Designation: D 6982 – 03

Standard Practice for
Detecting Hot Spots and Buried Objects Using Point-Net (Grid) Search Patterns[1]

This standard is issued under the fixed designation D 6982; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice provides equations and nomographs, and a reference to a computer program, for calculating probabilities of detecting hot spots (that is, localized areas of soil or groundwater contamination) and buried objects using point-net (that is, grid) search patterns. Hot spots, more generally referred to as targets, are presumed to be invisible on the ground surface. Buried objects may include former surface impoundments, waste disposal pits, and utilities that have been covered by soil or paving materials. Hot spots may also include contaminant plumes in ground water or soil gas.

1.2 For purposes of calculating detection probabilities, hot spots or buried objects are presumed to be elliptically shaped when projected vertically to the ground surface, and search patterns are square, rectangular, or rhombic. Assumptions about the size and shape of suspected hot spots are the primary limitations of this practice, and must be judged by historical information. A further limitation is that hot spot boundaries are assumed to be clear and distinct. Alternative approaches to hot spot detection using discrete sampling should also be considered where feasible, such as surface geophysical measurements (see Guide D 6429).

1.3 Search sampling would normally be conducted during preliminary investigations of hazardous waste sites or hazardous waste management facilities (see Guide D 5730). Sampling may be conducted via drilling or by direct-push methods. In contrast, guidance on sampling for the purpose of making statistical inferences about population characteristics (for example, contaminant concentrations) can be found in Guide D 6311.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:* [2]

D 5730 Guide for Site Characterization for Environmental Purposes with Emphasis on Soil, Rock, the Vadose Zone and Ground Water

D 6051 Guide for Composite Sampling and Field Subsampling for Environmental Waste Management Activities

D 6311 Guide for Generation of Environmental Data Related to Waste Management Activities: Selection and Optimization of Sampling Design

D 6429 Guide for Selecting Surface Geophysical Methods

3. Terminology

3.1 *Definitions:*

3.1.1 *hot spot*—a localized area of soil or groundwater contamination.

3.1.1.1 *Discussion*—A hot spot may be considered as a discrete volume of buried waste or contaminated soil where the concentration of a contaminant of interest exceeds some prespecified threshold value. Although elliptically shaped hot spots or targets are assumed for the purposes of calculating probabilities of detecting hot spots, hot spots are more likely to have variable sizes and shapes and not have clear and distinct boundaries. However, the concept of hot spots is consistent with known historical patterns of contaminant distributions.

3.1.2 *sampling density*—the number of borings (that is, sampling points) per unit area.

3.1.3 *semi-major axis, a*—one-half the length of the long axis of an ellipse. For a circle, this distance is simply the radius.

3.1.4 *semi-minor axis, b*—one-half the length of the short axis of an ellipse.

3.1.5 *target*—the object or "hot spot" that is being searched for.

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[1] This practice is under the jurisdiction of ASTM Committee D34 on Waste Management and is the direct responsibility of Subcommittee D34.01 on Sampling and Monitoring.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For Annual Book of ASTM Standards volume information, refer to the standard's Document Summary page on the ASTM website.

3.1.6 *threshold concentration*—the concentration of a contaminant above which a hot spot is considered to be detected.

3.1.7 *unit cell*—the smallest area into which a grid can be divided so that these areas have the same shape, size and orientation. For a triangular grid, the unit cell is a 60°/120° rhombus comprised of two equilateral triangles with a common side.

3.2 *Symbols*:

a = length of the semi-major axis of an ellipse

b = length of the semi-minor axis of an ellipse

A_T = area of target or hot spot. For an ellipse, $A_T = \pi ab$.

A_S = search area

S = the "shape" of an elliptical target (that is, the ratio of the length of the semi-minor axis to the length of the semi-major axis of an ellipse, b/a)

G = the distance between nearest grid nodes of a unit cell

Q = the ratio of the length of the long side of a rectangular grid cell to the length of the short side

A_C = the area of the unit cell. For a square, $A_{sq} = G^2$. For a rectangle $A_{re} = Q \cdot G^2$. For a 60°/120° rhombus, $A_{rh} = [(\sqrt{3})/2]G^2$. The inverse of A_C is the sampling density

β = the probability of not detecting a hot spot (that is, "consumer's risk")

$P(hit)$ = probability of detection (that is, $1 - \beta$)

4. Significance and Use

4.1 Search sampling strategies have found wide utility in geologic exploration where drilling is required to detect subsurface mineral deposit, such as when drilling for oil. Using such strategies to search for buried wastes, subsurface contaminants, and underground structures is a logical extension of these strategies.

