Pr 3

68/70

At steady state, a refridgeration cycle driven by a 1 hp motor removes 200 Btu/min of energy by heat transfer from a space maintained at 20° F and discharges energy by heat transfer to surroundings at 75° F. Determine

a) The COP of the refridgerator and the rate at which energy is discharged to the surrounding, in Btu/min

b) The minimum theoretical net power input, in hp for any refridgeration cycle operating between reservoirs at these two temperatures

a) coefficient of performance (COP)

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{200\ \frac{Btu}{min}\left(\frac{60\ min}{1\ hr}\right)}{1\ hp\left(\frac{2545\ \frac{Btu}{hr}}{1\ hp}\right)} = \boxed{4.72} = \beta$$

$$\dot{Q}_{net} = \dot{Q}_{in} + \dot{W}_{cycle}$$
$$= 200\ \frac{Btu}{min} + 1\ hp\left(\frac{2545\ \frac{Btu}{hr}}{1\ hp}\right)\left(\frac{1\ hr}{60\ min}\right)$$

$$\boxed{\dot{Q}_{net} = 242.4\ \frac{Btu}{min}}$$

b) The COP must be < or = to the COP of a reversible refrigeration cycle

$T_c = 480°R$

$T_H = 535°R$

$$\frac{\dot{Q}_{in}}{\dot{W}_{cycle}} \le \frac{T_c}{T_H - T_c} \Rightarrow \frac{(\dot{W}_{cycle})\,T_c}{T_c} \ge \frac{\dot{Q}_{in}\,(T_H - T_c)}{T_c}$$

$$\dot{W}_{cycle} \ge \dot{Q}_{in}\left(\frac{T_H - T_c}{T_c}\right)$$

$$\ge 200\ \frac{Btu}{min}\left(\frac{60\ min}{1\ hr}\right)\left(\frac{1\ hp}{2545\ \frac{Btu}{hr}}\right)\left(\frac{535-480}{480}\right)$$

$$\boxed{\dot{W}_{cycle} \ge 0.54\ hp}$$

## Pr 4

A building for which the heat-transfer rate through the walls and roof is 1400 $\frac{Btu}{hr}$ per °F difference between inside & outside is to be mentioned at 68°F. For a day when the outside temperature is 38°F, determine the power required at steady state, in hp, to heat the building using electrical resistance elements and compare with the minimum theoretical power that would be required by a heat pump. Repeat the comparison using typical manufacturer's data for the heat pump COP (Use 3.0 for typical COP)

$T_H = 38°F = 498°R$

$T_C = -30°F = 430°R$

$$\dot{Q}_H = 1400 \frac{Btu}{hr} (T_H - T_C) = 1400 \frac{Btu}{hr} \left( \frac{1 \text{ hp}}{2546.7 \frac{Btu}{hr}} \right) (498 - 430)$$

$$\dot{Q}_H = 37.4 \text{ hp}$$

$$\frac{Q_H}{Q_{cycle}} \le \frac{T_H}{T_H - T_C} \implies \frac{T_H(\dot{W}_{cycle})}{T_H} \ge \frac{\dot{Q}_H (T_H - T_C)}{T_H}$$

$$\dot{W}_{cycle} \ge 37.4 \text{ hp} \left( \frac{498°R - 430°R}{498°R} \right)$$

$$\boxed{\dot{W}_{cycle} = 5.10 \text{ hp}}$$

$$\beta = \frac{Q_H}{W_{cycle}} \longrightarrow \dot{W}_{cycle} = \frac{\dot{Q}_H}{\beta} = \frac{37.4 \text{ hp}}{3.0}$$

$$\boxed{W_{cycle} = 12.46 \text{ hp}}$$

Ideal : $\frac{37.4 \text{ hp}}{5.10 \text{ hp}} = 7.33$

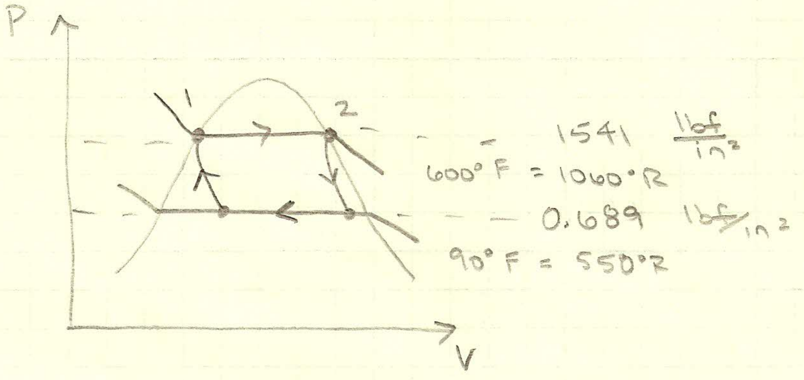
Real : $\frac{37.4 \text{ hp}}{12.46 \text{ hp}} = 3.0$

## 5.76

One-half pound of $H_2O$ executes a carnot power cycle. During the isothermal expansion, the $H_2O$ is heated at 600°F from a saturated liquid to a saturated vapor. The vapor then expands adiabatically w/ No heat to a temperature of 90°F and a quality of 64.3%

(a) Sketch the cycle on P-V coordinates
(b) evaluate the heat and work for each process, in Btu
(c) evaluate the thermal efficiency

(a)




(b) From 1 to 2

$$\frac{W_{12}}{m} = \int_1^2 p\,dV = p(V_2 - V_1)$$

$$\frac{Q_{12}}{m} = (u_2 - u_1) + \frac{W_{12}}{m} \Rightarrow (u_2 - u_1) + p(V_2 - V_1) = h_2 - h_1$$

From Table

$$Q_{12} = 0.5 \text{ lb} \left(549.7 \frac{\text{Btu}}{\text{lb}}\right) = \boxed{274.85 \text{ Btu}} \; Q_{12}$$

$$W_{12} = 0.5 \text{ lb} \left(1541 \times 144 \frac{\text{lbf}}{\text{ft}^2}\right)(0.2677 - 0.02363)\frac{\text{ft}^3}{\text{lb}}\left(\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right)$$

$$= \boxed{34.8 \text{ Btu}} \; W_{12}$$

From 2 to 3

$$\boxed{Q_{23} = 0}$$

$$W_{23} = m(u_2 - u_3)$$

From table

$$u_2 = 1090 \frac{\text{Btu}}{\text{lb}}$$

$$u_3 = 58.07 + 0.643(1040.2 - 58.07)$$

$$= 689.58 \frac{\text{Btu}}{\text{lb}}$$

5.76 continued

$$W_{23} = 0.5\,(1090 - 689.58)$$
$$= \boxed{200.2\ \text{Btu}}\ W_{23}$$

From 3-4

$$W_{34} = mp\,(V_4 - V_3)$$
$$Q_{34} = m\,(h_4 - h_3)$$
$$\frac{|Q_{34}|}{500} = \frac{Q_{12}}{1060} \implies \boxed{Q_{34} = -142.61\ \text{Btu}}$$
$$h_4 = \frac{Q_{34}}{m} + h_3 = -285.22 + (58.07 + 0.643\,(1042.7))$$
$$= 443.31\ \frac{\text{Btu}}{\text{lb}}$$
$$x_4 = \frac{(h_4 - h_f)}{h_{fg}} = \frac{443.31 - 58.07}{1042.7} = 0.369$$
$$V_4 - V_3 = (V_f + x_4 X_{fg}) - (V_f + x_3 V_{fg}) = V_{fg}\,(x_4 - x_3)$$
$$= (467.7 - 0.01610)\,(0.369 - 0.643) = -128.15\ \frac{\text{ft}^3}{\text{lb}}$$
$$W_{34} = 0.5\ \text{lb}\left(0.6988 \times 144\ \frac{\text{lbf}}{\text{ft}^2}\right)\left(-128.15\ \frac{\text{ft}^3}{\text{lb}}\right)\left(\frac{1\ \text{Btu}}{778\ \text{ft}^2\text{-lbf}}\right)$$
$$= \boxed{-8.29\ \text{Btu}}\ W_{34}$$

From 4 to 1

$$\boxed{Q_{41} = 0}$$
$$W_{41} = m\,(u_4 - u_1)$$
$$u_4 = 58.07 + 0.369\,(1040.2 - 58.07)$$
$$= 420.48\ \text{Btu/lb}$$
$$u_1 = 609.9\ \text{Btu/lb}$$
$$W_{41} = 0.5\,(420.48 - 609.9)$$
$$= \boxed{-94.71\ \text{Btu}} = W_{41}$$

(c) $\eta = \frac{1 - T_L}{T_H} = \frac{1 - 550}{1060} = 0.48 = \boxed{48\%}$

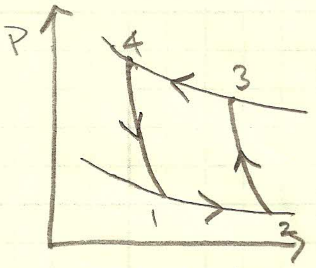
## 5.80

One tenth kilogram of air as an ideal gas with $k = 1.4$ executes a carnot refrigeration cycle, as shown in Fig 5.13. The isothermal expansion occurs at $-23°C$ with a heat transfer to the air of 3.4 kJ. The isothermal compression occurs at $27°C$ to a final volume of $0.01 m^3$. Using the results of prob 5.78 as needed

