Chapter 5
FORCE AND MOTION —— I

In this, the most fundamental chapter in the mechanics section of the text, you start to study how objects influence the motion of each other. The fundamental problem of mechanics is to find the acceleration of an object, given the object and its environment. The problem is split into two parts, connected by the idea of a force: the environment of an object exerts forces on the object and the net force on it causes it to accelerate. In this chapter, you concentrate on the relationship between the net force on an object and its acceleration.

Important Concepts

- ☐ Newton's first law
- ☐ inertial reference frame
- ☐ force
- ☐ mass
- ☐ inertia
- ☐ Newton's second law
- ☐ force of gravity (weight)
- ☐ tension (in a string)
- ☐ normal force
- ☐ frictional force
- ☐ Newton's third law
- ☐ free-body diagram

Overview

5–3 Newton's First Law

- ☐ If the net force on a particle is zero, then the acceleration of the particle, as measured relative to an inertial frame, is also zero. Such a frame might be attached to a particle on which zero total force acts. Clearly the acceleration of the particle, as measured in that frame, is zero.
- ☐ Newton's laws as stated in this text are valid only for inertial frames.
- ☐ For phenomena that take place over short times we may take a reference frame attached to Earth to be an inertial frame, but strictly it is not.

5–4 Force

- ☐ A **force** is a push or pull exerted by one object on another. It is measured, in principle, by applying the push or pull (alone) to the standard (1 kg) mass and measuring the acceleration of the standard mass. If SI units are used, the magnitudes of these quantities are numerically equal. They are vectors in the same direction. That forces obey the laws of vector addition can be checked by simultaneously applying two forces in different directions and verifying that the result is the same as when the vector sum of the forces is applied as a single force. All measurements must be made using an inertial reference frame.
- ☐ The SI unit of force is the newton and is abbreviated N. In terms of the SI base units, $1\,\text{N} = 1\,\text{kg}\cdot\text{m/s}^2$.

5–5 Mass

- ☐ The **mass** of an object is measured, in principle, by comparing the accelerations of the object and the standard mass when the same force is applied each to them. In particular, the mass of the object is given by $m = (1\,\text{kg})(a_0/a)$, where a is the magnitude of the acceleration of the object and a_0 is the magnitude of the acceleration of the mass standard. The accelerations must be measured using an inertial frame.
- ☐ An object with a small mass acquires a greater acceleration than an object with a large mass when the same force is applied to each of them. Mass is said to measure **inertia** or resistance to changes in motion.
- ☐ Mass is a scalar and is always positive. The mass of two objects in combination is the sum of the individual masses.

5–6 Newton's Second Law

- ☐ This is the central law of classical mechanics. It gives the relationship between the net force $\vec{F}_{\text{net}}$ on an object and the acceleration $\vec{a}$ of the object:

$$\vec{F}_{\text{net}} = m\vec{a}\,,$$

where m is the mass of the object.
- ☐ The Newton's second law equation is a vector equation. It is equivalent to the three component equations

$$F_{\text{net}\ x} = ma_x, \qquad F_{\text{net}\ y} = ma_y, \qquad \text{and} \qquad F_{\text{net}\ z} = ma_z.$$

- ☐ In these equations, $\vec{F}_{\text{net}}$ is the *vector* sum of all the individual forces. This means that in any given situation you must identify all the forces acting on the object and then sum them *vectorially*.
- ☐ Note that $\vec{F}_{\text{net}} = 0$ implies $\vec{a} = 0$. If the net force vanishes, then the object does not accelerate; its velocity as observed in an inertial reference frame is constant in both magnitude and direction. The resultant force may vanish because there are no forces on the object or because the forces on it sum to zero.

