

E702.1

## Designing Concrete Structures:

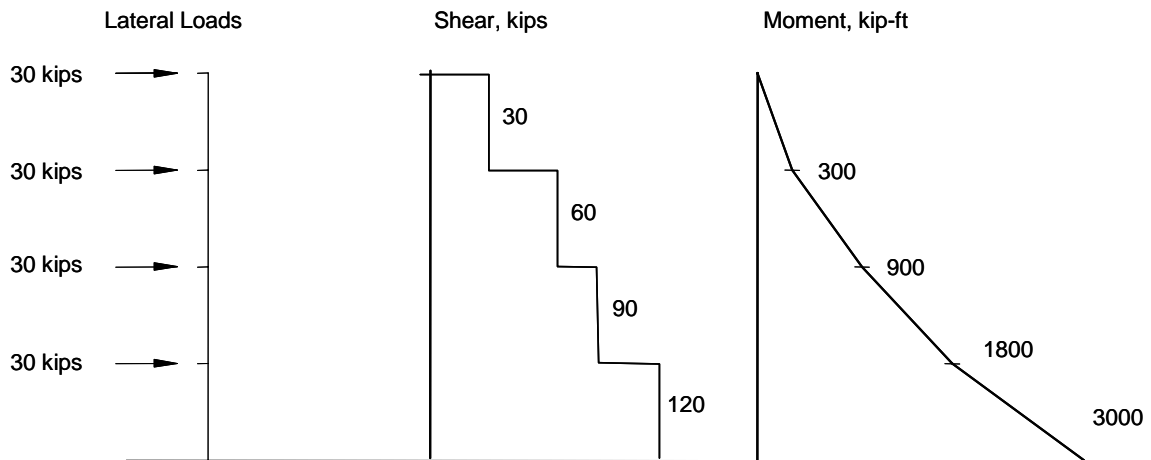
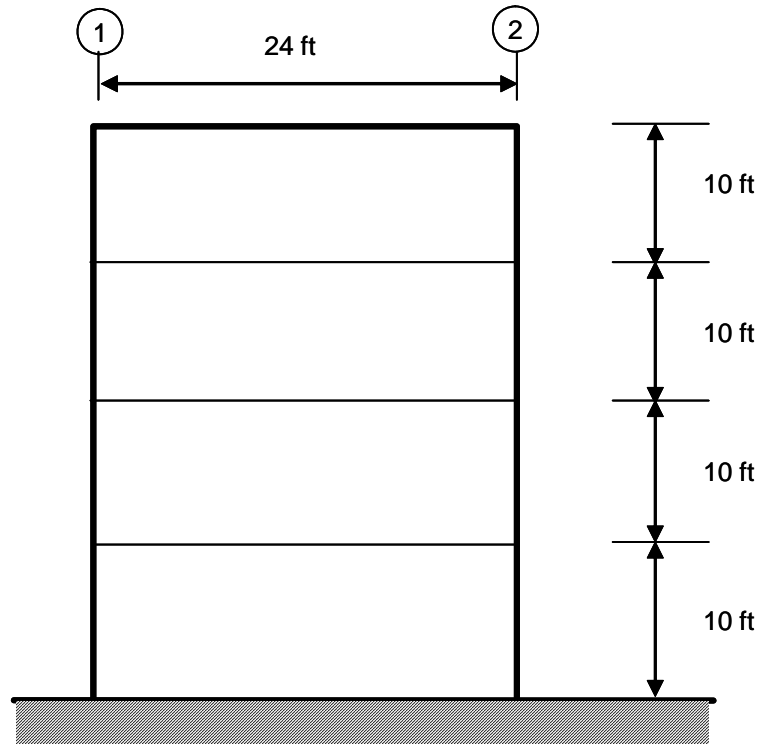
### Allowable-Stress Design of Reinforced Concrete Masonry Shear Wall



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**Example #1: Allowable-Stress Design of Reinforced Concrete Masonry Shear Wall**

Using the allowable-stress provisions of the 2005 MSJC Code (ACI 530), design the reinforced concrete masonry shear wall shown below. In the context of an entire building design, design loads would be obtained using the appropriate provisions of the legally adopted building code (which usually references ASCE 7). For this facet of the design, unfactored loads are shown below, and are assumed to be due to earthquake. Corresponding shear and moment diagrams are also shown.



Assume an 8-in. nominal concrete masonry wall, grouted solid, with Type S PCL mortar. The total plan length of the wall is 24 ft (288 in.), and its specified thickness is 7.63 in. Assume an effective depth  $d$  of 285 in.

	<i>Concrete Masonry</i>
<i>Unit Strength (C90)</i>	<i>1900 psi</i>
<i>Mortar</i>	<i>Type S</i>
$f'_m$ or $f'_g$	<i>1,500 psi</i>
$E_m$ or $E_g$	<i><math>1.35 \times 10^6</math> psi</i>
<i>Reinforcement = Grade 60; <math>E_s = 29 \times 10^6</math> psi</i>	

Unfactored axial loads on the wall are given in the table below.

<b>Level</b>	<b>DL</b>	<b>LL</b>
(Top of Wall)	(kips)	(kips)
4	90	15
3	180	35
2	270	55
1	360	75

Check shear for the assumed wall thickness.