4.2 Systematic sampling strategies are often the most cost-effective method for searching for hot spots or buried objects.

4.3 Search sampling patterns may also be used to optimize the locations of ground water monitoring wells or to optimize the location of vadose zone monitoring devices.

4.4 This practice may be used to determine the risk of missing a target of specified size and shape given a specified sampling pattern and sampling density.

4.5 This practice may be used to determine the smallest target that can be detected with a specified probability and given sampling density.

4.6 This practice may be used to select the optimum grid sampling strategy (that is, sampling pattern and density) for a specified risk of not detecting a hot spot or buried object.

4.7 By using the algorithms given in this practice, one can balance the cost of sampling versus the risk of missing a hot spot or buried object.

5. Assumptions

5.1 One or more targets (for example, hot spots) exist and are equally likely to occur in any part of the search area.

5.2 When projected vertically upward to a level ground surface, the target appears as an ellipse or a circle (Fig. 1). The probable size and shape of a hot spot can only be guessed from past site or facility records, known layout of the site or facility, and personal knowledge.

5.3 The search pattern is either a square, a rectangular, or an equilateral triangular grid. Borings are made at the intersections of grid lines (that is, nodes) (Fig. 2).

5.4 Borings or direct-push devices are directed downward vertically and the detection of the target is unambiguous. For detection of buried solid objects, this should present little difficulty. However, for buried contaminants, such an assumption presumes that the full depth of a boring would be subject to analysis as contiguous intervals of the boring. If sampling intervals are discontinuous, then contamination might be missed if it occurred between sampled intervals. If sampling

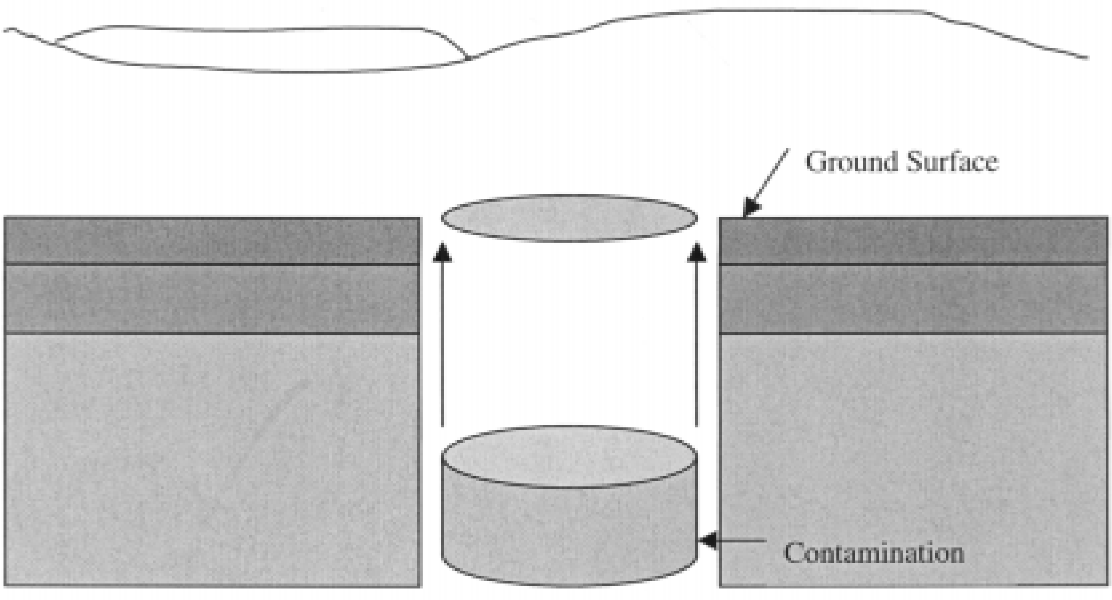

FIG. 1 Projection of Boundaries of Subsurface Contamination to the Ground Surface

