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ENCE 3300
TEST 4

1. Newton's law of cooling says that the temperature of a body changes at a rate proportional to the difference between its temperature and that of the surrounding medium (the ambient temperature),

$$dT/dt = -k(T - T_a)$$

where T = temperature of the body (°C), t = time (min), k = the proportionality constant (per minute), and T_a = the ambient temperature (°C). Suppose that a cup of coffee originally has a temperature of 70°C. Use Euler's method (iterative method) to compute the temperature from t = 0 to 20 min using a step size of 2 min if T_a = 25°C and k = .015/min.

$t_0 = 70°C$ $\quad \frac{dT}{dt} = (-)\frac{.015}{min}(70° - 25°)$ $\quad T(\ ) = t + (h)\left(\frac{dT}{dt}\right)$

$h = 2min$

$t_{2min} = 70° - (2min)\frac{.015}{min}(45°) = 70° - 1.25° = 68.6°C$

$t_{4min} = 68.5° - (2min)\frac{.015}{min}(68.6° - 25°) = 68.6° 1.3° = 67.3°C$

$t_{6min} = 67.3° - (2min)\frac{.015}{min}(67.3° - 25°) = 67.3° - 1.3° = 66.1°C$

$t_{8min} = 66.1° - (2min)\frac{.015}{min}(66.1° - 25°) = 66.1° - 1.3° = 64.7°C$

$t_{10min} = 64.8° - (2min)\frac{.015}{min}(64.8° - 25°) = 64.8° - 1.2° = 63.6°C$

$t_{12min} = 63.6° - (2min)\frac{.015}{min}(63.6° - 25°) = 63.6° - 1.2° = 62.4°C$

$t_{14min} = 62.4° - (2min)\frac{.015}{min}(62.4° - 25°) = 62.4° - 1.2° = 61.2°C$

$t_{16min} = 61.2° - (2min)\frac{.015}{min}(61.2° - 25°) = 61.2° - 1.1° = 60.1°C$

$t_{18min} = 60.1° - (2min)\frac{.015}{min}(60.1° - 25°) = 60.1° - 1.1° = 59.0°C$

$t_{20min} = 59.0° - (2min)\frac{.015}{min}(59.0° - 25°) = 59.0° - 1.0° = 58.0°C$

2. Use zero- through fourth-order Taylor series expansions to predict f(2.25) for

$$f(x) = \ln x$$

using a base point at x = 1. Compute the true percent relative error for each approximation.

$x_i = 1$

$h = (x_{i+1} - x_i) = 2.25 - 1 = 1.25$

$f(x) = \ln x \qquad f(1) = \ln 1 = 0$

$f'(x) = 1/x \qquad f'(1) = 1/1 = 1$

$f''(x) = -1/x^2 \qquad f''(1) = -\frac{1}{1^2} = -1$

$f'''(x) = 2/x^3 \qquad f'''(1) = 2/1^3 = 2$

$f''''(x) = -\frac{2\cdot 3}{x^4} \qquad f''''(1) = -\frac{2\cdot 3}{1^4} = -6$

0th - Order

$f(x_{i+1}) \approx f(x_i) = \ln(1) = \underline{0}$

$|E_t| = |\ln 2.25 - 0| = \underline{0.810930216}$

1st - Order

$f(x_{i+1}) \approx f(x_i) + f'(x_i)h$

$\approx 0 + (1)(1.25) = \underline{1.25}$

$|E_t| = |0.8109... - 1.25| = \underline{0.439069784}$

2nd - Order

$f(x_{i+1}) \approx f(x_i) + f'(x_i)h + f''(x_i)h^2/2!$

$\approx 0 + 1.25 + (-1)(1.25)^2/2 = \underline{0.46875}$

$|E_t| = |0.8109... - 0.46875| = \underline{0.342180216}$

3rd - Order

$f(x_{i+1}) \approx f(x_i) + f'(x_i)h + f''(x_i)h^2/2! + f'''(x_i)h^3/3!$

$\approx 0 + 1.25 - 0.78125 + (2)(1.25)^3/6 = \underline{1.119791667}$

$|E_t| = |0.8109... - 1.11979...| = \underline{0.30886145}$

4th - Order

$f(x_{i+1}) \approx f(x_i) + f'(x_i)h + f''(x_i)h^2/2! + f'''(x_i)h^3/3! + f''''(x_i)h^4/4!$

$\approx 0 + 1.25 - 0.78125 + 0.651041667 + (-6)(1.25)^4/24$

$= \underline{0.509440104}$

$|E_t| = |0.8109... - 0.50944...| = \underline{0.301490112}$

3. Find the algebraic (module) solution for

$$\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2 + \partial h/\partial x + \partial h/\partial y + h = 0$$

$$h_{j,k}\left\{ 1 \quad -2 \quad 1 \right\}\frac{1}{\Delta x^2} + h_{j,k}\left\{\begin{matrix}1\\-2\\1\end{matrix}\right\}\frac{1}{\Delta y^2}$$

$$+ h_{j,k}\left\{ -1 \quad 0 \quad 1 \right\}\frac{1}{2\Delta x} + h_{j,k}\left\{\begin{matrix}1\\0\\-1\end{matrix}\right\}\frac{1}{2\Delta y} + h = 0$$