Use ASCE 7-05 ASD Load Combination 5:  $0.6D + 0.7E$

$$V = 0.7 \times 120,000 \text{ lb} = 84,000 \text{ lb}$$

$$M = 0.7 \times 3000 \text{ kip-ft} = 2100 \text{ kip-ft}$$

$$P = 0.6 \times 360 \text{ kips} = 216 \text{ kips}$$

By Code Section 2.3.5.2.1,

$$f_v = \frac{V}{bd} = \frac{84,000 \text{ lb}}{7.63 \times 285} = 38.6 \text{ psi}$$

$$\frac{M}{Vd} = \frac{3000 \text{ kip-ft} \times 12 \text{ in./ft}}{120 \text{ kips} \times 285 \text{ in.}} = 1.05$$

By Code Section 2.3.5.2.2(b), where shear reinforcement is not provided to resist the calculated shear:

$$F_v = \frac{1}{3} \left( 4 - \frac{M}{Vd} \right) \sqrt{f'_m} = \frac{1}{3} (4 - 1.05) \sqrt{1500} = 38.1 \text{ psi}$$

but  $F_v \leq [80 - 45(M/Vd)] = 80 - 45(1.05) = 32.8 \text{ psi} \quad \Leftarrow \quad \text{GOVERNS}$

$$f_v = 38.6 \text{ psi} > F_v \therefore \text{Shear reinforcement is needed}$$

Now check against the higher allowable shear stresses of Code Section 2.3.5.2.3:

$$F_v = \frac{1}{2} \left( 4 - \frac{M}{Vd} \right) \sqrt{f'_m} = \frac{1}{2} (4 - 1.05) \sqrt{1500} = 57.13 \text{ psi} \quad \Leftarrow \quad \text{GOVERNS}$$

but  $F_v \leq [120 - 45(M/Vd)] = 72.75 \text{ psi} > 38.6 \text{ psi} \quad \text{OK}$

Design shear reinforcement using Code Section 2.3.2.1 and Code Section 2.3.5.3:

$$F_s = 24,000 \text{ psi}$$

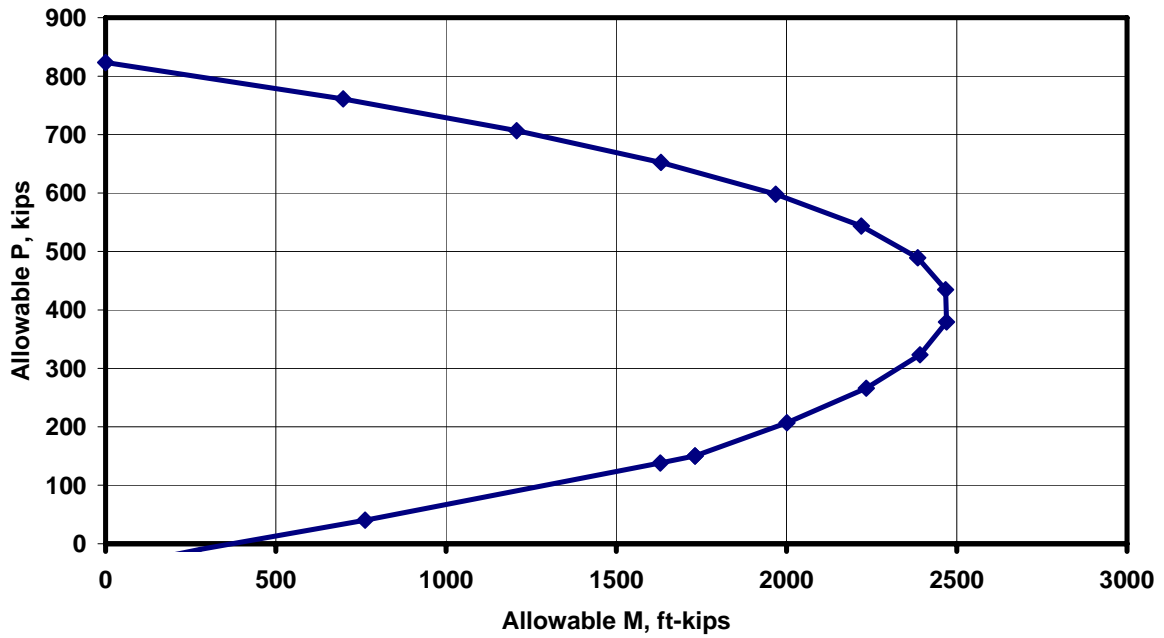
$$A_v = \left( \frac{V_s}{F_s d} \right) = \frac{84,000 \text{ kips} \times 24 \text{ in.}}{24,000 \text{ psi} \times 285 \text{ in.}} = 0.29 \text{ in.}^2$$

Use No. 5 bars horizontally at 24 inches on center

Now consider the flexural design. The attached spreadsheet for this problem illustrates how to calculate an allowable-stress moment-axial force interaction diagram for this wall. That spreadsheet is first used to check the wall with reinforcement consisting of #5 bars @ 4 ft. Because the space between extreme bars is not quite a multiple of 48 in., the spacing of the interior bars is adjusted slightly to fit the 16-in. nominal length of the concrete masonry units.

Depending on the building's Seismic Design Category, prescriptive seismic provisions might require more reinforcement or a closer spacing of reinforcement than those calculated above.

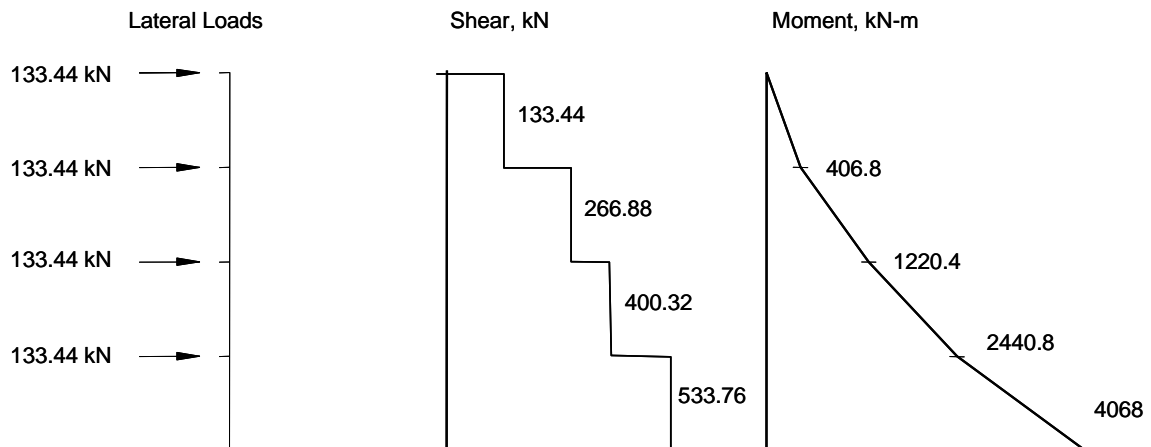
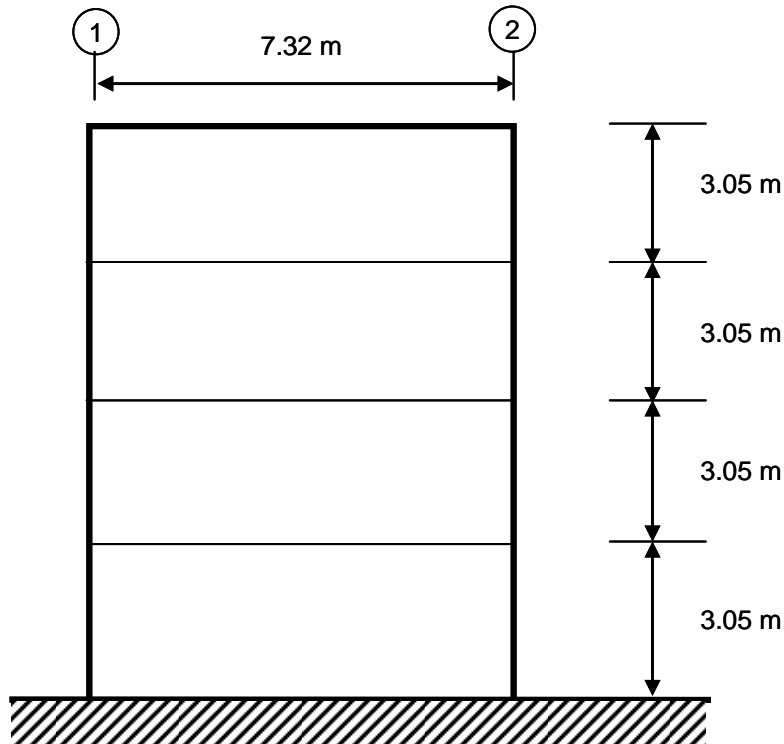
**Allowable-Stress Interaction Diagram by Spreadsheet  
Concrete Masonry Shear Wall of Example Problem  
f'm=1500 psi, 24 ft long, 7.63 in. thick, #5 bars @ 4'**



The interaction diagram shows that the wall, as designed above, can resist the combination of axial load and moment (216 kips, 2100 kip-ft).

**Example #2: Allowable-Stress Design of Reinforced Concrete Masonry Shear Wall (SI Units)**

Using the allowable-stress provisions of the 2005 MSJC Code (ACI 530), design the reinforced concrete masonry shear wall shown below. In the context of an entire building design, design loads would be obtained using the appropriate provisions of the legally adopted building code (which usually references ASCE 7). For this facet of the design, unfactored loads are shown below, and are assumed to be due to earthquake. Corresponding shear and moment diagrams are also shown.



Assume an 203.2-mm nominal concrete masonry wall, grouted solid, with Type S PCL mortar. The total plan length of the wall is 7.315 m, and its specified thickness is 193.8 mm. Assume an effective depth  $d$  of 7.239 m.

	<i>Concrete Masonry</i>
<i>Unit Strength (C90)</i>	<i>13.10 MPa</i>
<i>Mortar</i>	<i>Type S</i>
$f'_m$ or $f'_g$	<i>10.34 MPa</i>
$E_m$ or $E_g$	<i>9308 MPa</i>
<i>Reinforcement = Grade 60; <math>E_s = 199,955</math> MPa</i>	

Unfactored axial loads on the wall are given in the table below.

<b>Level</b>	<b>DL</b>	<b>LL</b>
(Top of Wall)	(kN)	(kN)
4	400.32	66.72
3	800.64	155.68
2	1200.96	244.64
1	1601.28	333.60

Check shear for the assumed wall thickness.

Use ASCE 7-05 ASD Load Combination 5:  $0.6D + 0.7E$

$$V = 0.7 \times 533.76 \text{ kN} = 373.63 \text{ kN}$$

$$M = 0.7 \times 4068 \text{ kN-m} = 2847.6 \text{ kN-m}$$

$$P = 0.6 \times 1601 \text{ kN} = 960.77 \text{ kN}$$

By Code Section 2.3.5.2.1,

$$f_v = \frac{V}{bd} = \frac{373.63 \text{ kN}}{193.8 \times 7239 \text{ mm}^2} = 0.26632 \text{ MPa}$$

$$\frac{M}{Vd} = \frac{2847 \text{ kN-m}}{373.63 \text{ kN} \times 7.239 \text{ m}} = 1.05$$

By Code Section 2.3.5.2.2(b), where shear reinforcement is not provided to resist the calculated shear:

$$F_v = 0.028 \left( 4 - \frac{M}{Vd} \right) \sqrt{f'_m} = 0.028(4 - 1.05) \sqrt{10.34} = 0.26 \text{ MPa}$$

but  $F_v \leq [0.55 - 0.31 (M/Vd)] = 0.55 - 0.31 (1.05) = 0.22 \text{ MPa} \quad \Leftarrow \text{ GOVERNS}$

$$f_v = 0.266 \text{ MPa} > F_v \therefore \text{Shear reinforcement is needed}$$

Now check against the higher allowable shear stresses of Code Section 2.3.5.2.3:

$$F_v = 0.042 \left( 4 - \frac{M}{Vd} \right) \sqrt{f'_m} = 0.42 (4 - 1.05) \sqrt{10.34} = 0.40 \text{ MPa} \quad \Leftarrow \text{ GOVERNS}$$

but  $F_v \leq [0.82 - 0.31(M/Vd)] = 0.49 \text{ MPa} > f_v = 0.266 \text{ MPa} \quad \text{OK}$

Design shear reinforcement using Code Section 2.3.2.1 and Code Section 2.3.5.3:

$$F_s = 165.48 \text{ MPa}$$

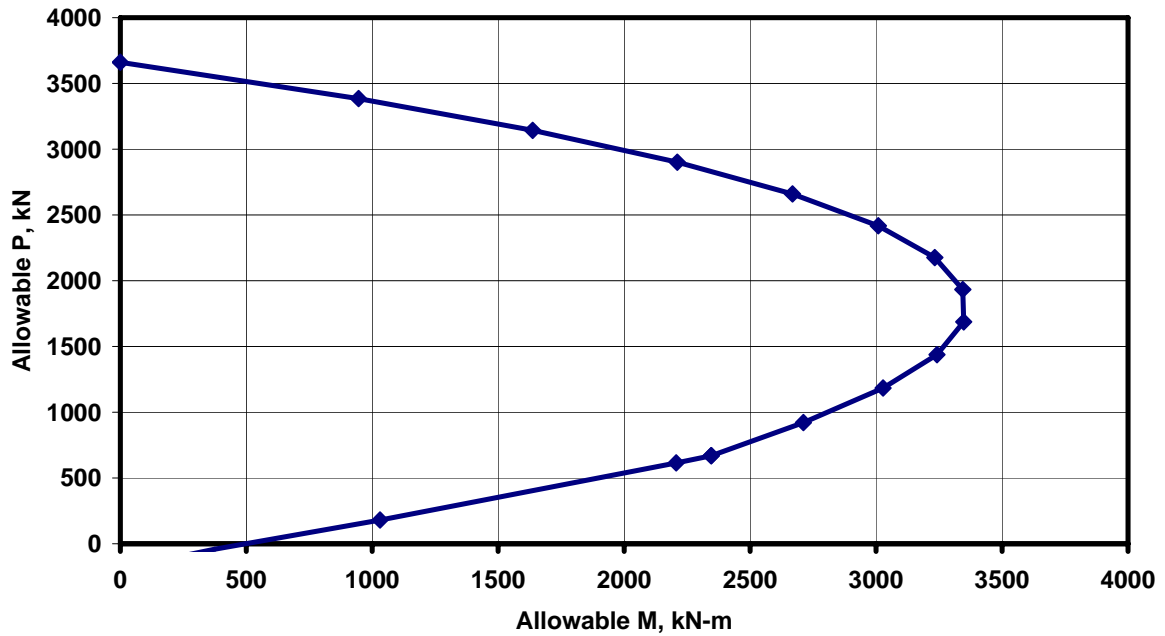
$$A_v = \left( \frac{Vs}{F_s d} \right) = \frac{373.63 \text{ kN} \times 609.6 \text{ mm}}{165.48 \text{ MPa} \times 7239 \text{ mm}} = 190 \text{ mm}^2$$

Use 16-mm bars horizontally at 609.6 mm on center

Now consider the flexural design. The attached spreadsheet for this problem illustrates how to calculate an allowable-stress moment-axial force interaction diagram for this wall. That spreadsheet is first used to check the wall with reinforcement consisting of 16-mm bars @ 1.22 m. Because the space between extreme bars is not quite a multiple of 1.22 m., the spacing of the interior bars is adjusted slightly to fit the 406.4-mm nominal length of the concrete masonry units.

Depending on the building's Seismic Design Category, prescriptive seismic provisions might require more reinforcement or a closer spacing of reinforcement than those calculated above.

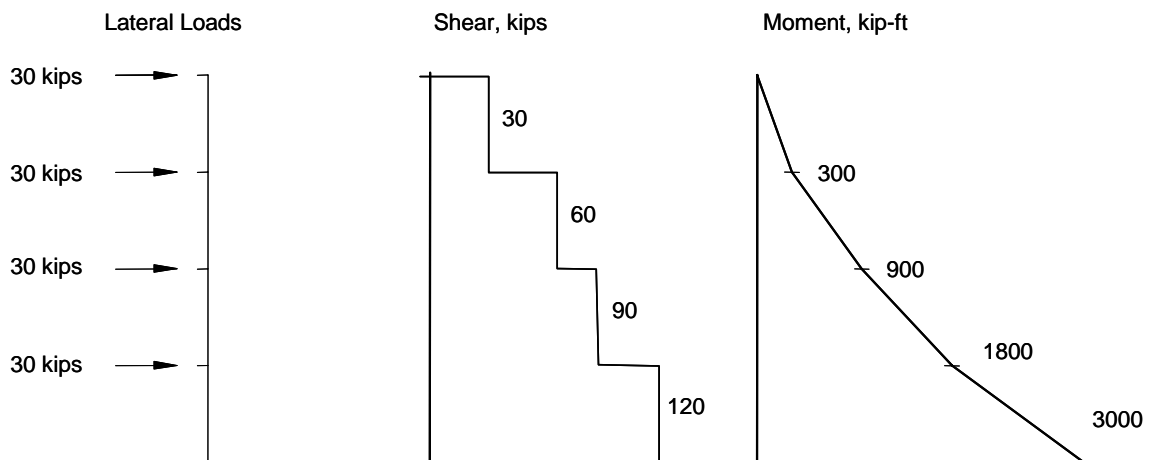
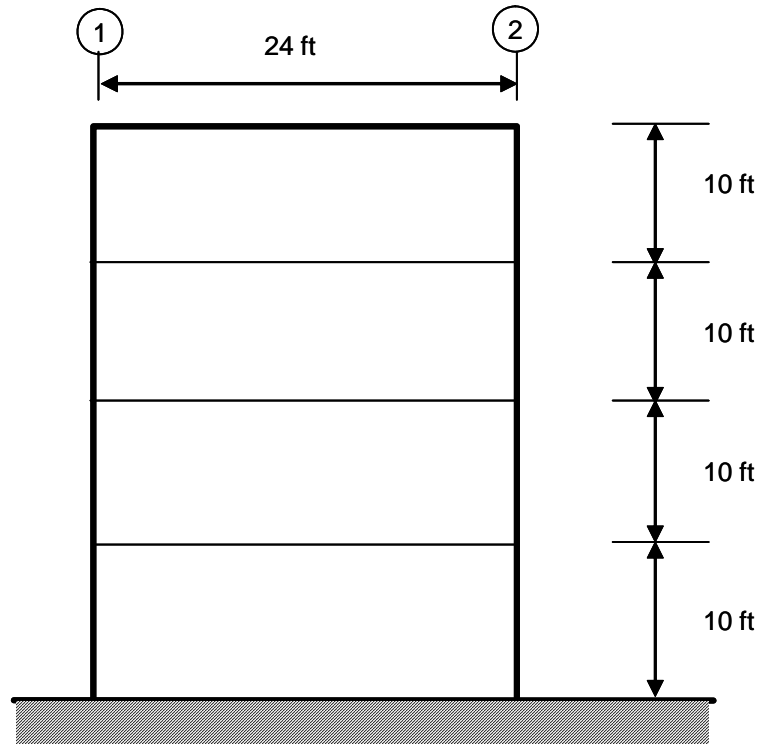
**Allowable-Stress Interaction Diagram by Spreadsheet (SI)**  
**Concrete Masonry Shear Wall of Example Problem**  
 $f'_m=10.34$  MPa, 7.32 m long, 193.8 mm thick, 16-mm bars @ 1.22 m



The interaction diagram shows that the wall, as designed above, can resist the combination of axial load and moment (960.77 kN, 2847.6 kN-m).

**Example #3: Strength Design of Reinforced Concrete Masonry Shear Wall**

Using the strength provisions of the 2005 MSJC Code (ACI 530), design the reinforced concrete masonry shear wall shown below. In the context of an entire building design, design loads would be obtained using the appropriate provisions of the legally adopted building code (which usually references ASCE 7). For this facet of the design, unfactored loads are shown below, and are assumed to be due to earthquake. Corresponding shear and moment diagrams are also shown.



Assume an 8-in. nominal concrete masonry wall, grouted solid, with Type S PCL mortar. The total plan length of the wall is 24 ft (288 in.), and its specified thickness is 7.63 in. Assume an effective depth  $d$  of 285 in.

	<i>Concrete Masonry</i>
<i>Unit Strength</i>	1,900
<i>Mortar</i>	Type S
$f'_m$ or $f'_g$ (psi)	1,500
<i>Reinforcement = Grade 60; <math>E_s = 29 \times 10^6</math> psi</i>	

Unfactored axial loads on the wall are given in the table below.

Level	DL	LL
(Top of Wall)	(kips)	(kips)
4	90	15
3	180	35
2	270	55
1	360	75

Use ASCE 7-05 Basic Strength Load Combination 7:  $0.9D + 1.0E$

Check shear for assumed wall thickness. By Section 3.3.4.1.2 of the 2005 MSJC Code,

$$V_n = V_m + V_s$$

$$M_u = 3,000 \times 12 \times 1,000 \text{ in.-lb} = 36.0 \times 10^6 \text{ in.-lb}$$

$$V_u = 120,000 \text{ lb} \quad d_v = 285 \text{ in.}$$

$$M_u / V_u d_v = \frac{36 \times 10^6 \text{ in. - lb}}{120,000 \text{ lb} (285 \text{ in.})} = 1.05$$

By Code 3.3.4.1.2.1,  $(M_u / V_u d_v)$  need not be taken greater than 1.0.

$$V_m = \left[ 4.0 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_n \sqrt{f'_m} + 0.25P$$

$$V_m = [4.0 - 1.75(1.0)] 7.63 \text{ in.} \times 285 \text{ in.} (\sqrt{1500 \text{ psi}}) + 0.25 (360,000 \text{ lb})$$

$$V_m = 189.5 + 90.0 \text{ kips} = 279.5 \text{ kips}$$

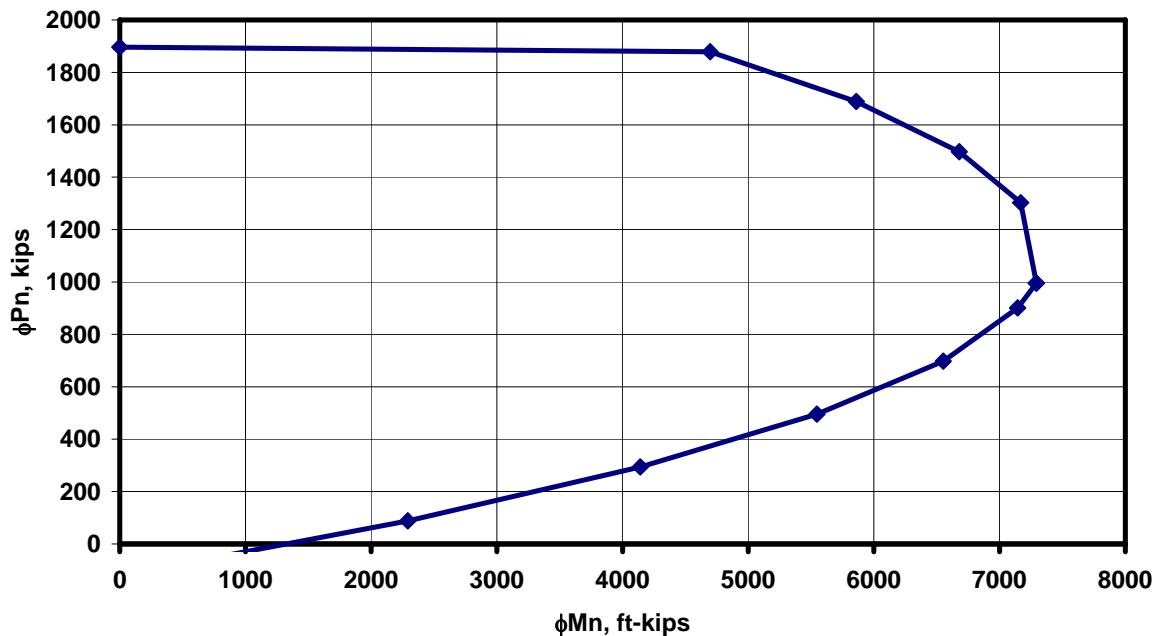
$$\phi V_n > V_u \quad \phi = 0.80 \quad V_n = V_m = 279.5 \text{ kips}$$

$$0.80(279.5 \text{ kips}) = 223.6 \text{ kips} \geq V_u = 120 \text{ kips}$$

Shear design is satisfactory so far, even without shear reinforcement. Code Section 3.1.3 will be checked later.

Now check flexural capacity using a spreadsheet-generated moment-axial force interaction diagram. Try #5 bars @ 4 ft.

**Strength Interaction Diagram by Spreadsheet**  
**Concrete Masonry Shear Wall of Example Problem**  
**f'm=1500 psi, 24 ft long, 7.63 in. thick, #5 bars @ 4'**



At a factored axial load of 0.9D, or  $0.9 \times 360 \text{ kips} = 324 \text{ kips}$ , the design flexural capacity of this wall is about 4000 ft-kips, and the design is satisfactory for flexure.

Now check Code Section 3.1.3. First try to meet the capacity design provisions of that section. At an axial load of 324 kips, the nominal flexural capacity of this wall is 4000 ft-kips, divided by the strength reduction factor of 0.9, or 4,444 ft-kips. The ratio of this nominal flexural capacity to the factored design moment is 4,444 divided by 3,000, or 1.48. Including the additional factor of 1.25, that gives a ratio of 1.85.

$$\phi V_n \geq 1.85 V_u$$

$$V_n \geq \frac{1.85}{\phi} V_u = \frac{1.85}{0.8} V_u = 2.31 V_u = 2.31 \times 120 = 277.5 \text{ kips}$$

The wall requires shear reinforcement. Prescriptive seismic reinforcement (for example, for SDC C) will probably be satisfied with #5 bars horizontally @ 24 in.

$$V_n = V_m + V_s = 379.5 \text{ kips} + \left(\frac{1}{2}\right) A_v f_y \frac{d}{s} = 279.5 + \left(\frac{1}{2}\right) 0.31 \text{ in.}^2 \times 60 \text{ ksi} \frac{285 \text{ in.}}{24 \text{ in.}}$$

$$V_n = 329.5 + 110.4 \text{ kips} = 389.9 \text{ kips}$$

Prescriptive seismic reinforcement is sufficient for shear. Use #5 bars at 2 ft.

Check  $\rho_{\max}$ , assuming that the wall is classified as an intermediate reinforced masonry shear wall.

$$\rho_{\max} = \frac{0.64 f'_m \left( \frac{\epsilon_{mu}}{\epsilon_{ma} + 3 \epsilon_y} \right) - \frac{P_u}{bd}}{f_y \left( \frac{3 \epsilon_y}{\epsilon_{mu} + 3 \epsilon_y} \right) - 0.5 \left( \frac{\epsilon_{mu}}{\epsilon_{mu} + 3 \epsilon_y} \right) \epsilon_{mu} E_s}$$

In accordance with MSJC Code Section 3.3.3.5.1(d), the governing axial load combination is  $D + 0.75 L + 0.525 Q_E$ , and the axial load is  $(360,000 + 0.75 \times 75,000 \text{ lb})$ , or 416,250 lb.

$$\rho_{\max} = \frac{0.64(1,500 \text{ psi}) \left[ \frac{0.0025}{0.0025 + 3(0.00207)} \right] - \frac{416,250 \text{ lb}}{(7.63 \text{ in.}) 285 \text{ in.}}}{(60,000 \text{ psi}) \left[ \frac{3(0.00207)}{0.0025 + 3(0.00207)} \right] - 0.5 \left[ \frac{0.0025}{0.0025 + 3(0.00207)} \right] (0.0025 \times 29 \times 10^6 \text{ psi})}$$

$$\rho_{\max} = 0.00127$$

Check maximum area of flexural reinforcement per 48 in. of wall length

$$A_{s \max} = \rho_{\max} b \times 48 \text{ in.} = 0.00127 (7.63 \text{ in.}) 48 \text{ in.} = 0.46 \text{ in.}^2$$

We have 0.31 in.<sup>2</sup> every 48 in., and the design is satisfactory.