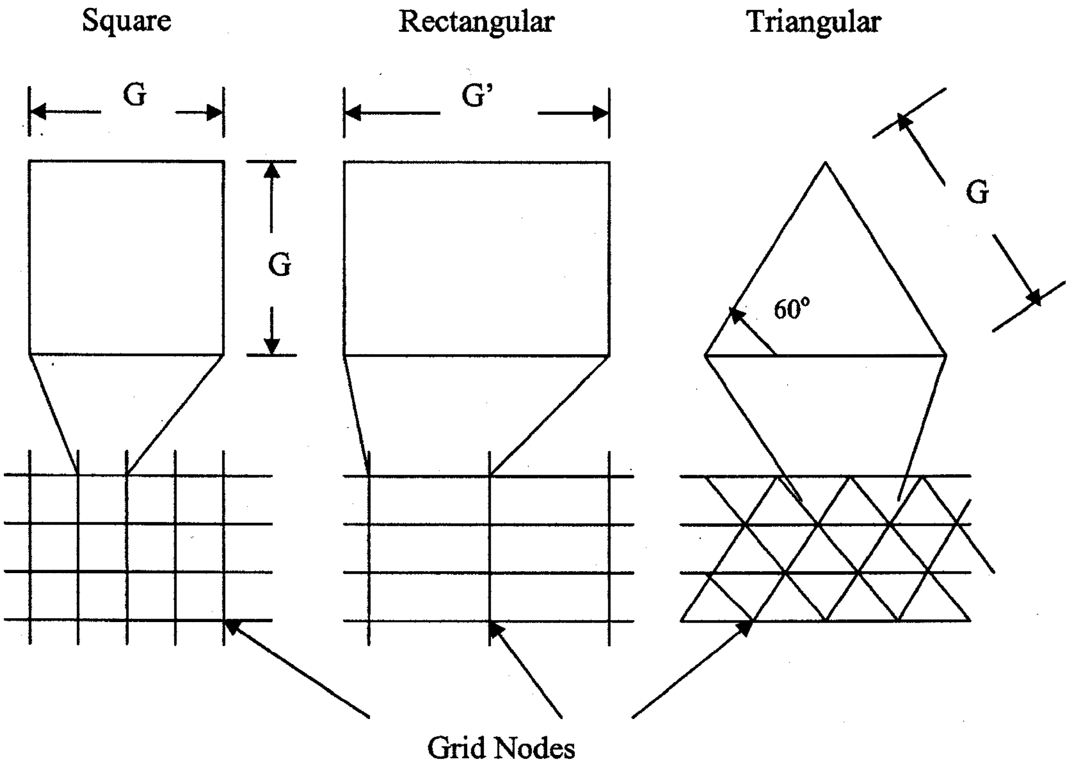

FIG. 2 Grid Patterns for Detecting Hot Spots. Borings are Made at the Grid Nodes

intervals are too long, then a hot spot may not be detected because of dilution of a hot spot with less contaminated portions of the sampled interval. The criteria for detection of contaminants may be prespecified threshold concentrations (for example, screening levels) that would trigger further investigation of sites or facilities.

5.5 The area of the borehole or direct-push device is infinitely small compared to the target area. The algorithms used in this practice assume that borehole or direct-push devices have no area, but rather are treated as a vertical line.

6. Preliminary Considerations

6.1 Before designing a hot spot detection strategy, a preliminary investigation of the area containing possible hot spots or targets should be conducted. From historical records, physical layout of buildings and equipment, known transportation pathways, landscape features, and eyewitness accounts, one may be able to identify areas with a high probability of subsurface contamination or the presence of buried waste or objects such as underground storage tanks. Areas with different expected probabilities of detection of a hot spot or other target should be clearly mapped.

6.2 Within areas of relatively uniform expected probability of hot spot or target detection, sampling grids of prespecified grid spacing G and type (for example, square, rectangular, or triangular) may be overlain. Areas with higher expected occurrence of hot spots or buried objects should have correspondingly higher sampling densities compared to areas with lower expected occurrence. However, areas with greater hazard from missing a hot spot should also have correspondingly higher sampling densities than areas with a lesser hazard. Ideally, the starting point for each grid and its orientation should be randomly determined.

6.3 When searching for hot spots, threshold concentrations for detection may be established by a regulatory authority. Whether or not a threshold concentration is exceeded will depend upon the physical distribution of the contaminant, the volume of the sampling device, the sampling intervals selected, and the sensitivity of the analysis. If contamination occurs in a discrete layer, then the probability of detecting a hot spot will decrease with increasing volume of material sampled in a bore hole or if the sampling interval exceeds the depth of the discrete hot spot layer. The analytically determined contaminant concentration may then be less than the threshold concentration because of the dilution of the hot spot layer with other layers of soil or waste. Further, a hot spot confined to a discrete layer may be missed entirely by not sampling that layer. For this reason, continuous sampling is recommended.