Let $\Delta x = \Delta y$

$$\boxed{\begin{aligned} h_{j,k} = & -\frac{1}{\Delta^2}\left(h_{j-1,k} - 4h_{j,k} + h_{j+1,k} + h_{j,k+1} + h_{j,k-1}\right) \\ & -\frac{1}{2\Delta}\left(-h_{j-1,k} + h_{j+1,k} + h_{j,k+1} - h_{j,k-1}\right)\end{aligned}}$$

4. A rectangular region is shown below. Along the upper boundary the value is 100. Along the left side the values go from 100 to 20 as shown. The other two boundaries satisfy the condition that the derivative normal to the region is zero.

$$\frac{dh}{dn} = 0$$

The points in the interior and along the boundary that are not fixed (values of 100, 80, etc.) are governed by the equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Compute the solution using finite difference for two iterations. Start with a value of 60 at the unknown points (does not count as an iteration). Assume Δx = Δy.

100	100	100	100	100
80	60	60	60	60
60	60	60	60	60
40	60	60	60	60
20	60	60	60	60

$$h_{j,k}\{1 \;\; -2 \;\; 1\}\frac{1}{\Delta x^2} + h_{j,k}\begin{Bmatrix}1\\-2\\1\end{Bmatrix}\frac{1}{\Delta y^2} = 0$$

Let $\Delta x = \Delta y = \Delta$

$$\begin{array}{c|ccc} j+1 & & 1 & \\ h_{j,k} & 1 & -4 & 1 \\ j-1 & & 1 & \\ \hline & k-1 & k & k+1 \end{array}$$

$$0 = -4h_{j,k} + 1h_{j+1,k} + 1h_{j-1,k} + 1h_{j,k-1} + 1h_{j,k+1}$$

$$h_{j,k} = \frac{1}{4}\left[h_{j+1,k} + h_{j-1,k} + h_{j,k-1} + h_{j,k+1}\right]$$

1	100	100	100	100	100
2	80	75	74	73	76
3	60	64	64	64	66
4	40	56	60	61	62
5	20	48	57	60	61

100				
80	79	79	80	81
60	65	67	68	70
40	53	59	62	64
20	46	56	60	62

test #4

Problem #4

1st iteration

2nd row				
$h_{x+1,y}$	60	60	60	73
$h_{x-1,y}$	80	75	74	73
$h_{x,y+1}$	100	100	100	100
$h_{x,y-1}$	60	60	60	60
Σ	$4\overline{)300}$ 75	$4\overline{)285}$ 74	$4\overline{)294}$ 73	$4\overline{)306}$ 76

3rd row				
$h_{x+1,y}$	60	60	60	64
$h_{x-1,y}$	60	64	64	64
$h_{x,y+1}$	75	74	73	76
$h_{x,y-1}$	60	60	60	60
	$4\overline{)255}$ 64	$4\overline{)258}$ 64	$4\overline{)257}$ 64	$4\overline{)264}$ 66

4th row				
$h_{x+1,y}$	60	60	60	61
$h_{x-1,y}$	40	56	60	61
$h_{x,y+1}$	64	64	64	66
$h_{x,y-1}$	60	60	60	60
	$4\overline{)224}$ 56	$4\overline{)240}$ 60	$4\overline{)244}$ 61	$4\overline{)248}$ 62

5th row				
$h_{x+1,y}$	60	60	60	60
$h_{x-1,y}$	20	48	57	60
$h_{x,y+1}$	56	60	61	62
$h_{x,y-1}$	56	60	61	62
	$4\overline{)192}$ 48	$4\overline{)228}$ 57	$4\overline{)239}$ 60	$4\overline{)244}$ 61

Test 4
Problem #4

2nd iteration

2nd row				
$h_{x+1,y}$	74	73	76	80
$h_{x-1,y}$	80	79	79	80
$h_{x,y+1}$	100	100	100	100
$h_{x,y-1}$	64	64	64	66
	$4\overline{)318}\ 79$	$4\overline{)316}\ 79$	$4\overline{)319}\ 80$	$4\overline{)326}\ 81$

3rd row				
$h_{x+1,y}$	64	64	66	68
$h_{x-1,y}$	60	65	67	68
$h_{x,y+1}$	79	79	80	81
$h_{x,y-1}$	56	60	61	62
	$4\overline{)259}\ 65$	$4\overline{)268}\ 67$	$4\overline{)274}\ 68$	$4\overline{)279}\ 70$

4th row				
$h_{x+1,y}$	60	61	62	62
$h_{x-1,y}$	40	53	59	62
$h_{x,y+1}$	65	67	68	70
$h_{x,y-1}$	48	57	60	61
	$4\overline{)213}\ 53$	$4\overline{)238}\ 59$	$4\overline{)249}\ 62$	$4\overline{)255}\ 64$

5th row				
$h_{x+1,y}$	57	60	61	60
$h_{x-1,y}$	20	46	56	60
$h_{x,y+1}$	53	59	62	64
$h_{x,y-1}$	53	59	62	64
	$4\overline{)183}\ 46$	$4\overline{)224}\ 56$	$4\overline{)241}\ 60$	$4\overline{)248}\ 62$