MATH 2107-601 **EXAM #1** NAME: ______________________ (Print)

MONDAY

FEBRUARY 19, 2001 NAME: ______________________ (Signature)

[6:00 p.m. - 7:15 p.m.] STUDENT I. D.: ______________________

1. (a) By definition, $f'(x) =$

[3 points]

(b) Geometrically, $f'(x)$ is equal to

[4 points]

(c) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function and that $a \in \mathbb{R}$. Then, by definition,

$$\lim_{x \to a} f(x) = L$$

if and only if

[6 points]

(d) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function and that $a \in \mathbb{R}$. Then, by definition, f is **continuous at** a if and only if

[3 points]

2. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function. What characteristic of the graph of f enables you to tell at a glance

(a) that f is everywhere continuous?

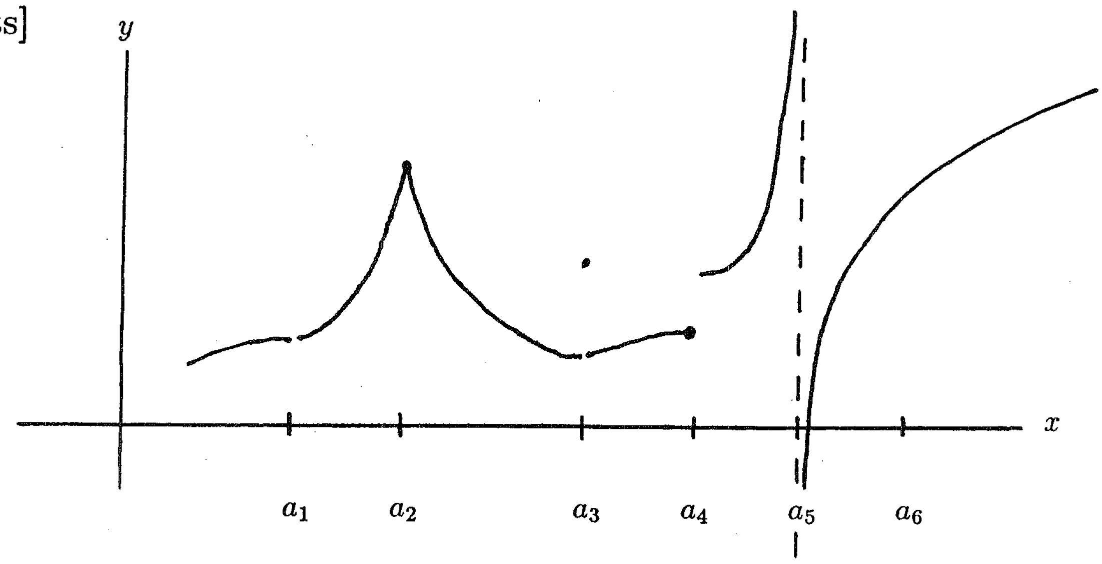
[1 point]

(b) that f is everywhere differentiable?

[3 points]

3. With reference to the graph below,

[11 points]



list all points p from the set $\{a_1, a_2, a_3, a_4, a_5, a_6\}$ for which it is true that

(a) f is **continuous** at p. **ANSWER:**

(b) f is **differentiable** at p. **ANSWER:**

(c) $\lim_{x \to p} f(x)$ **exists**. **ANSWER:**

(d) f is **defined** at p. **ANSWER:**

NOTE: Some points may possibly belong to more than one category. **BUT**, *Nota Bene*, the number of wrong answers will be subtracted from the number of right answers. This is to discourage "padded answers."

4. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ and a point $a \in \mathbb{R}$ such that f is continuous at a but $f'(a)$ does not exist.

[To get credit, you must give at least some indication of why f is continuous at a and why $f'(a)$ does not exist.]

[4 points]

5. Compute $f'(x)$ directly from the definition in case

$$f(x) = \sqrt{8x+4}\ .$$

NOTE: No credit will be given for just the answer.

[10 points]

6. Evaluate the following limits. **[No work, no credit.]**

[55 points: 5 each]

(a) $\lim_{x \to a} \frac{x^4 - a^4}{x - a} =$

(b) $\lim_{x \to -1} \frac{x^4 - 1}{x + 1} =$

(c) $\lim_{x \to 0} \frac{1 + \sin x}{\cos x} =$

(d) If $f(x) = |x - 3|$, then

$$\lim_{h \to 0+} \frac{f(3+h) - f(3)}{h} =$$

(e) $\lim_{x \to 0} \frac{\tan 9x}{3x} =$

(f) $\lim_{x \to 0} \frac{\sec x - 1}{x^2} =$

6. (Continued): Evaluate the following limits. **[No work, no credit.]**

(g) $\lim_{x \to 5} \frac{x-5}{x^2-6x+5} =$

(h) $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2-4x}} =$

(i) $\lim_{x \to +\infty} \frac{4x}{x-1} =$

(j) $\lim_{x \to 1^-} \frac{4x}{x-1} =$

(k) $\lim_{x \to 4} \frac{4-\sqrt{x+12}}{4-x} =$

EXTRA-CREDIT: [**10 points**: **But**, **any score over 100 will be truncated to 100**.]

1. Given $\varepsilon = .01$, find $\delta > 0$ so that $|(4x+5) - 17| < .01$ whenever $0 < |x-3| < \delta$.

[5 points]

SOLUTION:

2. Find the equation of the straight line that is tangent to the graph of the function

$$f(x) = x^4 + x^3 + x^2 + x + 2$$

at the point (1, 6).

[5 points]

SOLUTION: