Pr 2

70/70

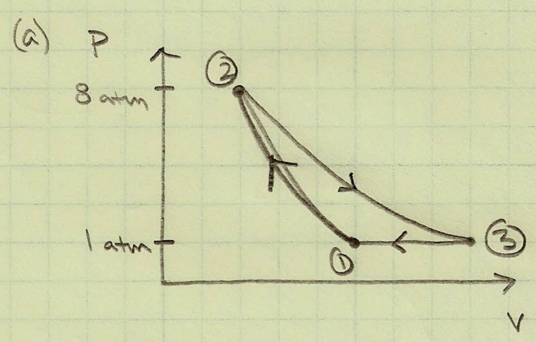
A quantity of air as an ideal gas, initially at 1 atm, -40°F, executes a power cycle consisting of three internally reversible processes in series.

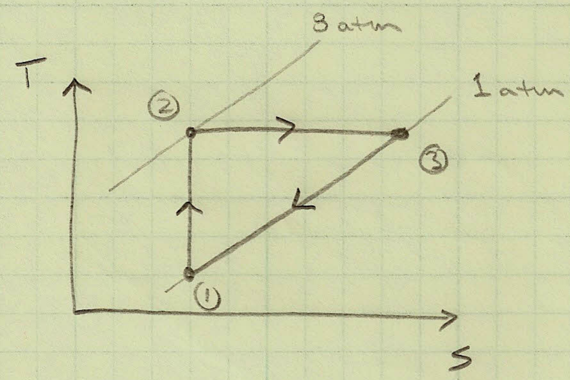
Process 1-2 : Adiabatic compression to 8 atm
2-3 : Isothermal expansion to 1 atm
3-1 : Isobaric compression

Find (a) sketch the cycle on P-v and T-s coordinates
(b) determine the temperature at state 3, in °F
(c) determine the net work, in Btu/lbm
(d) determine the thermal efficiency

$T_1 = -40°F = 419.67\ °R$

(a)




(b) Process 1-2, adiabatic & reversible = isentropic $(s_2 = s_1)$

$$s°(T_2) = s°(T_1) + R \ln\left(\frac{P_2}{P_1}\right)$$
$$= 0.54058 + \frac{1.986}{28.97} \ln\left(\frac{8}{1}\right)$$
$$= 0.6831 \ \frac{\text{Btu}}{\text{lb}}$$

$T_2 = 760°R$

$u_2 = 129.99$ Btu/lb $\qquad u_1 = 71.52 \ \frac{\text{Btu}}{\text{lb}}$

$$\frac{W_{12}}{m} = \cancel{\frac{Q_{12}}{m}} - (u_2 - u_1) = -(129.99 - 71.52) = -58.47 \ \frac{\text{Btu}}{\text{lb}}$$

Process 2-3, isothermal

$$T_3 = T_2 = 760°R = \boxed{300°F}$$

(c) $$\frac{W_{23}}{m} = \int_2^3 p\,dv = \int_2^3 \frac{RT}{v}\,dv = RT \ln\left(\frac{v_3}{v_2}\right) = RT \ln\left(\frac{P_2}{P_3}\right)$$

$P_2 v_2 = RT_2 \quad \& \quad P_3 v_3 = RT_3$

so $\frac{P_2}{P_3} = \frac{v_3}{v_2}$

$$\frac{W_{23}}{m} = \frac{1.986}{28.97}(760°R) \ln\left(\frac{8}{1}\right) = 108.34 \ \frac{\text{Btu}}{\text{lb}}$$

Pr 2 continued

Process 3-1, isobaric

$$\frac{W_{31}}{m} = \int_3^1 p\,dv = p(v_1 - v_3) = R(T_1 - T_3)$$

$$= \left(\frac{1.986}{28.97}\ \frac{Btu}{lb \cdot {}^\circ R}\right)(420 - 760\,{}^\circ R) = -23.3\ \frac{Btu}{lb}$$

$$\frac{W_{cycle}}{m} = \frac{W_{12}}{m} + \frac{W_{23}}{m} + \frac{W_{31}}{m}$$

$$= -58.5 + 108.3 - 23.3$$

$$= \boxed{26.5\ \frac{Btu}{lb}} = W_{net}$$

(d) $$\eta = \frac{\frac{W_{cycle}}{m}}{\frac{Q_{23}}{m}} = \frac{26.5}{108.3} = 0.245$$

$$\boxed{\eta = 24.5\%}$$

## Pr 3

1 lbm of air in a piston-cylinder assembly is compressed adiabatically from 40°F, 1 atm to 5 atm.
(a) If the air is compressed without internal irreversibilities, determine the temperature (°F) at the final state and the work (Btu) required.
(b) If the air is compressed requires 20% more work than found in part a, determine the temperature (°F) at the final state and the amount of entropy (Btu/°R) produced.
(c) Show the processes of parts a & b on T-s coordinates.

$\Delta U = \cancel{Q} - W$

$(-W) = -m(u_2 - u_1)$

$\Delta s = \cancel{\frac{dQ}{T}} + \sigma \Rightarrow \sigma = m(s_2 - s_1)$

(a) $\sigma = 0 \Rightarrow s_1 = s_2$

$(-W)_s = m(u_{2s} - u_1)$

$u_1 = 85.2 \frac{Btu}{lb}$

$s^\circ_{2s} = s_1^\circ + R \ln \frac{P_2}{P_1} = 0.58233 + \frac{1.986}{28.97} \ln\left(\frac{5}{1}\right)$

$= 0.692666$

$u_{2s}$

| s | u |
|---|---|
| 0.68942 | 133.47 |
| 0.692666 | $u_{2s}$ |
| 0.69558 | 136.97 |

$\frac{0.69558 - 0.68942}{0.69558 - 0.692666} = \frac{136.97 - 133.47}{136.97 - u_{2s}}$

$0.00616(136.97 - u_{2s}) = 0.00292(3.5)$

$u_{2s} = 135.31 \ Btu/lb$

$T_{2s}$

| s | T |
|---|---|
| 0.68942 | 780 |
| 0.692666 | $T_{2s}$ |
| 0.69558 | 800 |

$\frac{0.00616}{0.00292} = \frac{800 - 780}{800 - T_{2s}}$

$20(0.00292) = 0.00616(800 - T_{2s})$

$T_{2s} = 791°R = 331.33°F$

$(-W)_s = (1 \ lb)(135.3 - 85.2 \frac{Btu}{lb}) = \boxed{50.1 \ Btu}$

(b) $(-W) = 0.2(50.1) = 10.02 \Rightarrow 50.1 + 10.02 = 60.12 \ Btu$

$u_2 = \frac{-W}{m} + u_1 = \frac{60.12 \ Btu}{1 \ lb} + 85.2 \frac{Btu}{lb}$

$= 145.32 \frac{Btu}{lb}$

## Pr 3 continued

$s^\circ_2$

| $u$ | $s$ |
|---|---|
| 143.98 | 0.70747 |
| 145.32 | $s^\circ_2$ |
| 147.50 | 0.71323 |

$$\frac{147.5 - 143.98}{147.5 - 145.32} = \frac{0.71323 - 0.70747}{0.71323 - s_2}$$

$$0.0125568 = 3.52\,(0.71323 - s^\circ_2)$$

$$s^\circ_2 = 0.70966$$

$T_2$

| $u$ | $T$ |
|---|---|
| 143.98 | 840 |
| 145.32 | $T_2$ |
| 147.50 | 860 |

$$\frac{3.52}{2.18} = \frac{860 - 840}{860 - T_2}$$

$$43.6 = 3.52\,(860 - T_2)$$

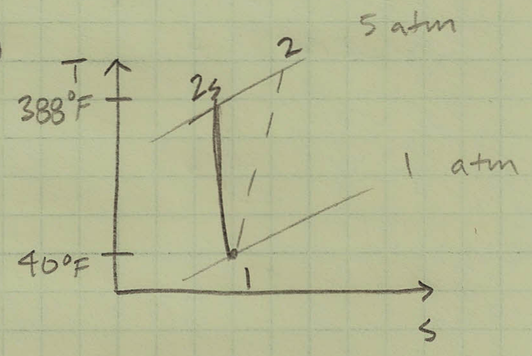
$$T_2 = 847.6\,^\circ R = \boxed{387.93\ ^\circ F}$$

$$\sigma = m\left(s^\circ_2 - s^\circ_1 - R \ln {}^{P_2}/_{P_1}\right)$$

$$= (1\,lb)\left(0.70966 - 0.58233 - \frac{1.986}{28.97} \ln\left(\frac{5}{1}\right)\right)$$

$$= \boxed{0.0116997\ {}^{Btu}/_{^\circ R}} = \sigma$$

(c)

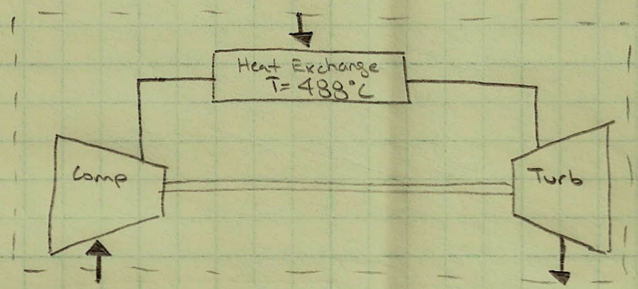


## Pr 5

Figure shows a gas turbine power plant operating at steady state consisting of a compressor, a heat exchanger, and a turbine. Air enters the compressor with a mass flow rate of 3.9 kg/s at 0.95 bar, 22°C and exits the turbine at 0.95 bar, 421°C. Heat transfer to the air as it flows through the heat exchanger occurs at an average temperature of 488°C. The compressor and turbine operate adiabatically. Using the ideal gas model for the air, and neglecting KE & PE effects,

Determine the max theoretical value for the net power that can be developed by the power plant, in MW

STEADY STATE




Air 0.95 bar
22°C
3.9 kg/s

Air 0.95 bar
421°C

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$$

at steady state, entropy rate balanced

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1) - T_b\dot{\sigma}_{cv} \implies \dot{Q}_{cv} = \dot{m}T_b(s_2 - s_1) - T_b\dot{\sigma}_{cv}$$

$$\dot{W}_{cv} = \dot{m}T_b(s_2 - s_1) - T_b\dot{\sigma}_{cv} + \dot{m}(h_1 - h_2)$$

$$= \dot{m}\left[(h_1 - h_2) + T_b(s_2 - s_1)\right] - T_b\dot{\sigma}_{cv}$$

at max value $\dot{W}_{cv}$ ; $\dot{\sigma}_{cv} = 0$

$$(\dot{W}_{cv})_{max} = \dot{m}\left[(h_1 - h_2) + T_b(s_2 - s_1)\right]$$

$$= \dot{m}\left[h(T_1) - h(T_2) + T_b\left(s^\circ(T_2) - s^\circ(T_1) - \cancel{R\ln}\right)\right]$$

$$= 3.9\,\frac{kg}{s}\left((295.17 - 706.8) + 761\,(2.5635 - 1.68515)\right)$$

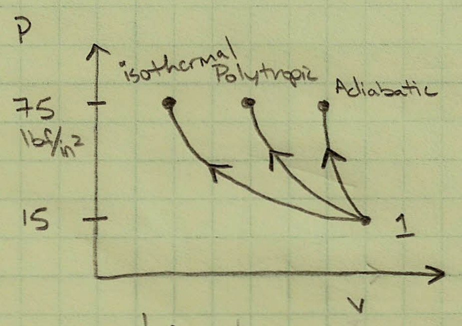
$$= 1001.5\,\frac{kJ}{s}\left(\frac{1\ MW}{10^3\ kJ/s}\right)$$

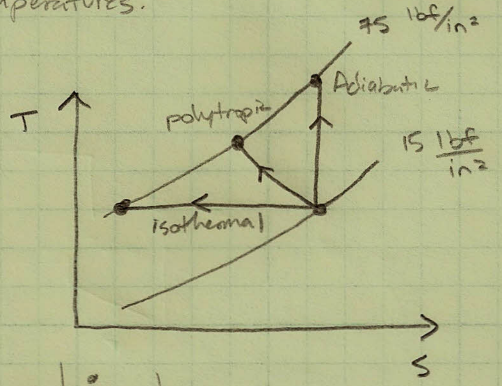
$$= \boxed{1\ MW} \quad \dot{W}_{cv}\ max$$

## Pr 6

Air enters a compressor operating at steady state at 15 $lbf/in^2$, 60°F and exits at 75 $lbf/in^2$. KE & PE changes can be ignored. If there are no internal irreversibilities, evaluate the work and heat transfer, each in Btu / lbm of air flowing, for the following cases: (a) Isothermal compression, (b) polytropic compression with $n = 1.3$, (c) adiabatic compression.

Sketch the processes on P-v and T-s coordinates and associate areas on the diagrams with work and heat transfer of each case. Compare the magnitudes of the work, heat transfer, and final temperatures.




$\left|\frac{\dot{W}_{cv}}{\dot{m}}\right|$ - increases $\qquad$ $\left|\frac{\dot{Q}_{cv}}{\dot{m}}\right|$ decreases

The area below the curve on T-s diagram represent heat transfer.
The area behind the curve of the P-v diagram represent the work per unit mass

(a) isothermal: energy rate balance at steady state

$$0 = \frac{Q_{cv}}{m} - \frac{W_{cv}}{m} + (h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}$$

temp is constant, so $h_1 = h_2$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}}$$

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\int_1^2 v\,dp = -\int_1^2 \frac{RT}{P}\,dp = -RT \ln\frac{P_2}{P_1}$$

$$= -\left(\frac{1.986}{28.97}\right) 520°R \ \ln\frac{75}{15} = \boxed{-57.37 \ \frac{Btu}{lb}}$$

(b) Polytropic $n = 1.3$

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{nR}{(n-1)}(T_2 - T_1)$$

polytropic for ideal gas

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\wedge \frac{n-1}{n}} \Rightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\wedge \frac{n-1}{n}} = 520\left(\frac{75}{15}\right)^{\frac{0.3}{1.3}}$$

$T_2 = 753.88°R$

Pr 6 continued

$$\frac{W_{cv}}{m} = -\left(\frac{1.3}{0.3}\right)\left(\frac{1.986}{28.97}\right)(753.89-520) = -69.48\ \frac{Btu}{lb}$$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 - h_1 = -69.48 + 180.6 - 124.27$$

$$= -13.14\ \frac{Btu}{lb}$$

(c) Adiabatic (isentropic)

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

$$P_{r_2} = P_{r_1}\left(\frac{P_2}{P_1}\right) = 1.2147\left(\frac{75}{15}\right) = 6.0735$$

$$h_2 = 197.06\ \frac{Btu}{lb}$$

$$\frac{\dot{W}_{cv}}{\dot{m}} = 124.27 - 197.06 = \boxed{-72.79\ \frac{Btu}{lb}}$$

The work & heat magnitude are associated with the areas on a P-V & T-s diagram.
The areas behind the curve on the P-v diagram increase from a-b-c, same as the work.
The areas below the curves on the T-s diagram decrease from a-b-c the same way the work decreases.