6.4 Detection of contaminant levels in samples above threshold concentrations or the detection of buried objects may trigger further action requiring possibly more detailed drilling and sampling to better define spatially the location of hot spots or buried objects. Again, a grid sampling strategy will be the most efficient. If new boring locations are centered at the midpoints of the unit cells, the total number of borings will be exactly doubled.

7. Computing Hot Spot Detection Probabilities

7.1 *Case I*—If the longest dimension of an elliptical target is less than or equal to the grid spacing (that is, $2a \leq G$), then the target can only be hit once and the probability P of detecting the hot spot is simply equal to the ratio of the area of the target A_T to the area of the unit cell A_C (that is, $P = A_T/A_C$).

7.2 *Case 2*—If the longest dimension of an elliptical target is greater than the grid spacing (that is, $2a > G$), then the target may be hit more than once. In this case, algorithms developed by Singer and Wickman **(1)**[3] employing affine transformations and programmed in FORTRAN by Singer **(2)** are required to calculate the exact probability of detecting the target. This program is limited to ellipses having a shape S between 0.05 and 1.0 and the ratio a/G between 0.05 and 1.0. Singer's algorithms have been adapted by J. R. Davidson **(3)** to the personal computer (PC) running under the MS DOS operating system. Supporting documentation for this program, ELIPGRID-PC, is available from Oak Ridge National Laboratory **(4, 5)**.

7.3 *Randomly Oriented Elliptical Target*—The probability of detecting a target, *P(hit)*, of a specified size a shape S and for a specified grid G spacing can be obtained from nomographs shown in Figs. 3 and 4 for square and equilateral triangular grid sampling patterns, respectively. Data for these nomographs were generated using the ELIPGRID-PC program. To use these graphs, first calculate the ratio a/G. Then draw a vertical line from the point represented by the ratio a/G on the x-axis of the graph to the curve representing the prespecified shape of the ellipse. Then draw a horizontal line to the y-axis. For shapes other than those shown on the graphs, one must interpolate between curves with closest values of S. The value on the y-axis represents the probability of at least one hit of the target. Using these same graphs, one can also determine the required grid spacing to detect an elliptical target of shape at a prespecified probability of detection. In this case, draw a horizontal line from the prespecified probability of a hit to the curve representing the prespecified shape of the ellipse. Then draw a vertical line down to the x-axis. From the ratio a/G at the point of intersection with the x-axis, one can determine the minimum required grid spacing. Similarly, one can also determine the smallest sized hot spot of a given shape that can be detected for a given grid spacing and probability of detection by calculating a from the ratio a/G and grid spacing G. Alternatively, one can use the computer program ELIPGRID-PC.

7.4 *Oriented Elliptical Target*—If the orientation of the elliptical target with respect to the grid lines is specified, then the probability of detecting the target must be determined using the computer program ELIPGRID-PC.

8. Comparing the Relative Efficiencies of Search Patterns

8.1 The efficiency of a search pattern is measured as the probability that a target (for example, hot spot) will be hit at least once. Given the same sampling density, a sampling pattern with a higher probability of hitting a target will be more efficient than a sampling pattern with a lower probability of