Determine (a) The pressure, in kPa, at each of the four principal states

(b) the work, in kJ, for each of the

(c) the coefficient of performance




$T_H = 300 K$

$T_C = 250 K$

$$\frac{T_4}{T_1} = \left(\frac{P_4}{P_1}\right)^{\frac{k-1}{k}}$$

$$V_4 V_2 = V_3 V_1$$

(a) $$P_4 = \frac{mRT_4}{V_4} = \frac{(0.1 kg)\left(\frac{8.314 kJ}{28.97 kgK}\right)(300 K)}{0.01 m^3}\left(\frac{1 kN \cdot m}{1 kJ}\right)\left(\frac{1 kPa}{1 kN/m^2}\right)$$

$$= \boxed{861 kPa} \; P_4$$

From 4 to 1

$$\frac{T_4}{T_1} = \left(\frac{P_4}{P_1}\right)^{\frac{k-1}{k}} \Longrightarrow P_1 = \left(\frac{T_1}{T_4}\right)^{\frac{k}{k-1}} P_4$$

$$P_1 = \left(\frac{250}{300}\right)^{\wedge \frac{1.4}{1.4-1}}\left(861 \frac{lbf}{in^2}\right) = \boxed{454.9 kPa} \; P_1$$

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.1\left(\frac{8.314}{28.97}\right)(250)}{454.9} = 0.01577 m^3$$

From 1 to 2

$$m(u_2 - u_1) = Q_{12} - W_{12}$$

$$T_1 = T_2 = T_C \; ; \; u_2 = u_1$$

$$Q_{12} = W_{12} = \int_1^2 p dV = \int_1^2 \frac{mRT}{V} dV = mRT_C \ln \frac{V_2}{V_1}$$

5.80 continued

$$\frac{V_2}{V_1} = \exp\left[\frac{W_{12}}{mRT_L}\right] = \exp\left(\frac{3.4 \text{ kJ}}{0.1\left(\frac{8.314}{28.97}\right)(250)}\right) = 1.6062$$

therefore $V_2 = 1.6062\,(0.01577 \text{ m}^3) = 0.02533 \text{ m}^3$

$$P_2 = \frac{mRT_2}{V_2} = \frac{0.1\left(\frac{8.314}{28.97}\right)(250)}{0.02533} = \boxed{283.2 \text{ kPa}} \quad P_2$$

$$V_4V_2 = V_3V_1 \implies V_3 = \left(\frac{V_4}{V_1}\right)V_2 = \left(\frac{0.01}{0.01577}\right)0.02533 = 0.01606 \text{ m}^3$$

$$P_3 = \frac{mRT_3}{V_3} = \frac{0.1\left(\frac{8.314}{28.97}\right)(300)}{0.01606 \text{ m}^3} = \boxed{536.1 \text{ kPa}} \quad P_3$$

(b) From 1 to 2

$$Q_{12} = W_{12} = \boxed{3.4 \text{ kJ}} \quad W_{12}$$

From 2 to 3

$$Q_{23} = 0$$

$$W_{23} = -m(u_3 - u_2) = -mc_v(T_3 - T_2) = -m\left(\frac{R}{k-1}\right)(T_3 - T_2)$$

$$= -0.1\left(\frac{8.314}{28.97}\right)\left(\frac{1}{1.4-1}\right)(300 - 250) = \boxed{-3.585 \text{ kJ}} \quad W_{23}$$

From 3 to 4

$$W_{34} = \int_3^4 p\,dV = mRT_H \ln\left(\frac{V_4}{V_3}\right)$$

$$= 0.1\left(\frac{8.314}{28.97}\right)(300)\ln\left(\frac{0.01}{0.01606}\right) = \boxed{-4.079 \text{ kJ}} \quad W_{34}$$

$Q_{34}$ $-2$

From 4 to 1

$$Q_{41} = 0$$

$$W_{41} = -m(u_1 - u_4) = -mc_v(T_1 - T_4)$$

$$= -0.1\,(0.717)(250 - 300) = \boxed{3.585 \text{ kJ}} \quad W_{41}$$

(c) Carnot Cycle

$$\beta = \beta_{max} = \frac{T_C}{T_H - T_C} = \frac{250}{300 - 250} = \boxed{5.0} \quad \beta$$