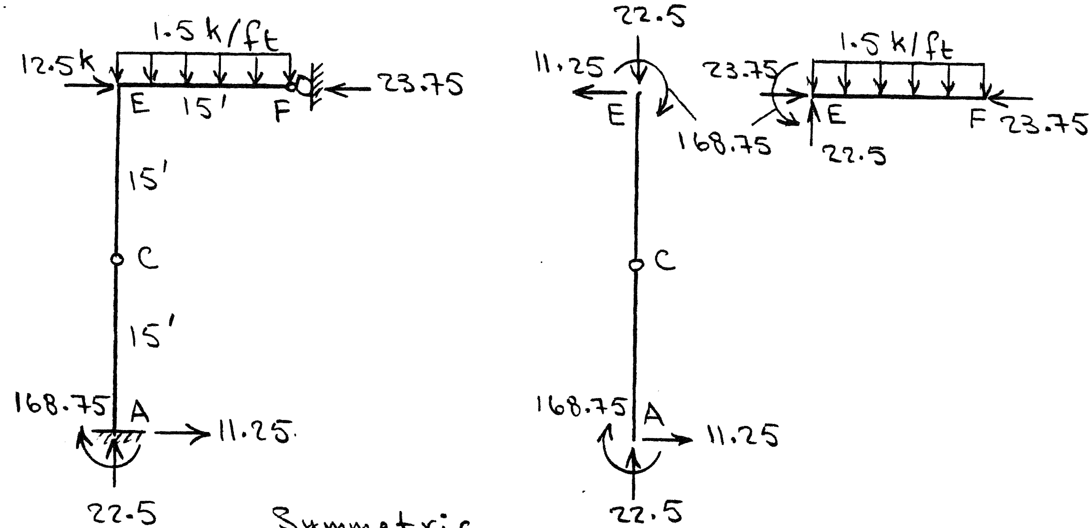


Antisymmetric

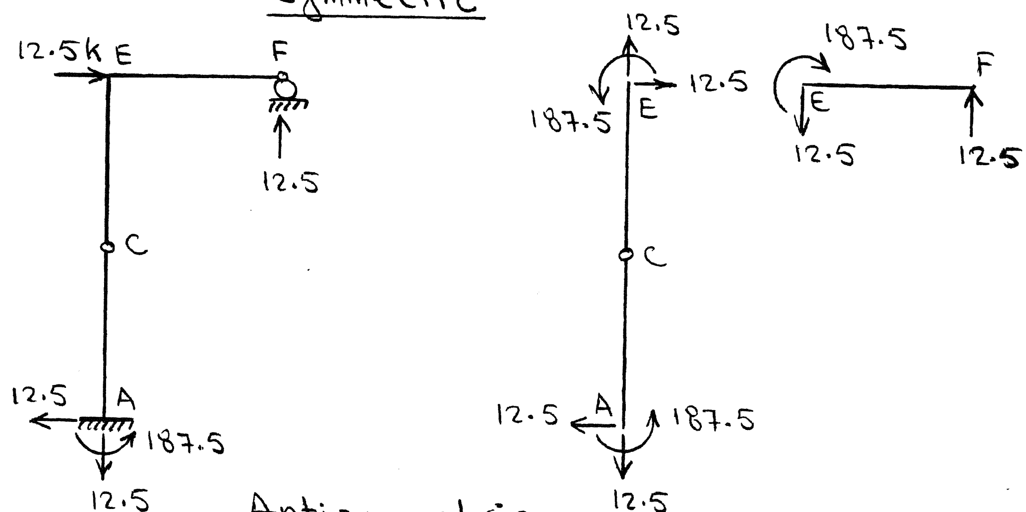


Member End Forces

10.23 See solution of Problem 10.8.



Symmetric



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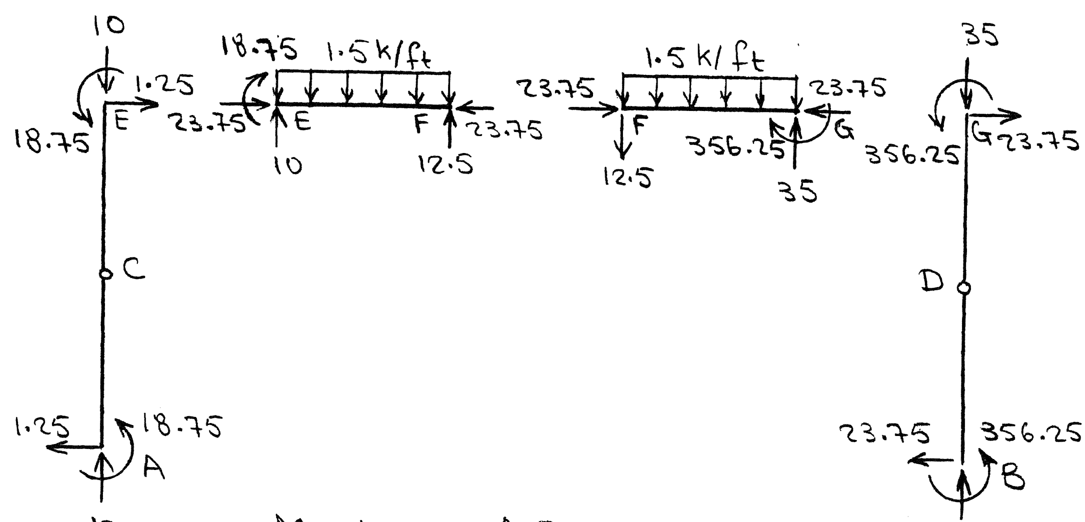
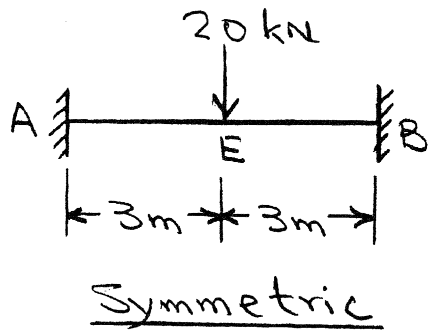
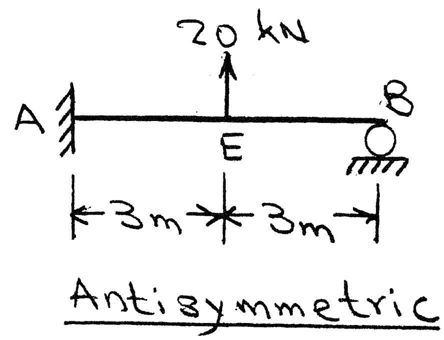


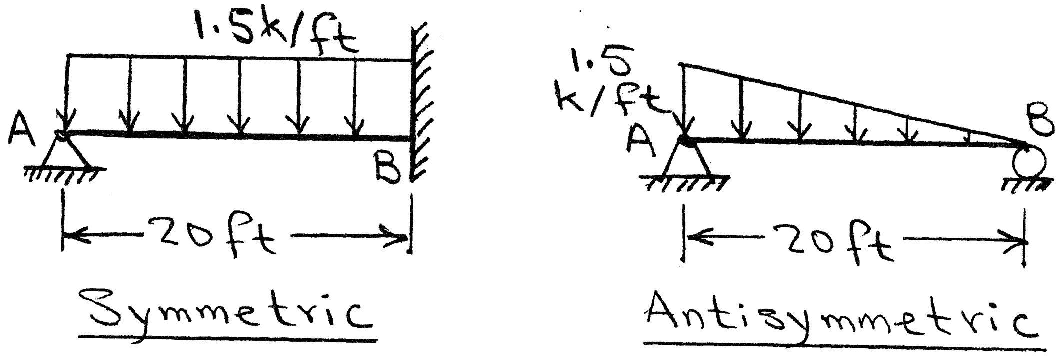
Member End Forces

10.24 See solution of Problem 10.9.

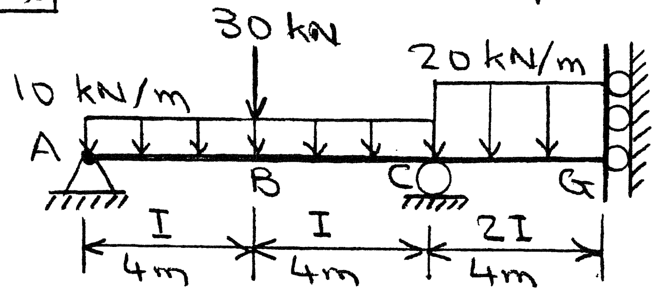




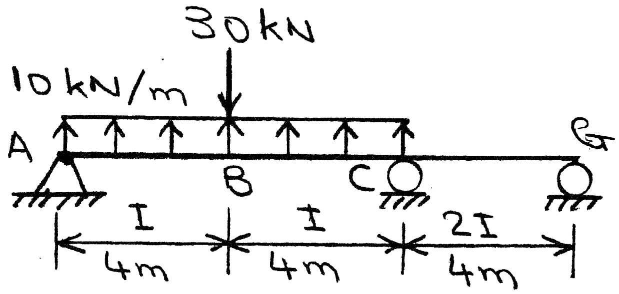
10.25 See solution of Problem 10.10.



10.26 See solution of Problem 10.11

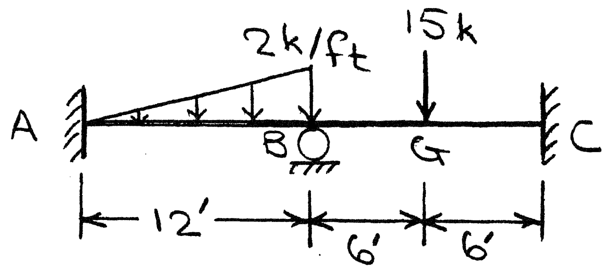


Symmetric

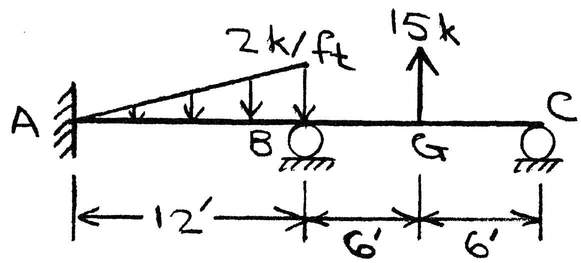


Antisymmetric

10.27 See solution of Problem 10.12.

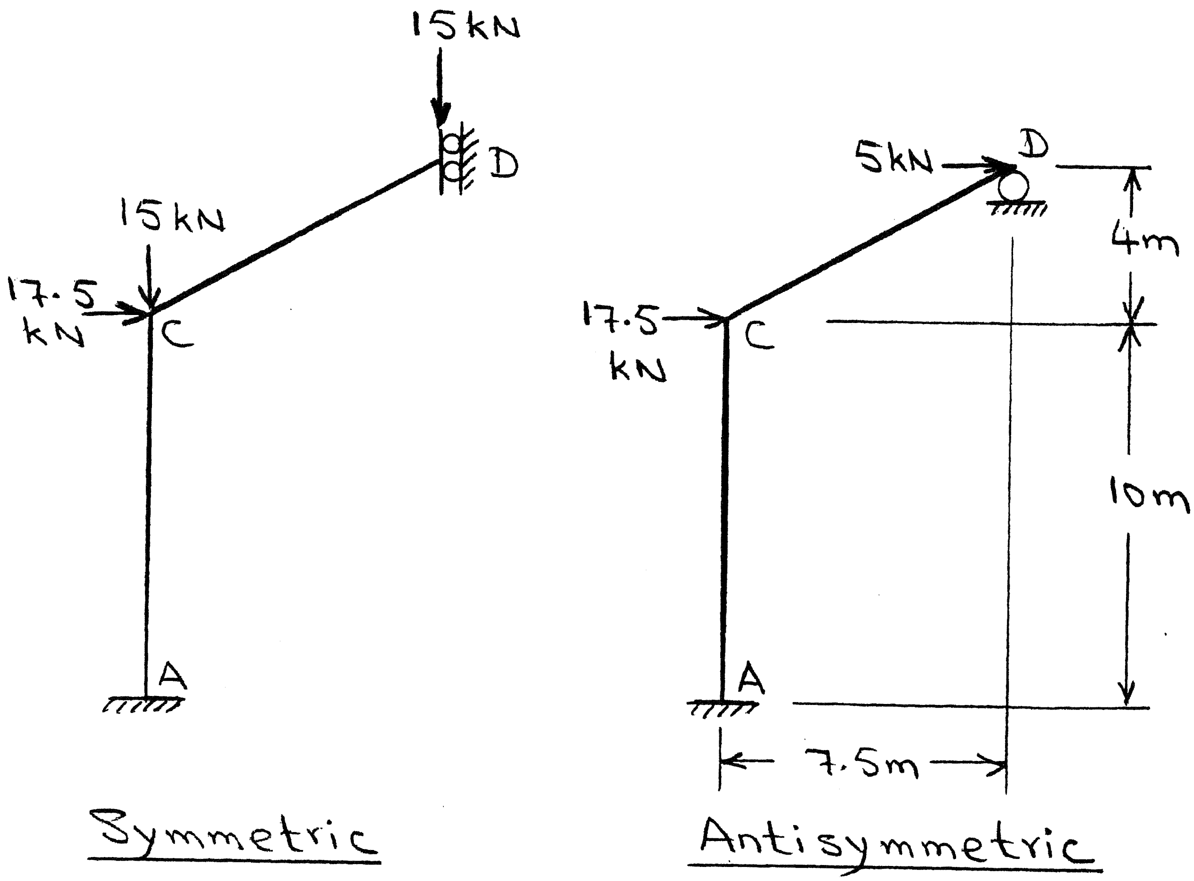


Symmetric

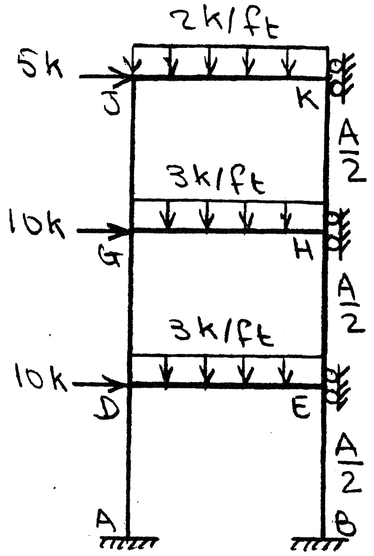


Antisymmetric

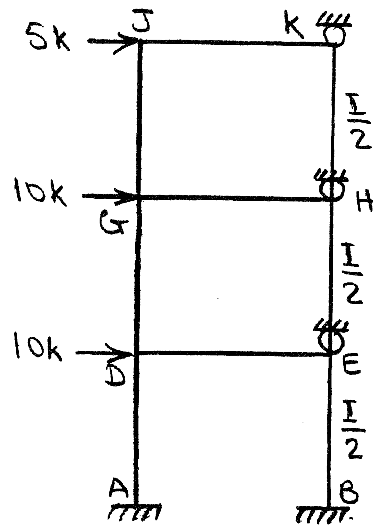
10.28 See solution of Problem 10.13.



10.29 See solution of Problem 10.14.

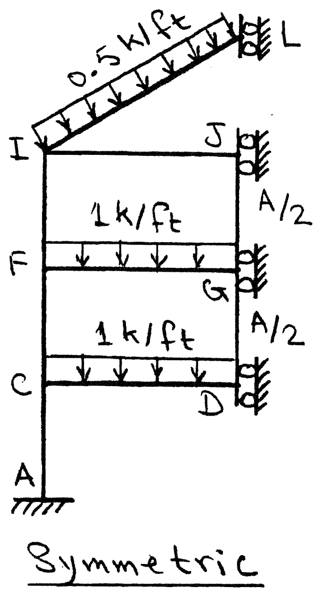


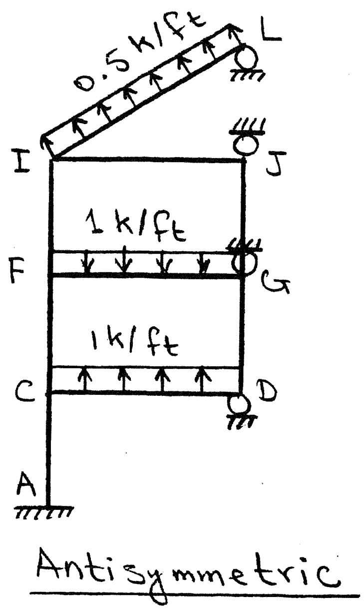
Symmetric



Antisymmetric

10.30 See solution of Problem 10.15.

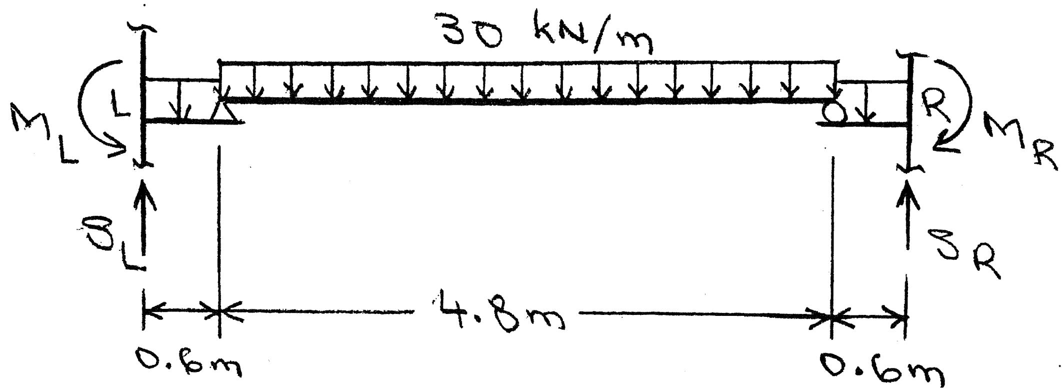


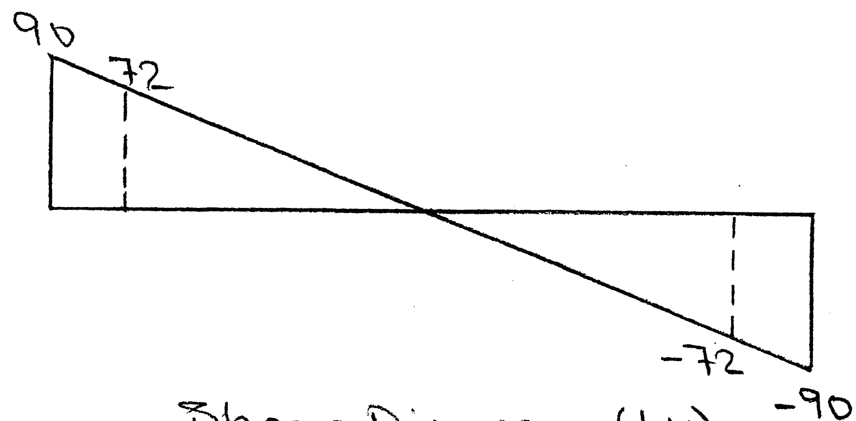


Chapter Twelve
Approximate Analysis of Rectangular Building Frames

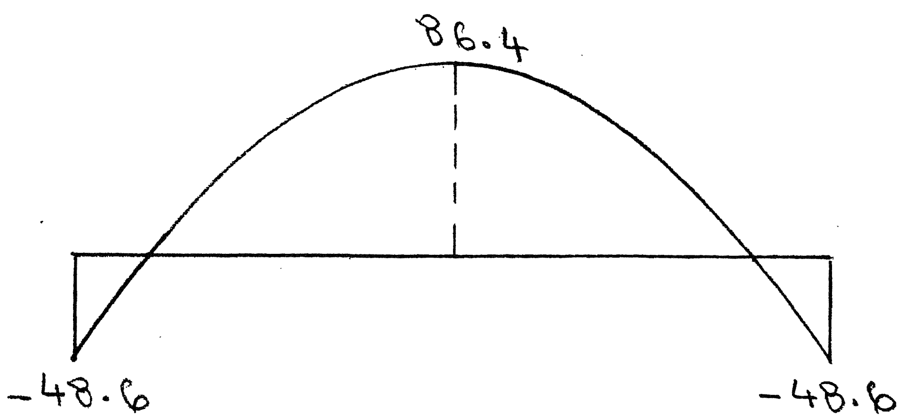
CHAPTER 12

12.1



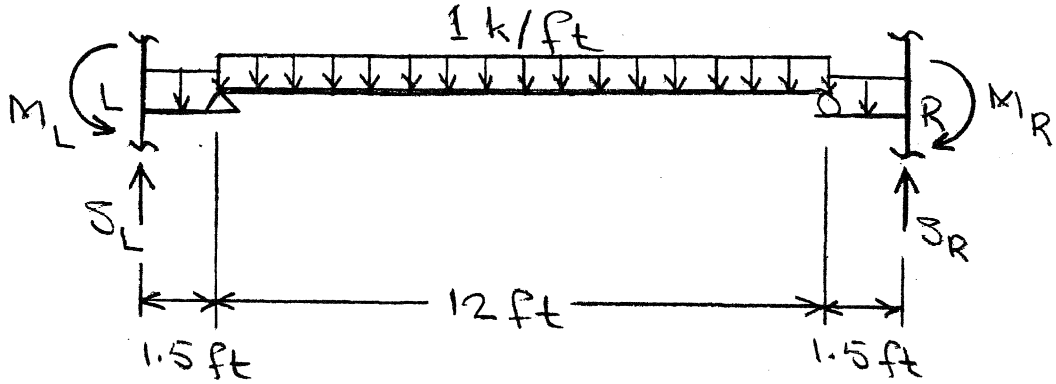


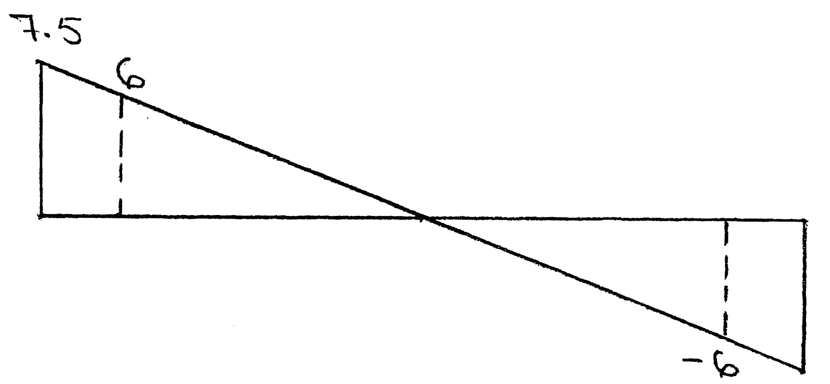
Shear Diagram (kN)



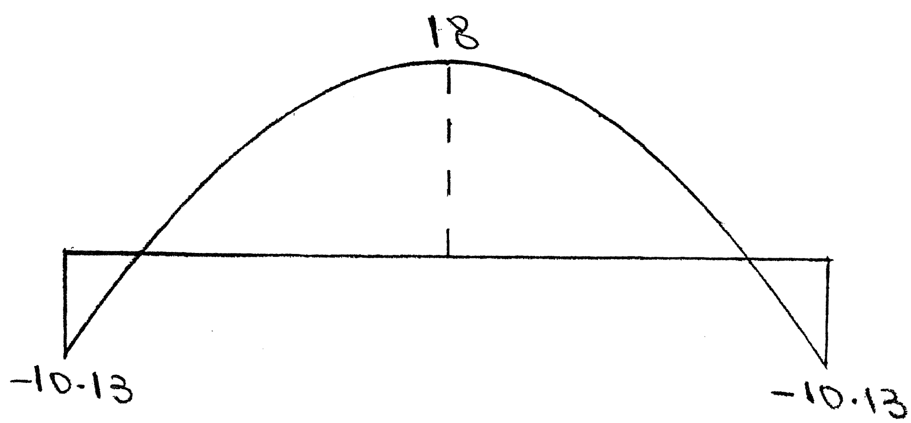
Bending Moment Diagram (kN.m)

12.2



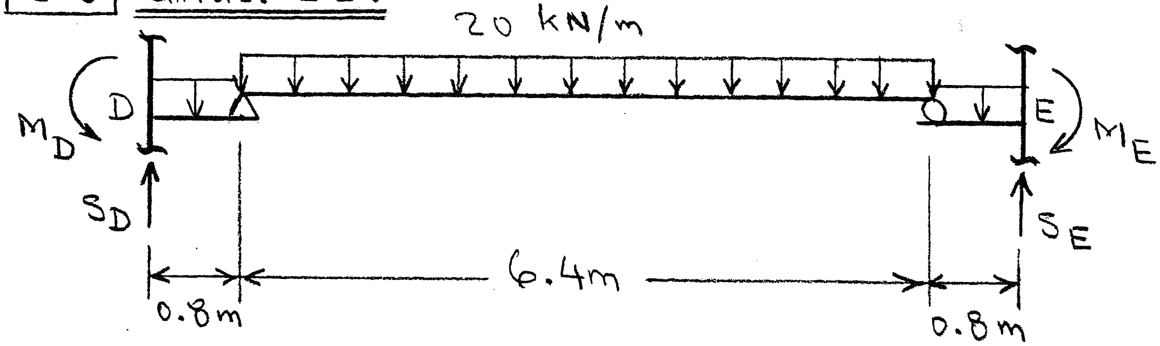


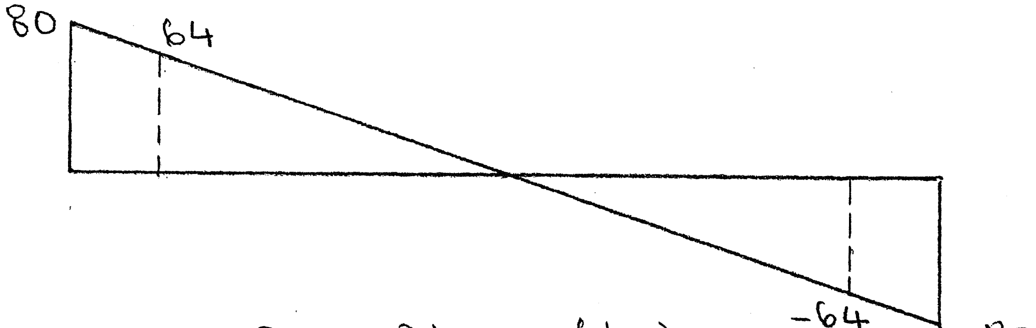
Shear Diagram (k)



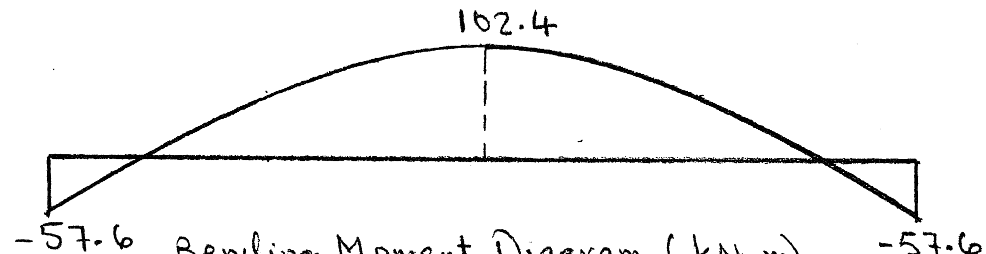
Bending Moment Diagram (k-ft)

12.3 Girder DE:



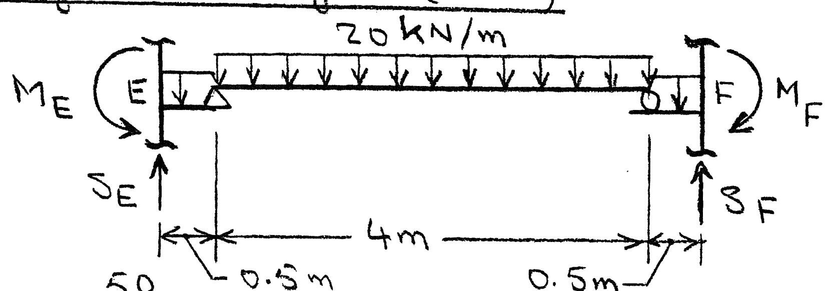


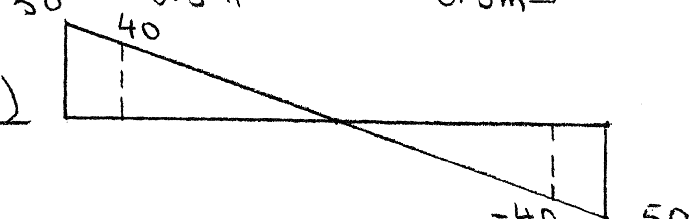
Shear Diagram (kN)



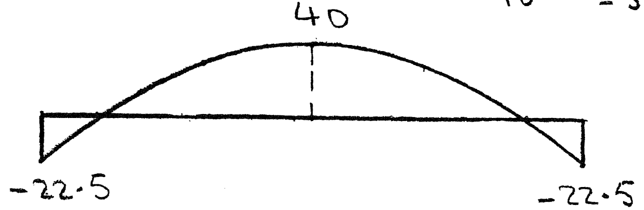
Bending Moment Diagram (kN.m)

Girder EF:



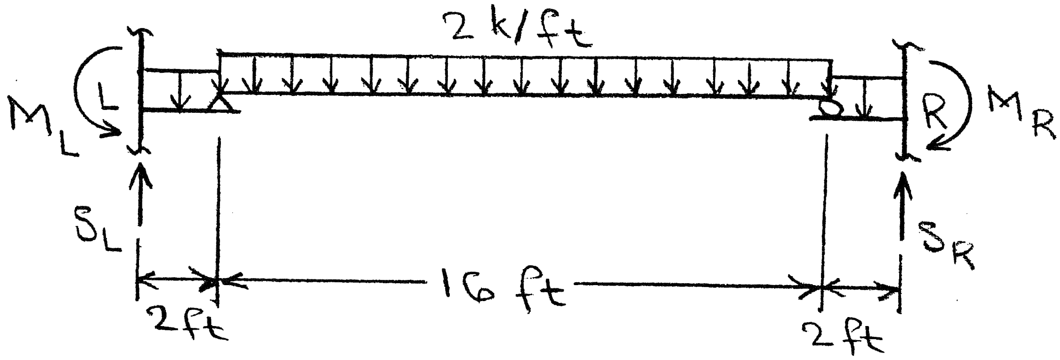


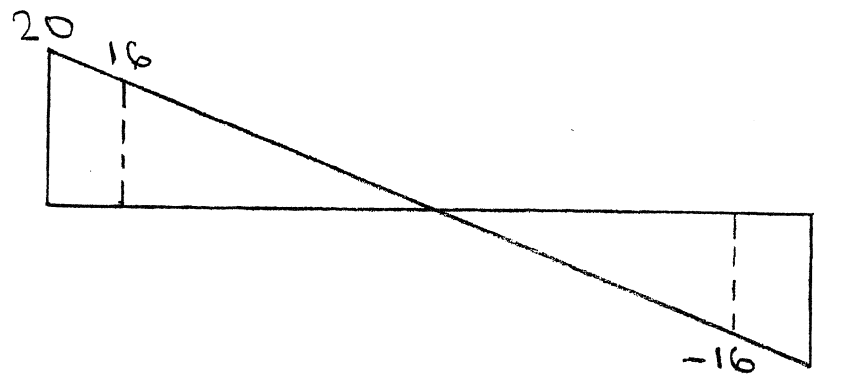
Shear Diagram (kN)



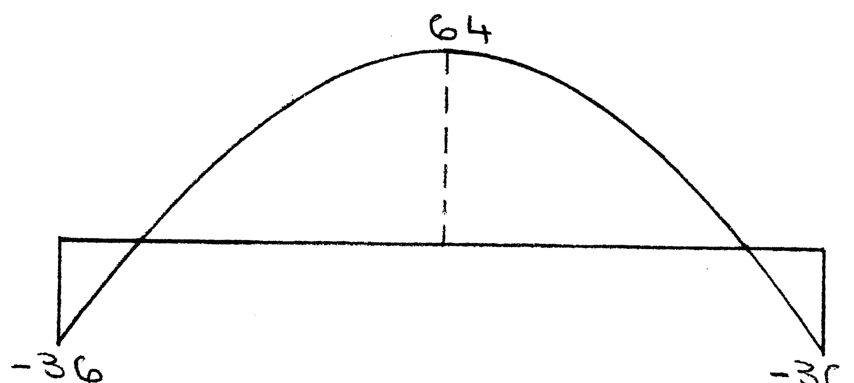
Bending Moment Diagram (kN.m)

12.4



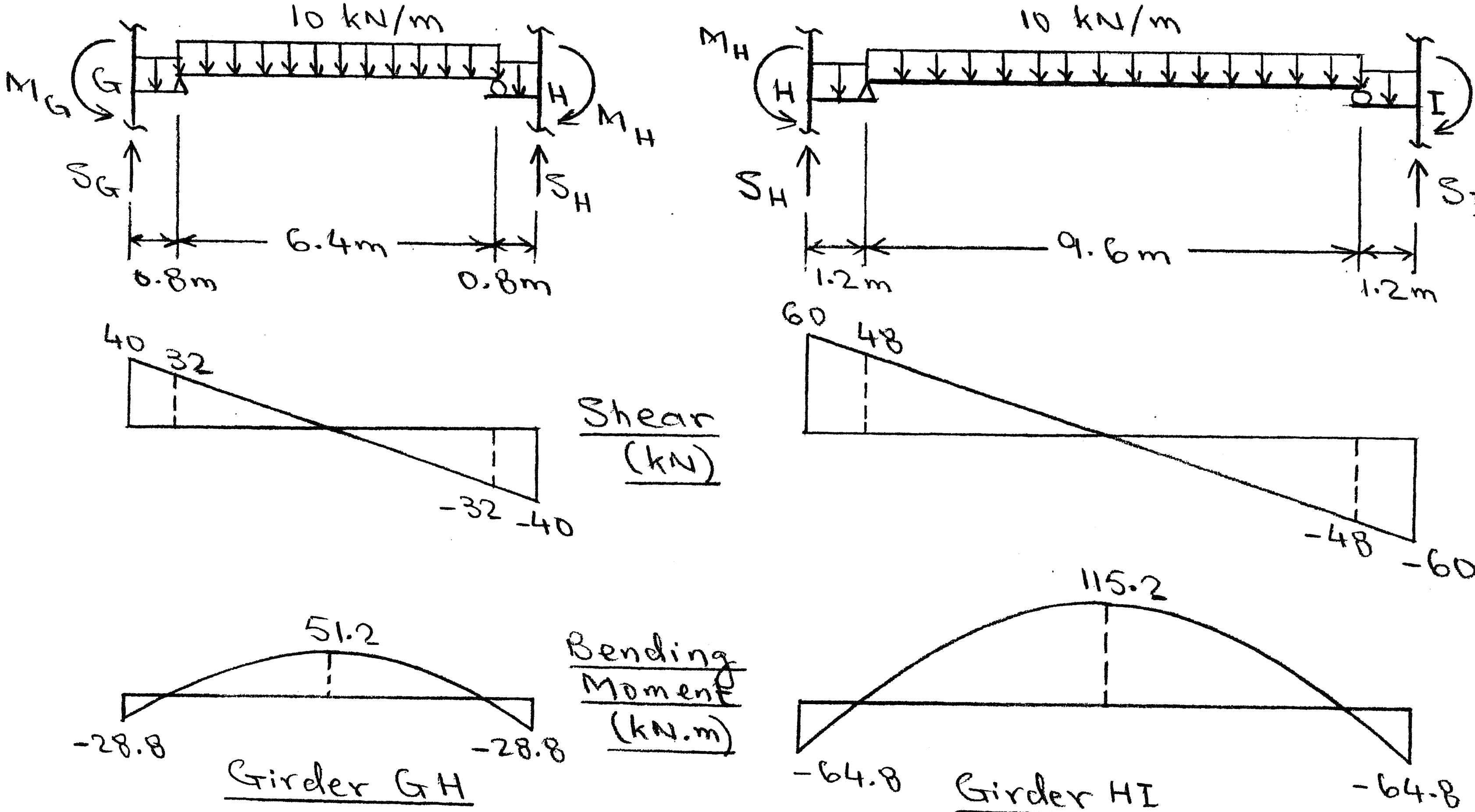


Shear Diagram (k)



Bending Moment Diagram (k-ft)

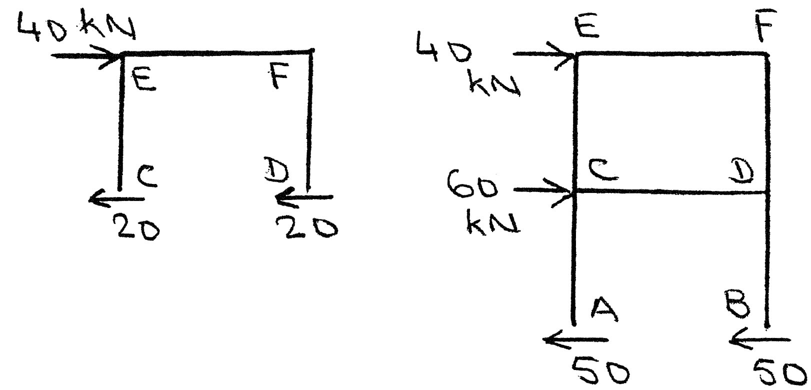
12.5



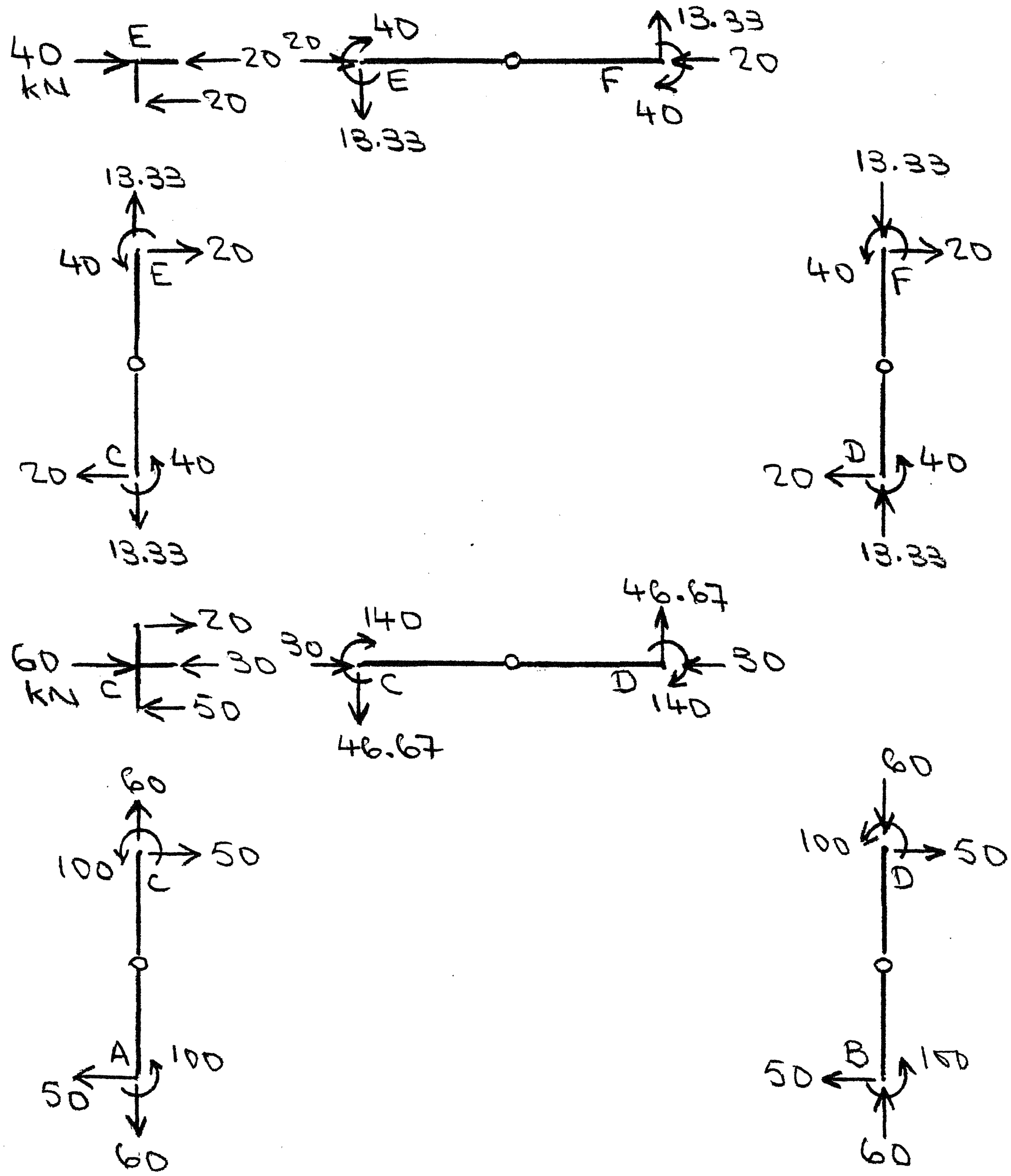
Shear in Girder DE = 2 x Shear in Girder GH.
Moment in Girder DE = 2 x Moment in Girder GH.

Shear in Girder EF = 2 x Shear in Girder HI.
Moment in Girder EF = 2 x Moment in Girder HI.

12.6

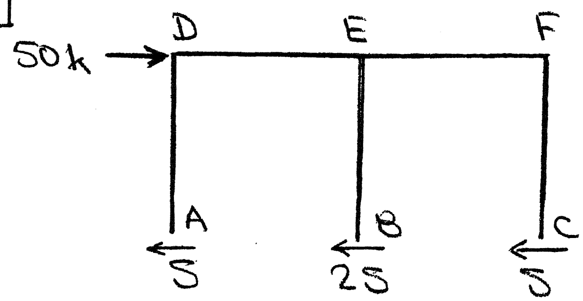


Column Shears



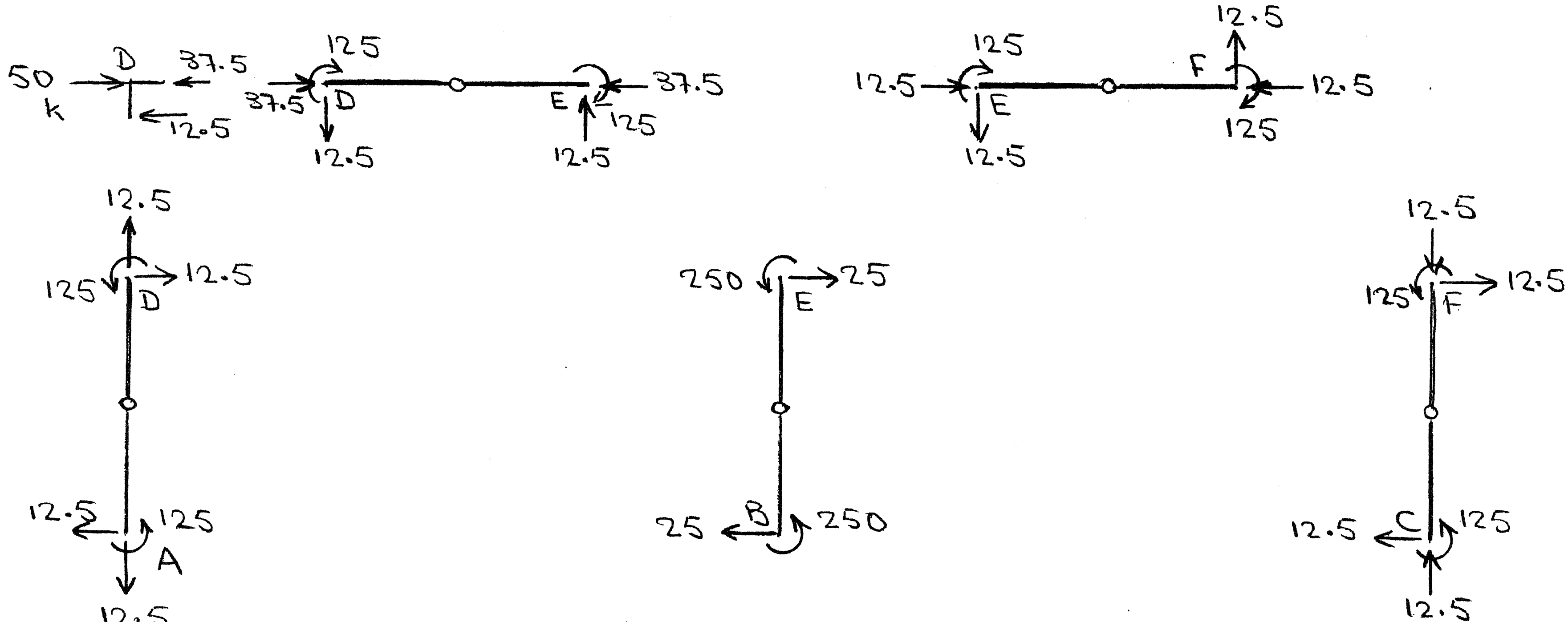
Member Axial Forces, Shears, and Moments

12.7



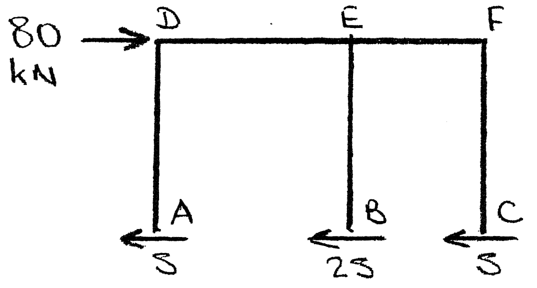
Column Shears

$4S = 50 \qquad S = 12.5\ k$



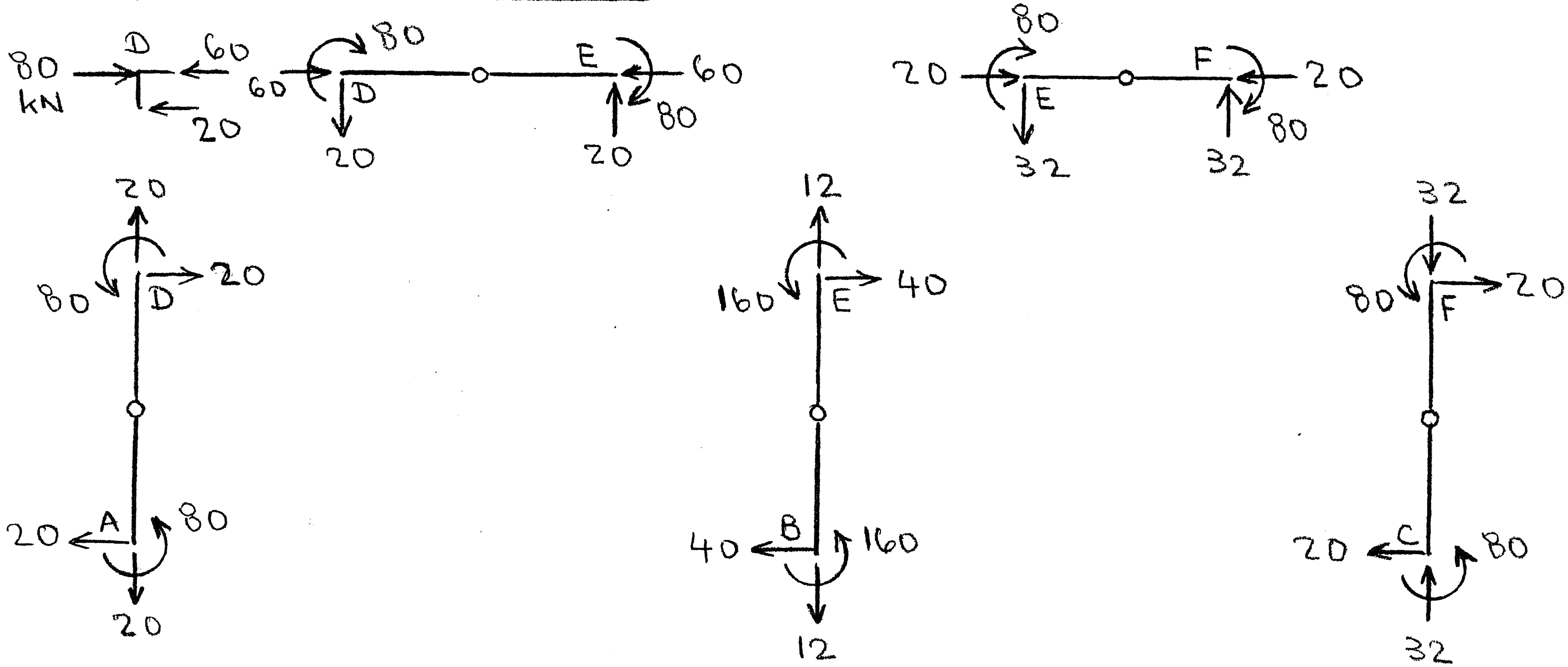
Member Axial Forces, Shears, and Moments

12.8

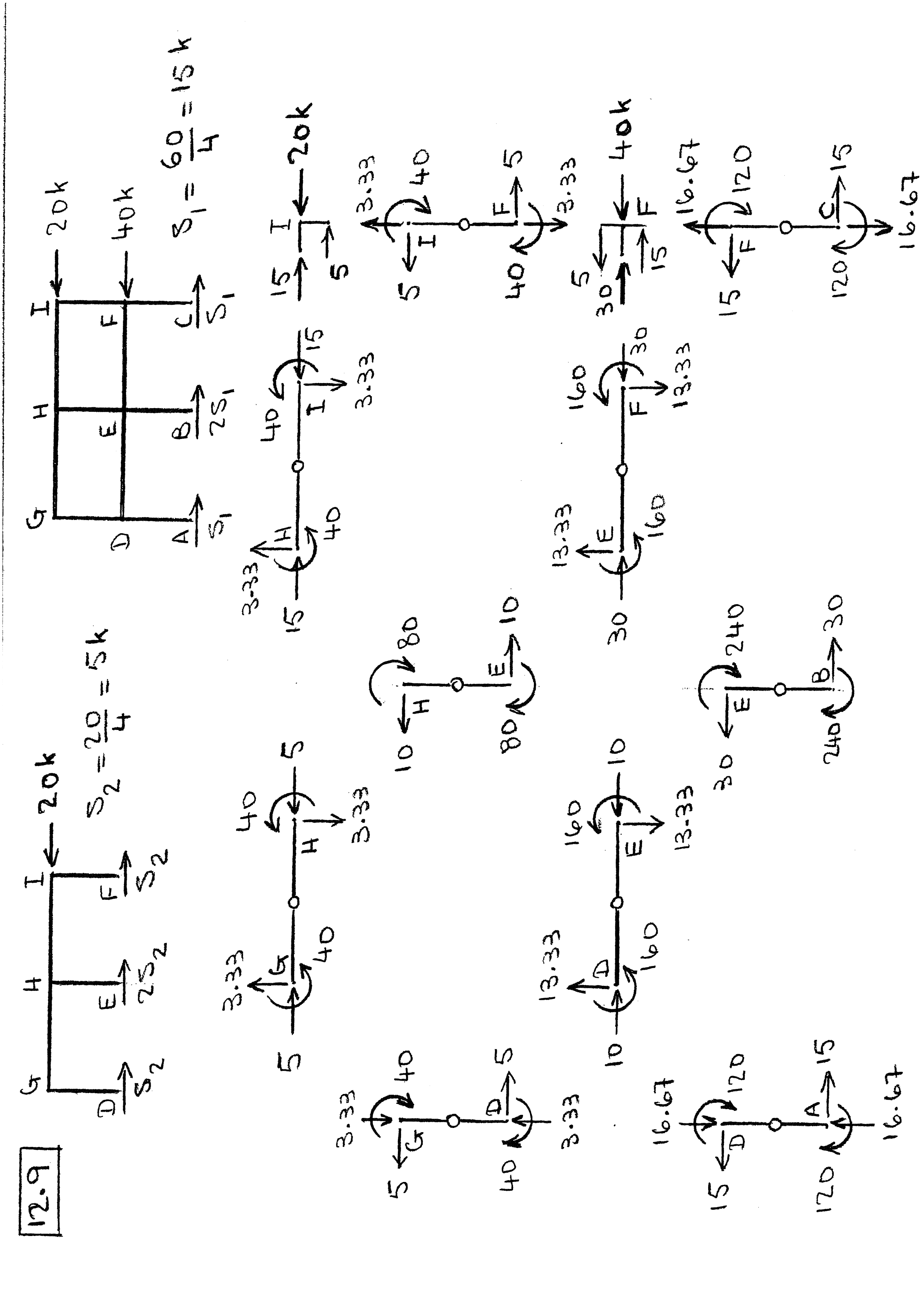


$4S = 80 \qquad S = 20\ \text{kN}$

Column Shears



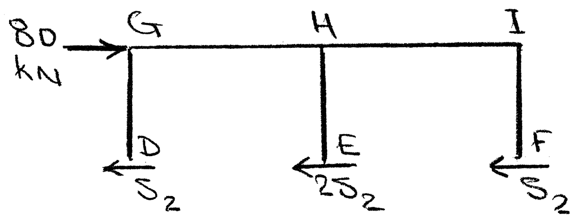
12.9



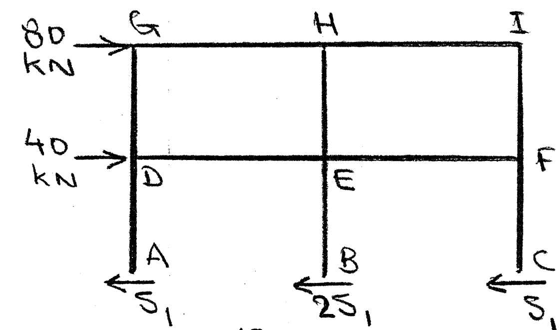
$S_1 = \frac{60}{4} = 15\text{ k}$

$S_2 = \frac{20}{4} = 5\text{ k}$

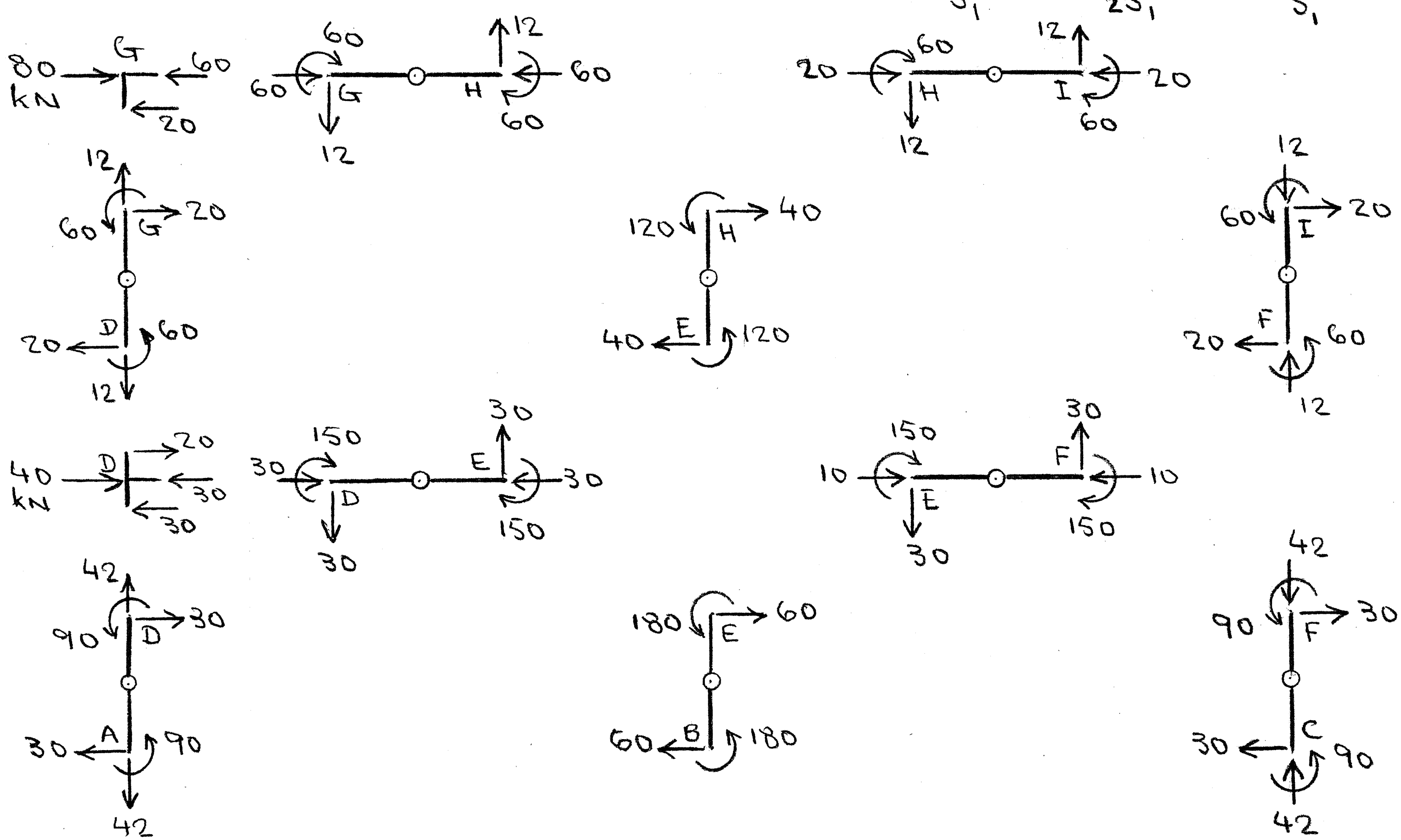
12.10



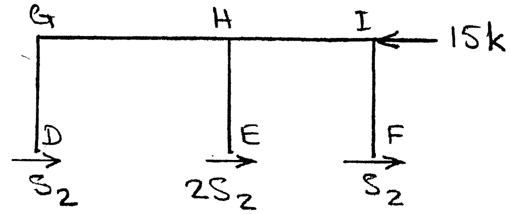
$$S_2 = \frac{80}{4} = 20 \text{ kN}$$



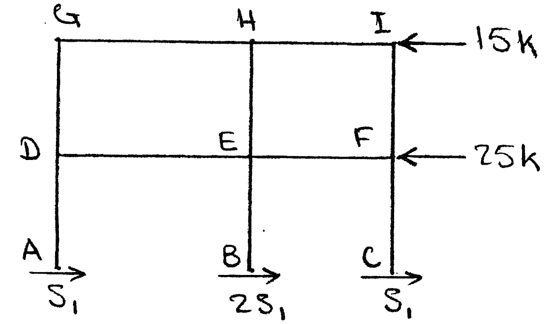
$$S_1 = \frac{(80+40)}{4} = 30 \text{ kN}$$



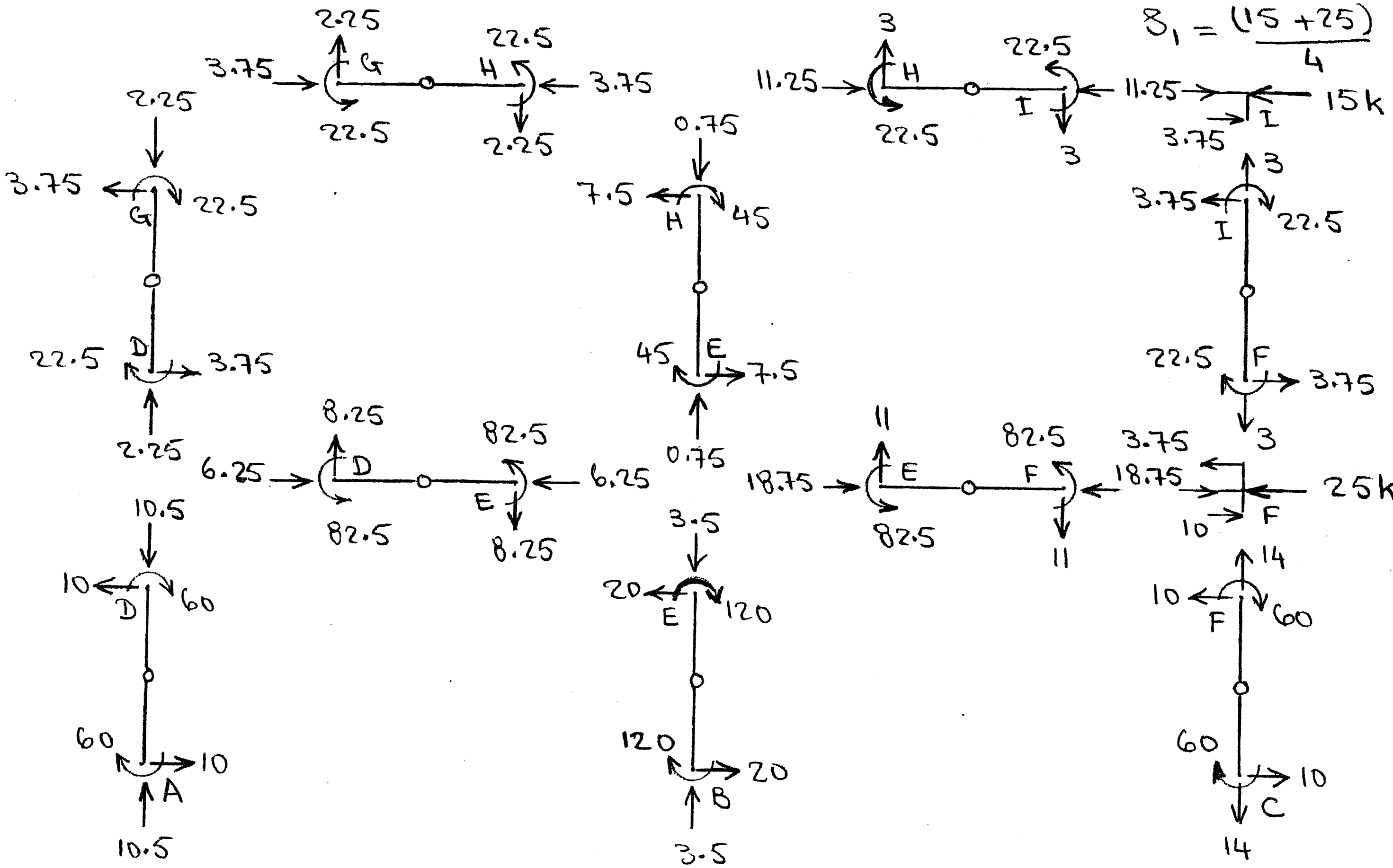
12.11



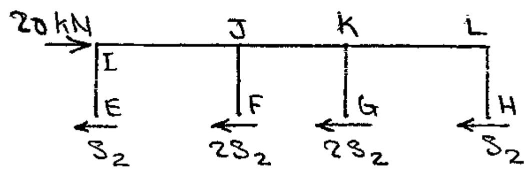
$$S_2 = \frac{15}{4} = 3.75\ k$$



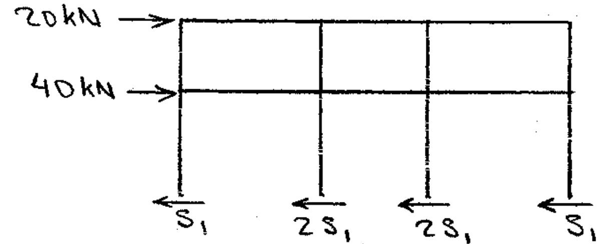
$$S_1 = \frac{(15+25)}{4} = 10\ k$$



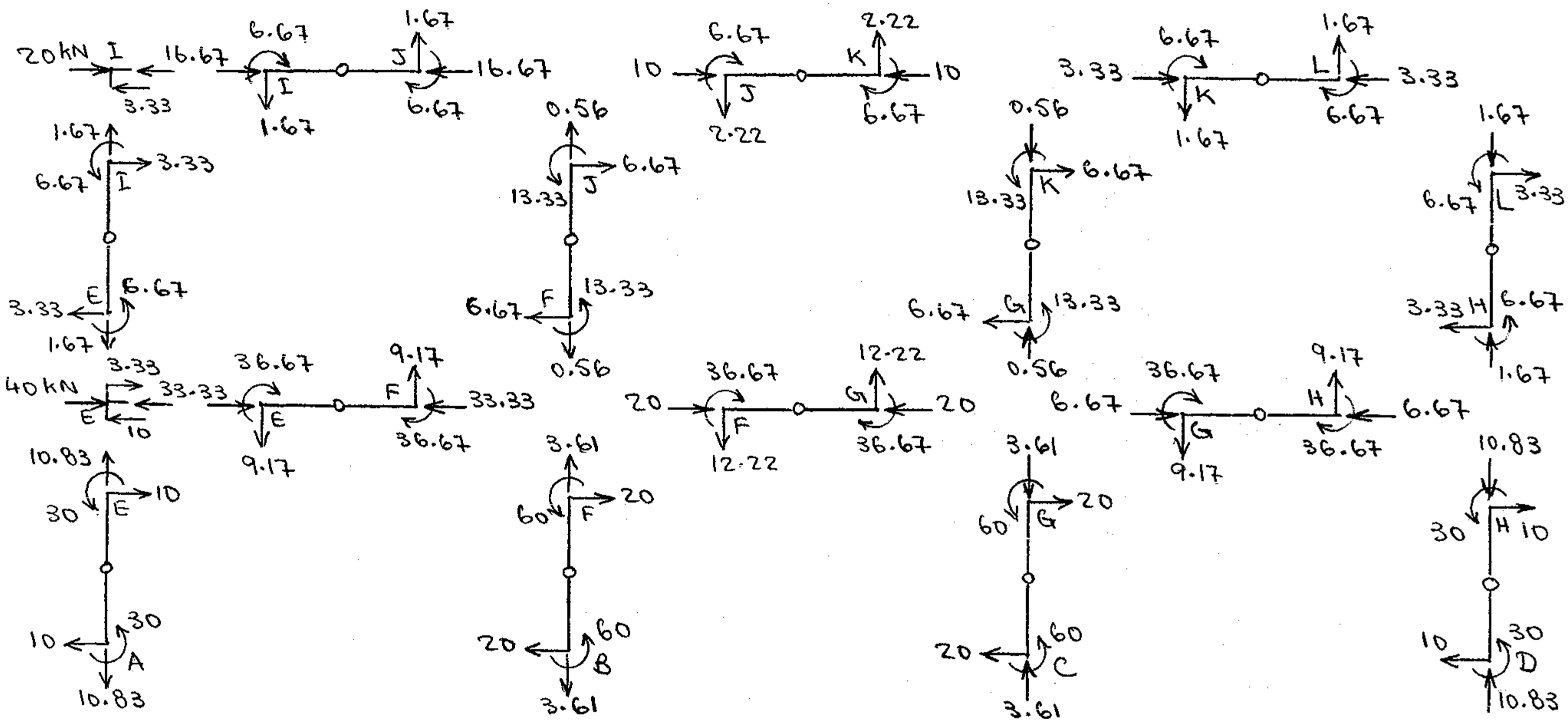
12-12



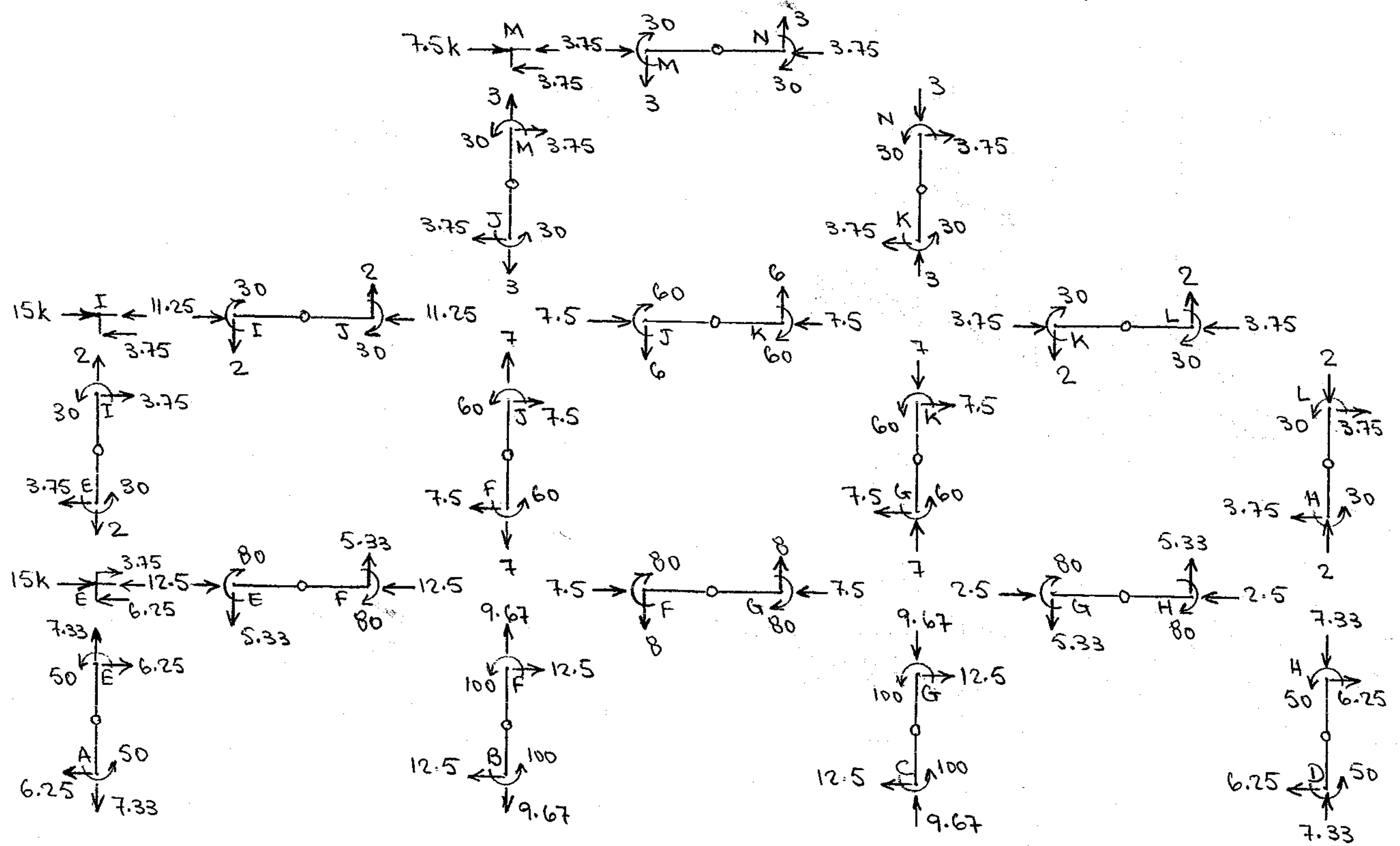
$S_2 = \frac{20}{6} = 3.33$ kN



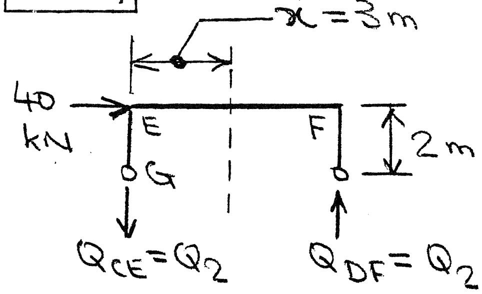
$S_1 = \frac{(20+40)}{6} = 10$ kN



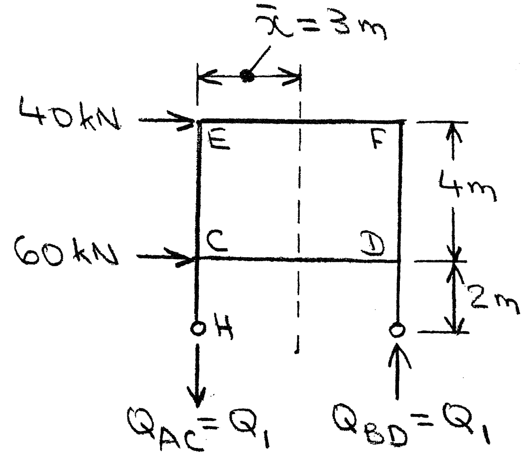
12.13 $S_3 = \frac{7.5}{2} = 3.75\text{k}$; $S_2 = \frac{(7.5+15)}{6} = 3.75\text{k}$; $S_1 = \frac{(7.5+15+15)}{6} = 6.25\text{k}$



12.14

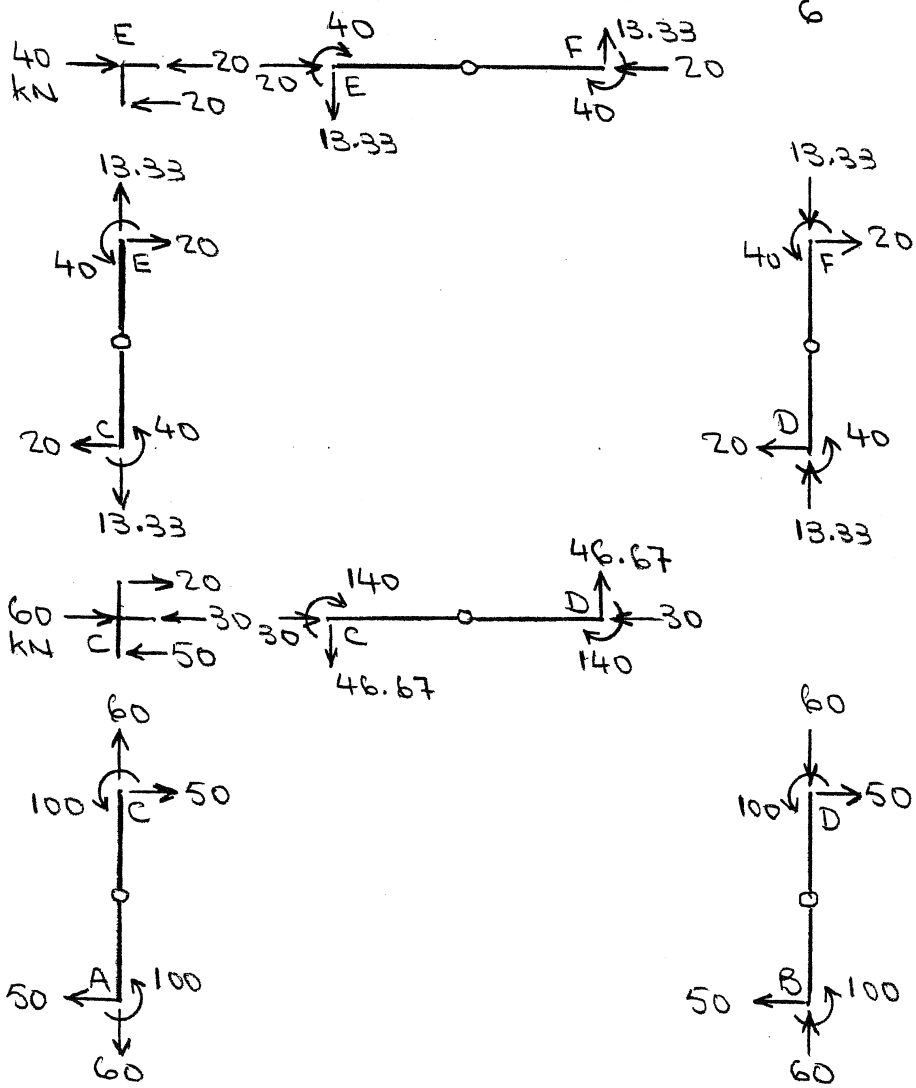


$\Sigma M_G = 0 \quad Q_2 = \frac{40(2)}{6} = 13.33 \text{ kN}$

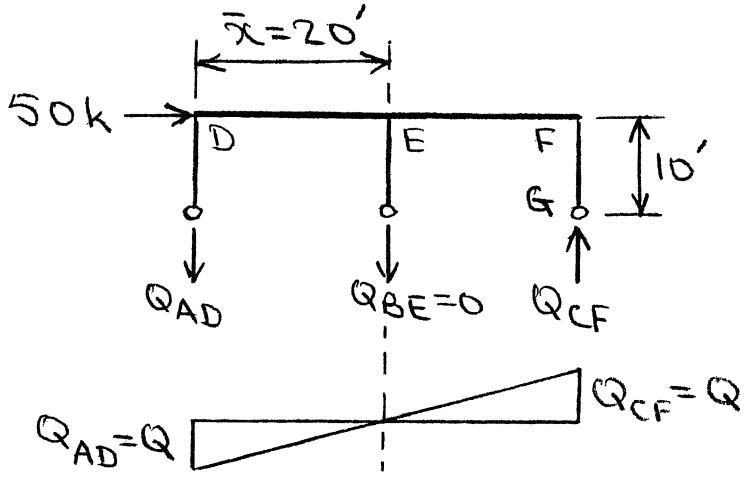


$\Sigma M_H = 0$

$Q_1 = \frac{40(6) + 60(2)}{6} = 60 \text{ kN}$



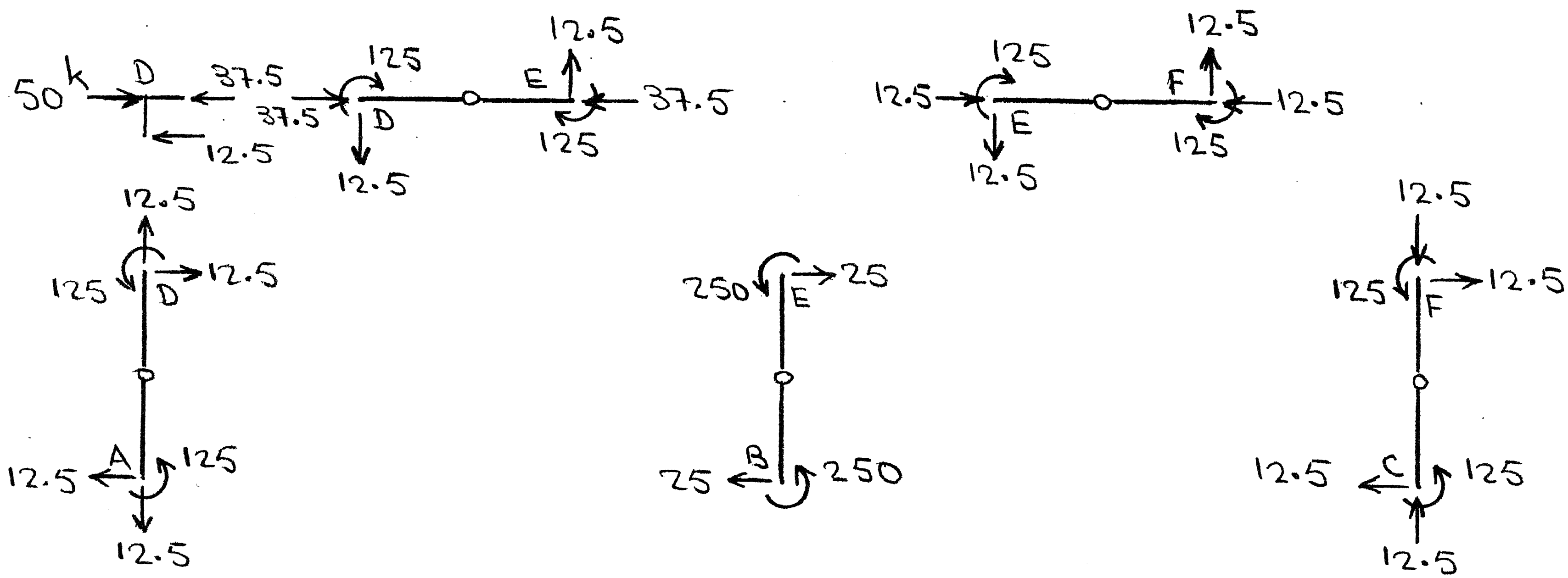
12.15



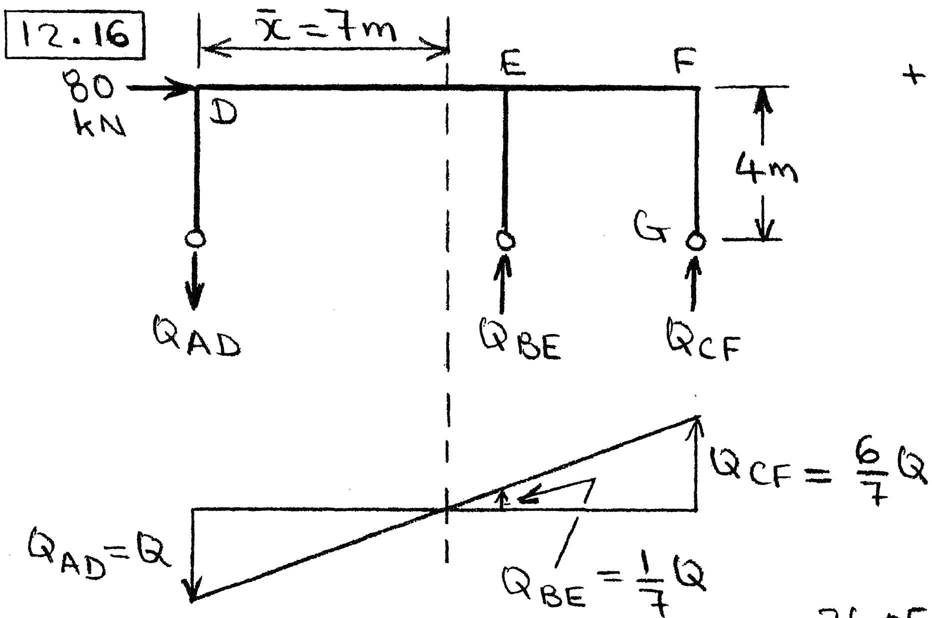
$\circlearrowleft \Sigma M_G = 0$

$-50(10) + Q(40) = 0$

$Q = 12.5\ k$



12.16



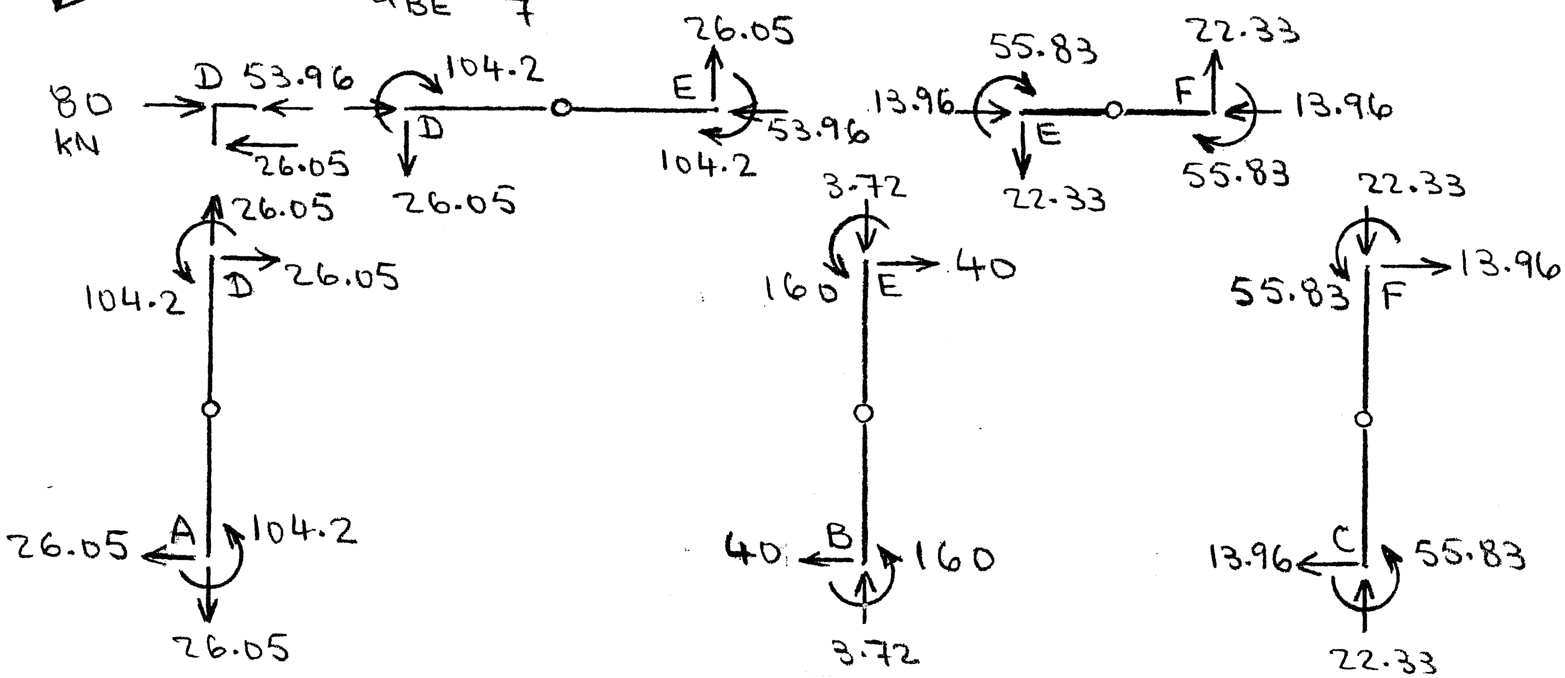
$+\circlearrowleft \Sigma M_G = 0$

$-80(4) + Q(13) - \frac{1}{7}Q(5) = 0$

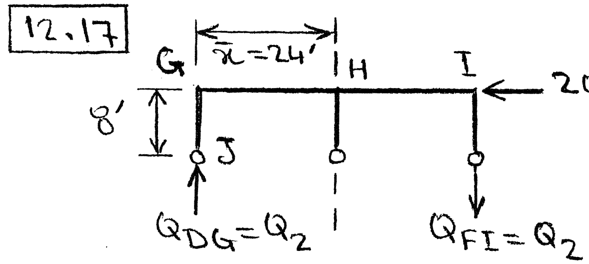
$Q = 26.05 \text{ kN}$

$Q_{AD} = 26.05 \text{ kN}, \quad Q_{BE} = 3.72 \text{ kN}$

$Q_{CF} = 22.33 \text{ kN}$



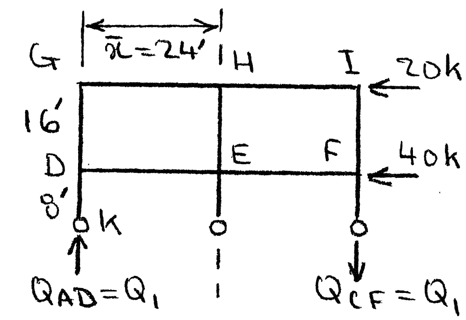
12.17



$\Sigma M_J = 0$

$Q_2 = \dfrac{20(8)}{48}$

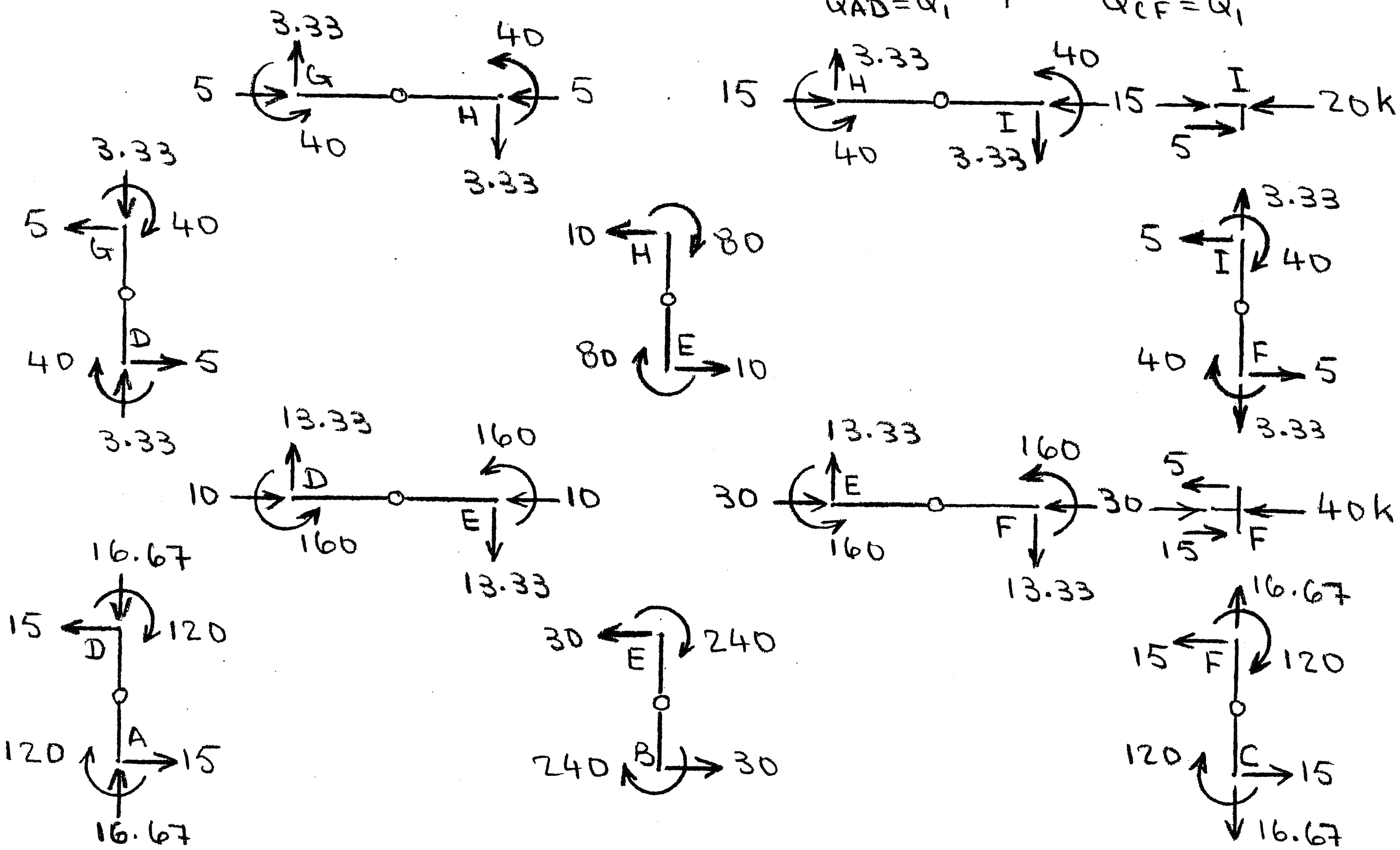
$= 3.33k$



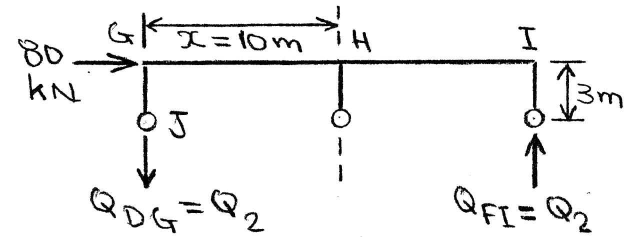
$\Sigma M_K = 0$

$Q_1 = \dfrac{20(24)+40(8)}{48}$

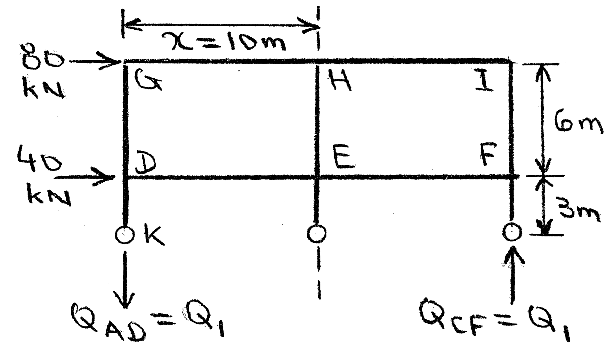
$= 16.67k$



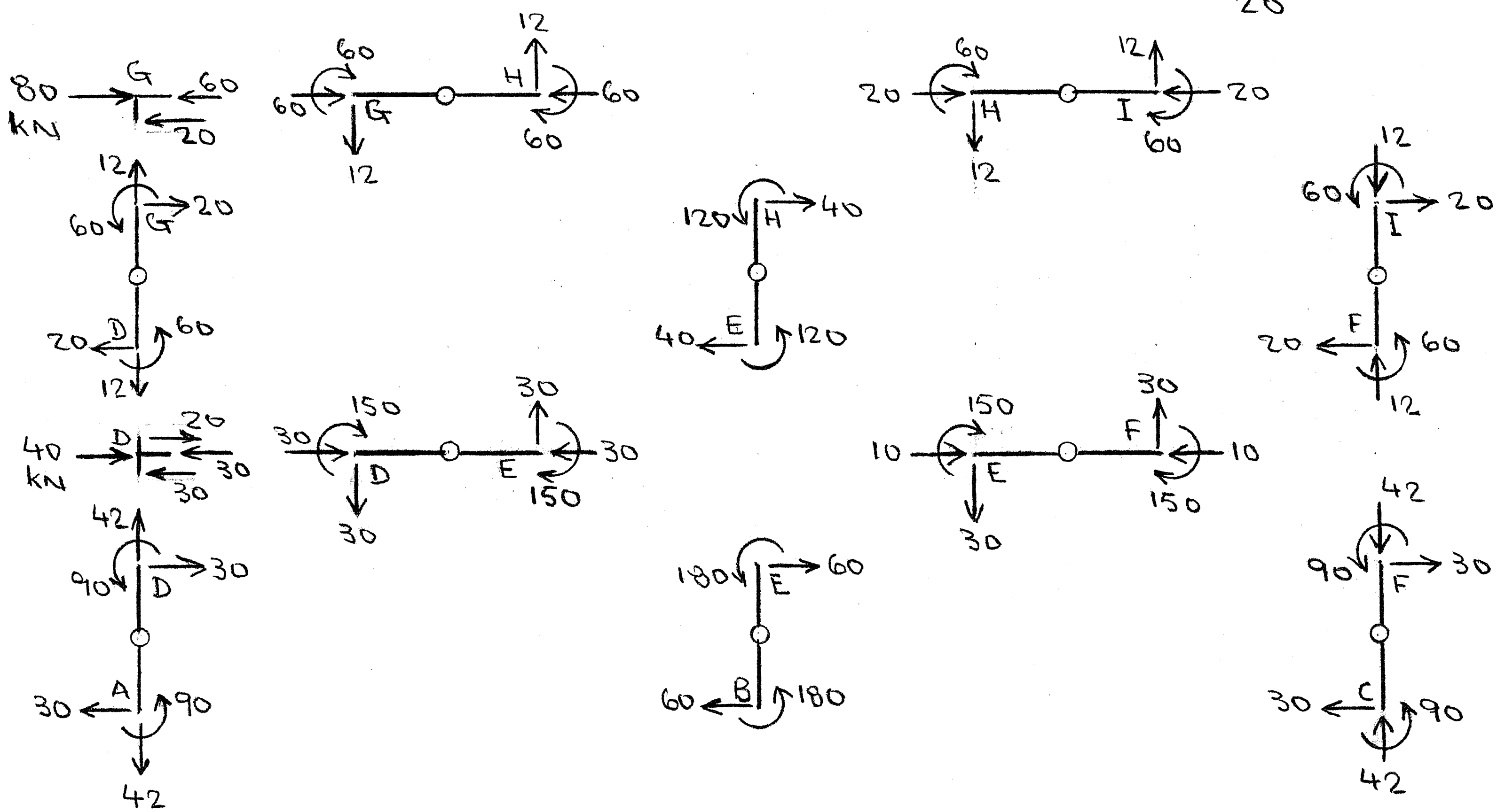
12.18



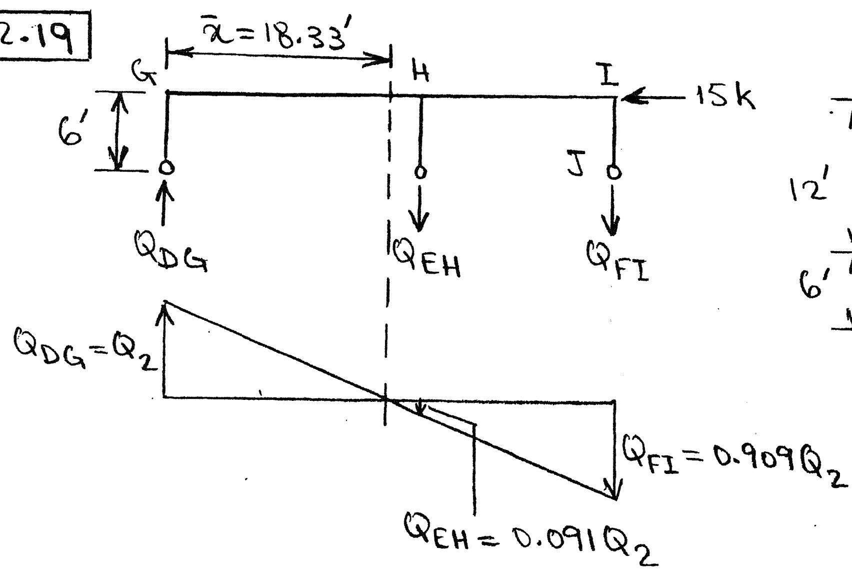
$\Sigma M_J = 0 \quad Q_2 = \dfrac{80(3)}{20} = 12 \text{ kN}$

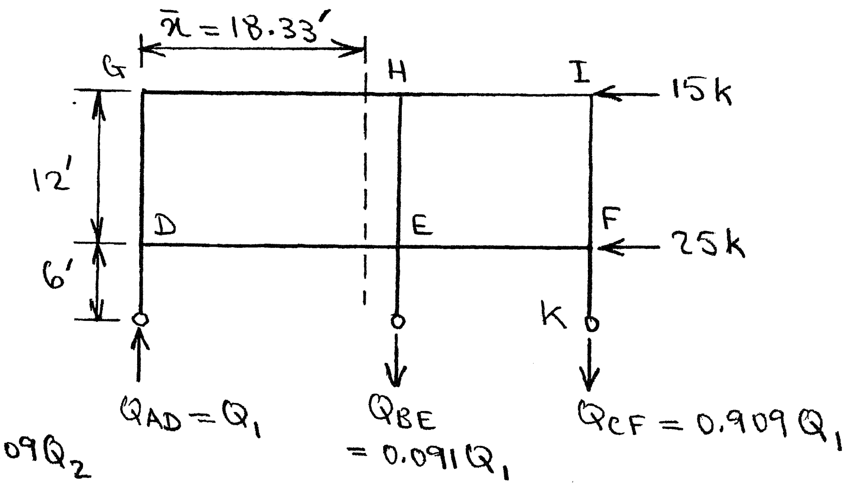


$\Sigma M_K = 0 \quad Q_1 = \dfrac{80(9) + 40(3)}{20} = 42 \text{ kN}$



12.19





$$+\circlearrowleft \sum M_J = 0$$

$$15(6) - Q_2(35) + 0.091Q_2(20) = 0$$

$$Q_2 = 2.71\text{k}$$

$$Q_{DG} = 2.71\text{k}, \; Q_{EH} = 0.247\text{k},$$

$$Q_{FI} = 2.46\text{k}$$

$$+\circlearrowleft \sum M_K = 0$$

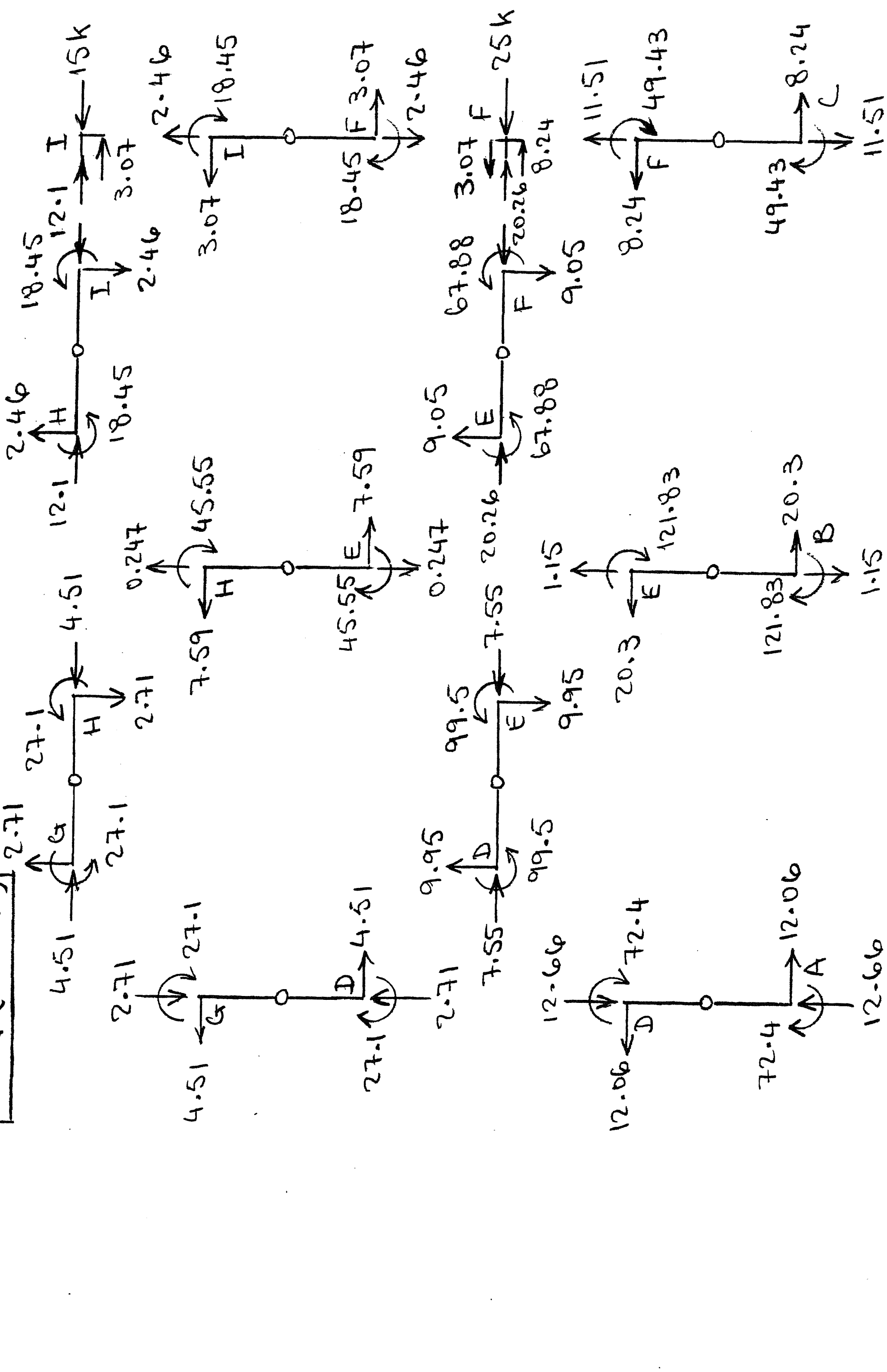
$$15(18) + 25(6) - Q_1(35) + 0.091Q_1(20) = 0$$

$$Q_1 = 12.66\text{k}$$

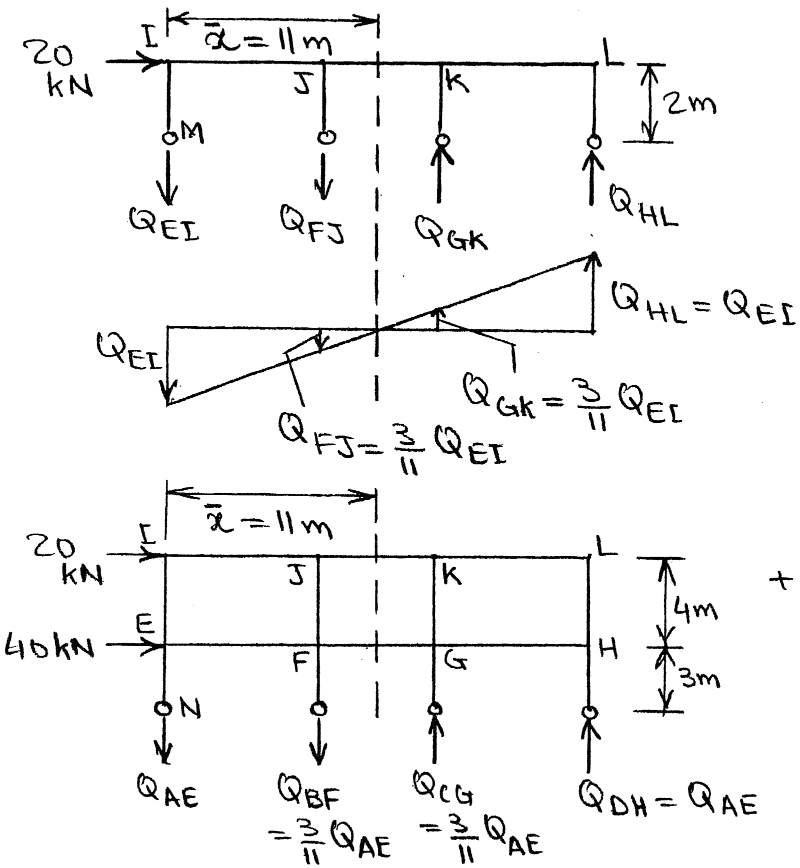
$$Q_{AD} = 12.66\text{k}, \; Q_{BE} = 1.15\text{k}$$

$$Q_{CF} = 11.51\text{k}$$

12.19 (Contd.)



12.20



$+\circlearrowleft \Sigma M_M = 0$

$$-20(2) - \frac{3}{11} Q_{EI}(8) + \frac{3}{11} Q_{EI}(14) + Q_{EI}(22) = 0$$

$Q_{EI} = Q_{HL} = 1.69$ kN

$Q_{FJ} = Q_{GK} = \frac{3}{11}(1.69) = 0.46$ kN

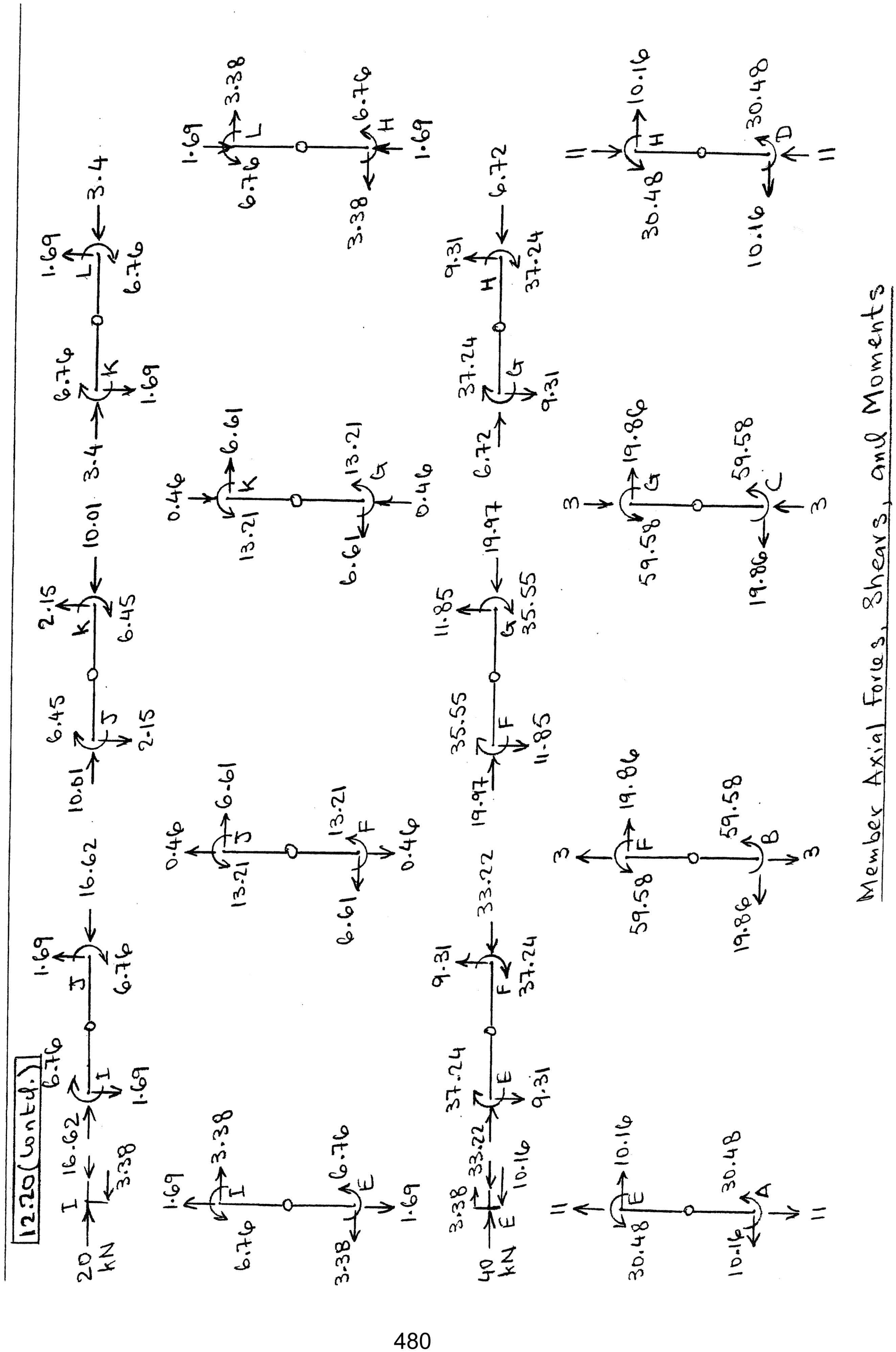
$+\circlearrowleft \Sigma M_N = 0$

$$-20(7) - 40(3) - \frac{3}{11} Q_{AE}(8) + \frac{3}{11} Q_{AE}(14) + Q_{AE}(22) = 0$$

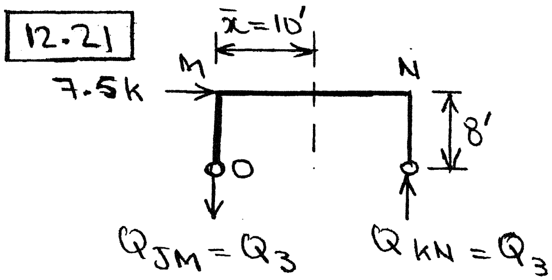
$Q_{AE} = Q_{DH} = 11$ kN

$Q_{BF} = Q_{CG} = 3$ kN

Column Axial Forces


12.20 (Contd.)
Member Axial Forces, Shears, and Moments

12.21

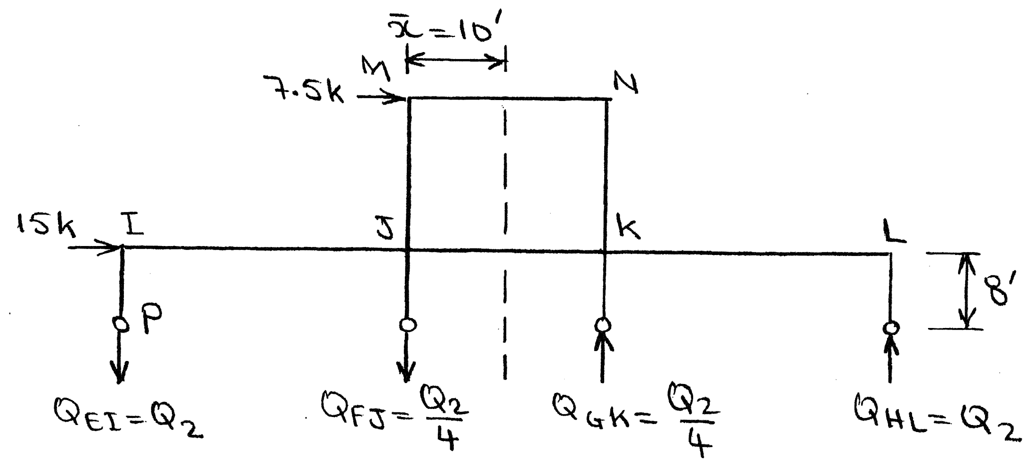


$+\circlearrowright \Sigma M_O = 0$

$-7.5(8) + Q_3(20) = 0$

$Q_3 = 3\,k$

$Q_{JM} = Q_{KN} = 3\,k$

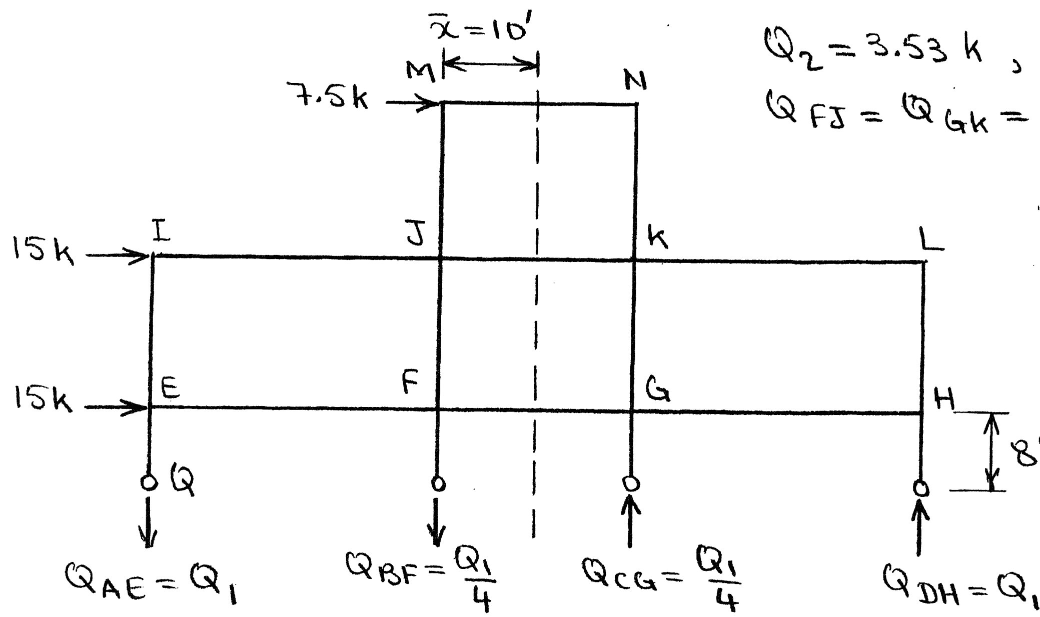


$+\circlearrowright \Sigma M_P = 0$

$-7.5(24) - 15(8) - \frac{Q_2}{4}(30) + \frac{Q_2}{4}(50) + Q_2(80) = 0$

$Q_2 = 3.53\,k$, $\quad Q_{EI} = Q_{HL} = 3.53\,k$

$Q_{FJ} = Q_{GK} = 0.882\,k$



$+\circlearrowright \Sigma M_Q = 0$

$-7.5(40) - 15(24) - 15(8) - \frac{Q_1}{4}(30)$

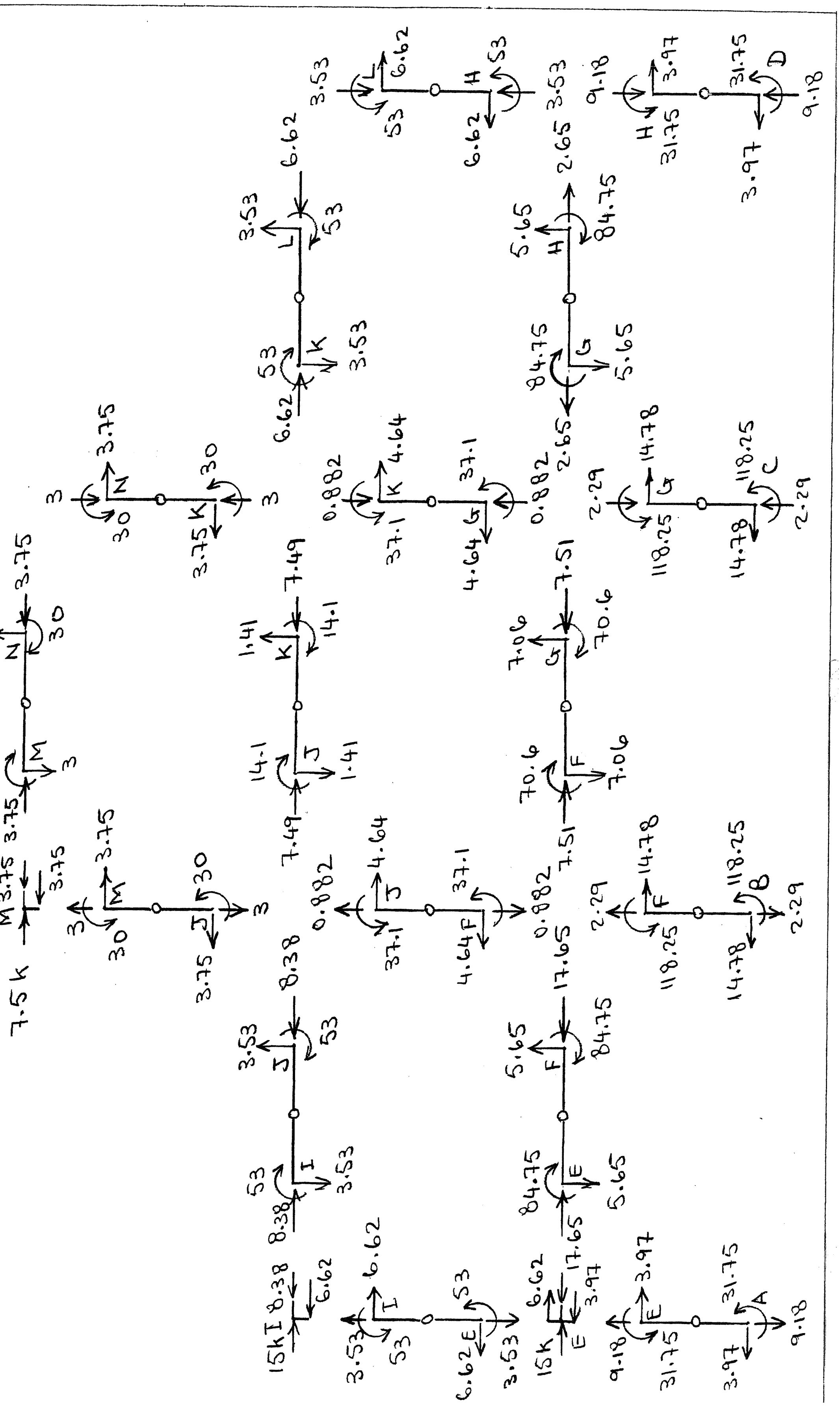
$+ \frac{Q_1}{4}(50) + Q_1(80) = 0$

$Q_1 = 9.18\,k$

$Q_{AE} = Q_{DE} = 9.18\,k$

$Q_{BF} = Q_{CG} = 2.29\,k$

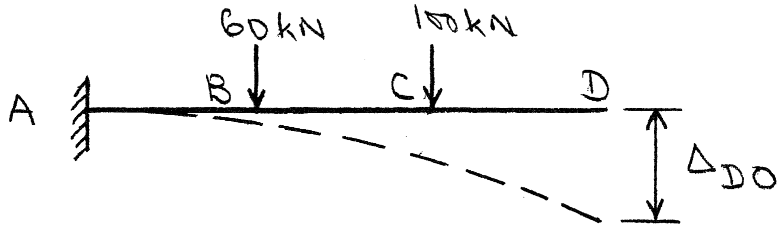
12.21 (Contd.)

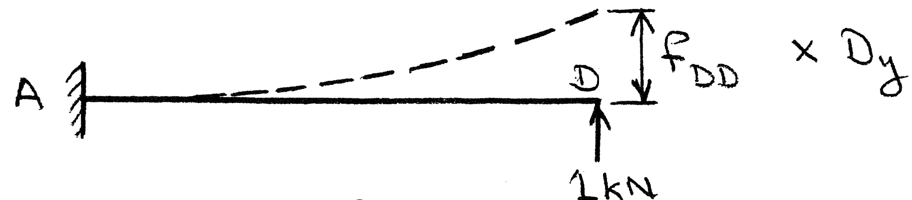


Chapter Thirteen
Method of Consistent Deformations – Force Method

CHAPTER 13

13.1



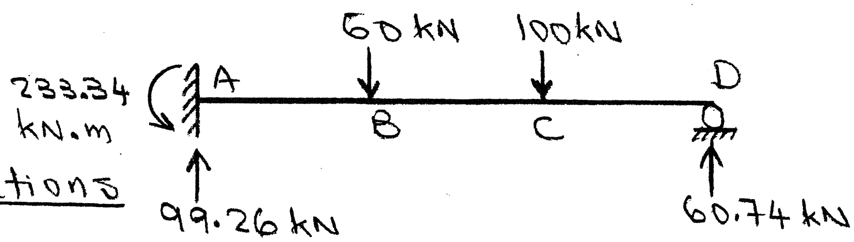


Using beam deflection formulas:

$$\Delta_{DO} = -\frac{14760 \text{ kN}\cdot\text{m}^3}{EI} \qquad f_{DD} = \frac{243 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

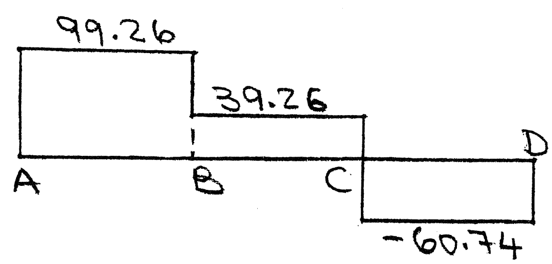
Compatibility Equation:

$$\Delta_{DO} + f_{DD} D_y = 0 \qquad D_y = \frac{14760}{243} = 60.74 \text{ kN} \uparrow$$

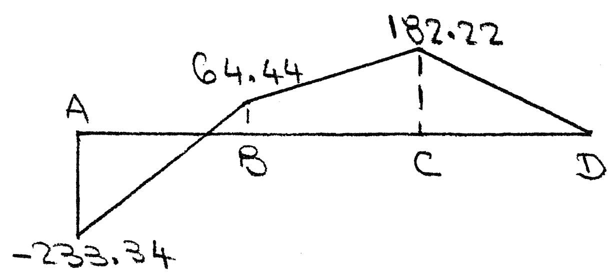


Reactions

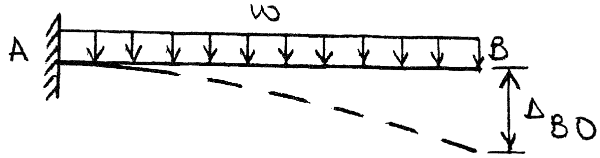
Shear Diagram (kN)



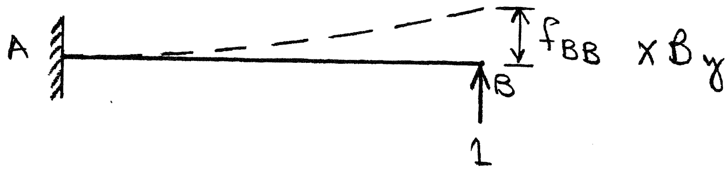
Bending Moment Diagram (kN.m)



13.2



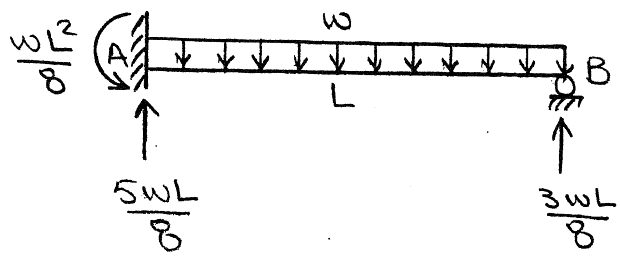
$$\Delta_{BO} = -\frac{wL^4}{8EI}$$



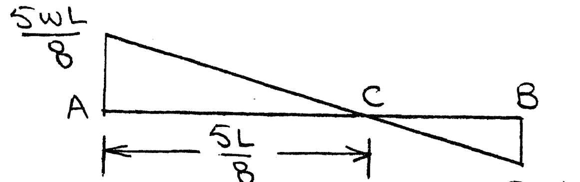
$f_{BB} \times B_y$

$$f_{BB} = \frac{L^3}{3EI}$$

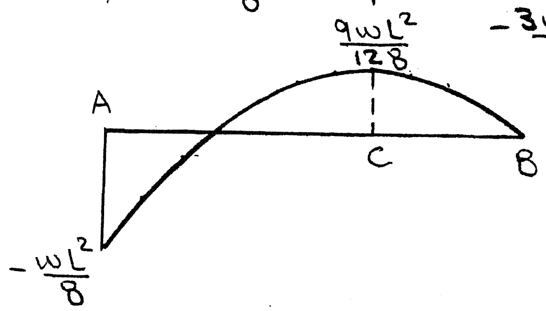
$$\Delta_{BO} + f_{BB} B_y = 0 \qquad B_y = \left(\frac{wL^4}{8EI}\right)\frac{3EI}{L^3} = \frac{3wL}{8}\uparrow$$



Reactions



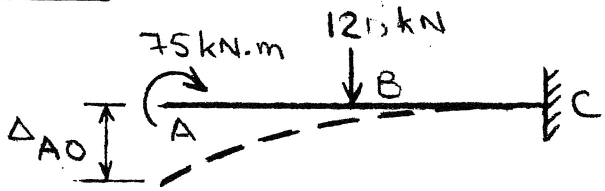
Shear Diagram



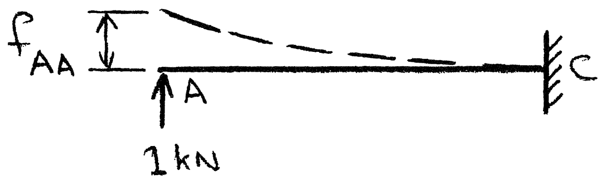
Bending Moment Diagram

13.3

Using beam deflection formulas:



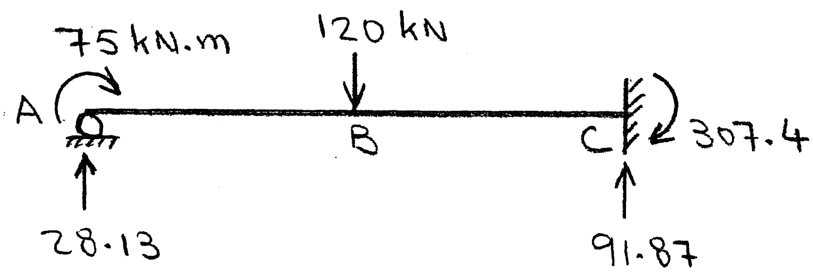
$$\Delta_{AO} = \frac{5400}{EI} - \frac{21600}{EI} = -\frac{16200 \text{ kN.m}^3}{EI}$$



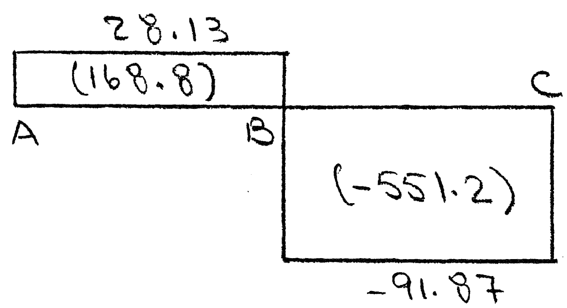
$$f_{AA} = \frac{576 \text{ kN.m}^3/\text{kN}}{EI}$$

$$\Delta_{AO} + f_{AA} A_y = 0 \qquad A_y = \frac{16200}{576} = \underline{28.13 \text{ kN} \uparrow}$$

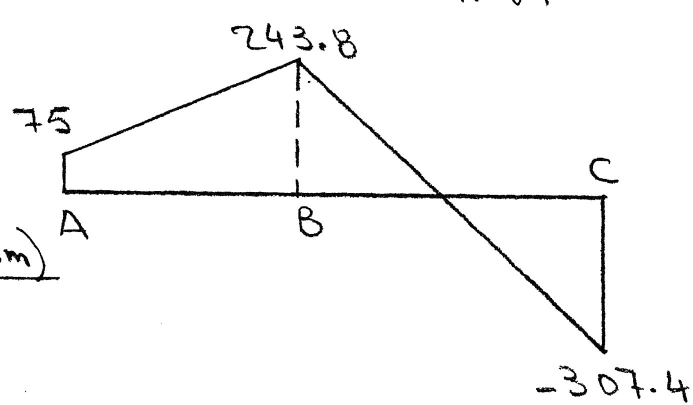
Reactions



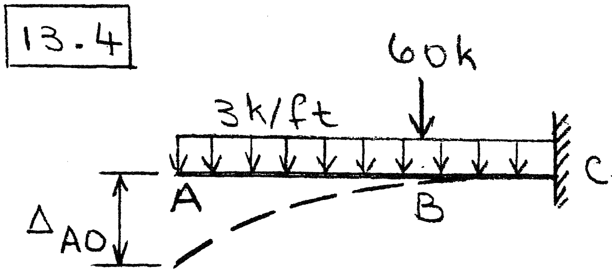
Shear Diagram (kN)



Bending Moment Diagram (kN.m)



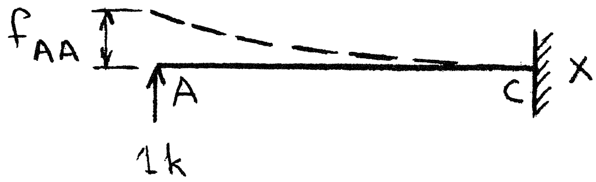
13.4



Using beam deflection formulas:

$$\Delta_{AO} = -\frac{303750}{EI} - \frac{80000}{EI}$$

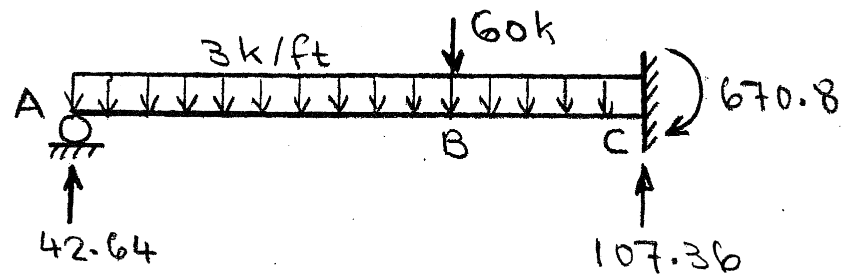
$$= -\frac{383750 \text{ k-ft}^3}{EI}$$



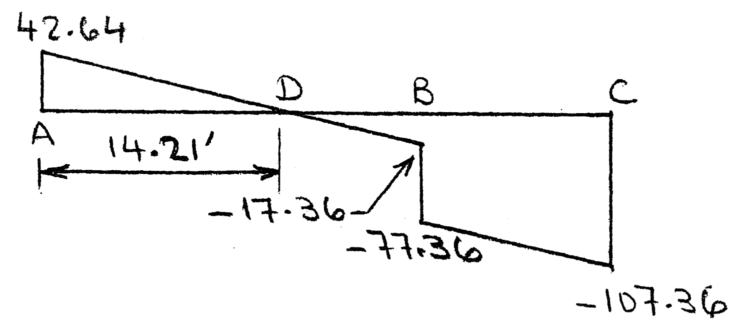
$$f_{AA} = \frac{9000 \text{ k-ft}^3/\text{k}}{EI}$$

$$\Delta_{AO} + f_{AA} A_y = 0 \qquad A_y = \frac{383750}{9000} = \underline{42.64\text{k} \uparrow}$$

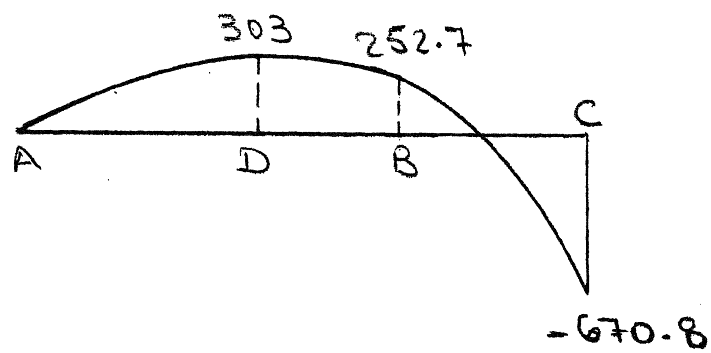
<u>Reactions</u>



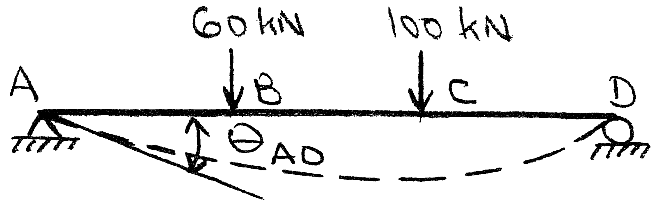
<u>Shear Diagram (k)</u>

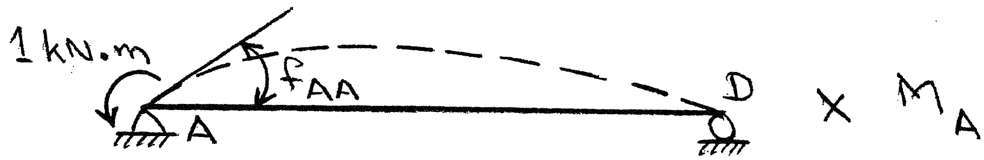


<u>Bending Moment Diagram (k-ft)</u>



13.5





Using beam deflection formulas:

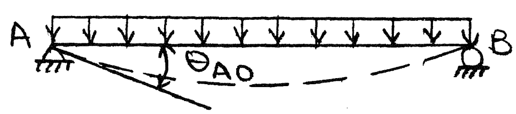
$$\theta_{AD} = -\frac{700 \text{ kN.m}^2}{EI} \qquad f_{AA} = \frac{3 \text{ kN.m}^2/\text{kN.m}}{EI}$$

Compatibility Equations:

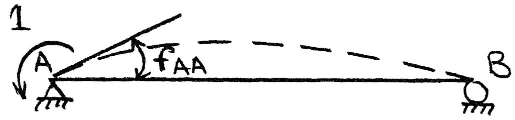
$$\theta_{AD} + f_{AA} M_A = 0 \qquad M_A = \frac{700}{3} = \underline{233.33 \text{ kN.m}}$$

For reactions, and shear and moment diagrams, see solution of Problem 13.1.

13.6



$$\theta_{AO} = -\frac{wL^3}{24EI}$$



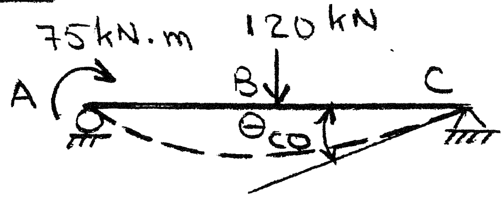
$\times M_A$

$$f_{AA} = \frac{L}{3EI}$$

$$\theta_{AO} + f_{AA} M_A = 0 \qquad M_A = \frac{wL^3}{24EI}\left(\frac{3EI}{L}\right) = \frac{wL^2}{8} \circlearrowleft$$

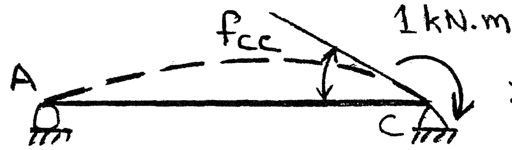
For reactions, and shear and moment diagrams, see solution of Problem 13.2.

13.7



Using beam deflection formulas:

$$\theta_{CO} = -\frac{150}{EI} - \frac{1080}{EI}$$
$$= -\frac{1230 \text{ kN.m}^2}{EI}$$

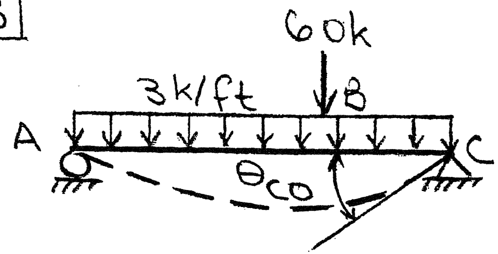


$\times M_C$ $\quad f_{CC} = \frac{4 \text{ kN.m}^2/\text{kN.m}}{EI}$

$$\theta_{CO} + f_{CC} M_C = 0 \qquad M_C = \frac{1230}{4} = \underline{307.5 \text{ kN.m}}$$

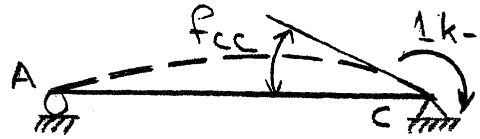
For reactions, and shear and moment diagrams, see solution of Problem 13.3.

13.8



Using beam deflection formula:

$$\theta_{CO} = -\frac{3375}{EI} - \frac{3333}{EI} = -\frac{6708 \text{ k-ft}^2}{EI}$$

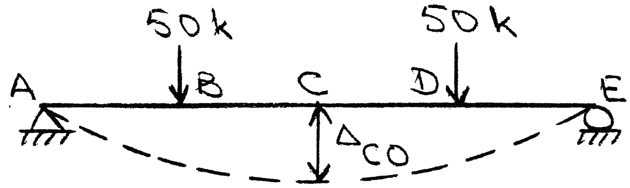


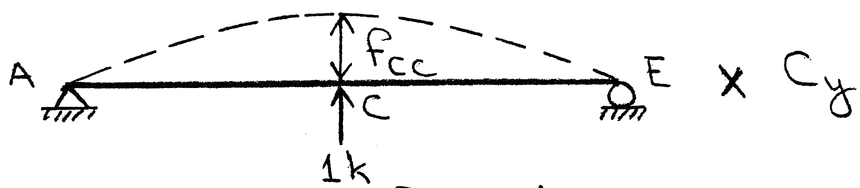
$\times M_C$ $\quad f_{CC} = \frac{10 \text{ k-ft}^2/\text{k-ft}}{EI}$

$$\theta_{CO} + f_{CC} M_C = 0 \qquad M_C = \frac{6708}{10} = \underline{670.8 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.4.

13.9



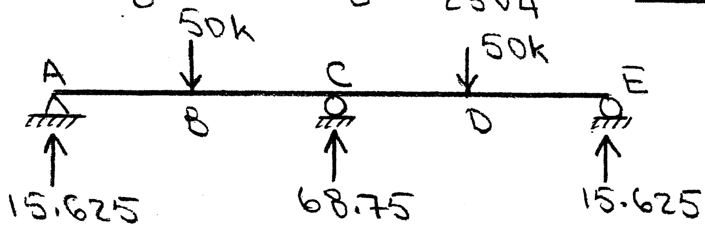


Using beam deflection formulas:

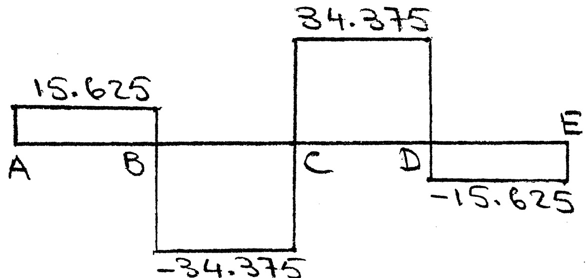
$$\Delta_{CO} = -\frac{158400 \text{ k-ft}^3}{EI} \qquad f_{CC} = \frac{2304 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

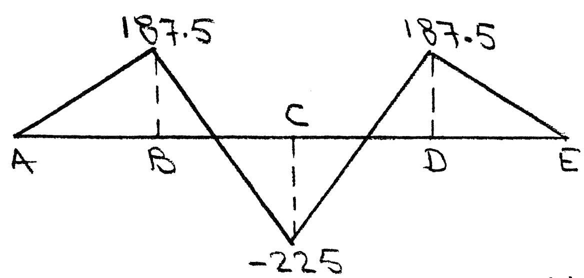
$$\Delta_{CO} + f_{CC} C_y = 0 \qquad C_y = \frac{158400}{2304} = 68.75 \text{ k} \uparrow$$



Reactions

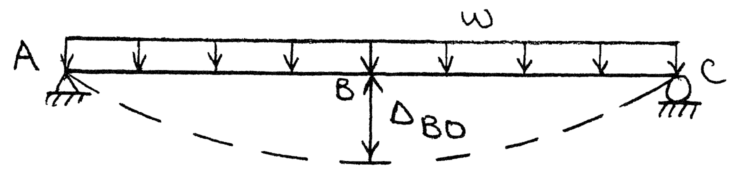


Shear Diagram (k)

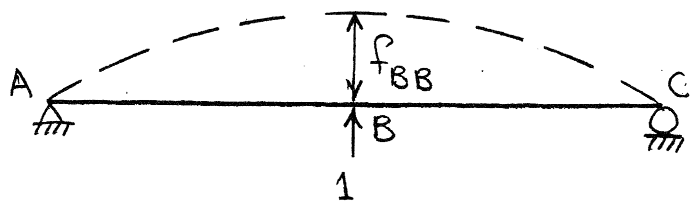


Bending Moment Diagram (k-ft)

13.10

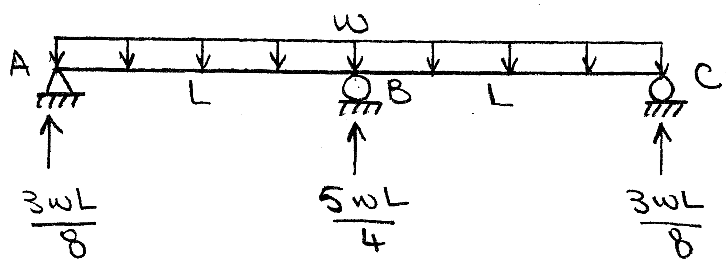


$$\Delta_{BO} = -\frac{5wL^4}{24EI}$$

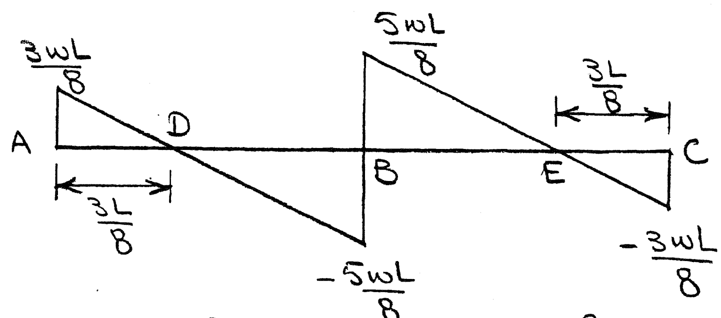


$\times B_y$ $\qquad f_{BB} = \frac{L^3}{6EI}$

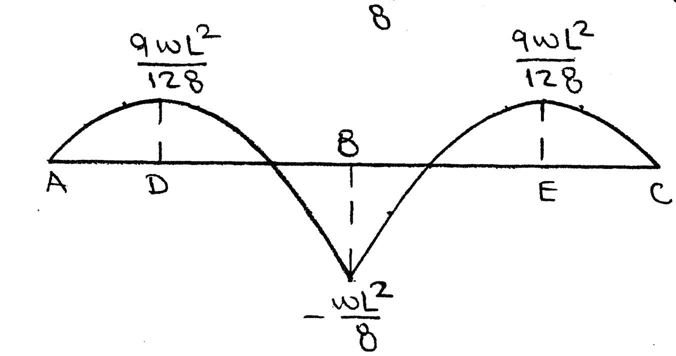
$$\Delta_{BO} + f_{BB} B_y = 0 \qquad B_y = \frac{5wL^4}{24EI}\left(\frac{6EI}{L^3}\right) = \frac{5wL}{4} \uparrow$$



Reactions

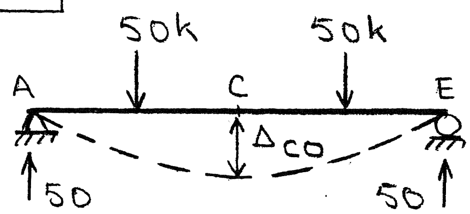


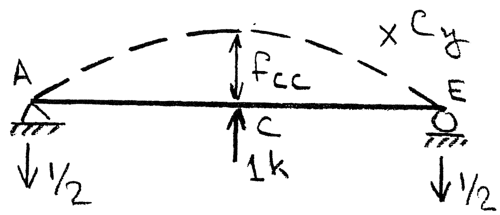
Shear Diagram

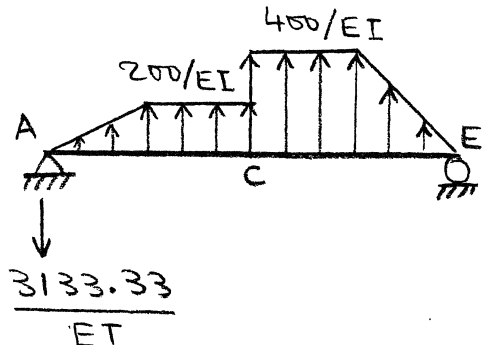


Bending Moment Diagram

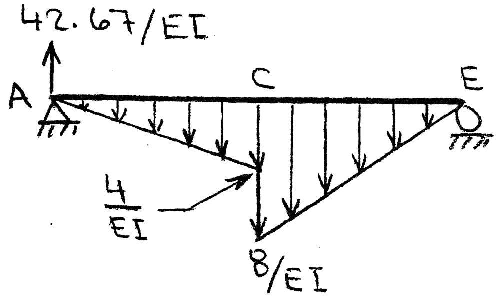
13.11







Conjugate Beam for External Loading



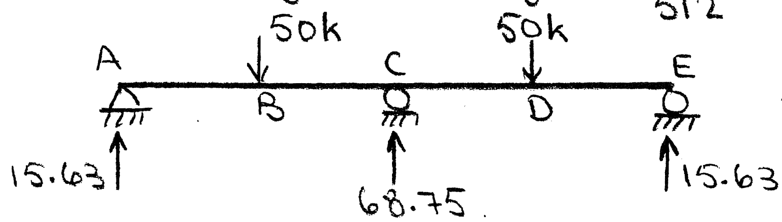
Conjugate Beam for Unit Value of Redundant C_y

$$\Delta_{CO} = -\frac{3133.33}{EI}(16) + \frac{1}{2}(8)\left(\frac{200}{EI}\right)\left(\frac{8}{3}+8\right) + \left(\frac{200}{EI}\right)8(4)$$

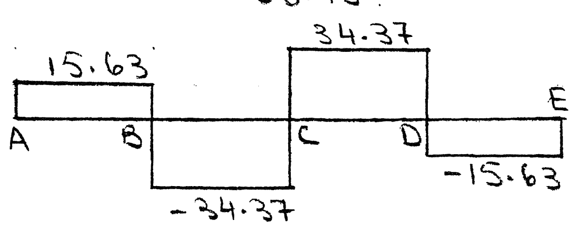
$$= -\frac{35200}{EI}$$

$$f_{CC} = \frac{42.67}{EI}(16) - \frac{1}{2}(16)\left(\frac{4}{EI}\right)\left(\frac{16}{3}\right) = \frac{512}{EI}$$

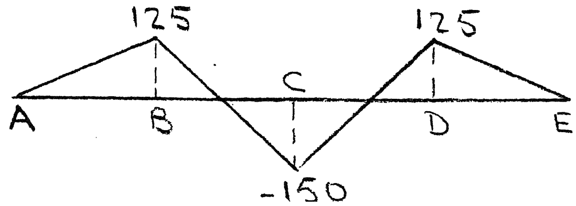
$$\Delta_{CO} + f_{CC}C_y = 0 \qquad C_y = \frac{35200}{512} = 68.75\text{ k}\uparrow$$



Reactions

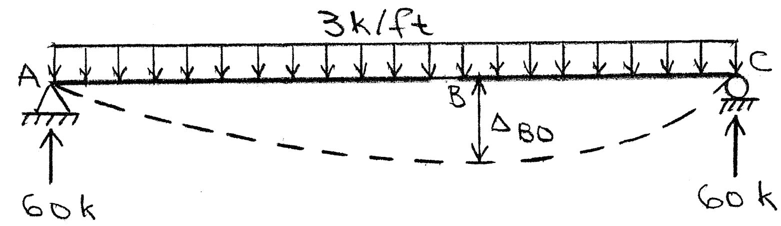


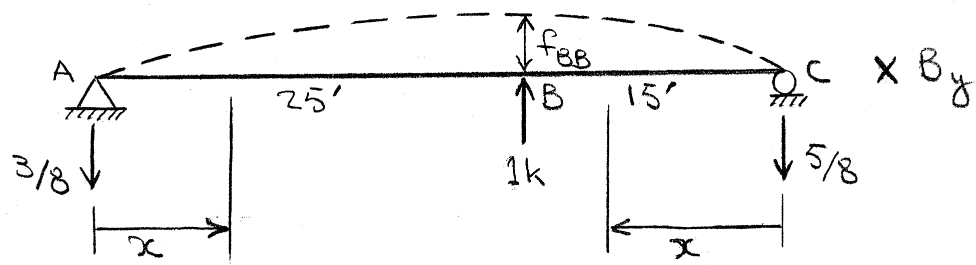
Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.12





Using the virtual work method:

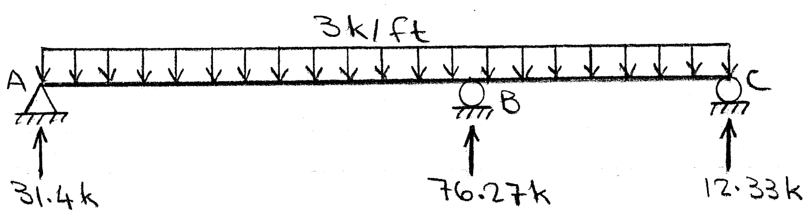
$$\Delta_{BO} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{25}\left(60x - \frac{3x^2}{2}\right)\left(-\frac{3x}{8}\right)dx + \int_0^{15}\left(60x - \frac{3x^2}{2}\right)\left(-\frac{5x}{8}\right)dx\right]$$

$$= -\frac{61450.2 \text{ k-ft}^3}{EI}$$

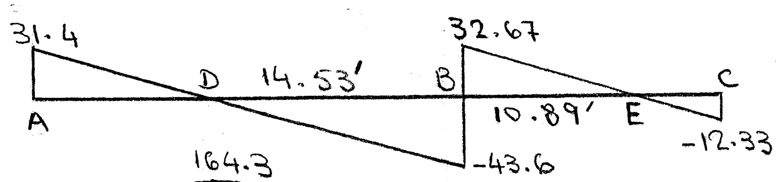
$$f_{BB} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{25}\left(-\frac{3x}{8}\right)^2 dx + \int_0^{15}\left(-\frac{5x}{8}\right)^2 dx\right] = \frac{805.66 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

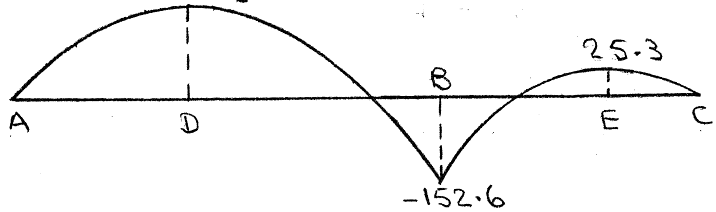
$$\Delta_{BO} + f_{BB} B_y = 0 \qquad B_y = \frac{61450.2}{805.66} = \underline{76.27 \text{ k}\uparrow}$$



Reactions

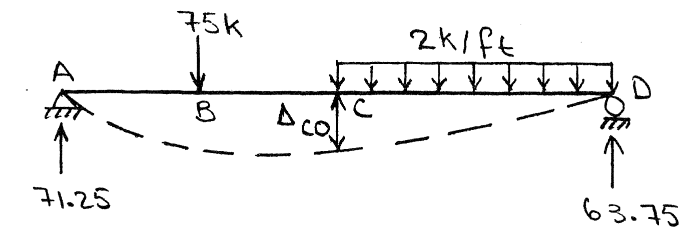


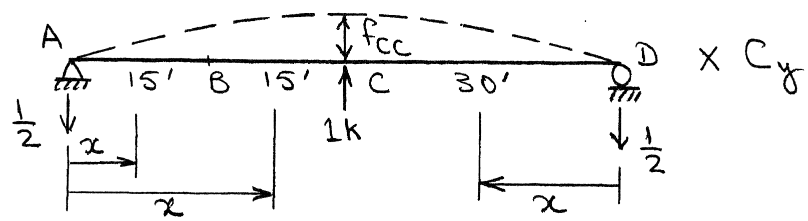
Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.13





Using the method of virtual work:

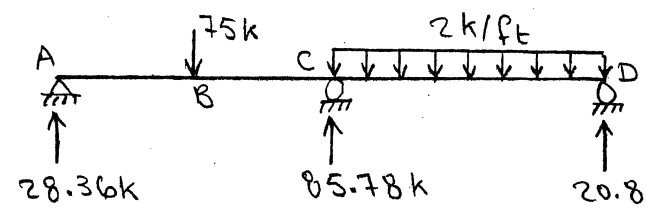
$$\Delta_{CO} = \frac{1}{EI}\left[\frac{1}{3}\int_0^{15}(71.25x)\left(-\frac{x}{2}\right)dx + \frac{1}{3}\int_{15}^{30}\{71.25x - 75(x-15)\}\left(-\frac{x}{2}\right)dx + \int_0^{30}(63.75x - x^2)\left(-\frac{x}{2}\right)dx\right]$$

$$= -\frac{257343.75 \text{ k-ft}^3}{EI}$$

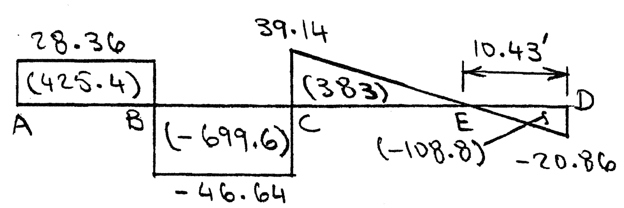
$$f_{CC} = \frac{1}{EI}\left[\frac{1}{3}\int_0^{30}\left(-\frac{x}{2}\right)^2 dx + \int_0^{30}\left(-\frac{x}{2}\right)^2 dx\right] = \frac{3000 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation: $\Delta_{CO} + f_{CC}C_y = 0$

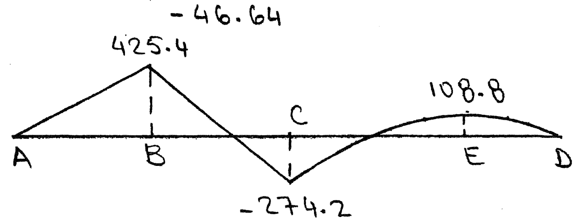
$$C_y = \frac{257343.75}{3000} = \underline{85.78 \text{ k} \uparrow}$$



Reactions

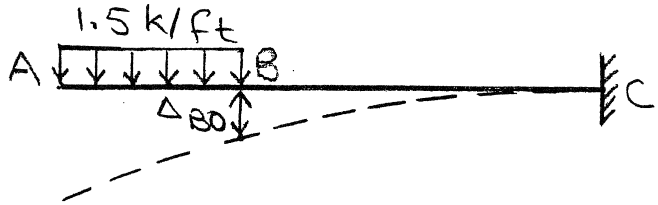


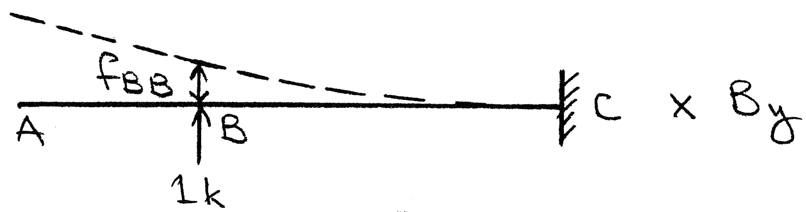
Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.14



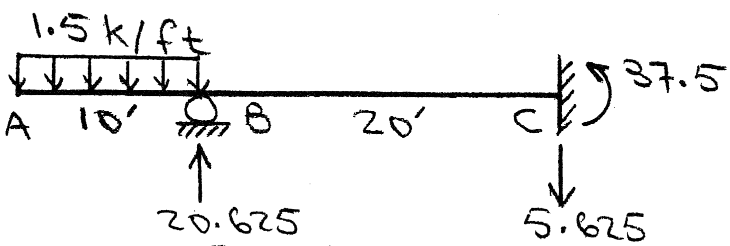


Using beam deflection formulas:

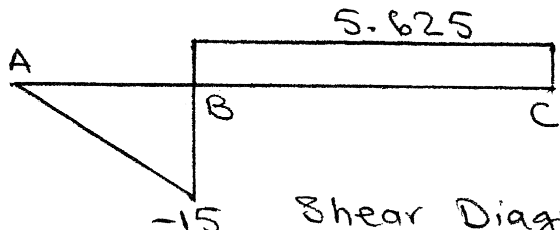
$$\Delta_{BO} = -\frac{55000 \text{ k-ft}^3}{EI} \qquad f_{BB} = \frac{2666.67 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

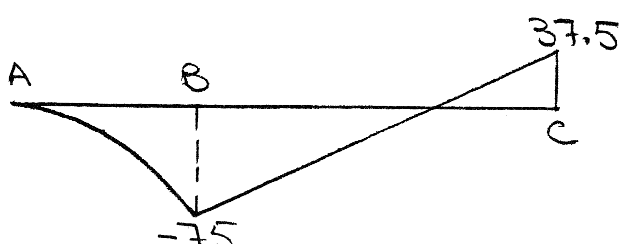
$$\Delta_{BO} + f_{BB} B_y = 0 \qquad B_y = \frac{55000}{2666.67} = \underline{20.625 \text{ k} \uparrow}$$



Reactions

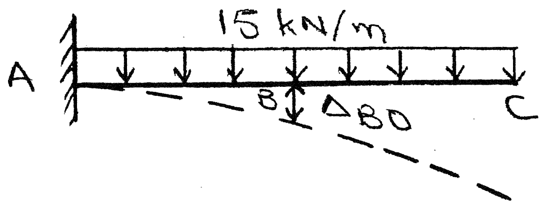


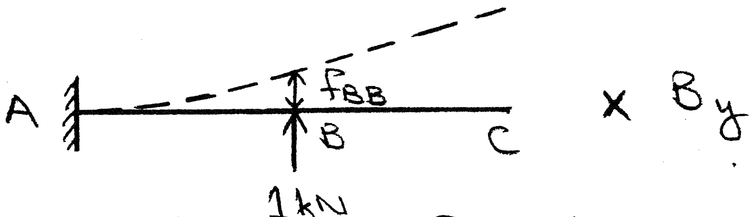
Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.15



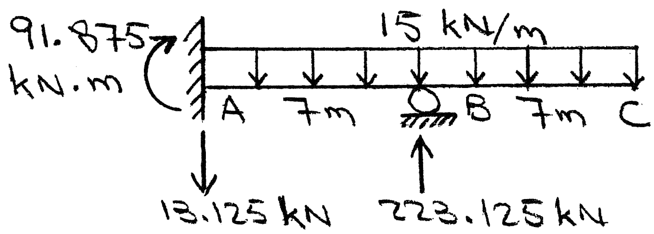


Using beam deflection formulas:

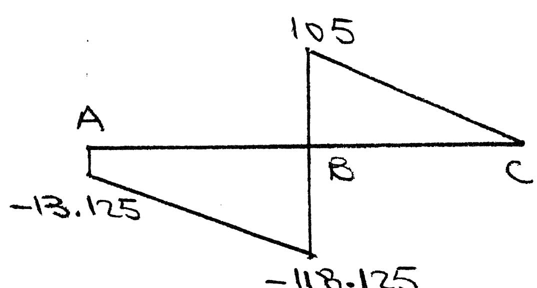
$$\Delta_{BO} = -\frac{25510.625 \text{ kN}\cdot\text{m}^3}{EI} \qquad f_{BB} = \frac{114.333 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB} B_y = 0$

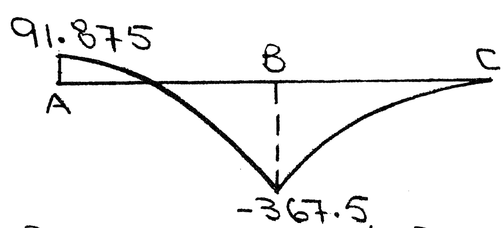
$$B_y = \frac{25510.625}{114.333} = \underline{223.125 \text{ kN} \uparrow}$$



Reactions

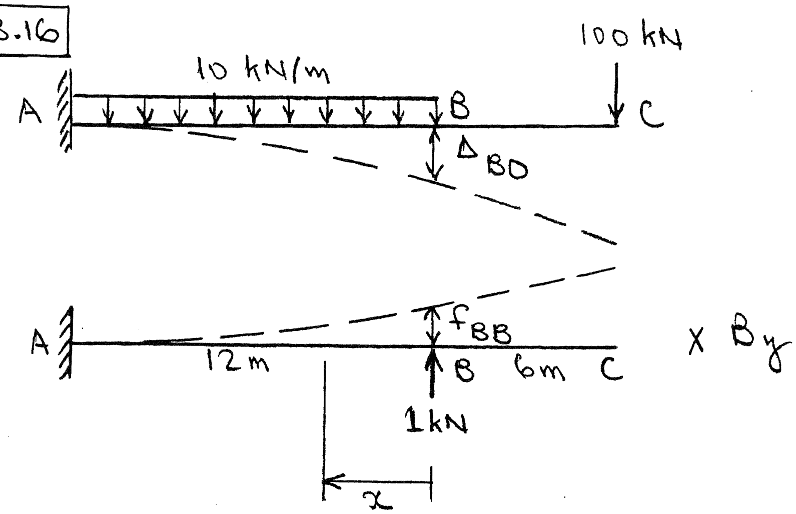


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

B.16



Using the virtual work method:

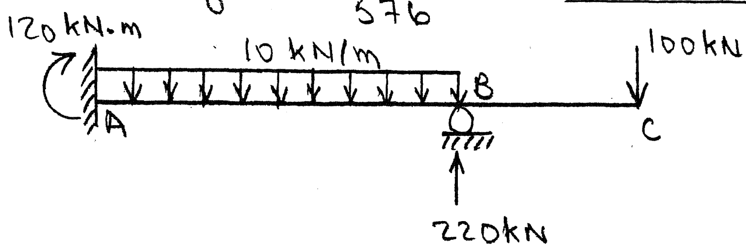
$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^{12}\left\{-100(6+x) - 5x^2\right\} x \, dx\right]$$

$$= -\frac{126720 \text{ kN-m}^3}{EI}$$

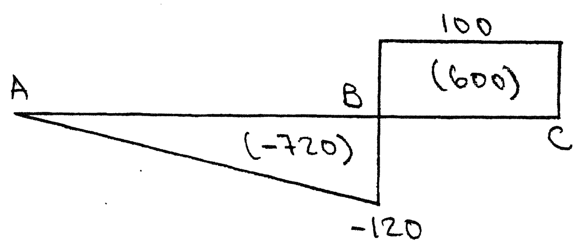
$$f_{BB} = \frac{1}{EI}\left[\int_0^{12} x^2 \, dx\right] = \frac{576 \text{ kN-m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB} B_y = 0$

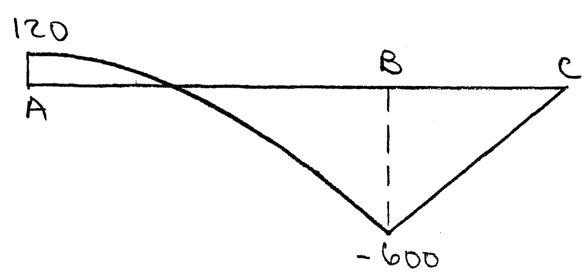
$$B_y = \frac{126720}{576} = 220 \text{ kN} \uparrow$$



Reactions

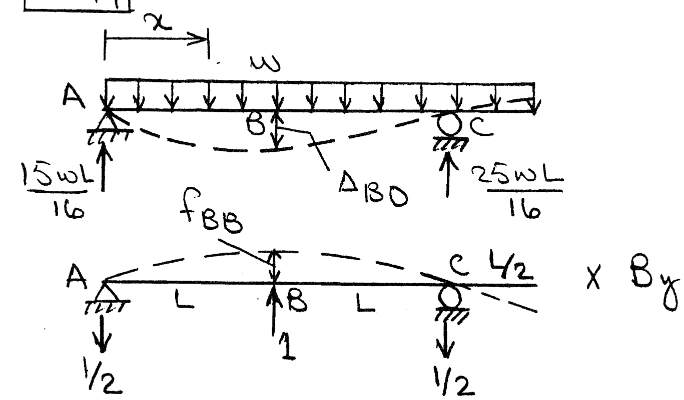


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

13.17



Using the virtual work method:

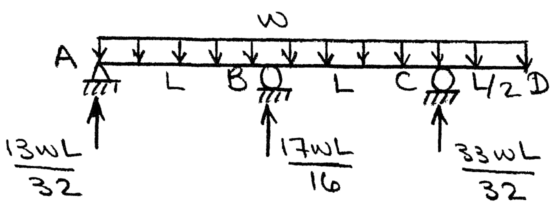
$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^L \left(\frac{15}{16}wLx - \frac{wx^2}{2}\right)\left(-\frac{x}{2}\right)dx + \int_L^{2L}\left(\frac{15}{16}wLx - \frac{wx^2}{2}\right)\left\{-\frac{x}{2} + 1(x-L)\right\}dx\right]$$

$$= -\frac{17wL^4}{96EI}$$

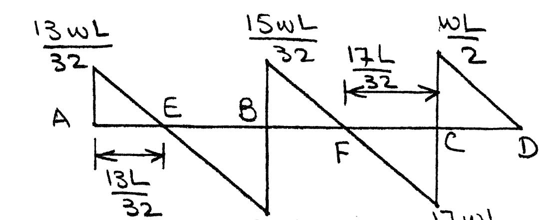
$$f_{BB} = \frac{2}{EI}\int_0^L \left(-\frac{x}{2}\right)^2 dx = \frac{L^3}{6EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB}B_y = 0$

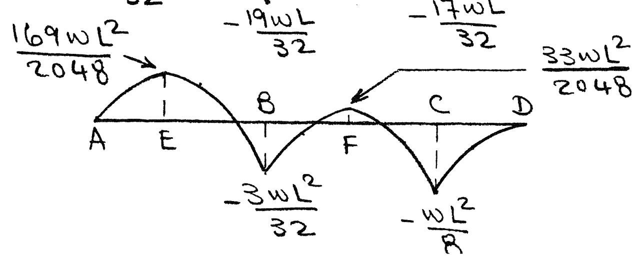
$$B_y = \frac{17wL^4}{96EI}\left(\frac{6EI}{L^3}\right) = \underline{\frac{17wL}{16}} \uparrow$$



