Flexure I

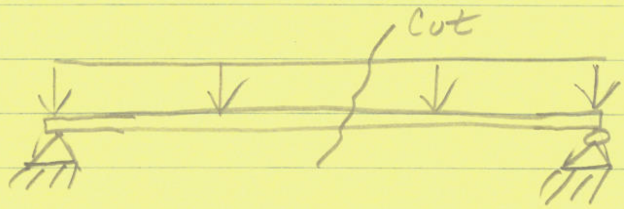
How a

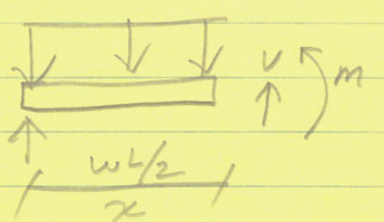
Internal Forces

Flexure II

Chap 2

How a beam works





V = Shear due to applied Load

$$V = \frac{wL}{L} - wx$$

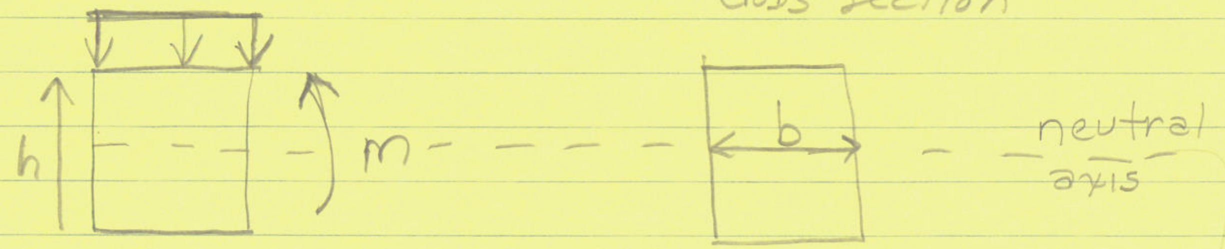
M = Moment due to applied Load

$$M = \frac{wL}{2}x (-) \frac{wx^2}{2}$$

But how does a beam resist moment?

Internal Moment

Cross Section



continued below

Assumptions: - plane section remain plane

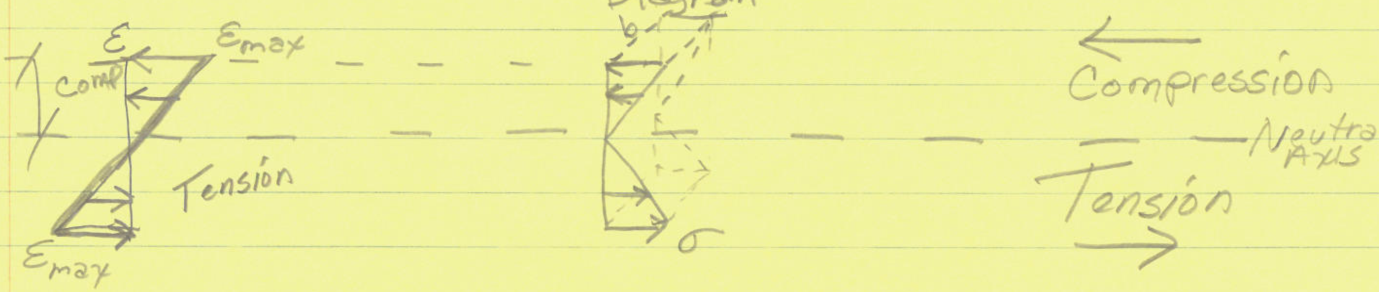
- linear elastic

- homogeneous

Strain Diagram

shear Diagram

Forces



$$\sigma = \frac{My}{I}$$

Resultant Forces

$$C = \frac{1}{2}\sigma_{max}(b)\left(\frac{h}{2}\right) = \frac{\sigma_{max}\,bh}{4}$$

$T = C$ (for equalibrium)

Internal Moment

$m = C \cdot$ Moment Arm

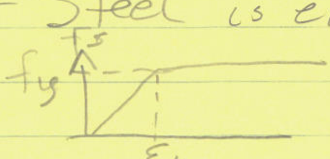
$$m = \frac{\sigma_{max}\,bh}{4}\left(\frac{2}{3}h\right) = \frac{2bh^2}{12}\,\sigma_{max}\,\frac{(h/2)}{(h/2)}$$

$$m = \frac{\sigma_{max}\,\frac{bh^3}{12}}{h/2} = \frac{\sigma_{max}\,I}{c}$$

How does RC member behave

Assumptions : Plane sections remain Plane

- ε_{cu} (ultimate) $= 0.003$ [ACI 10.2.3]
- Ignore tensil strength of concrete [ACI 10.2.3]
- Steel is elasto-plastic [10.2.4]



If $\varepsilon_s \le \varepsilon_y$: $f_s = E\varepsilon_s$

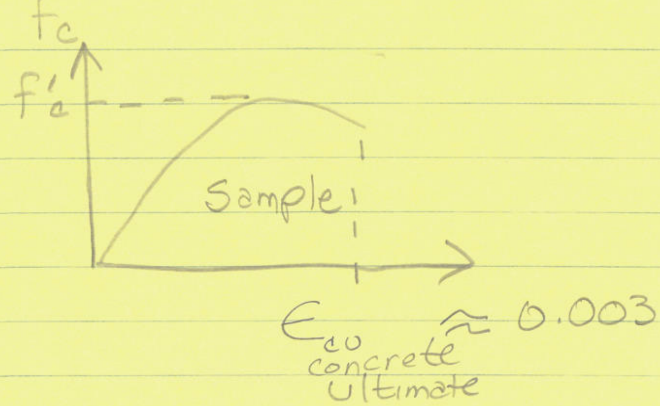
If $\varepsilon_s > \varepsilon_y$: $f_s = f_y$

Assumptions contineed

- perfect bond between steel & concrete
- Stress can be obtained from a stress-strain curve
- The rectangular stress block not 

Strain sub t ϵ_t

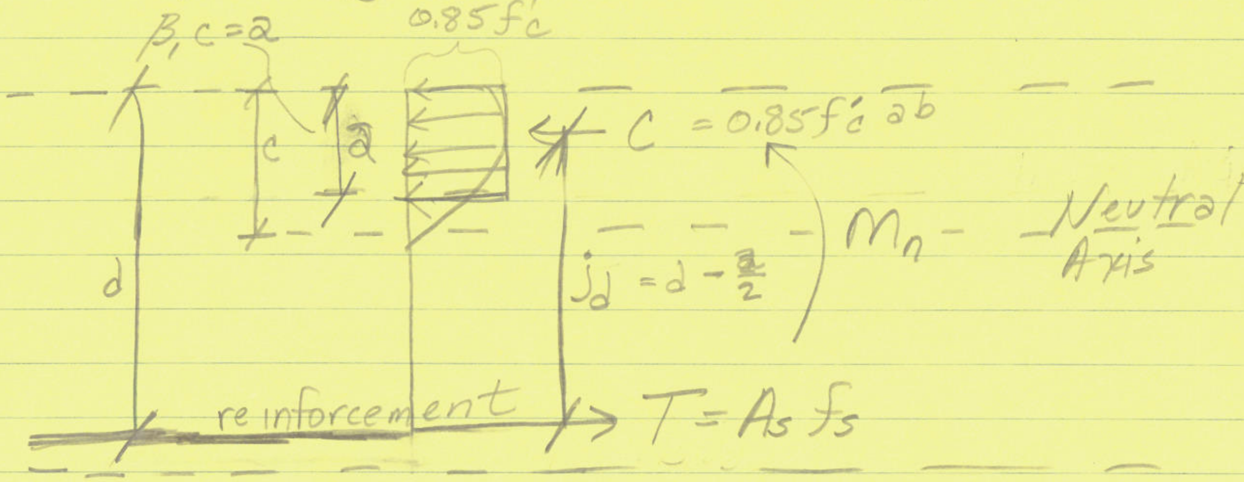
Concrete Stress - Strain Curves



Modeling the Compressive Stress Distribution

Equivalent Rectangular Stress Block (R58)

Whitney stress block [10.2.7]

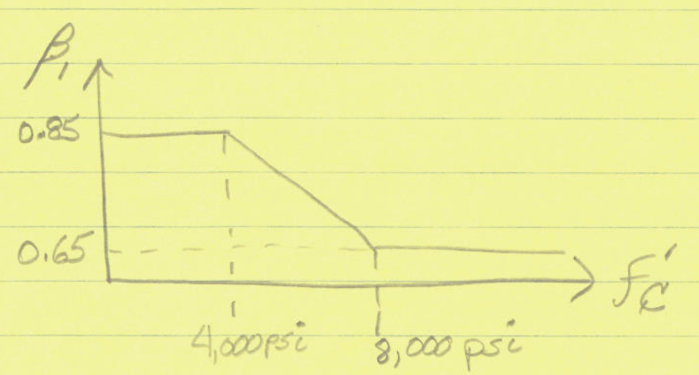


$a = \beta_1 c$

$jd = d - a/2$

β_1 factor [10.2.7]