5–7 Some Particular Forces

- ☐ Every object attracts every other object with a **gravitational force**. In this part of the course you consider only the mutual gravitational attraction of Earth and some object of interest.
- ☐ The gravitational force Earth on an object is equal to the **weight** of the object and its magnitude is given by $W = mg$, where m is the mass of the object and g is the magnitude of the acceleration due to gravity at the position of the object. Near the surface of Earth the direction of the weight is toward the center of Earth.
- ☐ Be sure you understand that mass and weight are quite different concepts. Mass is a property of an object and does not change as the object is moved from place to place or even into outer space. It is a scalar. Weight, on the other hand, is a force. It varies as the object moves from place to place and vanishes when the object is far from all other objects, as in outer space. This is because $\vec{g}$, not the mass, varies from place to place.

- ☐ Remember that the weight of an object is $m\vec{g}$ regardless of its acceleration. Weight is a force and, if appropriate, is included in the sum of all forces on the object. This sum equals $m\vec{a}$ and if other forces act, then $\vec{a}$ is different from $\vec{g}$.
- ☐ The SI unit of weight is the newton.
- ☐ A surface may exert a force on an object in contact with it. If the surface is frictionless, that force must be perpendicular to the surface. It is called a **normal force**. Unless adhesive glues the object to the surface a normal force can only push on the object. It must be directed away from the surface toward the interior of the object.
- ☐ If the surface is at rest or moving with constant velocity, the normal force adjusts until the component of the object's acceleration perpendicular to the surface vanishes. We often use this condition to solve for the normal force. Set the sum of the normal components of the forces on the object equal to zero. Since the normal force is one of these the resulting equation can be solved for it in terms of the normal components of the other forces.
- ☐ A surface may also exert a **frictional force** on an object in contact with it. This force is parallel to the surface.
- ☐ If a string with negligible mass connects two objects, it pulls on each with a force of the same magnitude, called the **tension** in the string. You may think of the string as simply transmitting a force from one object to the other; the situation is exactly the same if the objects are in contact and exert forces on each other. Strings pull, not push, along their lengths, so a string serves to define the direction of the force.

5–8 Newton's Third Law

- ☐ Newton's third law tells us something about forces. If the force of object A on object B is $\vec{F}_{\mathrm{BA}}$, then according to the third law, the force of object B on object A is given by $\vec{F}_{\mathrm{AB}} = -\vec{F}_{\mathrm{BA}}$. Compared to the force of A on B, the force of B on A is the same in magnitude and opposite in direction. You should also be aware that these two forces are of the same type. That is, if the force of A on B is gravitational then the force of B on A is also gravitational.
- ☐ The third law is useful in solving problems involving more than one object. If two objects exert forces on each other, we use the same symbol to represent their magnitudes and we remember the forces are in opposite directions when we write the second-law equations. In addition, we remember that the forces act on different objects. When we write Newton's second law for object A, one of the forces we include is the force of B on A, but emphatically *NOT* the force of A on B.

5–9 Applying Newton's Laws

- ☐ A definite procedure has been devised to solve Newton's second law problems. It ensures that you consider only one object at a time, reminds you to include all forces acting on the object you are considering, and guides you in writing Newton's second law in an appropriate form. Follow it closely.

  1. Identify the object to be considered. It is usually the object on which the given forces act or about which a question is posed.

2. Represent the object by a dot on a diagram. Do not include the environment of the object since this is replaced by the forces it exerts on the object.

3. On the diagram draw arrows to represent the forces exerted by the environment on the object. Try to draw them in roughly the correct directions. The tail of each arrow should be at the dot. Label each arrow with an algebraic symbol to represent the magnitude of the force, even if a numerical value is given in the problem statement.

The hard part is getting all the forces. If appropriate, don't forget to include the weight of the object, the normal and frictional forces of a surface on the object, and the forces of any strings or rods attached to the object. Carefully go over the sample problems in the text to see how to handle these forces.

Some students erroneously include forces that are not acting on the object. For each force you include, you should be able to point to something in the environment that is exerting the force. This simple procedure should prevent you from erroneously including a normal force, for example, when the object you are considering is not in contact with a surface.

4. Draw a coordinate system on the diagram. In principle, the placement and orientation of the coordinate system do not matter as far as obtaining the correct answer is concerned, but some choices reduce the work involved. If you can guess the direction of the acceleration, place one of the axes along that direction. The acceleration of an object sliding on a surface such as a table top or inclined plane, for example, is parallel to the surface. Once the coordinate system is drawn, label the angle each force makes with a coordinate axis. This will be helpful in writing down the components of the forces later.