Reactions

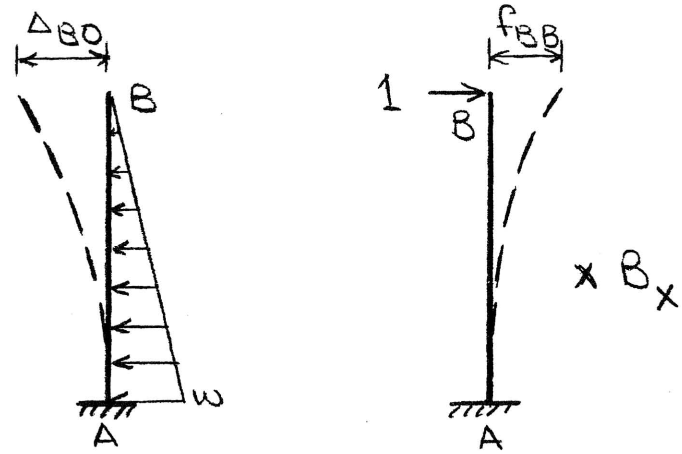


Shear Diagram



Bending Moment Diagram

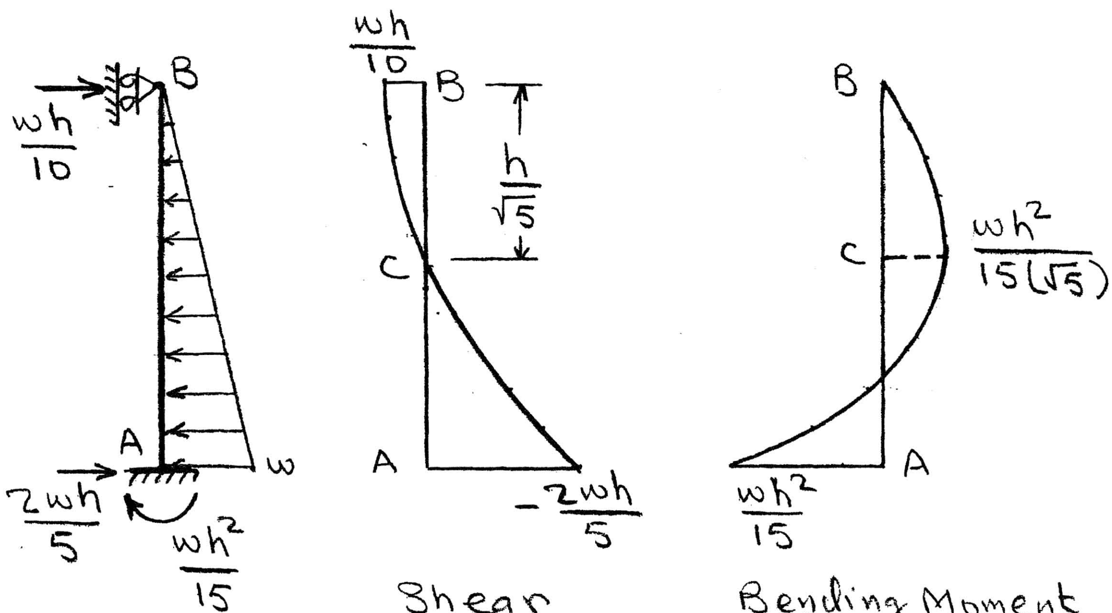
13.18



Using beam deflection formulas:

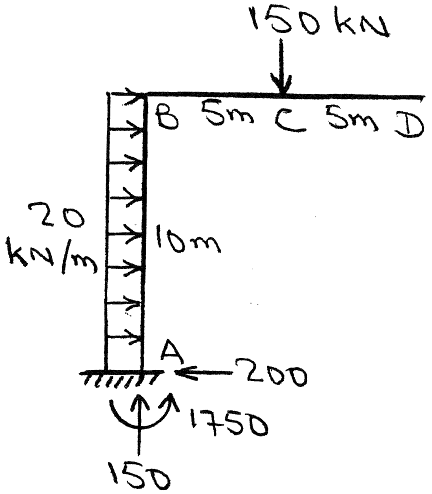
$$\Delta_{BO} = -\frac{wh^4}{30EI} \; ; \qquad f_{BB} = \frac{h^3}{3EI}$$

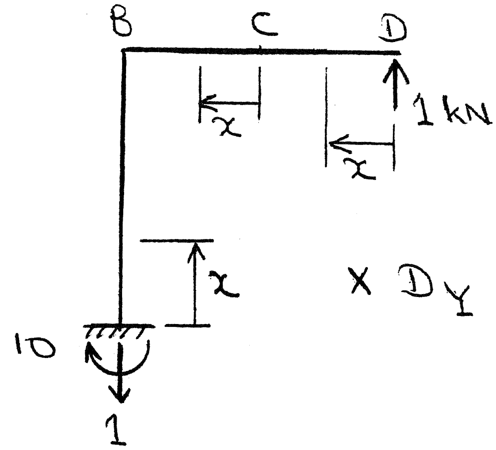
$$\Delta_{BO} + f_{BB} B_x = 0 \qquad B_x = \left(\frac{wh^4}{30EI}\right)\left(\frac{3EI}{h^3}\right) = \frac{wh}{10} \rightarrow$$



Reactions   Shear Diagram   Bending Moment Diagram

13.19





Using the virtual work method:

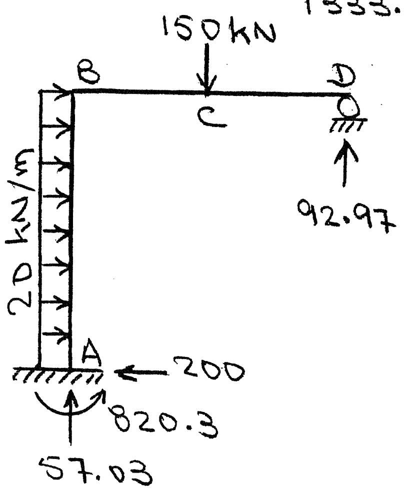
$$\Delta_{DO} = \frac{1}{EI}\left[\int_0^{10}(-1750 + 200x - 10x^2)(10)\,dx + \int_0^{5}(-150x)1(x+5)\,dx\right]$$

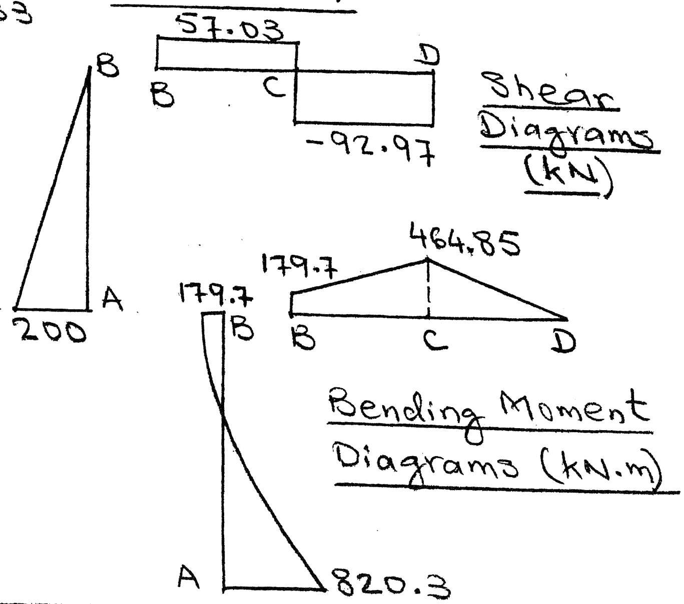
$$= -\frac{123958.33 \text{ kN.m}^3}{EI}$$

$$f_{DD} = \frac{1}{EI}\left[\int_0^{10}(10)^2 dx + \int_0^{10}(x)^2 dx\right] = \frac{1333.33 \text{ kN.m}^3/\text{kN}}{EI}$$

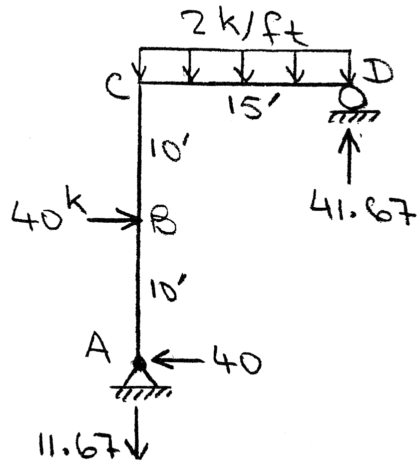
Compatibility Equation: $\Delta_{DO} + f_{DD} D_Y = 0$

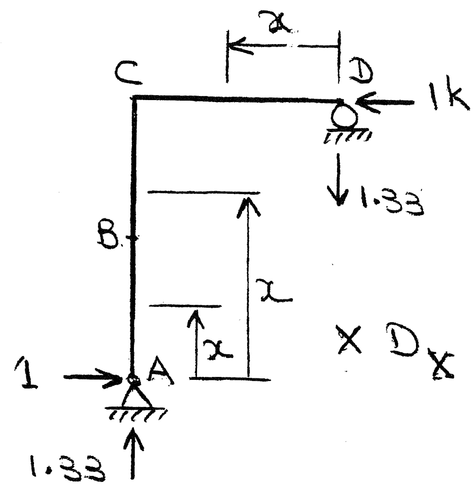
$$D_Y = \frac{123958.33}{1333.33} = 92.97 \text{ kN} \uparrow$$





13.20





Using the virtual work method:

$$\Delta_{DO} = \frac{1}{EI}\left[\int_0^{10}(40x)(-x)\,dx + \int_{10}^{20}(400)(-x)\,dx + \int_0^{15}(41.67x - x^2)(-1.33x)\,dx\right]$$

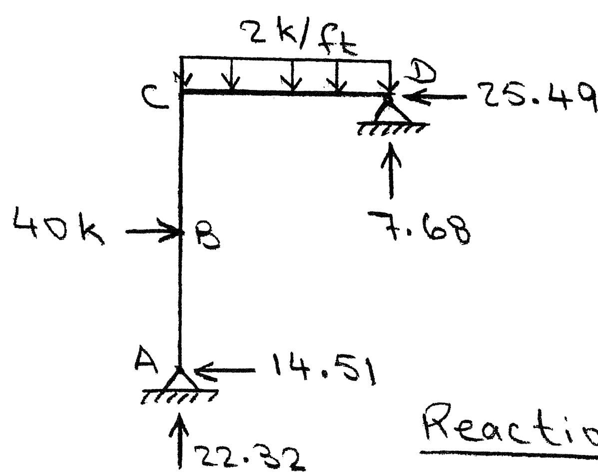
$$= -\frac{118958.33 \text{ k-ft}^3}{EI}$$

$$f_{DD} = \frac{1}{EI}\left[\int_0^{20}(-x)^2\,dx + \int_0^{15}(-1.33x)^2\,dx\right]$$

$$= \frac{4666.67 \text{ k-ft}^3/\text{k}}{EI}$$

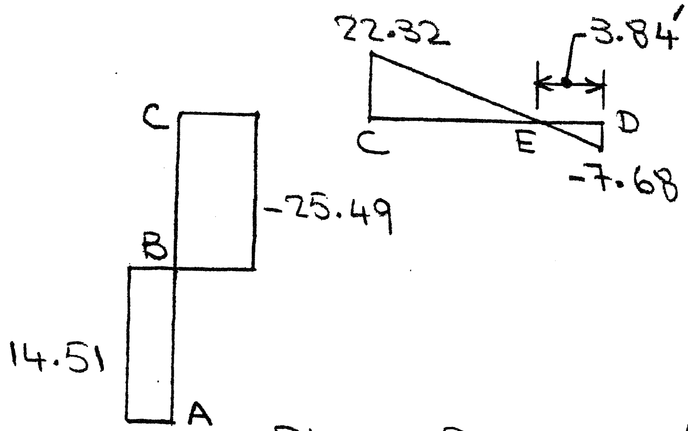
Compatibility Equation: $\Delta_{DO} + f_{DD}D_X = 0$

$$D_X = \frac{118958.33}{4666.67} = \underline{25.49\text{ k} \leftarrow}$$

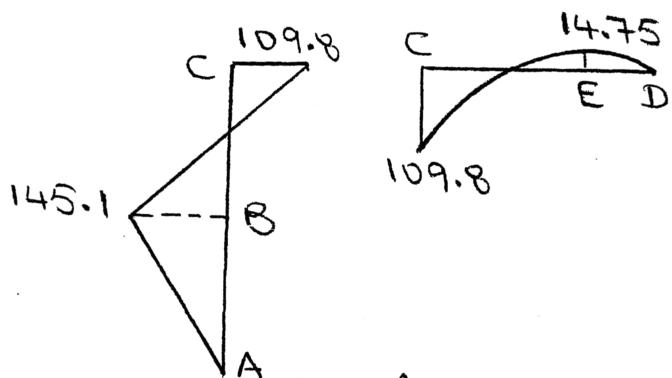


Reactions

13.20 (contd.)

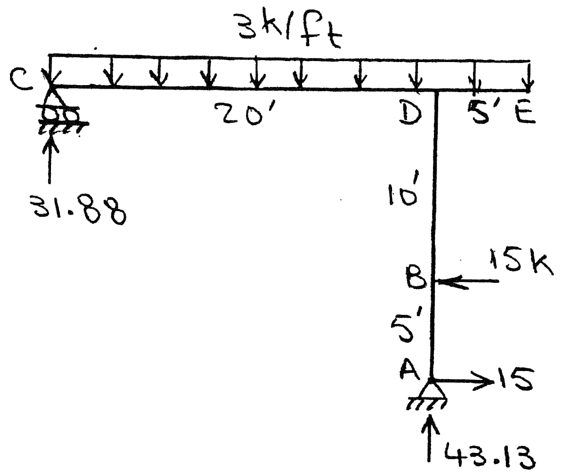


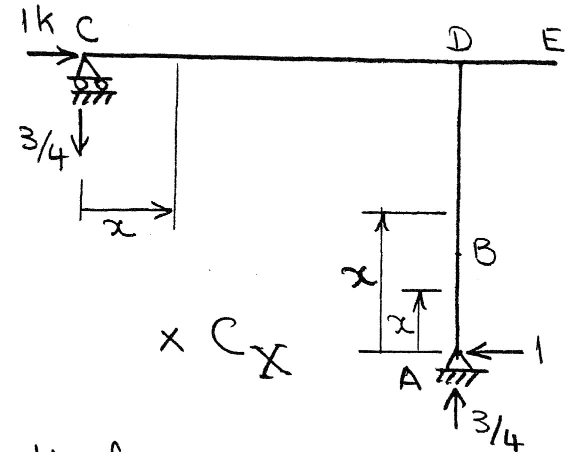
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.21





Using the virtual work method:

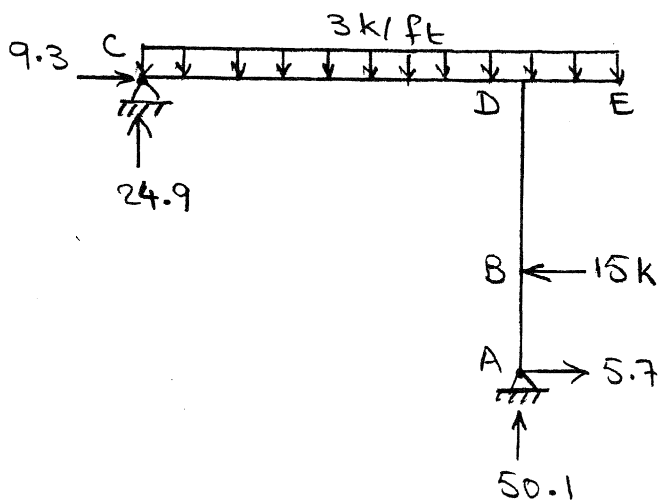
$$\Delta_{CO} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{20}(31.88x - 1.5x^2)\left(-\frac{3}{4}x\right)dx + \int_0^{5}(-15x)x\,dx + \int_5^{15}\{-15x+15(x-5)\}x\,dx\right]$$

$$= -\frac{17505\ \text{k-ft}^3}{EI}$$

$$f_{CC} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{20}\left(-\frac{3}{4}x\right)^2 dx + \int_0^{15}(x)^2 dx\right] = \frac{1875\ \text{k-ft}^3/\text{k}}{EI}$$

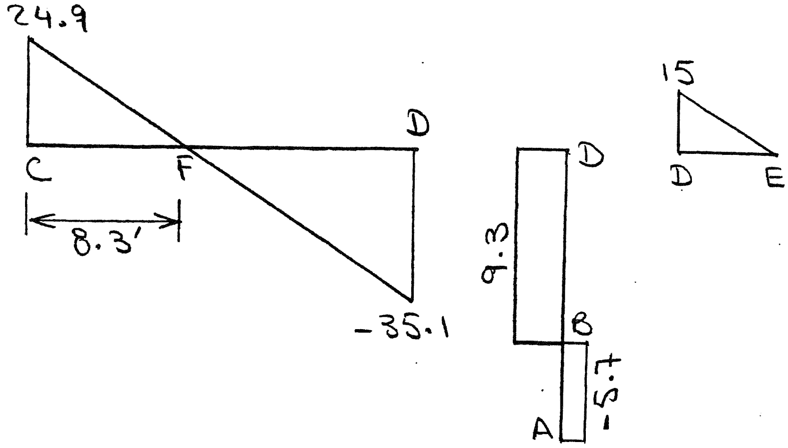
Compatibility Equation: $\Delta_{CO} + f_{CC}C_X = 0$

$$C_X = \frac{17505}{1875} = \underline{9.3\ \text{k} \rightarrow}$$

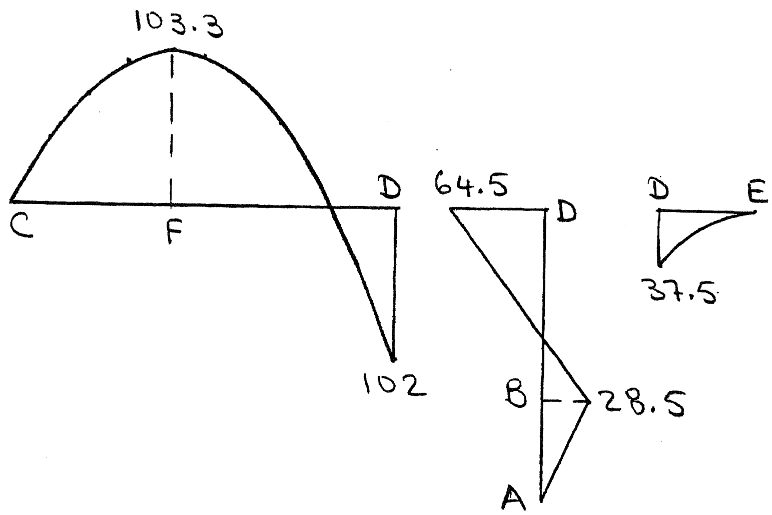


Reactions

13.21 (Contd.)

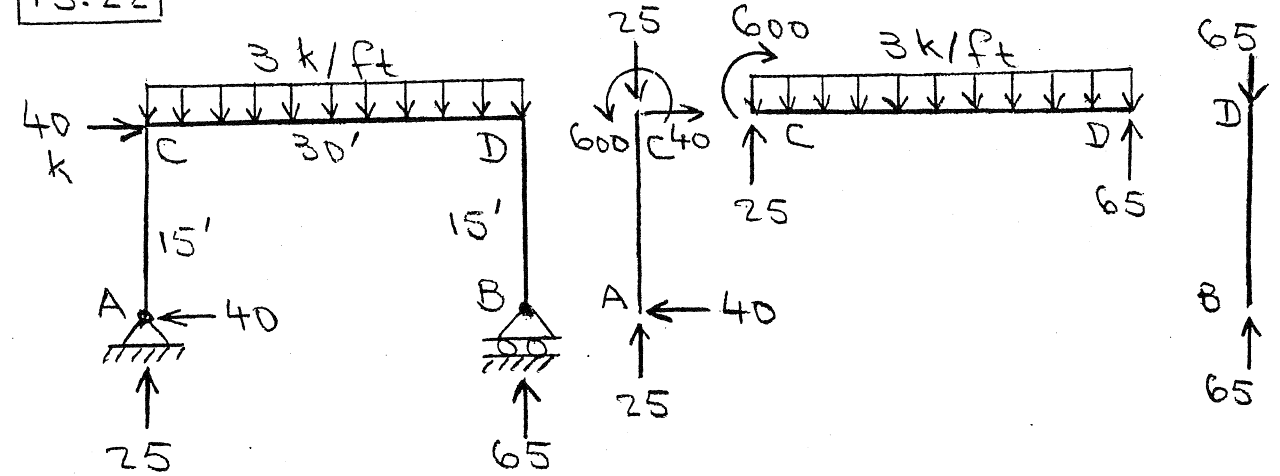


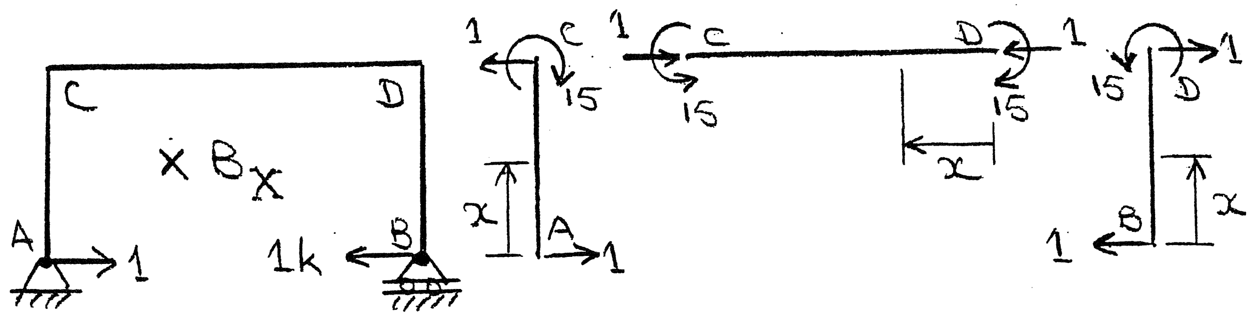
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.22





Using the virtual work method:

$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^{15}(40x)(-x)\,dx + \int_0^{30}(65x - 1.5x^2)(-15)\,dx\right]$$

$$= -\frac{281250\ \text{k-ft}^3}{EI}$$

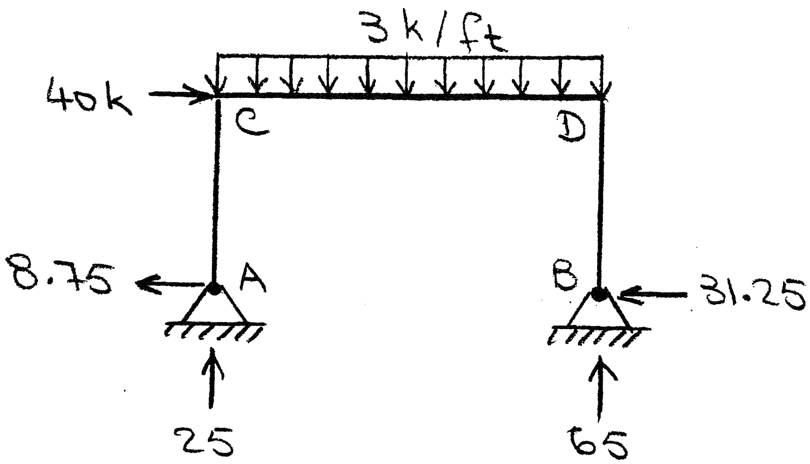
$$f_{BB} = \frac{1}{EI}\left[\int_0^{15}(-x)^2\,dx + \int_0^{30}(-15)^2\,dx + \int_0^{15}x^2\,dx\right]$$

$$= \frac{9000\ \text{k-ft}^3/\text{k}}{EI}$$

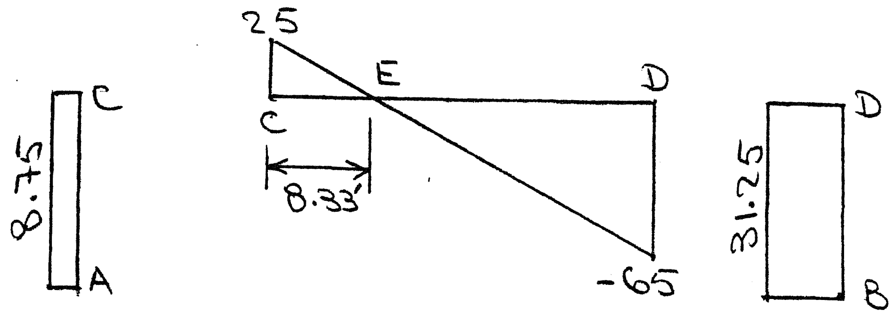
Compatibility Equation: $\Delta_{BO} + f_{BB}B_X = 0$

$$B_X = \frac{281250}{9000} = \underline{31.25\ \text{k} \leftarrow}$$

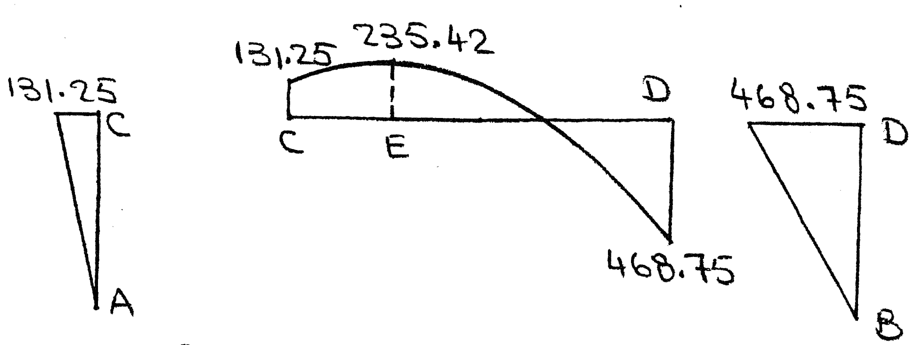
13.22 (Contd.)



Reactions

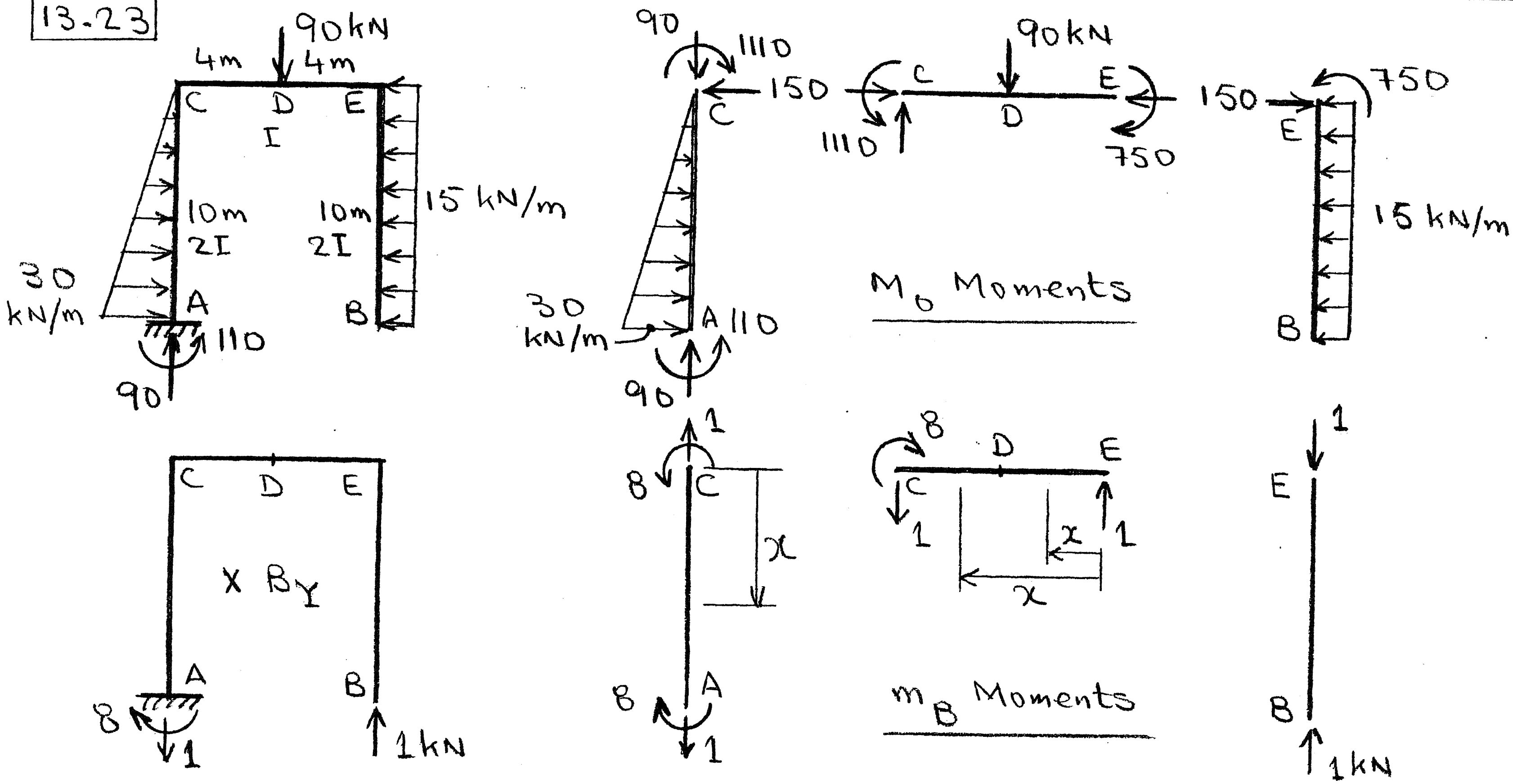


Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.23



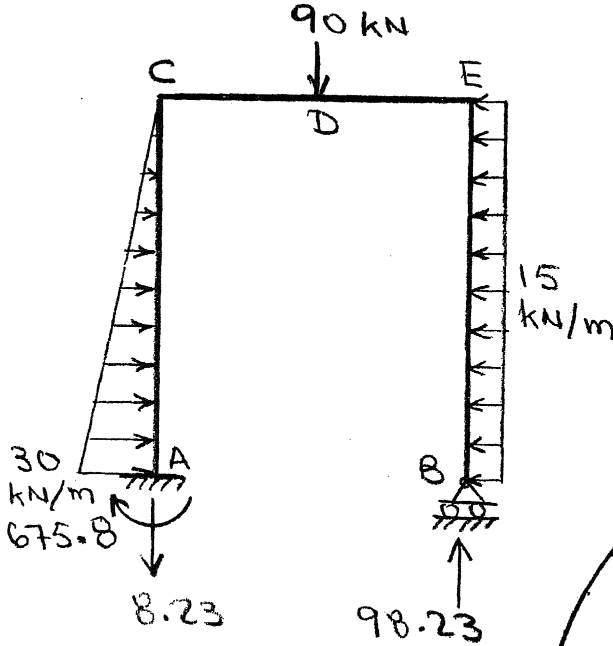
Segment	x Coordinate		M_o (kN.m)	m_B (kN.m/kN)
	Origin	Limits (m)		
CA	C	0–10	$150x - 1110 - \frac{1}{2}(x)3x\left(\frac{x}{3}\right)$	8
ED	E	0–4	-750	$1x$
DC	E	4–8	$-750 - 90(x-4)$	$1x$

13.23 (contd.) Using the virtual work method:

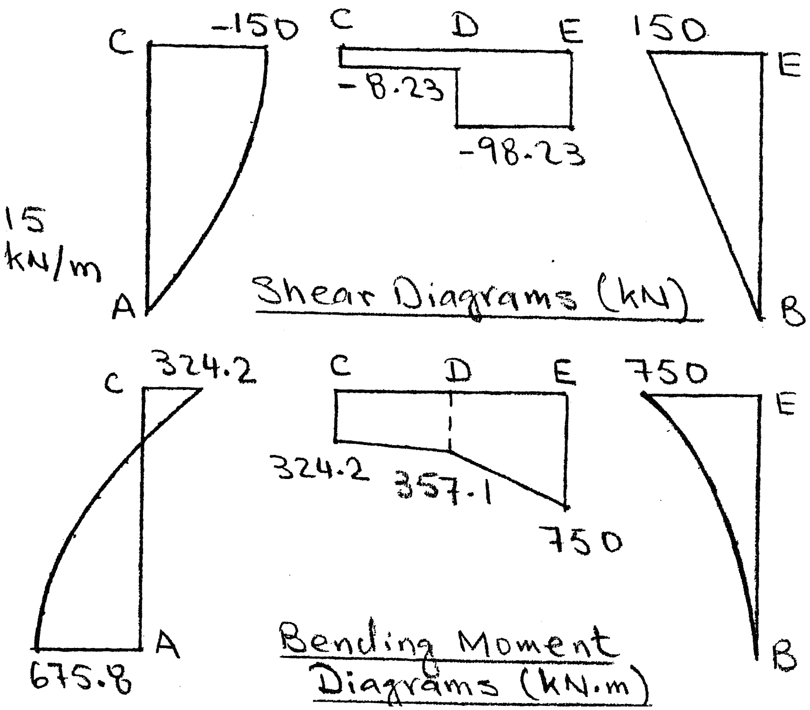
$$\Delta_{BO} = \Sigma \int \frac{M_o m_B}{EI} dx = \frac{1}{EI}\left[\frac{1}{2}\int_0^{10}\left(150x - 1110 - \frac{x^3}{2}\right)(8)\,dx + \int_0^4 (-750)x\,dx + \int_4^8 (-390-90x)x\,dx\right] = -\frac{48200\text{ kN}\cdot\text{m}^3}{EI}$$

$$f_{BB} = \Sigma \int \frac{m_B^2}{EI} dx = \frac{1}{EI}\left[\frac{1}{2}\int_0^{10} 64\,dx + \int_0^8 x^2\,dx\right] = \frac{490.67\text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB} B_Y = 0$ $\quad B_Y = \frac{48200}{490.67} = 98.23\text{ kN}\uparrow$



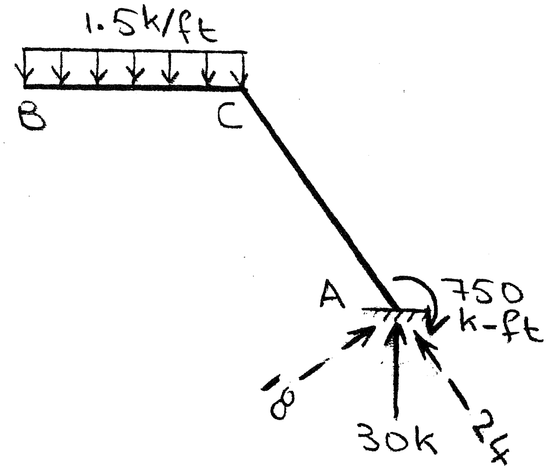
Reactions

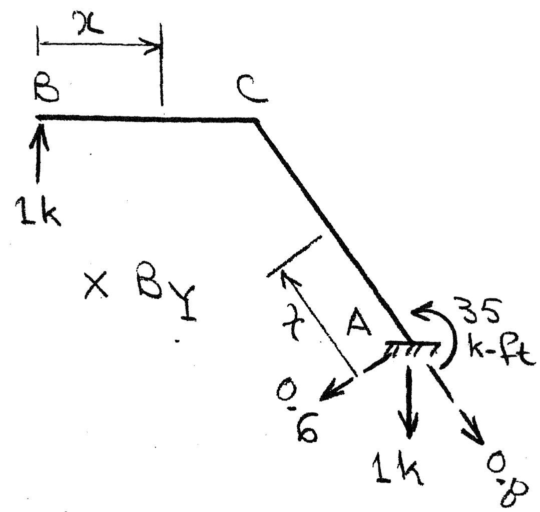


Shear Diagrams (kN)

Bending Moment Diagrams (kN.m)

13.24





Using the virtual work method:

$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^{20}\left(-\frac{1.5x^2}{2}\right)(1x)\,dx + \int_0^{25}(18x-750)(-0.6x+35)\,dx\right]$$

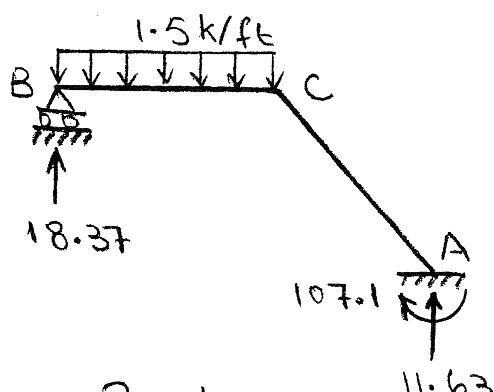
$$= -\frac{405000\ \text{k-ft}^3}{EI}$$

$$f_{BB} = \frac{1}{EI}\left[\int_0^{20}(1x)^2\,dx + \int_0^{25}(-0.6x+35)^2\,dx\right]$$

$$= \frac{22041.667\ \text{k-ft}^3/\text{k}}{EI}$$

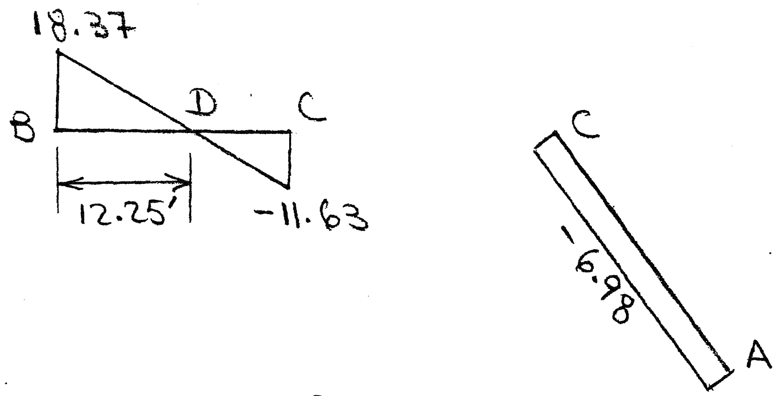
Compatibility Equation: $\Delta_{BO} + f_{BB}B_Y = 0$

$$B_Y = \frac{405000}{22041.667} = 18.37\ \text{k}\uparrow$$

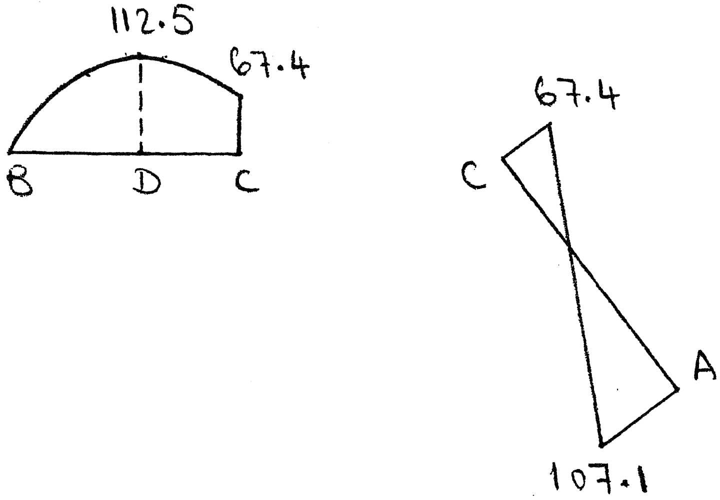


Reactions

13.24 (contd.)

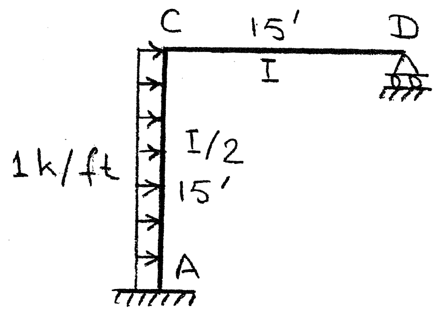


Shear Diagrams (k)

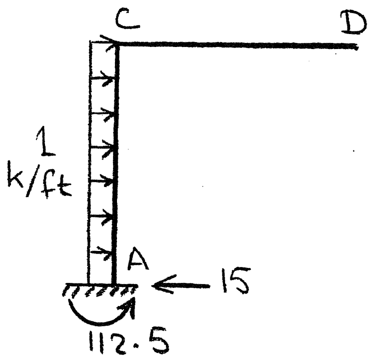


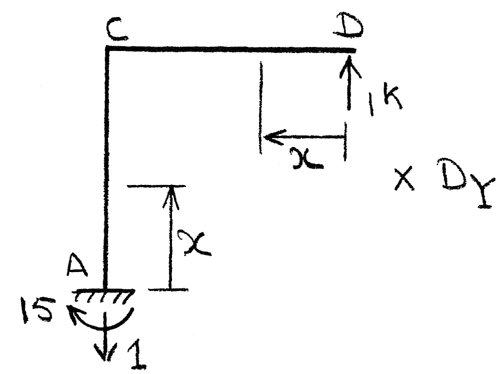
Bending Moment Diagrams (k-ft)

13-25 As the frame is symmetric subjected to antisymmetric loading, only a half of the structure needs to be analyzed.



Substructure for Analysis





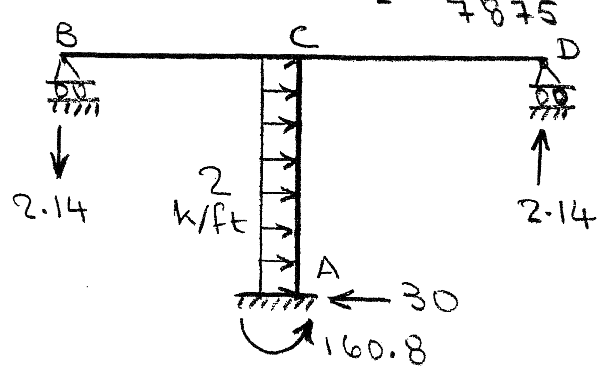
Using the virtual work method:

$$\Delta_{DO} = \frac{1}{EI}\left[2\int_0^{15}\left(-112.5 + 15x - \frac{x^2}{2}\right)(15)\,dx\right] = -\frac{16875}{EI}$$

$$f_{DD} = \frac{1}{EI}\left[2\int_0^{15}(15)^2\,dx + \int_0^{15}(1x)^2\,dx\right] = \frac{7875}{EI}$$

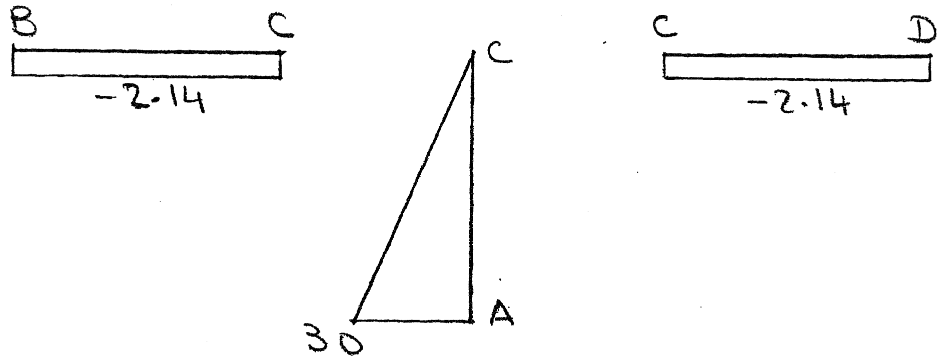
Compatibility Equation:

$$\Delta_{DO} + f_{DD}D_Y = 0 \qquad D_Y = \frac{16875}{7875} = 2.14\text{ k} \uparrow$$

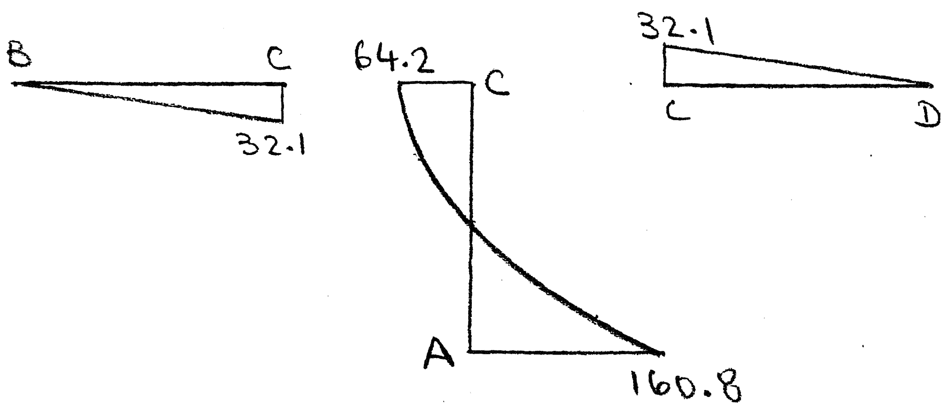


Reactions

13.25 (contd.)

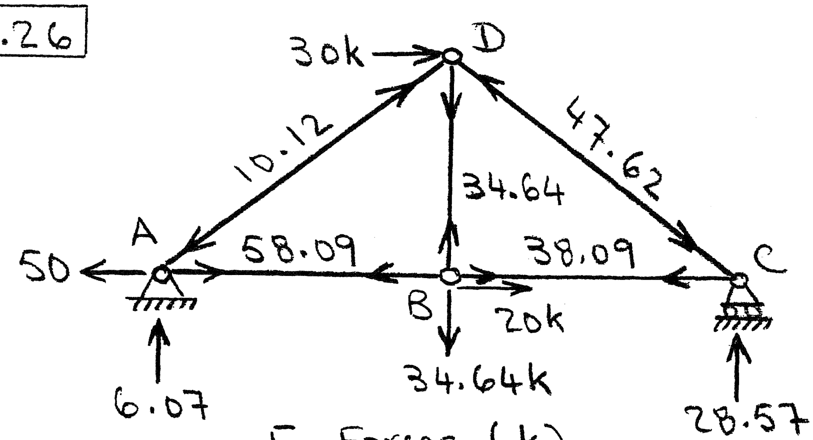


Shear Diagrams (k)

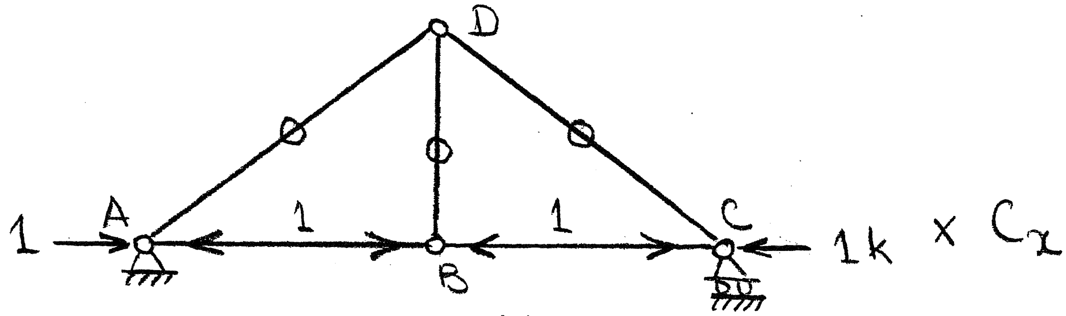


Bending Moment Diagrams (k-ft)

13.26



F_0 Forces (k)

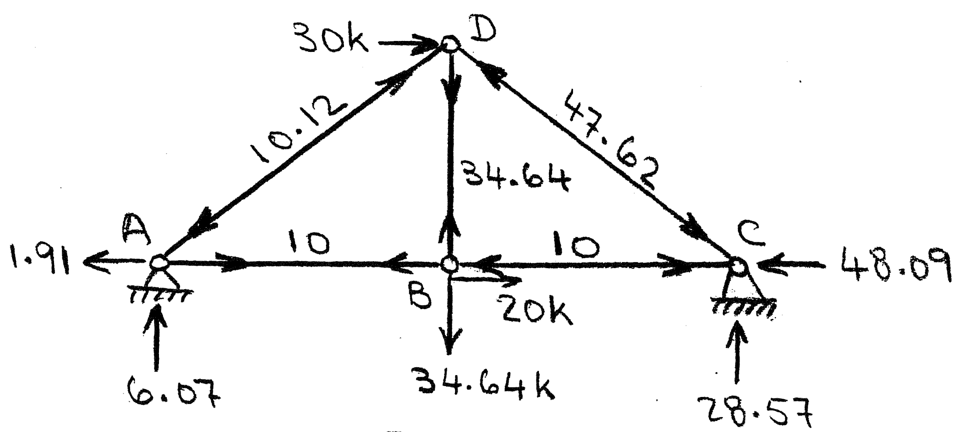


u_C Forces (k/k)

$$\Delta_{CO} = \frac{1}{EA}\sum F_0 u_C L = \frac{1}{EA}\left[58.09(-1)8 + 38.09(-1)8\right]$$
$$= -\frac{769.44\ \text{k-ft}}{EA}$$

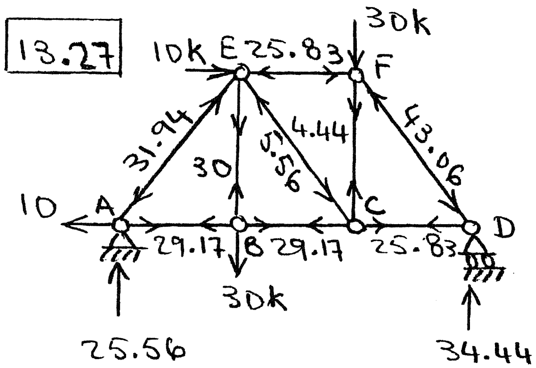
$$f_{CC} = \frac{1}{EA}\sum u_C^2 L = \frac{1}{EA}\left[2(-1)^2 8\right] = \frac{16\ \text{ft}}{EA}$$

$$C_x = -\frac{\Delta_{CO}}{f_{CC}} = \frac{769.44}{16} = \underline{48.09\ \text{k} \leftarrow}$$

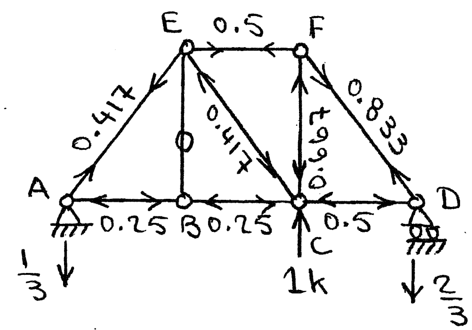


Member Forces and Reactions

13.27



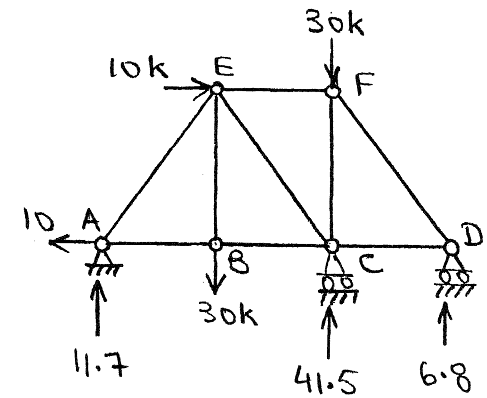
F_0 Forces



u_C Forces

$\times\ C_y$

Member	L (in.)	F_0 (k)	u_C (k/k)	$F_0 u_C L$ (k-in.)	$u_C^2 L$ (in.)	$F = F_0 + u_C C_y$ (k)
AB	180	29.17	−0.25	−1312.65	11.25	18.8
BC	180	29.17	−0.25	−1312.65	11.25	18.8
CD	180	25.83	−0.5	−2324.7	45	5.1
EF	180	−25.83	0.5	−2324.7	45	−5.1
AE	300	−31.94	0.417	−3995.69	52.17	−14.6
CE	300	−5.56	−0.417	695.56	52.17	−22.9
DF	300	−43.06	0.833	−10760.69	208.17	−8.5
BE	240	30	0	0	0	30
CF	240	4.44	−0.667	−710.76	106.77	−23.2
			Σ	−22046.28	531.78	

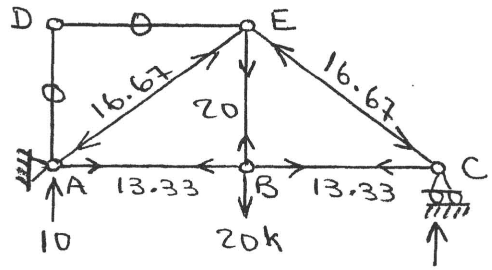


Reactions

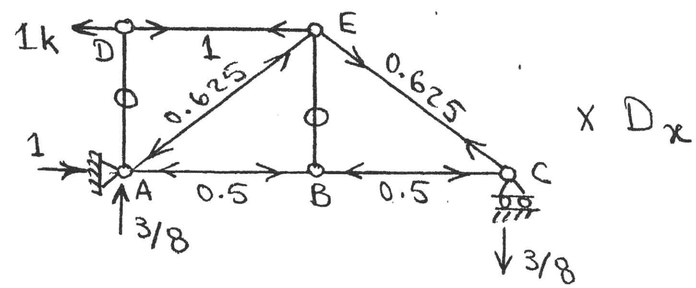
$$\Delta_{CO} = \frac{1}{EA}\Sigma F_0 u_C L = -\frac{22046.28\text{ (k-in.)}}{EA}\ ; \quad f_{CC} = \frac{1}{EA}\Sigma u_C^2 L = \frac{531.78\text{ (in.)}}{EA}$$

$$C_y = -\frac{\Delta_{CO}}{f_{CC}} = \frac{22046.28}{531.78} = \underline{41.5\text{ k}\uparrow}$$

13.28



F_o Forces

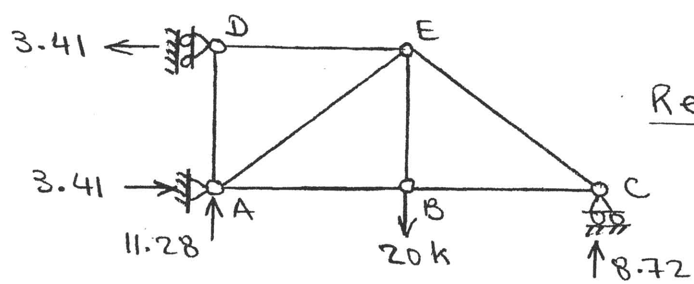


u_D Forces

Member	L (in.)	A (in.²)	F_o (k)	u_D (k/k)	$F_o u_D L/A$ (k/in.)	$u_D^2 L/A$ (1/in.)	$F = F_o + u_D D_x$ (k)
AB	192	8	13.33	-0.5	-159.96	6	11.63
BC	192	8	13.33	-0.5	-159.96	6	11.63
DE	192	8	0	1	0	24	3.41
AE	240	6	-16.67	-0.625	416.75	15.63	-18.8
CE	240	8	-16.67	0.625	-312.56	11.72	-14.54
AD	144	6	0	0	0	0	0
BE	144	6	20	0	0	0	20
				Σ	-215.73	63.35	

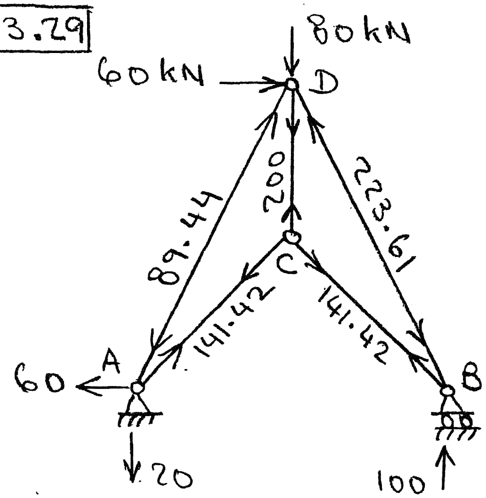
$$\Delta_{DO} = \frac{1}{E}\Sigma \frac{F_o u_D L}{A} = -\frac{215.73\ (\text{k/in.})}{E} \ ; \quad f_{DD} = \frac{1}{E}\Sigma \frac{u_D^2 L}{A} = \frac{63.35\ (1/\text{in.})}{E}$$

$$D_x = -\frac{\Delta_{DO}}{f_{DD}} = \frac{215.73}{63.35} = 3.41\ \text{k} \leftarrow$$

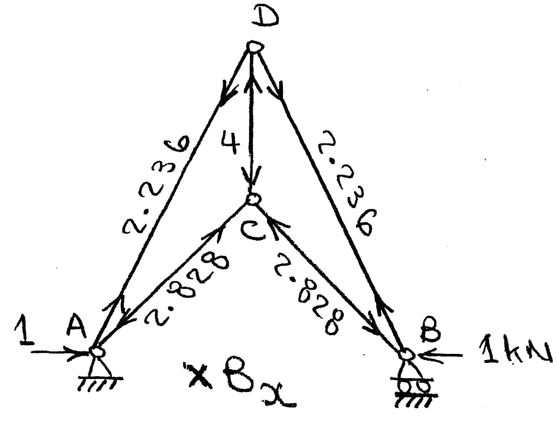


Reactions

13.29

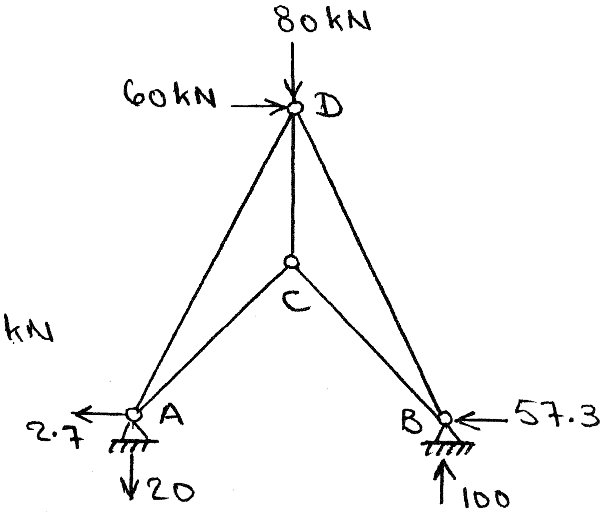


F_0 Forces



$\times B_x$

u_B Forces



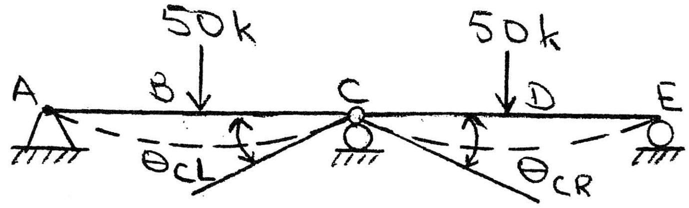
Reactions

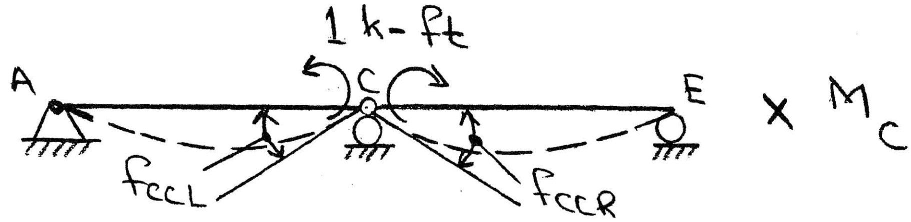
Member	L (m)	F_0 (kN)	u_B (kN/kN)	$F_0 u_B L$ (kN-m)	$u_B^2 L$ (m)	$F = F_0 + u_B B_x$ (kN)
AD	11.18	−89.44	2.236	−2235.86	55.9	38.7
BD	11.18	−223.61	2.236	−5589.91	55.9	−95.5
AC	7.07	141.42	−2.828	−2827.55	56.54	−20.6
BC	7.07	141.42	−2.828	−2827.55	56.54	−20.6
CD	5	200	−4	−4000	80	−29.2
			Σ	−17480.87	304.88	

$$\Delta_{BO} = \frac{1}{EA}\sum F_0 u_B L = -\frac{17480.87\ (\text{kN-m})}{EA} \quad ; \quad f_{BB} = \frac{1}{EA}\sum u_B^2 L = \frac{304.88\ (\text{m})}{EA}$$

$$B_x = -\frac{\Delta_{BO}}{f_{BB}} = \frac{17480.87}{304.88} = \underline{57.3 \text{ kN} \leftarrow}$$

13.30





Using beam deflection formulas:

$$\theta_{CL} = \theta_{CR} = \frac{1800 \text{ k-ft}^2}{EI}$$

$$\theta_{CO\,rel.} = \theta_{CL} + \theta_{CR} = \frac{3600 \text{ k-ft}^2}{EI}$$

$$f_{CCL} = f_{CCR} = \frac{8 \text{ ft}}{EI}$$

$$f_{CC\,rel.} = f_{CCL} + f_{CCR} = \frac{16 \text{ ft}}{EI}$$

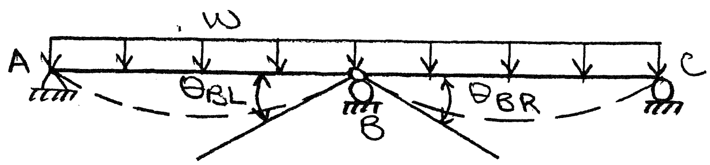
Compatibility Equation:

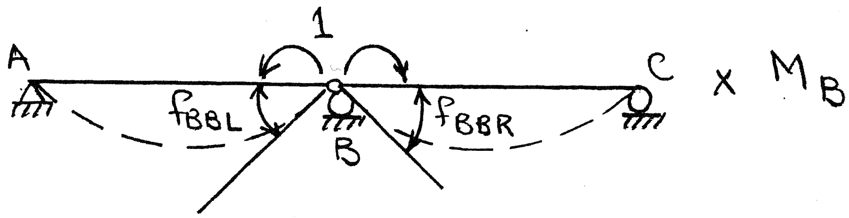
$$\theta_{CO\,rel.} + f_{CC\,rel.}\, M_C = 0$$

$$M_C = -\frac{3600}{16} = \underline{-225 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.9.

13.31





Using beam deflection formulas:

$$\theta_{BL} = \theta_{BR} = \frac{wL^3}{24EI}$$

$$\theta_{BO\,rel.} = \theta_{BL} + \theta_{BR} = \frac{wL^3}{12EI}$$

$$f_{BBL} = f_{BBR} = \frac{L}{3EI}$$

$$f_{BB\,rel.} = f_{BBL} + f_{BBR} = \frac{2L}{3EI}$$

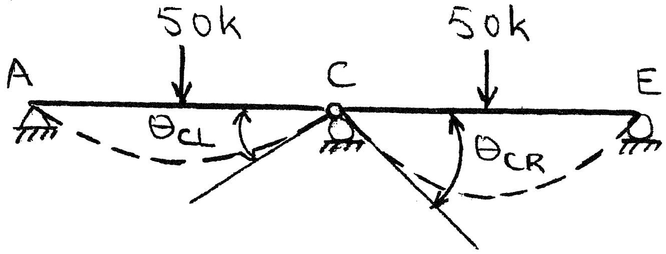
Compatibility Equation:

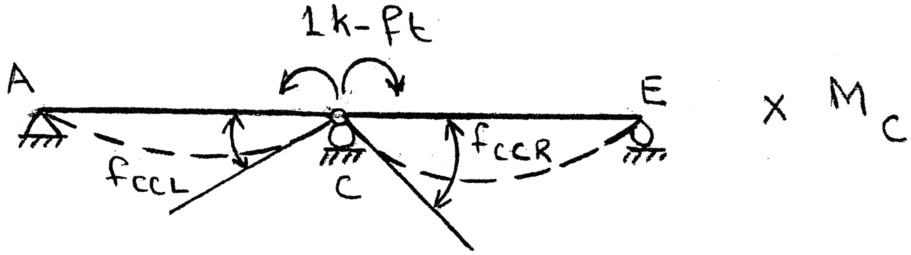
$$\theta_{BO\,rel.} + f_{BB\,rel.}\, M_B = 0$$

$$M_B = -\frac{wL^3}{12EI}\left(\frac{3EI}{2L}\right) = -\frac{wL^2}{8}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.10.

13.32





Using beam deflection formulas:

$$\theta_{CL} = \frac{400}{EI} \; ; \; \theta_{CR} = \frac{800}{EI}$$

$$\theta_{CO\,rel.} = \theta_{CL} + \theta_{CR} = \frac{1200\ \text{k-ft}^2}{EI}$$

$$f_{CCL} = \frac{2.67}{EI} \; ; \; f_{CCR} = \frac{5.33}{EI}$$

$$f_{CC\,rel.} = f_{CCL} + f_{CCR} = \frac{8\ \text{ft}}{EI}$$

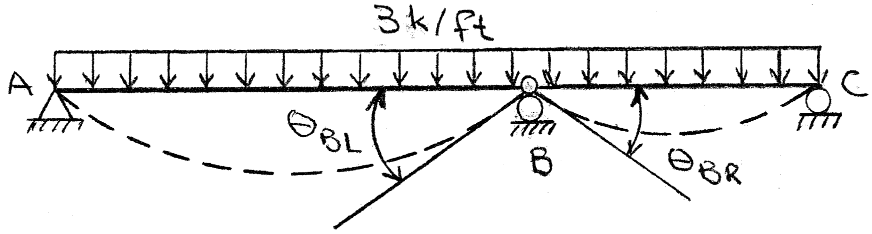
Compatibility Equation:

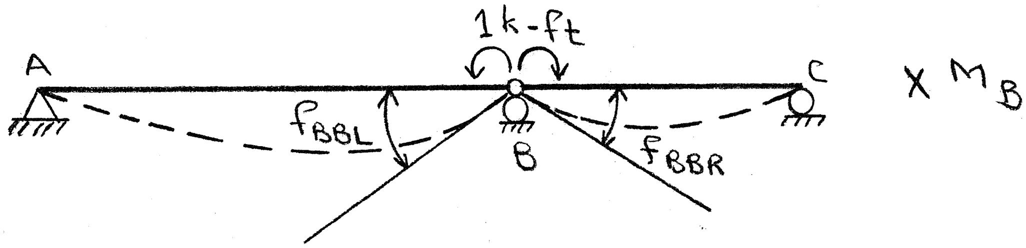
$$\theta_{CO\,rel.} + f_{CC\,rel.} M_C = 0$$

$$M_C = -\frac{1200}{8} = \underline{-150\ \text{k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.11.

13.33





Using beam deflection formulas:

$$\theta_{BL} = \frac{976.56 \text{ k-ft}^2}{EI} \; ; \qquad \theta_{BR} = \frac{421.88 \text{ k-ft}^2}{EI}$$

$$\theta_{BO\,rel.} = \theta_{BL} + \theta_{BR} = \frac{1398.44 \text{ k-ft}^2}{EI}$$

$$f_{BBL} = \frac{4.17 \text{ ft}}{EI} \; ; \qquad f_{BBR} = \frac{5 \text{ ft}}{EI}$$

$$f_{BB\,rel.} = f_{BBL} + f_{BBR} = \frac{9.17 \text{ ft}}{EI}$$

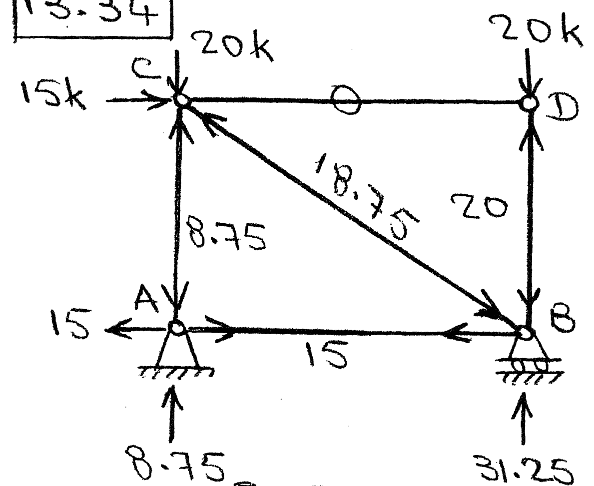
Compatibility Equation:

$$\theta_{BO\,rel.} + f_{BB\,rel.} M_B = 0$$

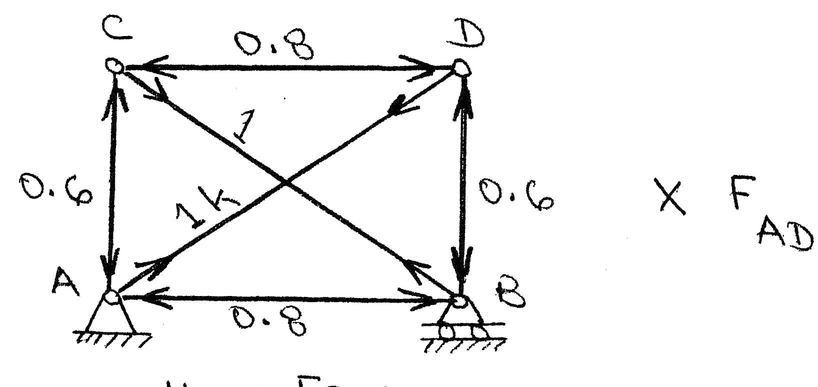
$$M_B = -\frac{1398.44}{9.17} = \underline{-152.6 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

13.34



F_o Forces



× F_{AD}

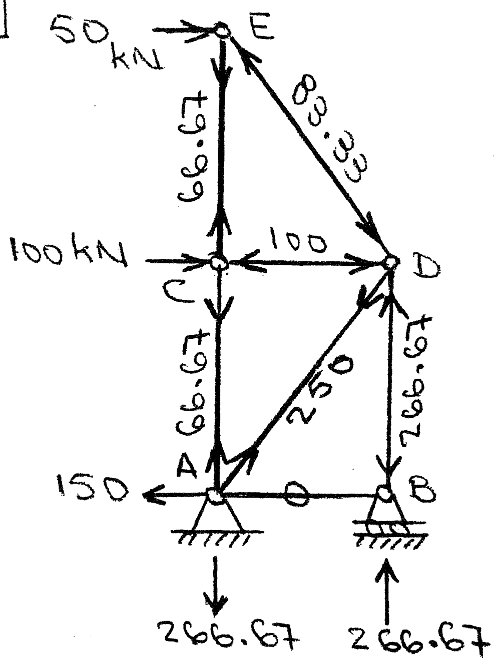
u_{AD} Forces

Member	L (in.)	A (in.²)	F_o (k)	u_{AD} (k/k)	$F_o u_{AD} L/A$ (k/in.)	$u_{AD}^2 L/A$ (1/in.)	$F = F_o + u_{AD} F_{AD}$ (k)
AB	192	6	15	−0.8	−384	20.48	11.4
CD	192	6	0	−0.8	0	20.48	−3.6
AC	144	6	−8.75	−0.6	126	8.64	−11.5
BD	144	6	−20	−0.6	288	8.64	−22.7
BC	240	8	−18.75	1	−562.5	30	−14.2
AD	240	8	0	1	0	30	4.5
				Σ	−532.5	118.24	

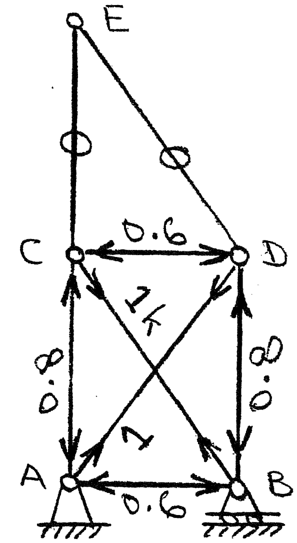
$$\Delta_{AD0} = \frac{1}{E}\sum \frac{F_o u_{AD} L}{A} = -\frac{532.5\ (\text{k-in.})}{E}\ ; \quad f_{AD,AD} = \frac{1}{E}\sum \frac{u_{AD}^2 L}{A} = \frac{118.24\ (1/\text{in.})}{E}$$

$$F_{AD} = -\frac{\Delta_{AD0}}{f_{AD,AD}} = \frac{532.5}{118.24} = \underline{4.5\ \text{k (T)}}$$

13.35



F_0 Forces



u_{BC} Forces

Member	L (m)	F_0 (kN)	u_{BC} (kN/kN)	$F_0 u_{BC} L$ (kN.m)	$u_{BC}^2 L$ (m)	$F = F_0 + u_{BC} F_{BC}$ (kN)
AB	3	0	−0.6	0	1.08	71.9
CD	3	−100	−0.6	180	1.08	−28.1
AC	4	66.67	−0.8	−213.33	2.56	162.5
BD	4	−266.67	−0.8	853.33	2.56	−170.8
CE	4	66.67	0	0	0	66.7
AD	5	250	1	1250	5	130.2
DE	5	−83.33	0	0	0	−83.3
BC	5	0	1	0	5	−119.8
			Σ	2070	17.28	

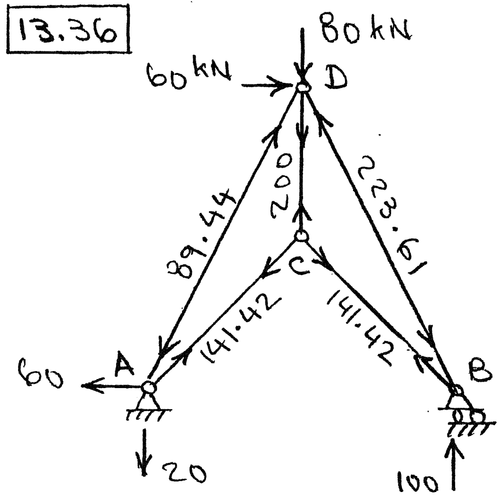
$$\Delta_{BCO} = \frac{1}{EA}\sum F_0 u_{BC} L = \frac{2070\ (\text{kN.m})}{EA}$$

$$f_{BC,BC} = \frac{1}{EA}\sum u_{BC}^2 L = \frac{17.28\ (\text{m})}{EA}$$

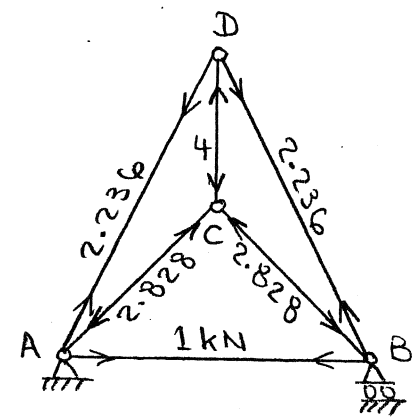
$$F_{BC} = -\frac{\Delta_{BCO}}{f_{BC,BC}} = -\frac{2070}{17.28} = -119.8\ \text{kN}$$

$$= \underline{119.8\ \text{kN (C)}}$$

13.36



F_0 Forces



u_{AB} Forces

Member	L (m)	F_0 (kN)	u_{AB} (kN/kN)	$F_0 u_{AB} L$ (kN-m)	$u_{AB}^2 L$ (m)	$F = F_0 + u_{AB} F_{AB}$
AD	11.18	−89.44	2.236	−2235.86	55.9	34.7
BD	11.18	−223.61	2.236	−5589.91	55.9	−99.5
AC	7.07	141.42	−2.828	−2827.55	56.54	−15.6
BC	7.07	141.42	−2.828	−2827.55	56.54	−15.6
CD	5	200	−4	−4000	80	−22
AB	10	0	1	0	10	55.5
			Σ	−17480.87	314.88	

$$\Delta_{AB0} = \frac{1}{EA} \Sigma F_0 u_{AB} L = -\frac{17480.87}{EA} \; ; \quad f_{AB,AB} = \frac{1}{EA} \Sigma u_{AB}^2 L = \frac{314.88}{EA}$$

$$F_{AB} = -\frac{\Delta_{AB0}}{f_{AB,AB}} = \frac{17480.87}{314.88} = \underline{55.5 \text{ kN (T)}}$$

13.37 Using beam deflection formulas:

$$\Delta_{BO} = -\frac{17wL^4}{384EI} = -\frac{17(25)(16)^4}{384\,EI} = -\frac{72533.33\ \text{kN.m}^3}{EI}$$

$$\Delta_{CO} = -\frac{wL^4}{8EI} = -\frac{25(16)^4}{8\,EI} = -\frac{204800\ \text{kN.m}^3}{EI}$$

$$f_{BB} = \frac{L^3}{24EI} = \frac{(16)^3}{24EI} = \frac{170.67\ \text{kN.m}^3/\text{kN}}{EI}$$

$$f_{BC} = f_{CB} = \frac{5L^3}{48EI} = \frac{5(16)^3}{48EI} = \frac{426.67\ \text{kN.m}^3/\text{kN}}{EI}$$

$$f_{CC} = \frac{L^3}{3EI} = \frac{(16)^3}{3EI} = \frac{1365.33\ \text{kN.m}^3/\text{kN}}{EI}$$

Compatibility Equations:

$$\Delta_{BO} + f_{BB}B_y + f_{BC}C_y = 0$$

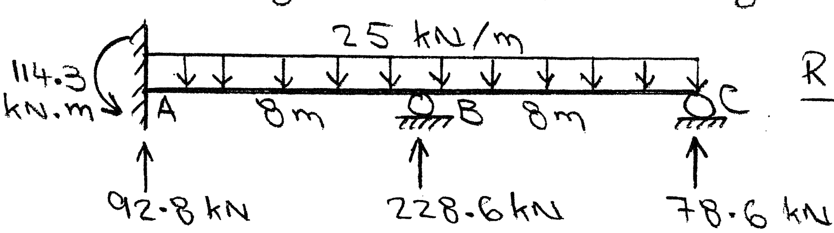
$$\Delta_{CO} + f_{CB}B_y + f_{CC}C_y = 0$$

$$-72533.33 + (170.67)B_y + (426.67)C_y = 0$$

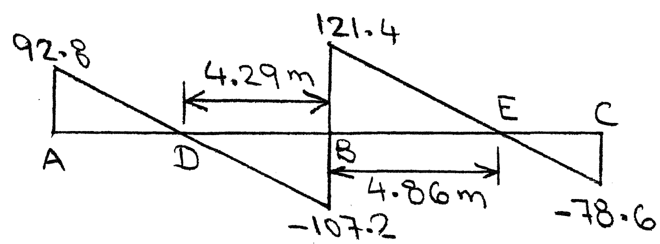
$$-204800 + (426.67)B_y + (1365.33)C_y = 0$$

Solving these equations, we obtain

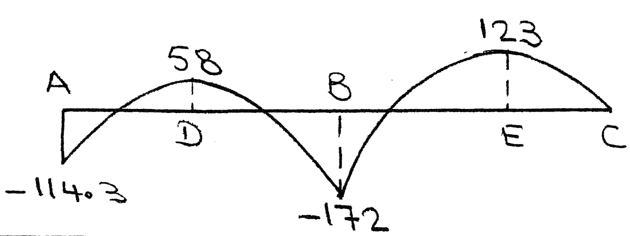
$B_y = 228.6$ kN ↑ $\quad C_y = 78.6$ kN ↑



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

13.38 Using beam deflection formulas:

$$\Delta_{BO} = -\frac{819166.67\ \text{k-ft}^3}{EI}; \quad \Delta_{DO} = -\frac{280000\ \text{k-ft}^3}{EI}$$

$$f_{BB} = \frac{21333.33\ \text{k-ft}^3/\text{k}}{EI}; \quad f_{BD} = f_{DB} = \frac{6666.67\ \text{k-ft}^3/\text{k}}{EI}$$

$$f_{DD} = \frac{2666.67\ \text{k-ft}^3/\text{k}}{EI}$$

Compatibility Equations:

$$\Delta_{BO} + f_{BB} B_y + f_{BD} D_y = 0$$

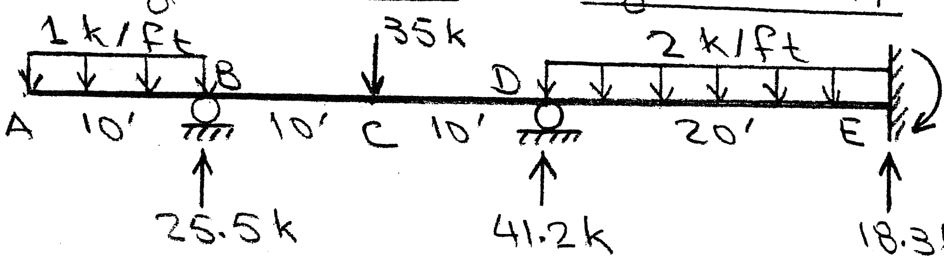
$$\Delta_{DO} + f_{DB} B_y + f_{DD} D_y = 0$$

$$-819166.67 + (21333.33) B_y + (6666.67) D_y = 0$$

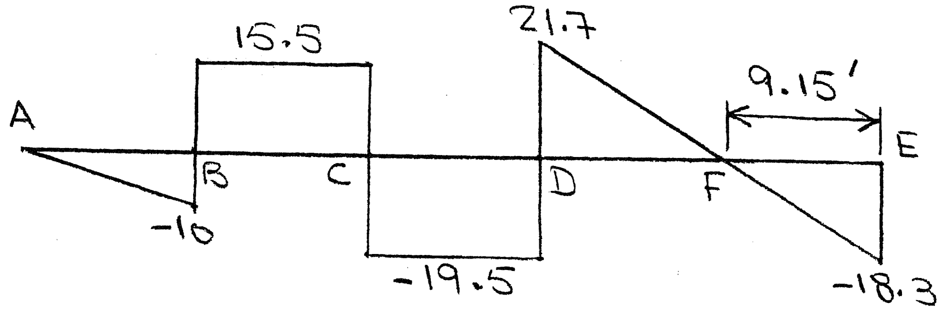
$$-280000 + (6666.67) B_y + (2666.67) D_y = 0$$

Solving these equations, we obtain

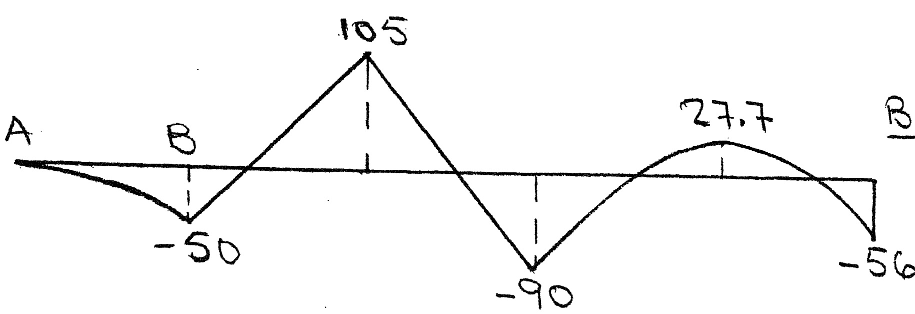
$B_y = 25.5\ \text{k} \uparrow$ $\quad D_y = 41.2\ \text{k} \uparrow$



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.39 The bending moments at C and E are selected as the redundants. Using beam deflection formulas, we obtain:

$$\theta_{CO\,rel.} = \theta_{CL} + \theta_{CR} = \frac{1}{EI}(768+336) = \frac{1104\ kN.m^2}{EI}$$

$$\theta_{EO\,rel.} = \theta_{EL} + \theta_{ER} = \frac{1}{EI}(384+600) = \frac{984\ kN.m^2}{EI}$$

$$f_{CC\,rel.} = f_{CCL} + f_{CCR} = \frac{1}{EI}\left(\frac{10}{3}+\frac{10}{6}\right) = \frac{5\ kN.m^2/kN.m}{EI}$$

$$f_{CE} = f_{EC} = \frac{10}{12EI} = \frac{0.833\ kN.m^2/kN.m}{EI}$$

$$f_{EE\,rel.} = f_{EEL} + f_{EER} = \frac{1}{EI}\left(\frac{10}{6}+\frac{8}{3}\right) = \frac{4.33\ kN.m^2/kN.m}{EI}$$

Compatibility Equations:

$$\theta_{CO\,rel.} + f_{CC\,rel.} M_C + f_{CE} M_E = 0$$

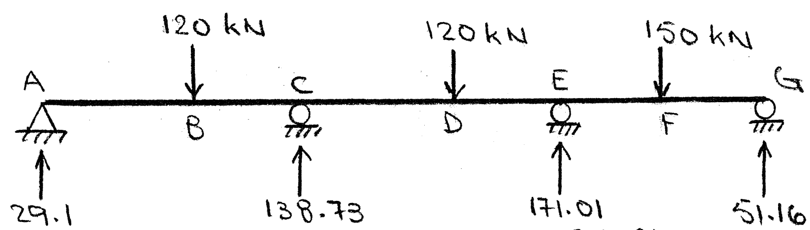
$$\theta_{EO\,rel.} + f_{EC} M_C + f_{EE\,rel.} M_E = 0$$

$$1104 + 5M_C + 0.833M_E = 0$$

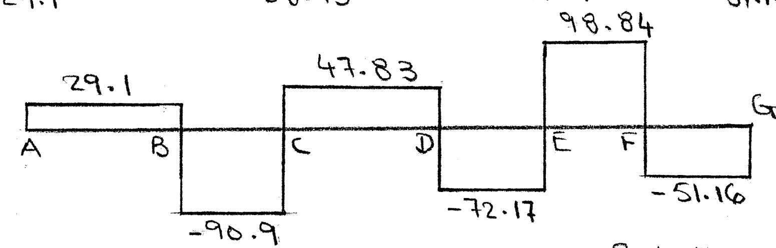
$$984 + 0.833M_C + 4.33M_E = 0$$

Solving these equations, we obtain

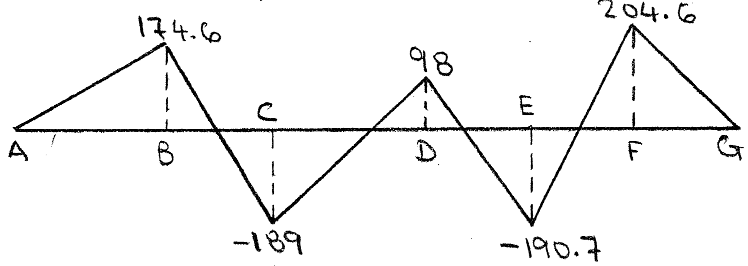
$M_C = -189$ kN.m $\qquad$ $M_E = -190.7$ kN.m



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN. m)

13.40 The bending moments at B and C are selected as the redundants. Using beam deflection formulas, we obtain:

$$\theta_{BOrel.} = \theta_{BL} + \theta_{BR} = \frac{1}{EI}(1440+720) = \frac{2160 \text{ k-ft}^2}{EI}$$

$$\theta_{COrel.} = \theta_{CL} + \theta_{CR} = \frac{1}{EI}(720+1440) = \frac{2160 \text{ k-ft}^2}{EI}$$

$$f_{BBrel.} = f_{BBL} + f_{BBR} = \frac{1}{EI}(8+4) = \frac{12 \text{ k-ft}^2/\text{k-ft}}{EI}$$

$$f_{CCrel.} = f_{CCL} + f_{CCR} = \frac{1}{EI}(4+8) = \frac{12 \text{ k-ft}^2/\text{k-ft}}{EI}$$

$$f_{BC} = f_{CB} = \frac{2 \text{ k-ft}^2/\text{k-ft}}{EI}$$

Compatibility Equations:

$$\theta_{BOrel.} + f_{BBrel.} M_B + f_{BC} M_C = 0$$

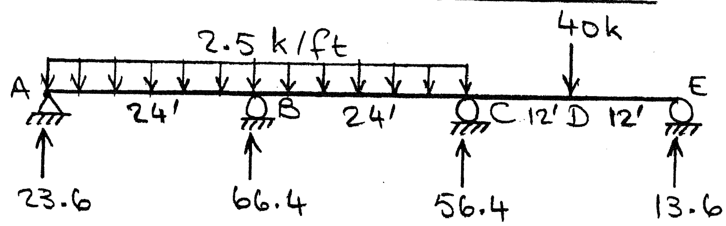
$$\theta_{COrel.} + f_{CB} M_B + f_{CCrel.} M_C = 0$$

$$2160 + 12 M_B + 2 M_C = 0$$

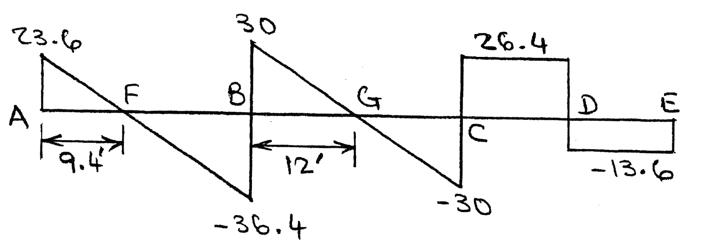
$$2160 + 2 M_B + 12 M_C = 0$$

Solving these equations, we obtain

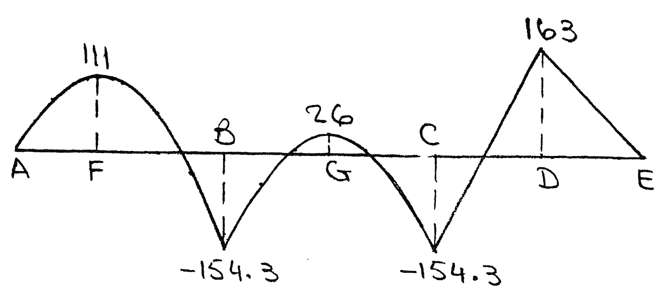
$$M_B = M_C = -154.3 \text{ k-ft}$$



Reactions

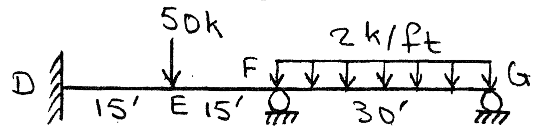


Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.41 As the beam and the loading are symmetric, we will analyze only the right half DG of the beam.