relates "a" & "c" based on strength



If $f'_c \le 4000$ psi, $\beta_1 = 0.85$

If $f'_c > 8000$ psi, $\beta_1 = 0.65$

If $4{,}000 < f'_c < 8{,}000$

$$\beta_1 = 0.85 - \left[\frac{(f'_c - 4000)\, 0.05}{1000}\right]$$

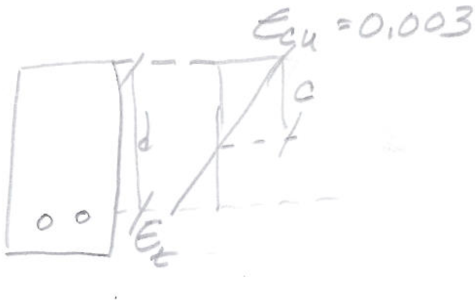
Normal Moment Capacity

$$\left.\begin{array}{l} C = 0.85 f'_c (a)(b) \\ T = A_s f_s \end{array}\right\} T = C$$

since $T=C$ $\therefore$ $a = \dfrac{A_s f_s}{0.85 f'_c (b)}$

$$M_n = jd(T) = \left(d - \frac{a}{2}\right) A_s f_s$$

$$M_n = jd(C) = \left(d - \frac{a}{2}\right)(0.85 f'_c\, a\, b)$$



Example Problem: RC Rectangular Beam
ENCE 4359 Structural Concrete Design
Dr. Lamanna

Compute the nominal moment capacity, M_n, of a beam having b = 10 in., d = 20 in., A_s = 4.74 in^2 (Four # 8 Bars), f'_c = 3,000 psi, and f_y = 60,000 psi.

① Assume Steel yields

$\therefore f_s = f_y$

② Calculate a, c, β_1

$$a = \frac{A_s f_y}{(0.85) f'_c (b)} = \frac{(4.74)(60{,}000\ \#/in^2)}{(0.85)(30{,}000\ \#/in^2)(10'')} = 11.2''$$

$$f'_c = 3\,ksi \le 4\,ksi \;\therefore\; \beta_1 = 0.85$$

$$c = a/\beta_1 = \frac{11.2''}{0.85} = 13.8''$$

③ Check Assumption

$$\frac{\epsilon_s}{d-c} = \frac{0.003}{c} \Rightarrow \epsilon_s = \frac{0.003\,(20-13.18)}{13.18} = 0.00155$$

$$\epsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29{,}000{,}000} = 0.00207$$

$\epsilon_s < \epsilon_y \;\therefore$ Steel does not yield

④ Start Over

$$\left.\begin{array}{l} C = 0.85 f'_c\, a\, b \\ T = A_s f_s \end{array}\right\} C = T \;\therefore f_s = E_s \epsilon_s \qquad c = \frac{a}{\beta_1}$$

$$\epsilon_s = \frac{0.003\,(d-c)}{c} = \frac{0.003\,(d - a/\beta_1)}{a/\beta_1} = \frac{0.003\,(d\beta_1 - a)}{a}$$

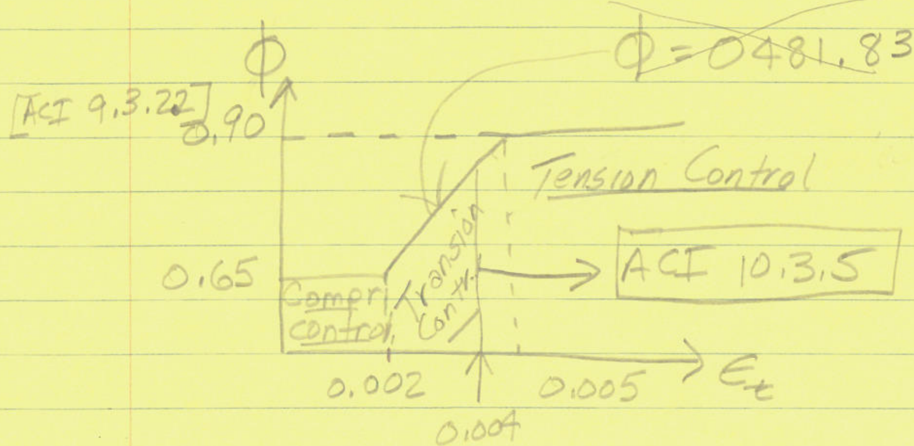
Substitute $f_s = E_s \epsilon_s$ into T

$$T = A_s E_s f_s$$

over

Straight Resistance Factor

Depends on strain in extreme tensil steel



SEE R9.3.2.2

$\boxed{\text{Flexure III}}$

Failure Modes
φ Factor
Example

$\boxed{\text{Flexure IV}}$

Reinforcement Ratio
Rules of Thumb
Example

$\boxed{\text{Homework}}$

Reinforcement Ratio, ρ (row)