The diagram, with all forces shown but without the coordinate system, is called a **free-body diagram** (or a force diagram). We add the coordinate system to help us carry out the next step in the solution of the problem.

5. Write the Newton's second law equation in component form: $F_{\text{net }x} = ma_x$, $F_{\text{net }y} = ma_y$, and, if necessary, $F_{\text{net }z} = ma_z$. The left sides of these equations should contain the appropriate components of the forces you drew on your diagram. You should be able to write the equations by inspection of your diagram. Use algebraic symbols to write them, not numbers; many problems give or ask for force magnitudes so you should usually write each force component as the product of a magnitude and the sine or cosine of an appropriate angle.

6. If more than one object is important, as when two objects are connected by a string, you might carry out the steps above separately for each object. Then, you consider additional conditions. When two objects are connected by a string, for example, the magnitudes of their accelerations might be the same. Be aware of these situations as you study the sample problems of the text.

7. Identify which quantities are known and which are unknown; solve for the unknowns.

Hints for Questions

1 For the block to remain stationary or move with constant velocity the vector sum of the forces must be zero. You can change the magnitudes of the forces but not their orientations.

[Ans: (a) 2 and 4; (b) 2 and 4]

3 To keep the lunch box sliding with constant velocity the vector sum of all forces on it must be zero. Since the gravitational force of Earth and the normal force of the floor are vertical the horizontal component of $\vec{F}_1$ must be equal to the magnitude of $\vec{F}_2$.

[Ans: increase]

5 (a) and (b) To see when the acceleration has an x component, add the x components of the forces. If they sum to zero, $a_x = 0$. Similarly, to see when the acceleration has a y component add the y components of the forces.
(c) If both the x and y components of the total force are positive the acceleration is in the first quadrant; if the x component is negative and the y component is positive the acceleration is in the second quadrant; if both components are negative the acceleration is in the third quadrant; if the x component is positive and the y component is negative the acceleration is in the fourth quadrant.

[Ans: (a) 2, 3, 4; (b) 1, 3, 4; (c) 1: in the positive y direction, 2: in the positive x direction, 3: in the fourth quadrant, 4: in the third quadrant]

7 The magnitude of the acceleration is proportional to the magnitude of the net force. Add the forces vectorially and compare their magnitudes.

[Ans: a, then b, c, and d tie]

9 Since the block is at rest, the vector sum of $\vec{F}$, the gravitational force of Earth, and the normal force of the floor must be zero. If $\vec{F}$ is downward $F + mg - F_N = 0$, where m is the mass of the block and F_N is magnitude of the normal force. If $\vec{F}$ is upward $F + F_N - mg = 0$. In each case tell what happens to F_N is F is increased.

[Ans: (a) increases from mg; (b) decreases from mg to zero]

11 (a), (b), and (c) $\vec{F}$ accelerates all three blocks. $\vec{F}_{21}$ accelerates blocks 2 and 3. $\vec{F}_{32}$ accelerates only block 3.
(d) The blocks move together.
(e) The force is proportional to the mass it accelerates and to the acceleration.

[Ans: (a) 17 kg; (b) 12 kg; (c) 10 kg; (d) all tie; (e) $\vec{F}$, $\vec{F}_{21}$, $\vec{F}_{32}$]

Hints for Problems

3 Use Newton's second law: $\vec{F}_{\text{net}} = m\vec{a}$, where $\vec{F}_{\text{net}}$ is the vector sum of all the forces on the block, m is the mass of the block, and $\vec{a}$ is its acceleration. Add the forces vectorially and divide by the mass.

[Ans: (a) 0; (b) $(4.0\,\text{m/s}^2)\,\hat{\text{j}}$; (c) $(3.0\,\text{m/s}^2)\,\hat{\text{i}}$]

15 Use Newton's second law in component form. The x component of the acceleration is the slope of the left-hand graph and the y component is the slope of the right-hand graph. Multiply each of these components by the mass of the package to obtain the components of the force. The magnitude is the square root of the sum of the squares of the components and the tangent of the angle that the force makes with the positive x axis is the y component divided by the x component.

[Ans: (a) 11.7 N; (b) $-59.0°$]

21 Use Newton's second law. The forces on the firefighter are the downward gravitational force of Earth and the upward force of the pole. Their vector sum must equal the product of the firefighter's mass and his acceleration, which is downward. Solve for the force of the pole on the firefighter. Don't forget to calculate his mass from the given value of his weight. According to Newton's third law the force of the firefighter on the pole is equal in magnitude and opposite in direction to the force of the pole on the firefighter.

[Ans: (a) 494 N; (b) up; (c) 494 N; (d) down]

25 Draw a free-body diagram for Tarzan and put in the axes. Remember that the vine pulls, not pushes, on him. The x component of the force of the vine on Tarzan is given by $T \sin\theta$ and the y component is given by $T \cos\theta$, where T is the tension in the vine and θ is the angle the vine makes with the vertical. The net force is the vector sum of the tension force and the gravitational force of Earth on Tarzan (his weight). According to Newton's second law his acceleration is the net force divided by his mass.

[Ans: (a) $(285\,\text{N})\hat{\imath}+(705\,\text{N})\hat{\jmath}$; (b) $(285\,\text{N})\hat{\imath}-(115\,\text{N})\hat{\jmath}$; (c) 307 N; (d) $-22.0°$; (e) $3.67\,\text{m/s}^2$; (f) $-22.0°$]

35 Draw a free-body diagram for the elevator cab. The forces on it are the tension force of the cable (upward) and gravitational force of Earth (downward). Solve the Newton's second law equation for the tension. In (a) the acceleration is upward and in (b) it is downward.

[Ans: (a) 31.3 kN; (b) 24.3 kN]

37 Draw a free-body diagram for the bundle. The forces on it are the upward tension force of the cable and the downward gravitational force of Earth. Take the tension to have its maximum value and solve the Newton's second law equation for the acceleration. Use $v^2 = 2ah$ to find the speed v of the bundle when it hits the ground. Here h is the starting height of the bundle above the ground.

[Ans: (a) $1.4\,\text{m/s}^2$; (b) $4.1\,\text{m/s}$]

45 (a) The tension force T_3 accelerates the three blocks, considered to be a single object. Use Newton's second law in the form $T_3 = (m_1 + m_2 + m_3)a$ to find the acceleration a.

(b) The tension force T_1 accelerates only block 1. Use Newton's second law in the form $T_1 = m_1 a$ to find T_1.

(c) The tension force T_2 accelerates blocks 1 and 2, considered to be a single object. Use Newton's second law in the form $T_2 = (m_1 + m_2)a$ to find T_2.

[Ans: (a) $0.970\,\text{m/s}^2$; (b) 11.6 N; (c) 19.1 N]

47 Draw a free-body diagram for each of the blocks. The forces on the left-hand block are the force of gravity $m_1 g$, down, and the tension force of the cord T, up. The forces on the right-hand block are the force of gravity $m_2 g$ and the tension force T, up. Let a_1 be the acceleration of block 1 and a_2 be the acceleration of block 2, then write a Newton's second law equation for each block. Note that the tension force on block 1 has the same magnitude as the tension force on block 2 and that the accelerations are actually vertical components.

The magnitudes of the accelerations are the same since the blocks are connected by the cord. Their signs, however, depend on the coordinate system used. If, for example, you take the upward direction to be positive for both blocks then $a_1 = -a_2$. Substitute $a_1 = a$ and $a_2 = -a$ into the second law equations and solve them simultaneously for a and the cord tension.

[Ans: (a) $3.6\,\mathrm{m/s^2}$; (b) $17\,\mathrm{N}$]

55 The acceleration of the block at any instant is given by $a_x = F_x/m$, where m is the mass of the block. At time t the velocity of the block is given by

$$v = v_0 + \int_0^{11\,\mathrm{s}} a\,dt = v_0 + \frac{1}{m}\int_0^{11\,\mathrm{a}} F_x\,dt\,.$$

The value of the integral is just the area under the curve. Don't forget you must subtract the magnitude of the area after $t = 6\,\mathrm{s}$ from the area before $t = 6\,\mathrm{s}$. If the result is positive the block is moving in the positive x direction; if it is negative the block is moving in the negative x direction.

[Ans: (a) $8.0\,\mathrm{m/s}$; (b) positive x direction]

61 If F_2+F_3 is less than F_1, the least force (and so the least acceleration) occurs if the two forces are both in the negative x direction. In part (c) the sum of the magnitudes is greater than F_1 but the forces have the same magnitude, so they can be oriented so their y components sum to zero and the sum of their x components cancel F_1.

[Ans: (a) 0; (b) $0.83\,\mathrm{m/s^2}$; (c) 0]

69 To answer (a) calculate what the tension should be to hold the performer stationary and see if it is greater or less than 425 N. To answer (b) take the tension force to have a magnitude of 425 N and use Newton's second law to compute the acceleration of the performer.

[Ans: (a) rope breaks; (b) $1.6\,\mathrm{m/s^2}$]

75 Draw a free-body diagram for each of the boxes. Take the positive x direction to be to the right for box 1 and up the incline for box 2. Write the x component of Newton's second law for each box. The tension force of the cord pulls to the right on box 1 and pulls down the incline on box 2. Use the same symbol for the magnitude of the tension force on each box. Solve the two second law equations simultaneously for the magnitude of the tension force.

[Ans: 4.6 N]

81 The mass is W/g, where W is the weight. The mass is the same no matter where the particle is but the weight depends on the local value of g.

[Ans: (a) 11 N; (b) 2.2 kg; (c) 0; (d) 2.2 kg]

85 Consider the person and parachute to be a single object. The forces on it are the downward force of gravity (which is the total weight of the person and parachute) and the upward force of the air. Solve Newton's second law for the force of the air. The forces on the parachute are the downward force of gravity (which is the weight of the parachute alone), the downward force of the person, and the upward force of the air. Solve Newton's second law for the force of the person.

[Ans: (a) 620 N; (b) 580 N]

87 Since the force is in the positive y direction, the x component of the velocity is constant and the x coordinate at the end of time t is $x = v_{0x}t$. The y component of the acceleration is

$a_y = F/m$, where F is the force of the wind and m is the mass of the armadillo. At the end of time t the y component of the velocity is $v_y = a_y t$ and the y coordinate is $y = \frac{1}{2}a_y t^2$.

[Ans: (a) $(5.0\,\text{m/s})\,\hat{\imath} + (4.3\,\text{m/s})\,\hat{\jmath}$; (b) $(15\,\text{m})\,\hat{\imath} + (6.4\,\text{m})\,\hat{\jmath}$]

93 Use $v = v_0 + at$ to compute the acceleration a. Here v is the velocity after time t and v_0 is the initial velocity. Use Newton's second law in the scalar form $F = ma$ to compute the force.

97 Use $v^2 = v_0^2 + 2a\,\Delta x$, where v_0 (= 0) is the initial speed, v is the speed after the electron travels a distance Δx, and a is its acceleration. Solve for a. The force on the electron is $F = ma$, where m is the electron's mass, and the weight of the electron is mg.

[Ans: (a) $1.1 \times 10^{-15}\,\text{N}$; (b) $8.9 \times 10^{-30}\,\text{N}$]

Quiz

Some questions might have more than one correct answer.