Using beam deflection formulas:

$$\Delta_{FO} = -\frac{1085625\ \text{k-ft}^3}{EI}\ ;\quad \Delta_{GO} = -\frac{3076875\ \text{k-ft}^3}{EI}$$

$$f_{FF} = \frac{9000\ \text{k-ft}^3/\text{k}}{EI}\ ;\quad f_{FG} = f_{GF} = \frac{22500\ \text{k-ft}^3/\text{k}}{EI}$$

$$f_{GG} = \frac{72000\ \text{k-ft}^3/\text{k}}{EI}$$

Compatibility Equations:

$$\Delta_{FO} + f_{FF}F_y + f_{FG}G_y = 0$$

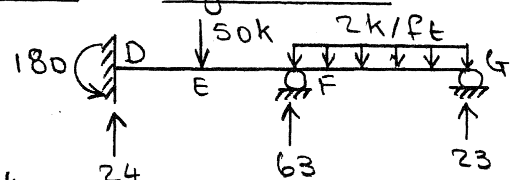
$$\Delta_{GO} + f_{GF}F_y + f_{GG}G_y = 0$$

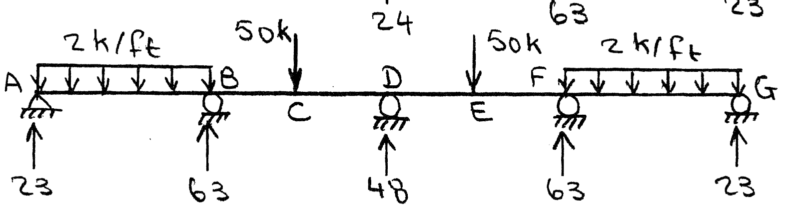
$$-1085625 + 9000\,F_y + 22500\,G_y = 0$$

$$-3076875 + 22500\,F_y + 72000\,G_y = 0$$

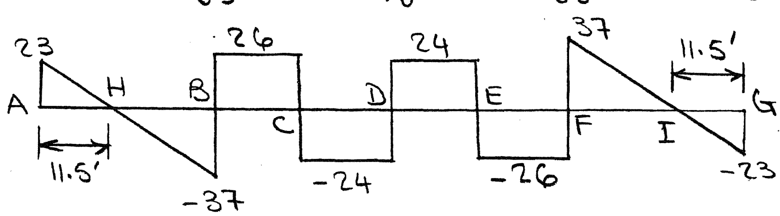
Solving these equations, we obtain

$F_y = 63\ \text{k}\uparrow$ $\quad G_y = 23\ \text{k}\uparrow$

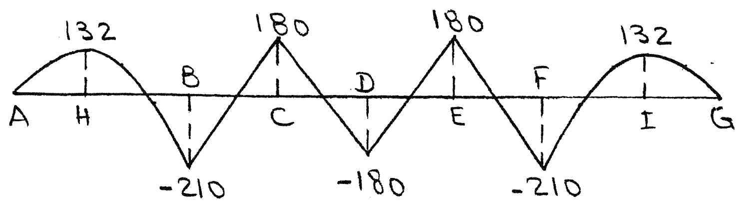




Reactions

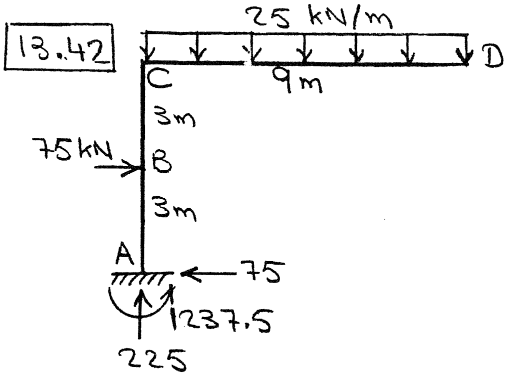


Shear Diagram (k)

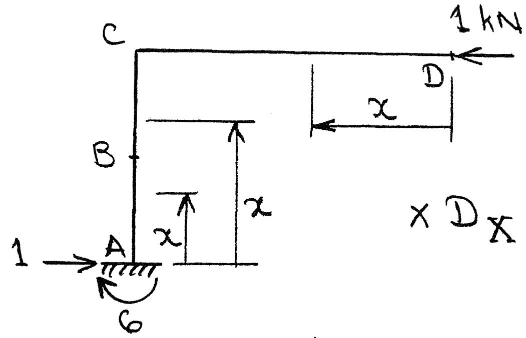


Bending Moment Diagram (k-ft)

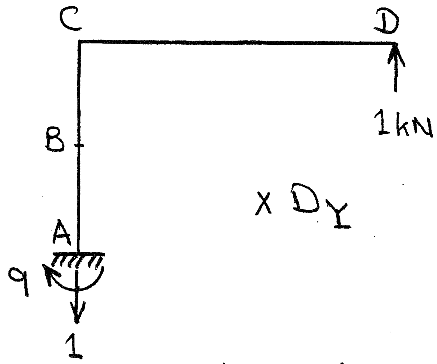
13.42



M_0 Moments



m_{DX} Moments



m_{DY} Moments

Segment	x Coordinate		M_0 (kN.m)	m_{DX} (kN.m/kN)	m_{DY} (kN.m/kN)
	Origin	Limits (m)			
AB	A	0–3	$-1237.5+75x$	$6-x$	9
BC	A	3–6	-1012.5	$6-x$	9
DC	D	0–9	$-12.5x^2$	0	x

$$\Delta_{DXO} = \Sigma\int \frac{M_0 m_{DX}}{EI}\,dx = -\frac{19912.5 \text{ kN-m}^3}{EI}$$

$$\Delta_{DYO} = \Sigma\int \frac{M_0 m_{DY}}{EI}\,dx = -\frac{78215.625 \text{ kN-m}^3}{EI}$$

$$f_{DX,DX} = \Sigma\int \frac{m_{DX}^2}{EI}\,dx = \frac{72 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DY,DY} = \Sigma\int \frac{m_{DY}^2}{EI}\,dx = \frac{729 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DX,DY} = f_{DY,DX} = \Sigma\int \frac{m_{DX} m_{DY}}{EI}\,dx = \frac{162 \text{ kN-m}^3/\text{kN}}{EI}$$

13.42 (contd.) Compatibility Equations:

$$\Delta_{DX0} + f_{DX,DX} D_X + f_{DX,DY} D_Y = 0$$

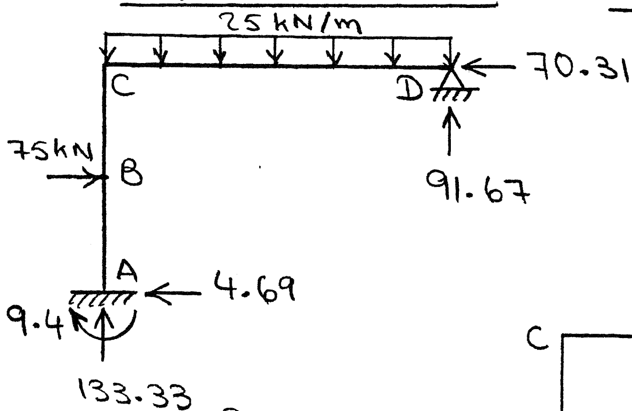
$$\Delta_{DY0} + f_{DY,DX} D_X + f_{DY,DY} D_Y = 0$$

$$-19912.5 + 72 D_X + 162 D_Y = 0$$

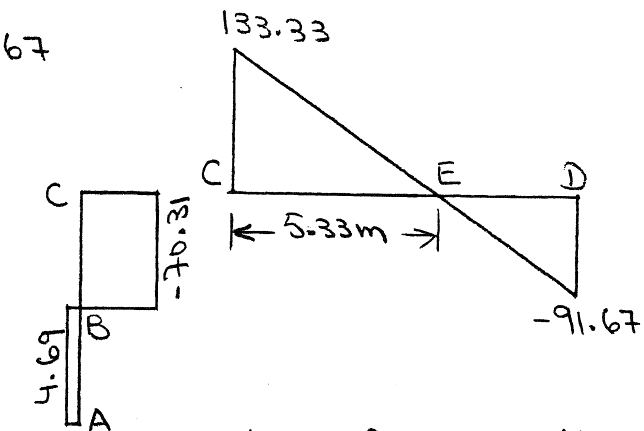
$$-78215.625 + 162 D_X + 729 D_Y = 0$$

Solving these equations, we obtain

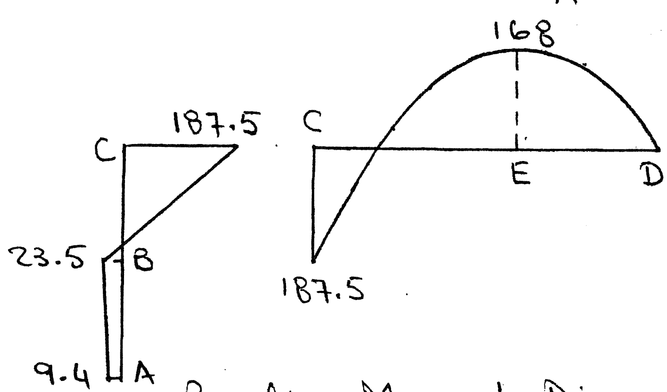
$D_X = 70.3$ kN ← $\qquad$ $D_Y = 91.7$ kN ↑



Reactions

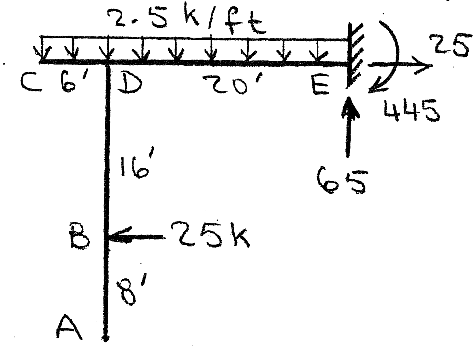


Shear Diagrams (kN)

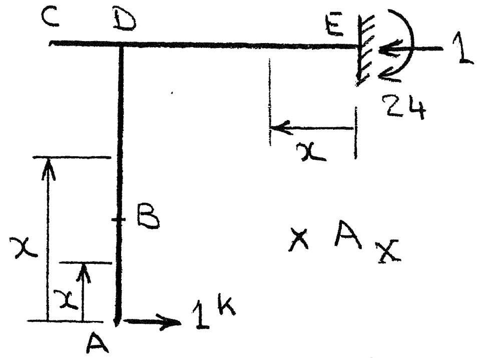


Bending Moment Diagrams (kN.m)

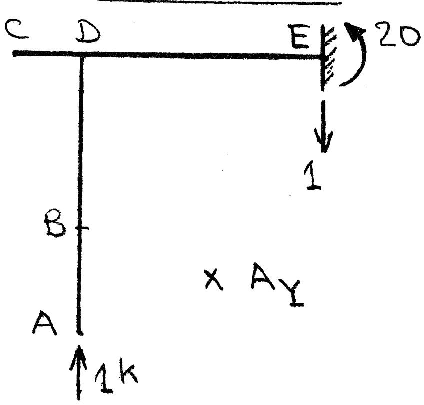
13.43 The reactions A_X and A_Y are selected as the redundants.



M_0 Moments



m_{AX} Moments



m_{AY} Moments

Segment	x Coordinate		M_0 (k-ft)	m_{AX} (k-ft/k)	m_{AY} (k-ft/k)
	Origin	Limits (ft)			
AB	A	0-8	0	$-1x$	0
BD	A	8-24	$25(x-8)$	$-1x$	0
ED	E	0-20	$-445+65x$ $-1.25x^2$	-24	$20-x$

$$\Delta_{AX0} = \sum \int \frac{M_0 m_{AX}}{EI} dx = -\frac{78133.33 \text{ k-ft}^3}{EI}$$

$$\Delta_{AY0} = \sum \int \frac{M_0 m_{AY}}{EI} dx = -\frac{19000 \text{ k-ft}^3}{EI}$$

$$f_{AX,AX} = \sum \int \frac{m_{AX}^2}{EI} dx = \frac{16128 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{AY,AY} = \sum \int \frac{m_{AY}^2}{EI} dx = \frac{2666.67 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{AX,AY} = f_{AY,AX} = \sum \int \frac{m_{AX} m_{AY}}{EI} dx = -\frac{4800 \text{ k-ft}^3/\text{k}}{EI}$$

13.43 (contd.) Compatibility Equations:

$$\Delta_{AX0} + f_{AX,AX} A_X + f_{AX,AY} A_Y = 0$$

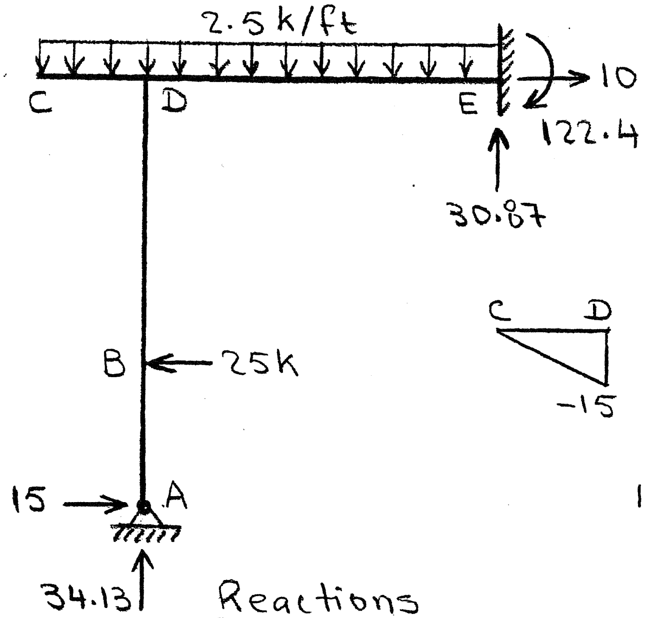
$$\Delta_{AY0} + f_{AY,AX} A_X + f_{AY,AY} A_Y = 0$$

$$-78133.33 + 16128 A_X - 4800 A_Y = 0$$

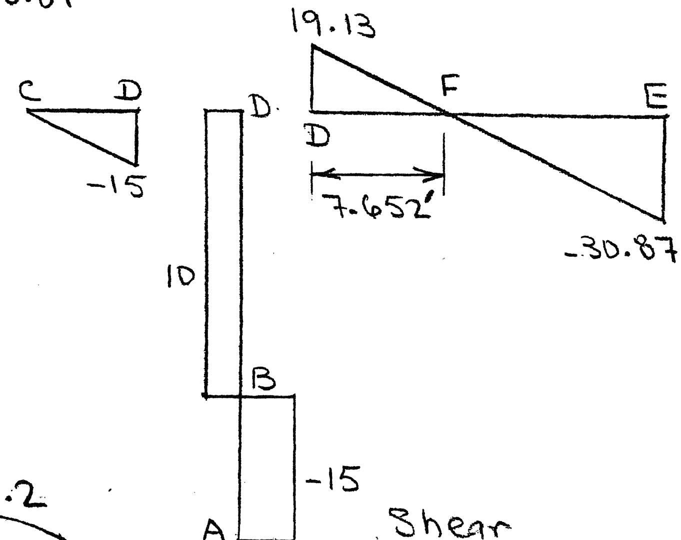
$$-19000 - 4800 A_X + 2666.67 A_Y = 0$$

Solving these equations, we obtain:

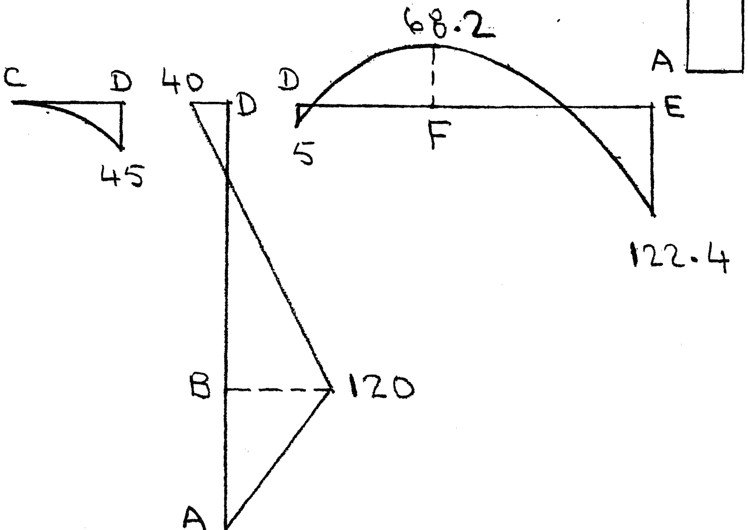
$A_X = 15\text{ k} \rightarrow$ $\qquad$ $A_Y = 34.13\text{ k} \uparrow$



Reactions

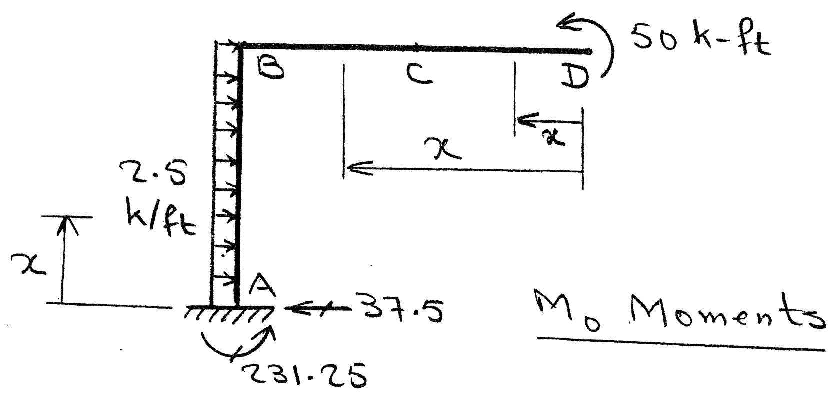


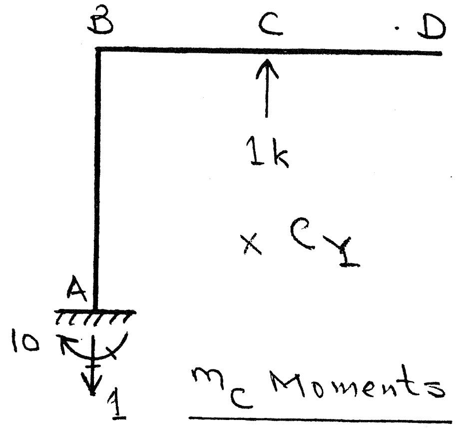
Shear Diagrams (k)

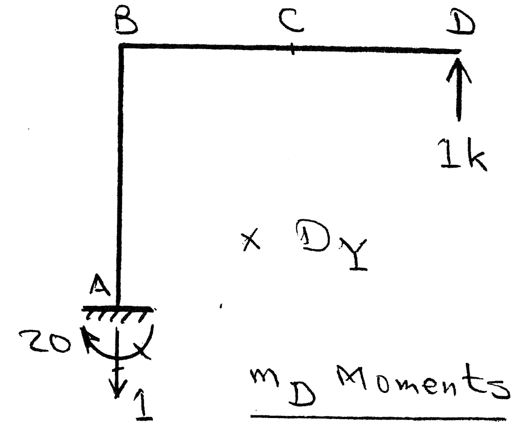


Bending Moment Diagrams (k-ft)

13.44







Member	x coordinate		M_o (k-ft)	m_C	m_D
	Origin	Limits (ft)			
AB	A	0 - 15	$-231.25+37.5x-1.25x^2$	10	20
CB	D	10 - 20	50	$1(x-10)$	$1x$
DC	D	0 - 10	50	0	$1x$

$$\Delta_{CO} = \Sigma\int \frac{M_o m_C}{EI}\,dx = -\frac{4062.5 \text{ k-ft}^3}{EI}$$

$$\Delta_{DO} = \Sigma\int \frac{M_o m_D}{EI}\,dx = -\frac{3125 \text{ k-ft}^3}{EI}$$

$$f_{CC} = \Sigma\int \frac{m_C^2}{EI}\,dx = \frac{1833.33 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{DD} = \Sigma\int \frac{m_D^2}{EI}\,dx = \frac{8666.67 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{CD} = f_{DC} = \Sigma\int \frac{m_C m_D}{EI}\,dx = \frac{3833.33 \text{ k-ft}^3/\text{k}}{EI}$$

13.44 (Contd.) Compatibility Equations:

$$\Delta_{CO} + f_{CC} C_Y + f_{CD} D_Y = 0$$

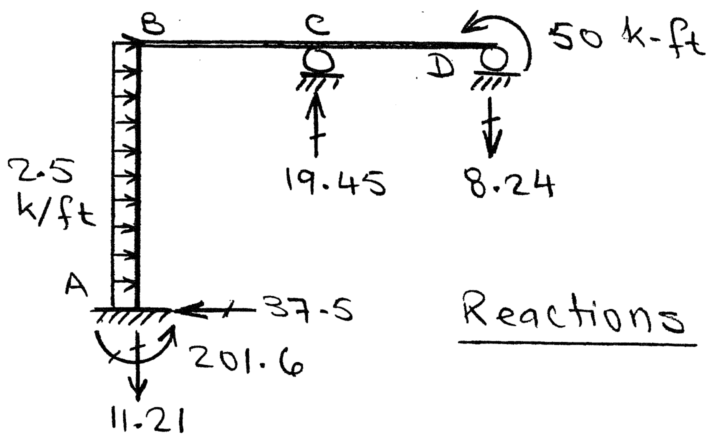
$$\Delta_{DO} + f_{DC} C_Y + f_{DD} D_Y = 0$$

$$-4062.5 + 1833.33 C_Y + 3833.33 D_Y = 0$$

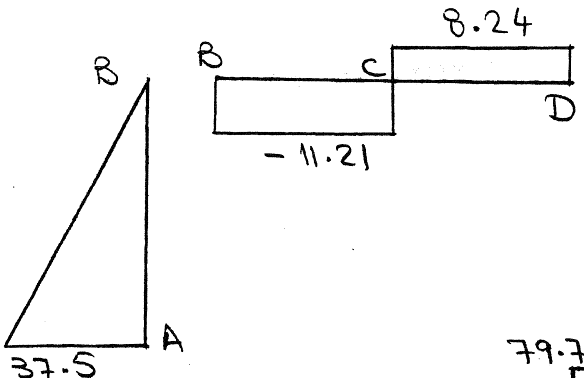
$$-3125 + 3833.33 C_Y + 8666.67 D_Y = 0$$

Solving these equations, we obtain:

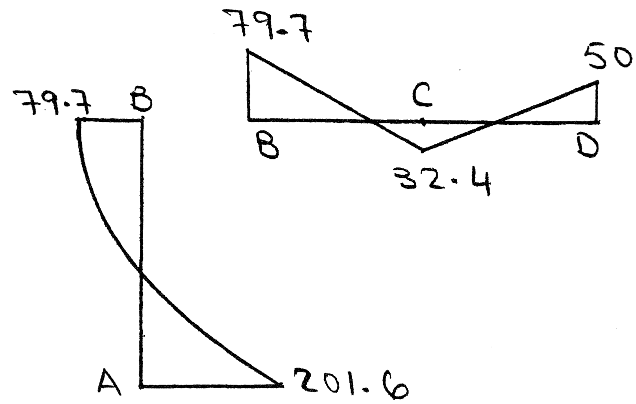
$C_Y = 19.45 \text{ k} \uparrow$ $\quad$ $D_Y = -8.24 \text{ k} = 8.24 \text{ k} \downarrow$



Reactions

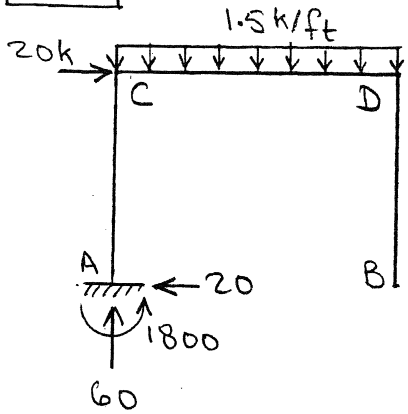


Shear Diagrams (k)

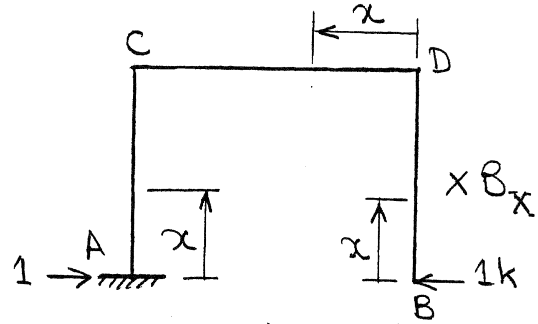


Bending Moment Diagrams (k-ft)

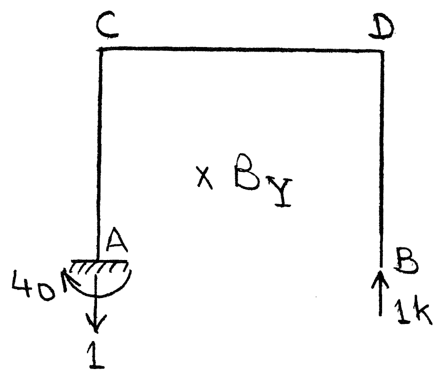
13.45



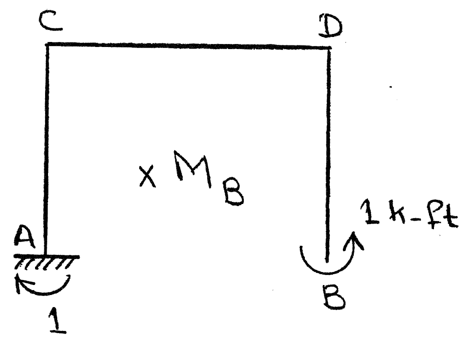
M_0 Moments



m_{BX} Moments



m_{BY} Moments



m_{MB} Moments

Member	x Coordinate		Moment of Inertia	M_0 (k-ft)	m_{BX}	m_{BY}	m_{MB}
	Origin	Limits (ft)					
AC	A	0-30	I	$-1800+20x$	$-x$	40	1
DC	D	0-40	2I	$-0.75x^2$	-30	x	1
BD	B	0-30	I	0	x	0	-1

$$\Delta_{BXO} = \Sigma\int \frac{M_0 m_{DX}}{EI}\,dx = \frac{870000 \text{ k-ft}^3}{EI}$$

$$\Delta_{BYO} = \Sigma\int \frac{M_0 m_{DY}}{EI}\,dx = -\frac{2040000 \text{ k-ft}^3}{EI}$$

$$\theta_{BO} = \Sigma\int \frac{M_0 m_{MB}}{EI}\,dx = -\frac{53000 \text{ k-ft}^2}{EI}$$

$$f_{BX,BX} = \Sigma\int \frac{m_{BX}^2}{EI}\,dx = \frac{36000 \text{ k-ft}^3/\text{k}}{EI}$$

13.45 (Contd.) $f_{BY,BY} = \sum \int \frac{m_{BY}^2}{EI} dx = \frac{58666.67}{EI}$

$$f_{MB,MB} = \sum \int \frac{m_{MB}^2}{EI} dx = \frac{80}{EI}$$

$$f_{BX,BY} = f_{BY,BX} = \sum \int \frac{m_{BX} m_{BY}}{EI} dx = -\frac{30000}{EI}$$

$$f_{BX,MB} = f_{MB,BX} = \sum \int \frac{m_{BX} m_{MB}}{EI} dx = -\frac{1500}{EI}$$

$$f_{BY,MB} = f_{MB,BY} = \sum \int \frac{m_{BY} m_{MB}}{EI} dx = \frac{1600}{EI}$$

Compatibility Equations:

$$\Delta_{BXO} + f_{BX,BX} B_X + f_{BX,BY} B_Y + f_{BX,MB} M_B = 0$$

$$\Delta_{BYO} + f_{BY,BX} B_X + f_{BY,BY} B_Y + f_{BY,MB} M_B = 0$$

$$\theta_{BO} + f_{MB,BX} B_X + f_{MB,BY} B_Y + f_{MB,MB} M_B = 0$$

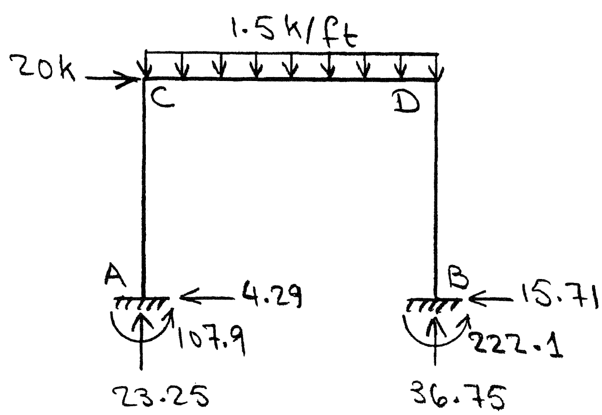
$$870000 + 36000 B_X - 30000 B_Y - 1500 M_B = 0$$

$$-2040000 - 30000 B_X + 58666.67 B_Y + 1600 M_B = 0$$

$$-53000 - 1500 B_X + 1600 B_Y + 80 M_B = 0$$

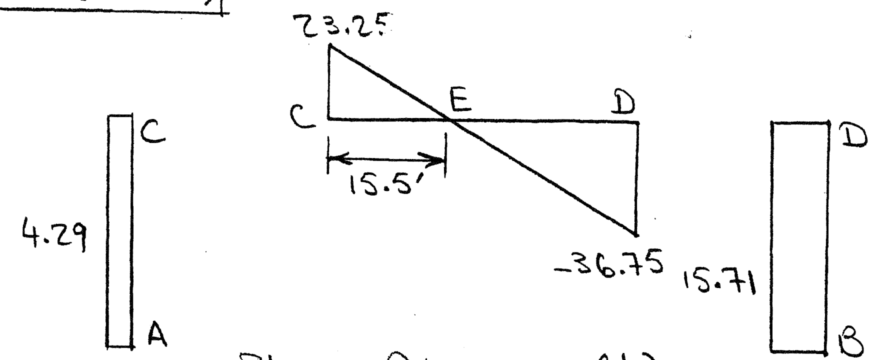
Solving these equations, we obtain:

$B_X = 15.71$ k ← $B_Y = 36.75$ k ↑ $M_B = 222.1$ k-ft ↺

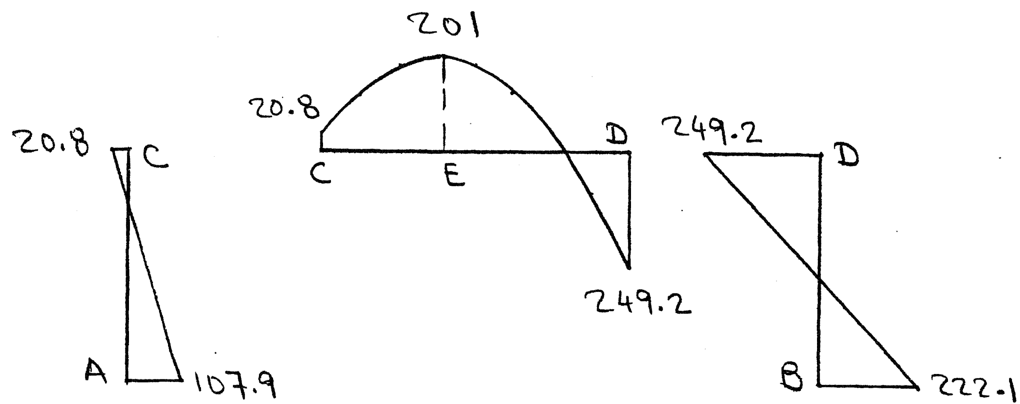


Reactions

13.45 (contd.)

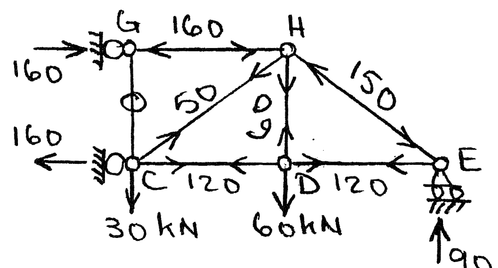


Shear Diagrams (k)

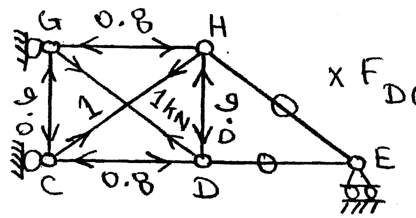


Bending Moment Diagrams (k-ft)

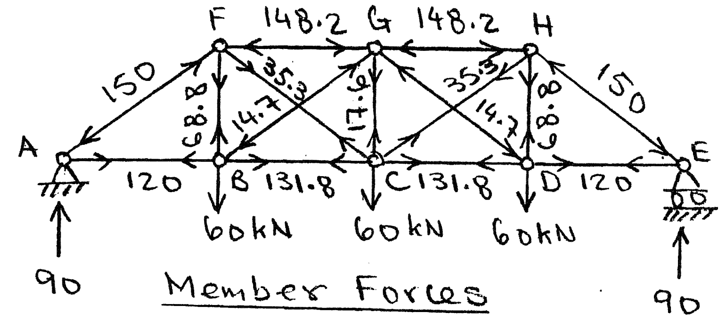
13.46 As the truss and the loading are symmetric, we will analyze only the right half CEGH of the truss.



F_0 Forces



u_{DG} Forces

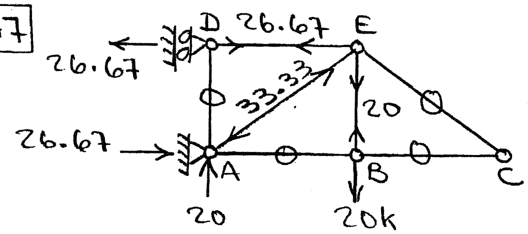


Member Forces

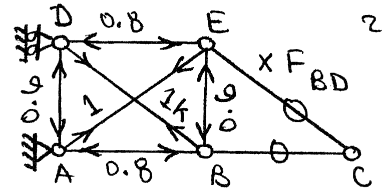
Member	L (m)	Relative A	F_0 (kN)	u_{DG} (kN/kN)	$\frac{F_0 u_{DG} L}{A}$	$\frac{u_{DG}^2 L}{A}$	$F = F_0 + u_{DG} F_{DG}$ (kN)
CD	8	1	120	−0.8	−768	5.12	131.8
DE	8	1	120	0	0	0	120
GH	8	1	−160	−0.8	1024	5.12	−148.2
CG	6	1/2	0	−0.6	0	4.32	8.8
DH	6	1	60	−0.6	−216	2.16	68.8
CH	10	1	50	1	500	10	35.3
EH	10	1	−150	0	0	0	−150
DG	10	1	0	1	0	10	−14.7
				Σ	540	36.72	

$$F_{DG} = -\frac{540}{36.72} = -14.7\ \text{kN} = \underline{14.7\ \text{kN (C)}}$$

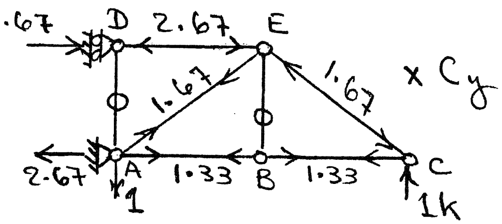
13.47



F_O Forces



u_{BD} Forces



u_C Forces

Member	L (in.)	A (in².)	F_O (k)	u_{BD} (k/k)	u_C (k/k)	$\frac{F_O u_{BD} L}{A}$	$\frac{F_O u_C L}{A}$	$\frac{u_{BD}^2 L}{A}$	$\frac{u_C^2 L}{A}$	$\frac{u_{BD} u_C L}{A}$	$F = F_O + u_{BD} F_{BD} + u_C C_y$ (k)
AB	192	8	0	-0.8	1.33	0	0	15.36	42.45	-25.54	-1.44
BC	192	8	0	0	1.33	0	0	0	42.45	0	8.29
DE	192	8	26.67	-0.8	-2.67	-512.06	-1709.01	15.36	171.09	51.26	0.31
AD	144	6	0	-0.6	0	0	0	8.64	0	0	-7.3
BE	144	6	20	-0.6	0	-288	0	8.64	0	0	12.7
AE	240	6	-33.33	1	1.67	-1333.2	-2226.44	40	111.56	66.8	-10.77
CE	240	8	0	0	-1.67	0	0	0	83.67	0	-10.4
BD	240	6	0	1	0	0	0	40	0	0	12.16
					Σ	-2133.26	-3935.45	128	451.22	92.52	

$\Delta_{BDO} = \frac{-2133.26}{E}$; $\Delta_{CO} = -\frac{3935.45}{E}$; $f_{BD,BD} = \frac{128}{E}$; $f_{CC} = \frac{451.22}{E}$

$f_{BD,C} = f_{C,BD} = \frac{92.52}{E}$

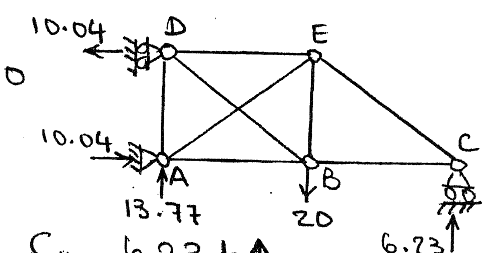
Compatibility Equations: $\Delta_{BDO} + f_{BD,BD} F_{BD} + f_{BD,C} C_y = 0$

$\Delta_{CO} + f_{C,BD} F_{BD} + f_{CC} C_y = 0$

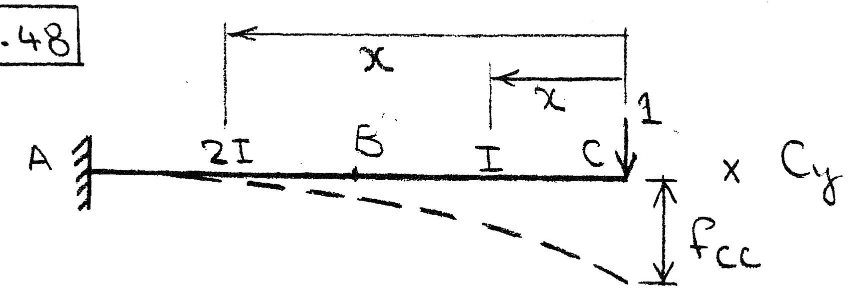
$-2133.26 + 128 F_{BD} + 92.52 C_y = 0$

$-3935.45 + 92.52 F_{BD} + 451.22 C_y = 0$

Solving these equations, we obtain: $F_{BD} = 12.16\text{ k (T)}$; $C_y = 6.23\text{ k}\uparrow$



13.48



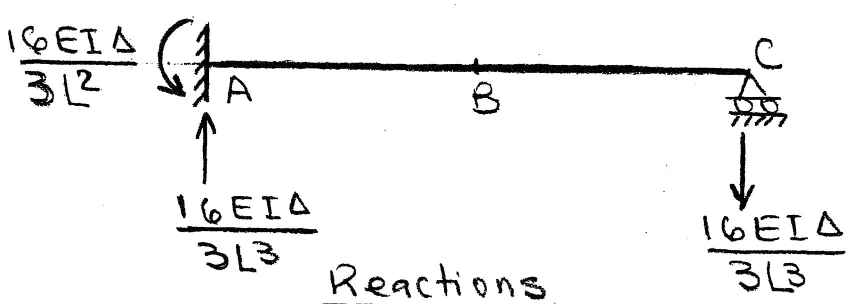
Using the virtual work method:

$$f_{CC} = \frac{1}{EI}\left[\int_0^{L/2}(-1x)^2\,dx + \frac{1}{2}\int_{L/2}^{L}(-1x)^2\,dx\right]$$

$$= \frac{3L^3}{16EI}$$

Compatibility Equation: $f_{CC}\,C_y = \Delta$

$$C_y = \frac{16EI\Delta}{3L^3}\downarrow$$



Reactions

13.49 From the solution of Problem 13.1

$$\Delta_{DO} = -\frac{14760 \text{ kN.m}^3}{EI} = -\frac{14760}{200(3250)} = -0.0227 \text{ m}$$

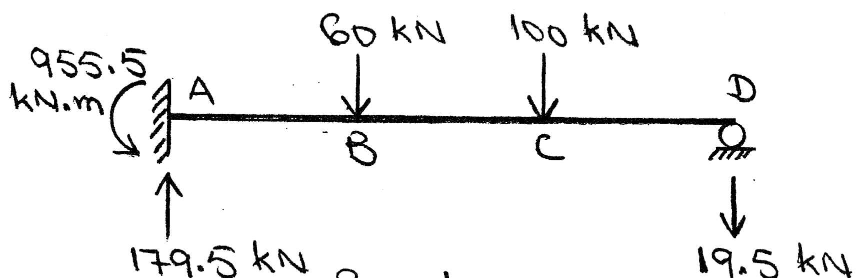
$$f_{DD} = \frac{243 \text{ kN.m}^3/\text{kN}}{EI} = \frac{243}{200(3250)} = 0.000374 \text{ m/kN}$$

Compatibility Equation:

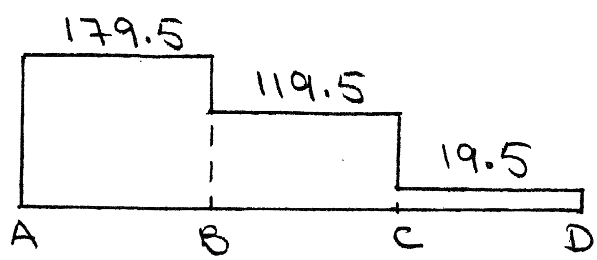
$$\Delta_{DO} + f_{DD} D_y = \Delta_D$$

$$-0.0227 + (0.000374) D_y = -0.03$$

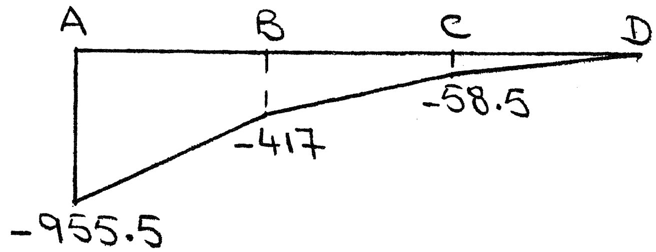
$$D_y = -\frac{0.0073}{0.000374} = -19.5 \text{ kN} = \underline{19.5 \text{ kN} \downarrow}$$



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

13.50 From the solution of Problem 13.9:

$$\Delta_{CO} = -\frac{158400\ \text{k-ft}^3}{EI} = -\frac{158400(12)^3}{29000(1500)} = -6.292\ \text{in.}$$

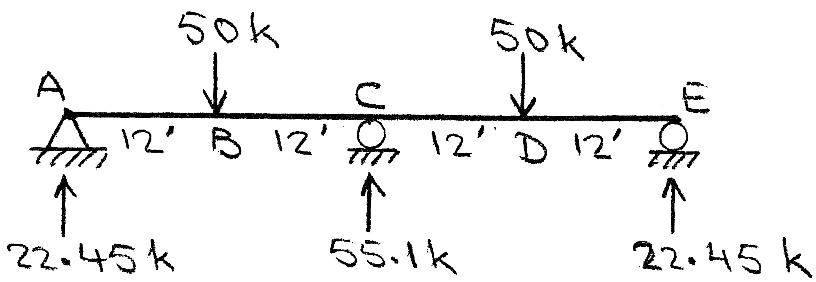
$$f_{CC} = \frac{2304\ \text{k-ft}^3/\text{k}}{EI} = \frac{2304(12)^3}{29000(1500)} = 0.0915\ \text{in./k}$$

Compatibility Equation:

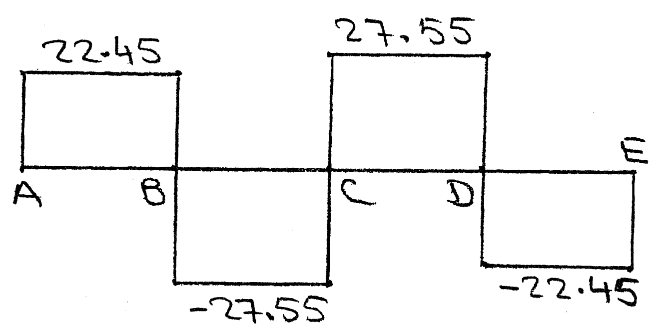
$$\Delta_{CO} + f_{CC}C_y = \Delta_C$$

$$-6.292 + (0.0915)C_y = -1.25$$

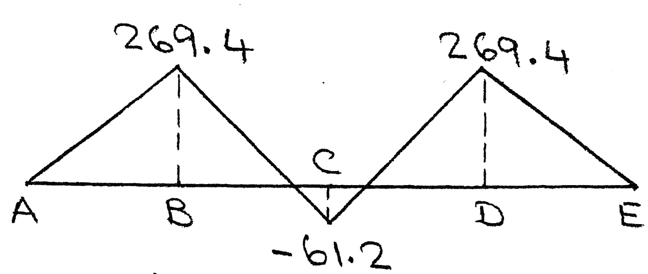
$$C_y = \frac{5.042}{0.0915} = \underline{55.1\ \text{k}\uparrow}$$



Reactions



Shear Diagram (k)

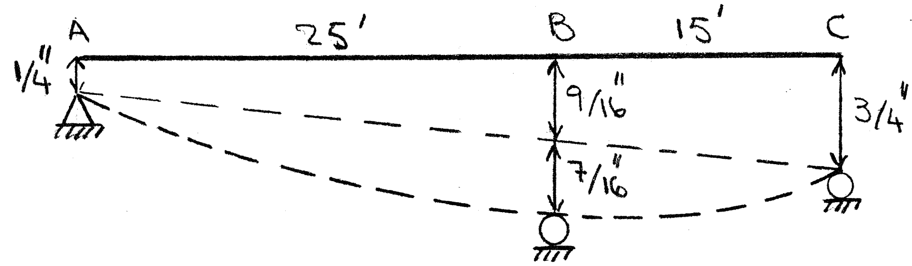


Bending Moment Diagram (k-ft)

13.51 From the solution of Problem 13.12:

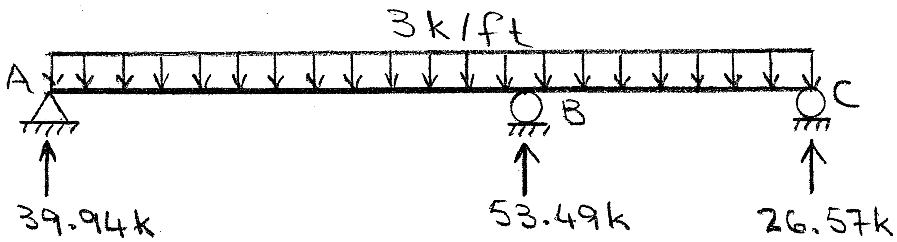
$$\Delta_{BO} = -\frac{61450.2\ \text{k-ft}^3}{EI} = -\frac{61450.2\,(12)^3}{29000\,(2500)} = -1.465 \text{ in.}$$

$$f_{BB} = \frac{805.66\ \text{k-ft}^3/\text{k}}{EI} = \frac{805.66\,(12)^3}{29000\,(2500)} = 0.0192 \text{ in./k}$$

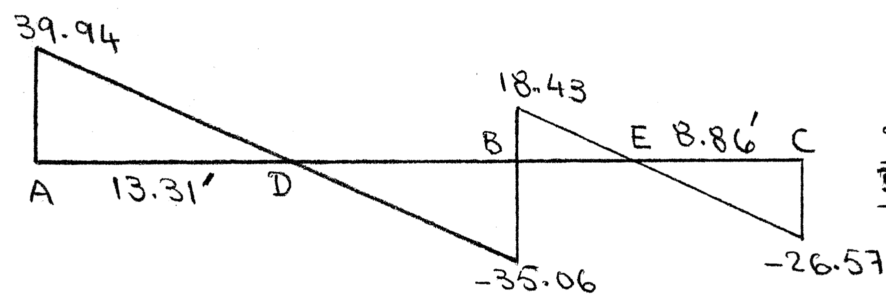


Compatibility Equation:

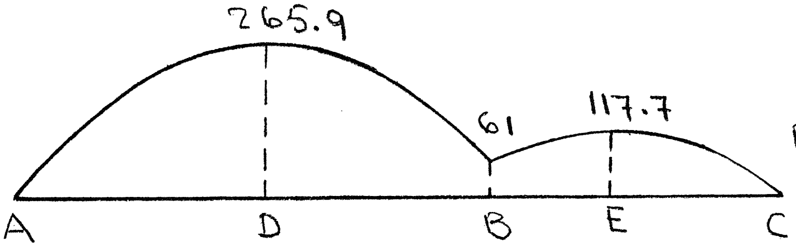
$$-1.465 + 0.0192\,B_y = -\frac{7}{16} \qquad \underline{B_y = 53.49\text{ k}\uparrow}$$



Reactions



Shear Diagram (k)

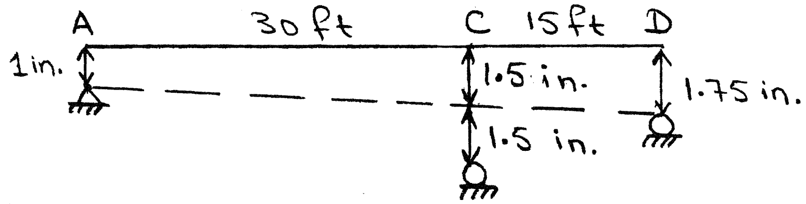


Bending Moment Diagram (k-ft)

13.52 From the solution of Problem 13.27:

$$\Delta_{CO} = -\frac{22046.28\ (\text{k-in})}{EA} = -\frac{22046.28}{29000(6)} = -0.127\ \text{in.}$$

$$f_{CC} = \frac{531.78\ (\text{in.})}{EA} = \frac{531.78}{29000(6)} = 0.00306\ \text{in./k}$$

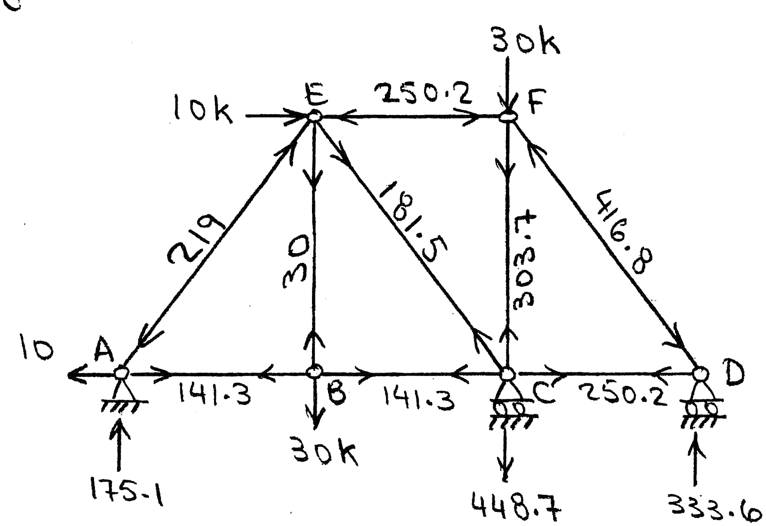


Compatibility Equation:

$$-0.127 + (0.00306)\,C_y = -1.5$$

$$C_y = -448.7\ \text{k} = \underline{448.7\ \text{k}\ \downarrow}$$

By using the F_O and M_C forces computed in the solution of Problem 13.17, we obtain the following member forces.



Member Forces and Reactions (k)

13.53 From the solution of Problem 13.37:

$$\Delta_{BO} = -\frac{72533.33\ kN.m^3}{EI} = -\frac{72533.33}{70(1300)} = -0.797\ m$$

$$\Delta_{CO} = -\frac{204800\ kN.m^3}{EI} = -\frac{204800}{70(1300)} = -2.251\ m$$

$$f_{BB} = \frac{170.67\ kN.m^3/kN}{EI} = \frac{170.67}{70(1300)} = 0.00188\ m/kN$$

$$f_{BC} = f_{CB} = \frac{426.67\ kN.m^3/kN}{EI} = \frac{426.67}{70(1300)} = 0.00469\ m/kN$$

$$f_{CC} = \frac{1365.33\ kN.m^3/kN}{EI} = \frac{1365.33}{70(1300)} = 0.015\ m/kN$$

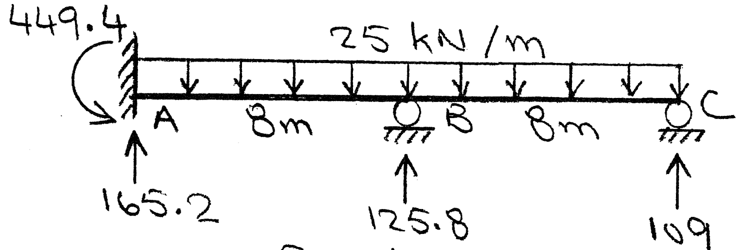
Compatibility Equations:

$$-0.797 + (0.00188)B_y + (0.00469)C_y = -0.05$$

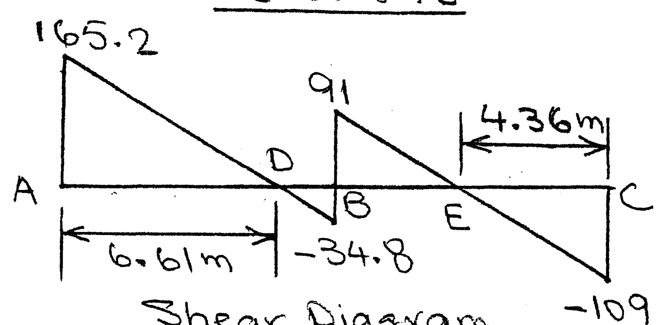
$$-2.251 + (0.00469)B_y + (0.015)C_y = -0.025$$

Solving these equations, we obtain

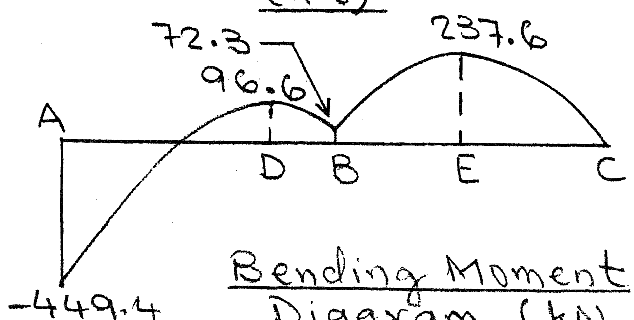
$B_y = 125.8\ kN \uparrow$ $\quad$ $C_y = 109\ kN \uparrow$



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

13.54 From the solution of Problem 13.39:

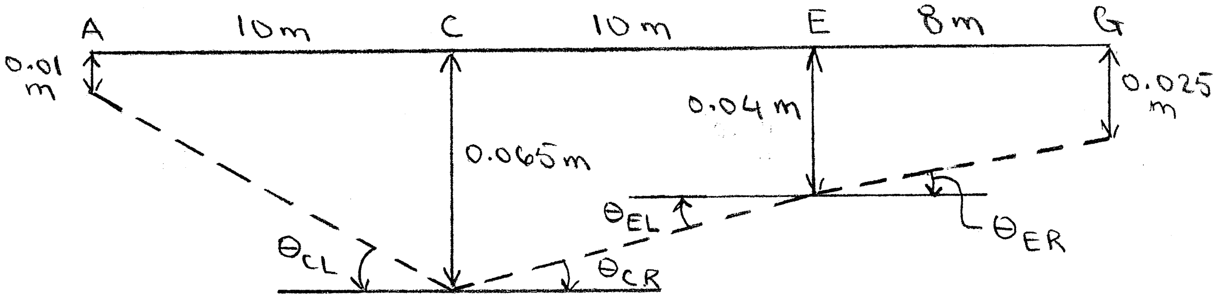
$$\theta_{CO rel.} = \frac{1104 \text{ kN.m}^2}{EI} = \frac{1104}{200(500)} = 0.01104 \text{ rad.}$$

$$\theta_{EO rel.} = \frac{984 \text{ kN.m}^2}{EI} = 0.00984 \text{ rad.}$$

$$f_{CC rel.} = \frac{5 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.00005 \text{ rad./kN.m}$$

$$f_{EE rel.} = \frac{4.33 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.0000433 \text{ rad./kN.m}$$

$$f_{CE} = f_{EC} = \frac{0.833 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.00000833 \text{ rad./kN.m}$$



$$\theta_{CL} = \frac{0.065 - 0.01}{10} = 0.0055 \text{ rad.}$$

$$\theta_{CR} = \frac{0.065 - 0.04}{10} = 0.0025 \text{ rad.}$$

$$\theta_C = \theta_{CL} + \theta_{CR} = 0.008 \text{ rad.}$$

$$\theta_{EL} = \frac{0.04 - 0.065}{10} = -0.0025 \text{ rad.}$$

$$\theta_{ER} = \frac{0.04 - 0.025}{8} = 0.001875 \text{ rad.}$$

$$\theta_E = \theta_{EL} + \theta_{ER} = -0.000625 \text{ rad.}$$

Compatibility Equations:

$$\theta_{CO rel.} + f_{CC rel.} M_C + f_{CE} M_E = \theta_C$$

$$\theta_{EO rel.} + f_{EC} M_C + f_{EE rel.} M_E = \theta_E$$

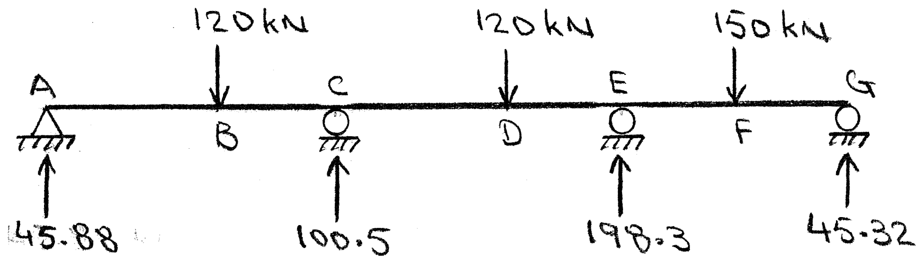
$$1104 + 5M_C + 0.833 M_E = 800$$

$$984 + 0.833 M_C + 4.33 M_E = -62.5$$

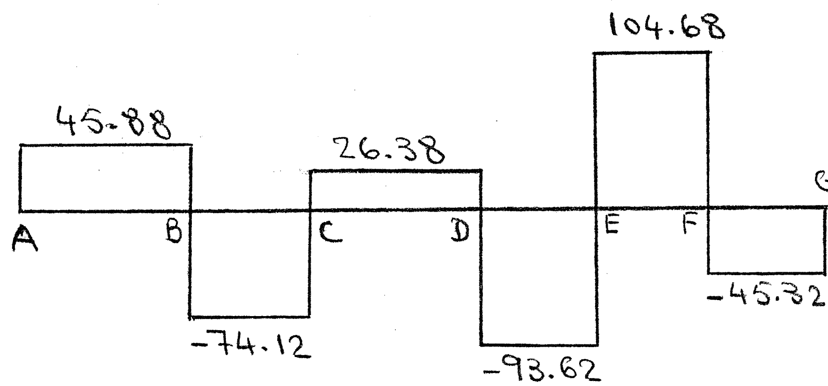
13.54 (contd.)

Solving these equations, we obtain

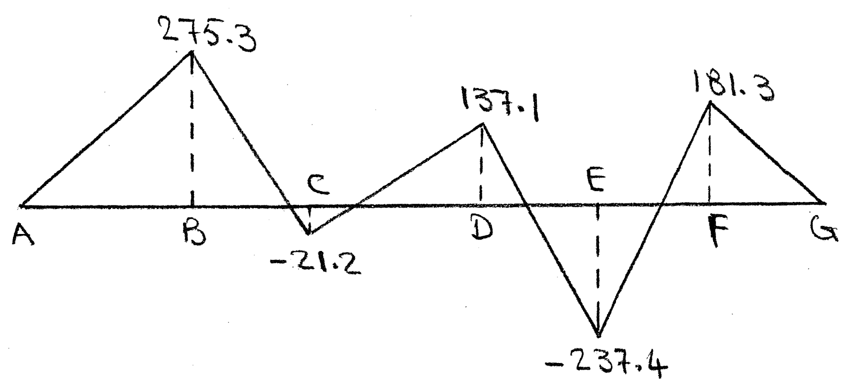
$M_C = -21.23$ kN.m $\quad M_E = -237.42$ kN.m



Reactions

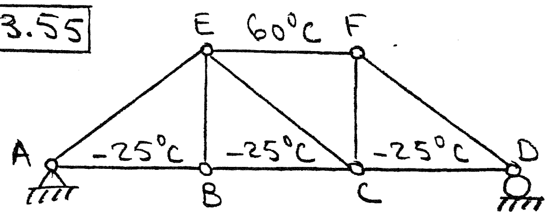


Shear Diagram (kN)

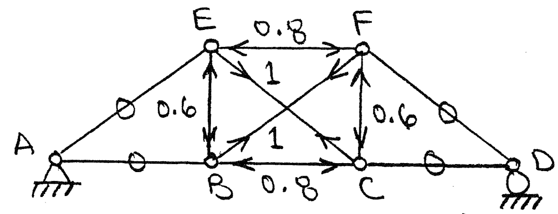


Bending Moment Diagram (kN.m)

13.55



Temperature Changes



u_{BF} Forces (kN)

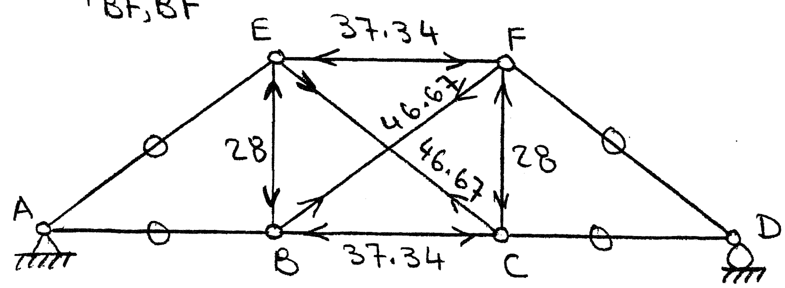
$$\Delta_{BFO} = \alpha \sum (\Delta T) L u_{BF} = 1.2 \times 10^{-5} (8)\left[-25(-0.8) + 60(-0.8)\right] = -0.002688 \text{ m} = -2.688 \text{ mm}$$

$$f_{BF,BF} = \frac{1}{EA} \sum u_{BF}^2 L$$

$$= \frac{1}{200(10^6)(0.003)}\left[2(1)^2(10) + 2(-0.8)^2(8) + + 2(-0.6)^2(6)\right] = 57.6(10^{-6}) \text{ m/kN}$$

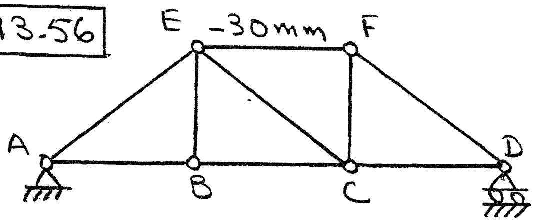
$$= 0.0576 \text{ mm/kN}$$

$$F_{BF} = -\frac{\Delta_{BFO}}{f_{BF,BF}} = \underline{46.67 \text{ kN (T)}}$$

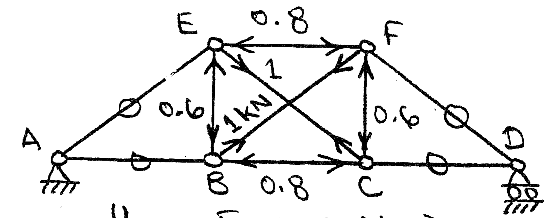


Member Forces (kN)

13.56



Fabrication Errors



u_{BF} Forces (kN)

$$\Delta_{BFO} = \Sigma \delta u_{BF} = -30(-0.8) = 24 \text{ mm}$$

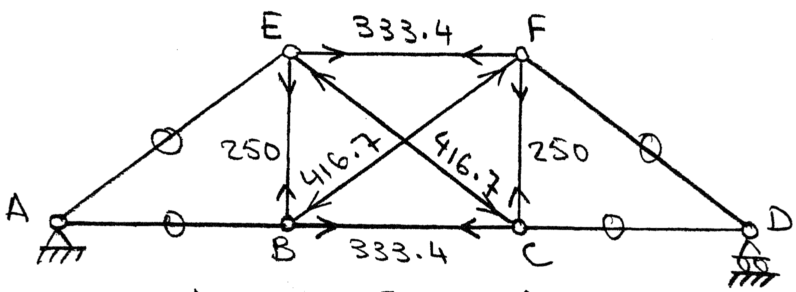
$$f_{BF,BF} = \frac{1}{EA} \Sigma u_{BF}^2 L$$

$$= \frac{1}{200(10^6)(0.003)} \left[2(1)^2 10 + 2(-0.8)^2 8 + 2(-0.6)^2 6 \right]$$

$$= 57.6(10^{-6}) \text{ m/kN} = 0.0576 \text{ mm/kN}$$

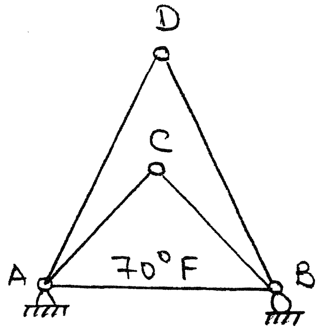
$$F_{BF} = -\frac{\Delta_{BFO}}{f_{BF,BF}} = -\frac{24}{0.0576} = -416.7 \text{ kN}$$

$$= \underline{416.7 \text{ kN (C)}}$$

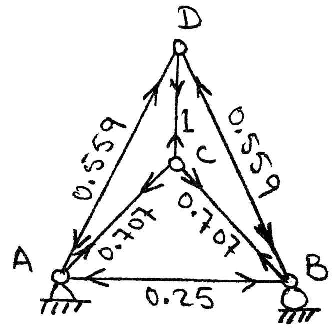


Member Forces (kN)

13.57



Temperature Change



u_{CD} Forces (k)

$$\Delta_{CD0} = \Sigma \alpha(\Delta T) L u_{CD} = 6.5\times10^{-6}(70)(20)(-0.25)$$

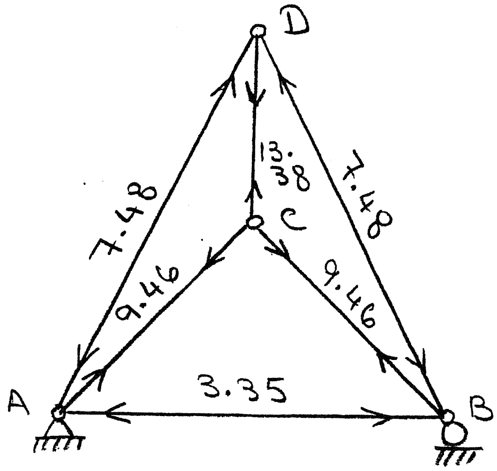
$$= -0.002275 \text{ ft} = -0.0273 \text{ in.}$$

$$f_{CD,CD} = \frac{1}{EA}\Sigma u_{CD}^2 L$$

$$= \frac{1}{29000(8)}\left[(1)^2(10) + (-0.25)^2(20) + 2(-0.559)^2 \times (22.36) + 2(0.707)^2(14.14)\right] = 0.00017 \text{ ft/k}$$

$$= 0.00204 \text{ in./k.}$$

$$F_{CD} = -\frac{\Delta_{CD0}}{f_{CD,CD}} = \underline{13.38 \text{ k (T)}}$$



Member Forces (k)

Chapter Fourteen
Three Moment Equation and the Method of Least Work

CHAPTER 14

14.1

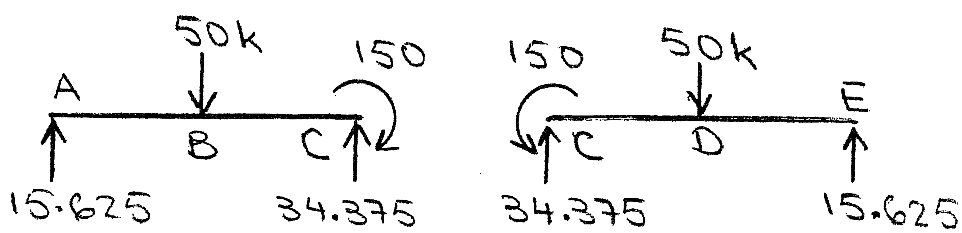
Three-Moment Equation at Joint C:

$$M_A = M_E = 0$$

$$2M_C\left(\frac{16}{2I} + \frac{16}{I}\right) = -\frac{50(16)^2(0.5)(1-0.25)}{2I} - \frac{50(16)^2(0.5)(1-0.25)}{I}$$

$$48M_C = -7200$$

$$\underline{M_C = -150 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.11.

14.2

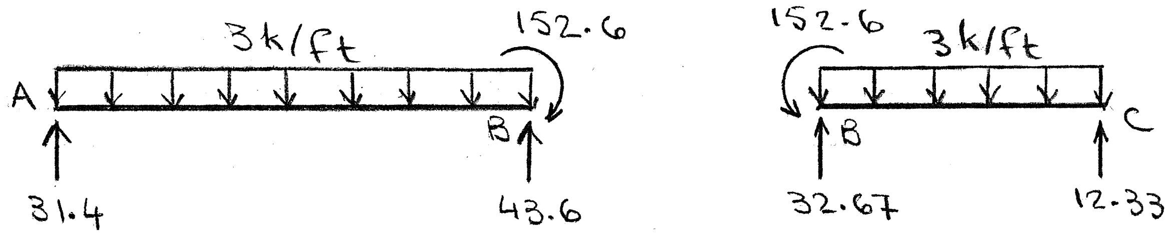
Three - Moment Equation at Joint B:

$$M_A = M_C = 0$$

$$2M_B\left(\frac{25}{2I}+\frac{15}{I}\right) = -\frac{3(25)^3}{4(2I)} - \frac{3(15)^3}{4I}$$

$$55M_B = -8390.625$$

$$\underline{M_B = -152.6 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

14.3

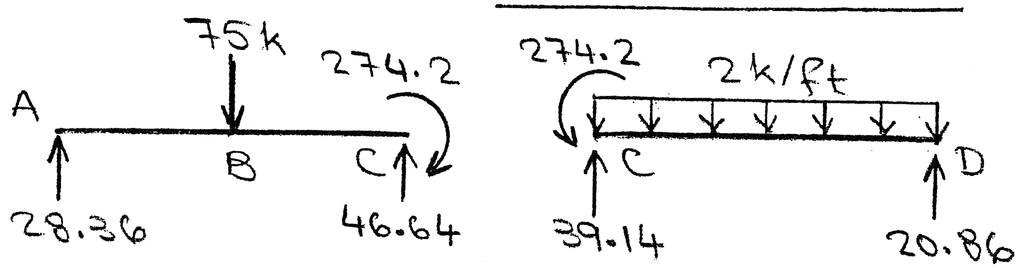
Three-Moment Equation at Joint C:

$$M_A = M_D = 0$$

$$2M_C\left(\frac{30}{3I} + \frac{30}{I}\right) = -\frac{75(30)^2(0.5)(1-0.25)}{3I} - \frac{2(30)^3}{4I}$$

$$80M_C = -21937.5$$

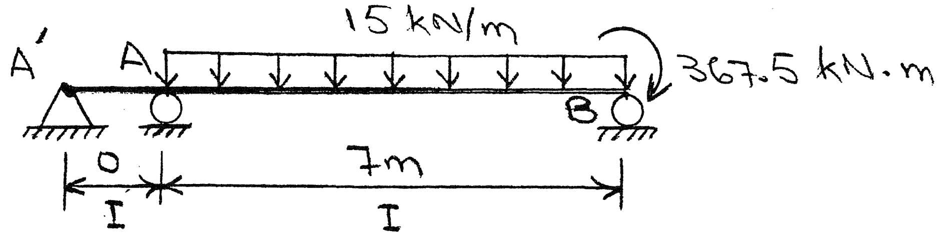
$$\underline{M_C = -274.2 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.13.

14.4



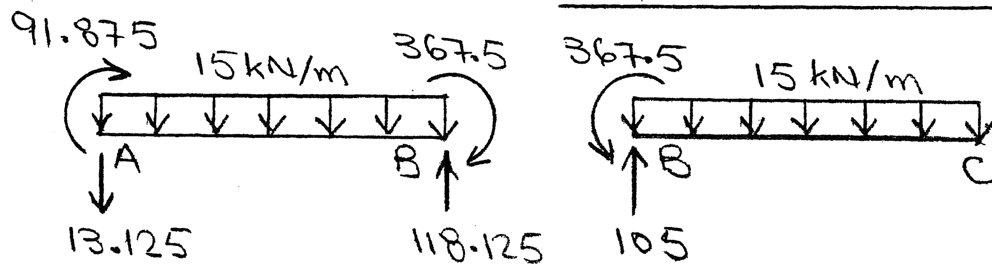
Three-Moment Equation at Joint A:

$M_{A'} = 0 \quad , \quad M_B = -367.5 \text{ kN.m}$

$$2M_A(0+7) - 367.5(7) = -\frac{1}{4}(15)(7)^3$$

$$14M_A = 1286.25$$

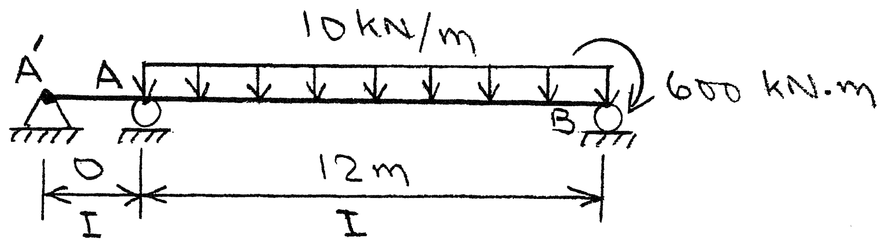
$$M_A = 91.875 \text{ kN.m}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.15.

14.5



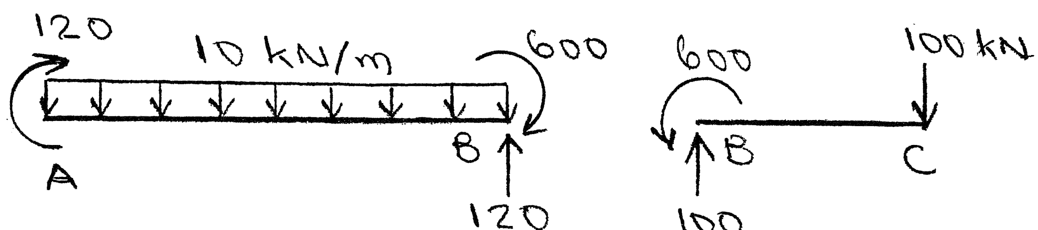
Three-Moment Equation at Joint A:

$M_{A'} = 0$, $M_B = -600$ kN.m

$$2M_A(0+12) - 600(12) = -\frac{1}{4}(10)(12)^3$$

$$24M_A = 2880$$

$$\underline{M_A = 120 \text{ kN.m}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.16.

14.6

Three-Moment Equation at Joint C:

$$M_A = 0$$

$$2M_C\left(\frac{10}{I}+\frac{10}{2I}\right)+M_E\left(\frac{10}{2I}\right) = -\frac{120(10)^2(0.6)(1-0.36)}{I}$$

$$-\frac{120(10)^2(0.4)(1-0.16)}{2I}$$

$$30M_C + 5M_E = -6624 \qquad (1)$$

Three-Moment Equation at Joint E:

$$M_G = 0$$

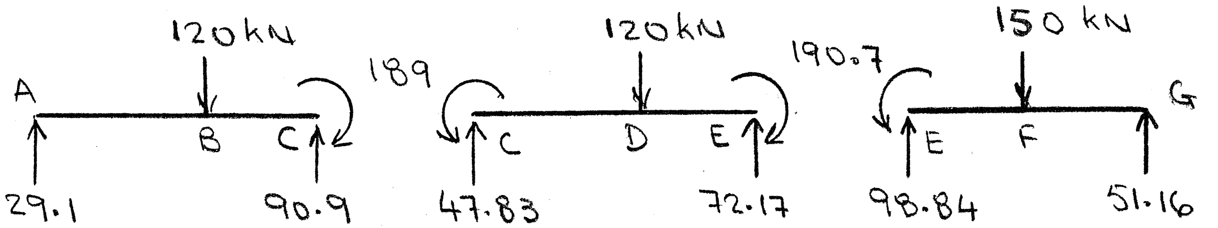
$$M_C\left(\frac{10}{2I}\right)+2M_E\left(\frac{10}{2I}+\frac{8}{I}\right) = -\frac{120(10)^2(0.6)(1-0.36)}{2I}$$

$$-\frac{150(8)^2(0.5)(1-0.25)}{I}$$

$$5M_C + 26M_E = -5904 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

$\underline{M_C = -189 \text{ kN.m}}$ $\quad\quad$ $\underline{M_E = -190.7 \text{ kN.m}}$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

14.7

Three-Moment Equation at Joint B:

$$M_A = 0$$

$$2M_B\left(\frac{24}{I}+\frac{24}{2I}\right)+M_C\left(\frac{24}{2I}\right)=-\frac{2.5(24)^3}{4I}-\frac{2.5(24)^3}{4(2I)}$$

$$72M_B + 12M_C = -12960 \qquad (1)$$

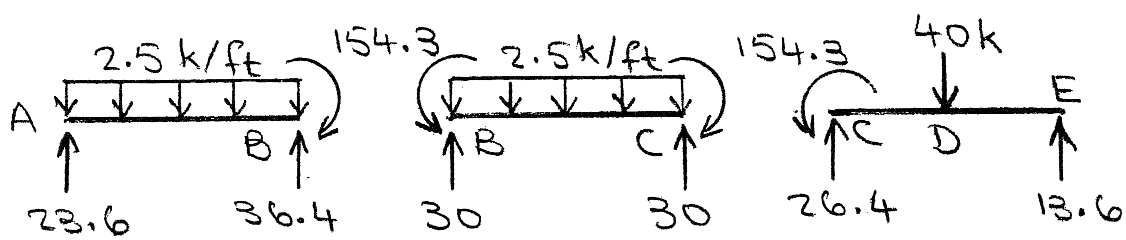
Three-Moment Equation at Joint C: $M_E = 0$

$$M_B\left(\frac{24}{2I}\right)+2\left(\frac{24}{2I}+\frac{24}{I}\right)=-\frac{2.5(24)^3}{4(2I)}-\frac{40(24)^2(0.5)(1-0.25)}{I}$$

$$12M_B + 72M_C = -12960 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

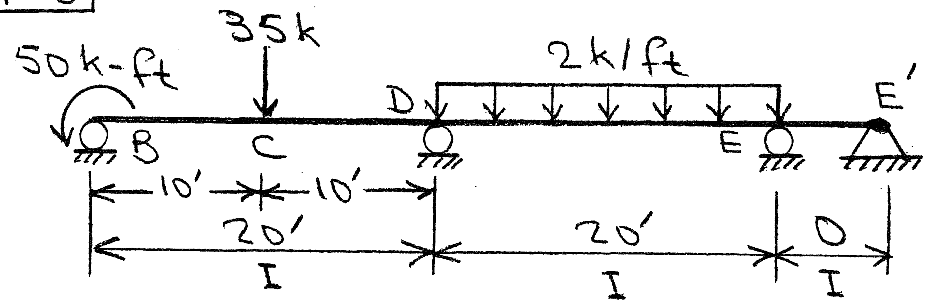
$$\underline{M_B = M_C = -154.3 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.40.

