Name___________________________
Section________________
Date________________

1 Two-dimensional Forces

Objectives:
- To add force vectors graphically.
- To add force vectors analytically.
- To add force vectors experimentally.

Equipment:
- Force table with four pulleys, ring, string, and pin
- Four weight hangers
- Set of slotted masses, including three 50 g and three 100 g masses
- Protractor
- Ruler
- Triangle
- Level

Safety:
- Observe ordinary laboratory precautions.
- Ensure that the heavy force table does not fall on your feet.

1.1 Introduction

A vector is a mathematical entity that has the properties of a magnitude and a direction. Vector quantities that we encounter in physics include velocity, displacement, acceleration, momentum, and force. Because the direction of a vector quantity is an essential part of it, ordinary scalar addition cannot be performed on vectors. A scalar is a quantity that only contains a magnitude, such as temperature, mass, distance, and speed.

In order to take the direction into account when performing the addition of vectors, either of two graphical methods or an analytical method may be employed. The sum of two or more vectors is called the resultant vector. The resultant may be found graphically by using the parallelogram method, the head-to-tail method, or by the analytical use of trigonometry. In this lab, we will practice all three methods, utilizing force vectors. The force vectors we will measure and draw represent actual forces directed at various angles on a two-dimensional force table.

1.2 Parallelogram Method

Your instructor will demonstrate the use of the parallelogram method for adding two vectors, $\vec{A}$ and $\vec{B}$, to obtain the resultant vector $\vec{R}$. You will need the protractor, triangle, and ruler. An arbitrary scale is used to represent the magnitude of the vector in your drawing. An example of a scale is 1 cm = 0.2 N.

1.3 Head-to-tail Method

Your instructor will also demonstrate the head-to-tail method for adding two vectors. You will need the protractor and ruler.

1.4 Analytical Method

This method is based on the three trigonometric ratios, sine, cosine, and tangent. We begin by memorizing the definition for each of these ratios.

- The sine of an acute angle in a right triangle is the ratio of the opposite side to the hypotenuse.
- The cosine of an acute angle in a right triangle is the ratio of the adjacent side to the hypotenuse.
- The tangent of an acute angle in a right triangle is the ratio of the opposite side to the adjacent side.
- One popular pneumonic device for remembering these definitions is to recall the expression SOHCAHTOA, pronounced much as Goldie Hawn did when she exclaimed "Sock it to yah,"in the television series *Laugh In*. Anyway, the letters in the pneumonic stand for <u>S</u>ine is <u>O</u>pposite over <u>H</u>ypotenuse, <u>C</u>osine is <u>A</u>djacent over <u>H</u>ypotenuse, <u>T</u>angent is <u>O</u>pposite over <u>A</u>djacent.

Any vector $\vec{A}$ can be drawn extending outward from the origin of a Cartesian coordinate system. This vector can be resolved into two perpendicular components, one along the horizontal or x-axis and the other in the direction of the vertical or y-axis. Let the vector $\vec{A}$ reside in the first quadrant of the coordinate system and measure the angle θ to the vector from the positive x-axis. Then, very clearly, the vector forms the hypotenuse of the right triangle that has the x-component as the adjacent leg and the y-component as the opposite leg.

By utilizing the definitions of the trigonometric functions, the magnitude of the x-component of the vector $\vec{A}$, A_x, is found by multiplying the magnitude of $\vec{A}$, denoted by A, by the cosine of theta. Similarly, the y-component of the vector $\vec{A}$, A_y, is found from $A\sin(\theta)$.

$$A_x = A\cos(\theta) \tag{1}$$

$$A_y = A\sin(\theta) \tag{2}$$

Now consider a second vector $\vec{B}$, also in the first quadrant. $\vec{B}$ also has x- and y-components, B_x and B_y. In order to add the vectors $\vec{A}$ and $\vec{B}$ and obtain the resultant vector $\vec{R}$, we need to add the x-components, A_x and B_x to get R_x, and then add the y-components A_y and B_y to get R_y.

$$R_x = A_x + B_x \tag{3}$$

$$R_y = A_y + B_y \tag{4}$$

Finally, the x- and y-components of $\vec{R}$ can be combined using the Pythagorean theorem to get the magnitude of $\vec{R}$ and the angle θ that $\vec{R}$ makes with the positive x-axis is found by using the tangent ratio.

$$R = \sqrt{R_x^2 + R_y^2} \tag{5}$$

$$\theta = \arctan\left(\frac{R_y}{R_x}\right) \tag{6}$$

The previous steps are equally valid for vectors in any of the other three quadrants. If the angle to $\vec{A}$ or $\vec{B}$ is measured from the positive x-axis, either using positive degrees (measuring counterclockwise) or using negative degrees (measuring clockwise), the sine and cosine functions of that angle measurement will yield the correct $+$ or $-$ sign of the components. On the other hand, when finding the direction of the resultant using the arctan function ($\tan^{-1}$), it is necessary to adjust the result obtained from Eq.(6) by 180° when the resultant is in the second and third quadrants.