1. Which of the following statements are true?

   A. an inertial frame cannot be accelerating relative to us
   B. a reference frame that is rotating relative to any other frame cannot be an inertial frame
   C. if a particle is not accelerating relative to a noninertial frame then the net force on the particle is not zero
   D. two inertial frames might be accelerating relative to each other
   E. the acceleration of a particle is different for different inertial frames

2. Object A has twice the mass of object B. For A to have the same acceleration as B the net force on it must be

   A. more than twice as great as the net force on B
   B. twice as great as the net force on B
   C. the same as the net force on B
   D. half as great as the net force on B
   E. less than half as great as the net force on B

3. The masses of four particles and the magnitudes of the net forces on them are:

   I. $m = 2\,\text{kg}$, $F_{\text{net}} = 6\,\text{N}$
   II. $m = 3\,\text{kg}$, $F_{\text{net}} = 4\,\text{N}$
   III. $m = 6\,\text{kg}$, $F_{\text{net}} = 3\,\text{N}$
   IV. $m = 10\,\text{kg}$, $F_{\text{net}} = 2\,\text{N}$

   Rank the magnitudes of their accelerations, least to greatest.

   A. I, III, IV, II
   B. I and II tied, III, IV
   C. IV, III, II, I
   D. III, II, IV, I
   E. I, II, III, IV

4. A block is sliding down an incline and you want to calculate its acceleration. Which of the following forces should you include in its free-body diagram?

   A. the gravitational force of Earth on the block
   B. the gravitational force of Earth on the incline
   C. the normal force of the incline on the block
   D. the normal force of the block on the incline
   E. the frictional force of the incline on the block

5. You are throwing a ball into the air and you want to calculate its acceleration while it is in your hand. Of the following forces, which should you include in the free-body diagram for the ball?

   A. the gravitational force of Earth on the ball
   B. the gravitational force of Earth on your hand
   C. the force of your hand on the ball
   D. the force of the ball on your hand
   E. the frictional force of the air on your hand

6. A man uses a rope to pull a child on a sled across a snowy field. To calculate the acceleration of man, child, and sled, taken together as an object which of the following forces should you include?

   A. the normal forces of the snow on the sled and man
   B. the frictional forces of the snow on the sled and man
   C. the force of the man on the rope
   D. the gravitational forces of Earth on the sled, man, and child
   E. the normal force of the snow on the child

7. A block is sliding with constant acceleration down an incline. Which of the following statements are true about the forces on the block?

   A. the gravitational and normal forces sum to zero
   B. the gravitational and frictional forces sum to zero
   C. the frictional and normal forces sum to zero
   D. the normal force is zero
   E. none of the above statements are true

8. Three forces of equal magnitude are applied to a particle. One is in the positive x direction, one is in the positive y direction, and one is in the negative y direction. The acceleration of the particle is

   A. zero
   B. in the positive y direction
   C. in the negative y direction
   D. in the positive x direction
   E. not parallel to either the x or y axis

9. Particle A, with a small mass, caroms off particle B, with a larger mass. Only the forces of the particles on each other are significant. During the time the blocks are in contact
   A. the acceleration of A is greater than the acceleration of B
   B. the acceleration of A is less than the acceleration of
   C. the acceleration of A is the same as the acceleration of B
   D. the force of particle A on B is less than the force of B on particle A
   E. the force of particle A on B is greater than the force of B on particle A
10. A block is sliding down an incline. Pairs of forces that are equal in magnitude and opposite in direction by virtue of Newton's third law are
   A. the normal force of the incline on the block and the gravitational force of Earth on the block
   B. the normal force of the incline on the block and the normal force of the block on the incline
   C. the gravitation force of Earth on the block and the normal force of the incline on the block
   D. the gravitational force of Earth on the block and the gravitational force of the block on Earth
   E. the frictional force of the incline on the block and the frictional force of the block on the incline
11. You hold a book motionless in your hand. Pairs of forces that are equal in magnitude and opposite in direction by virtue of Newton's third law are
   A. the gravitational force of Earth on the book and the gravitational force of the book on Earth
   B. the gravitational force of Earth on the book and the force of your hand on the book
   C. the force of your hand on the book and the force of the book on your hand
   D. the gravitational force of your hand on Earth and the force of the book on your hand
   E. the gravitation force of Earth on your hand and the force of your hand on the book

Answers: (1) B, C; (2) B; (3) C; (4) A, C, D; (5) A, C, E; (6) A, B, D; (7) E; (8) D; (9) A; (10) B, D, E; (11) A, C, E