14.8



Three-Moment Equation at Joint D:

$$M_B = -50 \text{ k-ft}$$

$$-50(20) + 2M_D(20+20) + M_E(20) = -35(20)^2(0.5)(0.75) - \frac{1}{4}(2)(20)^3$$

$$80M_D + 20M_E = -8250 \qquad (1)$$

Three-Moment Equation at Joint E:

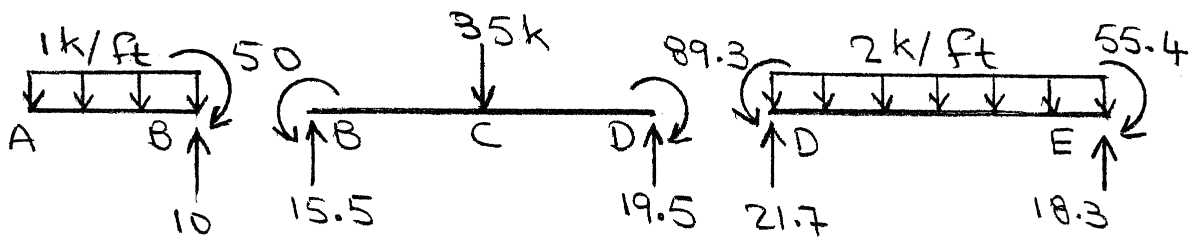
$$M_{E'} = 0$$

$$M_D(20) + 2M_E(20+0) = -\frac{1}{4}(2)(20)^3$$

$$20M_D + 40M_E = -4000 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

$M_D = -89.3$ k-ft $\qquad$ $M_E = -55.4$ k-ft



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

14.9

Three-Moment Equation at Joint B:

$$M_A = M_C = 0$$

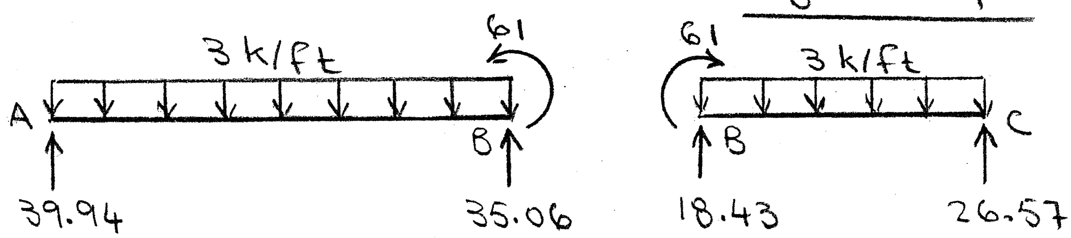
$$2M_B\left(\frac{25}{2I}+\frac{15}{I}\right) = -\frac{3(25)^3}{4(2I)} - \frac{3(15)^3}{4I} - 6E\left[\frac{0.25-1}{25(12)} + \frac{0.75-1}{15(12)}\right]$$

$$55M_B = -8390.625 + 0.0233EI$$

Substituting $EI = \frac{29000(2500)}{(12)^2} = 503472.2$ k-ft^2, we obtain:

$$55M_B = 3357$$

$$\underline{M_B = 61 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.51.

14.10

Three-Moment Equation at Joint C: $M_A = 0$

$$2M_C\left(\frac{10}{I}+\frac{10}{2I}\right)+M_E\left(\frac{10}{2I}\right)=\frac{-120(10)^2(0.6)(1-0.36)}{I}$$

$$-\frac{120(10)^2(0.4)(1-0.16)}{2I}-6E\left[\frac{0.01-0.065}{10}+\frac{0.04-0.065}{10}\right]$$

Substituting $EI = 200(500) = 100000 \text{ kN.m}^2$, we obtain:

$$30M_C + 5M_E = -1824 \qquad (1)$$

Three-Moment Equation at Joint E: $M_G = 0$

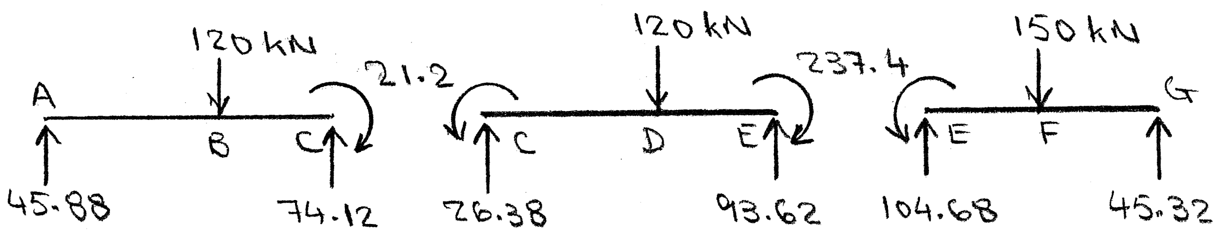
$$M_C\left(\frac{10}{2I}\right)+2M_E\left(\frac{10}{2I}+\frac{8}{I}\right)=-\frac{120(10)^2(0.6)(1-0.36)}{2I}$$

$$-\frac{150(8)^2(0.5)(1-0.25)}{I}-6E\left[\frac{0.065-0.04}{10}+\frac{0.025-0.04}{8}\right]$$

$$5M_C + 26M_E = -6279 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

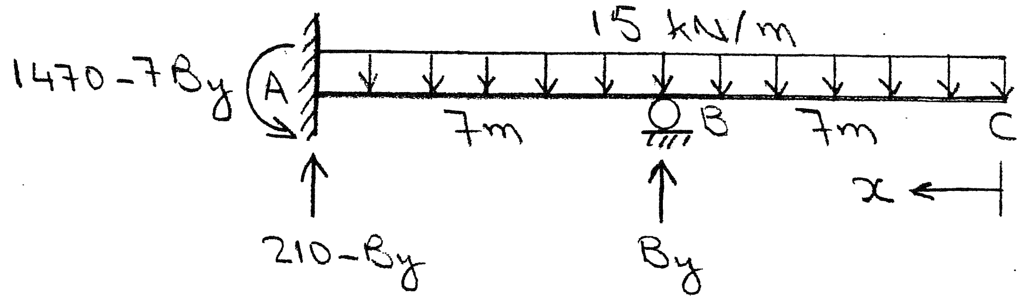
$M_C = -21.2 \text{ kN.m}$ $\quad$ $M_E = -237.4 \text{ kN.m}$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

14.11



Segment	x Coordinate		M	$\frac{\partial M}{\partial B_y}$
	Origin	Limits (m)		
CB	C	0 − 7	$-7.5x^2$	0
BA	C	7 − 14	$-7.5x^2+B_y(x-7)$	$x-7$

$$\frac{\partial U}{\partial B_y} = \int \left(\frac{\partial M}{\partial B_y}\right)\frac{M}{EI}\,dx = 0$$

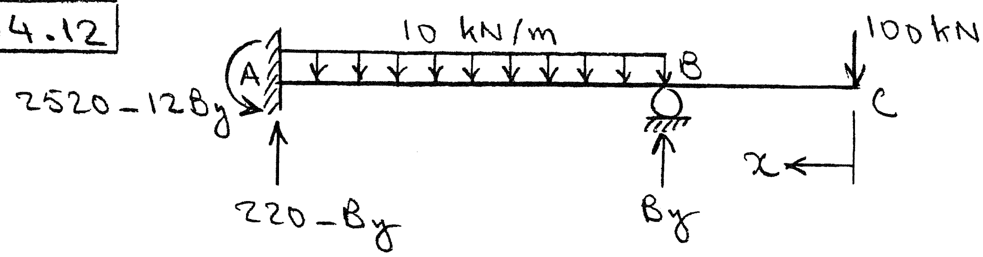
$$\frac{1}{EI}\int_7^{14} (x-7)\left[-7.5x^2 + B_y(x-7)\right] dx = 0$$

$$-25510.625 + (114.33)B_y = 0$$

$$\underline{B_y = 223.125 \text{ kN} \uparrow}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.15.

14.12



Segment	x Coordinate		M	$\frac{\partial M}{\partial B_y}$
	Origin	Limits (m)		
CB	C	0–6	$-100x$	0
BA	C	6–18	$-100x - 5(x-6)^2 + B_y(x-6)$	$x-6$

$$\frac{\partial U}{\partial B_y} = \int \left(\frac{\partial M}{\partial B_y}\right)\frac{M}{EI}\,dx = 0$$

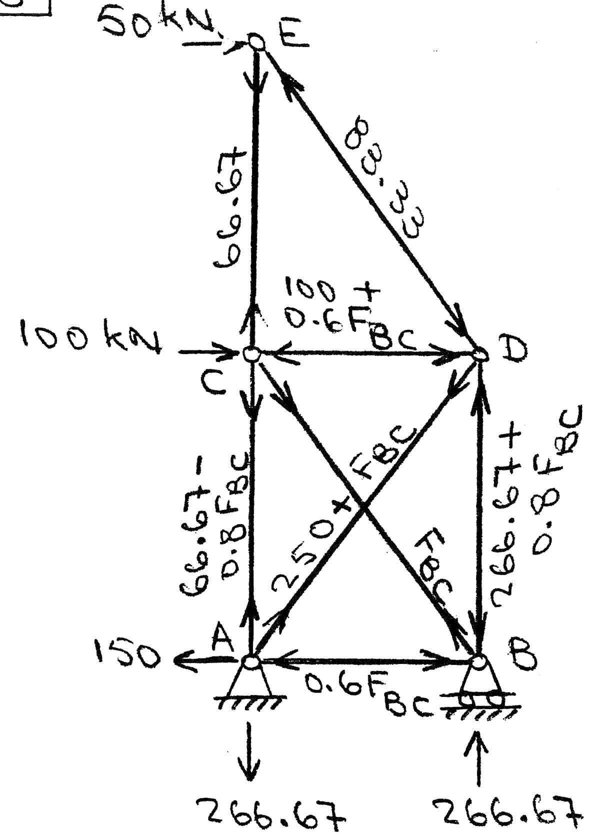
$$\frac{1}{EI}\int_6^{18}(x-6)\left[-100x - 5(x-6)^2 + B_y(x-6)\right]dx = 0$$

$$-126720 + 576B_y = 0$$

$$B_y = \underline{220 \text{ kN} \uparrow}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.16.

14.13



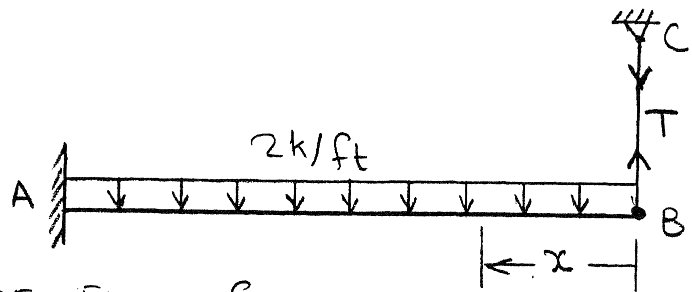
Member	L (m)	F	$\frac{\partial F}{\partial F_{BC}}$	$\left(\frac{\partial F}{\partial F_{BC}}\right) FL$	F (kN)
AB	3	$-0.6\,F_{BC}$	-0.6	$1.08\,F_{BC}$	71.9
CD	3	$-100-0.6\,F_{BC}$	-0.6	$180+1.08\,F_{BC}$	-28.1
AC	4	$66.67-0.8\,F_{BC}$	-0.8	$-213.33+2.56\,F_{BC}$	162.5
BD	4	$-266.67-0.8\,F_{BC}$	-0.8	$853.33+2.56\,F_{BC}$	-170.8
CE	4	66.67	0	0	66.7
AD	5	$250+F_{BC}$	1	$1250+5\,F_{BC}$	130.2
DE	5	-83.33	0	0	-83.3
BC	5	F_{BC}	1	$5\,F_{BC}$	-119.8
			Σ	$2070+17.28F_{BC}$	

$$\frac{\partial U}{\partial F_{BC}} = \frac{1}{EA}\sum\left(\frac{\partial F}{\partial F_{BC}}\right) FL = 0$$

$$\frac{1}{EA}\left(2070 + 17.28\,F_{BC}\right) = 0$$

$$F_{BC} = -119.8 \text{ kN} = \underline{119.8 \text{ kN (C)}}$$

14-14



$$\frac{\partial U}{\partial T} = \Sigma\left(\frac{\partial F}{\partial T}\right)\frac{FL}{AE} + \int\left(\frac{\partial M}{\partial T}\right)\frac{M}{EI}\,dx = 0$$

Cable BC: $F = T$; $\dfrac{\partial F}{\partial T} = 1$

Beam AB: $M = Tx - x^2$; $\dfrac{\partial M}{\partial T} = x$

$$\frac{\partial U}{\partial T} = \frac{1}{E}\left[\frac{T(5)(12)^2}{0.5} + \frac{(12)^4}{500}\int_0^{15} x\,(Tx - x^2)\,dx\right]$$

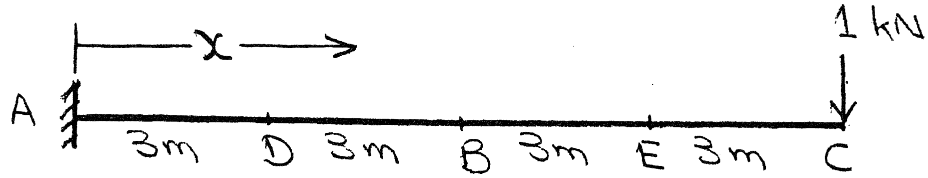
$$= 48096T - 524880 = 0$$

$$\underline{T = 10.9\text{ k (T)}}$$

Chapter Fifteen
Influence Lines for Statically Indeterminate Structures

CHAPTER 15

15.1 Select C_y (↑+) as the redundant.



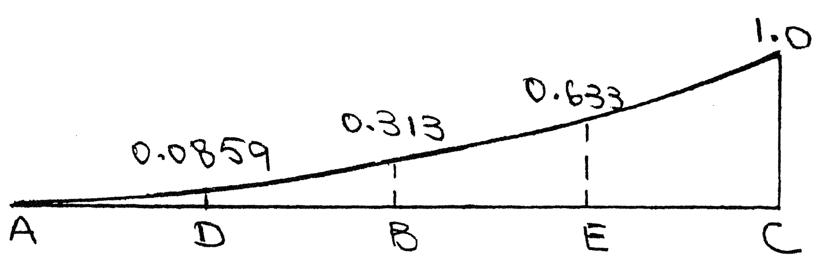
$$C_y = -\frac{f_{Cx}}{\bar{f}_{CC}} = -\frac{f_{xC}}{\bar{f}_{CC}}$$

Using beam deflection formulas:

$$f_{xC} = -\frac{1}{6EI}(x^3 - 36x^2)$$

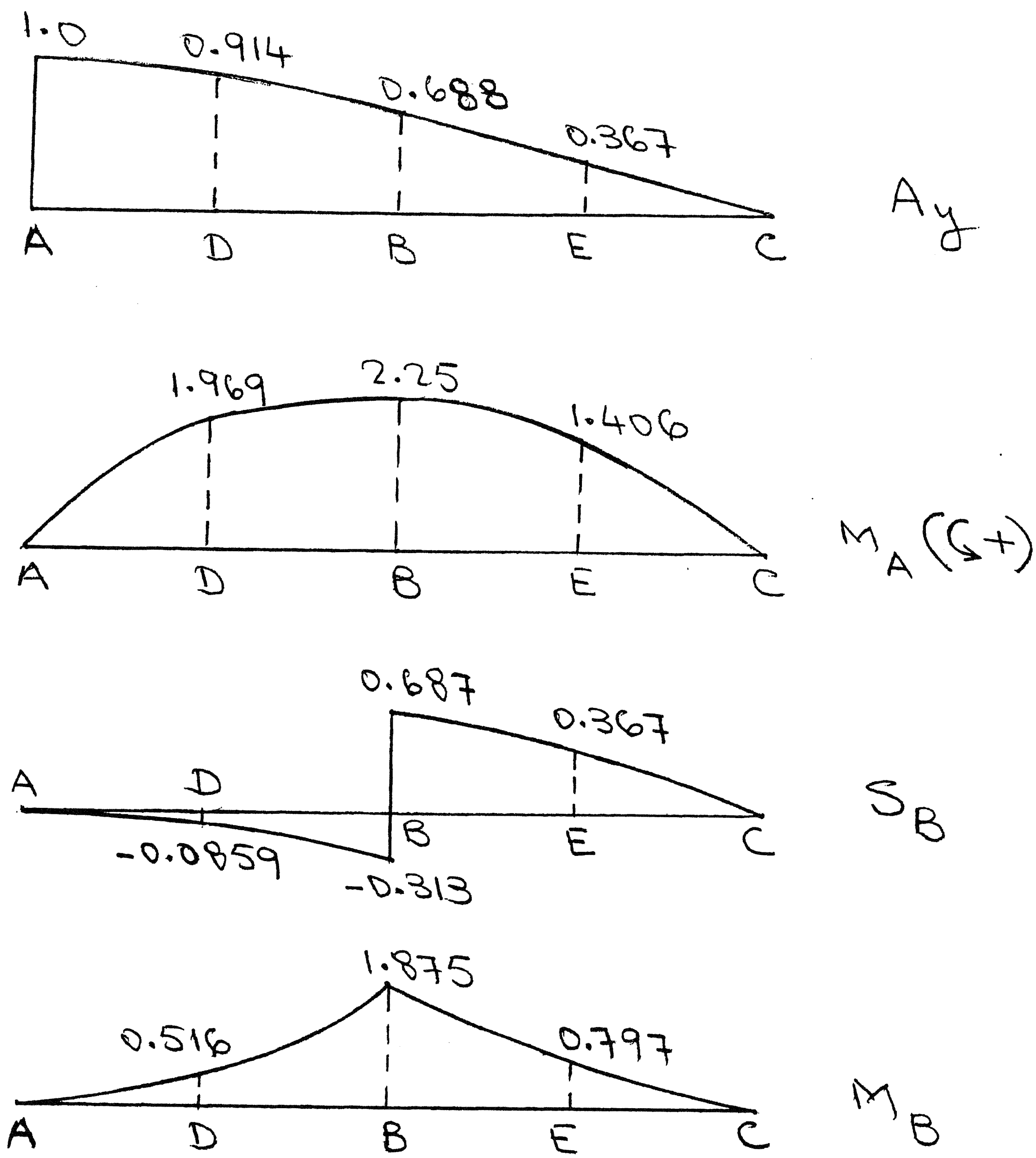
$$\bar{f}_{CC} = -f_{CC} = +\frac{576\ \text{kN.m}^3/\text{kN}}{EI}$$

x (m)	$EI f_{xC}$	C_y (kN/kN)	A_y (kN/kN)	M_A (kN.m/kN)	S_B (kN/kN)	M_B (kN.m/kN)
0	0	0	1	0	0	0
3	−49.5	0.0859	0.914	1.969	−0.0859	0.516
6	−180	0.313	0.688	2.25	−0.313 (L) 0.687 (R)	1.875
9	−364.5	0.633	0.367	1.406	0.367	0.797
12	−576	1	0	0	0	0

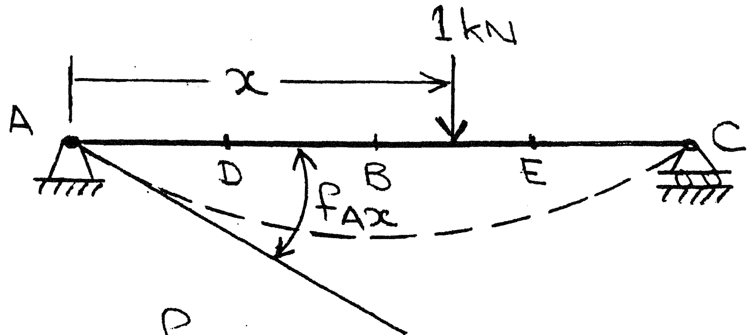


C_y

15.1 (Contd.)


1.0
0.914
0.688
0.367
A
D
B
E
C
A_y
1.969
2.25
1.406
A
D
B
E
C
M_A (↺+)
0.687
0.367
A
D
B
E
C
-0.0859
-0.313
S_B
1.875
0.516
0.797
A
D
B
E
C
M_B

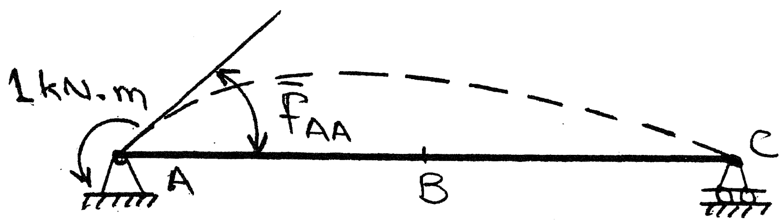
15.2 Select M_A (↻+) as the redundant.



$$M_A = -\frac{f_{Ax}}{\bar{f}_{AA}}$$

Using beam deflection formulas:

$$f_{Ax} = -\frac{1}{72EI}\left(x^3 - 36x^2 + 288x\right)$$

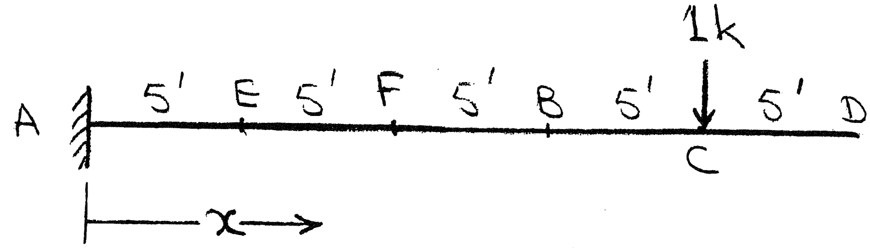


$$\bar{f}_{AA} = +\frac{4\text{ kN.m}^2/\text{kN.m}}{EI}$$

x (m)	$EI\,f_{Ax}$	M_A
0	0	0
3	-7.875	1.969
6	-9	2.25
9	-5.625	1.406
12	0	0

For influence lines, see solution of Problem 15.1.

15.3 Select C_y (↑+) as the redundant.



$$C_y = -\frac{f_{Cx}}{\bar{f}_{CC}} = -\frac{f_{xC}}{\bar{f}_{CC}}$$

Using beam deflection formulas:

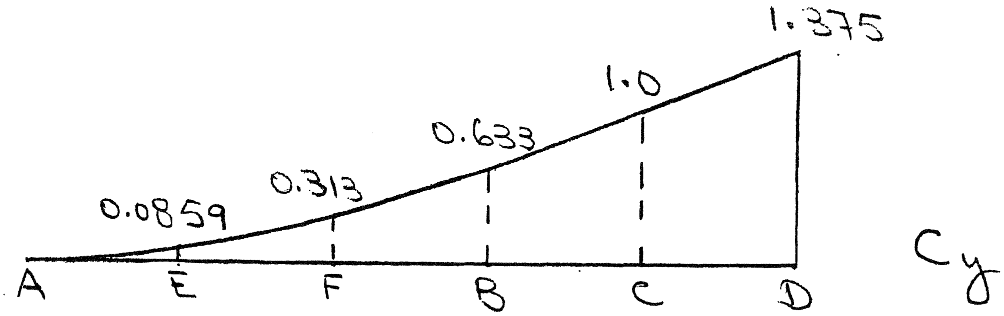
for $0 \leq x \leq 20'$ $\quad f_{xC} = \frac{1}{6EI}(x^3 - 60x^2)$

for $20' \leq x \leq 25'$ $\quad f_{xC} = \frac{200}{3EI}(20 - 3x)$

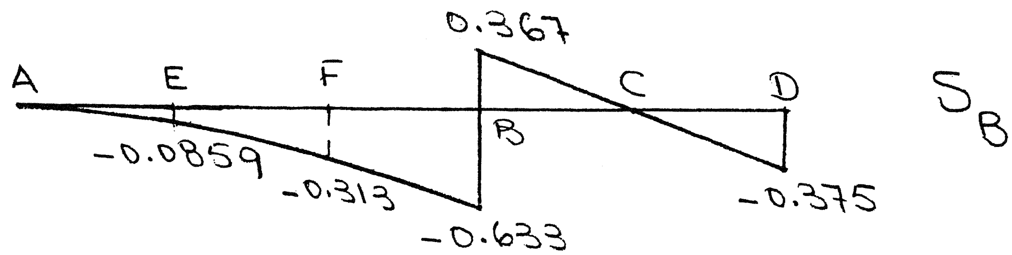
$$\bar{f}_{CC} = -f_{CC} = +\frac{2666.67 \text{ k-ft}^3/\text{k}}{EI}$$

x (ft)	$EI f_{xC}$	C_y (k/k)	S_B (k/k)	M_B (k-ft/k)
0	0	0	0	0
5	-229.17	0.0859	-0.0859	0.43
10	-833.33	0.313	-0.313	1.563
15	-1687.5	0.633	-0.633(L) 0.367(R)	3.164
20	-2666.67	1	0	0
25	-3666.67	1.375	-0.375	-3.125

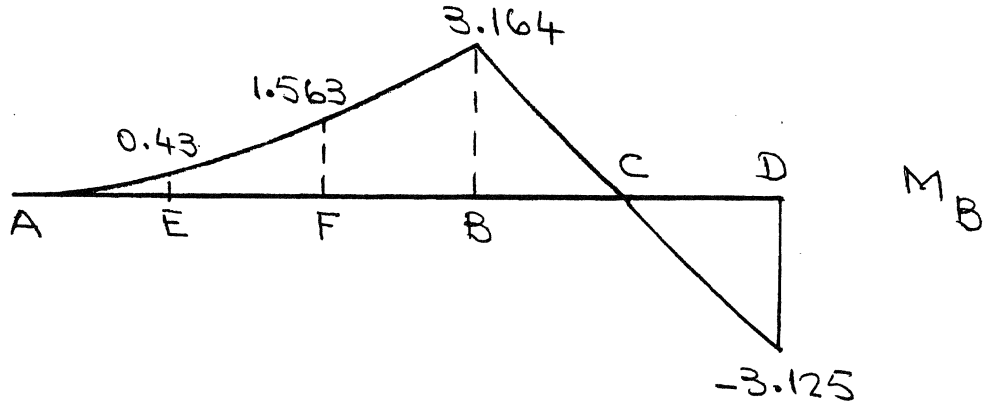
15.3 (Contd.)



1.375
1.0
0.633
0.313
0.0859
A
E
F
B
C
D
Cy

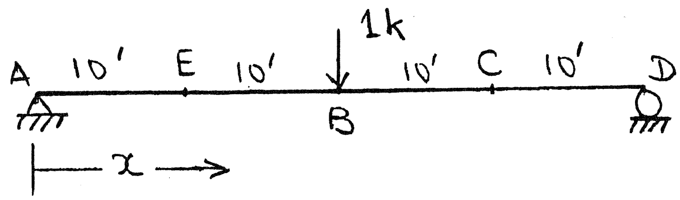


0.367
A
E
F
C
D
SB
B
-0.0859
-0.313
-0.633
-0.375



3.164
1.563
0.43
C
D
MB
A
E
F
B
-3.125

15.4 Select B_y (↑+) as the redundant.



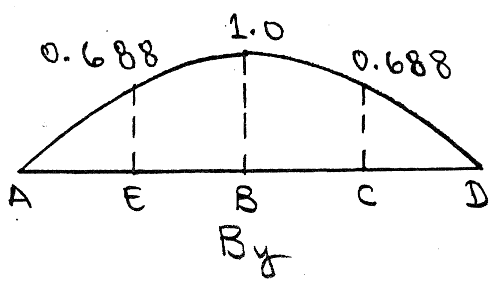
$$B_y = -\frac{f_{Bx}}{\bar{f}_{BB}} = -\frac{f_{xB}}{\bar{f}_{BB}}$$

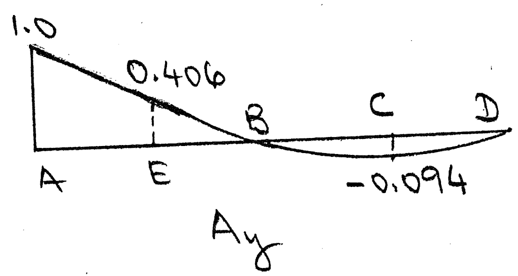
Using beam-deflection formulas:

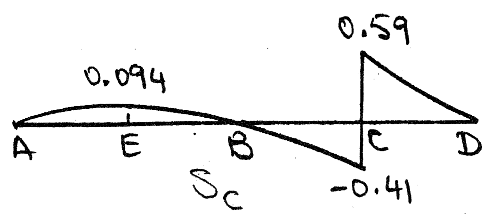
$$f_{EB} = f_{CB} = -\frac{916.7\ \text{k-ft}^3/\text{k}}{EI}$$

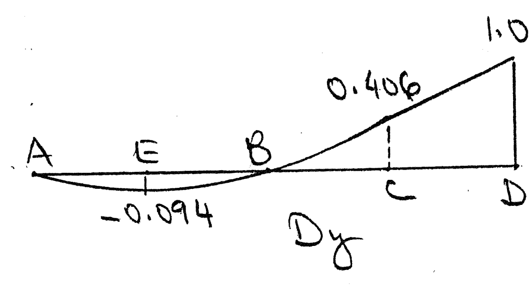
$$f_{BB} = -\frac{1333.3\ \text{k-ft}^3/\text{k}}{EI}$$

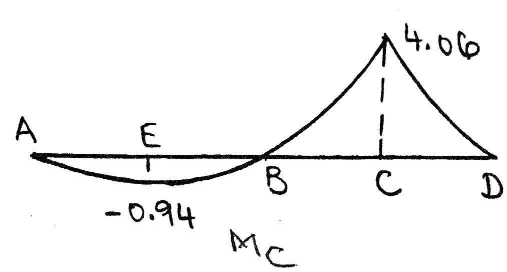
$$\bar{f}_{BB} = -f_{BB} = +\frac{1333.3\ \text{k-ft}^3/\text{k}}{EI}$$



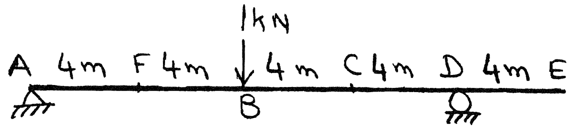


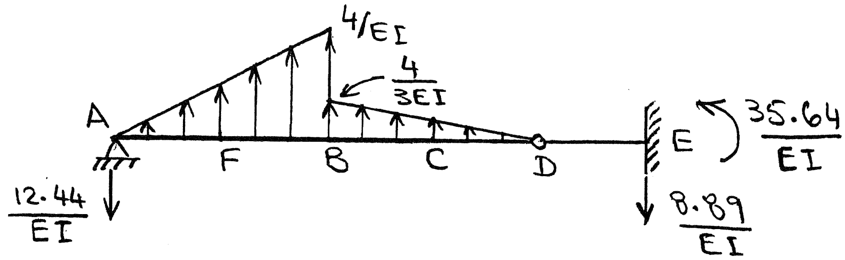






15.5 Select B_y (↑+) as the redundant.

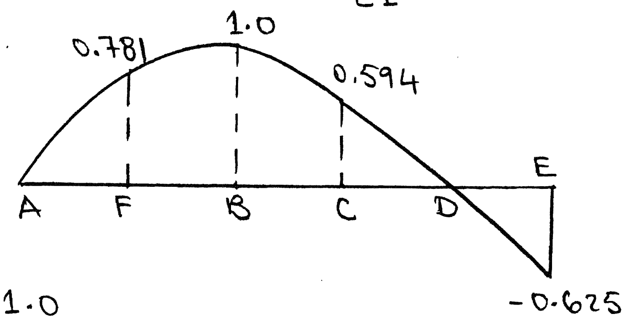




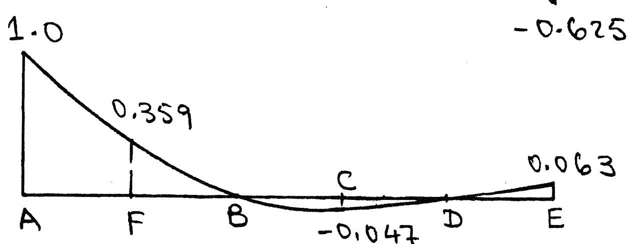
$$f_{BF} = f_{FB} = -\frac{44.43}{EI} \; ; \quad f_{BB} = -\frac{56.85}{EI}$$

$$f_{BC} = f_{CB} = -\frac{33.72}{EI} \; ; \quad f_{BE} = f_{EB} = +\frac{35.64}{EI}$$

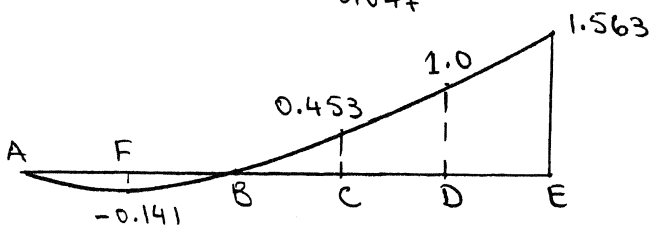
$$\bar{f}_{BB} = -f_{BB} = \frac{56.85}{EI}$$



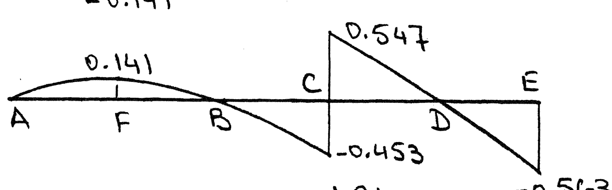
B_y



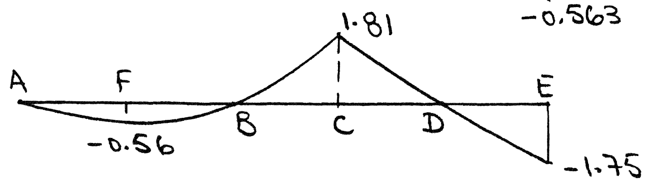
A_y



D_y

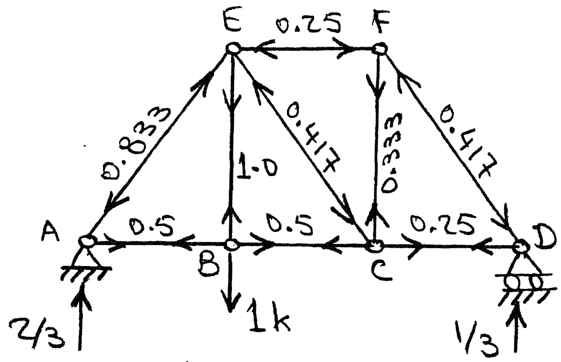


S_C

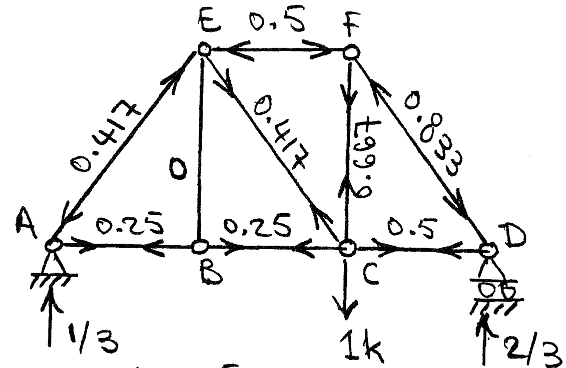


M_C

15.6 Select C_y (↑+) as the redundant.



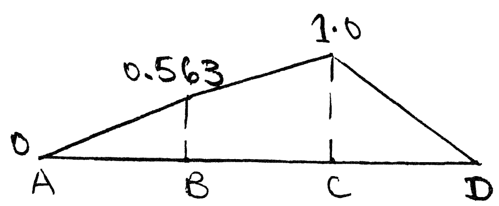
u_B Forces



u_C Forces

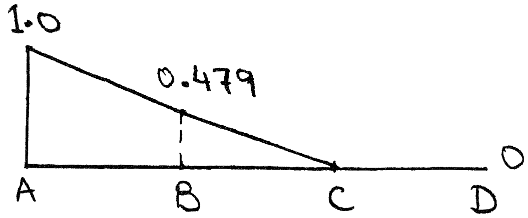
Member	L (ft)	u_B	u_C	$u_B u_C L$	$u_C^2 L$
AB	15	0.5	0.25	1.875	0.938
BC	15	0.5	0.25	1.875	0.938
CD	15	0.25	0.5	1.875	3.75
EF	15	−0.25	−0.5	1.875	3.75
BE	20	1.0	0	0	0
CF	20	0.333	0.667	4.442	8.898
AE	25	−0.833	−0.417	8.684	4.347
CE	25	−0.417	0.417	−4.347	4.347
DF	25	−0.417	−0.833	8.684	17.347
			Σ	24.963	44.315

Unit load at B: $C_y = -\dfrac{24.963}{44.315} = -0.563\text{k}$ or $0.563\text{k}\uparrow$

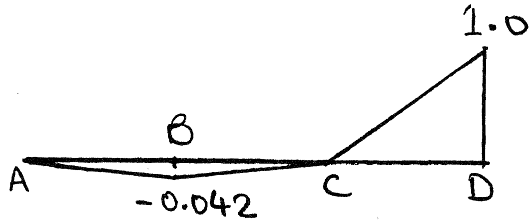


IL for C_y

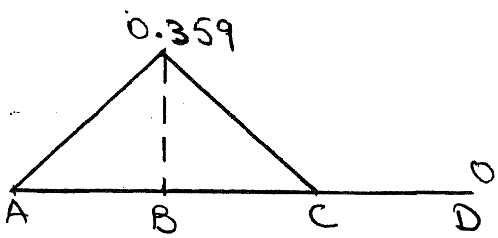
15.6 (cont'd.)



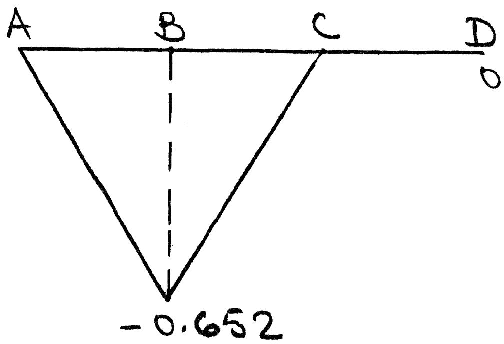
IL for A_y



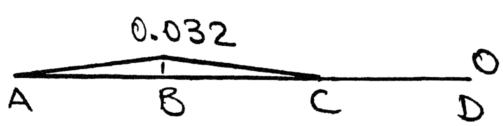
IL for D_y



IL for F_{BC}

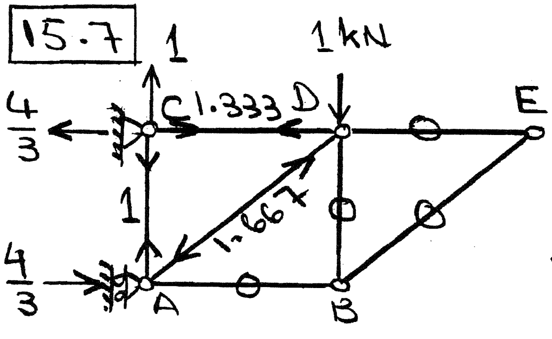


IL for F_{CE}

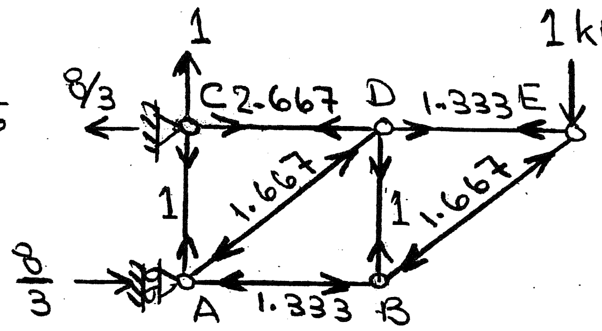


IL for F_{EF}

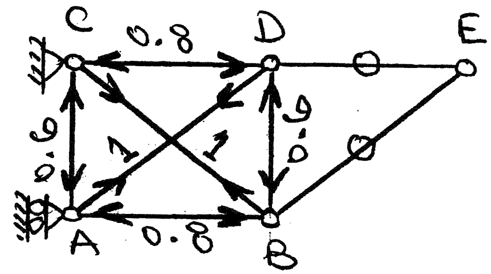
15.7



u_D Forces



u_E Forces



u_{BC} Forces

Member	L (m)	u_D	u_E	u_{BC}	$u_D u_{BC} L$	$u_E u_{BC} L$	$u_{BC}^2 L$
AB	8	0	-1.333	-0.8	0	8.533	5.12
CD	8	1.333	2.667	-0.8	-8.533	-17.067	5.12
DE	8	0	1.333	0	0	0	0
AC	6	1	1	-0.6	-3.6	-3.6	2.16
BD	6	0	1	-0.6	0	-3.6	2.16
AD	10	-1.667	-1.667	1	-16.667	-16.667	10
BE	10	0	-1.667	0	0	0	0
BC	10	0	0	1	0	0	10
				Σ	-28.8	-32.4	34.56

15.7 (contd.)

Unit load at D: $f_{BC,D} + f_{BC,BC} F_{BC} = 0$

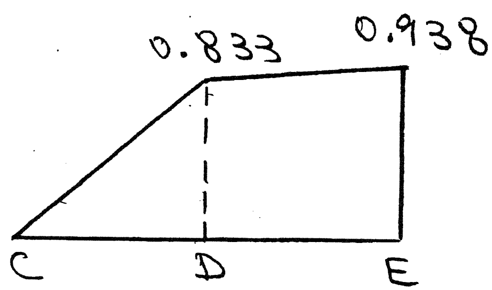
$$F_{BC} = \frac{28.8}{34.56} = \underline{0.833\ \text{kN/kN (T)}}$$

$$F_{CD} = u_D + u_{BC} F_{BC} = 1.333 - 0.8(0.833)$$
$$= \underline{0.667\ \text{kN/kN (T)}}$$

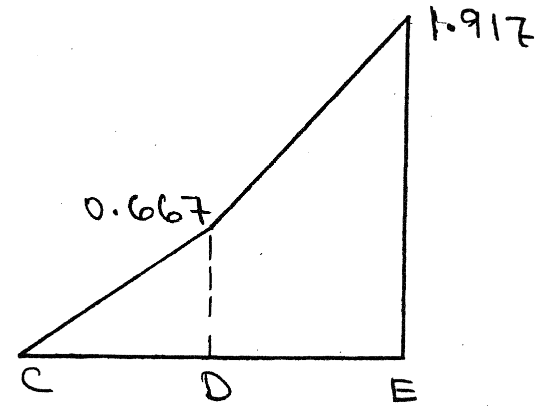
Unit load at E: $f_{BC,E} + f_{BC,BC} F_{BC} = 0$

$$F_{BC} = \frac{32.4}{34.56} = \underline{0.938\ \text{kN/kN (T)}}$$

$$F_{CD} = u_E + u_{BC} F_{BC} = 2.667 - 0.8(0.938)$$
$$= \underline{1.917\ \text{kN/kN (T)}}$$

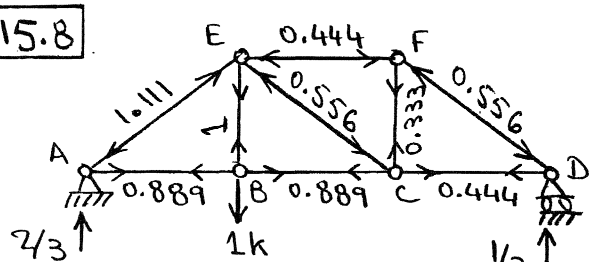


F_{BC}

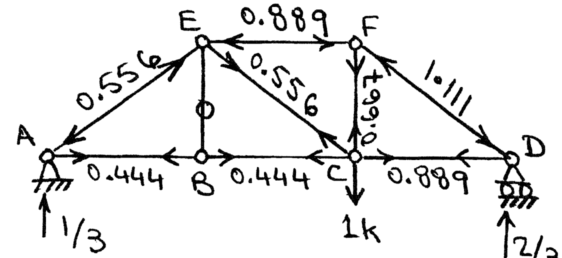


F_{CD}

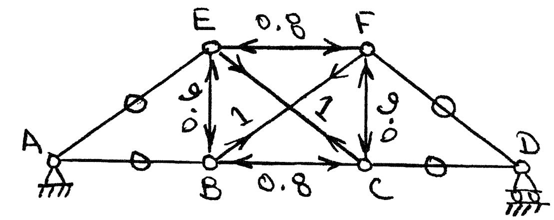
15.8



u_B Forces



u_C Forces



$\times F_{BF}$

u_{BF} Forces

Member	L (in.)	A (in.²)	u_B	u_C	u_{BF}	$\frac{u_B u_{BF} L}{A}$	$\frac{u_C u_{BF} L}{A}$	$\frac{u_{BF}^2 L}{A}$
AB	192	8	0.889	0.444	0	0	0	0
BC	192	8	0.889	0.444	-0.8	-17.07	-8.52	15.36
CD	192	8	0.444	0.889	0	0	0	0
EF	192	8	-0.444	-0.889	-0.8	8.52	17.07	15.36
BE	144	6	1	0	-0.6	-14.4	0	8.64
CF	144	6	0.333	0.667	-0.6	-4.8	-9.6	8.64
AE	240	8	-1.111	-0.556	0	0	0	0
DF	240	8	-0.556	-1.111	0	0	0	0
CE	240	6	-0.556	0.556	1	-22.24	22.24	40
BF	240	6	0	0	1	0	0	40
					Σ	-49.99	21.19	128

15.8 (contd.)

Unit load at B:

$$F_{BF} = \frac{49.99}{128} = 0.39\text{ k}$$

$$F_{BC} = 0.889 - 0.8(0.39) = 0.577\text{ k}$$

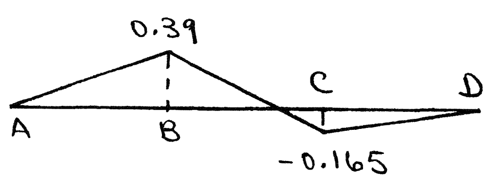
$$F_{CF} = 0.333 - 0.6(0.39) = 0.099\text{ k}$$

Unit load at C:

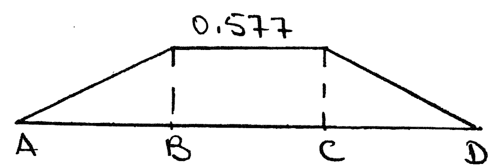
$$F_{BF} = -\frac{21.19}{128} = -0.165\text{ k}$$

$$F_{BC} = 0.444 - 0.8(-0.165) = 0.577\text{ k}$$

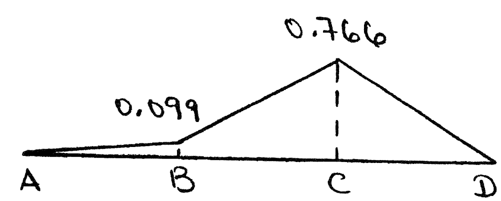
$$F_{CF} = 0.667 - 0.6(-0.165) = 0.766\text{ k}$$



F_{BF}

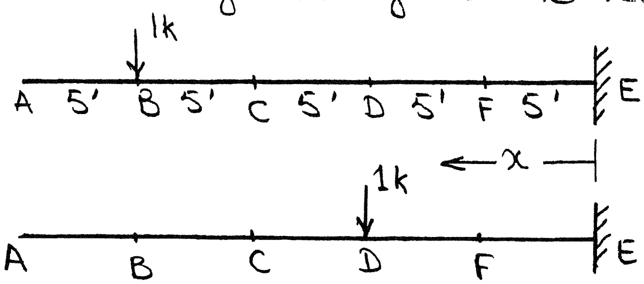


F_{BC}



F_{CF}

15.9 Select B_y and D_y as the redundants.



Compatibility Eqs.: $f_{Bx} + \bar{f}_{BB} B_y + \bar{f}_{BD} D_y = 0$

$$f_{Dx} + \bar{f}_{DB} B_y + \bar{f}_{DD} D_y = 0$$

Using beam deflection formulas:

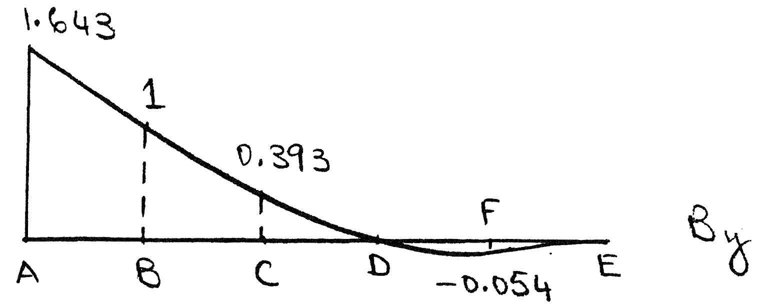
$$f_{Bx} = f_{xB} = \begin{cases} \frac{1}{6EI}(x^3 - 60x^2) & \text{for } 0 \leq x \leq 20' \\ \frac{200}{3EI}(20 - 3x) & \text{for } 20' \leq x \leq 25' \end{cases}$$

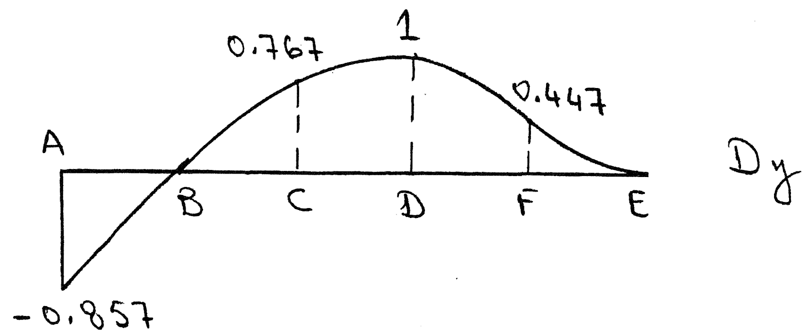
$$f_{Dx} = f_{xD} = \begin{cases} \frac{1}{6EI}(x^3 - 30x^2) & \text{for } 0 \leq x \leq 10' \\ \frac{50}{3EI}(10 - 3x) & \text{for } 10' \leq x \leq 25' \end{cases}$$

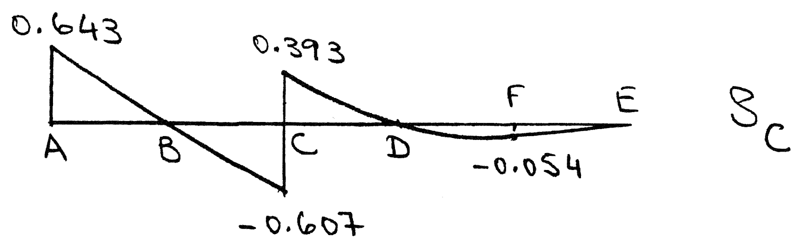
$$\bar{f}_{BB} = \frac{2666.7}{EI}; \quad \bar{f}_{BD} = \bar{f}_{DB} = \frac{833.3}{EI}; \quad \bar{f}_{DD} = \frac{333.3}{EI}$$

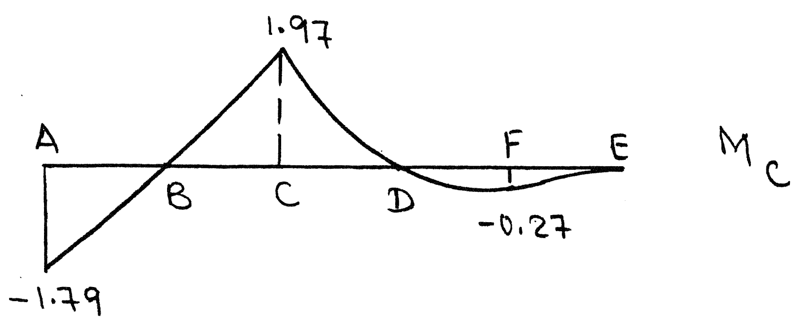
x (ft.)	$EI f_{Bx}$	$EI f_{Dx}$	B_y	D_y	δ_C	M_C
0	0	0	0	0	0	0
5	-229.2	-104.2	-0.054	0.447	-0.054	-0.27
10	-833.3	-333.3	0	1	0	0
15	-1687.5	-583.3	0.393	0.767	-0.607(L) 0.393(R)	1.97
20	-2666.7	-833.3	1	0	0	0
25	-3666.7	-1083.3	1.643	-0.857	0.643	-1.79

15.9 (Contd.)

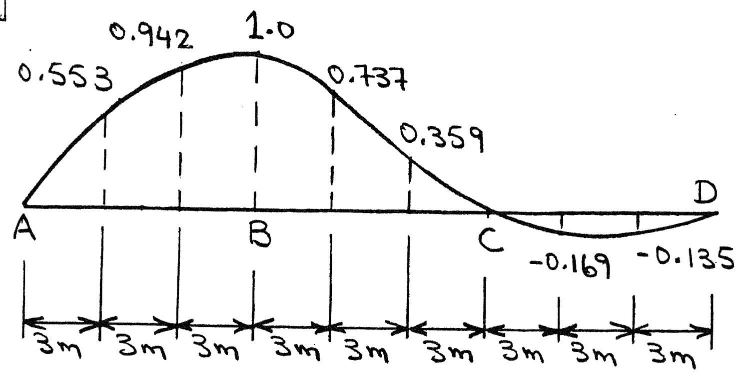

1.643
1
0.393
F
By
A
B
C
D
-0.054
E


0.767
1
0.447
A
Dy
B
C
D
F
E
-0.857

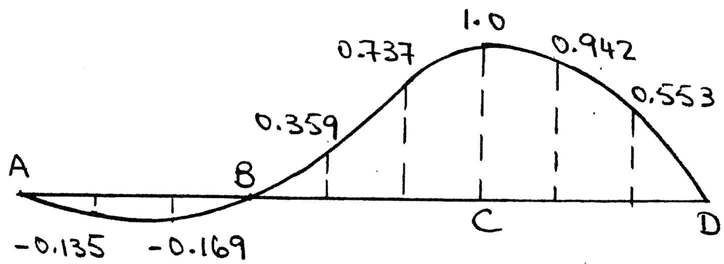

0.643
0.393
F
E
Sc
A
B
C
D
-0.054
-0.607


1.97
A
F
E
Mc
B
C
D
-0.27
-1.79

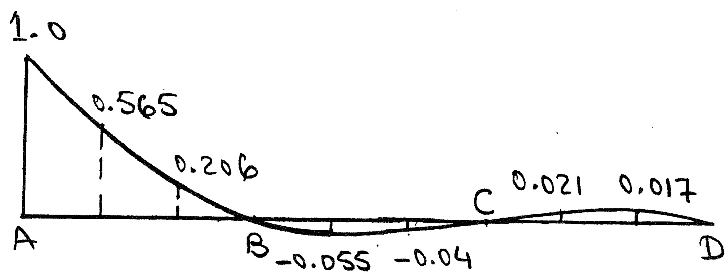
15.10



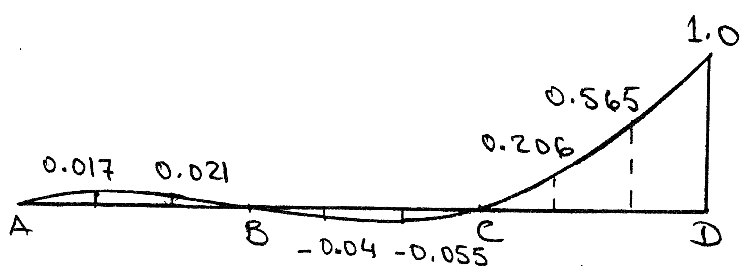
B_y



C_y

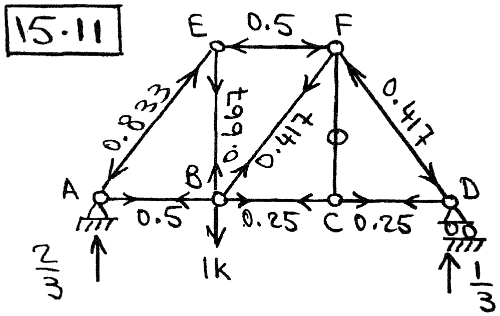


A_y

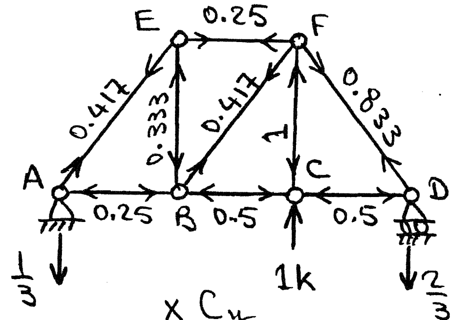


D_y

15.11

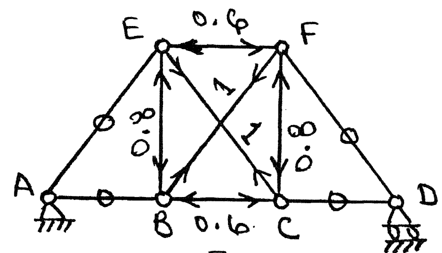


u_B Forces



$\times C_y$

u_C Forces



$\times F_{CE}$

u_{CE} Forces

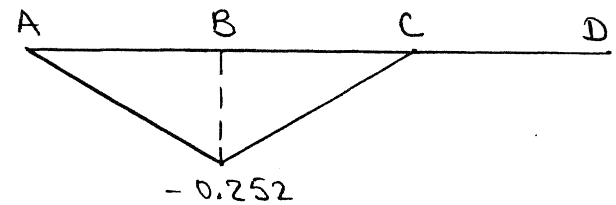
Member	L (ft)	u_B	u_C	u_{CE}	$u_B u_C L$	$u_B u_{CE} L$	$u_C^2 L$	$u_{CE}^2 L$	$u_C u_{CE} L$
AB	15	0.5	-0.25	0	-1.875	0	0.938	0	0
BC	15	0.25	-0.5	-0.6	-1.875	-2.25	3.75	5.4	4.5
CD	15	0.25	-0.5	0	-1.875	0	3.75	0	0
EF	15	-0.5	0.25	-0.6	-1.875	4.5	0.938	5.4	-2.25
BE	20	0.667	-0.333	-0.8	-4.442	-10.672	2.218	12.8	5.328
CF	20	0	-1	-0.8	0	0	20	12.8	16
AE	25	-0.833	0.417	0	-8.684	0	4.347	0	0
DF	25	-0.417	0.833	0	-8.684	0	17.347	0	0
BF	25	0.417	0.417	1	4.347	10.425	4.347	25	10.425
CE	25	0	0	1	0	0	0	25	0
				Σ	-24.963	2.003	57.635	86.4	34.003

Unit load at B: $-24.963 + 57.635\,C_y + 34.003\,F_{CE} = 0$

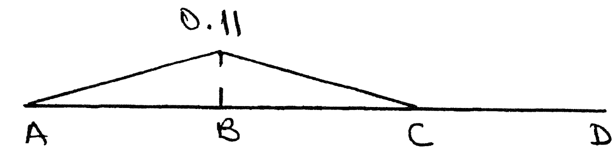
$2.003 + 34.003\,C_y + 86.4\,F_{CE} = 0$

Solving these equations, we obtain: $C_y = 0.582\text{ k}$ $\quad F_{CE} = -0.252\text{ k}$

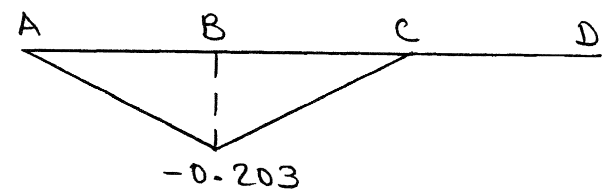
15-11 (Contd.)



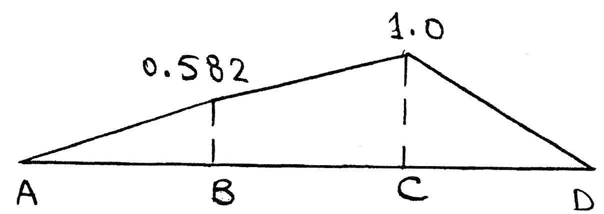
F_{CE}



F_{BC}

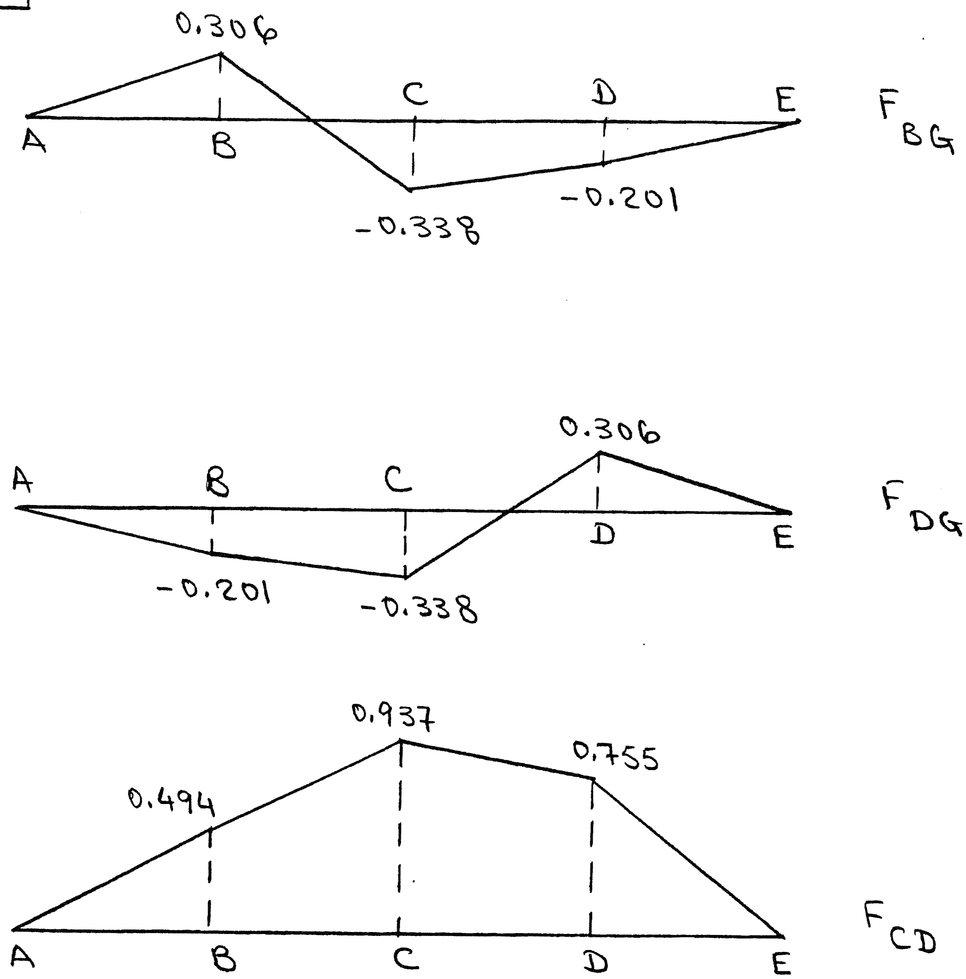


F_{EF}

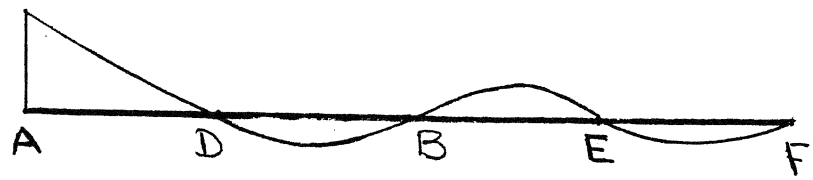


C_y

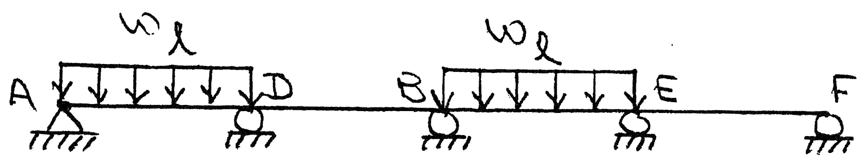
15.12


0.306
A
B
C
D
E
-0.338
-0.201
F BG
0.306
A
B
C
D
E
-0.201
-0.338
F DG
0.937
0.755
0.494
A
B
C
D
E
F CD

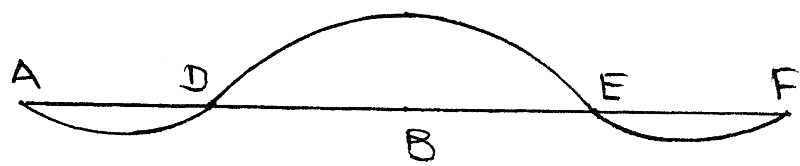
15.13



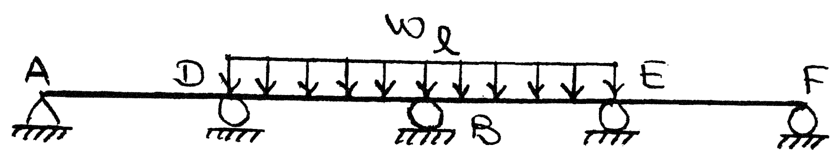
A_y



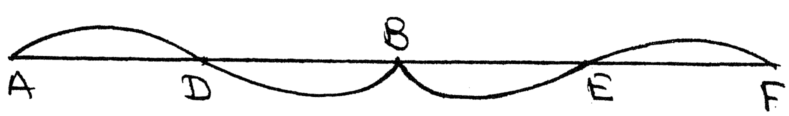
Live Load Arrangement for Maximum Positive A_y



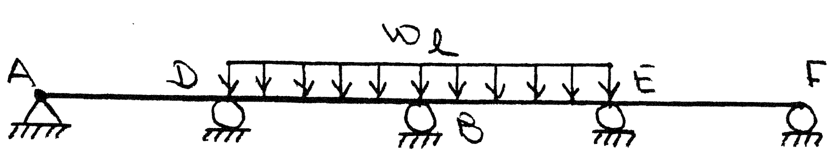
B_y



Live Load Arrangement for Maximum Positive B_y.

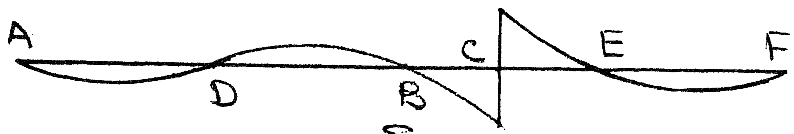


M_B

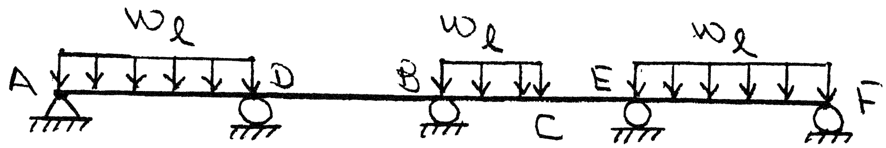


Live Load Arrangement for Maximum Negative M_B

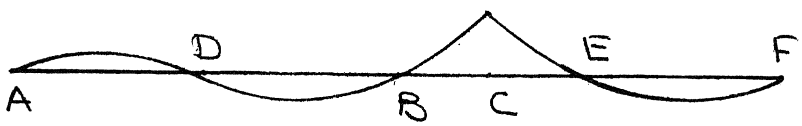
15.13 (Contd.)



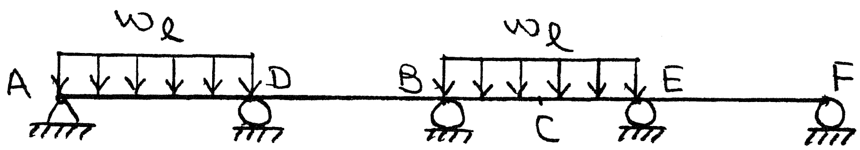
S_C



Live Load Arrangement for Maximum Negative S_C



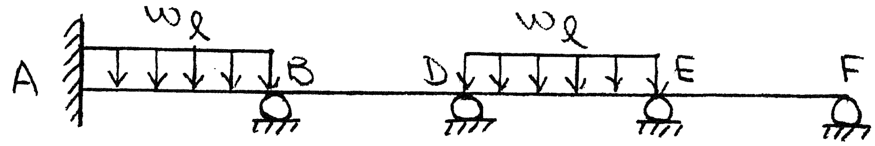
M_C



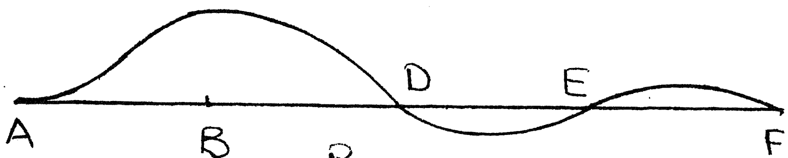
Live Load Arrangement for Maximum Positive M_C

15.14

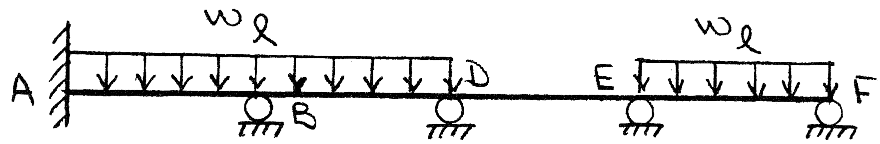
A_y



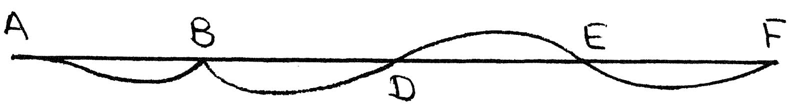
Live Load Arrangement for Maximum Positive A_y



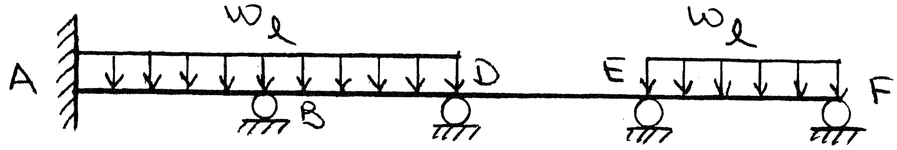
B_y



Live Load Arrangement for Maximum Positive B_y

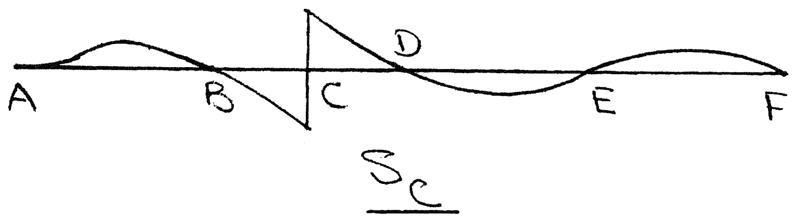


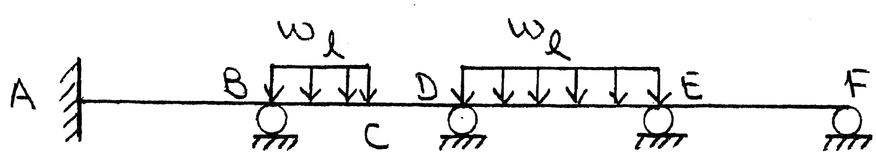
M_B



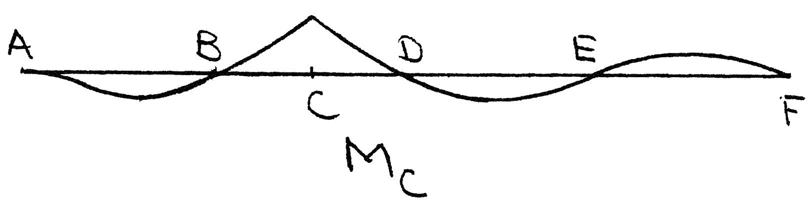
Live Load Arrangement for Maximum Negative M_B

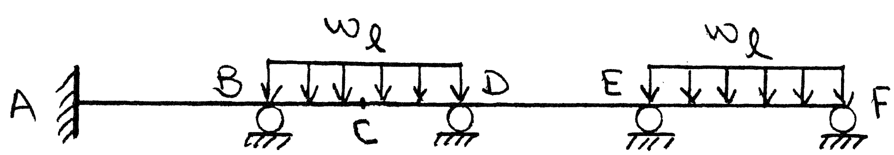
15.14 (Contd.)





Live Load Arrangement for Maximum Negative S_C

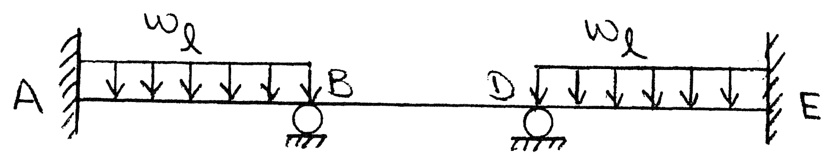




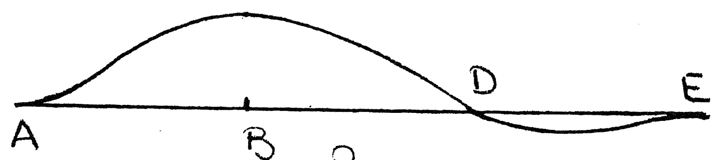
Live Load Arrangement for Maximum Positive M_C

15.15

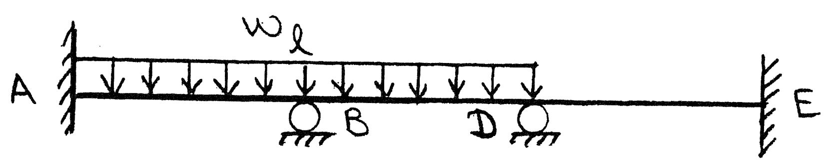
A_y



Live Load Arrangement for Maximum Positive A_y

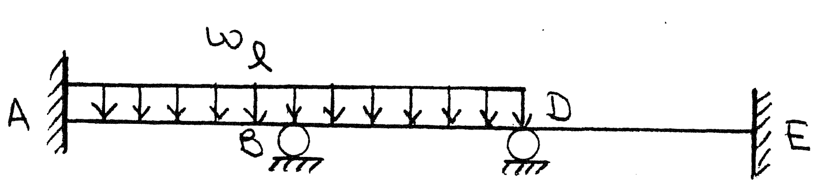


B_y



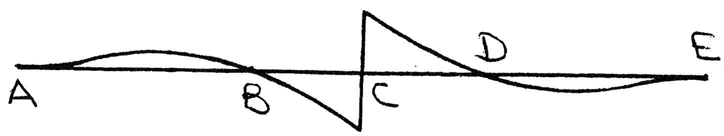
Live Load Arrangement for Maximum Positive B_y

M_B

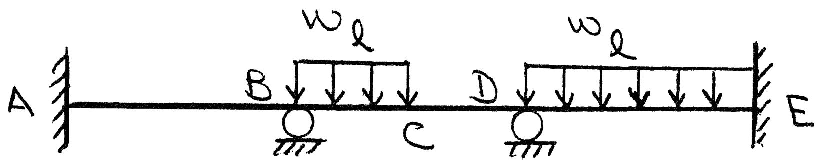


Live Load Arrangement for Maximum Negative M_B

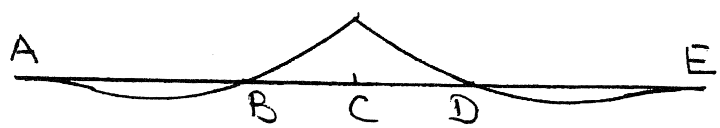
15.15 (Contd.)



