

PROBLEMS

3.29 The random variable X has a hypergeometric distribution with parameters $n = 4$, $N = 20$, $D = 6$.

(a) Compute $h(x)$, $x = 0, 1, 2, 3, 4$, using Eq. (3.27).

(b) Compute $h(x)$, $x = 0, 1, 2, 3, 4$, using the recurrence formula (3.30).

Which method is more efficient for computing the hypergeometric probabilities?

Done

3.30 A lot of 10 components contains 2 that are defective. A random sample of size 4 is drawn. Let X denote the number of defective components drawn.

(a) Show that X has a hypergeometric distribution and identify the parameters N , D , n .

Use the formula from part (a) to compute the following probabilities:

(b) $P(X = 0)$ (c) $P(X = 1)$ (d) $P(X > 1)$

3.31 Suppose the sampling plan is to sample 10 percent of a lot of N items with the acceptance number $c = 0$. Compute the acceptance probability in each of the following cases:

(a) $N = 50$, $D = 1$ (b) $N = 100$, $D = 2$ (c) $N = 200$, $D = 4$

3.32 Suppose a sample of size 15 is taken from a lot of 50 items, and the lot is accepted if the number X of defective items in the sample is less than or equal to 1 (i.e., we accept the lot if $X \leq 1$). Compute $P(\text{acceptance})$ for the following values of D :

(a) $D = 1$ (b) $D = 3$ (c) $D = 6$

3.33 A group of 10 items contains 2 that are defective. Articles are taken one at a time from the lot and tested. Let Y denote the number of articles tested in order to find the first defective item. Compute the pf of Y .

3.34 An urn contains two nickels and three dimes. We select two coins at random, without replacement. Let $X =$ monetary value of the coins in the sample; e.g., if the sample consists of a nickel and dime, $X = 15$ cents.

(a) Compute the pf of X . (b) Compute $E(X)$. (c) Compute $V(X)$.

3.35 An urn contains two white balls and four red balls. Balls are drawn one by one without replacement until two white balls have appeared. Denote by U the number of balls that must be drawn until both white balls have appeared. Compute the pf of U .

Problem 3.30

(a) This is a random sample from a dichotomous population (defective, non defective), so X is hypergeometric with $N = 10$, $D = 2$, $n = 4$.

$$(b) P(X = 0) = \frac{\binom{8}{4}}{\binom{10}{4}} = \frac{1}{3}$$

$$(c) P(X = 1) = \frac{\binom{2}{1} \binom{8}{3}}{\binom{10}{4}} = \frac{8}{15}$$

$$(d) P(X > 1) = 1 - P(X \leq 1) = \frac{2}{15}$$

Class

3.51

The probability that a disk drive lasts 6000 or more hours equals 0.8. In a batch of 20,000 disk drives, how many would be expected to last 6000 or more hours?

Problem 3.51 The number X of disk drives that last 6000 or more hours is binomial with $n = 20000$ and $p = 0.80$. Consequently, $E(X) = 20000 \times 0.8 = 16000$.

- 3.55 (a)** Compute the Poisson probabilities $p(x; 1.5)$, $x = 0, 1, 2, 3$, using Eq. (3.47).
- (b)** Compute the Poisson probabilities $p(x; 1.5)$, $x = 0, 1, 2, 3$, using the formula

$$p(x; \lambda) = P(x; \lambda) - P(x - 1; \lambda)$$

where the values of the distribution function

$$P(x; \lambda) = \sum_{0 \leq y \leq x} p(y; \lambda)$$

are provided in Table A.2 in the appendix.

- 3.56** A lot of 10,000 articles has 150 defective items. A random sample of 100 articles is selected. Let X denote the number of defective items in the sample. Give the formula for:
- (a)** The exact pf $f_X(x)$ of X .
- (b)** The binomial approximation of $f_X(x)$.
- (c)** The Poisson approximation of the binomial distribution of part (b).
- 3.57** If 99 percent of the students entering a junior high school are vaccinated against the measles, what is the probability that a group of 50 students includes:
- (a)** No unvaccinated students?
- (b)** Exactly one unvaccinated student?
- (c)** Two or more unvaccinated students?

Class

- 3.58** A batch of dough yields 500 cookies. How many chocolate chips should be mixed into the dough so that the probability of a cookie having no chocolate chips is ≤ 0.01 ?
- 3.59** A taxi company has a limousine with a seating capacity of N . The cost of each seat is \$5, and the price per seat is \$10. The number of reservations X is Poisson distributed with parameter $\lambda = 5$, i.e., $P(X = x) = e^{-5} 5^x / x!$, $x = 0, 1, \dots$. Compute the expected net revenue when:
- (a)** $N = 4$
- (b)** $N = 5$
- (c)** Which is more profitable to operate, a four- or five-seat limousine?
- 3.60** A sampling plan has $n = 100$, $c = 2$, and the lot size $N = 10,000$. Compute the approximate acceptance probabilities for the following values of D :
- (a)** $D = 50$ **(b)** $D = 100$ **(c)** $D = 200$
- (Hint: Use the Poisson approximation to the binomial.)

Problem 3.58 There are 500 cookies, so the probability that a chocolate chip is in a specific cookie is $p = 1/500$. Assume the cookie dough contains n chips. The probability that a cookie contains x chips is binomial with parameters n , $p = 1/500$. Using the Poisson approximation we see that the probability that a cookie contains no chips is given by

$P(X = 0) \approx \exp(-n/500)$. We choose n so that $\exp(-n/500) \leq 0.01$. Taking (natural) logs of both sides we obtain the inequality

$n \geq -500 \times \ln(0.01) = 2302.5$. The smallest integer satisfying this inequality is $n = 2303$.