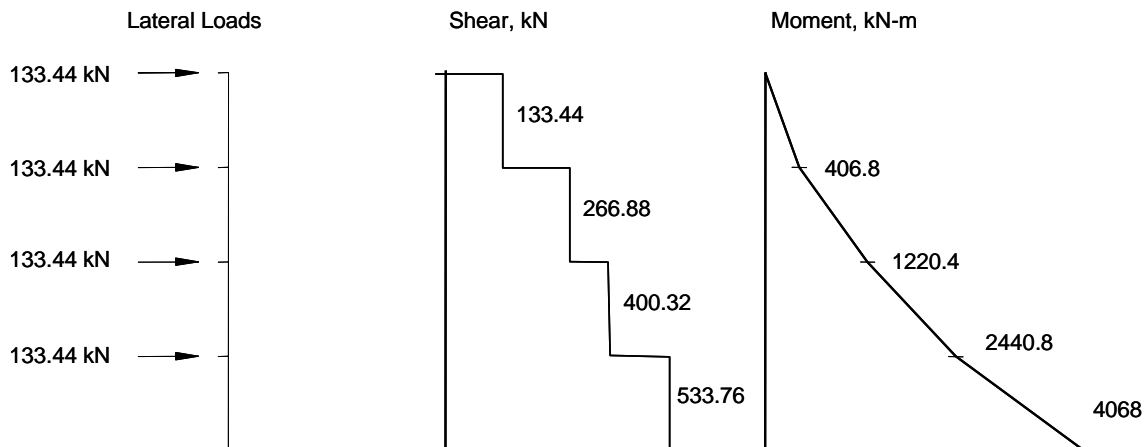
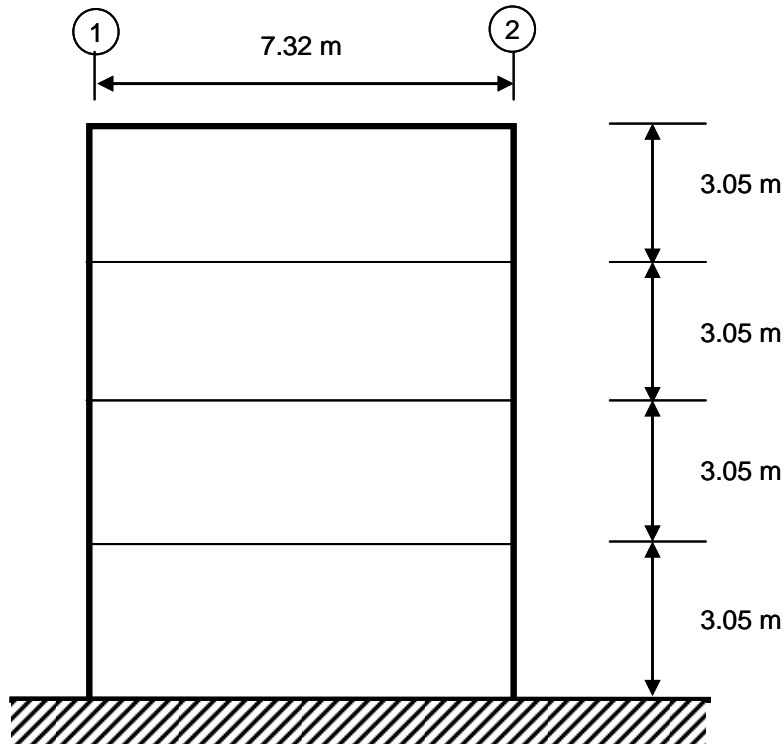
Summary:

Use #5 @ 4 ft vertically, #5 @ 2 ft horizontally.

Because the space between extreme bars is not quite a multiple of 48 in., the spacing of the interior bars is adjusted slightly to fit the nominal 16-in. length of the concrete masonry units.

**Example #4: Strength Design of Reinforced Concrete Masonry Shear Wall (SI Units)**

Using the strength provisions of the 2005 MSJC Code (ACI 530), design the reinforced concrete masonry shear wall shown below. In the context of an entire building design, design loads would be obtained using the appropriate provisions of the legally adopted building code (which usually references ASCE 7). For this facet of the design, unfactored loads are shown below, and are assumed to be due to earthquake. Corresponding shear and moment diagrams are also shown.



Assume an 203.2-mm nominal concrete masonry wall, grouted solid, with Type S PCL mortar. The total plan length of the wall is 7.315 m, and its specified thickness is 193.8 mm. Assume an effective depth  $d$  of 7.239 m.

	<i>Concrete Masonry</i>
<i>Unit Strength</i>	<i>13.10 MPa</i>
<i>Mortar</i>	<i>Type S</i>
<i>f' <sub>m</sub> or f<sub>g</sub></i>	<i>10.34 MPa</i>
<i>Reinforcement = Grade 60; E<sub>s</sub> = 199,955 MPa</i>	

Unfactored axial loads on the wall are given in the table below.

<b>Level</b>	<b>DL</b>	<b>LL</b>
(Top of Wall)	(kN)	(kN)
4	400.32	66.72
3	800.64	155.68
2	1200.96	244.64
1	1601.28	333.60

Use ASCE 7-05 Basic Strength Load Combination 7:  $0.9D + 1.0E$

Check shear for assumed wall thickness. By Section 3.3.4.1.2 of the 2005 MSJC Code,

$$V_n = V_m + V_s$$

$$M_u / V_u d_v = \frac{4068 \text{ kN} \cdot \text{m}}{533.76 \text{ kN} (7.239 \text{ m})} = 1.05$$

By Code 3.3.4.1.2.1,  $(M_u / V_u d_v)$  need not be taken greater than 1.0.

$$V_m = 0.083 \left[ 4.0 - 1.75 \left( \frac{M_u}{V_u d_v} \right) \right] A_n \sqrt{f'_m} + 0.25P$$

$$V_m = 0.083 [4.0 - 1.75(1.0)] 193.8 \text{ mm} \times 7239 \text{ mm} (\sqrt{10.34 \text{ MPa}}) + 0.25 (1601 \text{ kN})$$