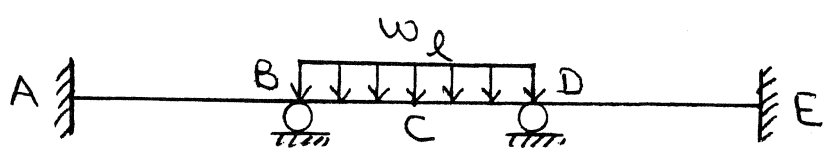
S_C



Live Load Arrangement for Maximum Negative S_C

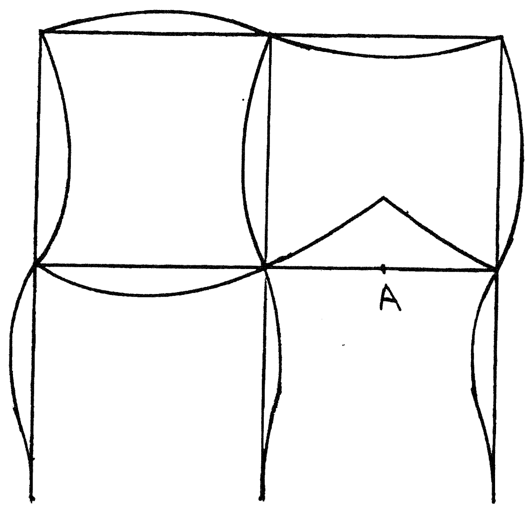


M_C

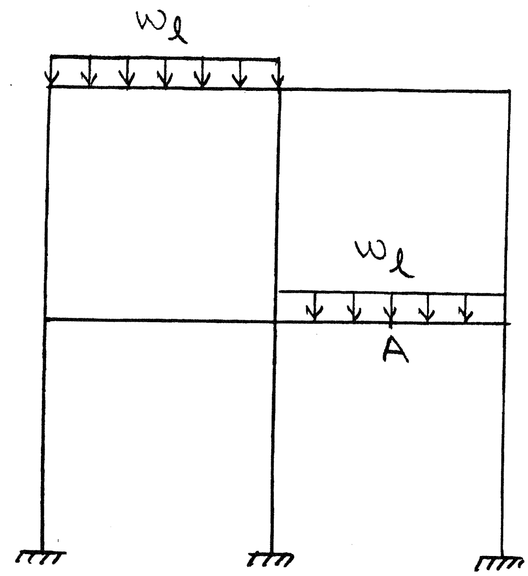


Live Load Arrangement for Maximum Positive M_C

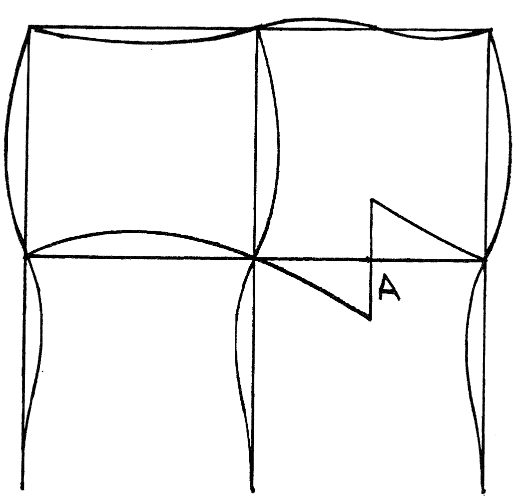
15.16



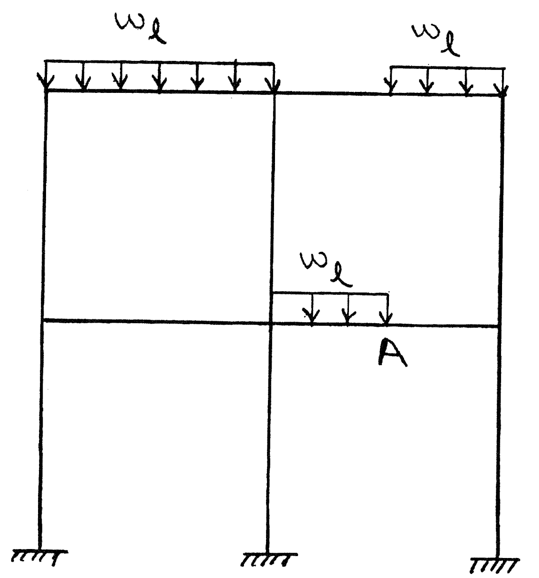
M_A



Live Load Arrangement for Max. Positive M_A

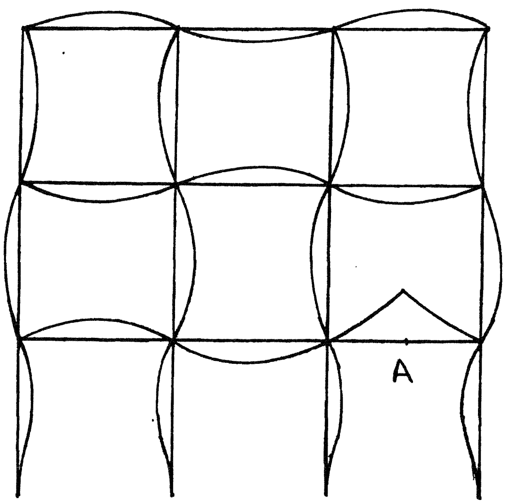


S_A

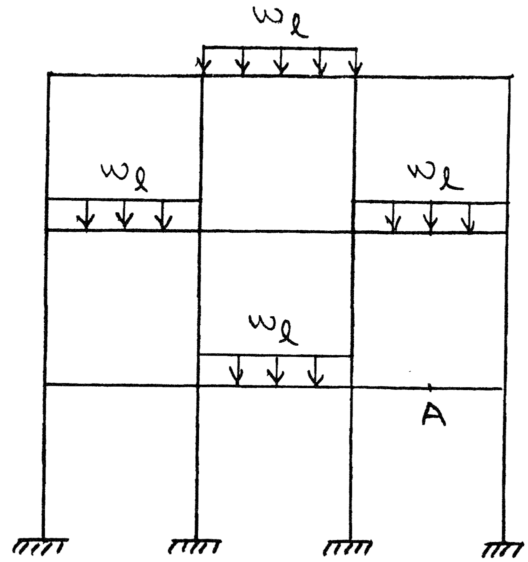


Live Load Arrangement for Max. Negative S_A

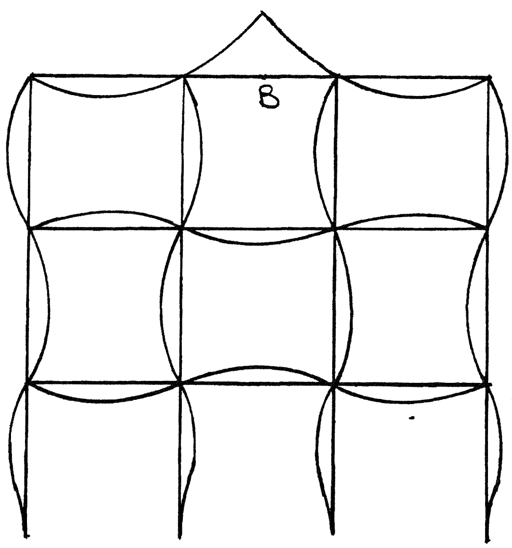
15.17



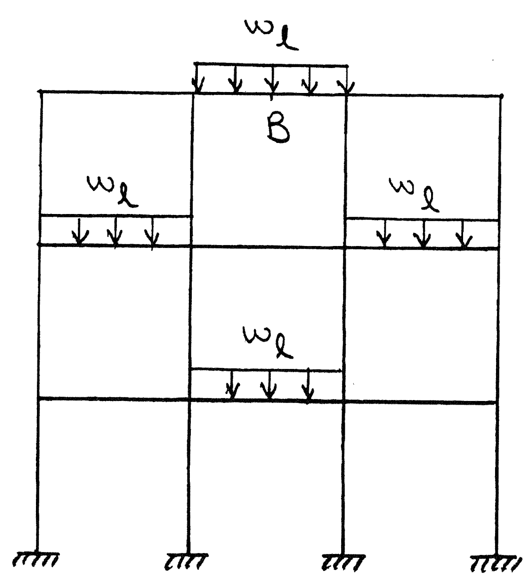
Qualitative I.L. for M_A



Live Load Arrangement for Max. Negative M_A



Qualitative I.L. for M_B



Live Load Arrangement for Max. Positive M_B

Chapter Sixteen
Slope-Deflection Method

CHAPTER 16

16-1 Fixed end moments:

$FEM_{AC} = +40$ k-ft ; $FEM_{CA} = -80$ k-ft

$FEM_{CE} = +37.5$ k-ft ; $FEM_{EC} = -37.5$ k-ft

Slope-deflection equations:

$M_{AC} = 0.0667 EI\theta_C + 40$; $M_{CA} = 0.133 EI\theta_C - 80$

$M_{CE} = 0.133 EI\theta_C + 37.5$; $M_{EC} = 0.0667 EI\theta_C - 37.5$

Equilibrium equation: $M_{CA} + M_{CE} = 0$

$$(0.133 EI\theta_C - 80) + (0.133 EI\theta_C + 37.5) = 0$$

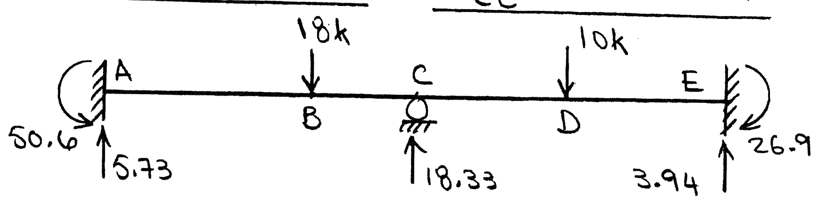
$$0.267 EI\theta_C - 42.5 = 0$$

$$EI\theta_C = 159.2 \text{ k-ft}^2$$

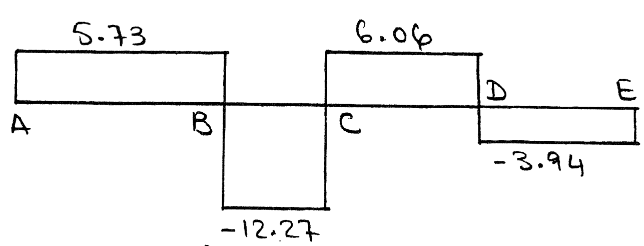
Member end moments. Substituting the numerical value of $EI\theta_C$ into the slope-deflection equations, we obtain:

$M_{AC} = 50.6$ k-ft ; $M_{CA} = -58.8$ k-ft

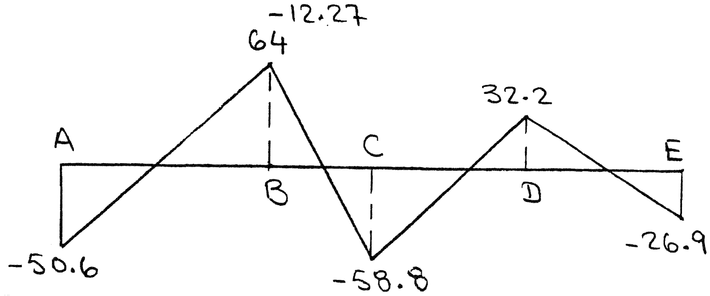
$M_{CE} = 58.8$ k-ft ; $M_{EC} = -26.9$ k-ft



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.2 Fixed-end Moments:

$$FEM_{AB} = \frac{1.5(30)^2}{12} + \frac{20(30)}{8} = 187.5 \text{ k-ft}$$

$$FEM_{BA} = -187.5 \text{ k-ft}; \quad FEM_{BC} = \frac{3(20)^2}{12} = 100 \text{ k-ft}$$

$$FEM_{CB} = -100 \text{ k-ft}.$$

Slope-deflection equations:

$$M_{AB} = 0.0667 EI\theta_B + 187.5; \quad M_{BA} = 0.133 EI\theta_B - 187.5$$

$$M_{BC} = 0.2 EI\theta_B + 100; \quad M_{CB} = 0.1 EI\theta_B - 100$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

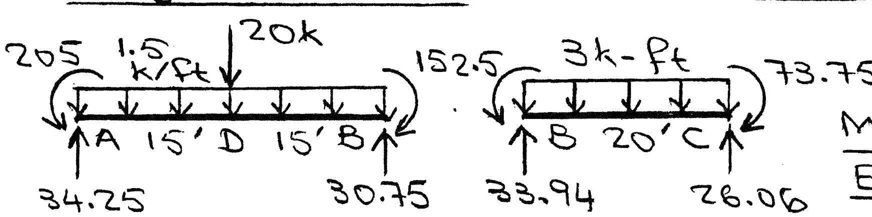
$$(0.133 EI\theta_B - 187.5) + (0.2 EI\theta_B + 100) = 0$$

$$EI\theta_B = 262.5 \text{ k-ft}^2$$

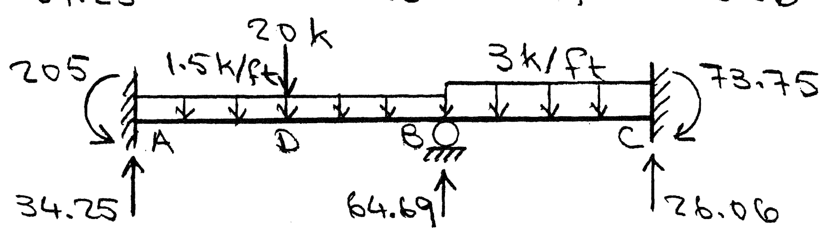
Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$M_{AB} = 205$ k-ft; $M_{BA} = -152.5$ k-ft

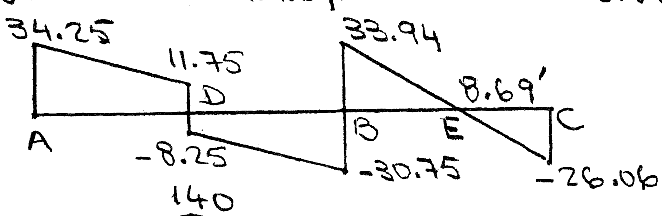
$M_{BC} = 152.5$ k-ft; $M_{CB} = -73.75$ k-ft



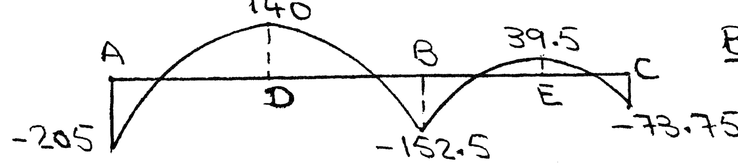
Member End Forces



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.3 Fixed end moments:

$$FEM_{BE} = +400 \text{ kN.m}; \quad FEM_{EB} = -400 \text{ kN.m}$$

Slope-deflection equations:

$$M_{AB} = 0.222\,EI\theta_B; \quad M_{BA} = 0.444\,EI\theta_B$$

$$M_{BE} = 0.444\,EI\theta_B + 400; \quad M_{EB} = 0.222\,EI\theta_B - 400$$

Equilibrium equation: $M_{BA} + M_{BE} = 0$

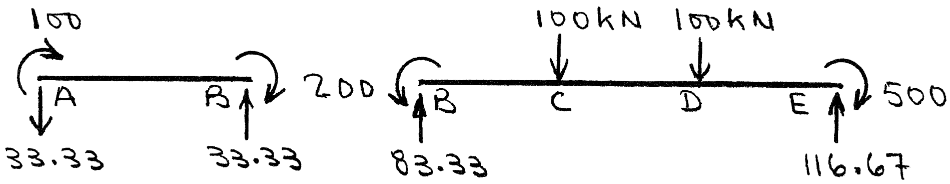
$$0.444\,EI\theta_B + (0.444\,EI\theta_B + 400) = 0$$

$$EI\theta_B = -450 \text{ kN.m}^2$$

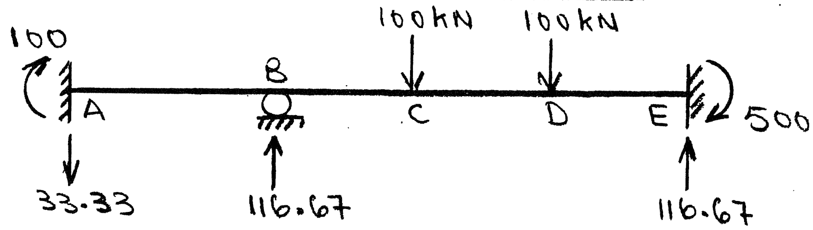
Member end moments: Substituting the numerical value of $EI\theta_C$ into the slope-deflection equations, we obtain

$$M_{AB} = -100 \text{ kN.m}; \quad M_{BA} = -200 \text{ kN.m}$$

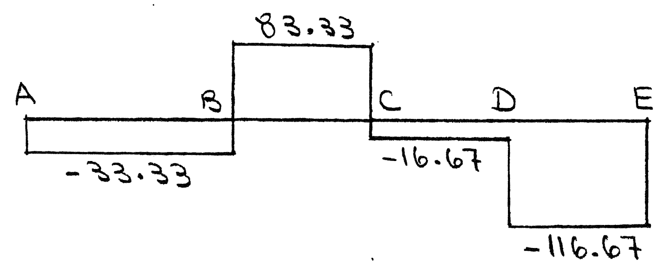
$$M_{BE} = 200 \text{ kN.m}; \quad M_{EB} = -500 \text{ kN.m}$$



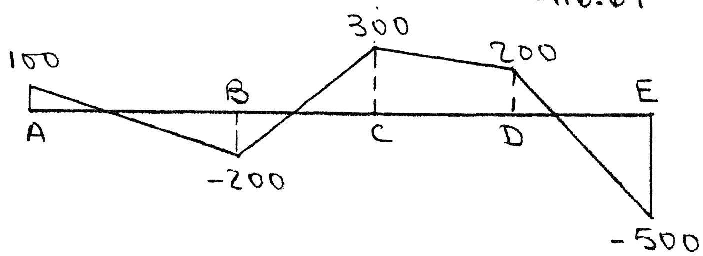
Member End Forces



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

16.4 Fixed-end moments:

$$FEM_{AB} = \frac{25(8)^2}{12} = 133.33 \text{ kN.m}; \quad FEM_{BA} = -133.33 \text{ kN.m}$$

$$FEM_{BC} = 133.33 \text{ kN.m}; \quad FEM_{CB} = -133.33 \text{ kN.m}$$

Slope-deflection equations: $M_{CB} = 0$

$$M_{AB} = 0.25 EI\theta_B + 133.33; \quad M_{BA} = 0.5 EI\theta_B - 133.33$$

$$M_{BC} = 0.375 EI\theta_B + 200$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.5 EI\theta_B - 133.33) + (0.375 EI\theta_B + 200) = 0$$

$$EI\theta_B = -76.19 \text{ kN.m}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$$M_{AB} = 114.3 \text{ kN.m}; \quad M_{BA} = -171.4 \text{ kN.m}$$

$$M_{BC} = 171.4 \text{ kN.m}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.37.

16.5

Fixed-end moments:

$$FEM_{AB} = \frac{3(25)^2}{12} = 156.25 \text{ k-ft}; \quad FEM_{BA} = -156.25 \text{ k-ft}$$

$$FEM_{BC} = \frac{3(15)^2}{12} = 56.25 \text{ k-ft}; \quad FEM_{CB} = -56.25 \text{ k-ft}$$

Slope-deflection equations: $M_{AB} = M_{CB} = 0$

$$M_{BA} = \frac{3E(2I)}{25}\theta_B - 156.25 - \frac{156.25}{2} = 0.24\,EI\theta_B - 234.38$$

$$M_{BC} = \frac{3EI}{15}\theta_B + 56.25 + \frac{56.25}{2} = 0.2\,EI\theta_B + 84.38$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.24EI\theta_B - 234.38) + (0.2EI\theta_B + 84.38) = 0$$

$$EI\theta_B = 340.91 \text{ k-ft}^2$$

Member end moments. Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain

$$M_{BA} = -152.6 \text{ k-ft}; \quad M_{BC} = 152.6 \text{ k-ft}$$

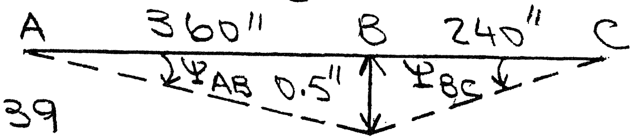
For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

16.6 Fixed-end moments:

$FEM_{AB} = 187.5$ k-ft; $FEM_{BA} = -187.5$ k-ft

$FEM_{BC} = 100$ k-ft; $FEM_{CB} = -100$ k-ft

Chord rotations:



$$\psi_{AB} = -\frac{0.5}{360} = -0.00139$$

$$\psi_{BC} = +\frac{0.5}{240} = +0.00208$$

Slope-deflection equations:

Using $EI = \frac{29000(1650)}{(12)^2}$ k-ft^2, we write

$$M_{AB} = 0.0667\,EI\theta_B + 92.3 + 187.5 = 0.0667\,EI\theta_B + 279.8$$

$$M_{BA} = 0.133\,EI\theta_B + 92.3 - 187.5 = 0.133\,EI\theta_B - 95.2$$

$$M_{BC} = 0.2\,EI\theta_B - 207.7 + 100 = 0.2\,EI\theta_B - 107.7$$

$$M_{CB} = 0.1\,EI\theta_B - 207.7 - 100 = 0.1\,EI\theta_B - 307.7$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.133\,EI\theta_B - 95.2) + (0.2\,EI\theta_B - 107.7) = 0$$

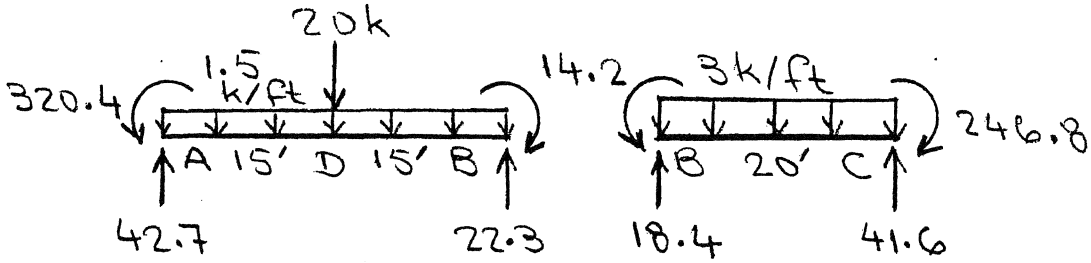
$$EI\theta_B = 609.31 \text{ k-ft}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

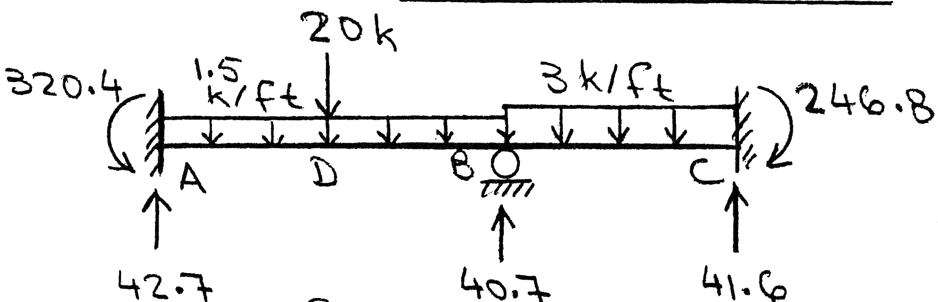
$M_{AB} = 320.4$ k-ft; $M_{BA} = -14.2$ k-ft

$M_{BC} = 14.2$ k-ft; $M_{CB} = -246.8$ k-ft

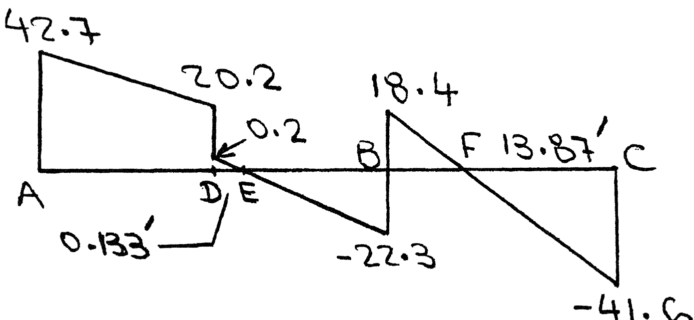
16.6 (Contd.)



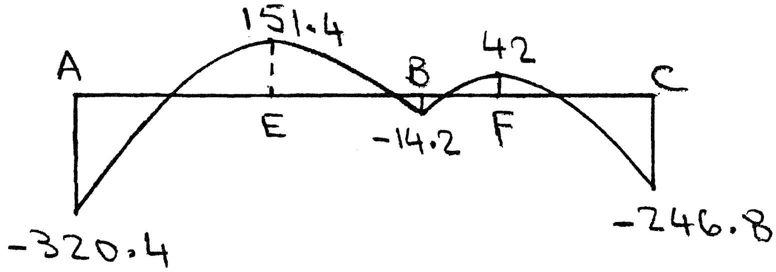
Member End Forces



Reactions



Shear Diagram (k)



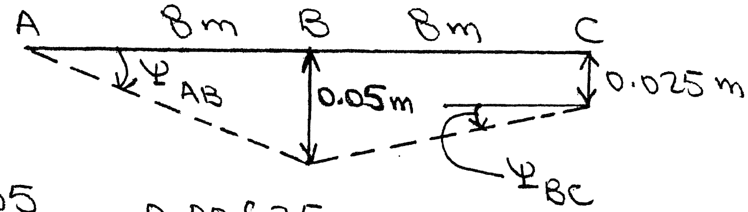
Bending Moment Diagram (k-ft)

16.7 Fixed-end moments:

$FEM_{AB} = 133.33$ kN.m ; $FEM_{BA} = -133.33$ kN.m

$FEM_{BC} = 133.33$ kN.m ; $FEM_{CB} = -133.33$ kN.m

Chord rotations:



$$\psi_{AB} = -\frac{0.05}{8} = -0.00625$$

$$\psi_{BC} = +\frac{0.025}{8} = +0.003125$$

Slope-deflection equations: $M_{CB} = 0$

Using $EI = 70(1300) = 91000$ kN.m^2, we write

$$M_{AB} = 0.25EI\theta_B + 426.56 + 133.33 = 0.25EI\theta_B + 559.89$$

$$M_{BA} = 0.5EI\theta_B + 426.56 - 133.33 = 0.5EI\theta_B + 293.23$$

$$M_{BC} = 0.375EI\theta_B - 106.64 + 133.33 + \frac{133.33}{2}$$

$$= 0.375EI\theta_B + 93.36$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.5EI\theta_B + 293.23) + (0.375EI\theta_B + 93.36) = 0$$

$$EI\theta_B = -441.82 \text{ kN.m}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$M_{AB} = 449.4$ kN.m ; $M_{BA} = 72.3$ kN.m

$M_{BC} = -72.3$ kN.m

For reactions, and shear and bending moment diagrams, see solution of Problem 13.53.

16.8 Fixed-end moments:

$FEM_{AB} = \frac{1.5(25)^2}{12} = 78.125$ k-ft ; $FEM_{BA} = -78.125$ k-ft

$FEM_{BC} = \frac{1.5(20)^2}{12} = 50$ k-ft ; $FEM_{CB} = -50$ k-ft.

$FEM_{CD} = 78.125$ k-ft ; $FEM_{DC} = -78.125$ k-ft.

Slope-deflection equations:

$M_{AB} = 0.08EI\theta_B + 78.125$; $M_{BA} = 0.16EI\theta_B - 78.125$

$M_{BC} = 0.2EI\theta_B + 0.1EI\theta_C + 50$; $M_{CB} = 0.1EI\theta_B + 0.2EI\theta_C - 50$

$M_{CD} = 0.16EI\theta_C + 78.125$; $M_{DC} = 0.08EI\theta_C - 78.125$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

$0.36EI\theta_B + 0.1EI\theta_C = 28.125$

$0.1EI\theta_B + 0.36EI\theta_C = -28.125$

By solving these equations, we obtain:

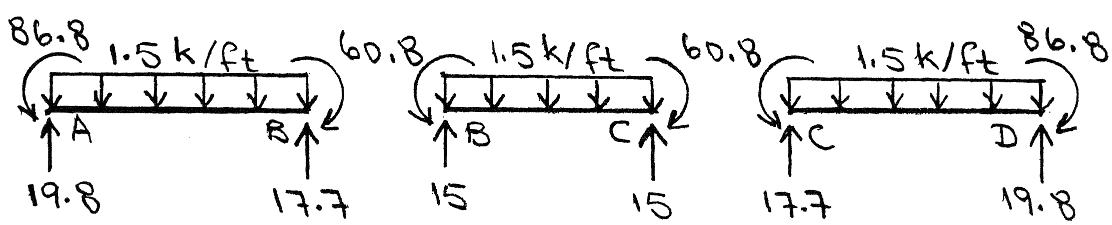
$EI\theta_B = 108.173$ k-ft^2 ; $EI\theta_C = -108.173$ k-ft^2

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain:

$M_{AB} = 86.8$ k-ft ; $M_{BA} = -60.8$ k-ft

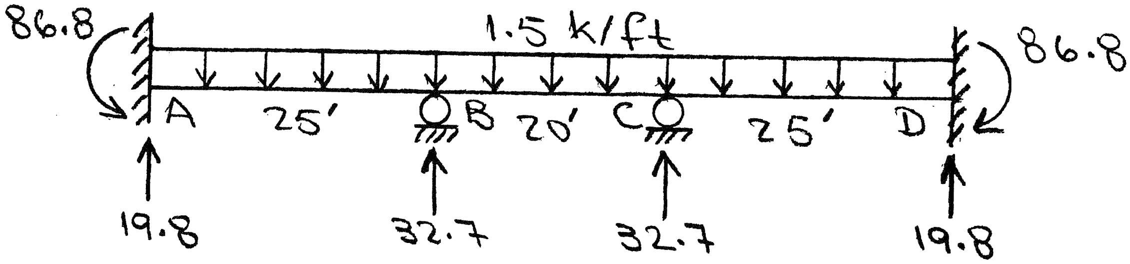
$M_{BC} = 60.8$ k-ft ; $M_{CB} = -60.8$ k-ft

$M_{CD} = 60.8$ k-ft ; $M_{DC} = -86.8$ k-ft

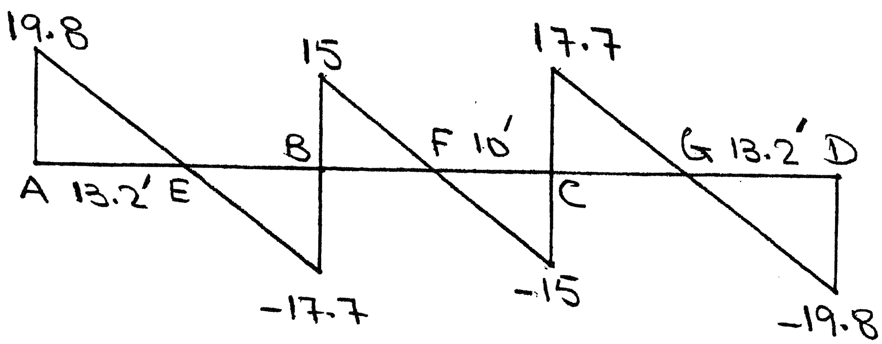


Member End Moments and Shears

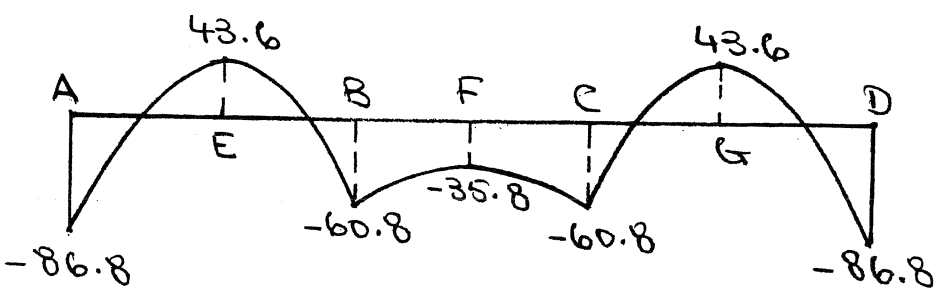
16.8 (Contd.)



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.9 Fixed end moments:

$$FEM_{AB} = \frac{20(8)^2}{12} = 106.7\ kN.m;\quad FEM_{BA} = -106.7\ kN.m$$

$$FEM_{BC} = 106.7\ kN.m;\quad FEM_{CB} = -106.7\ kN.m$$

$$FEM_{CE} = \frac{60(8)}{8} = 60\ kN.m;\quad FEM_{EC} = -60\ kN.m$$

Slope-deflection equations:

$$M_{AB} = 0.25EI\theta_B + 106.7;\quad M_{BA} = 0.5EI\theta_B - 106.7$$

$$M_{BC} = 0.5EI\theta_B + 0.25EI\theta_C + 106.7$$

$$M_{CB} = 0.25EI\theta_B + 0.5EI\theta_C - 106.7$$

$$M_{CE} = 0.5EI\theta_C + 60;\quad M_{EC} = 0.25EI\theta_C - 60$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$M_{CB} + M_{CE} = 0$$

$$EI\theta_B + 0.25EI\theta_C = 0$$

$$0.25EI\theta_B + EI\theta_C = 46.7$$

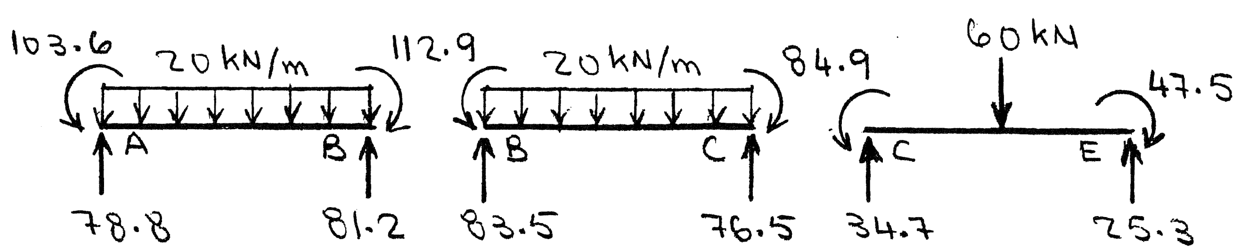
By solving these equations, we obtain

$$EI\theta_B = -12.45\ kN.m^2;\quad EI\theta_C = 49.81\ kN.m^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

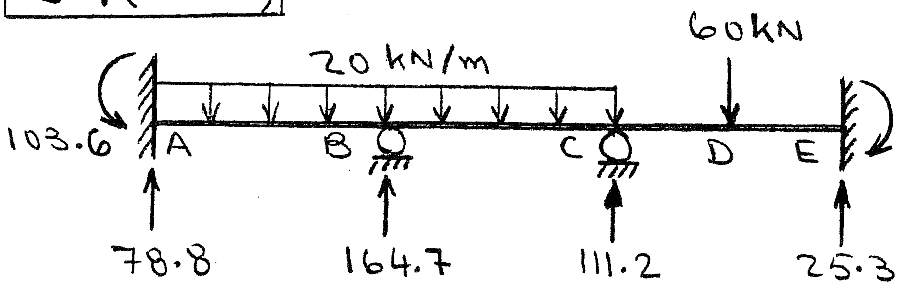
$M_{AB} = 103.6\ kN.m$; $M_{BA} = -112.9\ kN.m$; $M_{BC} = 112.9\ kN.m$

$M_{CB} = -84.9\ kN.m$; $M_{CE} = 84.9\ kN.m$; $M_{EC} = -47.5\ kN.m$



Member End Moments and Shears

16.9 (contd.)



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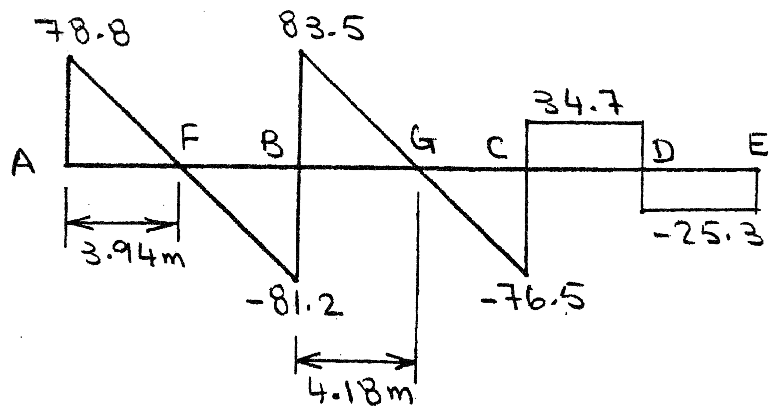
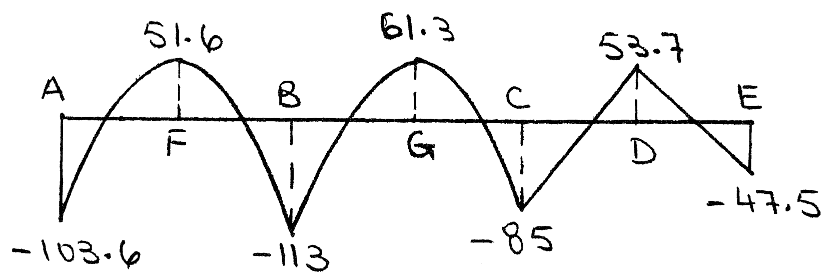


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

16.10 Fixed-end moments:

$FEM_{AB} = \frac{2(15)^2}{12} = 37.5$ k-ft; $FEM_{BA} = -37.5$ k-ft

$FEM_{BC} = \frac{3(10)^2}{30} = 10$ k-ft; $FEM_{CB} = -\frac{3(10)^2}{20} = -15$ k-ft

$FEM_{CD} = 15$ k-ft; $FEM_{DC} = -10$ k-ft

Slope-deflection equations:

$M_{AB} = 0.133EI\theta_B + 37.5$; $M_{BA} = 0.267EI\theta_B - 37.5$

$M_{BC} = 0.4EI\theta_B + 0.2EI\theta_C + 10$

$M_{CB} = 0.2EI\theta_B + 0.4EI\theta_C - 15$

$M_{CD} = 0.4EI\theta_C + 15$; $M_{DC} = 0.2EI\theta_C - 10$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

$0.667EI\theta_B + 0.2EI\theta_C = 27.5$

$0.2EI\theta_B + 0.8EI\theta_C = 0$

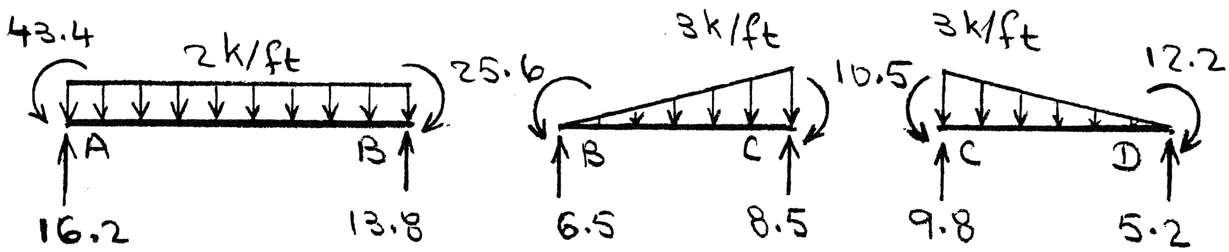
By solving these equations, we obtain

$EI\theta_B = 44.59$ k-ft²; $EI\theta_C = -11.15$ k-ft²

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

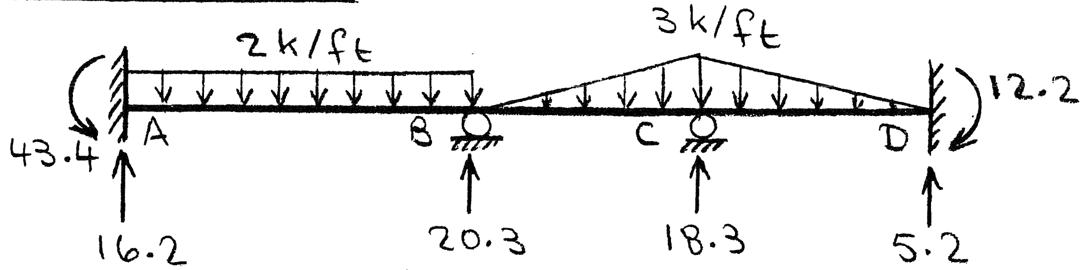
$M_{AB} = 43.4$ k-ft; $M_{BA} = -25.6$ k-ft; $M_{BC} = 25.6$ k-ft

$M_{CB} = -10.5$ k-ft; $M_{CD} = 10.5$ k-ft; $M_{DC} = -12.2$ k-ft

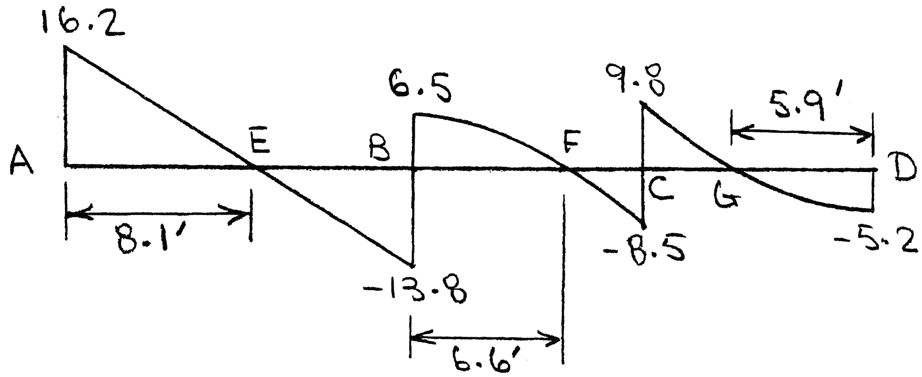


Member End Moments and Shears

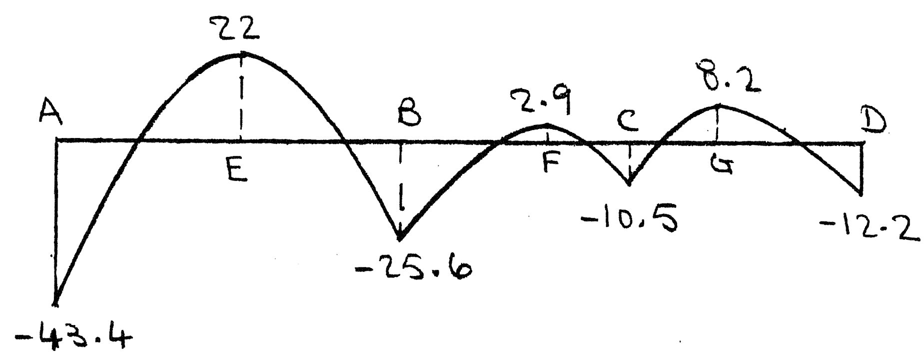
16.10 (contd.)



Reactions

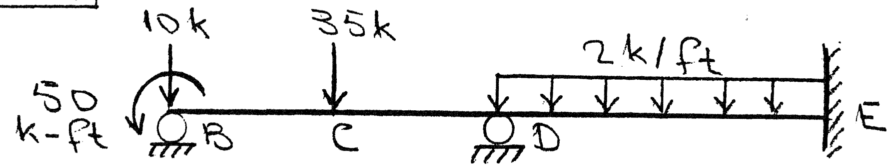


Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.11



Fixed-end moments:

$FEM_{BD} = \frac{35(20)}{8} = 87.5$ k-ft; $FEM_{DB} = -87.5$ k-ft

$FEM_{DE} = \frac{2(20)^2}{12} = 66.67$ k-ft; $FEM_{ED} = -66.67$ k-ft

Slope-deflection equations:

$M_{BD} = 0.2EI\theta_B + 0.1EI\theta_D + 87.5$

$M_{DB} = 0.1EI\theta_B + 0.2EI\theta_D - 87.5$

$M_{DE} = 0.2EI\theta_D + 66.67$; $M_{ED} = 0.1EI\theta_D - 66.67$

Equilibrium equations: $M_{BD} - 50 = 0$

$M_{DB} + M_{DE} = 0$

$0.2EI\theta_B + 0.1EI\theta_D = -37.5$

$0.1EI\theta_B + 0.4EI\theta_D = 20.83$

By solving these equations, we obtain:

$EI\theta_B = -244.05$ k-ft^2; $EI\theta_D = 113.1$ k-ft^2

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_D$ into the slope-deflection equations, we obtain:

$M_{BD} = 50$ k-ft; $M_{DB} = -89.3$ k-ft

$M_{DE} = 89.3$ k-ft; $M_{ED} = -55.4$ k-ft

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

16.12 Fixed-end moments:

$$FEM_{AC} = FEM_{CE} = \frac{120(6)(4)^2}{(10)^2} = 115.2 \text{ kN.m};$$

$$FEM_{CA} = FEM_{EC} = -\frac{120(4)(6)^2}{(10)^2} = -172.8 \text{ kN.m}$$

$$FEM_{EG} = \frac{150(8)}{8} = 150 \text{ kN.m}; \quad FEM_{GE} = -150 \text{ kN.m}$$

Slope-deflection equations: $M_{AC} = M_{GE} = 0$

$$M_{CA} = \frac{3EI}{10}\theta_C - 172.8 - \frac{115.2}{2} = 0.3EI\theta_C - 230.4$$

$$M_{CE} = \frac{2E(2I)}{10}(2\theta_C + \theta_E) + 115.2 = 0.4EI(2\theta_C + \theta_E) + 115.2$$

$$M_{EC} = 0.4EI(\theta_C + 2\theta_E) - 172.8$$

$$M_{EG} = \frac{3EI}{8}\theta_E + 150 + \frac{150}{2} = 0.375EI\theta_E + 225$$

Equilibrium equations: $M_{CA} + M_{CE} = 0$

$M_{EC} + M_{EG} = 0$

$$1.1EI\theta_C + 0.4EI\theta_E = 115.2$$

$$0.4EI\theta_C + 1.175EI\theta_E = -52.2$$

By solving these equations, we obtain

$$EI\theta_C = 137.96 \text{ kN.m}^2; \quad EI\theta_E = -91.39 \text{ kN.m}^2$$

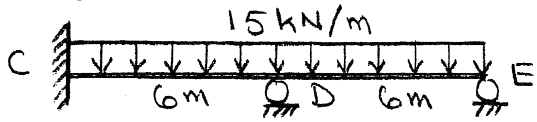
Member end moments. Substituting the numerical values of $EI\theta_C$ and $EI\theta_E$ into the slope-deflection equations, we obtain

$M_{CA} = -189$ kN.m ; $M_{CE} = 189$ kN.m

$M_{EC} = -190.7$ kN.m; $M_{EG} = 190.7$ kN.m

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

16.13 As the beam and the loading are symmetric, we will analyze only the right half CE of the beam.



Fixed-end moments: $FEM_{CD} = 45\ kN.m$; $FEM_{DC} = -45\ kN.m$

$FEM_{DE} = 45\ kN.m$; $FEM_{ED} = -45\ kN.m$

Slope-deflection equations: $M_{ED} = 0$

$M_{CD} = 0.333\,EI\theta_D + 45$; $M_{DC} = 0.667\,EI\theta_D - 45$

$$M_{DE} = \frac{3EI}{6}(\theta_D) + 45 + \frac{45}{2} = 0.5\,EI\theta_D + 67.5$$

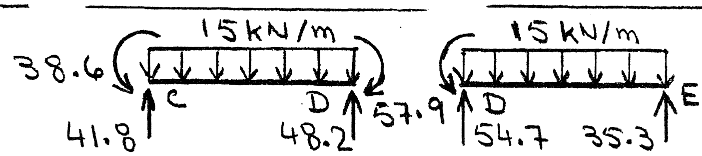
Equilibrium equation: $M_{DC} + M_{DE} = 0$

$1.167\,EI\theta_D + 22.5 = 0$; $EI\theta_D = -19.28\ kN.m^2$

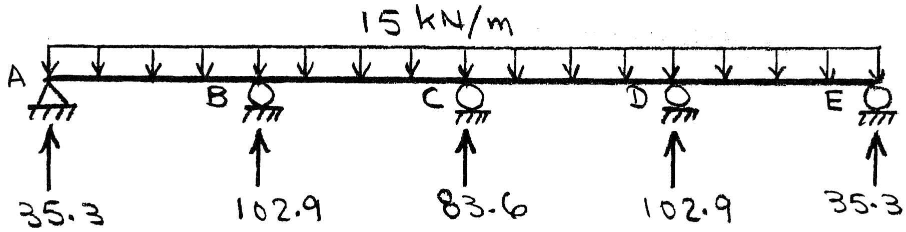
Member end moments: Substituting the numerical value of $EI\theta_D$ into the slope-deflection equations, we obtain

$M_{CD} = 38.6\ kN.m$; $M_{DC} = -57.9\ kN.m$; $M_{DE} = 57.9\ kN.m$

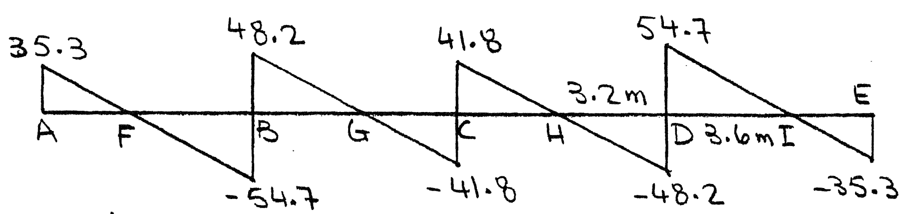
Member End Moments and Shears



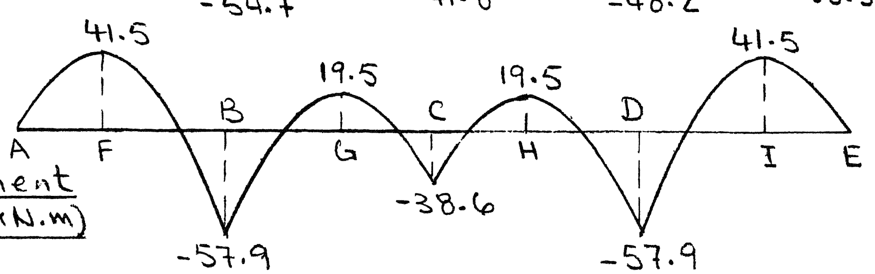
Reactions



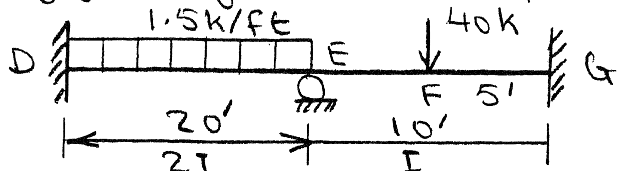
Shear Diagram (kN)



Bending Moment Diagram (kN.m)



16.14 As the beam and loading are symmetric, we will analyze only the right half DG of the beam.



<u>Fixed end moments:</u> $FEM_{DE} = 50$ k-ft ; $FEM_{ED} = -50$ k-ft

$FEM_{EG} = 100$ k-ft ; $FEM_{GE} = -100$ k-ft

<u>Slope deflection equations:</u>

$M_{DE} = 0.2EI\theta_E + 50$; $M_{ED} = 0.4EI\theta_E - 50$

$M_{EG} = 0.2EI\theta_E + 100$; $M_{GE} = 0.1EI\theta_E - 100$

<u>Equilibrium equation:</u> $M_{ED} + M_{EG} = 0$

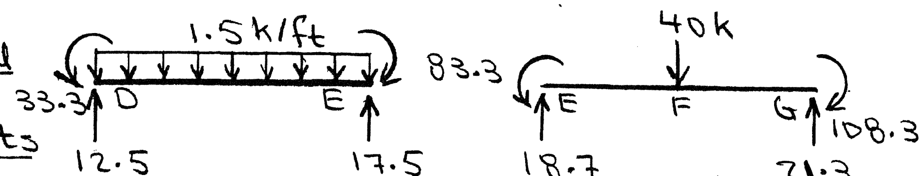
$0.6EI\theta_E + 50 = 0$ $\qquad EI\theta_E = -83.33$ k-ft^2

<u>Member end moments:</u> Substituting the numerical value of $EI\theta_E$ into the slope-deflection equations, we obtain

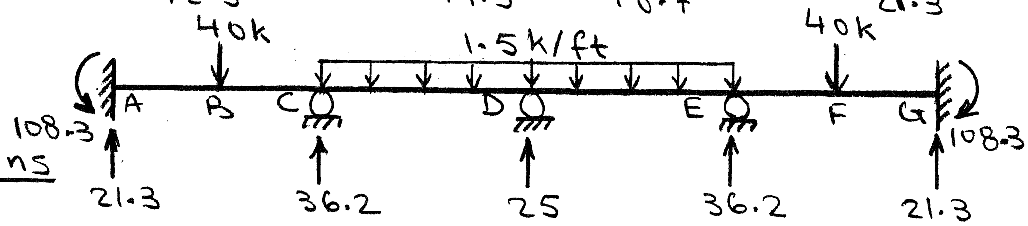
$M_{DE} = 33.3$ k-ft ; $M_{ED} = -83.3$ k-ft

$M_{EG} = 83.3$ k-ft ; $M_{GE} = -108.3$ k-ft

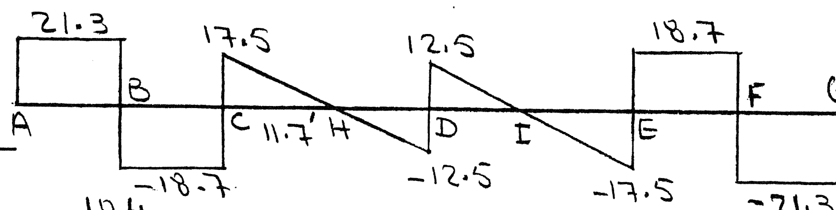
<u>Member End Shears and Moments</u>



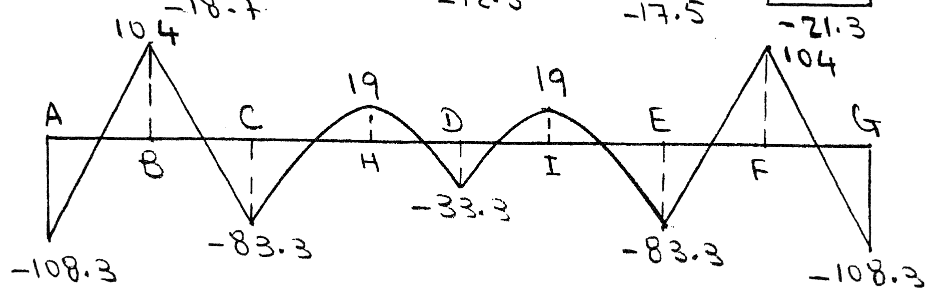
<u>Reactions</u>



<u>Shear Diagram (k)</u>



<u>Bending Moment Diagram (k-ft)</u>



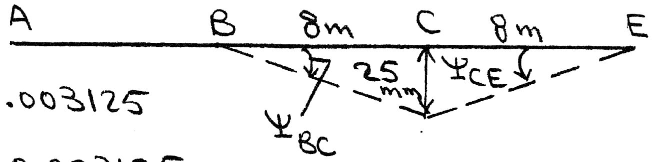
16.15 <u>Fixed end moments:</u>

$FEM_{AB} = 106.7$ kN.m; $FEM_{BA} = -106.7$ kN.m

$FEM_{BC} = 106.7$ kN.m; $FEM_{CB} = -106.7$ kN.m

$FEM_{CE} = 60$ kN.m; $FEM_{EC} = -60$ kN.m

<u>Chord rotations:</u>



$\Psi_{BC} = -\frac{0.025}{8} = -0.003125$

$\Psi_{CE} = +\frac{0.025}{8} = +0.003125$

<u>Slope-deflection equations:</u>

$M_{AB} = 0.25 EI\theta_B + 106.7$; $M_{BA} = 0.5 EI\theta_B - 106.7$

$M_{BC} = 0.5 EI\theta_B + 0.25 EI\theta_C + 238$

$M_{CB} = 0.25 EI\theta_B + 0.5 EI\theta_C + 24.6$

$M_{CE} = 0.5 EI\theta_C - 71.3$; $M_{EC} = 0.25 EI\theta_C - 191.3$

<u>Equilibrium equations:</u> $M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CE} = 0$

$EI\theta_B + 0.25 EI\theta_C = -131.3$

$0.25 EI\theta_B + EI\theta_C = 46.7$

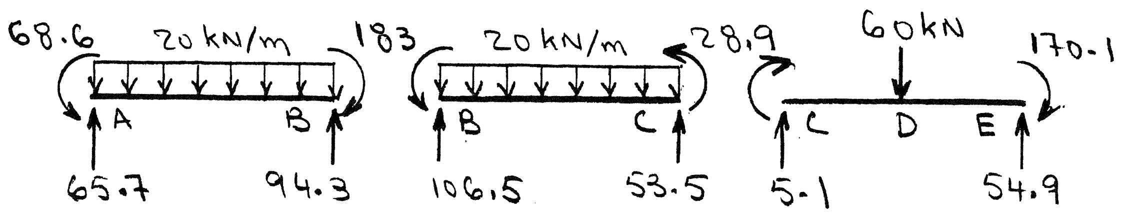
By solving these equations, we obtain

$EI\theta_B = -152.51$ kN.m^2; $EI\theta_C = 84.83$ kN.m^2

<u>Member end moments:</u> Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

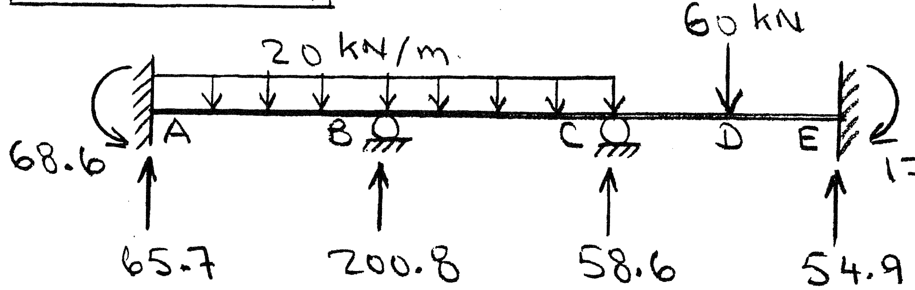
$M_{AB} = 68.6$ kN.m; $M_{BA} = -183$ kN.m; $M_{BC} = 183$ kN.m

$M_{CB} = 28.9$ kN.m; $M_{CE} = -28.9$ kN.m; $M_{EC} = -170.1$ kN.m

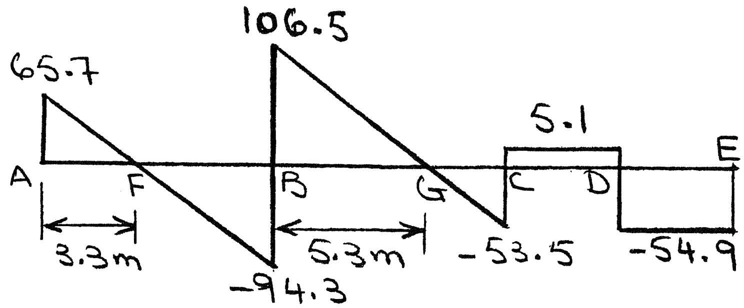


<u>Member End Moments and Shears</u>

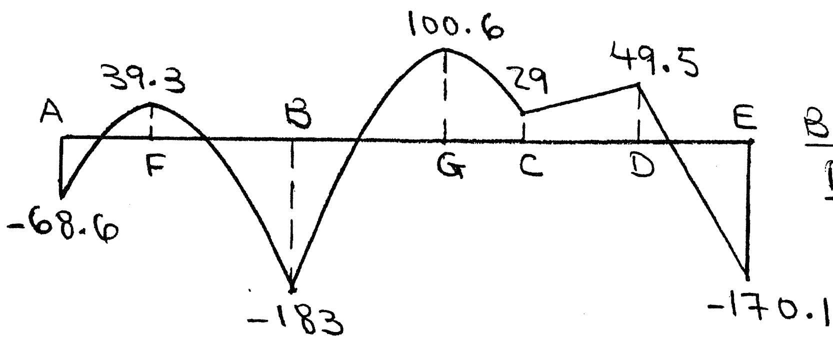
16.15 (contd.)



Reactions



Shear Diagram (kN)



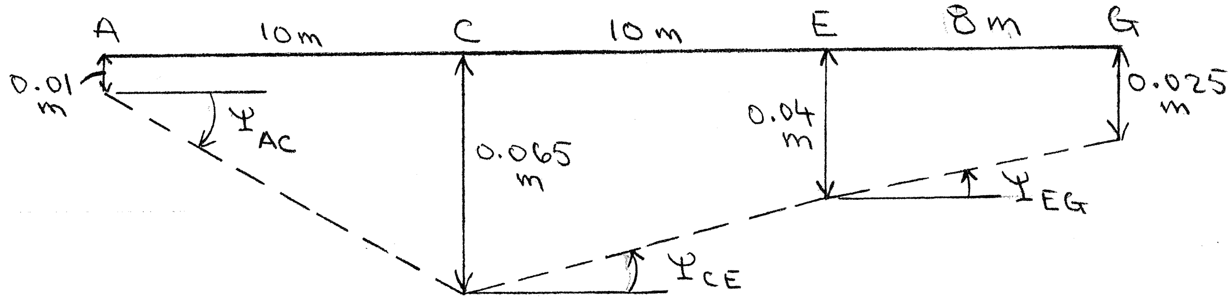
Bending Moment Diagram (kN.m)

16.16 Fixed-end moments: $FEM_{AC} = FEM_{CE} = 115.2$ kN·m

$FEM_{CA} = FEM_{EC} = -172.8$ kN·m; $FEM_{EG} = 150$ kN·m

$FEM_{GE} = -150$ kN·m

Chord rotations:



$\psi_{AC} = -\frac{0.055}{10} = -0.0055$; $\psi_{CE} = \frac{0.025}{10} = 0.0025$

$\psi_{EG} = \frac{0.015}{8} = 0.001875$

Slope-deflection equations: $M_{AC} = M_{GE} = 0$

Using $EI = 200(500) = 100000$ kN·m², we write:

$M_{CA} = 0.3EI\theta_C + 165 - 230.4 = 0.3EI\theta_C - 65.4$

$M_{CE} = 0.4EI(2\theta_C + \theta_E) - 300 + 115.2 = 0.4EI(2\theta_C + \theta_E) - 184.8$

$M_{EC} = 0.4EI(\theta_C + 2\theta_E) - 300 - 172.8 = 0.4EI(\theta_C + 2\theta_E) - 472.8$

$M_{EG} = 0.375EI\theta_E - 70.31 + 225 = 0.375EI\theta_E + 154.69$

Equilibrium equations:

$M_{CA} + M_{CE} = 0 \Rightarrow 1.1EI\theta_C + 0.4EI\theta_E = 250.2$

$M_{EC} + M_{EG} = 0 \Rightarrow 0.4EI\theta_C + 1.175EI\theta_E = 318.11$

By solving these equations, we obtain

$EI\theta_C = 147.23$ kN·m²; $EI\theta_E = 220.61$ kN·m²

Member end moments:

$M_{CA} = -21.2$ kN·m; $M_{CE} = 21.2$ kN·m

$M_{EC} = -237.4$ kN·m; $M_{EG} = 237.4$ kN·m

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

16.17 Fixed-end moments:

$FEM_{AC} = \frac{75(6)}{8} = 56.25$ kN.m; $FEM_{CA} = -56.25$ kN.m

$FEM_{CD} = \frac{25(9)^2}{12} = 168.75$ kN.m; $FEM_{DC} = -168.75$ kN.m

Slope-deflection equations: $M_{DC} = 0$

$M_{AC} = 0.333 EI\theta_C + 56.25$; $FEM_{CA} = 0.667 EI\theta_C - 56.25$

$M_{CD} = \frac{3EI}{9}(\theta_C) + 168.75 + \frac{168.75}{2} = 0.333 EI\theta_C + 253.13$

Equilibrium equation: $M_{CA} + M_{CD} = 0$

$$EI\theta_C = -196.88 \text{ kN.m}^2$$

Member end moments. Substituting the numerical value of $EI\theta_C$ into the slope-deflection equations, we obtain:

$M_{AC} = -9.4$ kN.m; $M_{CA} = -187.5$ kN.m

$M_{CD} = 187.5$ kN.m

For reactions, see solution of Problem 13.42.

16.18 Fixed-end moments:

$FEM_{AC} = FEM_{CA} = FEM_{CD} = FEM_{DC} = 0$

$FEM_{BC} = \dfrac{2(15)^2}{12} = 37.5 \text{ k-ft}$; $FEM_{CB} = -37.5 \text{ k-ft}$

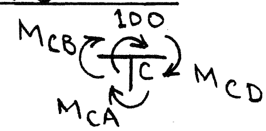
Slope-deflection equations:

$M_{AC} = 0.133 EI\theta_C$; $M_{CA} = 0.267 EI\theta_C$

$M_{BC} = 0.133 EI\theta_C + 37.5$; $M_{CB} = 0.267 EI\theta_C - 37.5$

$M_{CD} = 0.2 EI\theta_C$; $M_{DC} = 0$

Equilibrium equation:



$M_{CA} + M_{CB} + M_{CD} + 100 = 0$

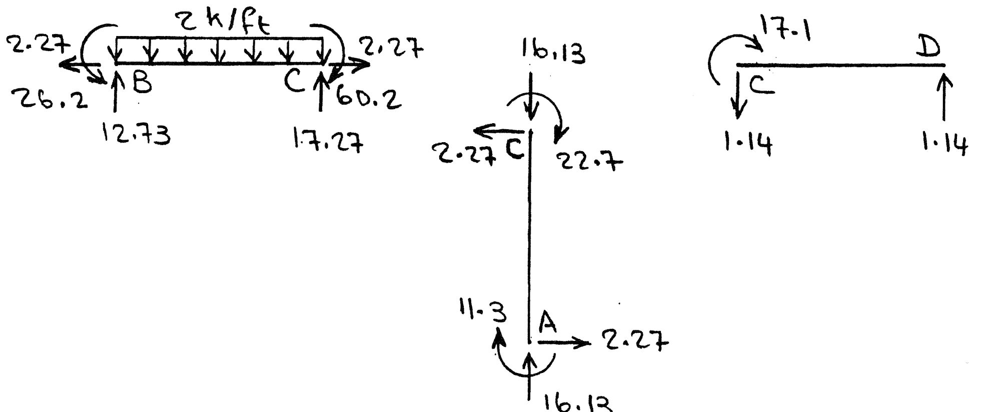
$(0.267 EI\theta_C) + (0.267 EI\theta_C - 37.5) + (0.2 EI\theta_C) + 100 = 0$

$0.734 EI\theta_C = -62.5 \qquad EI\theta_C = -85.15 \text{ k-ft}^2$

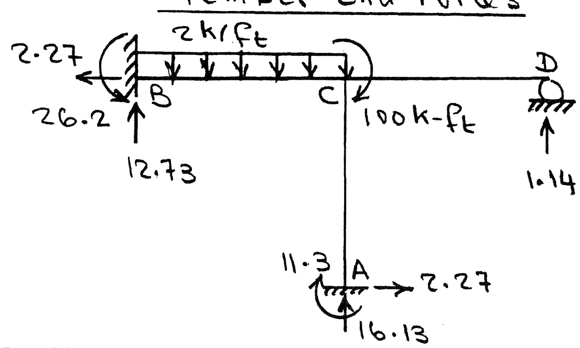
Member end moments: $M_{AC} = -11.3$ k-ft ;

$M_{CA} = -22.7$ k-ft ; $M_{BC} = 26.2$ k-ft ; $M_{CB} = -60.2$ k-ft;

$M_{CD} = -17.1$ k-ft .

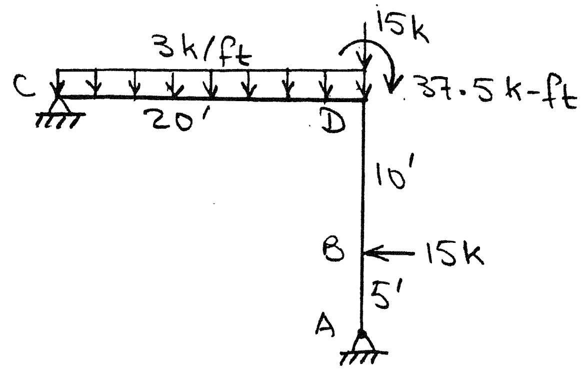


Member End Forces



Reactions

16.19



Fixed-end moments:

$$FEM_{AD} = -\frac{15(5)(10)^2}{(15)^2} = -33.33 \text{ k-ft}$$

$$FEM_{DA} = \frac{15(10)(5)^2}{(15)^2} = 16.67 \text{ k-ft}$$

$$FEM_{CD} = \frac{3(20)^2}{12} = 100 \text{ k-ft}; \quad FEM_{DC} = -100 \text{ k-ft}.$$

Slope-deflection equations: $M_{AD} = M_{CD} = 0$

$$M_{DA} = \frac{3EI}{15}(\theta_D) + 16.67 + \frac{33.33}{2} = 0.2EI\theta_D + 33.34$$

$$M_{DC} = \frac{3E(2I)}{20}(\theta_D) - 100 - \frac{100}{2} = 0.3EI\theta_D - 150$$

Equilibrium equation: $M_{DA} + M_{DC} + 37.5 = 0$

$$(0.2EI\theta_D + 33.34) + (0.3EI\theta_D - 150) + 37.5 = 0$$

$$0.5EI\theta_D = 79.16; \quad EI\theta_\theta = 158.32 \text{ k-ft}^2$$

Member end moments:

$M_{DA} = 65$ k-ft; $M_{DC} = -102.5$ k-ft

For reactions, see solution of Problem 13.21.

16.20 Fixed-end moments: $FEM_{AC} = FEM_{CA} = 0$

$FEM_{BD} = FEM_{DB} = 0$

$FEM_{CD} = \frac{30(10)^2}{12} = 250\ kN.m$; $FEM_{DC} = -250\ kN.m$

Slope-deflection equations:

$M_{AC} = 0.25\,EI\theta_C$; $M_{CA} = 0.5\,EI\theta_C$

$M_{BD} = 0.25\,EI\theta_D$; $M_{DB} = 0.5\,EI\theta_D$

$M_{CD} = 0.4EI\theta_C + 0.2\,EI\theta_D + 250$

$M_{DC} = 0.2EI\theta_C + 0.4\,EI\theta_D - 250$

Equilibrium equations: $M_{CA} + M_{CD} = 0$

$M_{DB} + M_{DC} = 0$

$0.9EI\theta_C + 0.2EI\theta_D = -250$

$0.2\,EI\theta_C + 0.9EI\,\theta_D = 250$

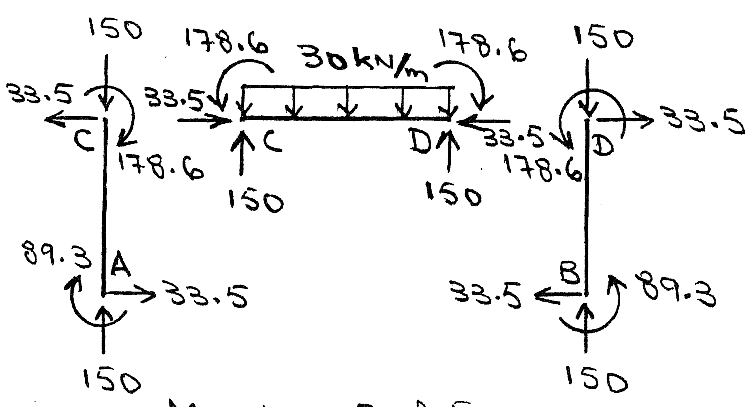
By solving these equations, we obtain:

$EI\theta_C = -357.14\ kN.m^2$; $EI\theta_D = 357.14\ kN.m^2$

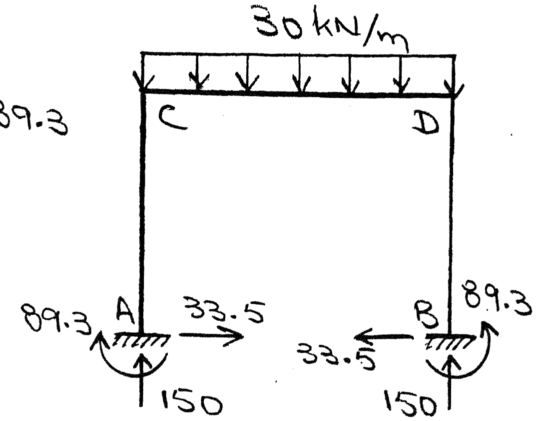
Member end moments: $M_{AC} = -89.3\ kN.m$;

$M_{CA} = -178.6\ kN.m$; $M_{BD} = 89.3\ kN.m$; $M_{DB} = 178.6\ kN.m$;

$M_{CD} = 178.6\ kN.m$; $M_{DC} = -178.6\ kN.m.$



Member End Forces

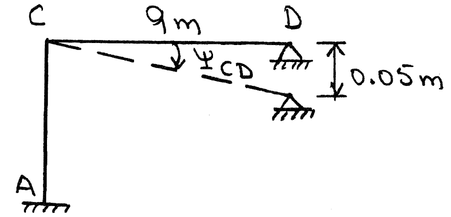


Reactions

16.21 Fixed-end moments:

$FEM_{AC} = 56.25$ kN.m ; $FEM_{CA} = -56.25$ kN.m

$FEM_{CD} = 168.75$ kN.m ; $FEM_{DC} = -168.75$ kN.m



Chord rotation:

$$\Psi_{CD} = -\frac{0.05}{9} = -0.00556$$

Slope-deflection equations: $M_{DC} = 0$

$M_{AC} = 0.333\,EI\theta_C + 56.25$; $M_{CA} = 0.667\,EI\theta_C - 56.25$

Using $EI = 200(400)$ kN-m², we write

$$M_{CD} = 0.333\,EI\theta_C + 148.27 + 168.75 + \frac{168.75}{2}$$

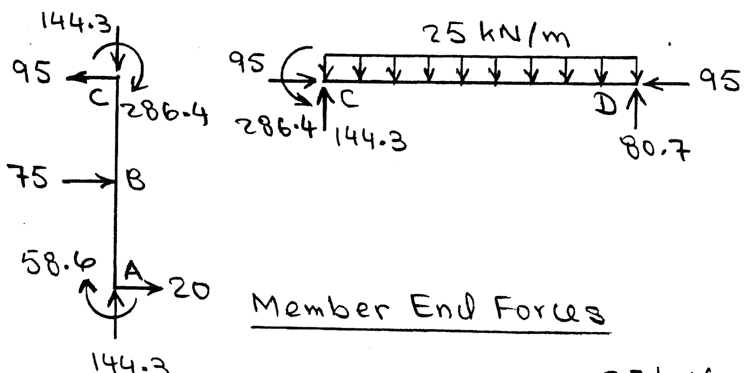
$$= 0.333\,EI\theta_C + 401.4$$

Equilibrium equation: $M_{CA} + M_{CD} = 0$

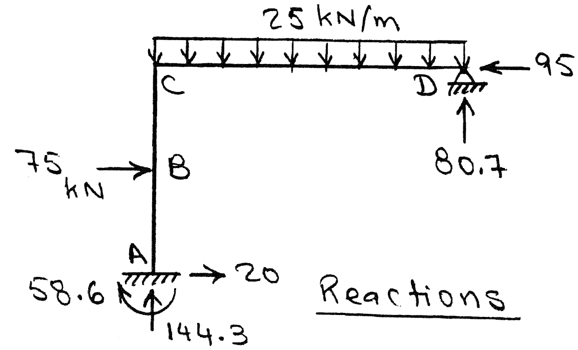
$$EI\theta_C = -345.15 \text{ kN-m}^2$$

Member end moments: $M_{AC} = -58.6$ kN.m

$M_{CA} = -286.4$ kN.m ; $M_{CD} = 286.4$ kN.m



Member End Forces



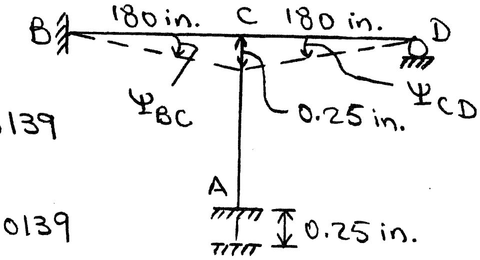
Reactions

16.22 Fixed-end moments:

$FEM_{AC} = FEM_{CA} = FEM_{CD} = FEM_{DC} = 0$

$FEM_{BC} = 37.5$ k-ft ; $FEM_{CB} = -37.5$ k-ft .

Chord rotations:



$\Psi_{BC} = -\frac{0.25}{180} = -0.00139$

$\Psi_{CD} = +\frac{0.25}{180} = +0.00139$

Slope-deflection equations. $M_{DC} = 0$

Using $EI = \frac{29000(3500)}{(12)^2}$ k-ft², we write

$M_{AC} = 0.133\,EI\theta_C$; $M_{CA} = 0.267\,EI\theta_C$

$M_{BC} = 0.133\,EI\theta_C + 391.9 + 37.5 = 0.133\,EI\theta_C + 429.4$

$M_{CB} = 0.267\,EI\theta_C + 391.9 - 37.5 = 0.267\,EI\theta_C + 354.4$

$M_{CD} = 0.2\,EI\theta_C - 196$

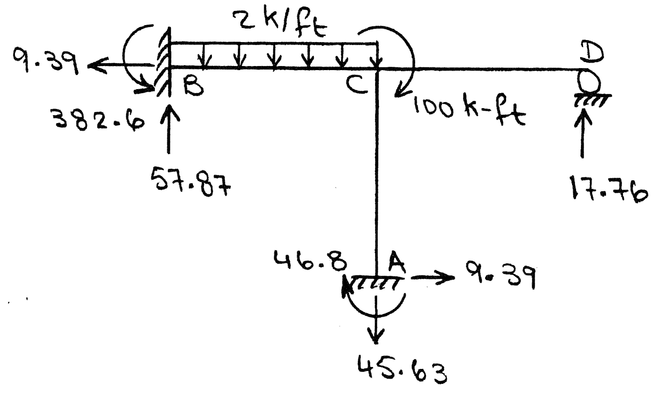
Equilibrium equation: $M_{CA} + M_{CB} + M_{CD} + 100 = 0$

$0.734\,EI\theta_C = -258.4 \qquad EI\theta_C = -352$ k-ft²

Member end moments: $M_{AC} = -46.8$ k-ft ;

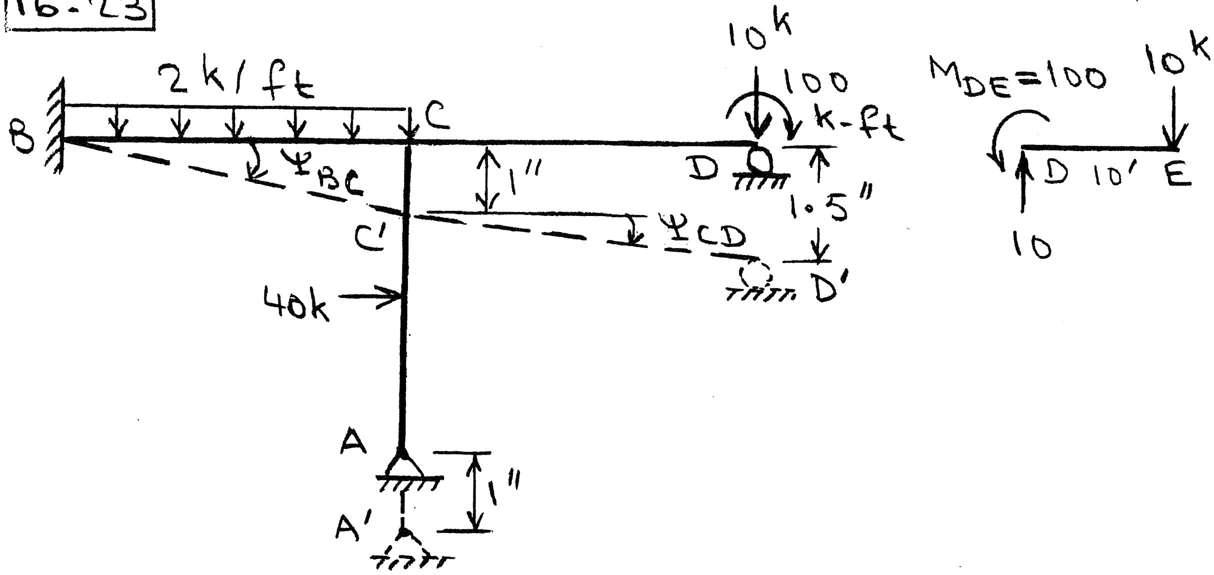
$M_{CA} = -94$ k-ft ; $M_{BC} = 382.6$ k-ft ;

$M_{CB} = 260.4$ k-ft ; $M_{CD} = -266.4$ k-ft.



Reactions

16-23



Fixed-end moments: $FEM_{CD} = FEM_{DC} = 0$

$FEM_{BC} = 150$ k-ft ; $FEM_{CB} = -150$ k-ft

$FEM_{AC} = 100$ k-ft ; $FEM_{CA} = -100$ k-ft

Chord rotations:

$$\Psi_{BC} = -\frac{1}{30(12)} = -0.00728 ; \quad \Psi_{CD} = -\frac{0.5}{30(12)} = -0.00139$$

Slope-deflection equations: $M_{AC} = 0$; $M_{DE} = 100$ k-ft

$M_{CA} = 0.15EI\theta_C - 150$; $M_{BC} = 0.0667EI\theta_C + 265.7$

$M_{CB} = 0.133EI\theta_C - 34.3$

$M_{CD} = 0.133EI\theta_C + 0.0667EI\theta_D + 57.9$

$M_{DC} = 0.0667EI\theta_C + 0.133EI\theta_D + 57.9$

Equilibrium equations: $M_{CA} + M_{CB} + M_{CD} = 0$

$M_{DC} + 100 = 0$

$0.417EI\theta_C + 0.0667EI\theta_D = 126.4$

$0.0667EI\theta_C + 0.133EI\theta_D = -157.9$

By solving these equations, we obtain:

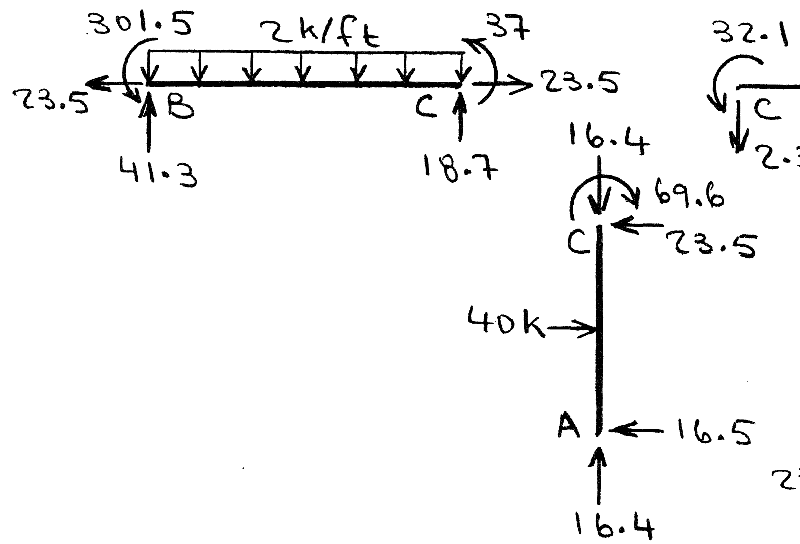
$EI\theta_C = 536$ k-ft^2; $EI\theta_D = -1456$ k-ft^2

16.23 (contd.)

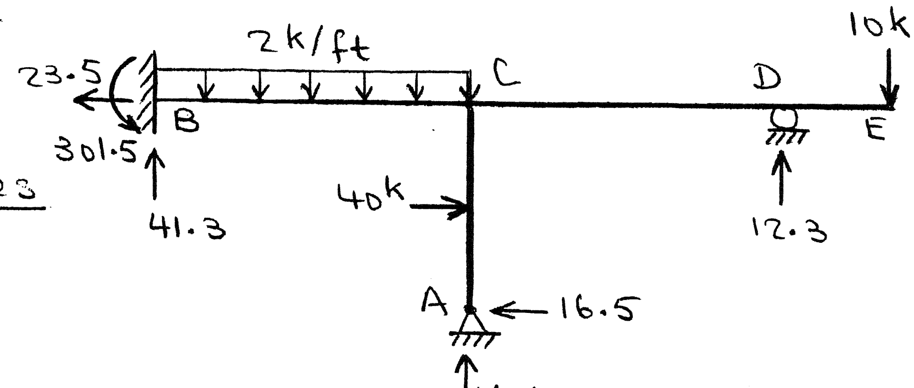
Member end moments: Substituting the numerical values of $EI\theta_C$ and $EI\theta_D$ into the slope-deflection equations, we obtain

$M_{CA} = -69.6$ k-ft; $M_{BC} = 301.5$ k-ft; $M_{CB} = 37$ k-ft; $M_{CD} = 32.1$ k-ft

$M_{DC} = -100$ k-ft.



Member End Forces

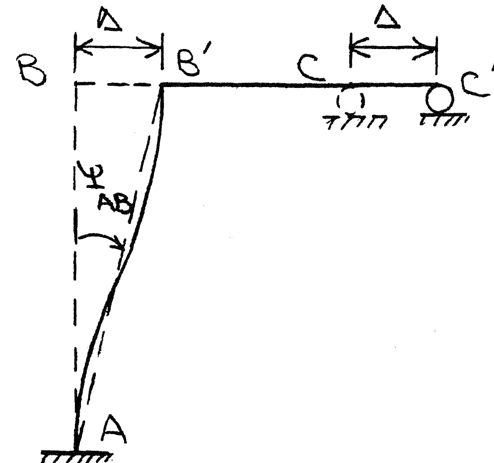


Reactions

16.24 Fixed-end moments: $FEM_{AB} = FEM_{BA} = 0$

$FEM_{BC} = \frac{2(15)^2}{12} = 37.5$ k-ft; $FEM_{CB} = -37.5$ k-ft

Chord rotations:



$\Psi_{AB} = -\frac{\Delta}{20}$

$\Psi_{BC} = 0$

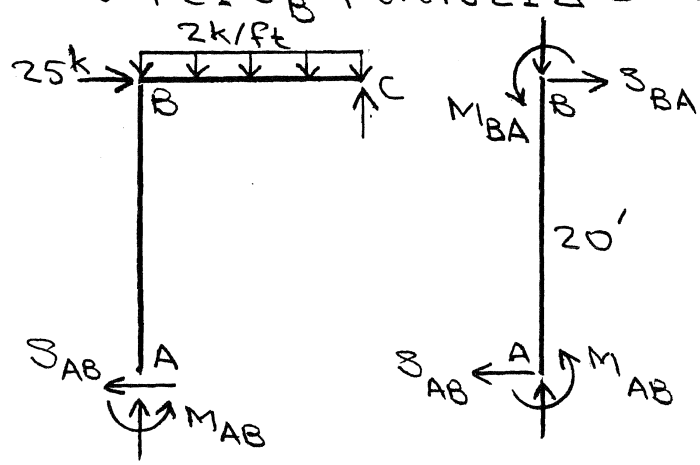
Slope-deflection equations:

$M_{AB} = 0.1EI\theta_B + 0.015EI\Delta$; $M_{BA} = 0.2EI\theta_B + 0.015EI\Delta$

$M_{BC} = 0.2EI\theta_B + 56.25$; $M_{CB} = 0$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$0.4EI\theta_B + 0.015EI\Delta = -56.25 \qquad (1)$$



$\Sigma F_x = 0 \qquad S_{AB} = 25 \qquad \frac{M_{AB} + M_{BA}}{20} = 25$

$$0.3EI\theta_B + 0.03EI\Delta = 500 \qquad (2)$$

By solving equations (1) and (2) simultaneously, we obtain:

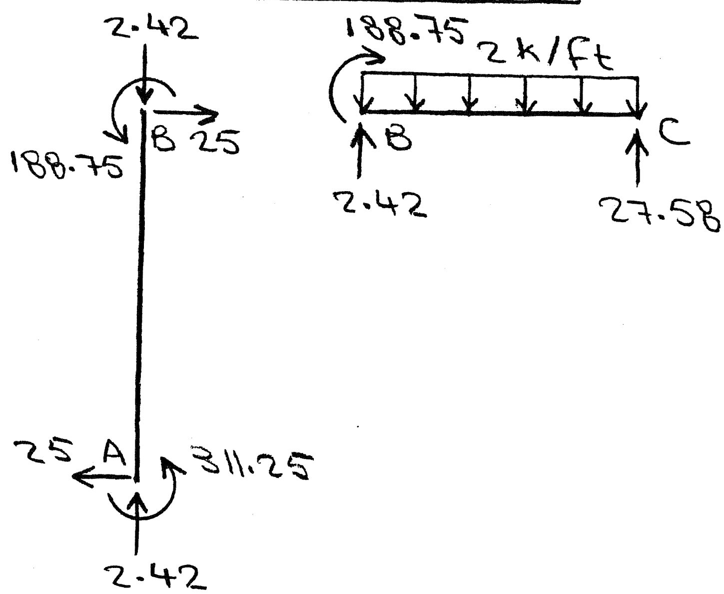
$EI\theta_B = -1225$ k-ft^2; $EI\Delta = 28916.67$ k-ft^3.

16.24 (contd.)

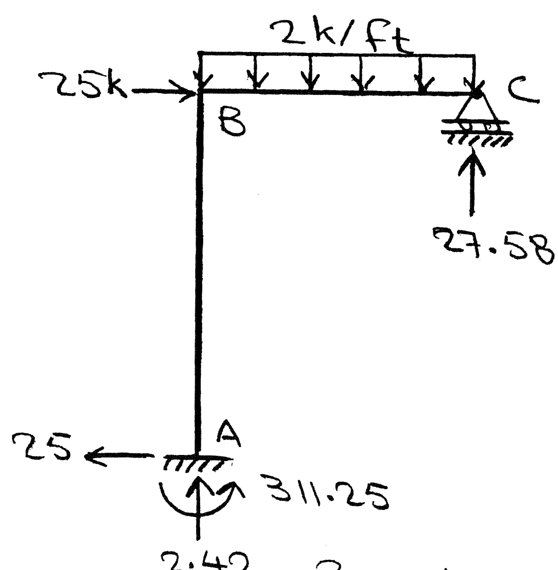
Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$M_{AB} = 311.25$ k-ft; $M_{BA} = 188.75$ k-ft;

$M_{BC} = -188.75$ k-ft.



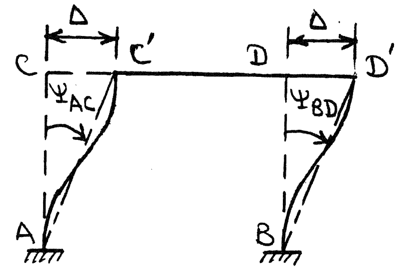
Member End Forces



Reactions

16.25 Fixed-end moments. The non-zero fixed-end moments are: $FEM_{CD} = \frac{1.5(40)^2}{12} = 200$ k-ft; and $FEM_{DC} = -200$ k-ft.

Chord rotations:



$$\Psi_{AC} = \Psi_{BD} = -\frac{\Delta}{30}$$

$$\Psi_{CD} = 0$$

Slope-deflection equations:

$M_{AC} = 0.0667EI(\theta_C + 0.1\Delta)$; $M_{CA} = 0.0667EI(2\theta_C + 0.1\Delta)$

$M_{BD} = 0.0667EI(\theta_D + 0.1\Delta)$; $M_{DB} = 0.0667EI(2\theta_D + 0.1\Delta)$

$$M_{CD} = 0.1\,EI(2\theta_C + \theta_D) + 200$$

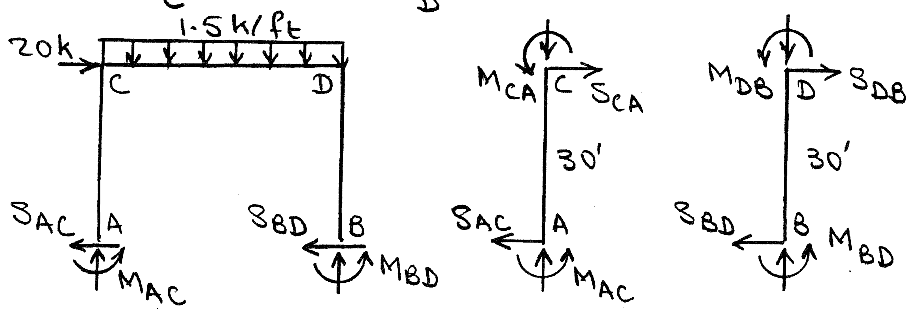
$$M_{DC} = 0.1\,EI(\theta_C + 2\theta_D) - 200$$

Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$0.333EI\theta_C + 0.1\,EI\theta_D + 0.00667EI\Delta = -200 \qquad (1)$$

$$M_{DC} + M_{DB} = 0$$

$$0.1EI\theta_C + 0.333EI\theta_D + 0.00667EI\Delta = 200 \qquad (2)$$



$\sum F_x = 0 \qquad S_{AC} + S_{BD} = 20$

$$\frac{M_{AC} + M_{CA}}{30} + \frac{M_{BD} + M_{DB}}{30} = 20$$

$$0.00667EI\theta_C + 0.00667EI\theta_D + 0.000889EI\Delta = 20 \qquad (3)$$

Solving Eqs. (1) thru (3) simultaneously, we obtain:

$EI\theta_C = -1308$ k-ft^2; $EI\theta_D = 408$ k-ft^2; $EI\Delta = 29242$ k-ft^3

Member end moments: $M_{AC} = 107.8$ k-ft; $M_{CA} = 20.8$ k-ft

$M_{BD} = 222.3$ k-ft; $M_{DB} = 249.2$ k-ft; $M_{CD} = -20.8$ k-ft

$M_{DC} = -249.2$ k-ft. For reactions, see solution of Problem 13.45.

16.26 Fixed-end moments: $FEM_{AC} = FEM_{CA} = 0$

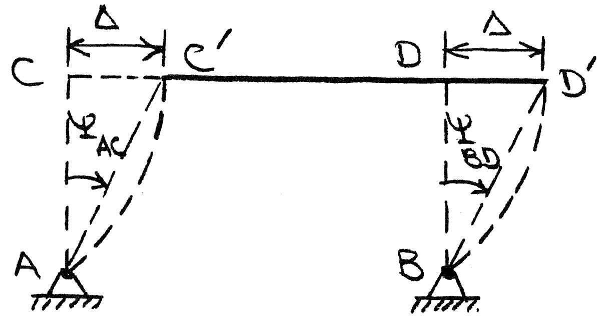
$FEM_{BD} = FEM_{DB} = 0$; $FEM_{CD} = \dfrac{3(30)^2}{12} = 225$ k-ft;

$FEM_{DC} = -225$ k-ft.

Chord rotations:

$\psi_{AC} = \psi_{BD} = -\dfrac{\Delta}{15}$

$\psi_{CD} = 0$



Slope-deflection equations: $M_{AC} = M_{BD} = 0$

$M_{CA} = 0.2EI\theta_C + 0.0133EI\Delta$

$M_{DB} = 0.2EI\theta_D + 0.0133EI\Delta$

$M_{CD} = 0.133EI\theta_C + 0.0667EI\theta_D + 225$

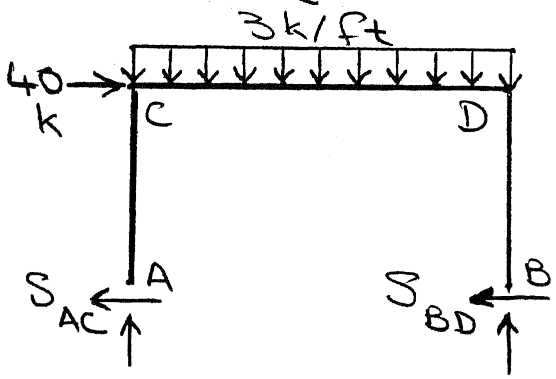
$M_{DC} = 0.0667EI\theta_C + 0.133EI\theta_D - 225$

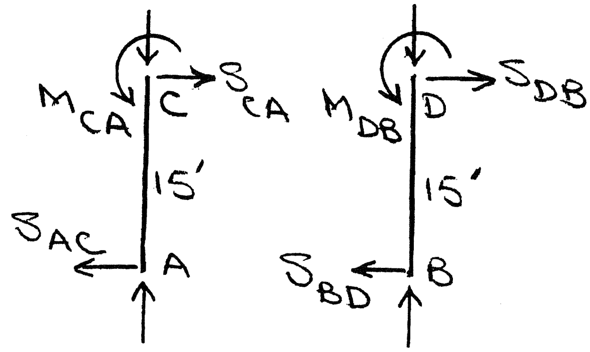
Equilibrium equations: $M_{CA} + M_{CD} = 0$

$0.333EI\theta_C + 0.0667EI\theta_D + 0.0133EI\Delta = -225 \quad (1)$

$M_{DB} + M_{DC} = 0$

$0.0667EI\theta_C + 0.333EI\theta_D + 0.0133EI\Delta = 225 \quad (2)$





$\sum F_x = 0 \qquad S_{AC} + S_{BD} = 40 \qquad \dfrac{M_{CA}}{15} + \dfrac{M_{DB}}{15} = 40$

$0.0133EI\theta_C + 0.0133EI\theta_D + 0.00178\Delta = 40 \quad (3)$

16.26 (Contd.)

By solving Eqs. (1) thru (3) simultaneously, we obtain:

$EI\theta_C = -2343.75 \text{ k-ft}^2$; $EI\theta_D = -656.25 \text{ k-ft}^2$;

$EI\Delta = 45000 \text{ k-ft}^3$.

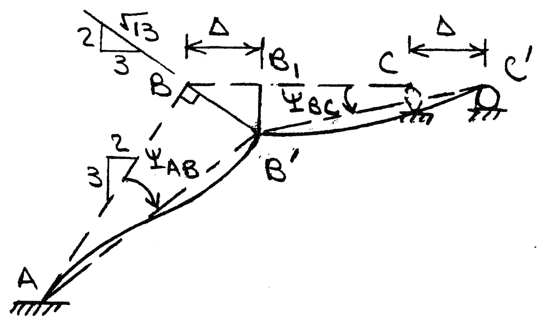
Member end moments:

$M_{CA} = 131.25$ k-ft; $M_{DB} = 468.75$ k-ft;

$M_{CD} = -131.25$ k-ft; $M_{DC} = -468.75$ k-ft.

For reactions, see solution of Problem 13.22.

15.27 Chord rotations:



$$\Psi_{AB} = -\frac{BB'}{L_{AB}} = -\frac{(\sqrt{13}/3)\Delta}{14.42} = -0.0833\,\Delta$$

$$\Psi_{BC} = \frac{B_1B'}{L_{BC}} = \frac{(2/3)\Delta}{12} = 0.0556\,\Delta$$

Slope-deflection equations: $M_{CB} = 0$

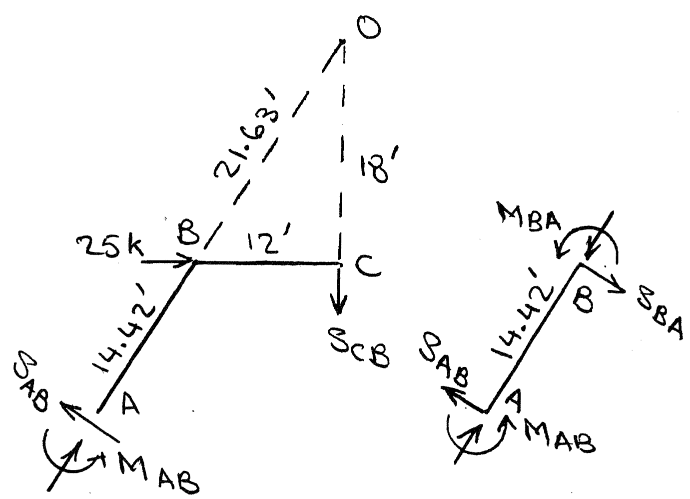
$$M_{AB} = 0.139 EI\,\theta_B + 0.0347\,EI\Delta$$

$$M_{BA} = 0.277 EI\,\theta_B + 0.0347\,EI\Delta$$

$$M_{BC} = 0.25 EI\,\theta_B - 0.0139\,EI\Delta$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$0.527 EI\theta_B + 0.0208 EI\Delta = 0 \qquad (1)$$



$$+\circlearrowleft \Sigma M_O = 0 \qquad M_{AB} - S_{AB}(36.05) + 25(18) = 0$$

$$M_{AB} - \left(\frac{M_{AB} + M_{BA}}{14.42}\right)(36.05) + 450 = 0$$

$$0.901 EI\theta_B + 0.139 EI\Delta = 450 \qquad (2)$$

16.27 (contd.)

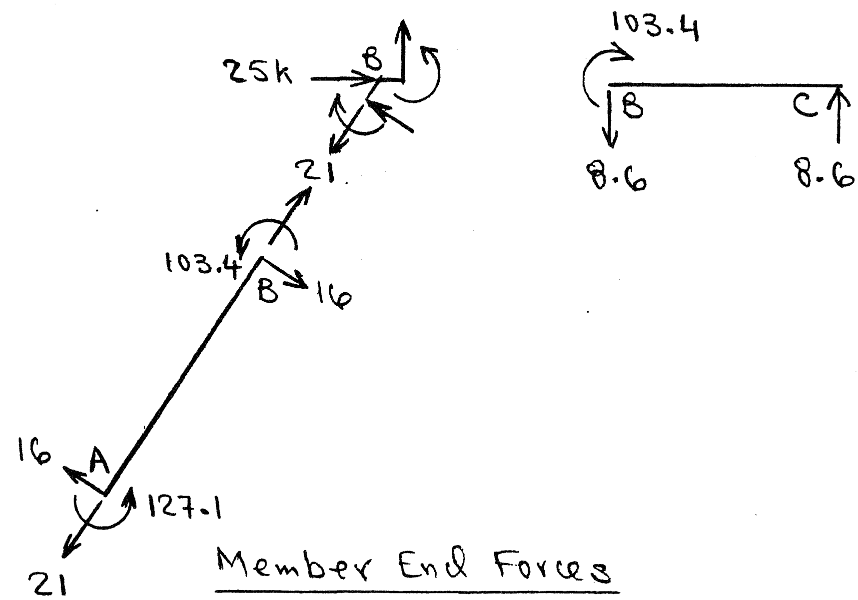
By solving Eqs. (1) and (2), we obtain:

$EI\theta_B = -171.7$ k-ft^2; $EI\Delta = 4350$ k-ft^3

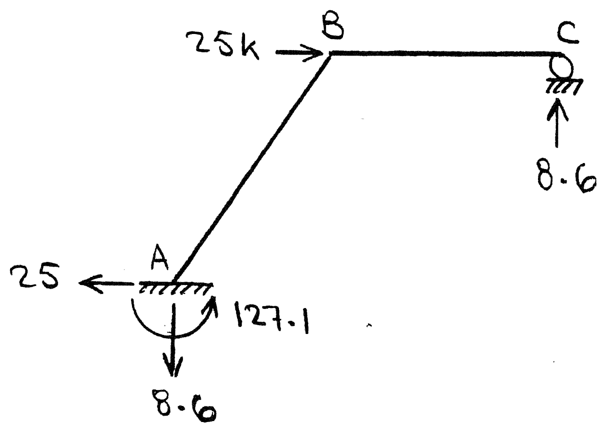
Member end moments. Substituting the numerical values of $EI\theta_B$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$M_{AB} = 127.1$ k-ft; $M_{BA} = 103.4$ k-ft;

$M_{BC} = -103.4$ k-ft.



Member End Forces

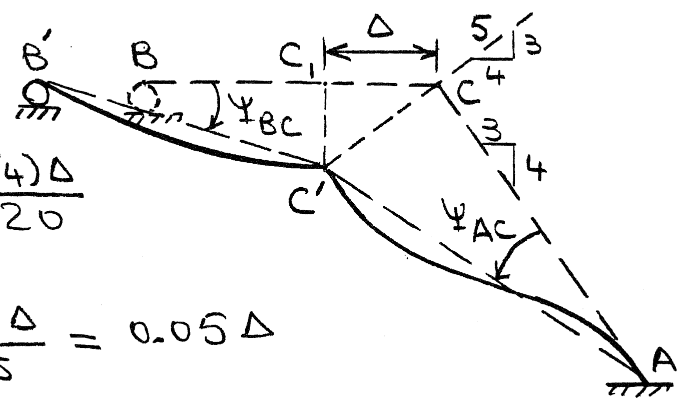


Reactions

16.28 Fixed end moments:

$$FEM_{BC} = \frac{1.5(20)^2}{12} = 50 \text{ k-ft}; \quad FEM_{CB} = -50 \text{ k-ft}$$

Chord rotations:



$$\Psi_{BC} = -\frac{C_1C'}{L_{BC}} = -\frac{(3/4)\Delta}{20}$$

$$= -0.0375\,\Delta$$

$$\Psi_{AC} = \frac{CC'}{L_{AC}} = \frac{(5/4)\Delta}{25} = 0.05\,\Delta$$

Slope-deflection equations: $M_{BC} = 0$

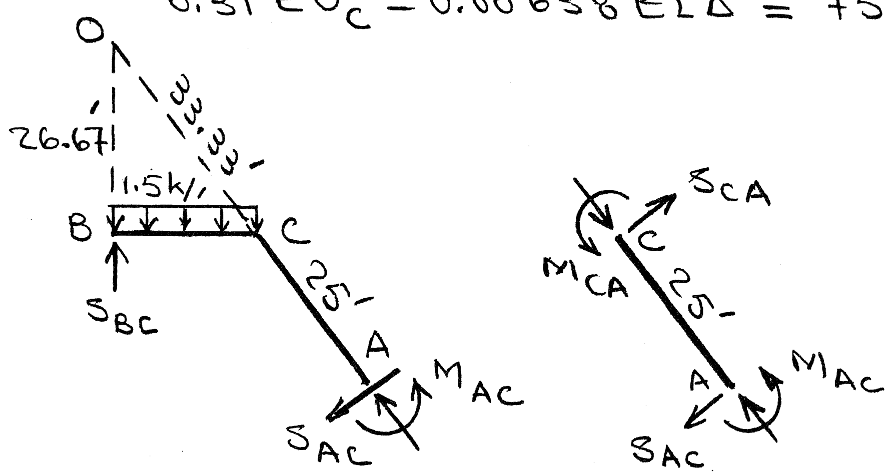
$$M_{CB} = 0.15EI(\theta_C + 0.0375\,\Delta) - 75$$

$$M_{CA} = 0.08\,EI\,(2\theta_C - 0.15\,\Delta)$$

$$M_{AC} = 0.08\,EI(\theta_C - 0.15\Delta)$$

Equilibrium equations: $M_{CB} + M_{CA} = 0$

$$0.31\,E\theta_C - 0.00638\,EI\Delta = 75 \qquad (1)$$



$$+\circlearrowright \Sigma M_O = 0 \qquad M_{AC} - S_{AC}(58.33) - 1.5(20)10 = 0$$

$$M_{AC} - \left(\frac{M_{AC} + M_{CA}}{25}\right)(58.33) - 300 = 0$$

$$-0.48\,EI\theta_C + 0.044\,EI\Delta = 300 \qquad (2)$$

By solving Eqs. (1) and (2), we obtain

$$EI\theta_C = 493 \text{ k-ft}^2; \quad EI\Delta = 12196 \text{ k-ft}^3.$$

16.28 (Contd.) **Member end moments:** Substituting the numerical values of $EI\theta_C$ and $EI\Delta$ into the slope-deflection equations, we obtain

$M_{CB} = 67.5$ k-ft ; $M_{CA} = -67.5$ k-ft

$M_{AC} = -106.9$ k-ft

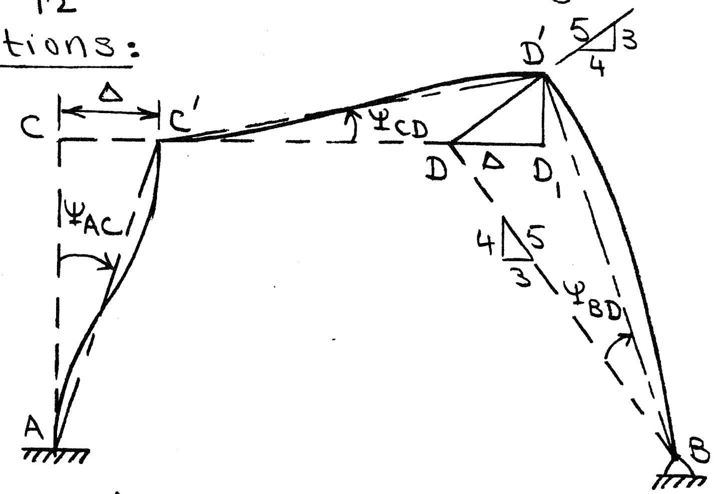
For reactions, see solution of Problem 13.24.

16.29 Fixed-end moments: $FEM_{BD} = FEM_{DB} = 0$

$FEM_{AC} = \frac{50(4)}{8} = 25\ kN.m$; $FEM_{CA} = -25\ kN.m$

$FEM_{CD} = \frac{18(5)^2}{12} = 37.5\ kN.m$; $FEM_{DC} = -37.5\ kN.m$

Chord rotations:



$\Psi_{AC} = -\frac{CC'}{L_{AC}} = -\frac{\Delta}{4} = -0.25\Delta$

$\Psi_{BD} = -\frac{DD'}{L_{BD}} = -\frac{(5/4)\Delta}{5} = -0.25\Delta$

$\Psi_{CD} = \frac{D_1D'}{L_{CD}} = \frac{(3/4)\Delta}{5} = 0.15\Delta$

Slope-deflection equations: $M_{BD} = 0$

$M_{AC} = 0.5EI(\theta_C + 0.75\Delta) + 25$

$M_{CA} = 0.5EI(2\theta_C + 0.75\Delta) - 25$

$M_{CD} = 0.4EI(2\theta_C + \theta_D - 0.45\Delta) + 37.5$

$M_{DC} = 0.4EI(\theta_C + 2\theta_D - 0.45\Delta) - 37.5$

$M_{DB} = 0.6EI(\theta_D + 0.25\Delta)$

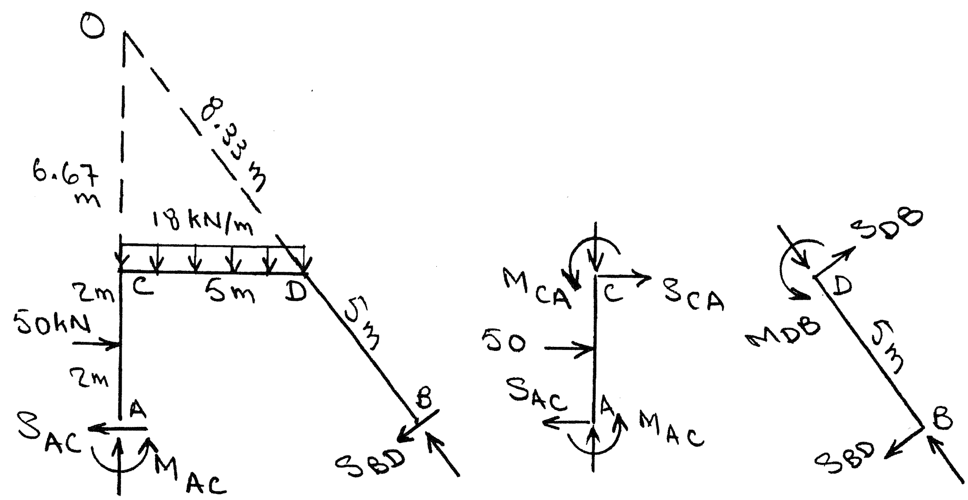
Equilibrium equations: $M_{CA} + M_{CD} = 0$

$1.8EI\theta_C + 0.4EI\theta_D + 0.195EI\Delta = -12.5$ (1)

$M_{DC} + M_{DB} = 0$

$0.4EI\theta_C + 1.4EI\theta_D - 0.03EI\Delta = 37.5$ (2)

16.29 (contd.)



$+\circlearrowleft \sum M_O = 0$

$$M_{AC} - S_{AC}(10.67) - S_{BD}(13.33) + 50(8.67) - 18(5)2.5 = 0$$

$$M_{AC} - \left(25 + \frac{M_{AC}+M_{CA}}{4}\right)10.67 - \left(\frac{M_{DB}}{5}\right)13.33 + 208.5 = 0$$

$$3.5EI\theta_C + 1.6EI\theta_D + 2.025EI\Delta = -33.25 \qquad (3)$$

By solving Eqs. (1) thru (3) simultaneously, we obtain:

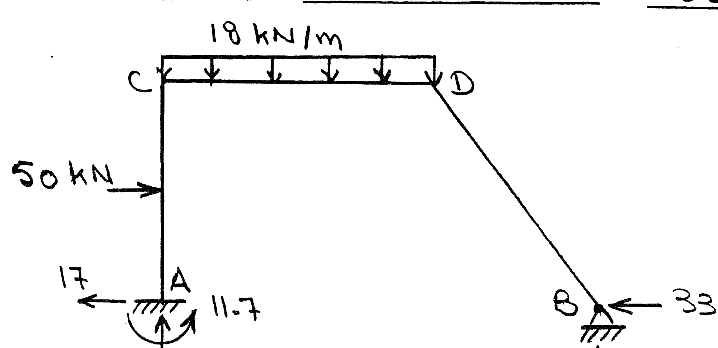
$EI\theta_C = -11.33$ kN-m^2; $EI\theta_D = 29.59$ kN-m^2

$EI\Delta = -20.22$ kN-m^3

Member end moments. Substituting the numerical values of $EI\theta_C$, $EI\theta_D$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$M_{AC} = 11.7$ kN.m; $M_{CA} = -43.9$ kN.m;

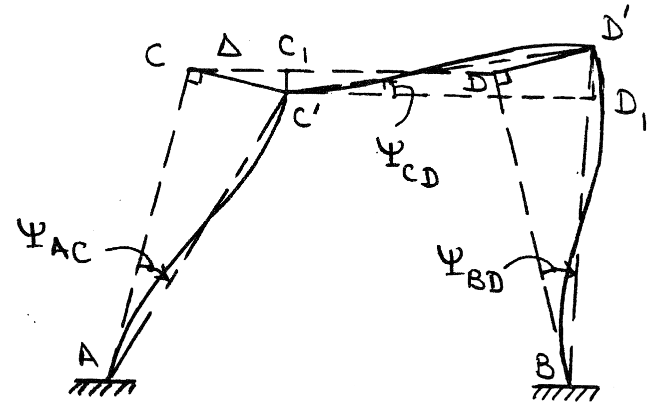
$M_{CD} = 43.9$ kN.m; $M_{DC} = -14.7$ kN.m; $M_{DB} = 14.7$ kN.m



Reactions

16.30 Fixed-end moments: The non-zero fixed-end moments are: $FEM_{CD} = \frac{3(16)^2}{12} = 64$ k-ft and $FEM_{DC} = -64$ k-ft.

Chord rotations:



$$\psi_{AC} = -\frac{CC'}{L_{AC}} = -\frac{(\sqrt{17}/4)\Delta}{16.49} = -0.0625\Delta$$

$$\psi_{BD} = -\frac{DD'}{L_{BD}} = -\frac{(\sqrt{17}/4)\Delta}{16.49} = -0.0625\Delta$$

$$\psi_{CD} = \frac{D_1D'}{L_{CD}} = \frac{2(1/4)\Delta}{16} = 0.03125\Delta$$

Slope-deflection equations:

$M_{AC} = 0.121EI(\theta_C + 0.188\Delta)$; $M_{CA} = 0.121EI(2\theta_C + 0.188\Delta)$

$M_{BD} = 0.121EI(\theta_D + 0.188\Delta)$; $M_{DB} = 0.121EI(2\theta_D + 0.188\Delta)$

$M_{CD} = 0.125EI(2\theta_C + \theta_D - 0.0938\Delta) + 64$

$M_{DC} = 0.125EI(\theta_C + 2\theta_D - 0.0938\Delta) - 64$

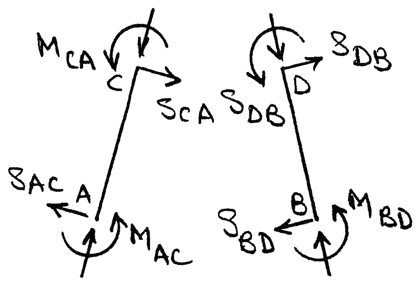
Equilibrium equations: $M_{CA} + M_{CD} = 0$

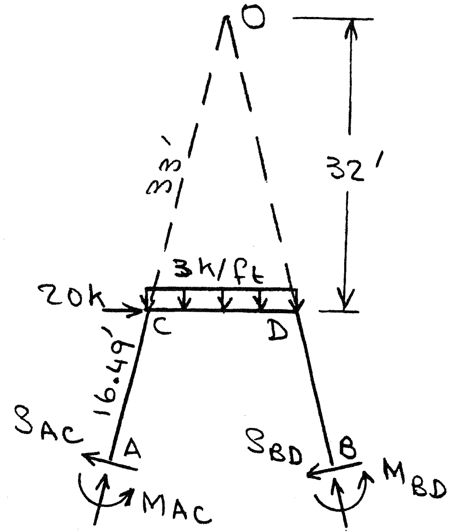
$$0.492EI\theta_C + 0.125E\theta_D + 0.011EI\Delta = -64 \quad (1)$$

$M_{DC} + M_{DB} = 0$

$$0.125EI\theta_C + 0.492EI\theta_D + 0.011EI\Delta = 64 \quad (2)$$

16.30 (Contd.)





$+\circlearrowleft \sum M_O = 0$

$$M_{AC} + M_{BD} - (S_{AC} + S_{BD})49.49 + 20(32) = 0$$

$$M_{AC} + M_{BD} - \frac{49.49}{16.49}(M_{AC} + M_{CA} + M_{BD} + M_{DB}) + 640 = 0$$

$$0.968EI\theta_C + 0.968EI\theta_D + 0.227EI\Delta = 640 \qquad (3)$$

By solving Eqs. (1) thru (3), we obtain:

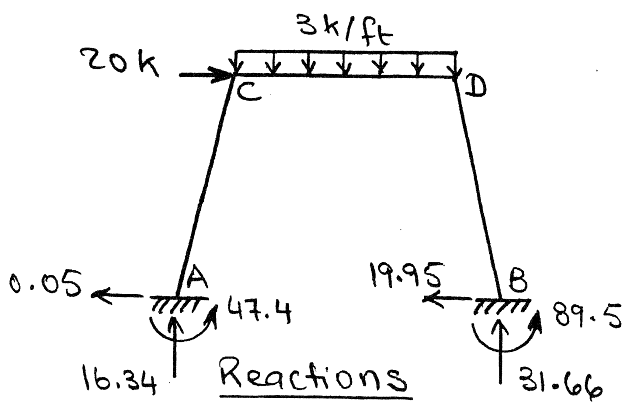
$EI\theta_C = -233.7$ k-ft², $EI\theta_D = 115.1$ k-ft²;

$EI\Delta = 3325$ k-ft³

Member end moments. Substituting the numerical values of $EI\theta_C$, $EI\theta_D$ and $EI\Delta$ into the slope-deflection equations, we obtain: $M_{AC} = 47.4$ k-ft;

$M_{CA} = 19$ k-ft; $M_{BD} = 89.5$ k-ft; $M_{DB} = 103.5$ k-ft;

$M_{CD} = -19$ k-ft; $M_{DC} = -103.5$ k-ft.

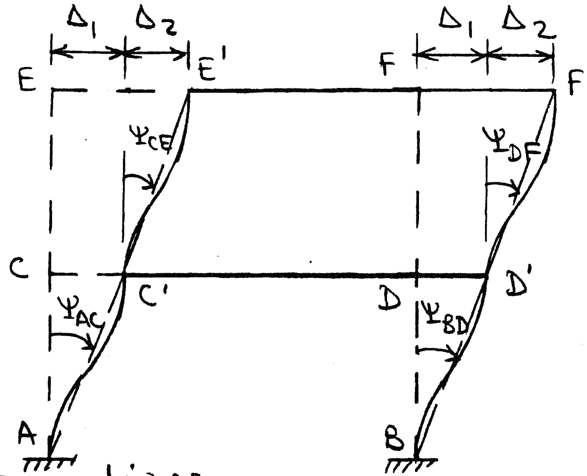


16.31 Chord rotations:

$$\Psi_{AC} = \Psi_{BD} = -\frac{\Delta_1}{15}$$

$$\Psi_{CE} = \Psi_{DF} = -\frac{\Delta_2}{15}$$

$$\Psi_{CD} = \Psi_{EF} = 0$$



Slope-deflection equations:

$$M_{AC} = 0.133EI(\theta_C + 0.2\Delta_1);\ M_{CA} = 0.133EI(2\theta_C + 0.2\Delta_1)$$

$$M_{BD} = 0.133EI(\theta_D + 0.2\Delta_1);\ M_{DB} = 0.133EI(2\theta_D + 0.2\Delta_1)$$

$$M_{CE} = 0.133EI(2\theta_C + \theta_E + 0.2\Delta_2)$$

$$M_{EC} = 0.133EI(\theta_C + 2\theta_E + 0.2\Delta_2)$$

$$M_{DF} = 0.133EI(2\theta_D + \theta_F + 0.2\Delta_2)$$

$$M_{FD} = 0.133EI(\theta_D + 2\theta_F + 0.2\Delta_2)$$

$$M_{CD} = 0.133EI(2\theta_C + \theta_D);\ M_{DC} = 0.133EI(\theta_C + 2\theta_D)$$

$$M_{EF} = 0.133EI(2\theta_E + \theta_F);\ M_{FE} = 0.133EI(\theta_E + 2\theta_F)$$

Equilibrium equations: $M_{CA} + M_{CD} + M_{CE} = 0$

$$EI(6\theta_C + \theta_D + \theta_E + 0.2\Delta_1 + 0.2\Delta_2) = 0 \quad (1)$$

$$M_{DB} + M_{DC} + M_{DF} = 0$$

$$EI(\theta_C + 6\theta_D + \theta_F + 0.2\Delta_1 + 0.2\Delta_2) = 0 \quad (2)$$

$$M_{EC} + M_{EF} = 0$$

$$EI(\theta_C + 4\theta_E + \theta_F + 0.2\Delta_2) = 0 \quad (3)$$

$$M_{FD} + M_{FE} = 0$$

$$EI(\theta_D + \theta_E + 4\theta_F + 0.2\Delta_2) = 0 \quad (4)$$

$$S_{CE} + S_{DF} = 9$$

$$\frac{1}{15}\left[(M_{CE} + M_{EC}) + (M_{DF} + M_{FD})\right] = 9$$

16.31 (Contd.)

$$EI(3\theta_C + 3\theta_D + 3\theta_E + 3\theta_F + 0.8\Delta_2) = 1015 \quad (5)$$

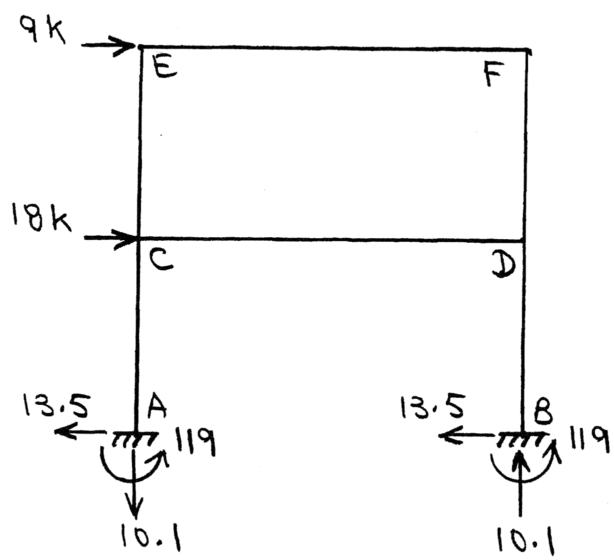
$$S_{AC} + S_{BD} = 27$$

$$\frac{1}{15}\left[(M_{AC} + M_{CA}) + (M_{BD} + M_{DB})\right] = 27$$

$$EI(3\theta_C + 3\theta_D + 0.8\Delta_1) = 3045 \quad (6)$$

By solving Eqs. (1) thru (6) simultaneously, we obtain: $EI\theta_C = EI\theta_D = -267.6$ k ft^2; $EI\theta_E = EI\theta_F = -110.7$ k-ft^2; $EI\Delta_1 = 5813$ k-ft^3; $EI\Delta_2 = 4106$ k-ft^3.

Member end moments: $M_{AC} = M_{BD} = 119$ k-ft; $M_{CA} = M_{DB} = 83.5$ k-ft; $M_{CE} = M_{DF} = 23.3$ k-ft; $M_{EC} = M_{FD} = 44.2$ k-ft; $M_{CD} = M_{DC} = -106.8$ k-ft; $M_{EF} = M_{FE} = -44.2$ k-ft.

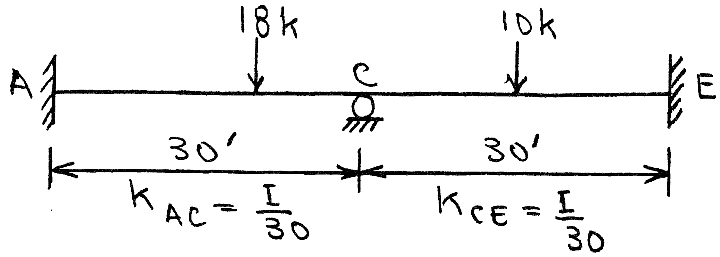


Reactions

Chapter Seventeen
Moment-Distribution Method

CHAPTER 17

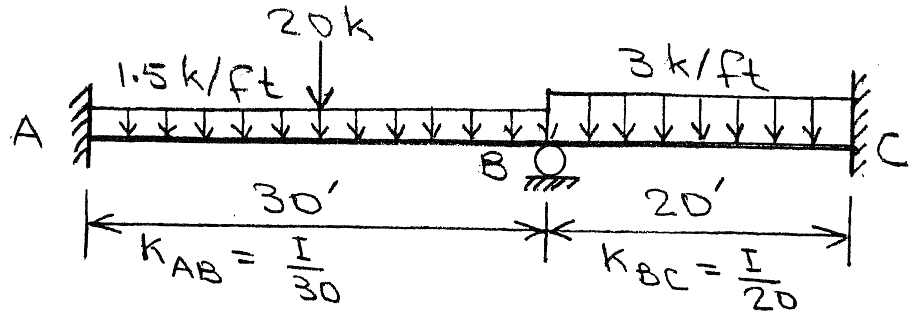
17-1



DF		0.5	0.5	
FEM	40	−80	37.5	−37.5
		21.3	21.3	
	10.6			10.6
Final Moments	50.6	−58.7	58.8	−26.9

For reactions, and shear and bending moment diagrams, see solution of Problem 16-1.

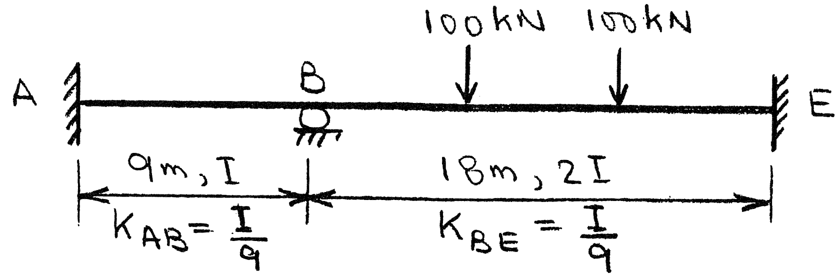
17.2



	AB	BA	BC	CB
DF		0.4	0.6	
FEM	187.5	−187.5	100	−100
		35	52.5	
	17.5			26.25
Final Moments	205	−152.5	152.5	−73.75

For reactions, and shear and bending moment diagrams, see solution of Problem 16.2.

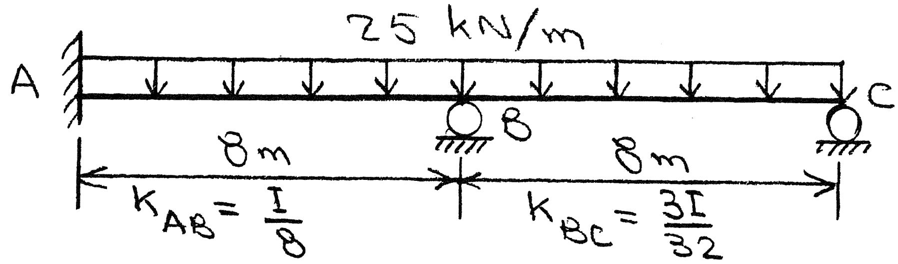
17.3



	AB	BA	BE	EB
DF		0.5	0.5	
FEM	0	0	400	-400
		-200	-200	
	-100			-100
Final Moments	-100	-200	200	-500

For reactions, and shear and bending moment diagrams, see solution of Problem 16.3.

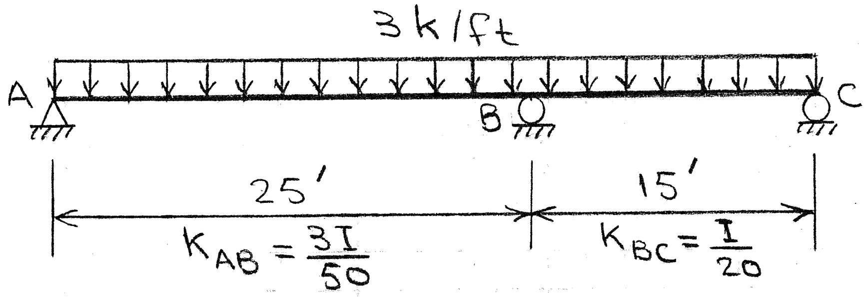
17.4



DF		0.571	0.429	
FEM	133.3	−133.3	133.3	−133.3
				133.3
			66.7	
		−38.1	−28.6	
	−19			
Final Moments	114.3	−171.4	171.4	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.37.

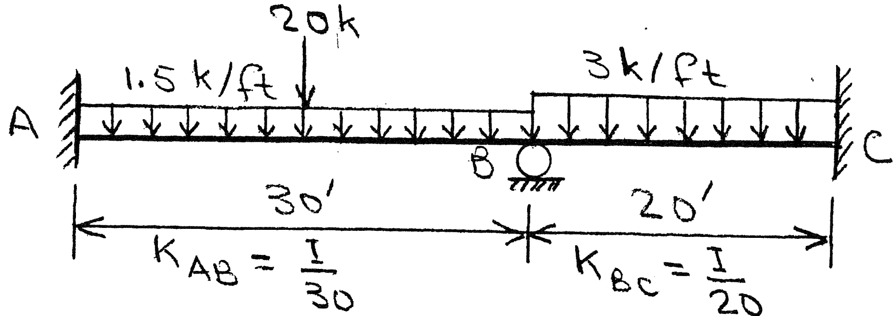
17-5



DF		0.545	0.455	
FEM	156.3	−156.3	56.3	−56.3
	−156.3	54.5	45.5	56.3
		−78.1	28.1	
		27.3	22.7	
Final Moments	0	−152.6	152.6	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

17.6



$$FEM_{AB} = \frac{1.5(30)^2}{12} + \frac{20(30)}{8} + \frac{6(29000)(1650)(0.5)}{(30)^2(12)^3}$$

$$= 112.5 + 75 + 92.3 = 279.8 \text{ k-ft}$$

$$FEM_{BA} = -112.5 - 75 + 92.3 = -95.2 \text{ k-ft}$$

$$FEM_{BC} = \frac{3(20)^2}{12} - \frac{6(29000)(1650)(0.5)}{(20)^2(12)^3}$$

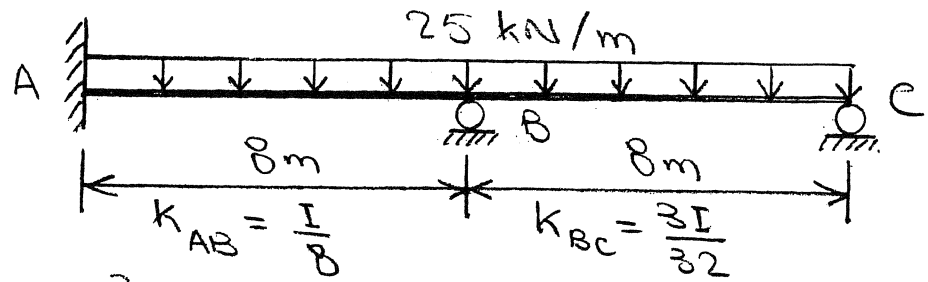
$$= 100 - 207.7 = -107.7 \text{ k-ft}$$

$$FEM_{CB} = -100 - 207.7 = -307.7 \text{ k-ft}$$

DF		0.4	0.6	
FEM	279.8	-95.2	-107.7	-307.7
		81.2	121.7	
	40.6			60.9
Final Moments	320.4	-14	14	-246.8

For reactions, and shear and bending moment diagrams, see solution of Problem 16.6.

17.7



$$FEM_{AB} = \frac{25(8)^2}{12} + \frac{6(70)(1300)(0.05)}{(8)^2}$$
$$= 133.3 + 426.6 = 559.9 \text{ kN.m}$$

$$FEM_{BA} = -133.3 + 426.6 = 293.3 \text{ kN.m}$$

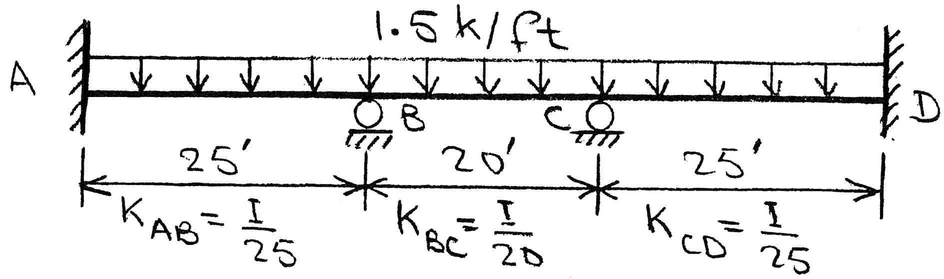
$$FEM_{BC} = 133.3 - \frac{6(70)(1300)(0.025)}{(8)^2}$$
$$= 133.3 - 213.3 = -80 \text{ kN.m}$$

$$FEM_{CB} = -133.3 - 213.3 = -346.6 \text{ kN.m}$$

	A (AB)	B (BA)	B (BC)	C (CB)
DF		0.571	0.429	
FEM	559.9	293.3	-80	-346.6
		-121.8	-91.5	346.6
	-60.9		173.3	
		-99	-74.3	
	-49.5			
Final Moments	449.5	72.5	-72.5	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.53.

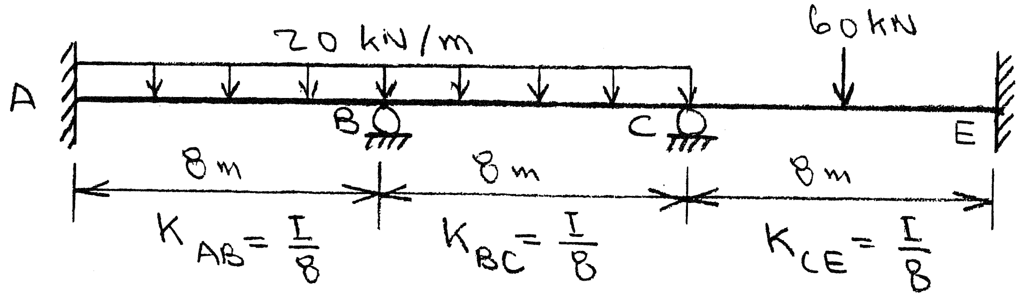
17.8



DF		4/9	5/9	5/9	4/9	
FEM	78.1	-78.1	50	-50	78.1	-78.1
		12.5	15.6	-15.6	-12.5	
	6.2		-7.8	7.8		-6.2
		3.5	4.3	-4.3	-3.5	
	1.7		-2.2	2.2		-1.7
		1	1.2	-1.2	-1	
	0.5		-0.6	0.6		-0.5
		0.3	0.3	-0.3	-0.3	
	0.1		-0.2	0.2		-0.1
		0.1	0.1	-0.1	-0.1	
Final Moments	86.6	-60.7	60.7	-60.7	60.7	-86.6

For reactions, and shear and bending moment diagrams, see solution of Problem 16.8.

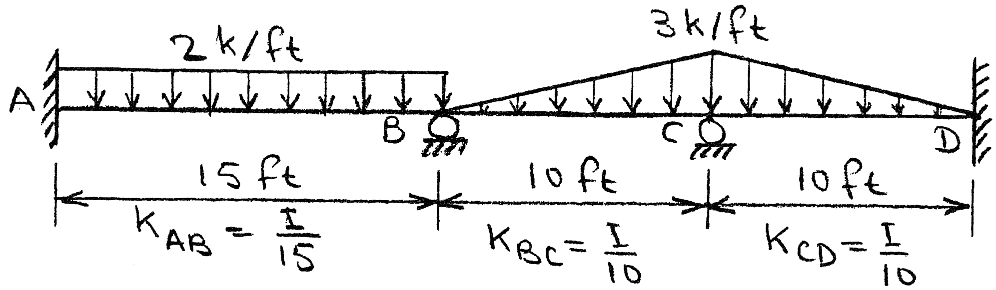
17.9



	AB	BA	BC	CB	CE	EC
DF		0.5	0.5	0.5	0.5	
FEM	106.7	−106.7	106.7	−106.7	60	−60
				23.4	23.4	
			11.7			11.7
		−5.9	−5.9			
	−3			−3		
				1.5	1.5	
			0.8			0.8
		−0.4	−0.4			
	−0.2			−0.2		
				0.1	0.1	
Final Moments	103.5	−113	112.9	−84.9	85	−47.5

For reactions, and shear and bending moment diagrams, see solution of Problem 16.9.

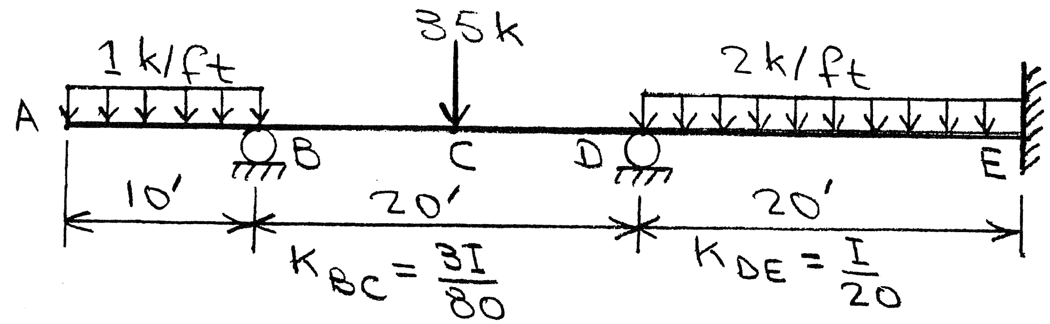
17.10



DF		0.4	0.6	0.5	0.5	
FEM	37.5	-37.5	10	-15	15	-10
		11	16.5			
	5.5			8.3		
				-4.2	-4.2	
			-2.1			-2.1
		0.8	1.3			
	0.4			0.7		
				-0.3	-0.3	
			-0.2			-0.2
		0.1	0.1			
Final Moments	43.4	-25.6	25.6	-10.5	10.5	-12.3

For reactions, and shear and bending moment diagrams, see solution of Problem 16.10.

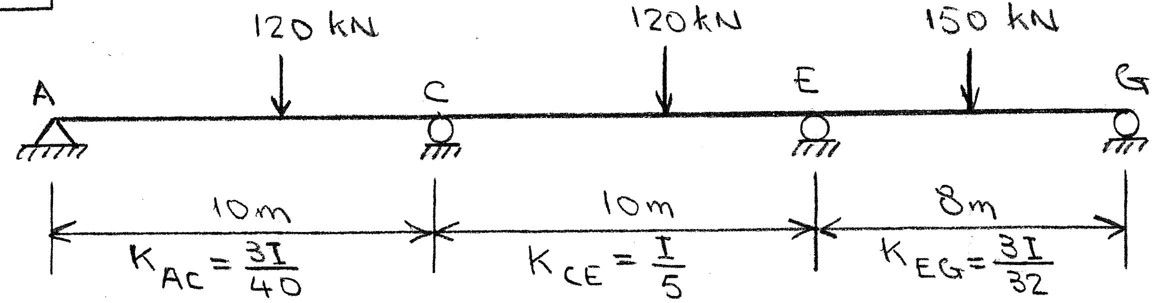
17.11



DF		1	0.429	0.571	
FEM	-50	87.5	-87.5	66.7	-66.7
		-37.5	8.9	11.9	
			-18.8		5.9
			8.1	10.7	
					5.4
Final Moments	-50	50	-89.3	89.3	-55.4

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

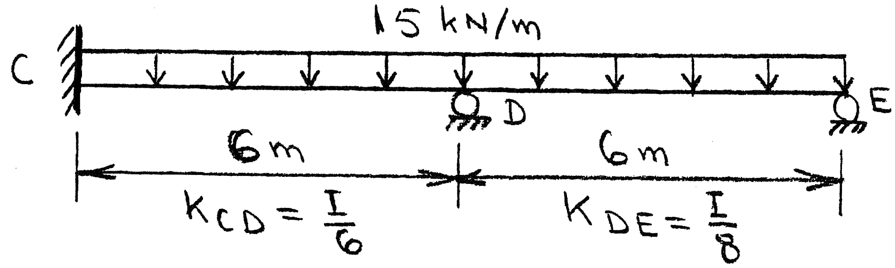
17.12



	AC	CA	CE	EC	EG	GE
DF		0.273	0.727	0.681	0.319	
FEM	115.2	−172.8	115.2	−172.8	150	−150
	−115.2	15.7	41.9	15.5	7.3	150
		−57.6	7.8	20.9	75	
		13.6	36.2	−65.3	−30.6	
			−32.7	18.1		
		8.9	23.8	−12.3	−5.8	
			−6.2	11.9		
		1.7	4.5	−8.1	−3.8	
			−4.1	2.3		
		1.1	3	−1.6	−0.7	
			−0.8	1.5		
		0.2	0.6	−1	−0.5	
			−0.5	0.3		
		0.1	0.4	−0.2	−0.1	
Final Moments	0	−189.1	189.1	−190.8	190.8	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

17.13 As the beam and loading are symmetric, we will analyze only the right half CE of the beam.



	CD	DC	DE	ED
DF		0.571	0.429	1
FEM	45	−45	45	−45
				45
			22.5	
		−12.8	−9.7	
	−6.4			
Final Moments	38.6	−57.8	57.8	0

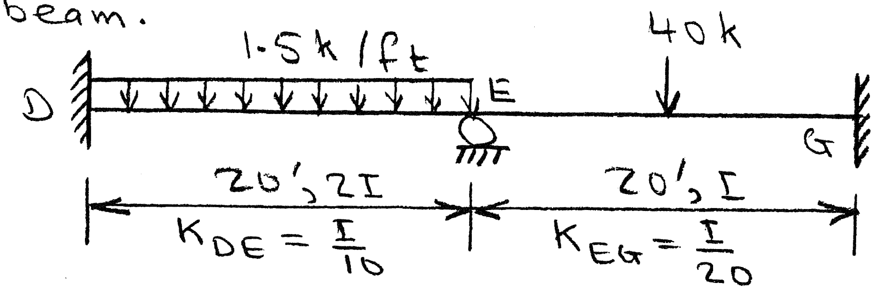
Because of symmetry, the member end moments for the left half of the beam are:

$M_{AB} = 0$; $M_{BA} = -57.8$ kN.m

$M_{BC} = 57.8$ kN.m; $M_{CB} = -38.6$ kN.m

For reactions, and shear and bending moment diagrams, see solution of Problem 16.13.

17.14 As the beam and loading are symmetric, we will analyze only the right half DG of the beam.



DF		2/3	1/3	
FEM	50	−50	100	−100
		−33.3	−16.7	
	−16.7			−8.3
Final Moments	33.3	−83.3	83.3	−108.3

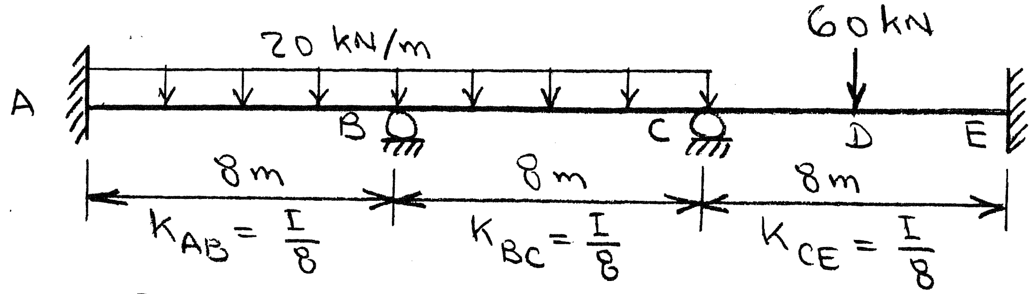
Because of symmetry, the member end moments for the left half of the beam are:

$M_{AC} = 108.3$ k-ft ; $M_{CA} = -83.3$ k-ft

$M_{CD} = 83.3$ k-ft ; $M_{DC} = -33.3$ k-ft

For reactions, and shear and bending moment diagrams, see solution of Problem 16.14.

17.15



$FEM_{AB} = \frac{20(8)^2}{12} = 106.7 \text{ kN.m}$; $FEM_{BA} = -106.7 \text{ kN.m}$

$FEM_{BC} = 106.7 + \frac{6(70)(800)(0.025)}{(8)^2} = 106.7 + 131.3$

$= 238 \text{ kN.m}$

$FEM_{CB} = -106.7 + 131.3 = 24.6 \text{ kN.m}$

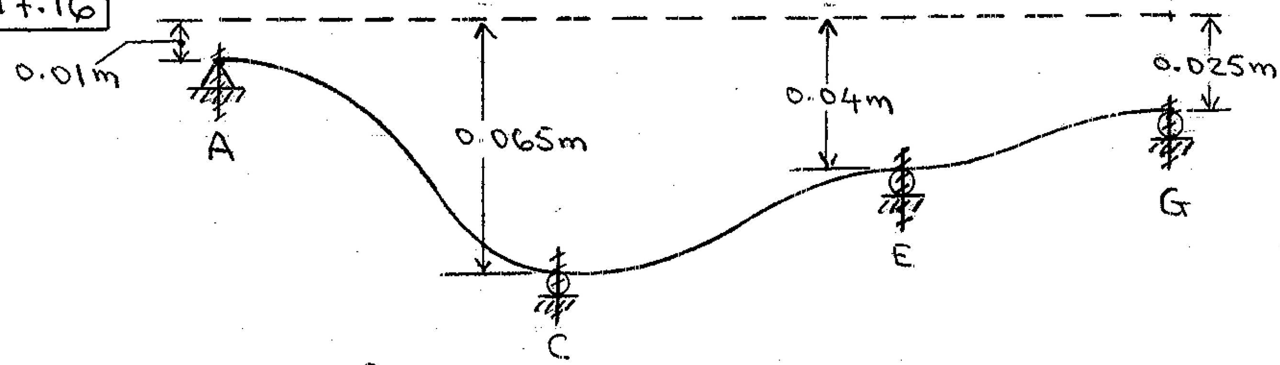
$FEM_{CE} = \frac{60(8)}{8} - 131.3 = 60 - 131.3 = -71.3 \text{ kN.m}$

$FEM_{EC} = -60 - 131.3 = -191.3 \text{ kN.m}$

	AB	BA	BC	CB	CE	EC
DF		0.5	0.5	0.5	0.5	
FEM	106.7	-106.7	238	24.6	-71.3	-191.3
		-65.7	-65.7	23.4	23.4	
	-32.8		11.7	-32.8		11.7
		-5.8	-5.8	16.4	16.4	
	-2.9		8.2	-2.9		8.2
		-4.1	-4.1	1.5	1.5	
	-2.1		0.7	-2.1		0.7
		-0.4	-0.4	1	1	
	-0.2		0.5	-0.2		0.5
		-0.2	-0.2	0.1	0.1	
Final Moments	68.7	-182.9	182.9	29	-28.9	-170.2

For reactions, and shear and bending moment diagrams, see solution of Problem 16.15.

17.16



$$FEM_{AC} = \frac{120(6)(4)^2}{(10)^2} + \frac{6(200)(500)(0.055)}{(10)^2} = 115.2 + 330 = 445.2 \text{ kN.m}$$

$$FEM_{CA} = -\frac{120(6)^2(4)}{(10)^2} + 330 = -172.8 + 330 = 157.2 \text{ kN.m}$$

$$FEM_{CE} = 115.2 - \frac{6(200)(1000)(0.025)}{(10)^2} = 115.2 - 300 = -184.8 \text{ kN.m}$$

$$FEM_{EC} = -172.8 - 300 = -472.8 \text{ kN.m}$$

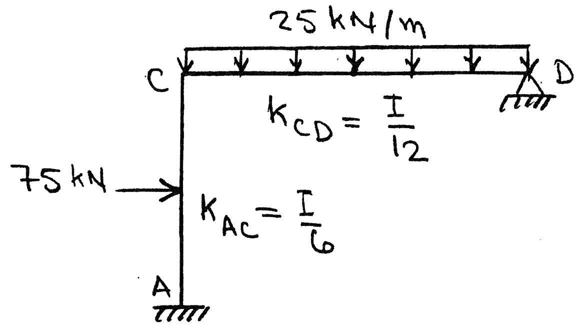
$$FEM_{EG} = \frac{150(8)}{8} - \frac{6(200)(500)(0.015)}{(8)^2} = 150 - 140.6 = 9.4 \text{ kN.m}$$

$$FEM_{GE} = -150 - 140.6 = -290.6 \text{ kN.m}$$

DF		0.273	0.727	0.681	0.319	
FEM	445.2	157.2	-184.8	-472.8	9.4	-290.6
	-445.2	7.5	20.1	315.6	147.8	290.6
		-222.6	157.8	10	145.3	
		17.7	47.1	-105.8	-49.5	
			-52.9	23.6		
		14.4	38.5	-16.1	-7.5	
			-8	19.2		
		2.2	5.8	-13.1	-6.1	
			-6.5	2.9		
		1.8	4.7	-2	-0.9	
			-1	2.4		
		0.3	0.7	-1.6	-0.8	
			-0.8	0.4		
		0.2	0.6	-0.3	-0.1	
Final Moments	0	-21.3	21.3	-237.6	237.6	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

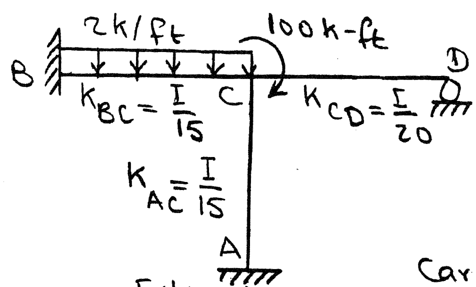
17.17



	AC	CA	CD	DC
DF		2/3	1/3	
FEM	56.3	-56.3	168.8	-168.8
		-75	-37.5	168.8
	-37.5		84.4	
		-56.3	-28.1	
	-28.2			
Final Moments	-9.4	-187.6	187.6	0

For reactions, see solution of Problem 13.42

17-18

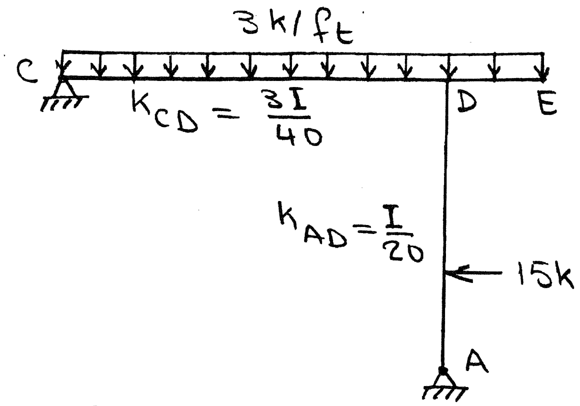


Carry Over (CA → AC)

	BC	CB	External Moment at Joint C	CA	CD	DC	AC
DF		0.364	—	0.364	0.272		
FEM	37.5	−37.5	100	0	0	0	0
		−22.7	—	−22.7	−17.1		
	−11.3						−11.3
Final Moments	26.2	−60.2	100	−22.7	−17.1	0	−11.3

For reactions, see solution of Problem 16.18.

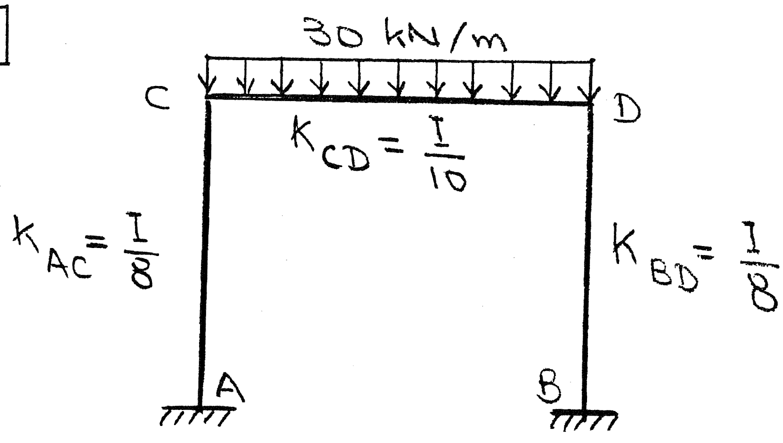
17.19



	CD	DC	DE	DA	AD
DF		0.6	—	0.4	
FEM	100	−100	37.5	16.7	−33.3
	−100	27.5		18.3	33.3
		−50		16.7	
		20		13.3	
Final Moments	0	−102.5	37.5	65	0

For reactions, see solution of Problem 13.21.

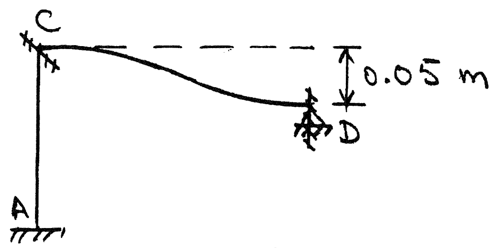
17.20



	AC	CA	CD	DC	DB	BD
DF		5/9	4/9	4/9	5/9	
FEM	0	0	250	-250	0	0
		-138.9	-111.1	111.1	138.9	
	-69.4		55.6	-55.6		69.4
		-30.9	-24.7	24.7	30.9	
	-15.4		12.3	-12.3		15.4
		-6.8	-5.5	5.5	6.8	
	-3.4		2.7	-2.7		3.4
		-1.5	-1.2	1.2	1.5	
	-0.8		0.6	-0.6		0.8
		-0.3	-0.3	0.3	0.3	
	-0.2		0.1	-0.1		0.2
Final Moments	-89.2	-178.4	178.5	-178.5	178.4	89.2

For reactions, see solution of problem 16.20.

17.21



$FEM_{AC} = \frac{75(6)}{8} = 56.3$ k-ft ; $FEM_{CA} = -56.3$ k-ft

$FEM_{CD} = \frac{25(9)^2}{12} + \frac{6(200)400(0.05)}{(9)^2} = 168.8 + 296.3$

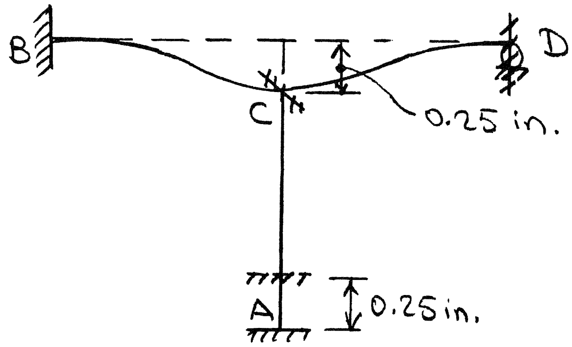
$= 465.1$ k-ft

$FEM_{DC} = -168.1 + 296.3 = 128.2$ k-ft

	AC	CA	CD	DC
DF		2/3	1/3	
FEM	56.3	-56.3	465.1	128.2
		-272.5	-136.3	-128.2
	-136.3		-64.1	
		42.7	21.4	
	21.4			
Final Moments	-58.6	-286.1	286.1	0

For reactions, see solution of Problem 16-21

17.22



$FEM_{AC} = FEM_{CA} = 0$

$$FEM_{BC} = \frac{2(15)^2}{12} + \frac{6(29000)(3500)(0.25)}{(15)^2(12)^3} = 37.5 + 391.6$$

$$= 429.1 \text{ k-ft}$$

$$FEM_{CB} = -37.5 + 391.6 = 354.1 \text{ k-ft}$$

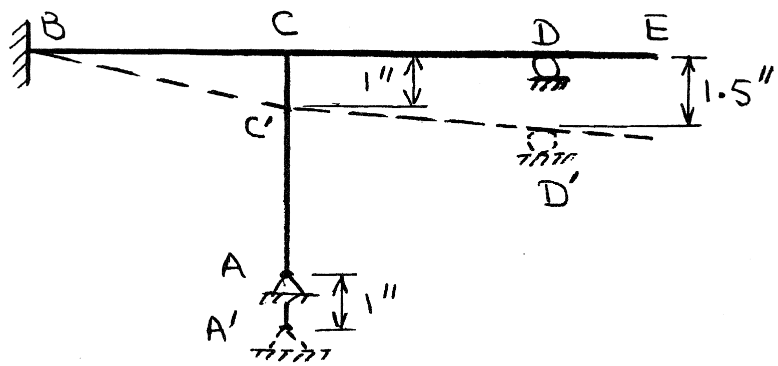
$$FEM_{CD} = FEM_{DC} = -391.6 \text{ k-ft}$$

Carry Over (CA → AC)

	BC	CB	External Moment at Joint C	CA	CD	DC	AC
DF		0.364	–	0.364	0.272		
FEM	429.1	354.1	100	0	-391.6	-391.6	0
		-22.7		-22.7	-17.1	391.6	
	-11.3				195.8		-11.3
		-71.3		-71.3	-53.2		
	-35.7						-35.7
Final Moments	382.1	260.1	100	-94	-266.1	0	-47

For reactions, see solution of Problem 16.22.

17.23



$K_{AC} = \frac{3}{4}\left(\frac{I}{20}\right) = \frac{3I}{80}$

$K_{BC} = \frac{I}{30}$; $K_{CD} = \frac{3}{4}\left(\frac{I}{30}\right) = \frac{I}{40}$

$FEM_{AC} = \frac{40(20)}{8} = 100$ k-ft; $FEM_{CA} = -100$ k-ft

$FEM_{BC} = \frac{2(30)^2}{12} + \frac{6(10000)(3000)(1)}{(30)^2(12)^3} = 150 + 115.7 = 265.7$ k-ft

$FEM_{CB} = -150 + 115.7 = -34.3$ k-ft; $FEM_{CD} = \frac{6(10000)(3000)(0.5)}{(30)^2(12)^3} = 57.9$ k-ft

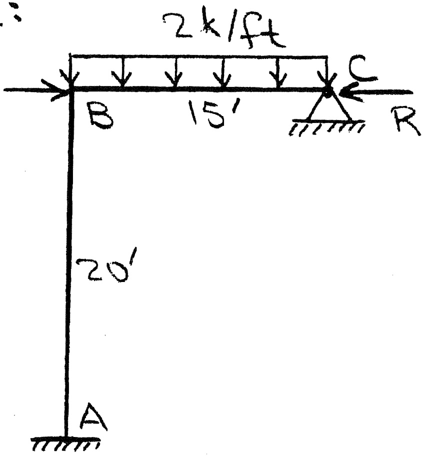
$FEM_{DC} = 57.9$ k-ft; $FEM_{DE} = 10(10) = 100$ k-ft.

	BC	CB	CA	CD	DC	DE	AC
DF		0.348	0.391	0.261	1	–	1
FEM	265.7	-34.3	-100	57.9	57.9	100	100
		26.6	29.9	19.9	-157.9		-100
	13.3		-50	-79			
		44.9	50.4	33.7			
	22.4						
Final Moments	301.4	37.2	-69.7	32.5	-100	100	0

For reactions, see solution of problem 16.23.

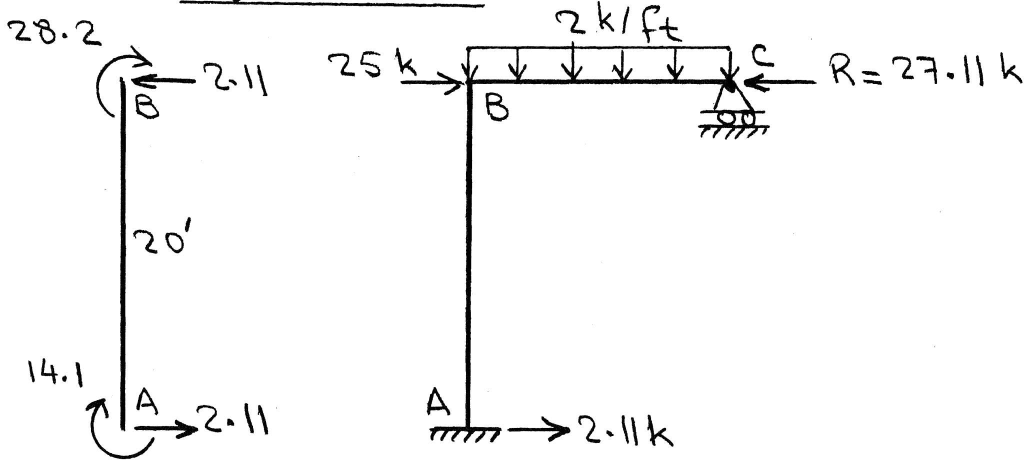
17.24 $K_{AB} = \frac{I}{20}$, $K_{BC} = \frac{3}{4}\left(\frac{I}{15}\right) = \frac{I}{20}$

<u>Part I - Sidesway prevented:</u>



AB	BA	BC	CB
	0.5	0.5	
		37.5	-37.5
	-18.8	-18.8	37.5
-9.4		18.8	
	-9.4	-9.4	
-4.7			
-14.1	-28.2	28.1	0

<u>M_O Moments</u>



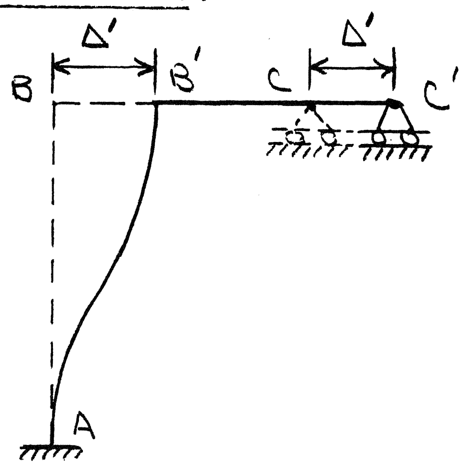
<u>Part II - Sidesway permitted:</u>

$FEM_{AB} = FEM_{BA}$

$= \frac{6EI\Delta'}{(20)^2}$

Let $FEM_{AB} = FEM_{BA}$

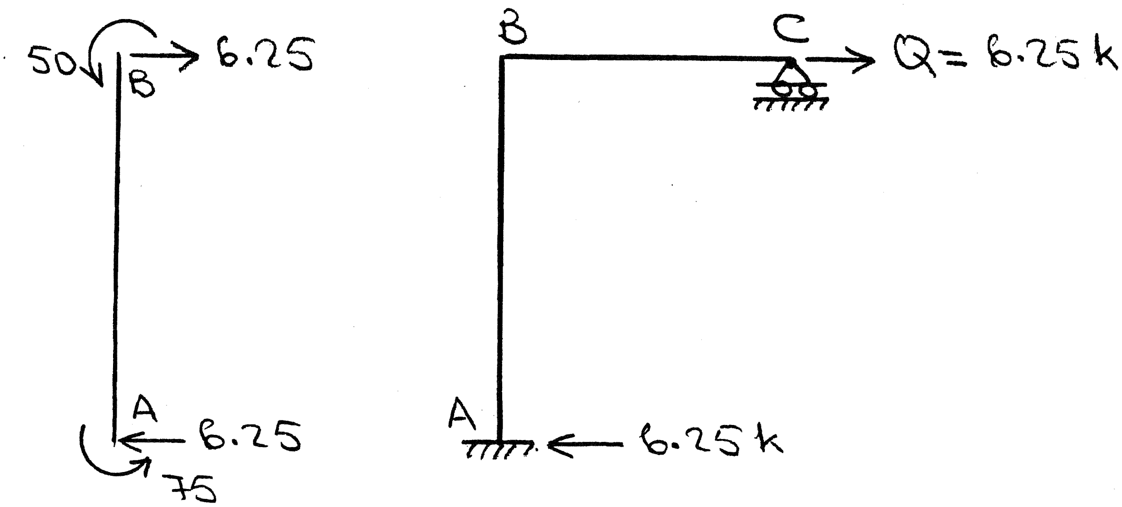
$= 100$ k-ft.



17.24 (Contd.)

AB	BA	BC	CB
	0.5	0.5	
100	100		
	−50	−50	
−25			
75	50	−50	0

M_Q Moments



Actual member end moments:

$M_{AB} = -14.1 + \left(\frac{27.11}{6.25}\right)75 = \underline{311.2 \text{ k-ft}}$

$M_{BA} = -28.2 + \left(\frac{27.11}{6.25}\right)50 = \underline{188.7 \text{ k-ft}}$

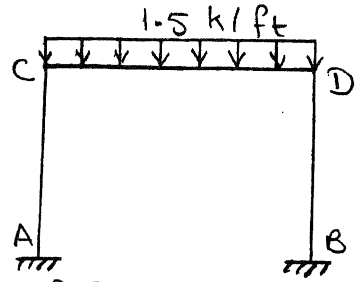
$M_{BC} = 28.1 - \left(\frac{27.11}{6.25}\right)50 = \underline{-188.8 \text{ k-ft}}$

$\underline{M_{CB} = 0}$

For reactions, see solution of Problem 16.24.

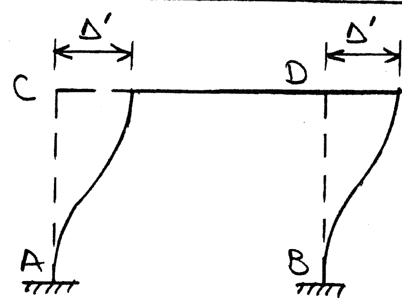
17.25 $K_{AC} = K_{BD} = \frac{I}{30}$; $K_{CD} = \frac{2I}{40} = \frac{I}{20}$

Part I- No Sidesway:



	AC	CA	CD	DC	DB	BD
DF		0.4	0.6	0.6	0.4	
FEM	0	0	200	-200	0	0
		-80	-120	120	80	
	-40		60	-60		40
		-24	-36	36	24	
	-12		18	-18		12
		-7.2	-10.8	10.8	7.2	
	-3.6		5.4	-5.4		3.6
		-2.2	-3.2	3.2	2.2	
	-1.1		1.6	-1.6		1.1
		-0.6	-1.0	1.0	0.6	
	-0.3		0.5	-0.5		0.3
		-0.2	-0.3	0.3	0.2	
	-0.1		0.2	-0.2		0.1
		-0.1	-0.1	0.1	0.1	
M_0 Moments	-57.1	-114.3	114.3	-114.3	114.3	57.1

Part II - Sidesway:



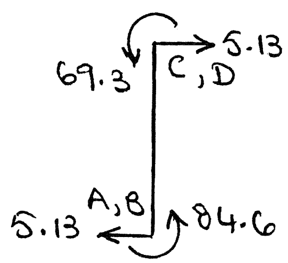
$FEM_{CD} = FEM_{DC} = 0$

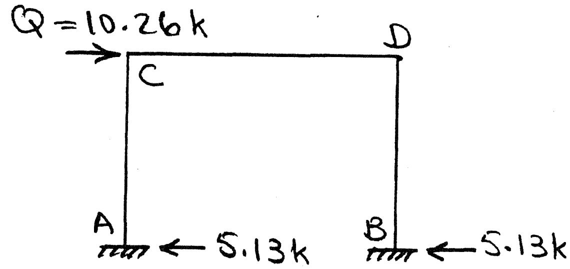
$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} = \frac{6EI\Delta'}{(30)^2}$

let $\frac{6EI\Delta'}{(30)^2} = 100$ k-ft

17.25 (contd.)

	AC	CA	CD	DC	DB	BD
DF		0.4	0.6	0.6	0.4	
FEM	100	100	0	0	100	100
		-40	-60	-60	-40	
	-20		-30	-30		-20
		12	18	18	12	
	6		9	9		6
		-3.6	-5.4	-5.4	-3.6	
	-1.8		-2.7	-2.7		-1.8
		1.1	1.6	1.6	1.1	
	0.6		0.8	0.8		0.6
		-0.3	-0.5	-0.5	-0.3	
	-0.2		-0.3	-0.3		-0.2
		0.1	0.2	0.2	0.1	
M_Q Moments	84.6	69.3	-69.3	-69.3	69.3	84.6





Actual member end moments: $M = M_0 + \left(\frac{20}{Q}\right) M_Q$

$M_{AC} = 107.8$ k-ft; $M_{CA} = 20.8$ k-ft;

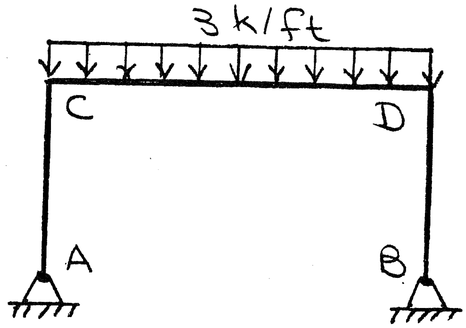
$M_{CD} = -20.8$ k-ft; $M_{DC} = -249.4$ k-ft;

$M_{DB} = 249.4$ k-ft; $M_{BD} = 222$ k-ft

For reactions, see solution of Problem 13.45

17.26 $K_{AC} = K_{BD} = \frac{3}{4}\left(\frac{I}{15}\right) = \frac{I}{20}$; $K_{CD} = \frac{I}{30}$

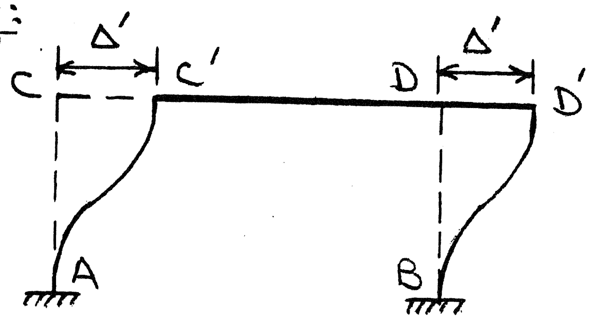
Part I – No Sidesway:



	AC	CA	CD	DC	DB	BD
DF		0.6	0.4	0.4	0.6	
FEM	0	0	225	−225	0	0
		−135	−90	90	135	
			45	−45		
		−27	−18	18	27	
			9	−9		
		−5.4	−3.6	3.6	5.4	
			1.8	−1.8		
		−1.1	−0.7	0.7	1.1	
			0.4	−0.4		
		−0.2	−0.2	0.2	0.2	
M_0 Moments	0	−168.7	168.7	−168.7	168.7	0

Part II – Sidesway:

$FEM_{CD} = FEM_{DC} = 0$

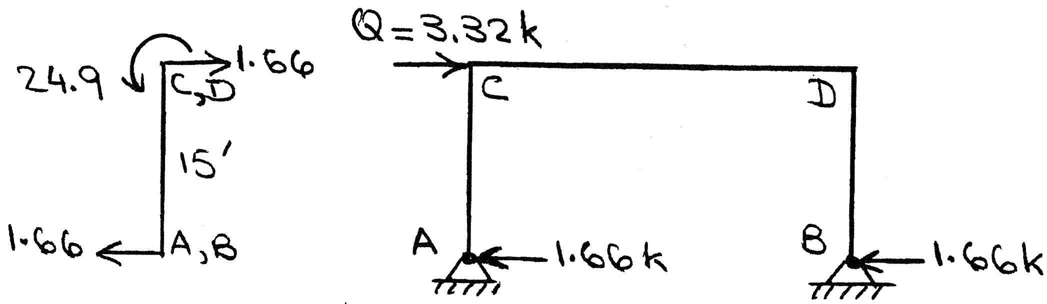


$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} = \frac{6EI\Delta'}{(15)^2}$

Let $\frac{6EI\Delta'}{(15)^2} = 100$ k-ft.