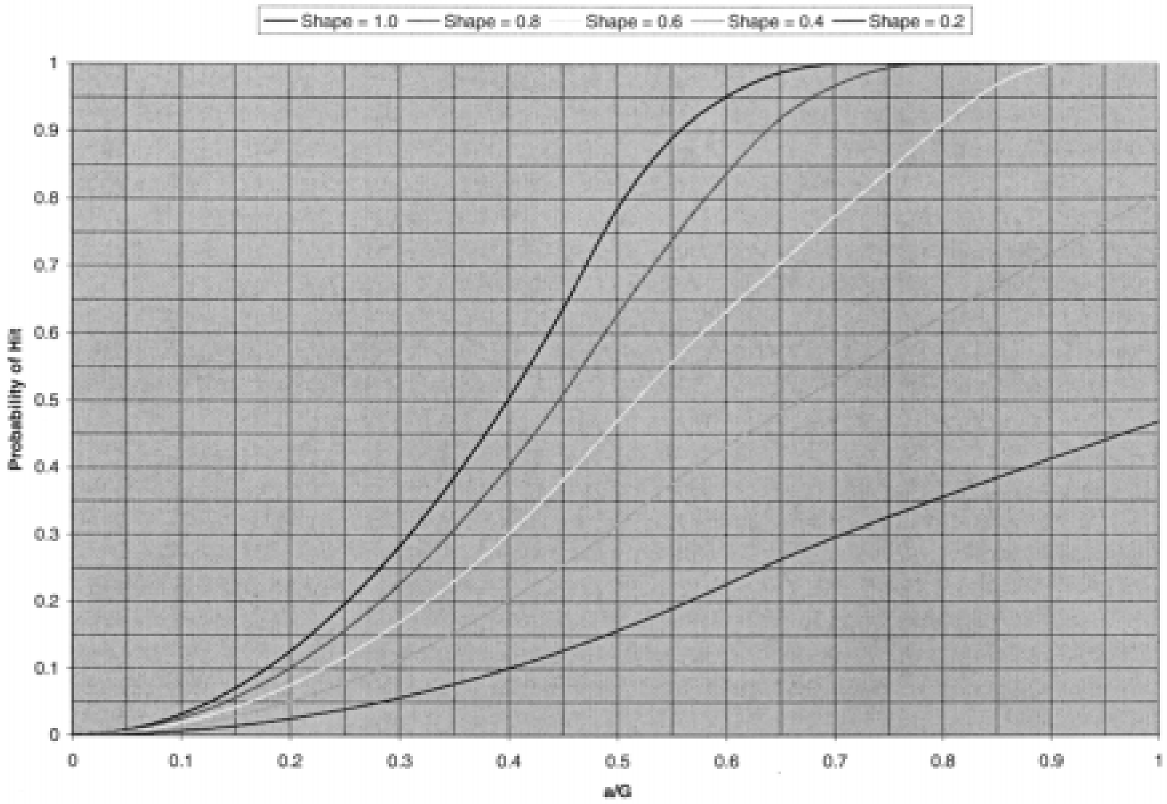

FIG. 3 Nomograph Relating the Probability of Detecting a Single Hot Spot to the Ratio *a/G* for Selected Shapes (*b/a*) Using a Square Grid with Grid Spacing *G*.

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[3] The boldface numbers in parentheses refer to the list of references at the end of this standard.

