

1)

$$y'' - 4y' + 4y = 0$$

$$e^{rt}(r^2 - 4r + 4) = 0$$

$$(r-2)^2 = 0$$

$r=2$ double root

$$y(0) = \frac{1}{2}; \quad y'(0) = 0$$

$$y = c_1 e^{rt} + c_2 t e^{rt}$$

$$y = c_1 e^{2t} + c_2 t e^{2t}$$

$$y' = 2c_1 e^{2t} + 2c_2 t e^{2t} + c_2 e^{2t}$$

$$y(0) = c_1 e^{2(0)} + c_2(0) e^{2(0)} = \frac{1}{2}$$

$$c_1$$

$$= \frac{1}{2}$$

$$y'(0) = 2c_1 e^{2(0)} + 2c_2(0) e^{2(0)} + c_2 e^{2(0)} = 0$$

$$2c_1 + 0 + c_2 = 0$$

$$2\left(\frac{1}{2}\right) + c_2 = 0$$

$$c_2 = -1$$

$$y(t) = \frac{1}{2} e^{2t} - t e^{2t}$$

$$2) \quad y^2 dx + (xy - ye^y) dy = 0$$

$$M_y = 2y \quad N_x = y$$

$$\text{test for } x \text{ dependancy} \quad \frac{2y - y}{xy - ye^y} = \frac{y(2-1)}{y(x - e^y)} = \frac{1}{x - e^y}$$

$$\text{test for } y \text{ dependancy} \quad \frac{y - 2y}{y^2} = \frac{y(1-2)}{y(y)} = -\frac{1}{y} \text{ or } -y^{-1}$$

$$\text{therefore } \mu(y) = e^{-\int \frac{1}{y} dy} = e^{-\ln|y|} = e^{\ln y^{-1}} = y^{-1}$$

$$y^{-1}(y^2) dx + y^{-1}(xy - ye^y) dy = 0(y^{-1})$$

$$y dx + (x - e^y) dy = 0$$

$$\text{test } \tilde{M}_y = 1 + \tilde{N}_x = 1$$

$$\int \Psi_y = \int (x - e^y) dy$$

$$\Psi = xy - e^y + h(x)$$

$$\Psi_x = y + h'(x) = y$$

$$h'(x) = 0$$

$$h(x) = 0$$

$$xy - e^y + 0 = C$$