17.26 (Contd.)

	AC	CA	CD	DC	DB	BD
DF		0.6	0.4	0.4	0.6	
FEM	100	100	0	0	100	100
	-100	-60	-40	-40	-60	-100
		-50	-20	-20	-50	
		42	28	28	42	
			14	14		
		-8.4	-5.6	-5.6	-8.4	
			-2.8	-2.8		
		1.7	1.1	1.1	1.7	
			0.6	0.6		
		-0.4	-0.2	-0.2	-0.4	
M_Q Moments	0	24.9	-24.9	-24.9	24.9	0



Actual member end moments: $M = M_O + \left(\frac{40}{Q}\right) M_Q$

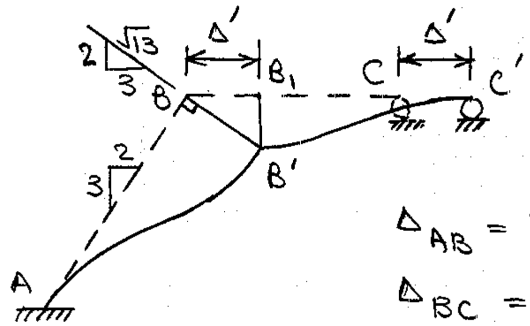
$M_{AC} = M_{BD} = 0$; $M_{CA} = 131.3$ k-ft

$M_{CD} = -131.3$ k-ft; $M_{DC} = -468.7$ k-ft

$M_{DB} = 468.7$ k-ft.

For reactions, see solution of Problem 13.22.

17.27 $K_{AB} = \frac{I}{14.42}$; $K_{BC} = \frac{3}{4}\left(\frac{I}{12}\right) = \frac{I}{16}$



$\Delta_{AB} = BB' = \frac{\sqrt{13}}{3}\Delta' = 1.202\Delta'$

$\Delta_{BC} = B_1B' = \frac{2}{3}\Delta' = 0.667\Delta'$

$$FEM_{AB} = FEM_{BA} = \frac{6EI(1.202\Delta')}{(14.42)^2}$$

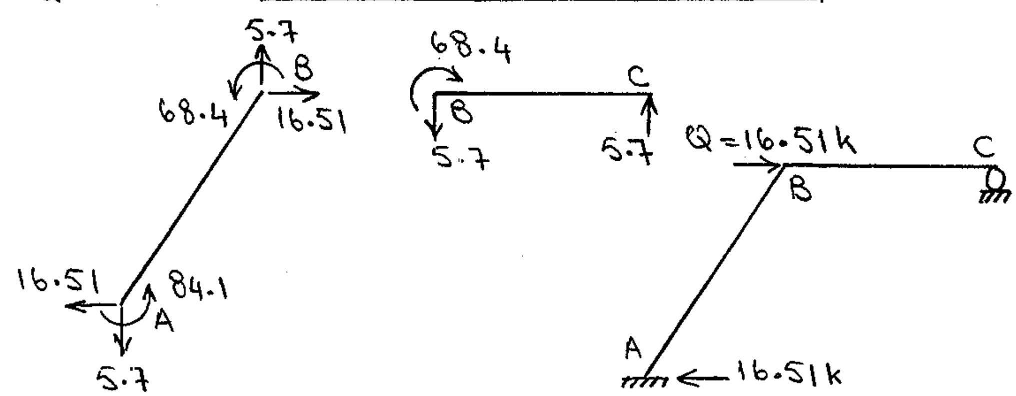
$$FEM_{BC} = FEM_{CB} = -\frac{6EI(0.667\Delta')}{(12)^2}$$

Let $FEM_{AB} = FEM_{BA} = \frac{6EI(1.202\Delta')}{(14.42)^2} = 100$ k-ft

$EI\Delta' = 2883.2$

$FEM_{BC} = FEM_{CB} = -80.1$ k-ft

	AB	BA	BC	CB
DF		0.526	0.474	
FEM	100	100	-80.1	-80.1
		-10.5	-9.4	80.1
	-5.3		40.1	
		-21.1	-19	
	-10.6			
M_Q Moments	84.1	68.4	-68.4	0



Actual member end moments: $M = \frac{25}{16.51} M_Q$

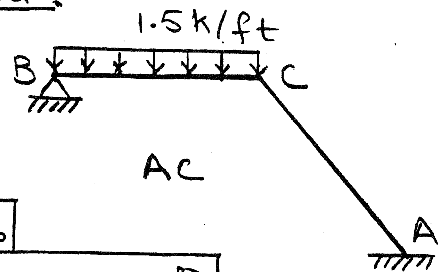
$M_{AB} = 127.3$ k-ft ; $M_{BA} = 103.6$ k-ft ;

$M_{BC} = -103.6$ k-ft , $M_{CB} = 0$

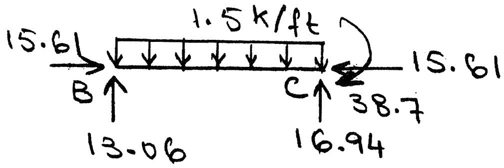
For reactions, see solution of Problem 16.27.

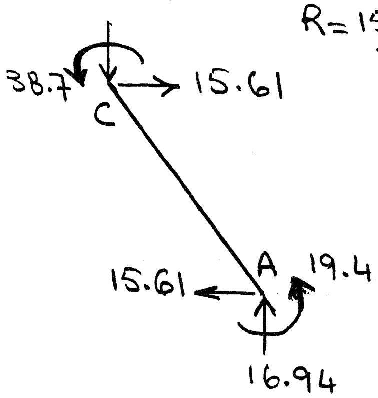
17.28 $K_{CB} = \frac{3}{4}\left(\frac{I}{20}\right) = \frac{3I}{80}$; $K_{CA} = \frac{I}{25}$

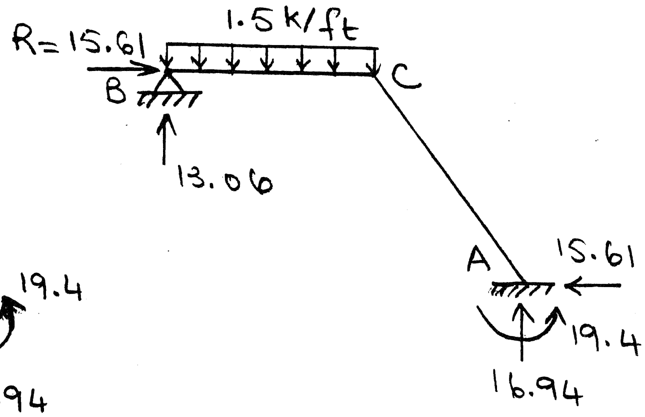
Part I – Sidesway prevented:



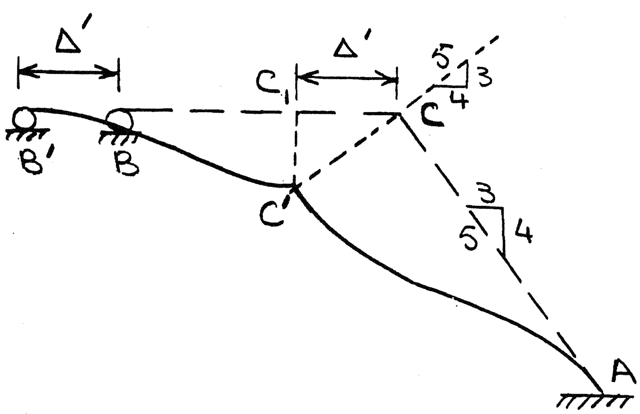
	BC	CB	CA	AC
DF		0.484	0.516	
FEM	50	−50	0	0
	−50	24.2	25.8	
		−25		12.9
		12.1	12.9	
				6.5
M_O Moments	0	−38.7	38.7	19.4







Part II – Sidesway permitted:



$\Delta_{BC} = \frac{3}{4}\Delta'$

$\Delta_{AC} = \frac{5}{4}\Delta'$

17.28 (contd.)

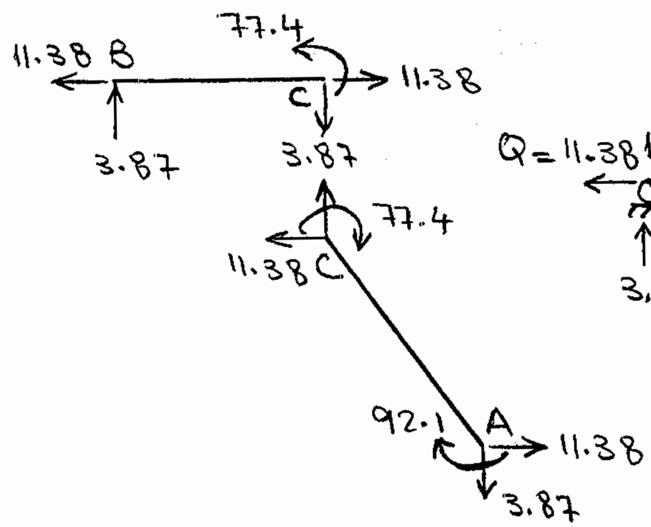
$$FEM_{BC} = FEM_{CB} = \frac{6EI(0.75\Delta')}{(20)^2}$$

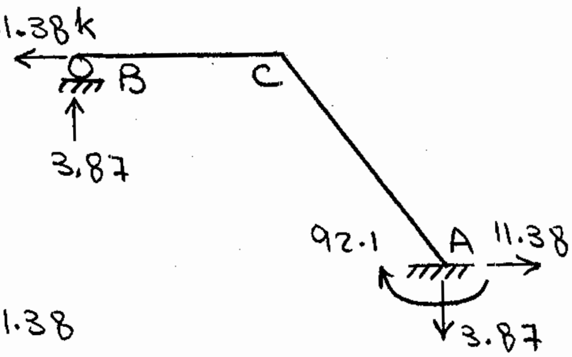
Let $FEM_{BC} = FEM_{CB} = 100$ k-ft

$$EI\Delta' = 8,888.89$$

$$FEM_{CA} = FEM_{AC} = -\frac{6EI(1.25\Delta')}{(25)^2} = -106.7 \text{ k-ft}$$

	BC	CB	CA	AC
DF		0.484	0.516	
FEM	100	100	−106.7	−106.7
	−100	3.2	3.5	
		−50		1.7
		24.2	25.8	
				12.9
M_Q Moments	0	77.4	−77.4	−92.1



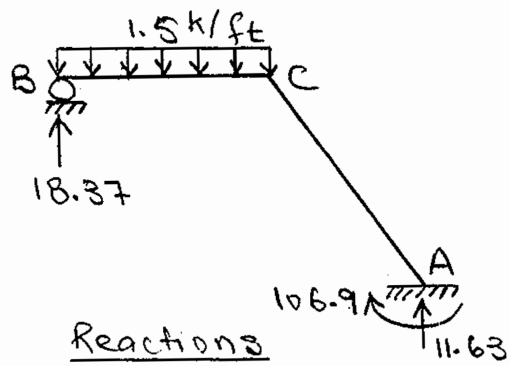


Actual Moments: $M = M_O + \left(\frac{R}{Q}\right)M_Q$

$M_{BC} = 0$; $M_{CB} = 67.5$ k-ft

$M_{CA} = -67.5$ k-ft

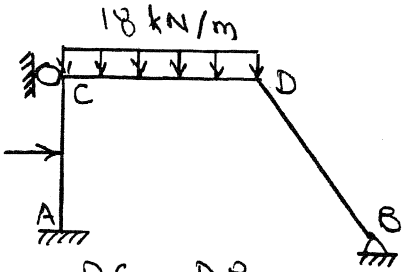
$M_{AC} = -106.9$ k-ft



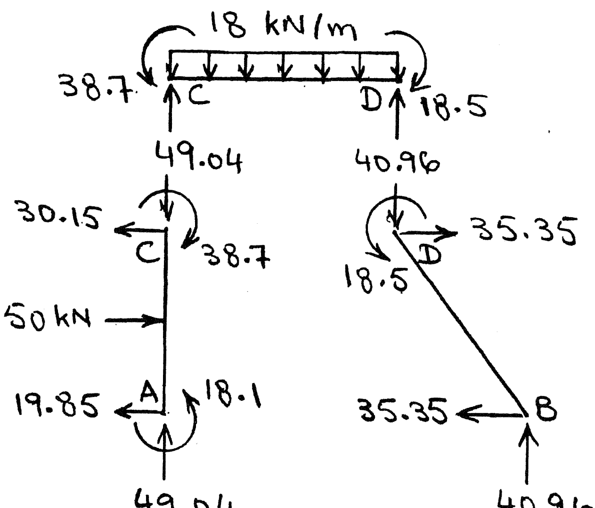
Reactions

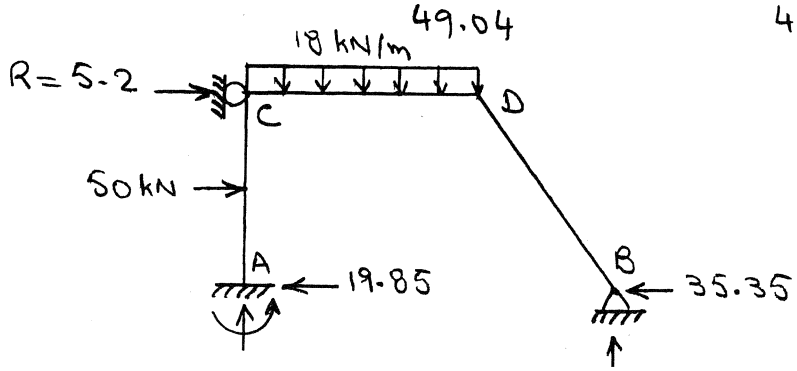
17.29 $K_{AC} = \frac{I}{4}$; $K_{CD} = \frac{I}{5}$; $K_{BD} = \frac{3}{4}\left(\frac{I}{5}\right) = \frac{3I}{20}$

Part I - Sidesway prevented:

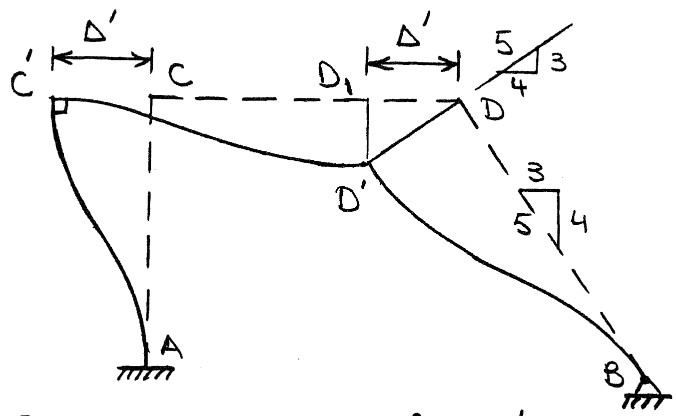


	AC	CA	CD	DC	DB	BD
DF		0.556	0.444	0.571	0.429	
FEM	25	-25	37.5	-37.5	0	0
		-7	-5.5	-21.4	16.1	
	-3.5		10.7	-2.8		
		-5.9	-4.8	1.6	1.2	
	-3		0.8	-2.4		
		-0.4	-0.4	1.4	1.0	
	-0.2		0.7	-0.2		
		-0.4	-0.3	0.1	0.1	
	-0.2			-0.2		
				0.1	0.1	
M_0 Moments	18.1	-38.7	38.7	-18.5	18.5	0





17.29 (contd.) Part II – Sidesway permitted:



$\Delta_{AC} = \Delta'$

$\Delta_{CD} = \frac{3}{4}\Delta' = 0.75\Delta'$

$\Delta_{BD} = \frac{5}{4}\Delta' = 1.25\Delta'$

$FEM_{AC} = FEM_{CA} = -\frac{6EI\Delta'}{(4)^2}$

Let $FEM_{AC} = FEM_{CA} = -100$ kN.m

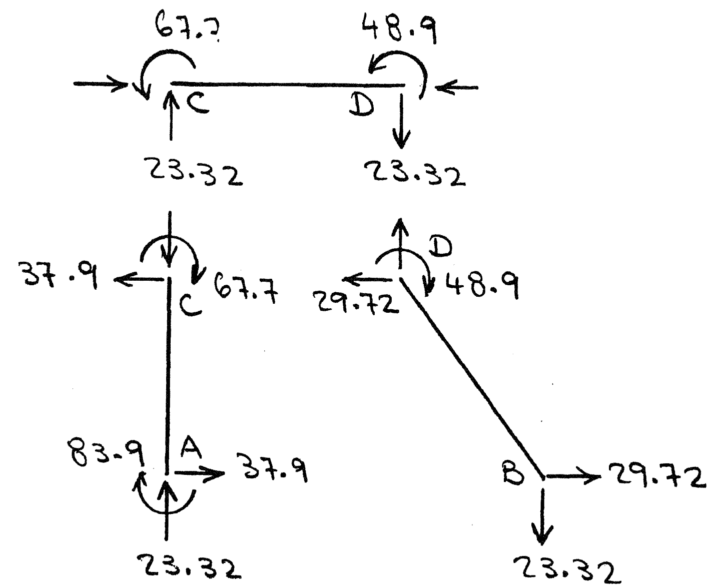
$EI\Delta' = 266.7$

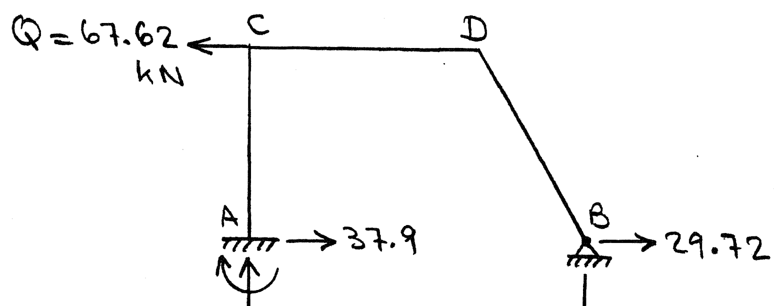
$FEM_{CD} = FEM_{DC} = \frac{6EI(0.75\Delta')}{(5)^2} = 48$ kN.m

$FEM_{BD} = FEM_{DB} = -\frac{6EI(1.25\Delta')}{(5)^2} = -80$ kN.m

	AC	CA	CD	DC	DB	BD
DF		0.556	0.444	0.571	0.429	
FEM	-100	-100	48	48	-80	-80
		28.9	23.1	18.3	13.7	80
	14.5		9.2	11.6	40	
		-5.1	-4.1	-29.5	-22.1	
	-2.6		-14.8	-2.1		
		8.2	6.6	1.2	0.9	
	4.1		0.6	3.3		
		-0.3	-0.3	-1.9	-1.4	
	-0.2		-1.0	-0.2		
		0.6	0.4	0.1	0.1	
	0.3			0.2		
				-0.1	-0.1	
M_Q Moments	-83.9	-67.7	67.7	48.9	-48.9	0

17.29 (contd.)



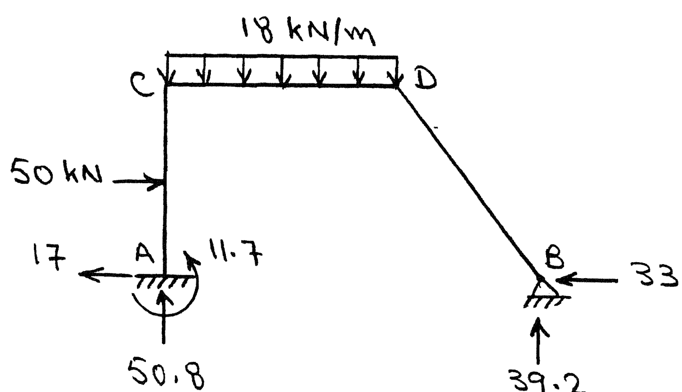


Actual member end moments: $M = M_0 + \left(\frac{R}{Q}\right) M_Q$

$M_{AC} = 11.7$ kN.m ; $M_{CA} = -43.9$ kN.m

$M_{CD} = 43.9$ kN.m ; $M_{DC} = -14.7$ kN.m

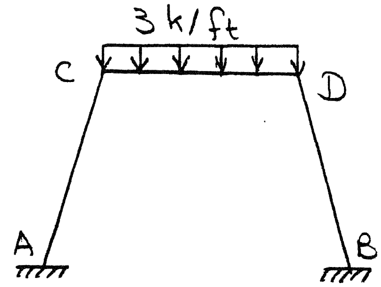
$M_{DB} = 14.7$ kN.m ; $M_{BD} = 0$



Reactions

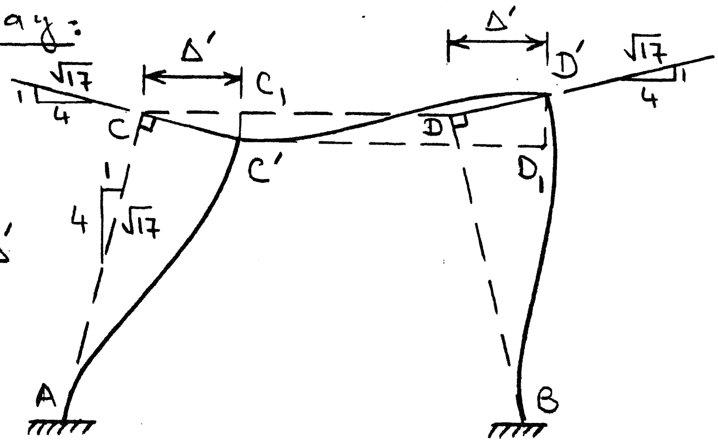
17.30 $K_{AC} = K_{BD} = \frac{I}{16.49}$; $K_{CD} = \frac{I}{16}$

Part I - No Sidesway:



	AC	CA	CD	DC	DB	BD
DF		0.492	0.508	0.508	0.492	
FEM	0	0	64	-64	0	0
		-31.5	-32.5	32.5	31.5	
	-15.8		16.3	-16.3		15.8
		-8	-8.3	8.3	8	
	-4		4.2	-4.2		4
		-2.1	-2.1	2.1	2.1	
	-1.1		1.1	-1.1		1.1
		-0.5	-0.6	0.6	0.5	
	-0.3		0.3	-0.3		0.3
		-0.1	-0.2	0.2	0.1	
M_0 Moments	-21.2	-42.2	42.2	-42.2	42.2	21.2

Part II - Sidesway:



$\Delta_{AC} = CC' = \frac{\sqrt{17}}{4}\Delta'$

$= 1.031\,\Delta'$

$\Delta_{CD} = D_1D' = 2\left(\frac{1}{4}\right)\Delta'$

$= 0.5\,\Delta'$

$\Delta_{BD} = DD' = \frac{\sqrt{17}}{4}\Delta'$

$= 1.031\,\Delta'$

$$FEM_{AC} = FEM_{CA} = \frac{6EI(1.031\Delta')}{(16.49)^2}$$

Let $FEM_{AC} = FEM_{CA} = 100$ k-ft

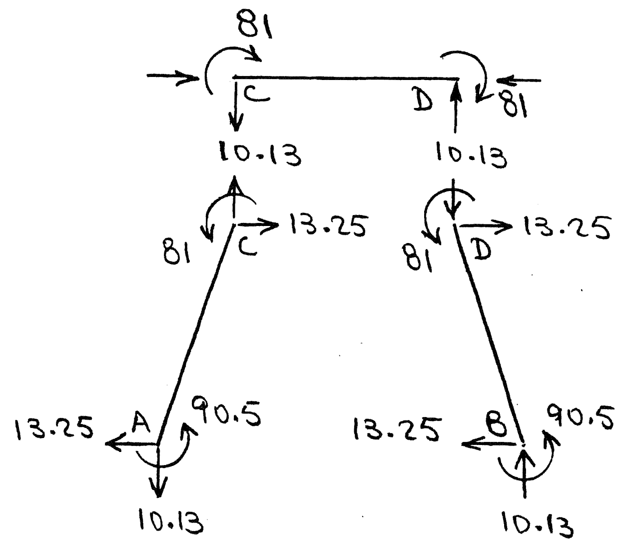
$EI\Delta' = 4395.7$

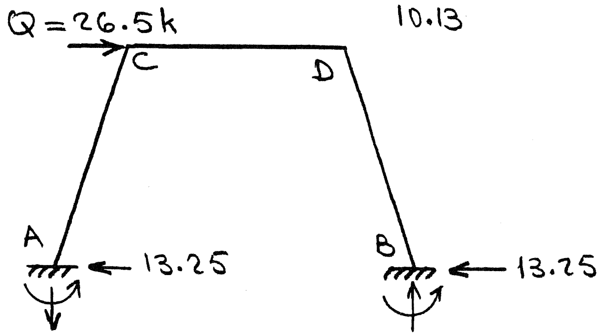
$$FEM_{BD} = FEM_{DB} = \frac{6EI(1.031\Delta')}{(16.49)^2} = 100 \text{ k-ft}$$

$$FEM_{CD} = FEM_{DC} = -\frac{6EI(0.5\Delta')}{(16)^2} = -51.5 \text{ k-ft.}$$

17.30 (contd.)

	AC	CA	CD	DC	DB	BD
DF		0.492	0.508	0.508	0.492	
FEM	100	100	-51.5	-51.5	100	100
		-23.9	-24.6	-24.6	-23.9	
	-12		-12.3	-12.3		-12
		6.1	6.2	6.2	6.1	
	3.1		3.1	3.1		3.1
		-1.5	-1.6	-1.6	-1.5	
	-0.8		-0.8	-0.8		-0.8
		0.4	0.4	0.4	0.4	
	0.2		0.2	0.2		0.2
		-0.1	-0.1	-0.1	-0.1	
M_Q Moments	90.5	81	-81	-81	81	90.5





Actual member end moments: $M = M_0 + \left(\frac{20}{26.5}\right) M_Q$

$M_{AC} = 47.1$ k-ft; $M_{CA} = 19$ k-ft; $M_{CD} = -19$ k-ft;

$M_{DC} = -103.3$ k-ft; $M_{DB} = 103.3$ k-ft; $M_{BD} = 89.5$ k-ft.

For reactions, see solution of Problem 16.30.

17-31 $K_{AC} = K_{BD} = K_{CE} = K_{DF} = \frac{I}{15}$; $K_{CD} = K_{EF} = \frac{2I}{30} = \frac{I}{15}$

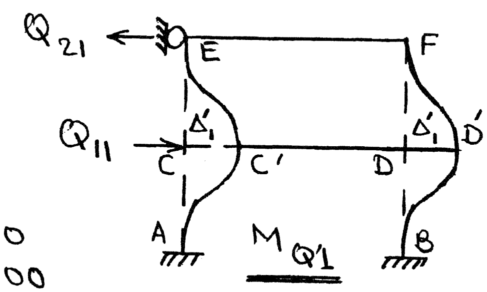
Let $\frac{6EI\Delta_1'}{(15)^2} = 100$ k-ft.

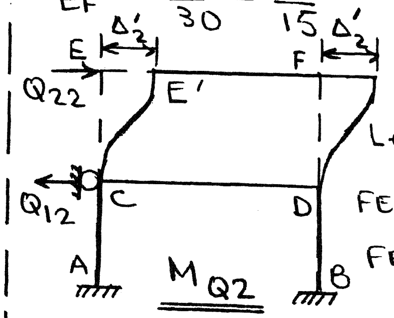
$FEM_{AC} = FEM_{CA} = 100$

$FEM_{BD} = FEM_{DB} = 100$

$FEM_{CE} = FEM_{EC} = -100$

$FEM_{DF} = FEM_{FD} = -100$





Let $\frac{6EI\Delta_2'}{(15)^2} = 100$

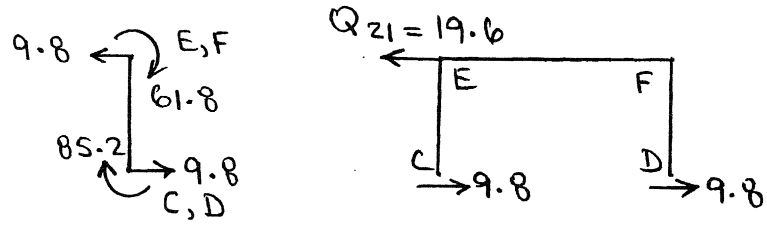
$FEM_{CE} = FEM_{EC} = 100$

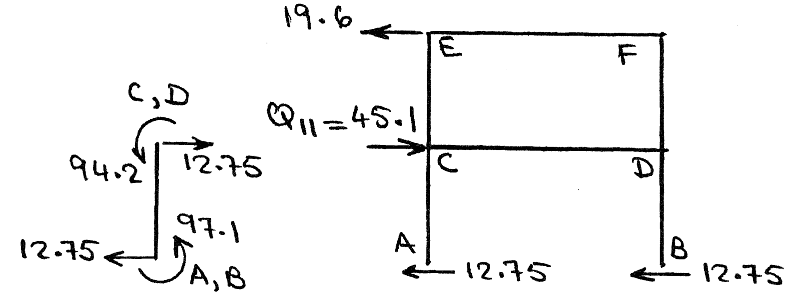
$FEM_{DF} = FEM_{FD} = 100$

Carry Over

	AC	CA	CD	CE	EC	EF	FE	FD	DF	DC	DB	BD
DF		1/3	1/3	1/3	1/2	1/2	1/2	1/2	1/3	1/3	1/3	
FEM	100	100	0	-100	-100	0	0	-100	-100	0	100	100
					50	50	50	50				
				25		25	25		25			
		-8.3	-8.4	-8.3	-12.5	-12.5	-12.5	-12.5	-8.3	-8.4	-8.3	
	-4.2		-4.2	-6.3	-4.2	-6.3	-6.3	-4.2	-6.3	-4.2		-4.2
		3.5	3.5	3.5	5.3	5.3	5.3	5.3	3.5	3.5	3.5	
	1.8		1.8	2.7	1.8	2.7	2.7	1.8	2.7	1.8		1.8
		-1.5	-1.5	-1.5	-2.3	-2.3	-2.3	-2.3	-1.5	-1.5	-1.5	
	-0.8		-0.8	-1.2	-0.8	-1.2	-1.2	-0.8	-1.2	-0.8		-0.8
		0.7	0.7	0.7	1	1	1	1	0.7	0.7	0.7	
	0.4		0.4	0.5	0.4	0.5	0.5	0.4	0.5	0.4		0.4
		-0.3	-0.3	-0.3	-0.5	-0.5	-0.5	-0.5	-0.3	-0.3	-0.3	
	-0.2		-0.2	-0.3	-0.2	-0.3	-0.3	-0.2	-0.3	-0.2		-0.2
		0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.2	0.2	0.2	
	0.1		0.1	0.2	0.1	0.2	0.2	0.1	0.2	0.1		0.1
		-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.2	-0.1	-0.1	-0.1	
MQ1 Moments	97.1	94.2	-8.8	-85.2	-61.8	61.7	61.7	-61.8	-85.2	-8.8	94.2	97.1

17-31 (contd.)

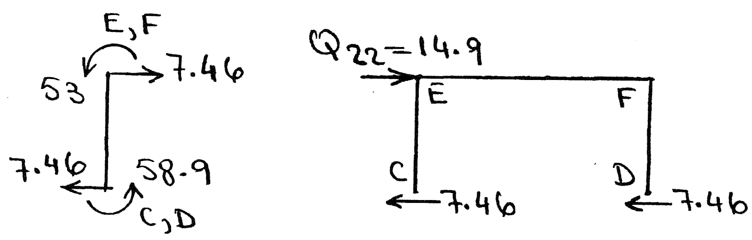


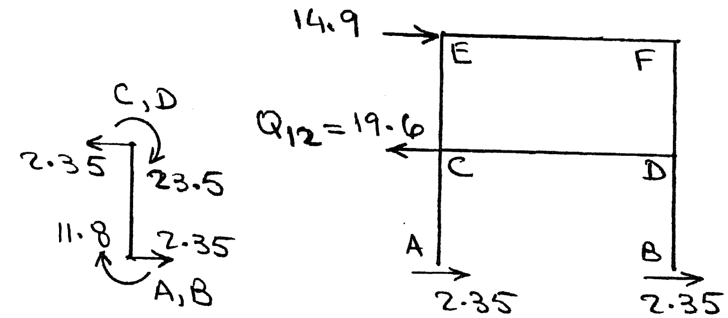


Carry Over

	AC	CA	CD	CE	EC	EF	FE	FD	DF	DC	DB	BD
DF		1/3	1/3	1/3	1/2	1/2	1/2	1/2	1/3	1/3	1/3	
FEM	0	0	0	100	100	0	0	100	100	0	0	0
		-33.3	-33.4	-33.3	-50	-50	-50	-50	-33.3	-33.4	-33.3	
	-16.7		-16.7	-25	-16.7	-25	-25	-16.7	-25	-16.7		-16.7
		13.9	13.9	13.9	20.9	20.9	20.9	20.9	13.9	13.9	13.9	
	7		7	10.5	7	10.5	10.5	7	10.5	7		7
		-5.8	-5.8	-5.8	-8.8	-8.8	-8.8	-8.8	-5.8	-5.8	-5.8	
	-2.9		-2.9	-4.4	-2.9	-4.4	-4.4	-2.9	-4.4	-2.9		-2.9
		2.4	2.4	2.4	3.7	3.7	3.7	3.7	2.4	2.4	2.4	
	1.2		1.2	1.9	1.2	1.9	1.9	1.2	1.9	1.2		1.2
		-1	-1	-1	-1.6	-1.6	-1.6	-1.6	-1	-1	-1	
	-0.5		-0.5	-0.8	-0.5	-0.8	-0.8	-0.5	-0.8	-0.5		-0.5
		0.4	0.4	0.4	0.7	0.7	0.7	0.7	0.4	0.4	0.4	
	0.2		0.2	0.4	0.2	0.4	0.4	0.2	0.4	0.2		0.2
		-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.3	-0.2	-0.2	-0.2	
	-0.1		-0.1	-0.2	-0.1	-0.2	-0.2	-0.1	-0.2	-0.1		-0.1
		0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	
M_{Q2} Moments	-11.8	-23.5	-35.4	58.9	53	-52.8	-52.8	53	58.9	-35.4	-23.5	-11.8

17.31 (contd.)





By superimposing the horizontal forces at joints C and E, we write

$$45.1c_1 - 19.6c_2 = 18$$

$$-19.6c_1 + 14.9c_2 = 9$$

By solving these equations, we obtain $c_1 = 1.545$ and $c_2 = 2.636$.

Member end moments: $M = c_1 M_{Q1} + c_2 M_{Q2}$

$M_{AC} = M_{BD} = 119$ k-ft; $M_{CA} = M_{DB} = 83.5$ k-ft

$M_{CD} = M_{DC} = -106.8$ k-ft; $M_{CE} = M_{DF} = 23.5$ k-ft

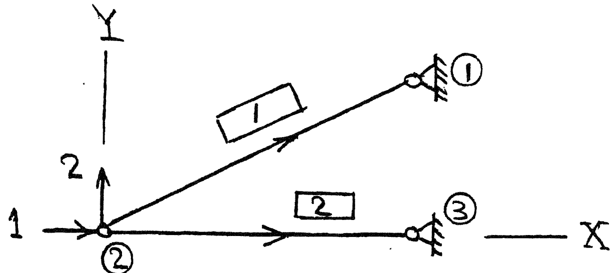
$M_{EC} = M_{FD} = 44$ k-ft; $M_{EF} = M_{FE} = -44$ k-ft.

For reactions, see solution of Problem 16.31.

Chapter Eighteen
Introduction to Matrix Structural Analysis

CHAPTER 18

18.1



Member 1: $L = 11.18$ ft; $\cos\theta = 0.894$; $\sin\theta = 0.447$

$$[K_1] = EA \begin{bmatrix} 0.0715 & 0.0357 & -0.0715 & -0.0357 \\ & 0.0179 & -0.0357 & -0.0179 \\ & \text{sym.} & 0.0715 & 0.0357 \\ & & & 0.0179 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

(columns: 1, 2, 0, 0)

Member 2: $L = 10$ ft; $\cos\theta = 1$; $\sin\theta = 0$

$$[K_2] = EA \begin{bmatrix} 0.1 & 0 & -0.1 & 0 \\ & 0 & 0 & 0 \\ & \text{sym.} & 0.1 & 0 \\ & & & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

(columns: 1, 2, 0, 0)

Structure stiffness matrix:

$$[S] = EA \begin{bmatrix} 0.1715 & 0.0357 \\ 0.0357 & 0.0179 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

(columns: 1, 2)

Joint load vector: $\{P\} = \begin{bmatrix} 0 \\ -24 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$ k

Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain: $\{d\} = \dfrac{1}{EA} \begin{bmatrix} 477.26 \\ -2292.3 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$

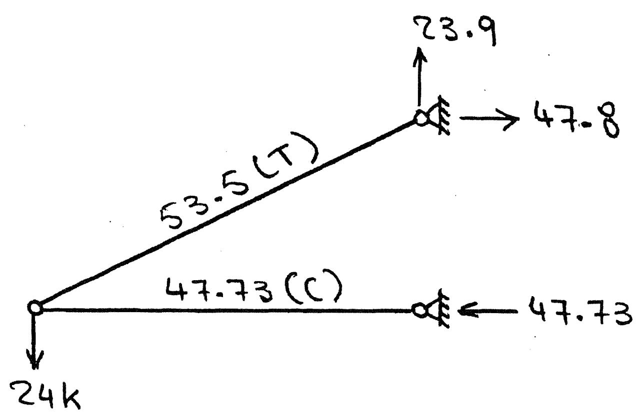
18.1 (contd.) Member forces:

$$\{v_1\} = \frac{1}{EA}\begin{bmatrix} 477.26 \\ -2292.3 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix} \qquad \{u_1\} = [T_1]\{v_1\} = \frac{1}{EA}\begin{bmatrix} -598 \\ 0 \end{bmatrix}$$

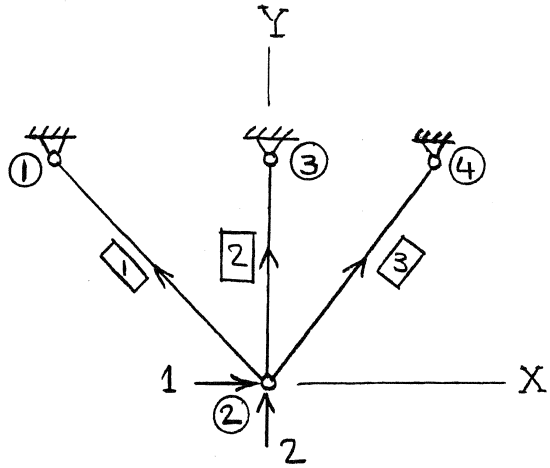
$$\{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} -53.5 \\ 53.5 \end{bmatrix} k \; ; \quad \{F_1\} = [T_1]^T\{Q_1\} = \begin{bmatrix} -47.8 \\ -23.9 \\ 47.8 \\ 23.9 \end{bmatrix} k$$

$$\{v_2\} = \frac{1}{EA}\begin{bmatrix} 477.26 \\ -2292.3 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix} \qquad \{u_2\} = [T_2]\{v_2\} = \frac{1}{EA}\begin{bmatrix} 477.26 \\ 0 \end{bmatrix}$$

$$\{Q_2\} = [k_2]\{u_2\} = \begin{bmatrix} 47.73 \\ -47.73 \end{bmatrix} k \; ; \quad \{F_2\} = [T_2]^T\{Q_2\} = \begin{bmatrix} 47.73 \\ 0 \\ -47.73 \\ 0 \end{bmatrix} k$$



$\boxed{18.2.}$



Member 1: L = 271.53 in.; cos θ = −0.707; sin θ = 0.707

$$[K_1] = 147.3 \begin{array}{cccc|c} 1 & 2 & 0 & 0 & \\ \hline 1 & -1 & -1 & 1 & 1 \\ & 1 & 1 & -1 & 2 \\ & \text{sym.} & 1 & -1 & 0 \\ & & & 1 & 0 \end{array}$$

Member 2: L = 192 in.; cos θ = 0; sin θ = 1.

$$[K_2] = 312.5 \begin{array}{cccc|c} 1 & 2 & 0 & 0 & \\ \hline 0 & 0 & 0 & 0 & 1 \\ & 1 & 0 & -1 & 2 \\ & & 0 & 0 & 0 \\ & & & 1 & 0 \end{array}$$

Member 3: L = 240 in.; cos θ = 0.6; sin θ = 0.8

$$[K_3] = 333.3 \begin{array}{cccc|c} 1 & 2 & 0 & 0 & \\ \hline 0.36 & 0.48 & -0.36 & -0.48 & 1 \\ & 0.64 & -0.48 & -0.64 & 2 \\ & & 0.36 & 0.48 & 0 \\ & & & 0.64 & 0 \end{array}$$

Structure stiffness matrix:

$$[S] = \begin{array}{cc|c} 1 & 2 & \\ \hline 267.3 & 12.7 & 1 \\ 12.7 & 673.1 & 2 \end{array}$$

Joint load vector: $\{P\} = \begin{bmatrix} 20 \\ -50 \end{bmatrix}$ k

18.2 (Contd.) <u>Joint displacements:</u> By solving the equations $\{P\} = [S]\{d\}$, we obtain:

$$\{d\} = \begin{bmatrix} 0.07842 \\ -0.07576 \end{bmatrix} \text{ in.}$$

<u>Member forces:</u>

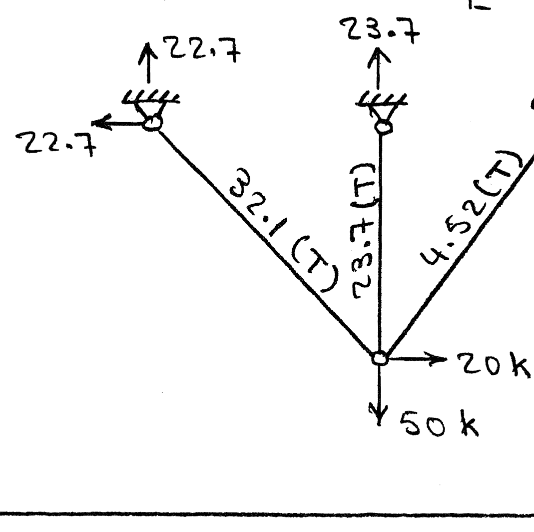
$$\{v_1\} = \{v_2\} = \{v_3\} = \begin{bmatrix} 0.07842 \\ -0.07576 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix} \text{ in.}$$

$$\{u_1\} = [T_1]\{v_1\} = \begin{bmatrix} -0.109 \\ 0 \end{bmatrix} \text{in.}; \quad \{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} -32.1 \\ 32.1 \end{bmatrix} \text{k}$$

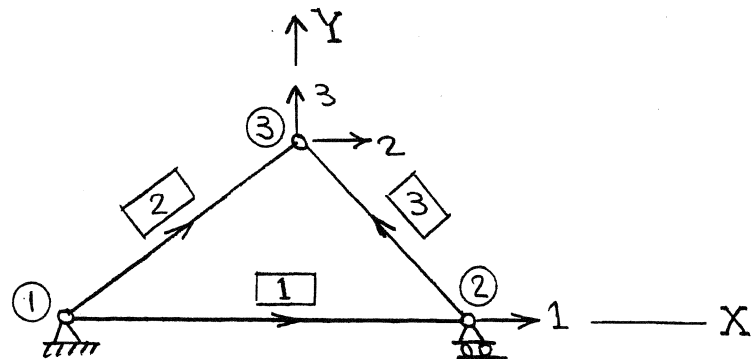
$$\{F_1\} = [T]^T\{Q_1\} = \begin{bmatrix} 22.7 \\ -22.7 \\ -22.7 \\ 22.7 \end{bmatrix} \text{k} \qquad \{u_2\} = \begin{bmatrix} -0.07576 \\ 0 \end{bmatrix} \text{in.}$$

$$\{Q_2\} = \begin{bmatrix} -23.7 \\ 23.7 \end{bmatrix} \text{k}; \qquad \{F_2\} = \begin{bmatrix} 0 \\ -23.7 \\ 0 \\ 23.7 \end{bmatrix} \text{k}$$

$$\{u_3\} = \begin{bmatrix} -0.01356 \\ 0 \end{bmatrix} \text{in.}; \quad \{Q_3\} = \begin{bmatrix} -4.52 \\ 4.52 \end{bmatrix} \text{k}; \quad \{F_3\} = \begin{bmatrix} -2.71 \\ -3.61 \\ 2.71 \\ 3.61 \end{bmatrix} \text{k}$$



18.3



Member 1: $L = 14\,m$; $\cos\theta = 1$; $\sin\theta = 0$

$$[K_1] = \frac{EA}{14}\begin{array}{c} \begin{matrix} 0 & 0 & 1 & 0 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix}$$

Member 2: $L = 10\,m$; $\cos\theta = 0.8$; $\sin\theta = 0.6$

$$[K_2] = \frac{EA}{10}\begin{array}{c} \begin{matrix} 0 & 0 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix} \end{array} \begin{matrix} 0 \\ 0 \\ 2 \\ 3 \end{matrix}$$

Member 3: $L = 8.485\,m$; $\cos\theta = -0.707$; $\sin\theta = 0.707$

$$[K_3] = \frac{EA}{16.97}\begin{array}{c} \begin{matrix} 1 & 0 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 0 \\ 2 \\ 3 \end{matrix}$$

Structure stiffness matrix:

$$[S] = EA\begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 0.1304 & -0.05893 & 0.05893 \\ -0.05893 & 0.1229 & -0.01093 \\ 0.05893 & -0.01093 & 0.09493 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Joint load vector:

$$\{P\} = \begin{bmatrix} 0 \\ 80 \\ -120 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \; kN$$

18.3 (Contd.) Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain:

$$\{d\} = \frac{1}{EA}\begin{bmatrix} 1439 \\ 1161 \\ -2024 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Member forces:

$$\{v_1\} = \frac{1}{EA}\begin{bmatrix} 0 \\ 0 \\ 1439 \\ 0 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix} \qquad \{u_1\} = [T_1]\{v_1\} = \frac{1}{EA}\begin{bmatrix} 0 \\ 1439 \end{bmatrix}$$

$$\{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} -102.8 \\ 102.8 \end{bmatrix} \text{kN}$$

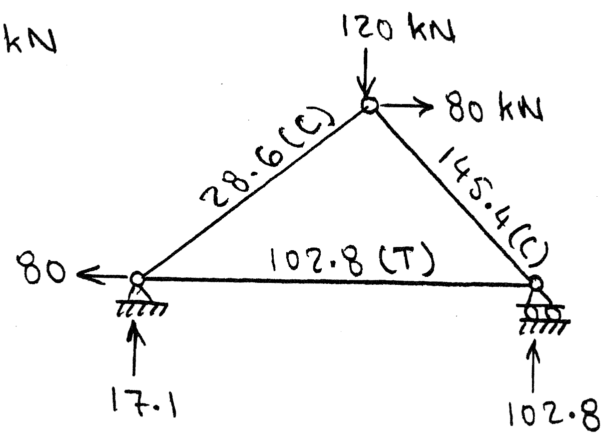
$$\{F_1\} = \begin{bmatrix} -102.8 \\ 0 \\ 102.8 \\ 0 \end{bmatrix} \text{kN}; \qquad \{v_2\} = \frac{1}{EA}\begin{bmatrix} 0 \\ 0 \\ 1161 \\ -2024 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 2 \\ 3 \end{matrix}$$

$$\{u_2\} = \frac{1}{EA}\begin{bmatrix} 0 \\ -285.6 \end{bmatrix}; \qquad \{Q_2\} = \begin{bmatrix} 28.56 \\ -28.56 \end{bmatrix} \text{kN}$$

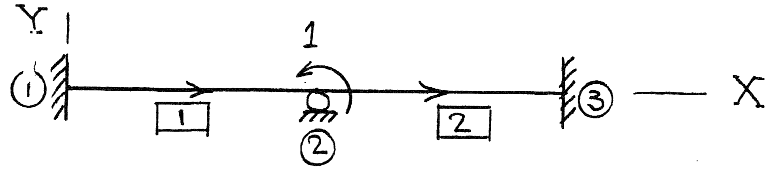
$$\{F_2\} = \begin{bmatrix} 22.85 \\ 17.14 \\ -22.85 \\ -17.14 \end{bmatrix} \text{kN}; \qquad \{v_3\} = \frac{1}{EA}\begin{bmatrix} 1439 \\ 0 \\ 1161 \\ -2024 \end{bmatrix} \begin{matrix} 1 \\ 0 \\ 2 \\ 3 \end{matrix}$$

$$\{u_3\} = \frac{1}{EA}\begin{bmatrix} -1017.4 \\ -2251.8 \end{bmatrix}; \qquad \{Q_3\} = \begin{bmatrix} 145.4 \\ -145.4 \end{bmatrix} \text{kN}$$

$$\{F_3\} = \begin{bmatrix} -102.8 \\ 102.8 \\ 102.8 \\ -102.8 \end{bmatrix} \text{kN}$$



18.4



Member 2 → 0 1 0 0

Member 1 → 0 0 0 1

$$[K_1]=[K_2]=\frac{EI}{L^3}\begin{bmatrix}12 & 180 & -12 & 180\\ & 3600 & -180 & 1800\\ & \text{sym.} & 12 & -180\\ & & & 3600\end{bmatrix}\begin{matrix}0 & 0\\ 0 & 1\\ 0 & 0\\ 1 & 0\end{matrix}$$

$$\{F_{f1}\}=\{Q_{f1}\}=\begin{bmatrix}4.67\\40\\13.33\\-80\end{bmatrix}\begin{matrix}0\\0\\0\\1\end{matrix}\qquad \{F_{f2}\}=\{Q_{f2}\}=\begin{bmatrix}5\\37.5\\5\\-37.5\end{bmatrix}\begin{matrix}0\\1\\0\\0\end{matrix}$$

$$[S]=\frac{EI}{L^3}[7200]1\;;\qquad \{P_f\}=[-42.5]1$$

Joint displacements: $[S]\{d\}=-\{P_f\}$

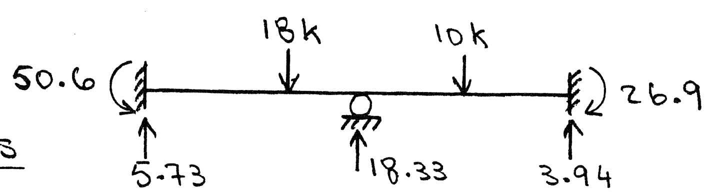
$$\{d\}=\frac{L^3}{EI}[0.005903]$$

Member forces:

$$\{u_1\}=\{v_1\}=\frac{L^3}{EI}\begin{bmatrix}0\\0\\0\\0.005903\end{bmatrix}\begin{matrix}0\\0\\0\\1\end{matrix}\;;\qquad \{u_2\}=\{v_2\}=\frac{L^3}{EI}\begin{bmatrix}0\\0.005903\\0\\0\end{bmatrix}\begin{matrix}0\\1\\0\\0\end{matrix}$$

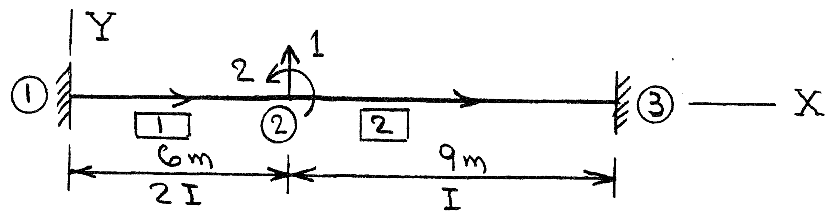
$$\{F_1\}=\{Q_1\}=[k_1]\{u_1\}+\{Q_{f1}\}=\begin{bmatrix}5.73\text{ k}\\50.6\text{ k-ft}\\12.27\text{ k}\\-58.8\text{ k-ft}\end{bmatrix}$$

$$\{F_2\}=\{Q_2\}=\begin{bmatrix}6.06\text{ k}\\58.8\text{ k-ft}\\3.94\text{ k}\\-26.9\text{ k-ft}\end{bmatrix}$$



Reactions

18.5



Member 1:

$$[K_1] = EI \begin{bmatrix} 0.111 & 0.333 & -0.111 & 0.333 \\ & 1.333 & -0.333 & 0.667 \\ & \text{sym.} & 0.111 & -0.333 \\ & & & 1.333 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix}$$

(column code numbers: 0, 0, 1, 2)

Member 2:

$$[K_2] = EI \begin{bmatrix} 0.0165 & 0.0741 & -0.0165 & 0.0741 \\ & 0.444 & -0.0741 & 0.222 \\ & \text{sym.} & 0.0165 & -0.0741 \\ & & & 0.444 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

(column code numbers: 1, 2, 0, 0)

Structure stiffness matrix:

$$[S] = EI \begin{bmatrix} 0.1275 & -0.2589 \\ -0.2589 & 1.777 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Joint load vector:

$$\{P\} = \begin{bmatrix} -150 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

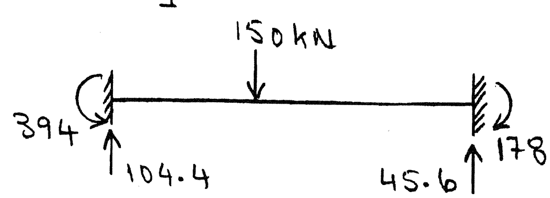
Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain: $\{d\} = \frac{1}{EI} \begin{bmatrix} -1671 \\ -243.4 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$

Member forces:

$$\{u_1\} = \{v_1\} = \frac{1}{EI} \begin{bmatrix} 0 \\ 0 \\ -1671 \\ -243.4 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix} \; ; \quad \{u_2\} = \{v_2\} = \frac{1}{EI} \begin{bmatrix} -1671 \\ -243.4 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

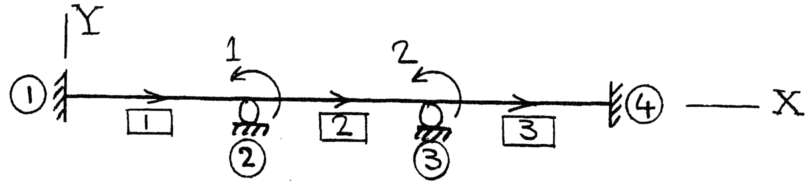
$$\{F_1\} = \{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} 104.4 \text{ kN} \\ 394 \text{ kN.m} \\ -104.4 \text{ kN} \\ 232 \text{ kN.m} \end{bmatrix}$$

$$\{F_2\} = \{Q_2\} = \begin{bmatrix} -45.6 \text{ kN} \\ -232 \text{ kN.m} \\ 45.6 \text{ kN} \\ -178 \text{ kN.m} \end{bmatrix}$$



Reactions

18.6



Code numbers: Member 3 → 0, 2, 0, 0; Member 2 → 0, 1, 0, 2; Member 1 → 0, 0, 0, 1

$$[K_1]=[K_2]=[K_3]=\frac{EI}{L^3}\begin{bmatrix}12 & 108 & -12 & 108\\ & 1296 & -108 & 648\\ & & 12 & -108\\ \text{Sym.} & & & 1296\end{bmatrix}$$

$$\{F_{f1}\}=\{Q_{f1}\}=\begin{bmatrix}27\\81\\27\\-81\end{bmatrix}\begin{matrix}0\\0\\0\\1\end{matrix}\;;\quad \{F_{f3}\}=\{Q_{f3}\}=\begin{bmatrix}13.5\\40.5\\13.5\\-40.5\end{bmatrix}\begin{matrix}0\\2\\0\\0\end{matrix}$$

Structure stiffness matrix:

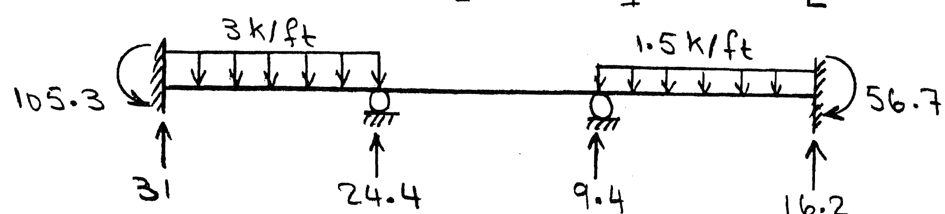
$$[S]=\frac{EI}{L^3}\begin{bmatrix}2592 & 648\\ 648 & 2592\end{bmatrix}\begin{matrix}1\\2\end{matrix}\qquad \{P_f\}=\begin{bmatrix}-81\\40.5\end{bmatrix}$$

Joint displacements: By solving the equations $-\{P_f\}=[S]\{d\}$, we obtain : $\{d\}=\frac{L^3}{EI}\begin{bmatrix}0.0375\\-0.025\end{bmatrix}\begin{matrix}1\\2\end{matrix}$

Member forces:

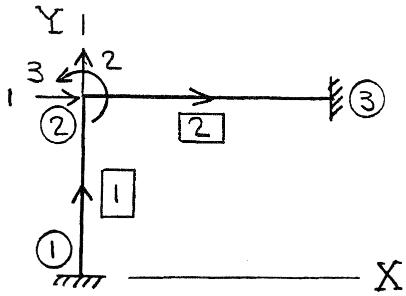
$$\{u_1\}=\frac{L^3}{EI}\begin{bmatrix}0\\0\\0\\0.0375\end{bmatrix};\quad \{u_2\}=\frac{L^3}{EI}\begin{bmatrix}0\\0.0375\\0\\-0.025\end{bmatrix};\quad \{u_3\}=\frac{L^3}{EI}\begin{bmatrix}0\\-0.025\\0\\0\end{bmatrix}$$

$$\{Q_1\}=\begin{bmatrix}31\text{ k}\\105.3\text{ k-ft}\\23\text{ k}\\-32.4\text{ k-ft}\end{bmatrix};\quad \{Q_2\}=\begin{bmatrix}1.4\text{ k}\\32.4\text{ k-ft}\\-1.4\text{ k}\\-8.1\text{ k-ft}\end{bmatrix};\quad \{Q_3\}=\begin{bmatrix}10.8\text{ k}\\8.1\text{ k-ft}\\16.2\text{ k}\\-56.7\text{ k-ft}\end{bmatrix}$$



Reactions

18.7



Member 1: $L = 15\text{ ft};\ \cos\theta = 0;\ \sin\theta = 1$

$$[k_1] = EI \times 10^{-3}\begin{bmatrix} 115.2 & 0 & 0 & -115.2 & 0 & 0 \\ & 3.556 & 26.67 & 0 & -3.556 & 26.67 \\ & & 266.7 & 0 & -26.67 & 133.3 \\ & \text{Sym.} & & 115.2 & 0 & 0 \\ & & & & 3.556 & -26.67 \\ & & & & & 266.7 \end{bmatrix}$$

$$\{Q_{f1}\} = \begin{bmatrix} 0 \\ 15 \\ 37.5 \\ 0 \\ 15 \\ -37.5 \end{bmatrix};\quad [T_1] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};\quad \{F_{f1}\} = \begin{bmatrix} -15 \\ 0 \\ 37.5 \\ -15 \\ 0 \\ -37.5 \end{bmatrix}\begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$

$$[K_1] = [T_1]^T[k_1][T_1] = EI \times 10^{-3}\begin{matrix} \quad 0 & \quad 0 & \quad 0 & \quad 1 & \quad 2 & \quad 3 \end{matrix}$$

$$\begin{bmatrix} 3.556 & 0 & -26.67 & -3.556 & 0 & -26.67 \\ & 115.2 & 0 & 0 & -115.2 & 0 \\ & & 266.7 & 26.67 & 0 & 133.3 \\ & \text{Sym.} & & 3.556 & 0 & 26.67 \\ & & & & 115.2 & 0 \\ & & & & & 266.7 \end{bmatrix}\begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$

Member 2: $L = 20\text{ ft};\ \cos\theta = 1;\ \sin\theta = 0$

$$[k_2] = [K_2] = EI \times 10^{-3}\begin{matrix} \quad 1 & \quad 2 & \quad 3 & \quad 0 & \quad 0 & \quad 0 \end{matrix}$$

$$\begin{bmatrix} 86.4 & 0 & 0 & -86.4 & 0 & 0 \\ & 1.5 & 15 & 0 & -1.5 & 15 \\ & & 200 & 0 & -15 & 100 \\ & & & 86.4 & 0 & 0 \\ & \text{Sym.} & & & 1.5 & -15 \\ & & & & & 200 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix}$$

$$\{F_{f2}\} = \{Q_{f2}\} = \begin{bmatrix} 0 \\ 10 \\ 50 \\ 0 \\ 10 \\ -50 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix}$$

18.7 (Contd.) <u>Structure matrices:</u>

$$[S] = EI \times 10^{-3} \begin{bmatrix} 89.96 & 0 & 26.67 \\ 0 & 116.7 & 15 \\ 26.67 & 15 & 466.7 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \quad (\text{columns } 1,\ 2,\ 3) \; ; \quad \{P_f\} = \begin{bmatrix} -15 \\ 10 \\ 12.5 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

<u>Joint displacements:</u> By solving the equations $-\{P_f\} = [S]\{d\}$, we obtain:

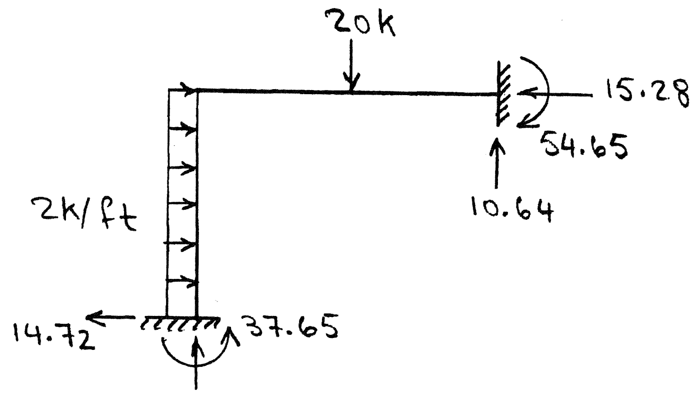
$$\{d\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0.1769 \\ -0.08129 \\ -0.03427 \end{bmatrix}$$

<u>Member forces:</u>

$$\{v_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1769 \\ -0.08129 \\ -0.03427 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \; ; \quad \{u_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.08129 \\ -0.1769 \\ -0.03427 \end{bmatrix}$$

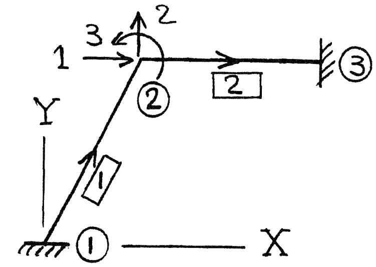
$$\{Q_1\} = \begin{bmatrix} 9.36 \text{ k} \\ 14.72 \text{ k} \\ 37.65 \text{ k-ft} \\ -9.36 \text{ k} \\ 15.29 \text{ k} \\ -41.92 \text{ k-ft} \end{bmatrix} \; ; \quad \{F_1\} = \begin{bmatrix} -14.72 \text{ k} \\ 9.36 \text{ k} \\ 37.65 \text{ k-ft} \\ -15.29 \text{ k} \\ -9.36 \text{ k} \\ -41.92 \text{ k-ft} \end{bmatrix}$$

$$\{u_2\} = \{v_2\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0.1769 \\ -0.08129 \\ -0.03427 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix} \; ; \quad \{F_2\} = \{Q_2\} = \begin{bmatrix} 15.28 \text{ k} \\ 9.36 \text{ k} \\ 41.92 \text{ k-ft} \\ -15.28 \text{ k} \\ 10.64 \text{ k} \\ -54.65 \text{ k-ft} \end{bmatrix}$$



<u>Reactions</u>

18.8



Member 1: $L = 11.18\text{ m}$; $\cos\theta = 0.447$; $\sin\theta = 0.894$

$$[k_1] = \begin{bmatrix} 71556 & 0 & 0 & -71556 & 0 & 0 \\ & 687 & 3840 & 0 & -687 & 3840 \\ & & 28623 & 0 & -3840 & 14311 \\ & & & 71556 & 0 & 0 \\ & \text{Sym.} & & & 687 & -3840 \\ & & & & & 28623 \end{bmatrix}$$

$$[T_1] = \begin{bmatrix} 0.447 & 0.894 & 0 & 0 & 0 & 0 \\ -0.894 & 0.447 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.447 & 0.894 & 0 \\ 0 & 0 & 0 & -0.894 & 0.447 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K_1] = [T_1]^T[k_1][T_1] = \begin{array}{c} \begin{matrix} 0 & \quad 0 & \quad 0 & \quad 1 & \quad 2 & \quad 3 \end{matrix} \\ \left[\begin{matrix} 14847 & 28320 & -3433 & -14847 & -28320 & -3433 \\ & 57327 & 1716 & -28320 & -57327 & 1716 \\ & & 28623 & 3433 & -1716 & 14311 \\ & & & 14847 & 28320 & 3433 \\ & \text{Sym.} & & & 57327 & -1716 \\ & & & & & 28623 \end{matrix}\right] \end{array} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$

Member 2: $L = 10\text{ m}$; $\cos\theta = 1$; $\sin\theta = 0$

$$[k_2] = [K_2] = \begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 & \quad 0 & \quad 0 & \quad 0 \end{matrix} \\ \left[\begin{matrix} 80000 & 0 & 0 & -80000 & 0 & 0 \\ & 960 & 4800 & 0 & -960 & 4800 \\ & & 32000 & 0 & -4800 & 16000 \\ & & & 80000 & 0 & 0 \\ & \text{Sym.} & & & 960 & -4800 \\ & & & & & 32000 \end{matrix}\right] \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix}$$

Structure matrices:

$$[S] = \begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 \end{matrix} \\ \left[\begin{matrix} 94847 & 28320 & 3433 \\ 28320 & 58287 & 3084 \\ 3433 & 3084 & 60623 \end{matrix}\right] \end{array} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} ; \quad \{P\} = \begin{bmatrix} 0 \\ 0 \\ -150 \end{bmatrix}$$

18.8 (contd.) Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain:

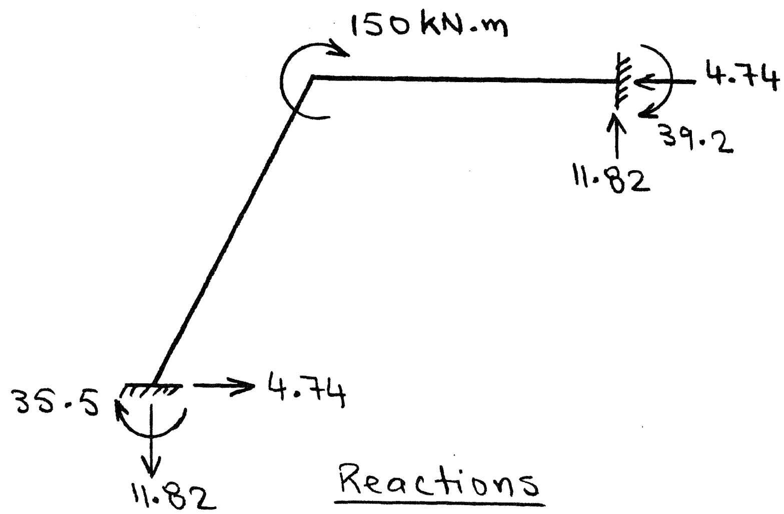
$$\{d\} = 10^{-4} \begin{bmatrix} 0.5924 \text{ m} \\ 1.0259 \text{ m} \\ -24.829 \text{ rad.} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Member forces:

$$\{v_1\} = 10^{-4} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5924 \\ 1.0259 \\ -24.829 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} ; \quad \{u_1\} = 10^{-4} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.182 \\ -0.07103 \\ -24.829 \end{bmatrix}$$

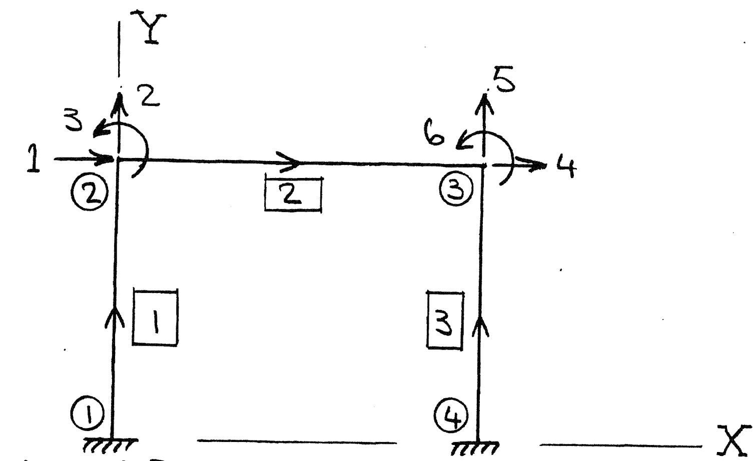
$$\{Q_1\} = \begin{bmatrix} -8.46 \text{ kN} \\ -9.53 \text{ kN} \\ -35.5 \text{ kN.m} \\ 8.46 \text{ kN} \\ 9.53 \text{ kN} \\ -71 \text{ kN.m} \end{bmatrix} ; \quad \{F_1\} = \begin{bmatrix} 4.74 \text{ kN} \\ -11.82 \text{ kN} \\ -35.5 \text{ kN.m} \\ -4.74 \text{ kN} \\ 11.82 \text{ kN} \\ -71 \text{ kN.m} \end{bmatrix}$$

$$\{u_2\} = \{v_2\} = 10^{-4} \begin{bmatrix} 0.5924 \\ 1.0259 \\ -24.829 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix} ; \quad \{F_2\} = \{Q_2\} = \begin{bmatrix} 4.74 \text{ kN} \\ -11.82 \text{ kN} \\ -79 \text{ kN.m} \\ -4.74 \text{ kN} \\ 11.82 \text{ kN} \\ -39.2 \text{ kN.m} \end{bmatrix}$$



Reactions

18.9



<u>Members 1 and 3:</u> $L = 30\text{ ft};\ \cos\theta = 0;\ \sin\theta = 1$

$$[k_1] = [k_3] = EI \times 10^{-3} \begin{bmatrix} 80 & 0 & 0 & -80 & 0 & 0 \\ & 0.444 & 6.67 & 0 & -0.444 & 6.67 \\ & & 133.3 & 0 & -6.67 & 66.67 \\ & \text{Sym.} & & 80 & 0 & 0 \\ & & & & 0.444 & -6.67 \\ & & & & & 133.3 \end{bmatrix}$$

$$[T_1] = [T_3] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Member 3 → 0 0 0 4 5 6

Member 1 → 0 0 0 1 2 3

$$[K_1] = [K_3] = EI \times 10^{-3} \begin{bmatrix} 0.444 & 0 & -6.67 & -0.444 & 0 & -6.67 \\ & 80 & 0 & 0 & -80 & 0 \\ & & 133.3 & 6.67 & 0 & 66.67 \\ & & & 0.444 & 0 & 6.67 \\ & \text{Sym.} & & & 80 & 0 \\ & & & & & 133.3 \end{bmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{matrix}$$

<u>Member 2:</u> $L = 40\text{ ft};\ \cos\theta = 1;\ \sin\theta = 0$

$$[k_2] = [K_2] = EI \times 10^{-3} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \begin{bmatrix} 120 & 0 & 0 & -120 & 0 & 0 \\ & 0.375 & 7.5 & 0 & -0.375 & 7.5 \\ & & 200 & 0 & -7.5 & 100 \\ & & & 120 & 0 & 0 \\ & \text{Sym.} & & & 0.375 & -7.5 \\ & & & & & 200 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

18.9 (Contd.)

$$\{F_{f2}\} = \{Q_{f2}\} = \begin{bmatrix} 0 \\ 30 \\ 200 \\ 0 \\ 30 \\ -200 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Structure matrices:

$$[S] = EI \times 10^{-3} \begin{bmatrix} 120.444 & 0 & 6.67 & -120 & 0 & 0 \\ & 80.375 & 7.5 & 0 & -0.375 & 7.5 \\ & & 333.3 & 0 & -7.5 & 100 \\ & & & 120.444 & 0 & 6.67 \\ & \text{Sym.} & & & 80.375 & -7.5 \\ & & & & & 333.33 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

(columns 1 2 3 4 5 6)

$$\{P\} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \{P_f\} = \{F_{f2}\}; \quad \{P\} - \{P_f\} = \begin{bmatrix} 20 \\ -30 \\ -200 \\ 0 \\ -30 \\ 200 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Joint displacements: By solving the equations $\{P\} - \{P_f\} = [S]\{d\}$, we obtain:

$$\{d\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 29.4204 \\ -0.2906 \\ -1.314 \\ 29.289 \\ -0.4594 \\ 0.4044 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Member forces:

$$\{v_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.4204 \\ -0.2906 \\ -1.314 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \qquad \{u_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.2906 \\ -29.4204 \\ -1.314 \end{bmatrix}$$

18.9 (contd.)

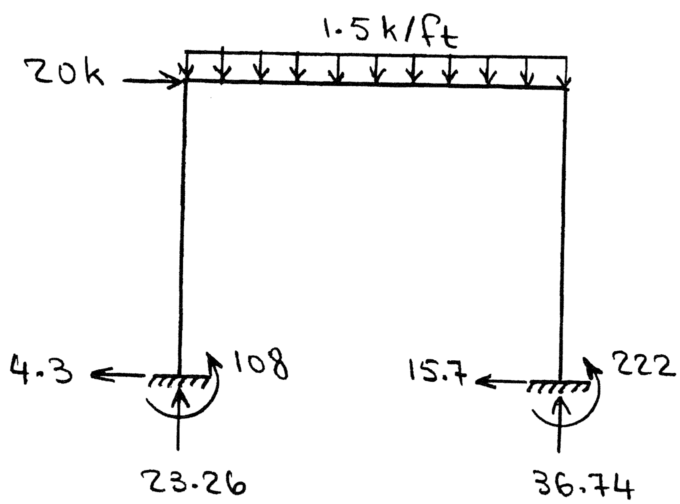
$$\{Q_1\} = \begin{bmatrix} 23.26 \text{ k} \\ 4.3 \text{ k} \\ 108 \text{ k-ft} \\ -23.26 \text{ k} \\ -4.3 \text{ k} \\ 21 \text{ k-ft} \end{bmatrix}$$

$$\{u_2\} = \{v_2\} = \{d\};$$

$$\{Q_2\} = \begin{bmatrix} 15.7 \text{ k} \\ 23.26 \text{ k} \\ -21 \text{ k-ft} \\ -15.7 \text{ k} \\ 36.74 \text{ k} \\ -249 \text{ k-ft} \end{bmatrix}$$

$$\{v_3\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.289 \\ -0.4594 \\ 0.4044 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{matrix} ; \quad \{u_3\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.4594 \\ -29.289 \\ 0.4044 \end{bmatrix}$$

$$\{Q_3\} = \begin{bmatrix} 36.74 \text{ k} \\ 15.7 \text{ k} \\ 222 \text{ k-ft} \\ -36.74 \text{ k} \\ -15.7 \text{ k} \\ 249 \text{ k-ft} \end{bmatrix}$$



Reactions

Appendix B
Review of Matrix Algebra
&
Appendix C
Computer Software

APPENDIX B

B.1

$$[C] = [A] + 3[B] = \begin{bmatrix} 18 & -11 & 18 \\ -11 & 19 & 28 \\ 18 & 28 & 4 \end{bmatrix}$$

B.2

$$[C] = 2[A] - [B] = \begin{bmatrix} 7 & 8 \\ 11 & 7 \\ 1 & 0 \end{bmatrix}$$

B.3

$$[C] = \begin{bmatrix} -6 & 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} = -12 - 4 - 10 = \underline{-26}$$

$$[D] = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \begin{bmatrix} -6 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -12 & 8 & -4 \\ 6 & -4 & 2 \\ -30 & 20 & -10 \end{bmatrix}$$

B.4

$$[C] = \begin{bmatrix} 2 & -5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -26 & 3 \\ 27 & -17 \end{bmatrix}$$

$$[D] = \begin{bmatrix} -3 & 4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} -26 & 27 \\ 3 & -17 \end{bmatrix}$$

B.5

$$[A][B] = \begin{bmatrix} 8 & -2 & 5 \\ 1 & -4 & 3 \\ 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & -5 \\ 7 & 0 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} (8-14) & (-40-15) \\ (1-28) & (-5-9) \\ (2) & (-10-18) \end{bmatrix} = \begin{bmatrix} -6 & -55 \\ -27 & -14 \\ 2 & -28 \end{bmatrix}$$

$$[AB]^T = \begin{bmatrix} -6 & -27 & 2 \\ -55 & -14 & -28 \end{bmatrix} \qquad (1)$$

$$[B]^T[A]^T = \begin{bmatrix} 1 & 7 & 0 \\ -5 & 0 & -3 \end{bmatrix} \begin{bmatrix} 8 & 1 & 2 \\ -2 & -4 & 0 \\ 5 & 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} (8-14) & (1-28) & (2) \\ (-40-15) & (-5-9) & (-10-18) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -27 & 2 \\ -55 & -14 & -28 \end{bmatrix} \qquad (2)$$

From Eqs. (1) and (2) we can see that:

$$\underline{[AB]^T = [B]^T[A]^T}$$

B-6

$$\left[\begin{array}{ccc|c} 2 & 5 & -1 & 15 \\ 5 & -1 & 3 & 27 \\ -1 & 3 & 4 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2.5 & -0.5 & 7.5 \\ 0 & -13.5 & 5.5 & -10.5 \\ 0 & 5.5 & 3.5 & 21.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.518 & 5.555 \\ 0 & 1 & -0.407 & 0.778 \\ 0 & 0 & 5.739 & 17.221 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Thus,

$$x_1 = 4 \qquad x_2 = 2 \qquad x_3 = 3$$

B.7

$$\left[\begin{array}{ccc|c} -12 & -3 & 6 & 45 \\ 5 & 2 & -4 & -9 \\ 10 & 1 & -7 & -32 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0.25 & -0.5 & -3.75 \\ 0 & 0.75 & -1.5 & 9.75 \\ 0 & -1.5 & -2 & 5.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & -2 & 13 \\ 0 & 0 & -5 & 25 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

Thus,

$$x_1 = -7 \quad x_2 = 3 \quad x_3 = -5$$

B.8

$$\left[\begin{array}{cccc|c} 5 & -2 & 6 & 0 & 0 \\ -2 & 4 & 1 & 3 & 18 \\ 6 & 1 & 6 & 8 & -29 \\ 0 & 3 & 8 & 7 & 11 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -0.4 & 1.2 & 0 & 0 \\ 0 & 3.2 & 3.4 & 3 & 18 \\ 0 & 3.4 & -1.2 & 8 & -29 \\ 0 & 3 & 8 & 7 & 11 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1.625 & 0.375 & 2.25 \\ 0 & 1 & 1.063 & 0.938 & 5.625 \\ 0 & 0 & -4.814 & 4.811 & -48.125 \\ 0 & 0 & 4.811 & 4.186 & -5.875 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -14 \\ 0 & 1 & 0 & 2 & -5 \\ 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 9 & -54 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right]$$

Thus,

$x_1 = -2 \quad x_2 = 7 \quad x_3 = 4 \quad x_4 = -6$

B.9

$$\left[\begin{array}{ccc|ccc} 4 & -3 & -1 & 1 & 0 & 0 \\ -2 & 5 & 1 & 0 & 1 & 0 \\ 6 & -4 & -5 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -0.75 & -0.25 & 0.25 & 0 & 0 \\ 0 & 3.5 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0.5 & -3.5 & -1.5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -0.143 & 0.357 & 0.215 & 0 \\ 0 & 1 & 0.143 & 0.143 & 0.286 & 0 \\ 0 & 0 & -3.572 & -1.572 & -0.143 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.42 & 0.22 & -0.04 \\ 0 & 1 & 0 & 0.08 & 0.28 & 0.04 \\ 0 & 0 & 1 & 0.44 & 0.04 & -0.28 \end{array}\right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} 0.42 & 0.22 & -0.04 \\ 0.08 & 0.28 & 0.04 \\ 0.44 & 0.04 & -0.28 \end{bmatrix}$$

B.10

$$\left[\begin{array}{cccc|cccc} 4 & 2 & 0 & -3 & 1 & 0 & 0 & 0 \\ 2 & 3 & -4 & 0 & 0 & 1 & 0 & 0 \\ 0 & -4 & 2 & -1 & 0 & 0 & 1 & 0 \\ -3 & 0 & -1 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0.5 & 0 & -0.75 & 0.25 & 0 & 0 & 0 \\ 0 & 2 & -4 & 1.5 & -0.5 & 1 & 0 & 0 \\ 0 & -4 & 2 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1.5 & -1 & 2.75 & 0.75 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & -1.125 & 0.375 & -0.25 & 0 & 0 \\ 0 & 1 & -2 & 0.75 & -0.25 & 0.5 & 0 & 0 \\ 0 & 0 & -6 & 2 & -1 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1.625 & 1.125 & -0.75 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -0.7917 & 0.2083 & 0.0833 & 0.1667 & 0 \\ 0 & 1 & 0 & 0.0833 & 0.0833 & -0.1667 & -0.3333 & 0 \\ 0 & 0 & 1 & -0.3333 & 0.1667 & -0.3333 & -0.1667 & 0 \\ 0 & 0 & 0 & 2.2917 & 0.7917 & -0.0833 & 0.3333 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0.4818 & 0.0545 & 0.2818 & 0.3455 \\ 0 & 1 & 0 & 0 & 0.0545 & -0.1636 & -0.3455 & -0.0364 \\ 0 & 0 & 1 & 0 & 0.2818 & -0.3455 & -0.1182 & 0.1455 \\ 0 & 0 & 0 & 1 & 0.3455 & -0.0364 & 0.1455 & 0.4364 \end{array}\right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} 0.4818 & 0.0545 & 0.2818 & 0.3455 \\ 0.0545 & -0.1636 & -0.3455 & -0.0364 \\ 0.2818 & -0.3455 & -0.1182 & 0.1455 \\ 0.3455 & -0.0364 & 0.1455 & 0.4364 \end{bmatrix}$$

APPENDIX C

C.1 $\frac{L}{360} = \frac{80}{360} = 0.222 \text{ ft} = 2.67 \text{ in.}$

(a) $A = 9.12 \text{ in}^2$

(b) $A = 6.33 \text{ in}^2$

(c) $A = 8.66 \text{ in}^2$

C.2 $\frac{L}{360} = \frac{24}{360} = 0.0667\text{ m}$

(a) $\underline{A = 8470\text{ mm}^2}$

(b) $\underline{A = 7620\text{ mm}^2}$

(c) $\underline{A = 6860\text{ mm}^2}$

C.3 $I = 1257 \text{ in}^4$