1.5 The Force Table

The force table is an apparatus that allows the experimental determination of the resultant of two or more force vectors. The rim of the table is calibrated in degrees. Special pulleys are clamped to the table such that the pointer indicates the degree reading. Weight forces are applied to a central ring by securely tying a string to the central ring, running the string over the pulley, and attaching a weight hanger to which slotted masses are added. The magnitude of the force is varied by adding or removing slotted masses. The direction of the force is changed by moving the pulley to a new position.

The resultant of two or more forces is found by balancing the forces with another force so that the ring is centered around the central pin. The balancing force is emphnot the resultant $\vec{R}$, but rather the equilibrant $\vec{E}$. The equilibrant is the force that balances the other forces and brings about a state of equilibrium. At equilibrium, the vector sum of all of the forces is zero. The equilibrant is the vector force of equal magnitude but in the opposite direction to the resultant. Hence,

$$\vec{R} = -\vec{E} \tag{7}$$

If two forces, $\vec{A}$ and $\vec{B}$ are added to produce the resultant $\vec{R}$, then

$$\vec{A} + \vec{B} = \vec{R} = -\vec{E} \tag{8}$$

Question. Why is it not possible to experimentally determine the resultant vector directly from the force table?

1.6 Procedure

1. Set up and level the force table with the center ring and three strings. Attach a weight hanger to each string. The knots on the strings at the center ring should be loose enough that the string can slip freely on the center ring and pull outward in a true radial direction. A good technique is to pull the center ring upward a short distance and release it. This helps adjust the friction in the pulleys as the ring vibrates up and down. When the forces are balanced, the pin may be carefully removed to see if the ring is centered around the center hole.

2. *Vector addition 1.* Given two vectors, $\vec{A}$ with magnitude 1.96 N and direction 30°, and $\vec{B}$ with magnitude 1.96 N at 120°, find their vector sum or resultant $\vec{R} = \vec{A} + \vec{B}$ by each of the following procedures. All angles are measured from the x-axis.

   (a) *Graphical.* Using the parallelogram method of vector addition, make a scale drawing that fills about half a sheet of paper. Indicate the scale you are using. An example of a scale is 1 cm = 0.2 N. Measure the magnitude with a ruler, measure the direction with a protractor. Use the triangle and ruler to draw a parallelogram. Draw the resultant and measure its magnitude and direction. Record the results in the data table. Save the drawing and attach it to your lab report.

   (b) *Analytical.* Using vector components, add the x-components of each vector to obtain R_x and add the y-components of each vector to obtain R_y. Use the Pythagorean theorem and the arctangent function to obtain the magnitude and direction of the resultant vector. Record the results in the data table. Save your calculations and attach them to the lab report.

   (c) *Experimental.* Attach two pulleys to the force table at the indicated degree markings. Add masses to the two weight hangers so that the total weights, including that of the weight hangers, are the specified amounts. Using a third pulley and weights, experimentally determine the magnitude and direction of the equilibrant force. Record the magnitude and direction of the resultant force in the data table. Remember, the resultant force has the same magnitude but the opposite direction as the equilibrant.

3. *Vector addition 2.* Repeat procedure 2 for $\vec{A} = 1.96$ N at 20° and $\vec{B} = 1.47$ N at 80°. Record the results from the graphical, analytical, and experimental procedures in the data table.

4. *Vector addition 3.* Repeat procedure 2 for $\vec{A} = 1.96$ N at 0° and $\vec{B} = 1.47$ N at 90°. Record the results from the graphical, analytical, and experimental procedures in the data table.

5. *Vector addition 4.* Get a fourth pulley, string, and weight hanger. Repeat procedure 2 for $\vec{A} = 0.98$ N at 30°, $\vec{B} = 1.96$ N at 90°, and $\vec{C} = 2.94$ N at 225°. Record the results from the graphical, analytical, and experimental procedures in the data table.

1.7 Data

Enter data in Table 1. Attach drawings and calculations to lab report.

1.8 Analysis

1. What possible sources of error can you identify for the graphical method of vector addition?

2. What possible sources of error can you identify for the analytical method of vector addition?

Table 1: Magnitude and Direction of Resultant

	Graphical	Analytical	Experimental
Vector addition 1			
Vector addition 2			
Vector addition 3			
Vector addition 4			

3. What possible sources of error can you identify for the experimental method of vector addition?

4. Of the graphical and analytic methods, which one do you consider to be more accurate? Why?

1.9 Homework

A picture hangs on a wall in the usual manner: a wire is attached to each side of the frame and passed over a nail on the wall. The tension T in each side of the wire is 3.5 N and the angle between the wire and the top edge of the picture frame is 45°.

1. What is the equilibrant or the upward reaction force of the nail?

2. What is the weight of the picture?