Duplicate in Bag if needed

76 +20

ENCE 3300
COMPUTATIONAL METHODS IN CIVIL ENGINEERING

TEST 2

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1. It is known that screws produced by a certain company will have 10 defective screws in a lot of 50. The company sells screws in packages of 10 and offers a money-back guarantee that at most 2 of the 10 screws is defective. What proportion of packages sold must the company replace? (What is the probability that a package will have to be replaced? P(X>2)=?) **USE THE HYPERGEOMETRIC DISTRIBUTION.**

-3 -3 -3

$P(X>2) = 1 - P(X \le 2) = 1 - P(X=0) - P(X=1) - P(X=$

$N - D - n + x$

$50 - 10 - 2 + 2$

$N = 50$ ✓
$D = 10$ ✓
$n = 2$ X -3
$x = 2$

$$n(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}} = \frac{\binom{10}{2}\binom{40}{0}}{\binom{50}{2}}$$

$$n(2) = \frac{\left(\frac{10!}{2!(8!)}\right)\left(\frac{40!}{(0!)(40!)}\right)}{\frac{50!}{2!(48!)}} = \frac{(45)\cdot 1}{1225} = 3.67\%$$

$$E(X) = 2 \times \frac{10}{50} = .4$$

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2. Assume automobile arrivals at a gasoline station are **POISSON DISTRIBUTED** and occur at an average rate of 20 per hour (20 per 60 minutes). The station has only one gasoline pump. If all cars are assumed to require one minute to obtain fuel, what is the probability that a waiting line will occur at the pump?
(P(X>1) = ?)

$np = (20)(1) = 20 = \lambda$ -3

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$P(X<1) = \frac{e^{-20}\, 20}{(1)!} = 4.122 \times 10^{-8}$$

$$P(X>1) = 1 - 4.122 \times 10^{-8} = .9999$$

$-P(x=0)$

-3

see Soral T2 #3

see Blakely T-2 #1 for four instead of five

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3. A certain type of component is packaged in lots of five. Let X represent the number of properly functioning components in a randomly chosen lot. Assume that the probability that exactly x components function is proportional to x; in other words, assume that the probability density function of X is given by:

$$f(x) = c\,x, \qquad x = 1, 2, 3, 4 \text{ or } 5$$
$$f(x) = 0, \qquad \text{elsewhere.}$$

where c is a constant.

a) Find the value of the constant c so that f(x) is a probability density function.

b) Find P(X = 2).

c) Find E(X).

d) Find V(X).

X	f(x)
1	0.067
2	.134
3	.201
4	.268
5	.335

a) $c(1) + c(2) + c(3) + c(4) + c(5) = 1$

$c(1+2+3+4+5) = 1$

$c = \frac{1}{15} = 0.067$

b) $P(X=2) = cx$

$P(X=2) = (0.067)(2) = .134$

c) $E(X) = \sum_x x f(x) = [1(0.067) + 2(.134) + 3(.201) + 4(.268) + 5(.335)] = 3.685$

$E(X) = 3.685$

d) $V(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \sum_x x^2 f(x)$

$= [1^2(0.067) + 2^2(.134) + 3^2(.201) + 4^2(.268) + 5^2(.335)]$

$E(X^2) = 15.075$

$V(X) = 15.075 - (3.685)^2 = 1.495775$

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4. (Binomial Distribution) A large industrial firm allows a discount on any invoice that is paid within 90 days. Of all invoices, 10% receive the discount. In a company audit, 12 invoices are sampled at random. What is the probability that more than 4 of the 12 sampled invoices receive the discount?

$P = 0.1$ $\quad P(X>4) = 1 - P(X \le 4) = 1 - P(X=0) - P(X=1)$

$n = 12$

$B(X; n, 1-P) = \sum_{0 \le y \le X} b(y; n, P)$ $\quad -3$ $\quad - P(X=2) - P(X=3)$

$-P(X=4)$

$P(X>4) = 1 - B(5; 12; 0.1)$

$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$P(X=x) = \binom{n}{p} P^x (1-P)^{n-x}$

$P(X=0) = \binom{12}{0} 0.1^0 (1-0.1)^{12-0} = \frac{12!}{0!(12-0)!} (0.1^0)(1-0.1)^{12-0} = (1)(1) =$

$P(X=0) = .2824$

$P(X=1) = \binom{12}{1}(0.1^1)(1-0.1)^{12-1} = 0.3765$

$P(X=2) = \binom{12}{2}(0.1^2)(1-0.1)^{12-2} = 0.2301$

$P(X=3) = \binom{12}{3}(0.1^3)(1-0.1)^9 = 0.0852$

$P(X=4) = \frac{12!}{4!(12-4)!} (.1^4)(1-0.1)^8 = (495)(.1^4)(.43046721) = .021308$

$P(X=5) = \frac{12!}{5!(12-5!)} (.1^5)(1-0.1)^7 = (792)(.1^5)(.4782969) = 0.0037881$

-3

$\Sigma (P<5) = .2824 + .3765 + .2301 + .0852 + .021308 + .0037881$

$= .9992961$

$P(X>4)$