ENME 3770 Engineering Thermodynamics
Homework # 3, Summer 2009

Jobaidur Khan

Pr 1. A rigid tank contains 0.5 kg of oxygen (O_2) initially at 30 bar and 200K. The gas is cooled and the pressure drops to 20 bar. Determine the volume of the tank, in m³, and the final temperature, in K. (10 points)

Sol. From Table A-1, for O_2, we get, $T_c = 154K$, $P_c = 50.5bar$

$$T_{R1} = \frac{T_1}{T_c} = \frac{200}{154} = 1.2987, \quad P_{R1} = \frac{P_1}{P_c} = \frac{30}{50.5} = 0.594$$

From Fig A-1, we get, $Z_1 = 0.92$. So, Specific volume,

$$v_1 = Z_1\frac{RT_1}{P_1} = Z_1\frac{\overline{R}T_1}{MP_1} = 0.92\times\frac{\left(8314\frac{N.m}{kmol.K}\right)\times(200K)}{\left(32\frac{kg}{kmol}\right)\times\left(30\times10^5\frac{N}{m^2}\right)} = 0.0159m^3/kg$$

So, the volume of the tank, $V_1 = mv_1 = 0.5 \times 0.0159 = 0.008m^3$.

Since both mass and volume remain constant, the water vapor cools at constant specific volume, and thus at constant v_R. Using the value of specific volume determined earlier, the constant v_R is,

$$v_R = \frac{v_1P_c}{RT_c} = \frac{v_1P_c}{RT_c} = \frac{\left(0.0159\frac{m^3}{kg}\right)\times\left(50.5\times10^5\frac{N}{m^2}\right)}{\left(\frac{8314}{32}\frac{N.m}{kg.K}\right)\times(154K)} = 2.0$$

For State 2, $P_2 = 20$ bar, so, $P_{R2} = \frac{P_2}{P_c} = \frac{20}{50.5} = 0.396$

From Fig. A-1, $T_{R2} = 0.96$, So, $T_2 = T_{R2} \times T_c = 0.96 \times 154 = 147.84K$

3.97 5 kg of C_4H_{10} in a piston-cylinder assembly undergo a process from P_1=5MPa, T_1=500K to P_2=3MPa, T_2=450K during which the relationship between pressure and specific volume is Pv^n = Constant. Determine the work, in kJ. (10 points)

Sol. From Table A-1, for C_4H_{10}, we get, $T_c = 425$ K, $P_c = 3.8$ MPa, M = 58.12 kg/kmol

State 1, P_1=5MPa, T_1=500K, $P_{R1} = \frac{P_1}{P_c} = \frac{5}{3.8} = 1.3158$, $\quad T_{R1} = \frac{T_1}{T_c} = \frac{500}{425} = 1.1765$

From Fig A-2, we get, $Z_1 = 0.67$, So, Specific volume,

$$v_1 = Z_1\frac{RT_1}{P_1} = Z_1\frac{\overline{R}T_1}{MP_1} = 0.67\times\frac{\left(8314\frac{N.m}{kmol.K}\right)\times(500K)}{\left(58.12\frac{kg}{kmol}\right)\times\left(5\times10^6\frac{N}{m^2}\right)} = 0.00958427m^3/kg$$

State 2, P_2=3MPa, T_2=450K, $P_{R2} = \frac{P_2}{P_c} = \frac{3}{3.8} = 0.7894$, $\quad T_{R2} = \frac{T_2}{T_c} = \frac{450}{425} = 1.0588$

From Fig A-1, we get, $Z_2 = 0.74$, So, Specific volume,

$$v_2 = Z_2\frac{RT_2}{P_2} = Z_2\frac{\overline{R}T_2}{MP_2} = 0.74\times\frac{\left(8314\frac{N.m}{kmol.K}\right)\times(450K)}{\left(58.12\frac{kg}{kmol}\right)\times\left(3\times10^6\frac{N}{m^2}\right)} = 0.015878424m^3/kg$$

To find the polytropic index, n, we get,

$P_1v_1^n = P_2v_2^n \Rightarrow 5 \times 0.00958427^n = 3 \times 0.01587842^n \Rightarrow 1.657^n = 5/3 \Rightarrow n = 1.01186$

$$\text{Sp. work, } w = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{3\times10^6\times0.01587842 - 5\times10^6\times0.00958427}{-0.01186} = 24122J/kg$$

So, W = mw = 5 × 24122 = 120611J = 120.6kJ

Pr 3. By integrating Cp(T) obtained from Table A-21, determine the change in specific enthalpy, in kJ/kg, or CH_4 from T_1 = 320K, P_1 = 2 bar to T_2 = 800K, P_2 = 10 bar. (10 points)

Sol. For CH_4, from Table A-21, we get, $\frac{\overline{C}p}{R} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4$

Where, $\alpha = 3.826$, $\beta = -3.979\times10^{-3}$, $\gamma = 24.558\times10^{-6}$, $\delta = -22.733\times10^{-9}$, $\varepsilon = 6.963\times10^{-12}$.

$$\text{So, } h_2 - h_1 = \int_{T_1}^{T_2}\overline{C}pdT = \int_{T_1}^{T_2}\frac{\overline{R}}{M}\left(\alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4\right)dT$$

$$= \frac{\overline{R}}{M}\left[\alpha(T_2 - T_1) + \tfrac{1}{2}\beta(T_2^2 - T_1^2) + \tfrac{1}{3}\gamma(T_2^3 - T_1^3) + \tfrac{1}{4}\delta(T_2^4 - T_1^4) + \tfrac{1}{5}\varepsilon(T_2^5 - T_1^5)\right]$$

$$= \frac{8.314\text{ kJ/kmole.K}}{16\text{ kg/kmole}}\Big[3.826\times(800-320) - \tfrac{1}{2}\times3.979\times10^{-3}\times(800^2 - 320^2)$$
$$+ \tfrac{1}{3}\times24.558\times10^{-6}\times(800^3 - 320^3) - \tfrac{1}{4}\times22.733\times10^{-9}\times(800^4 - 320^4)$$
$$+ \tfrac{1}{5}\times6.963\times10^{-12}\times(800^5 - 320^5)\Big]K$$

$= 1498.23$ kJ/kg

Pr 4. Air enters a control volume operating at steady state at 1.05 bar, 300K, with a volumetric flow rate of 12m³/min and exits at 12 bar, 400K. Heat transfer occurs at a rate of 20kW from the control volume to the surroundings. Neglecting KE and PE effects, determine the power, in kW. (10 points)

Sol. State 1, P_1 = 1.05 bar, T_1 = 300K, State 2, P_2 = 12 bar, T_2 = 400K

Using Table A-22, h_1 = 300.19 kJ/kg, h_2 = 400.98 kJ/kg

$$\text{Using the ideal gas law, } v_1 = \frac{RT_1}{P_1} = \frac{\left(287\frac{J}{kg.K}\right)\times300K}{1.05\times10^5\frac{N}{m^2}} = 0.82\ m^3/kg$$

Vol. flow rate, q = 12m³/min = 0.2 m³/s,

So, mass flow rate, $\dot{m} = qv_1 = 0.2/0.82 = 0.244kg/s$

Using the energy rate balance for the state 1 and 2, we get, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$

$$\Rightarrow \dot{W}_{cv} = -20kW + \left(0.244\frac{kg}{s}\right)\times(300.19 - 400.98)\frac{kJ}{kg} = -44.58kW$$