ENME 3770 Engineering Thermodynamics
Homework # 6 Solution, Summer 2009

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Pr 1. N_2 initially occupying 0.5m³ at 1.0 bar, 20°C undergoes an internally reversible compression during which $PV^{1.30}$ = Constant to a final state where the temperature is 200°C. Determine assuming the ideal gas model (15 points)

(a) The pressure at final state, in bar.
(b) The work and heat transfer, each in kJ.
(c) The entropy change, in kJ/K.

Sol. (a) Given, $V_1 = 0.5m^3$, $P_1 = 1.0$ bar, $T_1 = 20°C = 293K$, $T_2 = 200°C = 473K$

Using polytropic relationship, $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} \Rightarrow \frac{P_2}{1} = \left(\frac{473}{293}\right)^{\frac{1.3}{0.3}} \Rightarrow \underline{P_2 = 7.967\text{bars}}$

(b) From Table A-23

At $P_1 = 1.0$ bar, $T_1 = 293K$, $u_1 = 217.3$ kJ/kg, $s_1 = 6.82$ kJ/kg.K

$P_2 = 7.967$ bar, $T_2 = 473K$, $u_2 = 351.76$ kJ/kg, $s_2 = 7.32$ kJ/kg.K

Mass of the air, $m = \frac{P_1 V_1}{RT_1} = \frac{100 \times 0.5}{\frac{8.314}{28} \times 293} = 0.5747\text{kg}$

Work, $W_{1-2} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n} = \frac{0.5747 \times \frac{8.314}{28} \times (473-293)}{1-1.3} = \underline{-102.39\text{kJ}}$

$\Delta U = m\,(u_2 - u_1) = 0.5747 \times (351.76 - 217.3) = 77.27$ kJ

Heat transfer, $Q = W_{12} + \Delta U = -102.39 + 77.27 = \underline{-25.12\text{ kJ}}$

(c) Entropy change,

$$\Delta S_{12} = m\left[s°(T_2) - s°(T_1) - R\ln\left(\frac{P_2}{P_1}\right)\right]$$

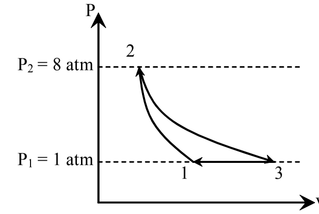
$$= 0.5747 \times \left[7.32 - 6.82 - \frac{8.314}{28}\ln\left(\frac{7.967}{1}\right)\right] = \underline{-0.116\text{kJ/K}}$$

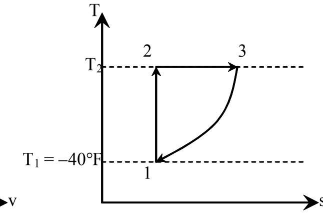
Pr. 2 A quantity of air as an ideal gas, initially at 1 atm, –40°F, executes a power cycle consisting of three internally reversible processes in series (20 points)

Process 1-2: Adiabatic compression to 8 atm.
Process 2-3: Isothermal expansion to 1 atm.
Process 3-1: Isobaric compression.

(a) Sketch the cycle on P-v and T-s coordinates.
(b) Determine the temperature at state 3, in °F.
(c) Determine the net work, in Btu/lbm.
(d) Determine the thermal efficiency.

Sol. (a) The P-v and T-s diagrams,





(b) $P_1 = 1$ atm, $T_1 = -40°F = 420°R$, $P_2 = 8$ atm,

As the system is internally reversible, so, $s_1 = s_2$, so, for air, $k = 1.4$

Using isentropic relationship, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow \frac{T_2}{420} = \left(\frac{8}{1}\right)^{\frac{0.4}{1.4}} \Rightarrow T_2 = 760.8°R$

For isothermal process $T_3 = T_2 = \underline{760.8°R}$

(c) Study of the area under the processes on the P-v diagram indicates that there is a net work input requirement. Accordingly, the cycle is a power cycle. Study of the areas under the processes of the T-s diagram suggest that process 2-3 involves a heat addition (entropy increases) and process 3-1 involves a heat rejection (entropy decreases). Thus,

$\eta = \frac{|W_{cycle}|}{Q_{23}}$, where, $W_{cycle} = W_{12} + W_{23} + W_{31} = Q_{12} + Q_{23} + Q_{31} = Q_{23} + Q_{31}$. $[Q_{12} = 0]$

So, $\eta = \frac{Q_{23} + Q_{31}}{Q_{23}}$

$$\frac{Q_{23}}{m} = \int_2^3 Tds = T(s_3 - s_2) = T\left[\underbrace{s(T_3) - s(T_2)}_{=0 \text{ as } T_3 = T_2} - R\ln\left(\frac{P_3}{P_2}\right)\right]$$

$$= -760.8\frac{1.986}{28.965}\ln\left(\frac{1}{8}\right) = 105.5\text{Btu/lbm.°R}$$

An energy balance for process 3-1 reads, $m(u_1 - u_3) = Q_{31} - W_{31}$,

Where, $\frac{W_{31}}{m} = \int_3^1 Pdv = P(v_1 - v_3)$,

Thus, $\frac{Q_{31}}{m} = (u_1 - u_3) + P(v_1 - v_3)$,

Which can be rewritten concisely as, $\frac{Q_{31}}{m} = h_1 - h_3$, with data from Table A-22E,

$\frac{Q_{31}}{m} = 100.32 - 182.27 = -81.95\text{Btu/lbm.°R}$, Net work $= Q_{23} + Q_{31} = 23.55$ Btu/lbm

(d) So, $\eta = \frac{105.5 - 81.95}{105.5} = 0.2232 = \underline{22.32\%}$

Pr. 3 1 lbm of air in a piston-cylinder assembly is compressed adiabatically from 40°F, 1 atm to 5 atm. (15 points)

(a) If the air is compressed without internal irreversibilities, determine the temperature (°F) at the final state and the work (Btu) required.

(b) If the air requires 20% more work than found in part (a), determine the temperature (°F) at the final state and the amount of entropy (Btu/°R) produced.

(c) Show the processes of parts (a) and (b) on T-s coordinates.

Employ the ideal gas model.

Sol. (a) P_1 = 1 atm, T_1 = 40°F = 500°R, P_2 = 5 atm,
From table 22-E, u_1 = 85.20 Btu/lbm, s_1 = 0.58233 Btu/lbm.°R
As the system is internally reversible, so, $s_1 = s_2$, so, for air, k = 1.4

Using isentropic relationship,

$$\frac{T_{2S}}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow \frac{T_2}{500}=\left(\frac{5}{1}\right)^{\frac{0.4}{1.4}} \Rightarrow T_2 = 791.91°R = \underline{331.91°F}$$

Isentropic work,

$$W_{1-2S}=\frac{mR(T_{2S}-T_1)}{1-n}=\frac{1\times\frac{1.99}{28.965}\times(791.91-500)}{1-1.4}=\underline{-50.14\text{Btu}}$$

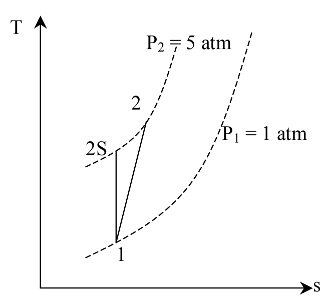
(b) For 20% more work, $W_{1\text{-}2}$ = 1.2 $W_{1\text{-}2S}$ = –1.2 × 50.14 = – 60.168 Btu

$$W_{1-2}=m(u_2-u_1)\Rightarrow 60.168=1\times(u_2-85.2)\Rightarrow u_2=145.368\text{Btu/lbm}$$

So, from Table A-22E, for u_2 = 145.368 Btu/lbm,
T_2 = 847.88°R = <u>387.88°F</u>, s_2 = 0.70947 Btu/lbm.°R

Entropy production,
$\sigma = -m\,(s_2 - s_1) = -1 \times [0.70947 - 0.58233 - 1.99\,(\ln 5)/28.965]$ = <u>0.017 Btu/°R</u>

(c)



Pr. 4 Steam at 140 lbf/in², 1000°F enters an insulated turbine operating at steady state with a mass flow rate of 3.24 lb/s and exits at 2 lbf/in². KE and PE effects are negligible. (10 points)

(a) Determine the maximum theoretical power (hp) that can be developed by the turbine and the corresponding exit temperature (°F)

(b) If the steam exits the turbine at 200°F, determine the isentropic turbine efficiency.

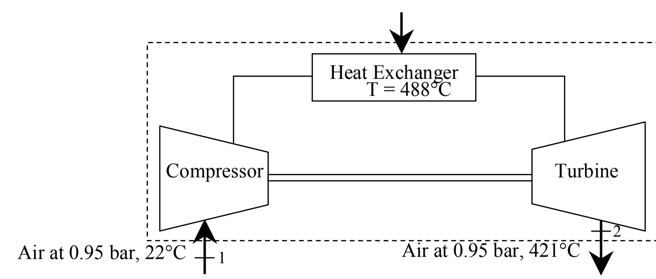
Sol. (a) P_1 = 140 lbf/in², T_1 = 1000°F, from Table A-4E,
s_1 = 1.883674 Btu/lbm.°R = s_2, v_1 = 6.217627 ft³/lbm, h_1 = 1535.5 Btu/lbm
$\dot{m}$ = 3.24 lb/s
P_2 = 2 lbf/in², s_2 = 1.883674 Btu/lbm.°R
From Table A-3E, T_{2S} = 125.8°F, h_{2S} = 1096.8 Btu/lbm
So, maximum theoretical power

$$=\dot{m}(h_{2S}-h_1)=3.24\frac{\text{lb}}{\text{s}}(1535.5-1096.8)\frac{\text{Btu}}{\text{lb}}=1421.388\text{Btu/s}=\underline{2009.3\text{hp}}$$

(b) When T_2 = 200°F, from Table A-4E, h_2 = 1153.3 Btu/lbm

Isentropic efficiency, $\eta_t=\dfrac{h_1-h_2}{h_1-h_{2S}}=\dfrac{1535.5-1153.3}{1535.5-1096.8}=\underline{87.12\%}$

Pr. 5 Figure shows a gas turbine power plant operating at steady state consisting of a compressor, a heat exchanger and a turbine. Air enters the compressor with a mass flow rate of 3.9 kg/s at 0.95 bar, 22°C and exits the turbine at 0.95 bar, 421°C. Heat transfer to the air as it flows through the heat exchanger occurs at an average temperature of 488°C. The compressor and turbine operate adiabatically. Using the ideal gas model for the air, and neglecting KE and PE effects, determine the maximum theoretical value for the net power that can be developed by the power plant, in MW. (15 points)



Sol. Mass and energy rate balances reduces as steady state gives,

$$0=\dot{Q}_{CV}-\dot{W}_{CV}+\dot{m}\left[(h_1-h_2)+\underbrace{\tfrac{1}{2}(V_1^2-V_2^2)}_{=0}+\underbrace{g(Z_1-Z_2)}_{=0}\right]$$

$$\Rightarrow \dot{W}_{CV}=\dot{Q}_{CV}+\dot{m}(h_1-h_2) \quad (1)$$

An entropy rate balance as steady state gives,

$$0 = \frac{\dot{Q}_{CV}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{CV}$$

$$\Rightarrow \dot{Q}_{CV} = \dot{m}T_b(s_2 - s_1) - T_b\dot{\sigma}_{CV}$$

Putting this into Eq. (1),

$$\dot{W}_{CV} = \underline{\dot{m}[(h_1 - h_2) + T_b(s_2 - s_1)]} - T_b\dot{\sigma}_{CV} \quad (2)$$

The underlined term of Eq. (2) is fixed by conditions on the boundary of the C.V. The last term involving entropy production is greater than or equal to zero, depending on the irreversibilities within the C.V. The maximum value of $\dot{W}_{CV}$ corresponds to $\dot{\sigma}_{CV} = 0$, so Eq. (2) becomes,

$$\dot{W}_{CV} = \dot{m}[(h_1 - h_2) + T_b(s_2 - s_1)]$$

$$= \dot{m}\left[(h_1 - h_2) + T_b\left(s°_2 - s°_1 - R\ln\frac{P_2}{P_1}\right)\right]$$