= relative amount of reinforcement

$$\rho = \frac{A_s}{bd}$$

reinforcement ratio @ balanced condition

$$\rho_{bal} \ \frac{0.85 f'_c}{f_y} \beta_1 \frac{87{,}000}{(87{,}000) f_y}$$

Maximum Reinforcement Ratio

$\rho_{max} = 0.75 \rho_b$ [10.3.3]

design aid to ensure $\epsilon_t > 0.005$

Rules of Thumb

#1 depth of beam $= \frac{l_n}{16}$ l_n = distance between support faces

#2 $\frac{b}{d} = r$ $(0.3 \le r \le 0.6)$ for efficiency

#3 $h \approx d + 2.5$ (1 Layer of steel)

#4 $\gamma \approx 150$ #/ft^3

#5 beams should be tension Controlled

Rule of Thumb continued

#6 $jd = \left(d - \frac{a}{2}\right)$ $(0.85d \le j \le 0.9d)$

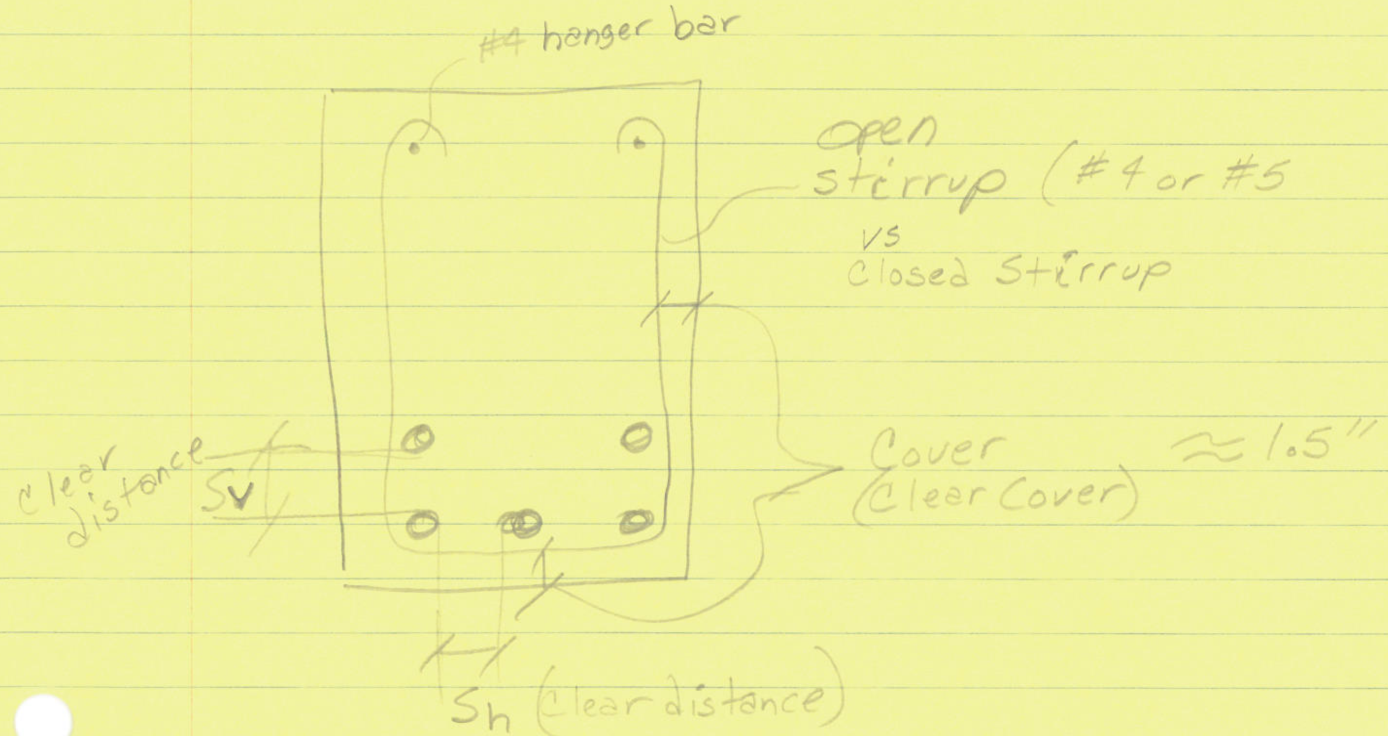
Code check

Minimum A_s [10.5.1]

$$A_{min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \ge 200 \frac{b_w d}{f_y}$$

#7 lots of trial & error in steel selection

#8 If mixing bar sizes; Quality Control becomes big issue



Example Problem: RC Rectangular Beam Design
ENCE 4359 Structural Concrete Design
Dr. Lamanna

Select an economical rectangular beam size and select bars using the ACI strength design method. The beam is a simply supported span of 40 ft and it is to carry a live load of 1.3 kips/ft and a dead load of 0.8 kip/ft (not including self-weight). Without actually checking deflection, use a reinforcement ratio ρ such that excessive deflection is unlikely. Use $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi.