Steam Quality Problems:

Find the steam quality for the following problems.

(a) H_2O; T = 20°C, v = 20 m³/kg,
From Table A-2, $v_f = 1.0018\times10^{-3}$ m³/kg, v_g = 57.791 m³/kg

$$\text{So, } x = \frac{v - v_f}{v_g - v_f} = \frac{20 - 1.0018\times10^{-3}}{57.791 - 1.0018\times10^{-3}} = 0.346 = 34.6\%$$

(b) C_3H_8; P = 15.00 bar, v = 0.02997 m³/kg, From Table A-17, v_g = 0.02997 m³/kg
So, x = 1 = 100%

(c) Refrigerant 134a; T = 60°C, v = 0.001 m³/kg
From Table A-10, $v_f = 0.9488\times10^{-3}$ m³/kg, v_g = 0.0114 m³/kg

$$\text{So, } x = \frac{v - v_f}{v_g - v_f} = \frac{0.001 - 0.9488\times10^{-3}}{0.0114 - 0.9488\times10^{-3}} = 0.0049 = 0.49\%$$

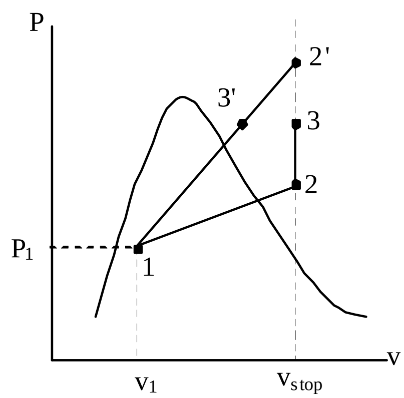
(d) NH_3; P = 1 MPa = 10 bar, v = 0.1 m³/kg
From Table A-14, $v_f = 1.6584\times10^{-3}$ m³/kg, v_g = 0.1285 m³/kg

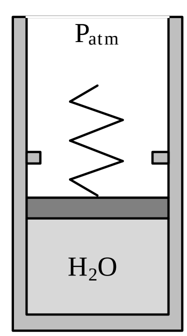
$$\text{So, } x = \frac{v - v_f}{v_g - v_f} = \frac{0.1 - 1.6584\times10^{-3}}{0.1285 - 1.6584\times10^{-3}} = 0.775 = 77.5\%$$

4.110 Book-Borgnakke

2 kg of water is contained in a piston/cylinder (Figure) with a mass less piston loaded with a linear spring and the outside atmosphere. Initially the spring force is zero and P_1 = P_0 = 100 kPa with a volume of 0.2m³. If the piston just hits the upper stops the volume is 0.8m³ and T = 600°C. Heat is now added until the pressure reaches 1.2 MPa. Find the final temperature, show the P-v diagram and find the work done during the process.

Sol. Schematic





State 1: $v_1 = V/m$ = 0.2/2 = 0.1 m³/kg

Process: 1-2-3 or 1-3'

State at stops: 2 or 2'; $v_2 = V_{stop}/m$ = 0.8/2 = 0.4 m³/kg and T_2 = 600°C, $P_2 \approx 1000$ kPa

Table A-4, P_{stop} < 1 MPa < P_3. Since T_{stop} < P_3 the process is as 1-2-3

State 3: P_3 = 1.2 MPa, $v_3 = v_2$ = 0.4 m³/kg,

$$\text{So, } \frac{T_3 - 700}{800 - 700} = \frac{0.4 - 0.37294}{0.41177 - 0.37294} \Rightarrow T_3 = 769.7°\text{C}$$

$W_{13} = W_{12} + W_{23} = ½ (P_1 + P_2) (V_2 – V_1) + 0$ = ½(100 + 1000)(0.8 – 0.2) = 330 kJ

5.73 A cylinder/piston arrangement contains 5 kg of water at 100°C with x = 20% and the piston, m_p = 75kg, resting on some stops like the figure. The outside pressure is 100 kPa and the cylinder area is A = 24.5 cm². Heat is now added until the water reaches a saturated vapor state. Find the initial volume, final pressure, work and heat transfer terms and show the P-v diagram.

Sol. C.V. the 5 kg water.

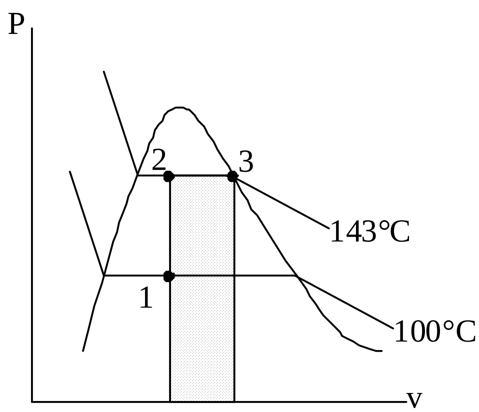
Mass conservation, $m_1 = m_2 = m$
Energy conservation, $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

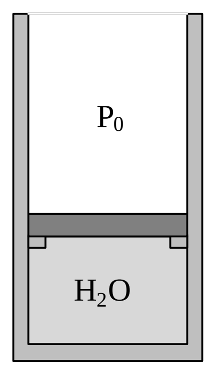
Process: V = Constant if $P < P_{lift}$ otherwise $P = P_{lift}$ in P-v diagram

$$P_3 = P_2 = P_{lift} = P_0 + \frac{m_p g}{A_p} = 100 + \frac{75 \times 9.807}{0.00245 \times 1000} = 400\text{kPa}$$

At 100°C, 101.3 kPa, v_g = 1.6729 m³/kg. If all H_2O becomes vapor, the volume would be,
$V = m\,v_g = 5 \times 1.6729 = 8.3645\ \text{m}^3 > V_1$,

So, $P_3 > P_1$ and the heating process must go through isometric process first with increased pressure until $P = P_{lift}$.





State 1: For T_1 = 100°C and x = 0.2, from Table A-2,
$v_1 = 0.001035 + 0.2 \times 1.6719 = 0.3354$ m³/kg, $V_1 = mv_1 = 5 \times 0.3354 = 1.677$ m³
$u_1 = 418.94 + 0.2 \times (2506.5 - 418.94) = 836.4$ kJ/kg

State 3: For P_3 = 400kPa and x = 1, from Table A-3,
$v_3 = v_{3g} = 0.4625$ m³/kg $> v_1$, $u_3 = u_{3g} = 2553.6$ kJ/kg

Work is seen in P-v diagram (if volume changes then $P = P_{lift}$)
${}_1W_3 = {}_2W_3 = P_{ext}\, m\,(v_3 - v_2) = 400 \times 5 \times (0.46246 - 0.3354) = 254.1$ kJ

Heat transfer is from the energy equation,
${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 5 \times (2553.6 - 836.4) + 254.1 = 8840$ kJ

4.111 Book-Borgnakke

A cylinder having an initial volume of 3m³ contains 0.1 kg of water at 40°C. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done and heat input in the process splitting it into two steps. Assume the water vapor in an ideal gas during the first step of the process.

Sol. C.V. Water.

State 2: 40°C and x = 1, from Table A-2, we get, P_{sat} = 7.384 kPa, v_g = 19.523 m³/kg

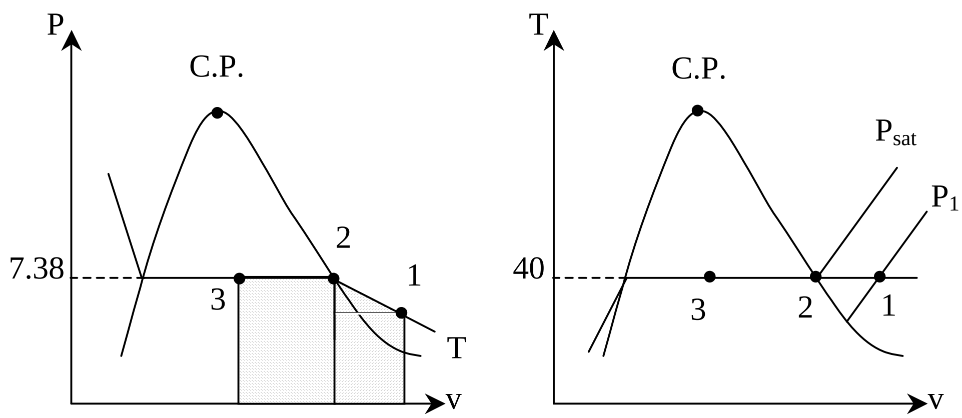
State 1: v_1 = ~~V_1~~/m = 3/0.1 = 30 m³/kg > v_g. (So superheated, u_1 = 2430 kJ/kg)

So, H_2O as an ideal gas (in superheated state),

From 1-2 through isothermal process (Pv = Constant)

$P_1 = P_{Sat}\, v_g/v_1 = 7.384 \times 19.523 / 30 = 4.8$ kPa

$V_2 = m\, v_2 = 0.1 \times 19.523 = 1.952$ m³



Process, T = C: and ideal gas gives work,

$$W_{12} = \int_1^2 PdV = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = 4.8 \times 3.0 \times \ln\left(\frac{1.952}{3}\right) = -6.19\text{kJ}$$

State 3: $T_3 = T_2 = T_1 = 40$°C

$v_3 = 0.0010078 + 0.5 \times (15.523 - 0.0010078) = 9.7605$ m³/kg, so, $V_3 = mv_3 = 0.976$m³

$u_3 = 167.56 + 0.5 \times (2430 - 167.56) = 1298.83$ kJ/kg

$P = P_{Sat}$ = Constant: This give a work term as,

$$W_{23} = \int_2^3 PdV = P_{Sat}(v_3 - v_2) = 7.384 \times (0.976 - 1.952) = -7.21\text{kJ}$$

So, total work, W13 = W12 + W23 = – 6.19 – 7.21 = – 13.4 kJ/kg

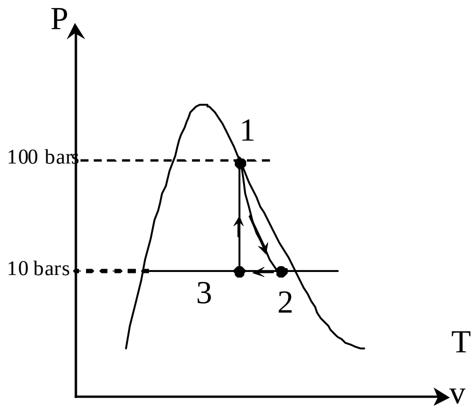
First law,

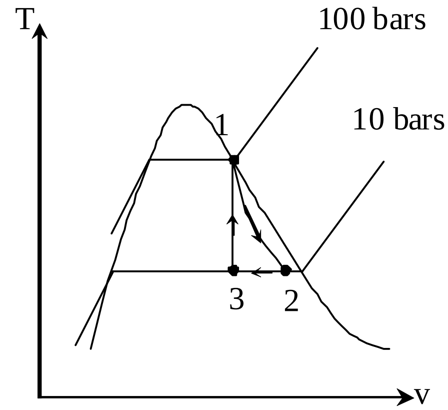
$Q_{in} = W_{out}$ + m(u3 – u1) = – 13.4 + 0.1 × (1298.83 – 2430) = – 126.517 kJ (cooling)

Known: A system consisting of $H2O$ undergoes a cycle composed of three processes. Sketch P-v and T-v diagrams. Determine the cycle net work and the heat transfer for each process.

Schematic and given data: The following data are given for each process.

Process 1-2: Expansion with Pv = Constant from saturated vapor at 100 bars t o10 bars
Process 2-3: Isobaric process $v_3 = v_1$
Process 3-1: Isometric heating





Assumptions: (i) Closed system
(ii) Neglect KE and PE.

Analysis: First, using data from Table A-2, fix each state.

State 1: P_1 = 100 bar, sat. vapor, thus $v_1 = v_g$ = 0.01803 m³/kg, $u_1 = u_g$ = 2544.4 kJ/kg
State 2: From Pv = Constant, $v_2 = (P_1v_1)/P_2$ = 0.1803 m³/kg,
This state is a two-phase state, so for v_2 = 0.1803 m³/kg, u_2 = 2450.7 kJ/kg
State 3: $v_3 = v_1$ = 0.01803m³/kg, so for v_3 = 0.01803 m³/kg, u_3 = 921.0 kJ/kg

Now, the work for each states.

$$W_{12} = mP_1v_1 \ln\left(\frac{v_2}{v_1}\right) = 2\times100\times10^2\times0.01803\times\ln\left(\frac{0.1803}{0.01803}\right) = 830.3\text{kJ}$$

$$W_{23} = mP_2(v_3 - v_2) = 2\times100\times10^2\times(0.1803 - 0.01803) = -324.5\text{kJ}$$

W_{31} = 0 (as isometric process and closed system)

So, the net work is, $W_{cycle} = W_{12} + W_{23} + W_{31}$ = 830.3 – 324.5 + 0 = 505.8 kJ

The heat transfer for each process can be found out as,
$Q_{12} = m (u_2 – u_1) + W_{12}$ = 2 × (2450.7 – 2544.4) + 830.3 = 642.9 kJ
$Q_{23} = m (u_3 – u_2) + W_{23}$ = 2 × (921.0 – 2450.7) – 324.5 = – 3383.9 kJ
$Q_{31} = m (u_1 – u_3) + W_{31}$ = 2 × (2544.4 – 921.0) + 0 = 3246.8 kJ

Comment: $Q_{cycle} = Q_{12} + Q_{23} + Q_{31}$ = 642.9 – 3383.9 + 3246.8 = 505.8 kJ
So, $W_{cycle} = Q_{cycle}$ as expected.

Known: Ethylene is compressed isobarically from P_1 = 213 Psia, T_1 = 612°R to T_2 = 460°R. Determine the work per mole of ethylene.

Given data: $P_1 = P_2$ = 213 lbf/in², T_1 = 612°R, T_2 = 460°R

Assumptions: (i) The ethylene is in a closed system
(ii) The process is isobaric.
(iii) Negligible KE and PE.

Analysis: The work is evaluated as, $W = nP(\bar{v}_2 - \bar{v}_1) \Rightarrow \frac{W}{n} = P(\bar{v}_2 - \bar{v}_1)$

The work is determine after $\bar{v}_1$ and $\bar{v}_2$ are found. First, using the data from Table A-1E.
P_c = 50.5 bars, T_c = 510°R

So, $P_{R1} = P_1/P_c$ = 213/(50.5 × 14.7) = 0.287
$T_{R1} = T_1/T_c$ = 612 / 510 = 1.2
And from Fig. A-1E, v'_{R1} = 4.00

$$\text{Thus, } \bar{v}_1 = v'_{R1}\left(\frac{\bar{R}T_c}{P_c}\right) = 4.00 \times \frac{\left(1545\frac{\text{ft.lbf}}{\text{lbmole.°R}}\right)\times(510°\text{R})}{(50.5\times14.7)\frac{\text{lbf}}{\text{in}^2}}\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) = 29.48\text{ft}^3/\text{lbmole}$$

Again, $P_{R2} = P_{R1}$ = 0.287
$T_{R2} = T_2/T_c$ = 460 / 510 = 0.9
And from Fig. A-1E, v'_{R2} = 2.62

$$\text{Thus, } \bar{v}_2 = v'_{R2}\left(\frac{\bar{R}T_c}{P_c}\right) = 2.62 \times \frac{\left(1545\frac{\text{ft.lbf}}{\text{lbmole.°R}}\right)\times(510°\text{R})}{(50.5\times14.7)\frac{\text{lbf}}{\text{in}^2}}\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) = 19.31\text{ft}^3/\text{lbmole}$$

Finally the work,

$$\frac{W}{n} = \left(213\frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{1\text{in}^2}{144\text{ft}^2}\right)(19.31 - 29.48)\frac{\text{ft}^3}{\text{lbmole}}\left(\frac{1\text{Btu}}{778\text{ft.lbf}}\right) = -400.9\text{Btu}/\text{lbmole}$$

Comment: The work for the compression process is negative, as expected.