$\frac{dy}{dx} = \frac{1-x^2}{y^2}$

$\int y^2\,dy = \int 1 - x^2\,dx$

$\frac{1}{3}y^3 = 1x - \frac{1}{3}x^3 + c$

$y = 3(x - \frac{1}{}$

$y = \sqrt[3]{3x - x^3 + c}$

MATH 2221 - EXAM I

July 1, 2008

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A

NAME:

$\frac{dy}{dx} = 4y^2 - 3y + 1$

$\frac{1}{4y^2-3y+1}\,dy = dx$ s

$s^2 + \frac{ds}{dt} = \frac{s+1}{st}$ $\frac{8}{s+1}$

$\frac{s}{s+1}\left(s^2 + \frac{ds}{dt}\right)$ $\frac{1}{t}$

$s^2 + \frac{ds}{dt} = \frac{s+1}{st}$

Not

$\frac{dy}{dx} = \frac{\sqrt{y}\,e^x e^y}{x^2+2}$

$\frac{1}{ye^y}\,dy = \frac{e^x}{x^2+2}\,dx$

$\frac{dy}{dx} = \frac{\sec^2 y}{1+x^2}$

$\int \cos^2 y\,dy = \tan^{-1} x + c$

$\frac{dy}{dx} = y(2 + \sin x)$

$\int \frac{1}{y}\,dy = \int 2 + \sin x\,dx$

$\ln y + c_1 = 2x + \cos x + c_2$

$\ln y = 2x + \cos x + c_2 + c_1$

$y = e^{2x + \cos x + c_2 + c_1}$

$y = e$

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1. (10pts) Determine the Laplace transform of the function

$$y(t) = \begin{cases} t, & 0 < t < 1, \\ e^{-2t}, & 1 < t < 3, \\ 1, & t = 3, \\ -2, & t > 3. \end{cases}$$

$$\mathcal{L}\{t\}\Big|_0^1 + \mathcal{L}\{e^{-2t}\}\Big|_1^3 + \mathcal{L}\{1\}\Big|^{=3} - 2\mathcal{L}\{1\}\Big|_3^\infty$$

$$\int t\cdot e^{-st}\,dt = \left(\frac{-t}{s} - \frac{1}{s^2}\right)\cdot e^{-ts}\Big|_{t=0}^{t=1} = \left[\left(\frac{-1}{s} - \frac{1}{s^2}\right)e^{-s} + \frac{1}{s^2}\right]$$

$$\int e^{-2t}e^{-st}\,dt = \frac{-e^{t(-s-2)}}{s+2}\Big|_1^3 = \left[\frac{-e^{-3s-6}}{s+2} - \left(\frac{-e^{-s-2}}{s+2}\right)\right]$$

$\mathcal{L}\{1\}$ doesn't matter, just a point

$$-2\{1\} = -2\int e^{-st} = \frac{-e^{-ts}}{s}\Big|_3^\infty = \left[\frac{e^{-3s}}{s}\right]$$

$$\boxed{Y(t) = \frac{1}{s^2} - \left(\frac{1}{s} + \frac{1}{s^2}\right)e^{-s} + \frac{e^{-s-2}}{s+2} - \frac{e^{-3s-6}}{s+2} + \frac{e^{-3s}}{s}}$$

2. (12pts) Solve the initial value problem

$$x^2\frac{dy}{dx} + 3x^3y = x^2, \quad y(2) = 0.$$

$$\frac{x^2}{x^2}\frac{dy}{dx} + \frac{3x^3}{x^2}y = \frac{x^2}{x^2} =$$

$$\mu(x) = e^{\int (3x)dx} = e^{\frac{3x^2}{2}}$$

$$f(x) = \frac{1}{e^{\frac{3x^2}{2}}}\int \left(e^{\frac{3x^2}{2}} \cdot 1\right)dx$$

10

3. (12pts) Find an implicit solution to the equation

$$\overset{M}{(3x^2y - 6x)}dx + \overset{N}{(x^3 + 1)}dy = 0$$

is: $\frac{x^3+1}{dx} = 3x^2 = \frac{3x^2y}{dy} = 3x^2$; yes

$\int(3x^2y-6x)dx = \frac{x^3y - 3x^2 + g(y)}{\partial y} =$

$x^3 + g'(y) = x^3 + 1$

$g'(y) = 1$ / $g(y) = y$

$$\boxed{f(x,y) = x^3y - 3x^2 + y + c}$$

solution ?

$x^3y - 3x^2 + y = c$

4. (12pts) Find the general solution of

$$\frac{dy}{dx} + 3y - y^2 = 0.$$

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$$\frac{dy}{dx} = -3y + y^2$$

$$\frac{dy}{y^2-3y} = dx \qquad \frac{dx}{y(y-3)} = \frac{A}{y} - \frac{B}{y-3} =$$

$$e \; \frac{\ln(y-3)}{3} - \frac{\ln y}{3} = x + c$$

$$e^{\frac{\ln(y-3)}{3}} + e^{-\frac{\ln y}{3}} = Ce^x$$

why did I get a point off ~~these~~ here

$$\boxed{(y-3)^{\frac{1}{3}} + y^{-\frac{1}{3}} = Ce^x}$$

10

5. (10pts) Determine the form of a particular solution for the differential equation (DO NOT EVALUATE COEFFICIENTS).

(a) $y'' + 8y' + 16y = 3e^{-4t} + t\cos 3t$

$(r+4)(r+4) = 0$

$r = -4$

$s = 2$

$y_p = At^2 e^{-4t} + \left[(B_1 t + B_0)\cos(3t) + (C_1 t + C_0)\sin(3t)\right]$

✓

(b) $y'' + 2y' + 2y = 2t\cos t - t^2 \sin 2t$

$(r+1)^2$

$r = -1$

$s = 0$

$y_p = (A_1 t + A_0)\cos t + (B_1 t + B_0)\sin t - \left[(C_2 t^2 + C_1 t + C_0)\sin 2t + (D_2 t^2 + D_1 t + D_0)\cos 2t\right.$

✓

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6. (12pts) Find a general solution to the equation

$$y'' - 5y' - 14y = -36e^t + +15\sin t + 5\cos t.$$

$(r-7)(r+2)=0$

$r_1=7 \quad r_2=-2 \qquad s=0$

$y_p = Ae^t + B\sin t + C\cos t + \cancel{D\sin t} + E\,\cancel{\cos t}$

$y_p' = Ae^t + B\cos t - C\sin t + \cancel{D\cos t} - \cancel{E\sin t}$

$y_p'' = Ae^t - B\sin t - C\cos t - \cancel{D\sin t} - \cancel{E\cos t}$

$Ae^t - B\sin t - C\cos t - D\sin t - E\cos t - 5Ae^t - 5B\cos t + 5E\sin t - 5D\cos t + 5E\sin t$

$-14Ae^t - 14B\sin t - 14C\cos t - 14D\sin t - 14E\cos t = -36e^t + 15\sin t + 5\cos t$

$-18Ae^t = -36e^t$

$A = 2$ ✓

$-B\sin t - D\sin t + 5C\sin t + 5E\sin t - 14B\sin t - 14D\sin t = 15\sin t$

$y_h = C_1e^{7t} + C_2e^{-2t}$

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7. (12pts) Find a general solution to the equation

$$t^2y'' + 3ty' + y = 3t \quad \rightarrow (g(t) = \frac{3}{t}$$

$ar^2 + (b-a)r + c$

$r^2 + 2r + 1 = (r+1)^2$

$y' = -\frac{1}{t^2} \quad y_2' = -\frac{1}{t^2}\ln t + \frac{1}{t^2}$

$r_{1,2} = -1$

$c_1 t^{-1} + c_2 t^{-1} \ln t$

$\frac{1}{t}\left(v_1' t^{-1} + v_2' t^{-1}\ln t = 0\right) = v' \frac{1}{t^2} + v_2' \frac{1}{t^2}\ln t = 0$

$-v_1'\frac{1}{t^2} + v_2'\left(\frac{1}{t^2} - \frac{\ln t}{t^2}\right) = 3t - v_1'\frac{1}{t^2} + v_2'\frac{1}{t^2} - v_2'\frac{\ln t}{t^2} = 3t$

$v_1 = \int -y_2$

$2v_2'\frac{1}{t^2} - v_2'\frac{\ln t}{t^2} = 3t$

$v_1 = \int \frac{-\frac{1}{t}\ln t \; \frac{3}{t}}{\frac{1}{t^3}} = -3\int t\ln t \, dt$

$2v_2' - v_2'\ln t = 3t^3 \quad t=1$

$v_2'(2 - \ln t) = 3t^2$

$v_2 = 3\int \frac{\frac{1}{t}\cdot\frac{1}{t}}{\frac{1}{t^3}} = \frac{3t^2}{2} \quad = \frac{t^2}{2}\ln t - \frac{t^2}{4}$

$v_2' = \frac{3t^3}{2-\ln t}$

$v_1'\frac{1}{t} = -\frac{3t^3}{2-\ln t}\left(\frac{1}{t}\ln t\right)$

$v_2 = \int \frac{3t^3}{2-\ln t}$

$v_1' = -\int\left[\left(\frac{3}{2-\ln t}\right)\left(\frac{1}{t}\ln t\right)\right]$

$$\boxed{y(t) = c_1 t^{-1} + c_2 t^{-1}\ln t + -\int\left[\left(\frac{3}{2-\ln t}\right)\left(\frac{1}{t}\ln t\right)dt\right] t^{-1} + \left[\int\left(\frac{3t^3}{2-\ln t}\right)dt\right] t \ln}$$

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8. (12pts) Solve the initial value problem

$$y'' + 4y' + 13 = 0, \quad y(0) = 2, \quad y'(0) = -1.$$

$$r = \frac{-4 \pm \sqrt{16 - 4(13)}}{2} = \frac{-4}{2} \pm \frac{6i}{2} = -2 \pm 3i$$

$$y(t) = c_1 e^{-2t} \cos 3t + c_2 e^{-2t} \sin 3t$$

$$y'(t) = (3c_2 - 2c_1) e^{-2t} \cdot \cos 3t + (-3c_1 - 2c_2) e^{-2t} \sin 3t$$

$$y(0) = c_1 (1)(1) + c_2 (1)(0) = 2$$

$$c_1 = 2$$

$$y'(0) = (3c_2 - 4)(1)(1) = -1$$

$$3c_2 - 4 = -1$$

$$3c_2 = 3 \qquad c_2 = 1$$

$$\boxed{y(t) = 2e^{-2t} \cos 3t + e^{-2t} \sin 3t}$$