$$V_m = 842.5 + 400 \text{ kN} = 1242.5 \text{ kN}$$

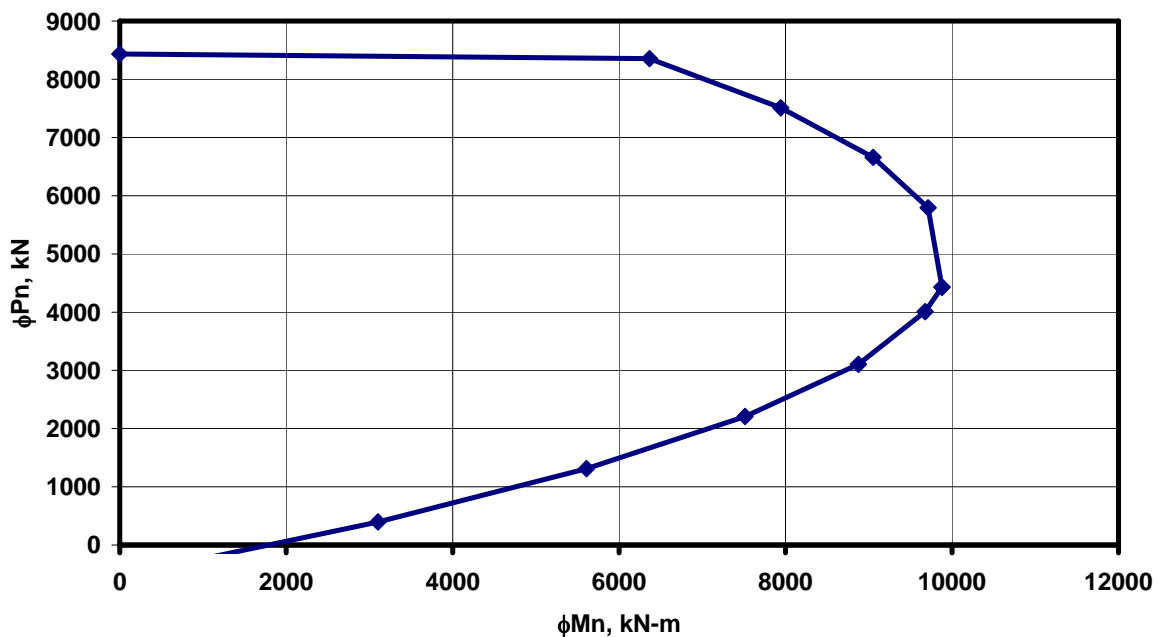
$$\phi V_n > V_u \quad \phi = 0.80 \quad V_n = V_m = 1242.5 \text{ kN}$$

$$0.80(1242.5 \text{ kN}) = 994.0 \text{ kN} \geq V_u = 533.76 \text{ kN}$$

Shear design is satisfactory so far, even without shear reinforcement. Code Section 3.1.3 will be checked later.

Now check flexural capacity using a spreadsheet-generated moment-axial force interaction diagram. Try 16-mm bars @ 1.22 m.

**Strength Interaction Diagram by Spreadsheet (SI units)**  
**Concrete Masonry Shear Wall of Example Problem**  
 f'm=10.34 MPa, 7.32 m long, 193.8 in. thick, 16-mm bars @ 1.22 m



At a factored axial load of  $0.9D$ , or  $0.9 \times 1601 \text{ kN} = 1441 \text{ kN}$ , the design flexural capacity of this wall is about  $5400 \text{ kN-m}$ , and the design is satisfactory for flexure.

Now check Code Section 3.1.3. First try to meet the capacity design provisions of that section. At an axial load of  $1441 \text{ kN}$ , the nominal flexural capacity of this wall is  $5424 \text{ kN-m}$ , divided by the strength reduction factor of  $0.9$ , or  $6,027 \text{ kN-m}$ . The ratio of this nominal flexural capacity to the factored design moment is  $6,027$  divided by  $4068$ , or  $1.48$ . Including the additional factor of  $1.25$ , that gives a ratio of  $1.85$ .

$$\phi V_n \geq 1.85 V_u$$

$$V_n \geq \frac{1.85}{\phi} V_u = \frac{1.85}{0.8} V_u = 2.31 V_u = 2.31 \times 533.76 = 1233 \text{ kN}$$

The wall requires shear reinforcement. Prescriptive seismic reinforcement (for example, for SDC C) will probably be satisfied with 16-mm bars horizontally @  $609.6 \text{ mm}$ .

$$V_n = V_m + V_s = 1242.5 \text{ kN} + \left(\frac{1}{2}\right) A_v f_y \frac{d}{s} = 1242.5 + \left(\frac{1}{2}\right) 197.9 \text{ mm}^2 \times 413.7 \text{ MPa} \frac{7239 \text{ mm}}{609.6 \text{ mm}}$$

$$V_n = 1242.5 + 491.1 \text{ kips} = 1734 \text{ kN}$$

Prescriptive seismic reinforcement is sufficient for shear. Use 16-mm bars at 609.6 mm.

Check  $\rho_{\max}$ , assuming that the wall is classified as an intermediate reinforced masonry shear wall.

$$\rho_{\max} = \frac{0.64 f'_m \left( \frac{\epsilon_{mu}}{\epsilon_{ma} + 3 \epsilon_y} \right) - \frac{P_u}{bd}}{f_y \left( \frac{3 \epsilon_y}{\epsilon_{mu} + 3 \epsilon_y} \right) - 0.5 \left( \frac{\epsilon_{mu}}{\epsilon_{mu} + 3 \epsilon_y} \right) \epsilon_{mu} E_s}$$

In accordance with MSJC Code Section 3.3.3.5.1(d), the governing axial load combination is  $D + 0.75 L + 0.525 Q_E$ , and the axial load is  $(1601 + 0.75 \times 333.6 \text{ kN})$ , or 1851.2 kN.

$$\rho_{\max} = \frac{0.64(10.34 \text{ MPa}) \left[ \frac{0.0025}{0.0025 + 3(0.00207)} \right] - \frac{1851.2 \text{ kN} \times 1000}{(193.8 \text{ mm}) 7239 \text{ mm}}}{(413.7 \text{ MPa}) \left[ \frac{3(0.00207)}{0.0025 + 3(0.00207)} \right] - 0.5 \left[ \frac{0.0025}{0.0025 + 3(0.00207)} \right] (0.0025 \times 199,995 \text{ MPa})}$$

$$\rho_{\max} = 0.00127$$

Check maximum area of flexural reinforcement per 1.22 m of wall length

$$A_{s \max} = \rho_{\max} b \times 1220 \text{ mm} = 0.00127 (193.8 \text{ mm}) 1220 \text{ mm} = 300.27 \text{ mm}^2$$

We have 197.9 mm<sup>2</sup> every 1.22 m, and the design is satisfactory.

Summary:

Use 16-mm bars @ 1.22 m vertically, 16-mm bars @ 606.9 mm horizontally.

Because the space between extreme bars is not quite a multiple of 1.22 m, the spacing of the interior bars is adjusted slightly to fit the nominal 406.4-mm length of the concrete masonry units.