Analysis of Doubly-Reinforced Beams

HW #5
4.28, 5.6, 5.26
5.28

- Doubly Reinforced Beams
- Cases
- Examples

Next: Shear

Case 1 - All Steel yields

$f_s = f_s' = f_y$

$C = 0.85 f_c' a b$

$C_s = A_s' f_y$

$T = A_s f_y$

$\Big\} \; T = C + C_s$

$A_s f_y = 0.85 f_c' a b + A_s' f_y$

$$a = \frac{(A_s - A_s') f_y}{0.85 f_c' b}$$

$$M_n = C\left(d - \frac{a}{2}\right) + C_s (d - d')$$

Case 2 - Tensile Steel Yields

Compression Steel has not yielded

$f_s = f_y$

$f_s' < f_y$

$C_s = A_s' \epsilon_s' E_s$

$C = 0.85 f_c' a b$

$T = A_s f_y$

$\Big\} \; T = C + C_s$

$A_s f_y = 0.85 f_c' a b + A_s' \epsilon_s' E_s$

$$a = \frac{A_s f_y - \epsilon_s' E_s A_s'}{0.85 f_c' b}$$

Let $\epsilon_s' = \dfrac{0.003 (c - d')}{c}$; $c = a / \beta_1$

Results in Quadratic

$$\underbrace{(0.85 f'_c\, b)}_{A} a^2 + \underbrace{(0.003 A'_s E_s - A_s f_y)}_{B} a$$
$$+ \underbrace{(A'_s E_s\, 0.003\, d'\, \beta_1)}_{C} = 0$$

Solve for "a" (positive root answer)

$$M_n = A'_s E_s \epsilon'_s (d - d') + (0.85 f'_c\, ab\left(d - \frac{a}{2}\right)$$

Case 3 No steel yields

Go Back & redesign!!

Solution Procedure

① Assume Case 1

② Check assumption

if $\epsilon_s > f_y$, $\epsilon_s' > \epsilon_y$ ⇒ Solve as case 1; done

if $\epsilon_s > f_y$, $\epsilon_s' < \epsilon_y$ ⇒ solve as case 2, recheck assumptions $\epsilon_s > f_y$, $\epsilon_s' < \epsilon_y$ → done

↑ tensile steel ↑ compression steel

③ If $\epsilon_s < \epsilon_y$, $\epsilon_s' < \epsilon_y$ → Stop Case 3 done

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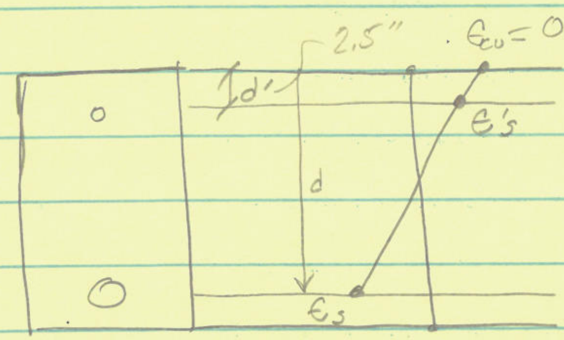
handout



$A_s = 10.12 \text{ in}^2$ Sometimes $A_1 = A_s$

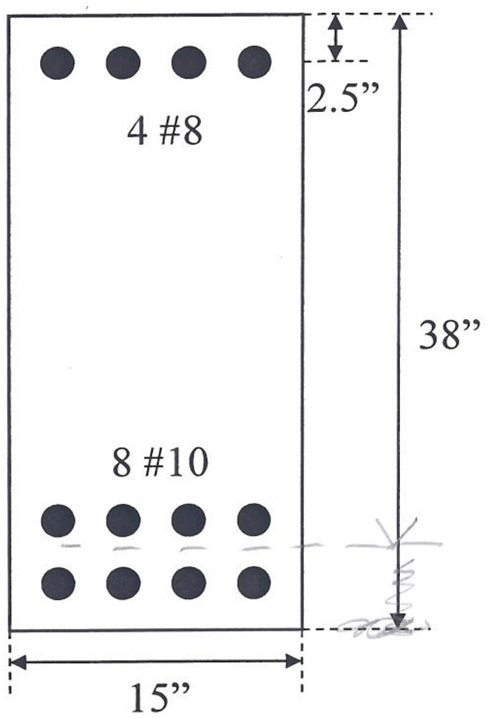
$A_s' = 3.14 \text{ in}^2$ $A_2 = A_s - A_s'$

① Assume all steel yields

$f_s = f_s' = f_y$

$$a = \frac{(A_s - A_s')}{0.85 f_c' b} + \frac{A_2 f_y}{0.85 f_c' b} = \frac{6.98 \text{ in}^2 (60)}{(0.85)(4)(15)} = 8.21'' = a$$

Design Strength of Doubly Reinforced Beams
ENCE 4359 Structural Concrete Design
Dr. Lamanna



$f'_c = 4{,}000$ psi
$f_y = 60{,}000$ psi

Compute the Design Strength.

Check the maximum permissible A_s to ensure tensile failure.

② check assumption that all steel yields

$c = a/\beta_1 = \frac{8.21}{0.85} = 9.66''$

$\epsilon_s = \frac{0.003\, d - c}{c} = \frac{(0.003)(38 - 9.66)}{9.66} = 0.0088 > 0.00207$ ✓

$\epsilon_s' = \frac{0.003\,(c - d')}{c} = \frac{(0.003)(9.66 - 2.50}{9.66} = 0.0022 > 0.00207$ ✓

∴ All Steel yields

ϵ_y for Gr 60

③ Calculate Mn

$\phi M_n = (A_s - A_s') f_y (d - a/2) + A_s' f_y (d - d')$

$= (10.12 - 3.14)(60)(38 - \frac{8.21}{2}) + (3.14)(60)(38 - 2.50)$

$= 18{,}795 \text{ in} \cdot k = 1{,}566 \text{ ft-k}$

④ Check tensile steel permitted

$A_{s,max} = \rho_{max}\, b d + A_s' \frac{f_s'}{f_y}$

$\rho_{max} = 0.0181$ [Table A.2]

$A_{s,max} = 0.0181(15)(38) + 3.14\left(\frac{60}{60}\right) = 10.12 \text{ in}^2$

✓ ok

$M_u > 1{,}566$ ft-k