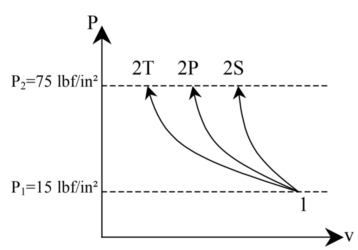
$$= 3.9[(295.17 - 706.8) + 761(2.5635 - 1.68515 - R\ln 1)]$$

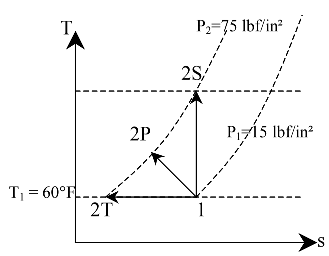
$$= 1001.5\text{kW} \approx \underline{1\text{MW}}$$

Pr. 6 Air enters a compressor operating at steady state at 15 lbf/in², 60°F and exits at 75 lbf/in². KE and PE changes can be ignored. If there are no internal irreversibilities, evaluate the work and heat transfer, each in Btu/lbm of air flowing, for the following cases: (20 points)

(a) Isothermal compression.
(b) Polytropic compression with n = 1.3.
(c) Adiabatic compression.

Sketch the processes on P-v and T-s coordinates and associate areas on the diagrams with work and heat transfer of each case. Referring to your sketches, compare for these cases the magnitudes of the work, heat transfer, and final temperatures, respectively.





Sol. The P-v and T-s diagrams are shown below:

(a) The isothermal process is shown in 1-2T.
For isothermal, P_1 = 15 lbf/in², T_1 = 60°F = 520°R, P_2 = 75 lbf/in², $T_{2T} = T_1$ = 520°R

$$v_1 = \frac{RT_1}{P_1} = \frac{53.34\times 520}{15\times 144} = 12.84\text{ft}^3/\text{lb}, \text{ so, } v_2 = \frac{P_1 v_1}{P_2} = \frac{15\times 12.84}{75} = 2.568\text{ft}^3/\text{lb}$$

$s°_1$ = 1.59 Btu/lbm.°R

$$\text{So, } W = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = 15\times 144\times 12.84\times\frac{1}{778.2}\times\ln(0.2) = \underline{-57.36\text{Btu/lbm}}$$

Q = W = <u>57.36Btu/lbm</u>

(b) The polytropic process is shown in 1-2P.
For isothermal, P_1 = 15 lbf/in², T_1 = 60°F = 520°R, P_2 = 75 lbf/in²,

$$\frac{T_{2P}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow \frac{T_{2P}}{520} = \left(\frac{75}{15}\right)^{\frac{0.3}{1.3}} \Rightarrow T_{2P} = 753.9°\text{R}$$

$$\text{So, } v_{2P} = \frac{RT_{2P}}{P_2} = \frac{53.34\times 753.9}{75\times 144} = 3.724\text{ft}^3/\text{lb}$$

h_1 = –5.78 Btu/lbm, h_{2P}, = 50.755 Btu/lbm

$$\text{So, } W = \frac{n(P_2 v_{2P} - P_1 v_1)}{1-n} = \frac{1.3(75\times 144\times 3.724 - 15\times 144\times 12.84)}{(1-1.3)\times 778.2} = \underline{-69.5\text{Btu/lbm}}$$

$Q = W + h_{2P} - h_1$ = – 69.5 + 50.755 – (–5.78) = <u>–13.15 Btu/lbm</u>

(c) The polytropic process is shown in 1-2S. So, Q = 0
For isothermal, P_1 = 15 lbf/in², T_1 = 60°F = 520°R, P_2 = 75 lbf/in²,

$$\frac{T_{2S}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow \frac{T_{2S}}{520} = \left(\frac{75}{15}\right)^{\frac{0.4}{1.4}} \Rightarrow T_{2S} = 823.6°\text{R}$$

$$\text{So, } v_{2S} = \frac{RT_{2S}}{P_2} = \frac{53.34\times 823.6}{75\times 144} = 4.07\text{ft}^3/\text{lb}$$

h_1 = –5.78 Btu/lbm, h_{2S}, = 67.77 Btu/lbm

$W = h_1 - h_{2S}$ = – 5.78 – 67.77 = <u>–73.55 Btu/lbm</u>