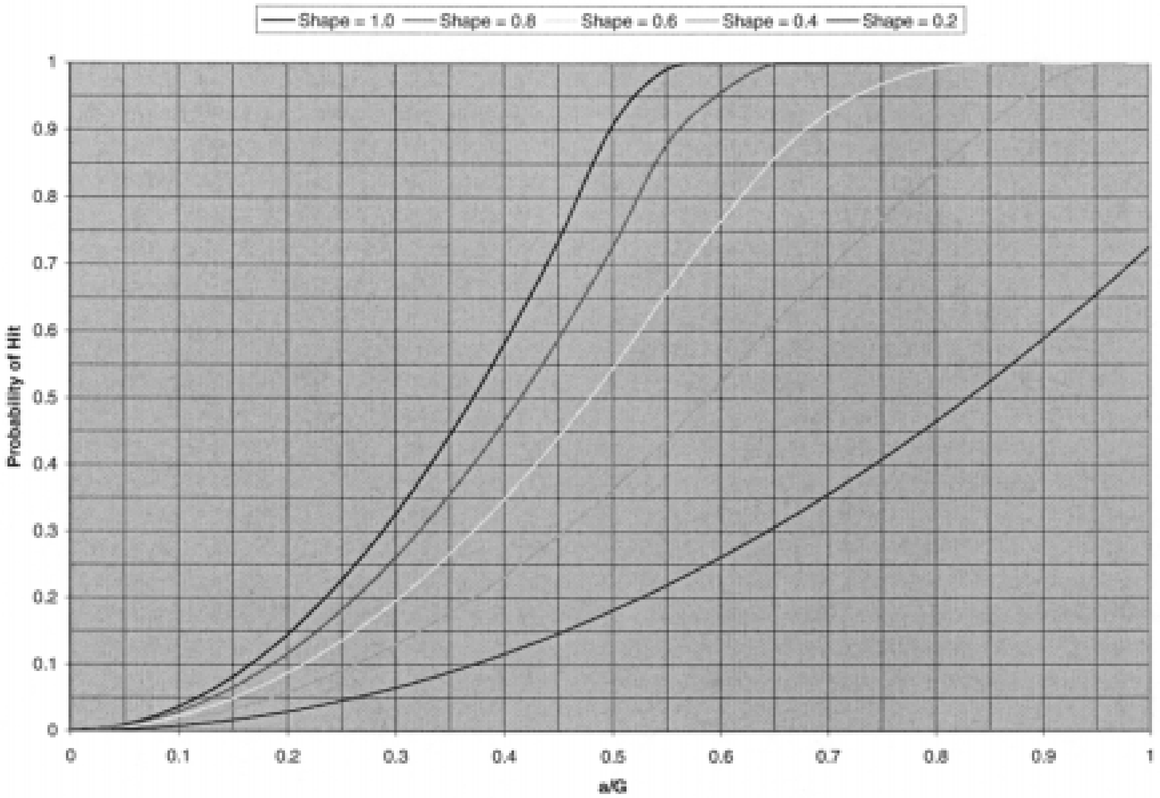

FIG. 4 Nomograph Relating the Probability of Detecting a Single Hot Spot to the Ratio *a/G* for Selected Shapes (*b/a*) Using a Triangular Grid with Grid Spacing *G*.

hitting the same target. The relative efficiency, *RE*, of one sampling pattern over another when searching for a target is measured as the percent difference in the efficiency of two equivalent density sampling patterns. For example, $RE = 100\,\%\ (P_{TRI} - P_{SQR})/P_{SQR}$ where P_{TRI} and P_{SQR} are the probabilities of detecting a target with a triangular grid and a square grid, respectively. By extension, for the same probability of detecting a target, a more efficient sampling pattern will require fewer borings, and will thus be more economical. In this section, the relative efficiencies of hitting randomly oriented (that is, orientation unknown) and oriented elliptical targets of prespecified size and shape are compared using different sampling patterns having equivalent sampling densities.

8.2 *Randomly Oriented Elliptical Targets*:

8.2.1 *Square versus Triangular Grid*—When the criterion for detection is one or more hits, the triangular grid is up to 6 % more efficient than the square grid (for a circular target where $a/G = 0.55$) while a square grid is never more than 0.2 % more efficient **(6)**. Efficiencies are the same for *a/G* ratios less than 0.5 since a target can be hit no more than once. For $a/G > 0.5$ and if two or more hits are required for detection, then a square grid is overall more efficient than a triangular grid.

8.2.2 *Point-net versus Random*—When one or more hits is required for detection, then point-net search sampling strategies are more efficient than random sampling strategies for detecting subsurface contamination or buried objects. This can be easily shown by comparing the probability of detecting a hot spot using a grid sampling approach to the probability of detecting a hot spot by random sampling (see Appendix X1). Where two or more hits are required for detection and $a/G < 0.5$, then a random search is more efficient **(6)**.

8.3 *Targets with Known Orientation*:

8.3.1 *Square versus Triangular Grid*—When one or more hits is required for detection, a triangular grid is generally more efficient when the angle of orientation is close to 30°, 90°, or 150° whereas a square grid is more efficient when the angle of orientation is between 25° and 65° or between 115° and 155°. Between 25° and 35°, efficiencies are nearly the same. These orientations minimize the probabilities of hitting the same target more than once which would result in less efficient sampling.

8.3.2 *Rhombic Grid versus Square and Triangular Grids*—A rhombus is a parallelogram having opposite sides equal in length. A rhombus is also a square if the inside angles are 90°. Two equilateral triangles having a common side become a rhombus with inside angles of 60° and 120°. If the angle of orientation of an elliptical target is known, then it has been shown that a rhombic search pattern is optimal if the long diagonal of the rhombus is oriented parallel to the long axis of the elliptical target **(7)**. Table 1 gives multiplication factors necessary to calculate the lengths of the diagonals of a rhombic grid for a desired probability of detecting an elliptical target of known size and shape **(8)**. For a given probability of detection *p*, the optimum diagonal distances are $d_1 = 2a \cdot f_1(p)$ and $d_2 = 2b \cdot f_2(p)$.

TABLE 1 Multiplication Factors to Calculate the Optimum Lengths of the Diagonals of a Rhombic Grid Oriented such that the Long Axis of the Elliptical Target is Parallel to the Longest Diagonal of the Rhombic Grid

p	$f_1(p)$	$f_2(p)$	p	$f_1(p)$	$f_2(p)$
0.05	7.37658	4.25888	0.55	2.22411	1.28409
0.10	5.21605	3.01148	0.60	2.12943	1.22943
0.15	4.25888	2.45886	0.65	2.04590	1.18120
0.20	3.68828	2.12943	0.70	1.97148	1.13823
0.25	3.29890	1.90462	0.75	1.90462	1.09964
0.30	3.01148	1.73868	0.80	1.84415	1.06472
0.35	2.78810	1.60971	0.85	1.78907	1.03292
0.40	2.60801	1.50574	0.90	1.73867	1.00382
0.45	2.45886	1.41962	0.95	1.67320	0.96602
0.50	2.33267	1.34676	1.00	1.50000	0.86602

8.3.3 *Example 1*—If there exists an elliptical target of known orientation with major axis ($2a$) of length 200 ft and minor axis ($2b$) of length 100 ft, what are the optimum lengths of the diagonals of a rhombic grid that would yield a 90 % probability of detecting this elliptical target? From Table 1, $d_1 = 200 \cdot f_1(0.90) = 200 \cdot 1.73867 = 347.7$ ft and $d_2 = 100 \cdot f_2(0.90) = 100 \cdot 1.00382 = 100.3$ ft.

9. Computing the Number of Borings and Grid Spacings for Specified Costs

9.1 Costs for conducting a search for hot spots or buried objects can be roughly split between the cost of mobilization and demobilization C_M and a cost of each boring and associated laboratory or field analysis, C_S, times the number of borings, n. The total cost would equal $C_M + nC_S$. With a known budget, C_T, and an estimate for C_M and C_S, one can determine the number of borings that can be taken for a given area of coverage, A_S. First calculate the number of borings, $n = C_T - C_M / C_S$. The required area of the unit cell A_C would then be equal to A_S/n. The appropriate grid spacing G can then be determined from the formula given for the different types of unit cells under the terminology section. Using these formulae, it can be shown that for the same sampling density, the grid spacing G for a triangular grid is slightly larger than that for a square grid by a factor of $\sqrt{2/\sqrt{3}} = 1.0746$.

9.2 *Example 2*—If a hot spot search was budgeted for \$50,000 with the cost for mobilization and demobilization set at \$25,000 and a per boring cost set at \$1000 and the area to be searched is 10 000 m^2 (one hectare), (*1*) what is the maximum number of borings that can be made, and (*2*) what grid spacing would be required for a square grid and for a triangular grid? The number of samples n would be $\$50,000 - \$25,000 / \$1000 = 25$. The unit cell size A_C would be $10\,000\ m^2/25 = 400\ m^2$. For a square grid, the grid spacing G would be $\sqrt{A_C} = \sqrt{400} = 20m$. For a triangular grid, the grid spacing G would be $\sqrt{2A_C / \sqrt{3}} = 21.49m$. For a given G, one can then use the nomographs to determine the probability of detecting a hot spot of a specified size a and shape S. Testing various values for A_C and S will reveal that a triangular grid is more efficient than a square grid for most values of a and S.

10. Probability of Detection with Multiple Hot Spots

10.1 The probability of detecting a hot spot can easily be extended to two or more hot spots if the number of hot spots is known, it is assumed that each hot spot has an equal probability of being detected, and the locations of the hot spots are independent of one-another. Because the probabilities of detection are assumed to be independent and equal, one can take advantage of the binomial probability distribution, $b(x;n;P) = \binom{n}{x} P^x (1 - P)^{n-x}$ which gives the probability of x successes (that is, detections) in n independent trials (that is, targets) with probability of success P. The quantity, $\binom{n}{x}$, the number of combinations of n distinct objects (for example, samples) taken x at a time, is equivalent to $n! / x!(n - x)!$. The advantages of using the binomial formula are that the following probabilities are easily determined: (*1*) the probability of detecting any number of x hot spots out of a total number of hot spots n, (*2*) the probability of not detecting any of the hot spots (that is, $P(no\ hit) = (1 - p)^n$), and (*3*) the probability of detecting at least one hot spot out of a total of n hot spots (that is, $P(hit) = 1 - (1 - p)^n$).

10.2 *Example 3*—If it is known that three identical objects are located randomly and independently within a search area and each has a probability of detection of 50 % for any search pattern, what is the probability of not detecting any of these objects? Since the number of hits, x, is 0 and $n = 3$, $b(x;n;P) = \binom{3}{0} 0.50^0 (0.50)^3 = 0.125$. The probability of hitting one, two or three targets can be similarly determined by appropriate substitution.

11. Effect of Composite Sampling and Sampling Interval on Hot-Spot Detection

11.1 Where the cost of analysis is high relative to the cost of sampling, it may be more economically advantageous to composite soil or waste samples. The same grid patterns would be used as previously described. However, individual samples would be composited from nearest neighbor boring locations. In composite sampling, samples to be composited should have the same size, shape and orientation. If the soil or waste material is horizontally layered, or will be removed in layers, compositing over similar horizons or layers may be most appropriate. Where contaminants of interest have vapor pressures that would result in loss of contaminant if exposed to the atmosphere, samples should not be composited and care should be taken to avoid losses by volatilization during sampling and shipping. Please refer to Guide D 6051 for additional guidance on composite sampling.

11.2 The threshold concentration for hot-spot detection would necessarily be lower for a composite sample given that individual (component) sample concentrations will have been physically averaged. In the most conservative approach, the threshold concentration for composite samples would be equal to the threshold concentration for single samples divided by the number of samples comprising the composite sample.

12. Keywords

12.1 preliminary investigation; sampling; site investigation; soil investigation; subsurface exploration; systematic sampling

APPENDIX

(Nonmandatory Information)

X1. COMPARING THE EFFICIENCY OF A POINT-NET SEARCH PATTERN TO A RANDOM SEARCH FOR DETECTING ELLIPTICAL HOT SPOTS

X1.1 The question often arises as to whether a random search or a systematic sampling pattern is the most efficient method for detecting targets. This section demonstrates that where only one hit is required for detection of a randomly oriented target that systematic sampling is more efficient. For samples obtained at random within a defined search area, the probability of hitting a target can be calculated from the binomial distribution as:

$$P(r) = \frac{n!}{(n-k)!r!} p^r (1-p)^{(n-r)} \tag{X1.1}$$

where p is the proportion of the search area, A_S, occupied by the target; r is the number of hits; and n is the total number of samples taken. However, since this equation would have to be evaluated for all possible number of hits, it is simpler to solve this equation for the probability of no hits:

$$P(0) = \frac{n!}{(n-0)!0!} p^0 (1-p)^{(n-0)} \tag{X1.2}$$

$$P(0) = (1-p)^n$$

where P (0) is the "consumer's risk" for a random sampling pattern. The complementary probability of one or more hits is therefore:

$$P(hit) = 1 - P(0) = 1 - (1-p)^n \tag{X1.3}$$

However, $p = A_T/A_S$ where A_T is the area of the target. Hence:

$$P(hit) = 1 - P(0) = 1 - (1 - A_T/A_S)^n \tag{X1.4}$$

X1.2 It can easily be shown that random sampling is less efficient than a square grid sampling design for detecting an elliptical hot spot. For an elliptical target, the proportion p of the total search area occupied by the target is:

$$p = \frac{A_T}{A_S} = \frac{\pi ab}{A_S} \tag{X1.5}$$

where πab is the area of the elliptical target and A_S is the search area, and a and b are the semi-major and semi-minor axes of the ellipse, respectively. Since the area of the square grid (D^2) equals A_S/n:

$$p = \frac{\pi ab}{nD^2} \tag{X1.6}$$

Since the shape of an ellipse $S = b/a$:

$$p = \frac{\pi}{n}\left(\frac{a}{D}\right)^2 S \tag{X1.7}$$

One can now directly compare the probability of detecting an elliptical target using a random search versus a systematic search for different values of a, S, and D.

X1.2.1 For comparable sampling densities, as the size of the circular target increases relative to the grid spacing, the probability of missing the target decreases more rapidly for the square grid sampling pattern than for the random sampling pattern. Further, given the same sampling densities, the probability of missing a target increases with increasing size of the search area.

X1.2.2 It was noted by Singer **(6)**, however, that if two or more hits on a single target are desirable for detection, then random sampling may be more efficient as the length of the semi-major axis increases relative to the grid spacing.

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